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A simplistic plasma dust removal model employing radiation pressure

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Abstract

A simple heuristic model is developed to examine the feasibility of using radiation pressure as a means to transport plasma dust out of the path of the forthcoming electron or photon beam. A slow electromagnetic surface wave coupled to a planar target or substrate exerts the required pressure in the removal process. The model is examined using data and parameters from single-shot radiography experiments. Optimal source requirements are identified for a typical radiography experiment. Source energies and powers are a minimum over an optimum band of frequencies where both conduction and plasma oscillation effects are mutually significant. Above the band of frequencies, dissipative losses in the surface supporting the surface wave increases exponentially with frequency. Below the optimal band, the energy concentration over the plume at the surface structure decreases significantly with frequency, thereby requiring higher source energies/powers for plasma removal.

Keywords: Lasers; Plasma; Radiation pressure; Radiography; Surface waves

1. INTRODUCTION

There is a growing interest in the plasma technology and radiography communities to control the plasma dust generated from beam–target interactions. The plasma dust or debris generated results in the scattering or altering of the forthcoming portion of a beam or beam pulse, affecting spatial resolutions, spot size, intensity, and, indirectly, time duration of the process applied in these technologies.

Plasma and laser processing techniques are employed to etch or transfer fine-line features on an electronic chip during fabrication. Clouds of fine dust particles are formed above the wafers (National Research Council, 1995). This dust is a critical source of defects in chip manufacturing (Duval, 1982a, 1982b; McCaughey & Kushner, 1991; Bouchoule & Boufendi, 1994; Chattopadhyay et al., 1994; Hosokawa et al., 1994; Chattopadhyay & Banerjee, 1995; Daryanani & Allen, 1995; National Research Council, 1995; Anderson & Radovanov, 1996; Koesen et al., 1996). Unlike ions, the dust particle charge is not fixed (Tsytovich & Havnes, 1993). In low pressure plasmas, dust particle charging may be quite high (Matsoukas & Russell, 1997). Along with relatively large dust densities (Anderson & Radovanov, 1996), the interparticle potential energy is greater than the thermal kinetic energy of the plasma. This is typical in fluids and in solids. Consequently, the dusty plasma is strongly coupled (Goree et al., 1995).

Present and futuristic, robust, radiography devices generate high intensity electron beams strong enough to thermalize typical X-ray generating targets. Upon electron beam–target interaction, an undesired target plasma is formed in the shape of a plume. The bremsstrahlung spent plasma debris experiences the large electromagnetic forces between the cathode and anode, resulting in a countflow of ions. Futuristic, robust radiography machines are to operate in multipulsed mode, generating a series of flash radiographs for three-dimensional movie imaging. The presence of the plasma debris degrades the X-ray generated spot size, dose, voltage, and, indirectly, the time duration between pulses. Minimizing the spot size is a critical feature when high degrees of spatial resolution are required. Denser materials require greater penetrating power. Increasing the electron voltage increases the probability of generating higher energy X rays. The dose of X rays generated is dependent on the number of high-energy electrons impinging on the bremsstrahlung converting target. The signal level at the radiographic detector is dose dependent. Scattering and/or various plasma instabilities (e.g., electron–ion streaming and ion
hose instabilities) degrade the electron beam quality and hence the quality of the radiographs. Further, between shots, the plasma debris may collect on various surfaces in the diode region. Desorption of these contaminates from surfaces in the harsh field environment due to forthcoming electron beam pulses adversely influences the electron beam’s properties.

One technique of moving the plasma dust out of the way of a forthcoming pulse is to use electromagnetic radiation pressure. It is desired to determine if practical pulsed lasers may impart enough radiation pressure to move the ionized plasma debris out of the beam’s path. Guided as a surface wave by way of the target or substrate, this plasma removal technique will not directly couple with the electron beam if excited after the electron beam has been expended in the target. Further, if nonlinear mixing effects are not present, wave–wave interactions should not be present if employed in laser etching schemes while the source beam is illuminating the substrate being etched. Optimal source frequencies would have to be chosen to minimize heating losses and to concentrate the laser energy near the target or substrate surface. Such waves have the advantage of being excited at a remote location. This is of importance in robust radiography design where field strengths are large enough to break down and ionize materials.

The specifics of the simple plasma removal model developed and examined are motivated by past pulsed power radiography experiments conducted at Sandia National Laboratory on SABRE (Sandia Accelerator Beam Research Experiment; Mazarakis et al., 1997) and by future radiography requirements. Because of the high melting temperature and large atomic number, tantalum is sometimes used as the target material. Tantalum has a high conductivity. The nearly free electron model is used to characterize the planar, good conducting target with quantum mechanical properties built into the mass of the electrons. Important conduction losses are built into the model. The surface wave characteristics are based on both simulation and experimental data. Future robust radiography will require a 1-mm or smaller spot size. Based on experiments, simulations, and guesstimates, a 50-ns 9-MeV electron beam with a 1-mm spot size is estimated to vaporize a 1-mm-in-diameter hole about 2 mm deep in the tantalum target. The dense plasma debris formed by the energetic electron beam on SABRE with a larger spot size is reported to be as high as 10^{23} ions/m^3 (P.R. Menge, pers. comm.). The plasma debris is treated as a moving, perfectly conducting, solid slab. Modeling the plume as a solid slab is based on typical densities of the dynamic pinch device, the dense plasma focus. Plasma focus densities as low as 10^{16} m^{-3} have been reported (Kies, 1988). The dense plasma focus is noted for its negligible electromagnetic field diffusion (skin depth) resulting from its large plasma sheath densities. One can argue that this, along with the high conductivity of tantalum, justifies treating that ionized portion of the target, the plasma debris, as a perfectly conducting solid slab. Heuristic arguments will verify such an approximation for pulse widths and frequencies of interest. Large potential differences between the cathode diode and anode target result in a plasma plume with an aspect ratio of four to one (the ratio of the perpendicular to parallel velocity relative to the target surface). Numerical codes indicate that debris velocities parallel to the target surface are on the order of 0.5 cm/μs to 1 cm/μs. Therefore, the height of the plasma plume may be estimated. Robust radiography machines will require about 8 to 10 radiograms during a single event. The allowed time duration between each radiogram is between 200 ns to 10 μs. The simplistic model presented in this paper does not consider the Doppler effects resulting from the moving modeled plume. For a first study, these effects are assumed not to give orders of magnitude differences in results. Experiments conducted on SABRE have employed a large focusing magnet. The model posed neglects all external magnetostatic field contributions. Energy losses due to leaky waves have also been neglected. Unequal wall pressures will result in torque effects. Such effects have been neglected since they are meaningless as far as the physical plume is concerned. Since thermal issues have not been examine, the conductivity of tantalum has been chosen about the melting temperature of the material. This value had to be extrapolated from a set of commonly found measured data provided at lower temperatures. If magnetic diffusion and thermal conduction can be minimized such that the laser beam can impose a temporary thermal gradient in the plume, plume acceleration due to thermal pressures may aid electromagnetic radiation pressures. Such effects have not been built into this model.

