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Feedback Control Solutions to Network Level User-Equilibrium Real-Time Dynamic Traffic Assignment Problems

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Abstract - A new method for performing Dynamic Traffic Assignment (DTA) is presented which is applicable in real time, since the solution is based on feedback control. This method employs the design of nonlinear H_∞ feedback control systems which is robust to certain class of uncertainties in the system. The solution aims at achieving user equilibrium on alternate routes in a network setting.

1.1 Introduction

The technique we propose solves the network-wide system optimal DTA/DTR (Routing) problem using real-time feedback control. We employ nonlinear H_∞ feedback control design methodology to produce the solution of the problem which also provides robustness against bounded disturbances. The modeling paradigm of nonlinear H_∞ approach is an exact match with the requirements of a network-wide DTA/DTR problem applicable to Advanced Traffic Management/Information Systems (ATMIS) of Intelligent Transportation Systems (ITS), because it solves the optimal dynamic routing problem by only performing simple algebraic operations in real-time, unlike existing techniques which rely on lengthy off-line/on-line mathematical operations.

2. System Description

In this section, we present a mathematical model for the traffic system which is in the form usable for the design of DTA/DTR feedback control. Many models have been proposed before, but the most appropriate model has been proposed by Papageorgiou [9]. In this section, we will present the same model with minor changes, which then will be used for feedback control design.

2.1 System Network

Following the general notation and development in [9], let N be the set of all the network nodes for the problem, M the set of all the networked links, I_n the set of links entering node n, O_n the set of links leaving node n, O the set of origin nodes, and D the set of destination nodes. Let d_{ij} denote the origin-destination demands. where $i \in O$ and $j \in D$. Traffic flow entering a link m is shown by q_m and that leaving the same link by Q_m. Let S^n denote the set of destination nodes which are reachable from a node n. There are ℓ_{nj} alternate routes from node n to destination node j, and $L^z_{nj}, z = 1, 2, \ldots, \ell_{nj}$ are the ordered sets of links included in alternate routes. Λ_{nj} is the set of output links which connect the node to the destination j using one of the alternate ℓ_{nj} routes. $q^m_{nj}, m \in \Lambda_{nj}$ is the flow in the link m belonging to one of the routes for destination j flowing out from node n. The sum of all $q^m_{nj}, m \in \Lambda_{nj}$ for a node n for destination j is given by q_{nj}.

There are two kinds of split factors which can be used in system dynamics. One is destination based splits which are given as ratio of the destination based flow on a link out of a node n and the total flow from the same node to the same destination. This is shown in equation (1). The constraint associated with this split factor inputs is given by (2).

$$\beta^m_{nj} = \frac{q^m_{nj}}{q_{nj}}, \quad m \in \Lambda_{nj} \tag{1}$$

$$\sum_{m \in \Lambda_{nj}} \beta^m_{nj} = 1 \tag{2}$$

Second kind of split factor input takes the ratio of the flow into a link m from a node n and the total flow from the same node, as shown in (3). The associated constraint is shown in (4).

$$\beta^m_n = \frac{q^m_n}{q_n}, \quad m \in \Lambda_n \tag{3}$$

$$\sum_{m \in \Lambda_n} \beta^m_n = 1 \tag{4}$$

The total number of independent split factor input variables, whether they are destination based or node based, is reduced by one because of the constraints (2) and (4).

Link variables can also be modeled based on either destination or independent of those. Let S_m be the

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set of destination nodes which are reachable through link m. The inflow into a link m is given by

$$q_m = \sum_{j \in S_m} \beta_{nj}^m q_{nj} \quad n \in N, m \in O_n \qquad (5)$$

Let q_{nj} denote the total flow leaving a node n for destination j. The composition rates on a link are given by (6) and the corresponding constraints are given by (7).

$$\gamma_{mj} = \beta_{nj}^m \frac{q_{nj}}{q_m}, \quad n \in N, m \in O_n, j \in S_m$$

(6)

$$\sum_{j \in S_m} \gamma_{mj} = 1$$

(7)

The destination based link flow is given by

$$q_{nj} = \sum_{m \in I_n} Q_m \Gamma_{mj} + d_{nj} \quad n \in N, j \in S^n$$

(8)

where Γ_{mj} is the fraction of traffic volume exiting a link m destined for destination j.

