Estimating Kelly Fraction
Gambling & Risk Taking
Las Vegas

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Diffusion Model

Estimating $F$

Estimating $\sigma^2_F$

Warren Buffett

Further Work
Geometric Brownian Motion

F times Kelly betting is modeled by

$$dB = \left( F \frac{\mu}{\sigma^2} \right) \mu Bdt + \left( F \frac{\mu}{\sigma^2} \right) \sigma BdW$$
Geometric Brownian Motion

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Geometric Brownian Motion

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- \( \mu \) is the “edge“ and \( F \frac{\mu}{\sigma^2} B \) is the “bet size“
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Letting \( \theta = \frac{\mu}{\sigma} \) we have

\[ dB = (F \theta^2) B dt + (F \theta) B dW \]
Itoh’s Lemma:

\[ \frac{dB}{B} = F \theta^2 dt + F \theta dW \]

yields

\[ d \ln B = (F - \frac{1}{2} F^2) \theta^2 dt + F \theta dW \]
Diffusion Model

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Further Work
Estimating $F$

Set $\rho = \left( F - \frac{1}{2} F^2 \right)^2$ and solving for $F$ yields $F = \frac{2\Sigma}{2\rho + \Sigma^2}$. 
Estimating $F$

- Set $\rho = (F - \frac{1}{2} F^2) \theta^2$ and

\[ F = 2\Sigma \theta^2 + \Sigma \rho \]
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- $\Sigma = F\theta$
- solving for $F$ yields

$$F = \frac{2\Sigma^2}{2\rho + \Sigma^2}$$
In order for our estimate to be useful in practice we need some idea of its statistical variability. Assume that for times $t_0, t_1, \cdots, t_N$ we know the bankrolls $B_0, B_1, \cdots, B_N$. Define $\Delta B_k = B_k - B_{k-1}$ and $\Delta t_k = t_k - t_{k-1}$. 
Estimating Drift and SD

We use standard estimators for the parameters \( \rho \) and \( \Sigma^2 = \text{Var}(\ln(B)) \):

\[
R = \frac{\ln(B_N) - \ln(B_0)}{t_N - t_0}
\]

\[
v = \sum_{k=1}^{N} \frac{[\ln(B_k) - \ln(B_{k-1}) - \Delta t_k R]^2}{t_N - t_0}
\]
Estimating $F$

Our estimator for the Kelly fraction $F$ is

$$\hat{F} = \frac{2\nu}{2R + \nu}$$

We estimate its standard deviation next.
Diffusion Model

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Further Work
Estimating $\hat{\sigma}^2_F$

Taylor approximation of $\hat{\sigma}^2_F$

$$\approx \left| \frac{\partial F}{\partial R} \right|^2 \sigma^2_R + \left| \frac{\partial F}{\partial v} \right|^2 \sigma^2_v$$
Estimating $\sigma^2_{\hat{F}}$

Taylor approximation of $\sigma^2_{\hat{F}}$

$$\approx \left| \frac{\partial F}{\partial R} \right|^2 \sigma^2_R + \left| \frac{\partial F}{\partial v} \right|^2 \sigma^2_v$$

$$= \left| \frac{-v}{(2R + v)^2} \right|^2 \left( \frac{v}{t_N - t_0} \right)$$
Estimating $\sigma^2_{\tilde{F}}$

Taylor approximation of $\sigma^2_{\tilde{F}}$

$$\tilde{\zeta} \approx \left| \frac{\partial F}{\partial R} \right|^2 \sigma_R^2 + \left| \frac{\partial F}{\partial v} \right|^2 \sigma_v^2$$

$$= \left| \frac{-v}{(2R + v)^2} \right|^2 \left( \frac{v}{t_N - t_0} \right)$$

$$+ \left| \frac{R}{(2R + v)^2} \right|^2 \left( \frac{2v^2}{(t_N - t_0)^2} \sum_{k=1}^{N} (\Delta t_k)^2 \right)$$
Estimating $\sigma^2_{\hat{F}}$

Taylor approximation of $\sigma^2_{\hat{F}}$ with $N$ equal time intervals
Estimating $\sigma_{\hat{F}}^2$

Taylor approximation of $\sigma_{\hat{F}}^2$ with $N$ equal time intervals

$$\frac{\nu^3 + 2R^2\nu^2}{N(R + \frac{1}{2}\nu)^4}$$
Diffusion Model

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Further Work
E. Thorp (2006) and W. Ziemba (2003) say that Warren Buffett seems to be a Full Kelly bettor. We find a Kelly Fraction of 
\[ F = .26 \pm .09 \ (\alpha = .05) \] for closing prices 1980-present.
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Further Work
Methods in discrete time