For the parameters under investigation, it is observed that the optimal laser frequency for the minimum energy required to move the plasma plume falls within the regime in which both the conduction loss and the nearly free electron plasma mechanisms are of equal importance. The neglect of either mechanism will result in gross one to two orders of magnitude deviations in the powers and energies required to move the plasma debris with increase or decrease in frequency about the optimal frequency. The optimal source wavelength range lies within the upper submillimeter to far infrared spectrum [roughly: 100 μm (3 THz) to 375 μm (800 GHz)] if the conductivity of the target is used at its melting temperature. Optically pumped far infrared (FIR) lasers exist (e.g., Edinburgh Instruments) in the spectrum range of interest. It is unclear if the current state of the art in FIR laser design can generate the required energies and powers. By decreasing the distance in which the target must guide the laser beam, the optimal source range tends to flatten out at the lower end of the wavelength range before growing exponentially. This increases the range of optimal source wavelengths. By maintaining a cold target temperature of 0°C, the conductivity of the target is about an order or so of magnitude greater than that at the melting temperature. This causes a significant up-shift in the minimum optimal source frequency, thereby decreasing the laser energies required at higher frequencies (lower wavelengths) to move
the plume the same distance. As a result, the pulsed carbon dioxide laser [i.e., transversely excited atmospheric (TEA) lasers and gas-dynamic lasers] then becomes a suitable but not an optimal source to move the dust plumes with densities on the order of $10^{23}$ ions/m$^3$ and lower. It is well known that CO$_2$ lasers are capable of generating gigawatts of peak power (Liao, 1988). The plume is moved a distance of at least two electron beam spot sizes from its original position in the counterdirection of its initial motion. It is anticipated that the source requirements to move a physical plasma plume as compared to the modeled solid plume may be somewhat relaxed because the physical plume is not a physical solid. Further, as a worst case scenario, the modeled solid plume slab is assumed to expand in one dimension. Allowing for a two-dimensional radial expansion will decrease the overall mass and density of the plume being moved. Therefore, the model presented is expected to provide an upper limit of source requirements needed for plasma removal. Energies and powers required to move the plasma plume modeled as a solid are substantial. Other areas of study such as contamination control in microelectronic fabrication processes where dust densities, temperatures, removal duration times, and initial dust velocities are not as stringent or extreme do not require significantly large sources.

The normalized dispersion curve is also presented. The nature of the waves both internal and external to the stopband is illustrated. As expected, strong absorption occurs in the stopband but the wave nature of the fields is present. The group and phase velocity dependencies on the collision frequency are drastically altered from the collisionless case when the collision frequency is comparable to or on the same order as the electron plasma frequency. This becomes significant for the case under consideration, since the collision frequency is between 10 and 20% of the electron plasma frequency. The temperature dependence of the conductivity dictates the size of the collision frequency.

This paper is organized in the following fashion. Section 2 describes surface wave fields supported by the target. The plume dynamics are built into the model in Section 3. Expressions for the required energy and power of the surface wave are established in Section 4. A numerical study is conducted in Section 5 with conclusions following in Section 6.

2. SUPPORTED SURFACE WAVE FIELDS

A planar good conducting target, extending throughout the lower half of space, supports a transverse magnetic (TM) surface wave as illustrated in Figure 1. For convenience, the $z$- and $x$-axes of a Cartesian coordinate system are, respectively, oriented parallel and normal to the planar surface with origin on the target surface. The nearly free electron model characterizes the electrical properties of the target. The number density of conduction electrons, $N_0$, is determined by the body center cubic formation of the crystal as in its natural, defect free, state. The effective mass of the electrons in the target, $m^*$, phenomenologically incorporates quantum mechanical binding force contributions due to the lattice ions in a modeled free space region. The surface wave, propagating in the $+z$-direction, impinges normally upon the plasma debris or dust plume modeled as a rectangular slab of mass $m_t$ projecting normal to the target with height $h$ and thickness $d$ moving with velocity $-v_{\perp}$. The plume slab velocity represents the expansion of the dust plume parallel to the target surface. It is anticipated that the expansion speed of the plume is many orders small compared to the phase and group velocities of the surface wave. Therefore, all moving medium effects have been neglected. The mass of the plume slab is determined by the amount of material ejected from the target when thermalized by the electron beam. Since the plume is expanding radially, half of the total material ejected from the target is lost due to material expansion coparallel to the incident surface wave. Therefore, all moving medium effects have been neglected. The mass of the plume slab is determined by the amount of material ejected from the target when thermalized by the electron beam. Since the plume is expanding radially, half of the total material ejected from the target is lost due to material expansion coparallel to the incident surface wave. Therefore, all moving medium effects have been neglected.

Fig. 1. Sketch of the surface wave guided by a flat planar target medium towards a plasma plume slab of height $h$ propagating counterparallel to the surface wave with velocity $-v_{z0}$. The plume slab velocity represents the expansion of the dust plume parallel to the target surface. It is anticipated that the expansion speed of the plume is many orders small compared to the phase and group velocities of the surface wave. Therefore, all moving medium effects have been neglected. The mass of the plume slab is determined by the amount of material ejected from the target when thermalized by the electron beam. Since the plume is expanding radially, half of the total material ejected from the target is lost due to material expansion coparallel to the incident surface wave. Therefore, all moving medium effects have been neglected.

The height, $h$, of the plume slab is determined by the time interval between shots times the average velocity of the debris moving perpendicular to the target surface. The thickness of the slab, $d$, is one half of the electron beam diameter bombarding the target. This is reasonable since the dust plume is assumed to expand uniformly in the radial direction relative to the electron beam axis. The length of the plume parallel to the target surface is considered to be of infinite extent. For simplicity, the origin of the coordinate system is located at the front surface of the plume slab at the time the surface wave impinges upon the plume surface. Since the target material is assumed to be a good conductor, the dense plasma debris formed is considered to be highly conducting and is modeled as a perfect conductor. The region external to the target and dust plume has the following permittivity and permeability properties: $\varepsilon_0 = \varepsilon_0 \varepsilon_r$ and $\mu_0 = \mu_0$, respectively. The quantities $\varepsilon_0$ and $\mu_0$ are, respectively, the permittivity and permeability of free space. A time harmonic solution of the form $\exp[-i\omega t]$ is assumed throughout this work.