2.2 System Dynamics

There are essentially two kinds of system dynamics which have been modeled for this problem : link based model, and route based model.

2.2.1 Link Based Model

In this scheme, the state variables are link density and composition rates. The link density dynamics are obtained from conservation equation, and are given by (9). The relationship between outflow and link density is shown by (10). There have been many other relationships shown in literature instead of (10).

$$\dot{\rho}_m(t) = \frac{1}{\delta_m}[q_m(t) - Q_m(t)] \qquad (9)$$

$$Q_m(t) = q_{max,m}[1 - e^{-\rho_m(t)/\kappa_m}] \qquad (10)$$

The composition rate dynamics can be represented as a time delay, where the amount of time delay is related to the travel time on the link. This dynamic relation is shown as

$$\Gamma_{mj}(t) = \gamma_{mj}(t - \chi_{mj}) \qquad (11)$$

An alternate method models the dynamics of composition rate propagation as a first order filter, given by

$$\dot{\Gamma}_{mj}(t) = \alpha_m[\gamma_{mj} - \Gamma_{mj}] + \Gamma_{mj} \qquad (12)$$

where α_m is either constant or a function of travel time.

2.2.2 Route Based Model

In the route based model, the state variables are the destination based densities on the links of the system. The dynamic equations are shown in (13). The exiting composition rates are given by the ratio of the destination based density to the total link density as shown in (14).

$$\dot{\rho}_{mj}(t) = \frac{1}{\delta_m}[\gamma_{mj}(t) q_m(t) - \Gamma_{mj}(t) Q_m(t)] \qquad (13)$$

$$\Gamma_{mj}(t) = \rho_{mj}(t) / \rho_m(t) \qquad (14)$$

3. User Equilibrium Dynamic Traffic Assignment Problem

The link based and route based models described in section 2.2 can be represented in the following form:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} \qquad (15)$$

For instance, a link based model with filter composition rate dynamics (12) and destination based split factors can be written in form (15) by using (1), (2), (5-10) and (12). A more complete form of model (15) would also represent disturbances, which come from uncertainties of the system. This new form is given by

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} + \mathbf{a}(\mathbf{x})\mathbf{w} \qquad (16)$$

where $\mathbf{w}$ is the disturbance to the system model and $\mathbf{a}(\mathbf{x})$ the corresponding gain.

$$G: \left\{ \begin{array}{c} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} + \mathbf{a}(\mathbf{x})\mathbf{w} \\ \mathbf{y} = \mathbf{x} \\ \mathbf{z} = \begin{bmatrix} h(\mathbf{x}) \\ \mathbf{u} \end{bmatrix} \end{array} \right\} \qquad (17)$$

The function $h(\mathbf{x})$ is very important in the network-wide user-equilibrium optimal dynamic traffic assignment control. The aim of the assignment is to reach user equilibrium by keeping cost on alternate routes the same. We can achieve that by taking the cost as the sum of the squares of the differences between the costs on the alternate routes. Then the controller will try to minimize the travel time keeping the controller cost low. In the actuation sense, which could be variable message sign, or direct communication with the vehicles, it makes sense to keep the variation of the split factors low with respect to time, so that the signs or commands don't change too rapidly [9]. In order to accomplish that, we add more states to the system which consist of the split factors also, and the control input becomes the derivative of the split factor values.

Note that due to the presence of origin-destination (OD) flows in the dynamics, the system dynamics are non-linear-time varying (NLTV). In order for us to use a stationary solution of the Hamilton-Jacobi (HJ) equation, we need to have a time invariant system. We can convert the NLTV system into a nonlinear-time-invariant (NLTI) system by introducing additional dynamic equations for OD flows using the assumptions from (13, 14), which consider an autonomous dynamic behavior of the OD flows. This extends the state variable vector by the additional OD flow dynamics.

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Now, in order to formulate the problem in state feedback H_∞ control, we need to identify $\mathbf{z}(t)$. For a system user-equilibrium, we can take $\mathbf{z}(t)$ as a cost function of the state variables with weights given to the function of states as well as to the variation of the split factors. That would in effect provide bounded variation of the split factor commands which is crucial for the actual implementation and effectiveness of the system. This further increases the size of the state variable vector by the number of split factor variables.

