Overview of Fuzzy Logic

Mohamed Trabia

University of Nevada, Las Vegas, mbt@me.unlv.edu

Follow this and additional works at: http://digitalscholarship.unlv.edu/me_presentations

Part of the Acoustics, Dynamics, and Controls Commons, Controls and Control Theory Commons, and the Control Theory Commons

Repository Citation


Available at: http://digitalscholarship.unlv.edu/me_presentations/1

This Presentation is brought to you for free and open access by the Mechanical Engineering at Digital Scholarship@UNLV. It has been accepted for inclusion in Presentations (ME) by an authorized administrator of Digital Scholarship@UNLV. For more information, please contact digitalscholarship@unlv.edu.
Overview of Fuzzy Logic

Mohamed B. Trabia, Professor
Department of Mechanical Engineering
University of Nevada, Las Vegas
Las Vegas, NV 89154-4027
USA
• Dr. Lotfi Zadeh, a professor of mathematics from U.C. Berkeley, proposed the fuzzy theory 1965/1967.

• Dr. Zadeh originally started his research within traditional control theory.
• He was unsatisfied with the failure of the traditional control theory to explain many phenomena such as, why a person can control a complex system that he/she cannot describe mathematically (Driving car is a good example).
Thesaurus:
- *Fuzzy*: uncertain / unclear / vague
- *Logic*: reason

Definition:
- Fuzzy logic is a mean to transform linguistic experience into mathematical information.
- Fuzziness may be related to possibility as opposed to probability.
**References**

<table>
<thead>
<tr>
<th>Author</th>
<th>Title</th>
<th>Publ info</th>
</tr>
</thead>
<tbody>
<tr>
<td>Driankov, Dimiter</td>
<td>An introduction to fuzzy control / Dimiter Driankov, Hans Hellendoorn, Michael Reinfrank ; with cooperation from Rainer Palm, Bruce Graham, and Anibal Ollero ; foreword by Lennart Ljung</td>
<td>Berlin ; New York : Springer, c1996</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Edition 2nd, rev. ed</td>
</tr>
</tbody>
</table>
**Language → Mathematics**

Example: Height

<table>
<thead>
<tr>
<th>Height (cm)</th>
<th>Truthfulness</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>short (sure)</td>
</tr>
<tr>
<td>150</td>
<td>short (sure)</td>
</tr>
<tr>
<td>165</td>
<td>short (?)</td>
</tr>
<tr>
<td></td>
<td>medium (sure)</td>
</tr>
<tr>
<td></td>
<td>tall (?)</td>
</tr>
<tr>
<td>170</td>
<td>short (no way)</td>
</tr>
<tr>
<td></td>
<td>medium (sure)</td>
</tr>
<tr>
<td></td>
<td>tall (?)</td>
</tr>
<tr>
<td>175</td>
<td>short (no way)</td>
</tr>
<tr>
<td></td>
<td>medium (?)</td>
</tr>
<tr>
<td></td>
<td>tall (?)</td>
</tr>
<tr>
<td>180</td>
<td>short (no way)</td>
</tr>
<tr>
<td></td>
<td>medium (no way)</td>
</tr>
<tr>
<td></td>
<td>tall (sure)</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
</tr>
</tbody>
</table>
Fuzzy Sets (Membership Functions):

- Membership function (fuzzy set) relates the degree of truthfulness $\mu$ (between 0,1) for a linguistic term.

$\mu=0$  No truth / Absolutely false / No possibility  
$\mu=1$  Full confidence / Absolutely correct / Sure thing  

- The same value of a variable can be represented using more than one membership function (check 165 cm).  
- Membership sets can be of any shape (trapezoid, triangle, gaussian, etc….)
Fuzzy Logic Controller

Fuzzy logic can be used to control systems that do not have well-defined models. Fuzzy logic controller is a feedback controller that looks like:

![Fuzzy Logic Controller Diagram]
Fuzzifier

- The fuzzifier uses the fuzzy membership sets to define the truthfulness of a variable as shown before.
Fuzzy Inference Rules

- These rules determine the actions of the controller. Each rule is in the form of:

\[
\text{IF } \{\text{<variable #1> is <fuzzy term}>\}
\]
\[
\text{and } \{\text{<variable #2> is <fuzzy term}>\}
\]
\[
\text{and } \{\text{<variable #3> is <fuzzy term}>\}
\]
\[
\text{and } \ldots
\]

\[
\text{THEN}
\]
\[
\text{<controller input #j> is <fuzzy term>}
\]
Defuzzification:
- Defuzzification starts by assigning a truth value for the control output as follows,

\[ \mu(\text{fuzzy term of control input } \#j) = \min(\mu(\text{fuzzy term of sensor output } \#1), \mu(\text{fuzzy term of sensor output } \#2), \ldots, \mu(\text{fuzzy term of sensor output } \#n)) \]
At this stage, we know two things:
1. What fuzzy term(s) control input \( j \) belongs to.
2. What is the truthfulness of each of these terms.

However, the controller input has to be a *crisp* (definitive) number.

Different processes of defuzzifications can achieve this goal.

We will present here the *moment of area method*:

\[
x = \frac{\sum_{i=1}^{n} A_i \cdot rcg_i}{\sum_{i=1}^{n} A_i}
\]
Example

Consider the control of a vehicle,

**Model:**

\[ F(\phi) = m \ddot{x} \]
\[ \dot{x} \geq 0 \]

**Objectives of the controller:**

- Reach a target point at \( x_d \) meters away from the starting point
- Velocity is equal to zero at the end of the motion
Variables:
Sensor Outputs:
  • Distance_to_target, \( d = (x_d - x) \)
  • Velocity, \( v = \ddot{x} \)

Controller input
  • Accelerator/Brake_angle, \( \phi \)
<table>
<thead>
<tr>
<th>Variable</th>
<th>Fuzzy Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>close, medium, far</td>
</tr>
<tr>
<td>$v$</td>
<td>zero, slow, medium</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Negative Medium, Negative Small, Zero, Positive Small, Positive Medium</td>
</tr>
</tbody>
</table>
Fuzzy membership sets
Fuzzy rules:

IF \(<\text{Distance} \text{ to target}> \text{ is} <\text{close}>>\) and \(<\text{Velocity} \text{ and} <\text{medium}>>\) THEN 
\(<\text{Accelerator/Break} \text{ angle}> \text{ is} <\text{Negative Medium}>>\)

All these rules can be combined in one Table as follows,

<table>
<thead>
<tr>
<th>Fuzzy Rules for the Autonomous Vehicle Controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance</td>
</tr>
<tr>
<td>Velocity</td>
</tr>
<tr>
<td>Z</td>
</tr>
<tr>
<td>S</td>
</tr>
<tr>
<td>M</td>
</tr>
</tbody>
</table>

Note:

C = close  B = big  S = small
M = medium  Z = zero
P = positive  N = negative
Results