The conduction and dielectric properties of the target slab are modeled within the limits of the cold, nearly free electron model, assuming the target material extends over all
space. Collisional loss contributions are incorporated in the force equation impeding free streaming motion of the electron. Neglecting magnetic field contributions, the nonrelativistic equation of motion governing the nearly free electron trajectories in the target is

\[ \frac{\partial \vec{v}}{\partial t} = -e\vec{E} - nm^*v\vec{u}, \]  

where \( \vec{v} \) is the drift velocity, \( \nu \) is the collision frequency, \( \vec{E} \) is the applied electric field, and \( q = -e \) for electrons. The cold-plasma dielectric constant (effective permittivity) of the planar target can be shown to yield (Stix, 1992)

\[ \varepsilon_{\text{meff}} = \varepsilon_0\left[1 - \frac{\omega_{pe}^2}{(\omega^2 + i\nu \nu)}\right] \varepsilon_{\text{meff}} \varepsilon_0^{m*}, \]  

where \( \omega_{pe}^2 = e^2N_0/e_0m^* \) is the angular electron plasma frequency of the conduction electrons in the target.

Consider the free streaming motion during the mean time between collisions. The second term on the right hand side of Eq. (1) is then set to zero. Using the point form of Ohm’s law, the collision frequency is expressed in terms of the material properties of the target medium as

\[ \nu = \frac{\varepsilon_0}{\sigma} \omega_{pe}^2, \]  

where \( \sigma \) is the conductivity properties of the target.

The planar target yields an inductive nature and thereby is limited to supporting the TM surface wave. The surface wave fields as obtained from Maxwell's equations are

\[ H_{\text{ao}}(x, z) = \frac{2ik}{\omega e_a} H_{\text{ao}}^+e^{a_z} \cos(k_a z) \]  

\[ E_{\text{ao}}(x, z) = \frac{2ik}{\omega e_a} H_{\text{ao}}^+e^{-a_z} \sin(k_a z) \]  

\[ E_{\text{oa}}(x, z) = \frac{2ik}{\omega e_a} H_{\text{oa}}^+e^{-a_z} \cos(k_a z) \]  

\[ \alpha_a = \left[ k_a^2 - \omega^2\mu_a\varepsilon_a \right]^{1/2} \]  

\[ H_{\text{ao}}(x, z) = H_{\text{ao}}^+e^{a_z}e^{ik_a z} + H_{\text{ao}}^-e^{-a_z}e^{-ik_a z} \]  

\[ E_{\text{ao}}(x, z) = \frac{k}{\omega \varepsilon_{\text{meff}}} \left[ H_{\text{oa}}^0e^{a_z}e^{ik_a z} - H_{\text{om}}^0e^{-a_z}e^{-ik_a z} \right] \]  

Here, the boundary conditions at the perfectly conducting, plume slab surface located at \( z = 0 \) have been taken into consideration in Eqs. (4a)–(4d).

Continuity of the tangential magnetic field component at the air–target interface requires the phase information at the interface to be equivalent, implying that for all \( z < 0, k_a = k_m = k \) and \( H_{\text{ao}}^0 = H_{\text{mo}}^0 \). Satisfying the boundary condition regarding the tangential electric field yields

\[ \alpha_a = -\frac{\varepsilon_a}{\varepsilon_{\text{meff}}} \alpha_m, \]  

or, consequently,

\[ k = \frac{\omega}{c} (\varepsilon_m)^{1/2} \left[ \omega^2 - \omega_{pe}^2 + i\nu \nu \omega \right]^{1/2} \]

\[ = |k| e^{i\phi} = k + i\nu. \]  

The wave solution requires the real part of the wavenumber in Eq. (6b) to be positive. Therefore, for surface waves to be supported by the air–target interface,

\[ \omega^2 \approx \frac{\left[ (1 + \varepsilon_m)\omega_{pe}^2 - \nu^2 \right]}{2(1 + \varepsilon_m)} \]  

or

\[ 0 \leq \omega^2 \approx \frac{\left[ (1 + \varepsilon_m)\omega_{pe}^2 - \nu^2 \right]}{2(1 + \varepsilon_m)} \]  

Further, the attenuation coefficients \( \alpha_a \) and \( \alpha_m \) expressed in terms of the frequency given by

\[ \alpha_a = \frac{\varepsilon_a}{c} \left[ \omega^2 + i\nu \nu \right]^{1/2} \]

\[ = \alpha_{ra} + i\alpha_{ia} = |\alpha_a| e^{i\phi_{\alpha_a}} \]  

\[ \alpha_m = \frac{\omega}{c} \left[ \frac{(\omega^2 + i\nu \nu)(\omega^2 - 2\omega_{pe}^2) + \omega_{pe}^4}{(\omega^2 + i\nu \nu)(\omega^2 - 2\omega_{pe}^2) + \omega_{pe}^4} \right]^{1/2} \]

\[ = \alpha_{rm} + i\alpha_{im} = |\alpha_m| e^{i\phi_{\alpha_m}} \]  

must have a positive real part. If Eq. (7b) is satisfied, the constraints on both attenuation coefficients are satisfied. As \( \nu \) approaches zero, the upper limit in Eq. (7b) approaches \( \omega_{pe}^2/2 \). As \( \nu \) approaches \( \omega_{pe} \sqrt{2} \) and \( \nu \) approaches zero, the attenuation coefficient in the air region approaches the attenuation coefficient in the target material.
The phase velocity, \( v_{ph} \), can be obtained by dividing the angular frequency by the real part of the wavenumber given in Eq. (6b). The associated group velocity, \( v_g \), with target losses included is

\[
v_g = \frac{2c}{(\varepsilon_n)^{1/2} [A + A^*]^{-1}}, \tag{9a}
\]

where

\[
A = \frac{(2\varepsilon_n)(\omega^2 + i\omega \varepsilon_0)^2 - 2\omega_p\omega_e(\omega^2 + i\omega \varepsilon_0) + \omega_e^2 - i0.5\varepsilon_n\omega_e \omega_p^2}{[(\omega^2 + i\omega \varepsilon_0 - \omega_p^2)^{1/2}][(1 + \varepsilon_n)(\omega^2 + i\omega \varepsilon_0 - \omega_p^2)^{1/2}].} \tag{9b}
\]

In the limit as the collision frequency approaches zero, Eq. (9a) simplifies to

\[
v_g = \frac{c}{(\varepsilon_n)^{1/2}} \frac{[\omega^2 - \omega_p^2]^{1/2}[(1 + \varepsilon_n)\omega^2 - \omega_p^2]^{3/2}}{(1 + \varepsilon_n)\omega^2 + \omega_p^4 - 2\omega_p^2 \omega^2}. \tag{9c}
\]