The DTA/DTR can be further solved in two ways depending on what we use for split factors i.e. destination based split factors or node based split factors. There are many subtle and apparent theoretical as well as practical implications of this choice. The theoretical implications are related to the controllability aspect when deciding on which split factor to use. Without any detailed analysis, it seems intuitive that the destination based split factor formulation will give a more controllable system dynamics than the node based split factors. However, the actual implementation of destination based split factors is not trivial. At present, VMS systems or other actuation methods can be used for node based splitting, and they would have to be either modified or designed in such a way that destination based splitting information can be provided to the drivers. In automated highway systems, or in general in a transportation system, where communication infrastructure is already present for infrastructure to vehicle communication (such as with in-vehicle route guidance system), the destination based split factors could be easily implemented, and be highly effective.

Definition 1: System G/K is said to have L_2 gain less than or equal to g for some $g > 0$ if

$$\int_0^T \|\mathbf{z}(t)\|^2 dt \le \gamma^2 \int_0^T \|\mathbf{w}(t)\|^2 dt \qquad (18)$$

$\forall\ T > 0$ and $d(t) \in L_2[0, T]$.

Nonlinear H_∞ Control Problem: Find an output feedback controller K if any, such that the closed loop system $\Omega(G, K)$ is asymptotically stable and has L_2-gain $\le g$, $\forall\ T \in R^+$.

Solution: The solution to this problem can be derived from the theory of dissipative systems [6], which also has implications to the theory of differential games [5]. In order to develop the solution, define a storage function as in [4] as:

$$V_s(\mathbf{x}) = \sup_{\mathbf{w}\in L_2[0,T],\mathbf{x}(o)=\mathbf{x}} \frac{1}{2}\int_0^T (\|\mathbf{z}(t)\|^2 - \gamma^2\|\mathbf{w}(t)\|^2)dt \qquad (19)$$

Condition (18) is equivalent to $V_s(x) < \infty$, which in turn is true if and only if there exists a solution to the following integral dissipation inequality.

$$V(\mathbf{x}(t_1)) - V(\mathbf{x}(t_0)) \le \frac{1}{2}\int_{t_0}^{t_1} (\gamma^2\|\mathbf{w}(t)\|^2 - \|\mathbf{z}(t)\|^2)dt,\ V(0) = 0 \qquad (20)$$

$\forall t_1 > t_0$ and $w \in L_2[t_0, t_1]$. Function $V(\mathbf{x})$ which satisfies (20) is called a storage function, and if such a storage function exists, then system(16) is called dissipative with respect to the supply rate $\frac{1}{2}(\gamma^2\|\mathbf{w}(t)\|^2 - \|\mathbf{z}(t)\|^2)$. If $V(\mathbf{x})$ is differentiable, we can rewrite (20) as

$$\dot{V}(\mathbf{x}) \le \frac{1}{2}(\gamma^2\|\mathbf{w}(t)\|^2 - \|\mathbf{z}(t)\|^2) \qquad (21)$$

which combined with (16) gives

$$\frac{\partial V}{\partial \mathbf{x}}^T(\mathbf{x})(\mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} + \mathbf{a}(\mathbf{x})\mathbf{w}) - \frac{1}{2}\gamma^2\|\mathbf{w}(t)\|^2 + \frac{1}{2}\|\mathbf{z}(t)\|^2 \le 0 \qquad (22)$$

We call the left hand side of (22) the energy Hamiltonian H. We perform a min-max operation on H following the differential game analogy. By solving $\frac{\partial H}{\partial \mathbf{w}} = 0$ and $\frac{\partial H}{\partial \mathbf{u}} = 0$ we obtain the optimum maximizing disturbance $\mathbf{w}^*$ and minimizing input $\mathbf{u}^*$ as:

$$\mathbf{w}^* = \frac{1}{\gamma^2}\mathbf{a}^T(\mathbf{x})\frac{\partial V}{\partial \mathbf{x}},\ \mathbf{u}^* = -\mathbf{g}^T(\mathbf{x})\frac{\partial V}{\partial \mathbf{x}} \qquad (23)$$

which provides the saddle point property

$$H(\mathbf{x}, \frac{\partial V}{\partial \mathbf{x}}^T, \mathbf{w}, \mathbf{u}^*) \le H(\mathbf{x}, \frac{\partial V}{\partial \mathbf{x}}^T, \mathbf{w}^*, \mathbf{u}^*) \le H(\mathbf{x}, \frac{\partial V}{\partial \mathbf{x}}^T, \mathbf{w}^*, \mathbf{u}) \qquad (24)$$