The dispersion relation based on Eq. (6b) is examined at source frequencies below \( \omega_{pe}/\sqrt{1 + \varepsilon_n} \) in Figure 2a for various normalized collisional frequencies \( \omega_n = \nu/\omega_{pe} \) where \( \varepsilon_n = 1 \). The dispersion relation depicted in Figure 2a is significantly altered from its collisionless case when the collision frequency is an order of magnitude small relative to the electron plasma frequency. The straight-line curve marked L.C. in the figure demarcates the Light Cone where \( \omega/k = d\omega/dk = c \). In the collisionless case, a stopband exists in the range \( \omega_{pe}/\sqrt{1 + \varepsilon_n} < \omega < \omega_{pe} \). Electromagnetic waves with frequencies in this range are evanescent in the collisionless case. When loss is introduced, propagating waves can exist with frequencies in the stopband but the nature of the wave is significantly altered. At any one frequency, the phase velocity monotonically decreases to \( c\sqrt{1 + \varepsilon_n}/\sqrt{\varepsilon_n} \) as the collision frequency increases. For a particular collision frequency, the minimum phase velocity is greater than zero excluding the special collisionless case. Near resonance (\( \omega \approx \omega_{pe} \)), the concept of group velocity tends to lose its physical meaning when the collision frequency is small. This is because a small spread in wave number does not yield a small spread in frequency as required for the group velocity to be meaningful in energy transport (Seshadri & Schill, 1984; Schill, 1991). The group velocity is associated with the speed of the wave packet, which is not necessarily related to the motion of the disturbance, especially in the case of anomalous dispersion (i.e., \( dv_{ph}/d\omega > 0 \) implying \( v_g > v_{ph} \); Seshadri, 1971). The dispersion diagrams in Figure 2a show this tendency as the slope of the curves changes from positive to negative with increase in frequency. For large collision frequencies as depicted in Figure 2a, the group velocity and the phase velocity are greater than the speed of light. Again, the physical interpretation of the group velocity loses its meaning. Near the melting temperature of tantalum, the normalized collision frequency lies between 0.1 and 0.2 in value. For source frequencies less than 65% of the electron plasma frequency, the phase and group velocities are greater than 25% of the speed of light. Even in the collisionless case, the phase and group velocities are greater than 10% of the speed of light. Decreasing the source frequency, both the phase and group velocities monotonically approach the speed of light. Consequently, it is a reasonable assumption to neglect the moving medium properties of the plume slab when \( \omega_{pe} \gg \sqrt{2} \omega \). Figure 2b shows the nature of the wave when the source frequency varies over the range between 0 and twice the electron plasma frequency for a collision frequency equal to 15% of the electron plasma frequency. In the stopband frequency range of the special collisionless case, wave propagation is apparent, but high attenuation is present. A smooth transition from slow wave to fast wave occurs as the source frequency increases from values below \( \omega_{pe}/\sqrt{1 + \varepsilon_n} \) to values above \( \omega_{pe} \). In the stopband region away from the transition points, the group velocity is in the opposite direction with respect to the phase velocity. This anomalous dispersion is representative of the flow of energy back toward the source generating the wave.

The loading effects in the target region behind the position of the front surface of the plume slab have been neglected in the above formulation. This is because the energy in the surface wave is concentrated near the target surface interface in front of the slab. Since the fields inside the plume are zero, the fields tangential to the plume slab-target interface just inside the target are zero. As a result, only evanescent waves can be supported in regions below the plume slab. Consequently, energy loss as a result of the leakage fields to the other side of the plasma plume has been neglected.

### 3. DYNAMICS OF THE PLUME SLAB

The electromagnetic pressure acting on the plume slab must be large enough to overcome the initial momentum of the slab and move it an appropriate distance beyond its initial position in the direction opposite to its initial motion. The vector pressure acting on the plume slab in terms of the charge and current densities on the surface of the slab is

\[
\vec{P}(x, z = 0, t) = \rho_s(x, t)\vec{E}(x, z = 0, t) + \vec{J}_s(x, t) \times \vec{B}(x, z = 0, t). \tag{10}
\]

The surface current density is dictated by the boundary conditions on the surface of the plume slab. Consequently,

\[
\vec{J}_s(x, y, t) = 2\left|H_{1,0}\right|e^{-\alpha_{uw}x} \cos(\omega t + \alpha_{uw}x - \theta_H)\hat{y}, \tag{11}
\]

where \( H_{1,0} = |H_{1,0}| \exp(i\theta_H) \). The continuity equation links the surface charge density directly to the surface current density. The surface charge density yields
Fig. 2. The dispersion diagram is presented for (a) various normalized collision frequencies below the electron plasma frequency, and (b) a single normalized collision frequency 0.15 relevant to the plasma removal parameters under consideration over frequencies well beyond the collisionless stopband region. In b, the real and imaginary parts of the wavenumber are presented showing a strong absorption in the stopband region. Collisional losses provide a smooth transition between the slow and fast propagating wave dispersion curves by way of the stopband. The expression for the normalized collision frequency in terms of the unnormalized collision frequency is given as $\nu_n = \nu/\omega_{pe}$. 
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\[
\rho(x, y, t) = \frac{2|\alpha_{\text{ai}}|}{\omega} |H_{\text{aw}}^*| e^{-\alpha_{\text{ai}} x} \sin(\omega t + \alpha_{\text{ai}} x - \theta_H - \theta_{\text{aw}}),
\]

(12)

which is in agreement with boundary conditions on the surface of the perfect conductor.

The instantaneous pressure and force exerted on the plume slab wall are, respectively,

\[
p(x, y, z, t) = 2\mu_0 |H_{\text{aw}}^*|^2 e^{-2\alpha_{\text{ai}} x} \\
\times \left[ \left( 1 - \frac{c^2 |\alpha_{\text{ai}}|^2}{\epsilon_{\text{ai}} \omega^2} \right) + \cos(2\omega t + 2\alpha_{\text{ai}} x - 2\theta_H) \right] \\
+ \frac{c^2 |\alpha_{\text{ai}}|^2}{\epsilon_{\text{ai}} \omega^2} \cos(2\omega t + 2\alpha_{\text{ai}} x - 2\theta_H - 2\theta_{\text{aw}}) \right]
\]

(13a)

\[
\hat{F}(x, y, z, t) = \int_0^h \int_0^{by} 2\mu_0 |H_{\text{aw}}^*|^2 e^{-2\alpha_{\text{ai}} x} \\
\times \left[ \left( 1 - \frac{c^2 |\alpha_{\text{ai}}|^2}{\epsilon_{\text{ai}} \omega^2} \right) + \cos(2\omega t + 2\alpha_{\text{ai}} x - 2\theta_H) \right] \\
+ \frac{c^2 |\alpha_{\text{ai}}|^2}{\epsilon_{\text{ai}} \omega^2} \cos(2\omega t + 2\alpha_{\text{ai}} x - 2\theta_H - 2\theta_{\text{aw}}) \right] \, dx \, dy.
\]

(13b)

The percent change in the time-averaged pressure at the top of the plume relative to the time-averaged pressure at the base of the plume as determined from Eq. (13a) is

\[
\% \Delta p_{\text{ave}} = 100 (1 - e^{-2\alpha_{\text{ai}} h}).
\]

(14)

The source frequency may be determined directly from Eq. (14) with the aid of Eq. (8a) based on the desired change in pressure over the height of the plume slab. This relation provides one design criteria on the type of source needed for moving the plume slab.

The reference time is chosen to be zero as the surface wave impinges on the plume surface crossing the \( z = 0 \) position with velocity \( \dot{v} = -v_\text{aw} \hat{z} \). Under these conditions, the position of the front surface of the plume, \( z(t) \), as a function of \( t \) may be determined from Eq. (13b). The amplitude and phase of the magnetic field, time, and the position of the plume surface are not known. The magnetic field amplitude required to move the plume of mass \( m_p \) a distance \( Z_s \) in a time \( T_s \) can be written as

\[
|H_{\text{aw}}^*| = \left[ \frac{[Z_s + v_\text{aw} T_s]}{2\omega T_s} \sin(2\omega T_s - 2\theta_H) \right] \\
\times \left[ \frac{A_2}{4\omega^2} + \frac{2\omega T_s \cos(2\theta_H) - \sin(2\theta_H)}{4\omega^2} \right]^{1/2}
\]

(15)

where

\[
A_1 = \frac{\mu_0 A_0 y}{\alpha_\text{aw} m_p} \left[ \left( 1 - \frac{c^2 |\alpha_{\text{ai}}|^2}{\epsilon_{\text{ai}} \omega^2} \right) (1 - e^{-2\alpha_{\text{ai}} h}) \right]
\]

(16)

\[
A_2 = \frac{\mu_0 A_0 y}{\alpha_\text{aw} m_p} \left( \frac{\alpha_{\text{ai}}}{\alpha_{\text{aw}}} \right)^2 + 1
\]

\[
\times \left[ \frac{\alpha_{\text{aw}}}{\alpha_{\text{ai}}} \left[ 1 - \cos(2\alpha_{\text{ai}} h) e^{-2\alpha_{\text{ai}} h} \right] \right. \\
+ \sin(2\alpha_{\text{ai}} h) e^{-2\alpha_{\text{ai}} h} + \frac{c^2 |\alpha_{\text{ai}}|^2}{\epsilon_{\text{ai}} \omega^2} \right. \\
\left. \times \left[ \frac{\alpha_{\text{aw}}}{\alpha_{\text{ai}}} \left[ \cos(2\theta_{\text{aw}}) - \cos(2\alpha_{\text{ai}} h - 2\theta_{\text{aw}}) e^{-2\alpha_{\text{ai}} h} \right] \right] \right. \\
\left. + \sin(2\theta_{\text{aw}}) + \sin(2\alpha_{\text{ai}} h - 2\theta_{\text{aw}}) e^{-2\alpha_{\text{ai}} h} \right] \right] \right]
\]

(17)

\[
A_3 = \frac{\mu_0 A_0 y}{\alpha_\text{aw} m_p} \left( \frac{\alpha_{\text{ai}}}{\alpha_{\text{aw}}} \right)^2 + 1
\]

\[
\times \left\{ \frac{\alpha_{\text{aw}}}{\alpha_{\text{ai}}} \sin(2\alpha_{\text{ai}} h) e^{-2\alpha_{\text{ai}} h} \\
- [1 - \cos(2\alpha_{\text{ai}} h) e^{-2\alpha_{\text{ai}} h}] + \frac{c^2 |\alpha_{\text{ai}}|^2}{\epsilon_{\text{ai}} \omega^2} \right. \\
\left. \times \left[ \frac{\alpha_{\text{aw}}}{\alpha_{\text{ai}}} \left[ \sin(2\alpha_{\text{ai}} h - 2\theta_{\text{aw}}) e^{-2\alpha_{\text{ai}} h} \\
- \sin(2\theta_{\text{aw}}) - \cos(2\theta_{\text{aw}}) \right] \right] \right. \\
\left. + \cos(2\alpha_{\text{ai}} h - 2\theta_{\text{aw}}) e^{-2\alpha_{\text{ai}} h} \right] \right\}. \]

(18)

In the limit when the conductivity of the target approaches infinity and \( \theta_H = 0 \), the amplitude of the magnetic field in Eq. (15) simplifies to
\[ |H_{\text{out}}^{\omega} \rangle = \left[ 2 \mu_0 \Delta y \alpha_0 m_e \left( 1 - e^{-2 \alpha_0 \Delta y} \right) \left( \frac{1}{\epsilon_{m_0} \omega_0^2} \right) \left( 1 - \cos(2 \omega_0 T_s) \right) \right]^{1/2}. \] (19)

4. POWER AND ENERGY

The time-averaged power and energy required from the source to drive such a surface wave is obtained from a Poynting vector calculation. The time-averaged power per unit width is

\[ \frac{P_{\text{ave}}}{\Delta y} = \frac{1}{4} \omega \left[ \epsilon_{e_0 a_{rel}} + \frac{|k|}{\alpha_{m_0} |\epsilon_{m_0}|} \cos(\theta_k - \theta_{e_m}) \right] |H_{\text{out}}^{\omega}|^2 e^{2k L}. \] (20)

where \( L = L_c + Z_s \) is the distance between where the laser beam is coupled to the target surface (\( z = -L_c \)) and the final position of the plume wall (\( z = Z_s \)). The minimum time-averaged power required to move the plume the desired distance in the time allotted is obtained if \( |H_{\text{out}}^{\omega}| = |H_{\text{min}}^{\omega}| \) as expressed in Eq. (15). On its own right, this expression is not very illuminating. In the limit when collisional losses approach zero and \( \epsilon_{e_0} = 1 \), Eq. (20) simplifies to

\[ \frac{P_{\text{ave}}}{\Delta y} = \frac{1}{4} \omega \left[ \frac{\omega_0^2 - \omega^2}{\omega^2} \left( \frac{\omega^4}{\omega_0^4 + \omega_0^2 (\omega_0^2 - 2 \omega^2)} \right)^{1/2} \right] \times \frac{|H_{\text{out}}^{\omega}|^2}{\epsilon_{e_0} (\omega_0^2 - \omega^2)^{1/2}}. \] (21)

The source energy per unit width required to move the conductor in a time interval \( T_s \) is

\[ \frac{E_{\text{ave}}}{\Delta y} = \frac{P_{\text{ave}}}{\Delta y} T_s. \] (22)

In the limit when the source frequency becomes small compared to the electron plasma frequency and collisional losses are forced to zero, the time-averaged power per unit length given by Eq. (21), simplifies to

\[ \frac{P_{\text{ave}}}{\Delta y} \approx \frac{1}{4} \omega \frac{\omega_0}{\epsilon_{e_0} \omega_0^2} |H_{\text{out}}^{\omega}|^2. \] (23)

Here, the amplitude of the field is determined by the properties of the source. The minimum time-averaged power required of a source to move the plume slab the desired distance in the allotted time is obtained when \( |H_{\text{out}}^{\omega}| = |H_{\text{min}}^{\omega}| \) [refer to Eq. (19)]. A critical angular frequency,

\[ \omega_{\text{critical}} = \left( \frac{c \omega_0}{2 h} \right)^{1/2}, \] (24)

is identified which demarcates the source frequency’s behavior on the minimum time-averaged power required to move the plume slab. Equation (24) is used to binomial expand the exponential term in the denominator of Eq. (19).