By substituting (23) in (22), we obtain the Hamilton-Jacobi inequality

$$\frac{\partial V}{\partial \mathbf{x}}(\mathbf{x})\mathbf{f}(\mathbf{x}) + \frac{1}{2}\frac{\partial V}{\partial \mathbf{x}}[\frac{1}{\gamma^2}\mathbf{a}(\mathbf{x})\mathbf{a}^T(\mathbf{x}) - \mathbf{g}(\mathbf{x})\mathbf{g}^T(\mathbf{x})]\frac{\partial V}{\partial \mathbf{x}}^T + \frac{1}{2}h(\mathbf{x})h^T(\mathbf{x}) \le 0 \qquad (25)$$

If the system is reachable from $\mathbf{x}_0$ then the storage function (19) is finite, and if it is also smooth then it is also a solution of the Hamilton-Jacobi-Isaac equation

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$$\frac{\partial V}{\partial \mathbf{x}}(\mathbf{x})\mathbf{f}(\mathbf{x})+\frac{1}{2}\frac{\partial V}{\partial \mathbf{x}}[\frac{1}{\gamma^2}\mathbf{a}(\mathbf{x})\mathbf{a}^T(\mathbf{x})-\mathbf{g}(\mathbf{x})\mathbf{g}^T(\mathbf{x})]\frac{\partial V^T}{\partial \mathbf{x}}$$

$$+\frac{1}{2}h(\mathbf{x})h^T(\mathbf{x})=0 \qquad (26)$$

In the game theoretic formulation, the objective function on which the players perform min-max is

$$J(\mathbf{u},\mathbf{w})=\int_0^T(\mathbf{z}'\mathbf{z}-\gamma^2\mathbf{w}'\mathbf{w})dt \qquad (27)$$

The solution of the game theoretic formulation is given by (23) in conjunction with (26).

The solution for the standard infinite time horizon optimal control problem can be obtained from this by eliminating the disturbance player $\mathbf{w}(t)$. This can be achieved by taking the limit $\gamma \to \infty$ in the Hamilton-Jacobi inequality. Hence an optimal control problem with a feedback solution for minimizing

$$J(\mathbf{u})=\int_0^T(\mathbf{z}'\mathbf{z})dt \qquad (28)$$

The solution of this is given by

$$\mathbf{u}^*=-\mathbf{g}^T(\mathbf{x})\frac{\partial V}{\partial \mathbf{x}} \qquad (29)$$

where $V(\mathbf{x})$ is the solution of the Hamilton Jacobi equation

$$\frac{\partial V}{\partial \mathbf{x}}(\mathbf{x})\mathbf{f}(\mathbf{x})+\frac{1}{2}\frac{\partial V}{\partial \mathbf{x}}[-\mathbf{g}(\mathbf{x})\mathbf{g}^T(\mathbf{x})]\frac{\partial V^T}{\partial \mathbf{x}}$$

$$+\frac{1}{2}h(\mathbf{x})h^T(\mathbf{x})=0 \qquad (30)$$

Polynomial Approximation Method for Solving Hamilton-Jacobi equation and inequality:

Set

$$V(\mathbf{X})=V^{[2]}(\mathbf{X})+V^{[3]}(\mathbf{X}) \qquad (31)$$

where $V^{[2]}(\mathbf{X})$ contains second order terms and $V^{[3]}(\mathbf{X})$ contains third order terms. For solving (30), we can substitute (31) in (30), and solve for similar order terms. The details of using the polynomial approximation technique which provides local results are shown in references [10] and [11].

Measurement Feedback Control: The above described solutions (23), (29) are valid when the full state is available for feedback, i.e. the full state is measured directly. On the other hand, in many cases, the full state is not available. In those cases, it needs to be found out if the partial measurement available renders the system observable; in other words, can the state variables be estimated from the measured outputs. If the system is observable, then we can design state observers which process the measured outputs and provide best (in some sense) estimates of the state variables, which then can be used in the controllers. There is a good amount of literature on the topic of linear and nonlinear observers. In linear systems, Luenberger observer and Kalman filters have been used effectively [12]. Reference [13] provides a good survey of nonlinear observers.

For dynamic traffic assignment problems using feedback control, normally state variables like traffic flow or traffic density are measured. Other variables like split parameters, origin-destination flows have to be estimated. If information about split factors and origin-destination flows is available through communication with vehicles (such as by using GPS, cellular communication, etc.) then it becomes a full state feedback control problem, but at the present level of applied technology, these variables have to be estimated. There has been some effort at building Kalman filter based observers for origin-destination trip table estimation [14, 15], but the authors have not seen any work in the area of estimating the composition rates. This will be area of further research by the authors, which would then in conjunction with the work presented in this paper provide an immediately deployable DTA scheme. In the meantime, however, this solution is highly attractive for off-line simulation studies also and for preliminary design for deployable systems, which would work with state observers for real time deployable feedback systems.

3.1 DTA Problem using Link Based Model

The dynamics of the link based model can be written in form (15) by using equations (1), (2), (5-10) and either (11) or (12). In this paper we will deal with equations of type (12) instead of (11) for composition rate dynamics. In the link based model, the link densities can be used directly to formulate the system cost. For instance, if we are trying to minimize the weighted cost of input and the user-equilibrium travel cost, we can write the variable $\mathbf{z}(t)$ as

$$\mathbf{z}(t)=\begin{bmatrix} w\,h(\mathbf{x}) \\ \mathbf{u} \end{bmatrix} \qquad (32)$$

where w is the relative weight on the input. If we assume that travel time on a link is given by the quotient of the division of its length with the average velocity on it. On that basis, $h(\mathbf{x})$ will be

$$h(\mathbf{x})=\sum_{i\in P}e_i, \quad \text{where } e_i=\sum_{k=1}^{k=\ell_{nj}}(\Delta_k-\Delta_{k+1})^2,$$

$$\text{and } \Delta_k=\sum_{i\in r_k}\delta_i\rho_i/Q_i \qquad (33)$$

We have taken $\ell_{nj}+1$ to be same as 1. The symbol Δ_k indicates the total travel time on the kth alternate route starting from the node I, P is the set of all the

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node-destination pairs nj, and r_k is the set of all links in the kth alternate route.

3.2 DTA Problem using Route Based Model

Route based system dynamics model is obtained by combining (1), (2), (5-8), (13) and (14). In this case also, the system cost is a weighted function of the input and state dependent cost. Since the state variables are different in this case, the state dependent cost will have to be written in a different form. We can take $\mathbf{z}(t)$ to be

$$\mathbf{z}(t) = \begin{bmatrix} w\, h(\mathbf{x}) \\ \mathbf{u} \end{bmatrix} \qquad (34)$$

where w is the relative weight on the input. For the route based model $h(\mathbf{x})$ can be written as

$$h(\mathbf{x}) = \sum_{i \in P} e_i, \quad \text{where } e_i = \sum_{k=1}^{k=\ell_{nj}} (\Delta_k - \Delta_{k+1})^2,$$

$$\text{and } \Delta_k = \sum_{i \in r_k} \delta_i [\sum_{j \in S_m} \rho_{ij}] / Q_i \qquad (35)$$

3.3. Feedback Control for the Traffic

When the complete extended system which includes the dynamics of the OD flows as well as those of the split factors, so that the input vector consists of all the rate change of split factors, is written in form (16), then the feedback control for the system is given by (23) combined with solution of the Hamilton-Jacobi inequality, where appropriate substitution of $h(\mathbf{x})$ is made from (32) or (34). Specifically, in (25) wherever $h(\mathbf{x})$ appears, it will be replaced by w $h(\mathbf{x})$.

Conclusions

In this paper we present a real-time on-line feedback control solution for the network wide dynamic traffic assignment problem using user equilibrium. The solutions, which are based on nonlinear H_∞ design, are shown for link based as well route based models, and it is also shown how the problem can be modeled and solved using destination based and route based split factors.

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