Under the constraints that \( \nu = 0 \), \( \omega_{\text{pe}} \gg \omega \sqrt{2} \), \( \omega \gg 1/T_s \), and \( \omega^2 \ll \omega_{\text{critical}}^2 \), the minimum time-averaged power required from the source as approximated from Eq. (21) is

\[ \frac{P_{\text{ave,min}}}{\Delta y} \approx \frac{1}{4} \omega \frac{\omega_{\text{pe}}}{\omega_0^2} \frac{c^2 m_e}{\epsilon_{e_0} \Delta y} \frac{(Z_s + v_{\text{c}} T_s)}{T_s^2}. \] (25)

In both Eqs. (23) and (25), the time-averaged power per unit length is inversely proportional to the source frequency squared. Consequently, as the source frequency is increased, the time-averaged power decreases by the inverse of the square of the source frequency. For Eq. (25) to be valid, the frequency must be less than some upper limit. This upper limit can be determined by the inequality \( (\omega_{\text{pe}} c)/(2 \hbar \omega_0^2) \gg 1 \) where the left-hand side of this relation is solely contained in Eq. (25). Further, for Eq. (25) to be valid, the parameters of the problem must satisfy the inequalities \( \omega_{\text{pe}} \gg (2 h)/(c T_s^2) \) and \( \omega_{\text{pe}} > (2)^{1/2}/T_s \). These expressions basically imply that the duration of time needed to move the plume slab must be long enough for the effective motion of the slab to be felt and communicated in the target material composed of the nearly free electrons. It is obvious from Eq. (25) that the minimum power decreases with increases in the time required to move the slab. For a long time duration, the minimum energy required to move the slab is independent of the time but is proportional to the velocity of the slab and inversely proportional to the source frequency squared. Observe that the minimum time-averaged power per unit length is also inversely proportional to the height of the slab. This is reasonable, since the greater the height of the slab, the more surface area available to push on and, consequently, the pressure and hence the power supplied by the source need not be as great. For the applications being considered, it is desirable to keep the height of the slab as small as possible.

Under the constraints \( \nu = 0 \), \( \omega_{\text{pe}} \gg \omega \sqrt{2} \), \( \omega \gg 1/T_s \), and \( \omega^2 \gg \omega_{\text{critical}}^2 \), the minimum time-averaged power required from the source is

\[ \frac{P_{\text{ave,min}}}{\Delta y} \approx \frac{c m_e}{2 \hbar} \frac{(Z_s + v_{\text{c}} T_s)}{T_s^4}. \] (26)

Equation (26) states that the minimum time-averaged power is no longer a function of the source frequency. The maximum value of the source frequency is only limited by the plasma frequency. The collective nature of the target electron plasma limits the validity of Eq. (26) as observed from combining the constraints to yield \( \omega_{\text{pe}} \gg (2)^{1/2}/T_s \) and \( \omega_{\text{pe}} > c/h \). For a long time duration to move the plume, the
minimum time averaged energy is independent of both the time duration to move the plume, the height of the plume, and source frequency.

5. NUMERICAL CALCULATION AND DISCUSSION

Tantalum is one high Z material typically used to generate high energy X rays. The tantalum target is assumed to be made of pure tantalum atoms with negligible imperfections. The tantalum ion mass is $3 \times 10^{-25}$ kg. The body centered cubic packing formation of the tantalum atom yields a $4.9 \times 10^{28}$ atoms/m$^3$ density. The effective mass of the conduction electron in the tantalum material is nearly equal to the rest mass of the electron. The conductivity of the metal is a function of temperature and has been determined over a range of temperatures elsewhere (Lide, 1994). This data has been used to extrapolate the conductivity of the metal at the melting point (3017°C) to be $6.83 \times 10^3$ S/m which is significantly different from its $8.1 \times 10^6$ S/m value for temperatures between 0°C and 20°C. Further, it is assumed that there is one conduction electron for every target atom when determining the plasma frequency and electron density in the target material.

The parameters used for the electron beam and generated plume are commensurate with single-shot radiography experiments performed with the SABRE device at Sandia National Laboratory and with the future goals of robust radiography. The electron beam spot size was chosen to be equal to the required spot size of the bremsstrahlung-generated X rays desired. It has been shown that the X-ray spot size is dependent on the electron beam diameter. A 1-mm electron beam diameter is assumed. The pulse duration of the electron beam is 50 ns and the time duration between pulses in the absence of the beam is chosen to be 450 ns. The tantalum plasma debris density as obtained from simulations is on the order of $10^{20}$ to $10^{23}$ ions/m$^3$. These densities are on the order of eight to five orders of magnitude smaller than the atomic density of the solid target. The maximum slab density of the plume is $7.7 \times 10^{22}$ ions/m$^3$. This is based on the spallation of half the number of atoms from the cylindrical hole equally redistributed in a slab of length equal to the electron beam diameter, of depth $d$ equal to half of the diameter of the electron beam, and of height $h$ equivalent to the time between shots times the plume velocity perpendicular to the target surface. Note that half of the generated ions are assumed to be moving in the same direction of the surface wave and therefore are not of interest. The plume velocity perpendicular to the target surface is 20 km/s, while that parallel to the surface is 5 km/s, yielding the four to one aspect ratio. The plasma debris is modeled as a solid perfectly conducting, finite slab. The height of the plume slab in a 500-ns window is $h = 10$ mm. Unless otherwise stated, these parameters are used in numerical simulation.

Except when explicitly indicated, the surface wave is assumed to travel about $L_c = 100$ mm along the hot (3017°C) tantalum target before reaching the original position of the plasma plume. Further, the plume is to be moved 2 mm ($Z_s = 2$ mm), which is twice the diameter of the spot size of the electron beam.

It is desired to confine most of the source energy near the surface of the target with a reasonable distribution over the plume slab height (10 mm) so to minimize leakage losses. The source requirements for a 90% pressure variation, as determined when target losses are neglected, over the plume height is examined. The power and energy from a 90.86 μm (3.302 THz) source required to move a 10-mm by 1-mm by 0.5-mm plume slab 2 mm from its original position in the direction of surface wave propagation in a 450-ns window are shown in Figure 3. A slab with a $7.7 \times 10^{20}$ m$^{-3}$ plume density will require about 11 J distributed over the plume surface when target conduction losses are considered. In the absence of target conduction losses, the required source energy decreases by about a factor of five. As suggested by Eqs. (19) and (20), the time-averaged power and time-averaged energy are linearly proportional to the mass of the slab. The slab mass, in turn, is linearly proportional to the plume density. Consequently, one may deduce the source energies required for various plume densities under the above constraints.

Referring to Figure 4, observe in the lossless case that the power and energy of the laser greatly exceeds that required by the case with loss as the frequency decreases. This is expected because the lossless model breaks down. As the frequency becomes small, the effective permittivity of the target based on the nearly free electron model as indicated from Eq. (2) approaches negative infinity when $\nu = 0$. As the permittivity becomes large in the negative sense, the target medium tends to expel the electric field in that medium as observed in Eqs. (5b) and (5c) with the aid of Eqs. (2), (6a), (6b), (8a), and (8b). Therefore, the fields in the low frequency limit tend to dominate in the air region. Due to boundary conditions, the tangential electric field must then vanish at the surface of the target and hence at every place in the air medium. Further, the attenuation coefficient in the air medium approaches zero as shown in Eqs. (6a) and (8b). The fields in the air region tend to approach that of a homogeneous TEM plane wave. The power and energy of the laser were determined by evaluating the total field over the infinite in-extent plane at a $z = \text{constant}$ position. Since the field in the low frequency limit approaches that of a homogeneous TEM plane wave, the power added up over the infinite in-extent plane becomes very large, approaching infinity. One might expect that the amplitude of the magnetic field intensity given by Eq. (19) would approach infinity in order to adjust for the decrease of the electric field as the source frequency approaches zero. This would be a reasonable expectation, but it is not the case because the plume slab was modeled as a perfect conductor and hence a surface current density was used to describe the electro-
magnetic characteristics of the plume. In the limit that the frequency approaches zero, physically the magnetic field penetrates the plume slab. This cannot be observed in the model due to the use of the surface current. The difficulty can be resolved by solving Maxwell’s equations inside the plume, assuming the plume to be a good conductor and matching boundary conditions. In the limit as the frequency approaches zero, a magnetostatic field will be allowed to penetrate the medium regardless of the plume conductivity. In practice, this is one case in which one must be careful of the order in which the limits are being applied, namely, frequency approaches zero first, then the conductivity may be allowed to approach infinity and not vice versa. Consequently, the validity of the theory is limited by the use of this surface current. The skin depth, \( \delta \), given by \( \delta = 1/[\sigma \pi \mu]^{1/2} \), must be examined to determine the low frequency limit of the model. Since the fields in the air region approach that of a plane wave, the concept of skin depth for a plane wave propagating normal to a good conducting medium (the plume slab) is used. If the penetration depth of the wave, which is approximately five skin depths, is very small compared to the thickness or depth of the slab, \( d \), then the surface current model for the plume slab is reasonable. Here, very small implies about one to preferably two orders of magnitude small. Consequently, the penetration depth \( 5 \delta < 100 \ d \) where \( d \) is the plume depth. The depth of the slab considered is one half of the spot size since only half of the slab is expanding in the undesired direction. Therefore for a 1-mm spot size, the depth of the slab is 0.5 mm. Consequently, using the conduction properties of tantalum at the melting temperature, \( \sigma = 6.83 \times 10^5 \ \text{S/m} \), the minimum source frequency in which the theory is valid is roughly 4 kHz. The extrapolated conductivity of the target metal about its boiling point is about half, thereby doubling the minimum frequency. An upper frequency limit for the theory also exists. The concept of skin depth and hence surface current begins to lose its meaning, as the frequency becomes very large, assuming a finite conductivity, since displacement currents become significant. The upper limit on frequency based on the loss tangent is roughly \( 5.6 \times 10^{16} \ \text{rad/s} \) (an engineering order of magnitude approximation is used), which is greater than the plasma frequency of the target and consequently is not exceeded. The concept of skin depth and hence surface current is valid throughout the range of frequencies between

Fig. 3. The laser power and energy required to move a plasma plume modeled as a perfectly conducting solid slab 2 mm in the direction counter to its initial motion within 450 ns with and without target loss contributions. Initially, the plume slab is moving at 5 km/s. If losses are neglected, the pressure distribution over the surface of the slab is 90%. In the loss case, the distribution is 100%. The source frequency is 3.30 THz (\( \lambda = 90.9 \ \mu\text{m} \)). The maximum density of the slab is \( N_{\text{max}} = 7.7 \times 10^{17} \ \text{ions/m}^3 \). The abscissa is the fraction of the maximum density, \( N_{\text{plume}}/N_{\text{max}} \).
10 kHz and one electron plasma frequency ($\sim 2 \times 10^{15}$ Hz $[1.25 \times 10^{16}$ rad/s]) for tantalum. Upon examining the effective permittivity of the target medium as given by Eq. (2), the magnitude of the effective permittivity in the case with loss dominates the case without loss when the source frequency is small compared to the collision frequency. Using a two order of magnitude approximation, the minimum source frequency that may be meaningfully examined in the case without loss is when $\omega_{\text{min lossless}} \approx \nu/100$, where the collision frequency is given by Eq. (3). Assuming the target temperature is at the melting point, the minimum angular frequency based on Eq. (3) is $\omega_{\text{min lossless}} \approx 1.3 \times 10^{-19} \omega_{pe}^2$, where the electron plasma frequency for tantalum is $1.25 \times 10^{16}$ rad/s. Consequently, the minimum frequency is $f_{\text{min lossless}} \sim 3$ THz. As observed in Figure 4 for frequencies lower than this value, the lossless case exhibits poor physics when compared to the case with loss.

The power and energy curves for the cases with and without target conduction losses are shown in Figure 4 over a range of optimal source frequencies. The plume density was assumed to be $7.7 \times 10^{20}$ ions/m$^3$. The critical frequency as determined by Eq. (24) is computed as 2.2 THz. Consequently, the lossless curve depicted in Figure 4 is weakly governed in part by Eqs. (25) and (26). Based on the calculations above, the no-loss case is meaningless at frequencies below 3 THz, where conduction effects dominate the plasma oscillation effects. The dominate contribution of the curve with loss contributions above 2 THz is exponential in nature and is due to target heating as the surface wave propagates to the plume. The exponential term in Eq. (20) is responsible for the exponential nature of the curve. Therefore, to decrease the required source energies and powers as the source frequency is increased, one needs to minimize the heating losses resulting from the target guiding the surface wave. This may be accomplished by minimizing the distance the wave must propagate over the target surface to the plume. It is interesting to note that at the very low frequency end of Figure 4, the power and energy required of the source begins to increase with decrease in frequency for the case with loss contribution. This will be explained later. As a result, a band of optimal frequencies exists between roughly 800 GHz ($375 \mu$m) and 3 THz ($100 \mu$m) where the source requirements are reasonably flat. The percent change in pressure distribution over the plume surface is shown in Figure 5 over a range of source frequencies. It is observed that the difference in the pressure distribution over the height of the modeled plume slab changes more rapidly when loss effects are included as compared to the no-loss case. This implies...
that a lower frequency source is sufficient in driving the surface wave to move the plume slab. This is significant. Figure 3 was based on a 90% change in pressure over the plume slab when conduction contributions were neglected. The required source frequency provided above agrees with what is obtained from Figure 5. If the 90% change in pressure is based on the target model with conduction loss included, the required source frequency is less than 1 THz and the source energy and power are significantly lower.

Curve a in Figures 6a and 6b extend the source frequency of Figure 4 over a five order of magnitude range for the case with loss contributions. Observe that the laser energy and power increase with a decrease in the source frequency beyond a certain value. The increase is very significant and is mainly a consequence of the surface wave height (1 e-fold) above the target. The surface wave height, $1/\alpha_{sv}$, is displayed in Figure 7. A five order of magnitude change in frequency results in about seven orders of magnitude change in distance. The amplitude of the magnetic field yields about a one order of magnitude change over this frequency range as shown in Figure 8. Figures 4, 6a, and 6b show that the minimum source power and energy occur at about 1 THz. In about 500 ns, the unilluminated plume slab will move about 2.5 mm in the $-z$ direction. By decreasing the distance that the surface wave must propagate to four times this 2.5 mm distance, $L_c = 10$ mm, the optimal band of frequencies (wavelengths) increases as shown by curve b in Figures 6a and 6b. Further, if the temperature of the target can on average be maintained at about 0°C, the optimal band of frequencies is frequency up-shifted as shown by curve c in Figures 6a and 6b. This up-shift in frequency is a consequence of the increase in conductivity of the material from its melting temperature value. Since few high power sources exist in the far infrared spectrum, an up-shifted source frequency is desirable, especially if the source requirements allow for the use of high power and high energy CO$_2$ laser technology. Another means of minimizing the required source energy is to move the target at a constant velocity parallel to its planar surface. The electron beam impinging on the target will generate a plume with nearly the same dynamic properties, but the with an overall drift velocity. This is a consequence of the momentum of the moving target. Significant changes in the source requirements appear to occur when the target velocity is on the same order as the modeled plume slab velocity. This may not be easily achieved due to the target mass. Figures 4, 6a, and 6b were generated assuming a dust density of $7.7 \times 10^{20}$ m$^{-3}$. Since the energy and power are proportional to the dust density, increasing or decreasing the dust density by an order of magnitude shifts each point on the curves by the same change in order of magnitude in

![Figure 5](image_url)

Fig. 5. The percent change in pressure distributed over the height of the modeled plasma plume is related to the laser frequency. When loss contributions are considered in the target, the surface wave has a tendency to be concentrated near the surface of the target as compared to the case when the target contains no conduction loss.
Fig. 6. The required (a) source powers and (b) source energies to move the modeled plasma plume 2 mm in 450 ns when target conduction loss contributions are retained over five magnitudes of laser frequencies. The following three combinations of target temperature ($T_T$) and distance from the point the source is coupled to the target to the original position of the plume slab ($L_c$) are depicted: curve a, $T_T = 3017^\circ C$, $L_c = 100$ mm; curve b, $T_T = 3017^\circ C$, $L_c = 10$ mm; curve c, $T_T = 0^\circ C$, $L_c = 10$ mm. The plume density is about $7.7 \times 10^{20}$ ions/m$^3$. Increasing or decreasing the plume density by an order of magnitude shifts the energy and power curves by an order of magnitude in the same direction.
power and energy. Consequently, at the optimal frequency, a
dust density of $7.7 \times 10^{22} \text{ m}^{-3}$ will require a source energy
of roughly 190 J.

Consider the situation when a 3.34-THz source is used to
move the plasma plume over a range of times between 400 ns
and 10 $\mu$s. The height of the plasma plume and the plume
density are, respectively, fixed at 10 mm and $7.7 \times 10^{20} \text{ m}^{-3}$.
About a factor of two change in source energy is observed in
Figure 9 over this range in time. Extending the time duration
of plume removal beyond 10 $\mu$s does not result in a signif-
icanct decrease in energy. This is reasonable when examining
the theory. For large times, the magnitude of the magnetic
field intensity approaches the inverse square root of the time
duration. The energy is related to the power times time. For
large times, the laser energy required to move the plasma
slab becomes independent of time duration of plasma re-
moval. The dynamic nature of the plume is responsible for
this effect. As observed in Eqs. (15) and (19), increasing the
time needed to move the plume slab also increases the dis-
tance the slab must be moved due to its initial velocity. If the
slab was initially stationary, then the energy would decrease
with increase in the time duration of plasma removal.

The energy and power appear to be insensitive to changes
in the phase of the magnetic field over the parameters
examined.

Other important justifications and considerations are ex-
amined in light of the simplicity of the model. First, the
plasma plume is assumed to be have an infinite conductivity
because it is very dense and highly collisional. For this
assumption to be valid, magnetic diffusion into the plume
must be small over the removal time. A skin depth argument
was provided assuming the dense plasma plume was a solid.
Here the plume is assumed to be fully ionized, allowing for
the plasma resistivity to be described by the Spitzer resis-
tivity. Using very conservative numbers, the plasma tem-
perature is assumed to be that at the melting point of tantalum
(3017°C; $KT = 0.28 \text{ eV}$) with a density of $7.7 \times 10^{20} \text{ m}^{-3}$.
The Spitzer resistivity, $\eta_S$ (Chen, 1984) and Coulomb loga-
-rithm of the plume are 1.5 mΩ and 4.4, respectively. The
Coulomb logarithm is insensitive to large changes in tem-
perature and density. Since the Spitzer resistivity is in-
versely proportional to temperature and linearly proportional
to the Coulomb logarithm, increases in temperature will
significantly decrease the plume resistance. The character-
istic time, $\tau$, for magnetic field penetration into the plume is
approximated by (Chen, 1984) $\tau = \mu_0 \delta^2 / \eta_S$, where $\delta$ is the
penetration depth. Assuming the laser beam is a rectangular
pulse with a 450 ns time duration, the penetration depth is
$\delta = 2.36 \text{ cm}$ which is much larger than the slab thickness
assumed. Within this time duration, an 800-GHz source will

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**Fig. 7.** The surface wave heights (1 e-folding or skin depth) in the air and target mediums are depicted against over five magnitudes of the source frequency. The height in the air region changes rapidly with frequency as compared to the skin depth in the target medium.
undergo $3.6 \times 10^5$ cycles in 450 ns. Consequently, one may assume the source to be in steady state. Employing the usual definition of skin depth using the Spitzer resistivity to characterize the electrical properties of the plume, the penetration depth $\delta = 22 \mu m$. The modeled plume slab thickness (0.5 mm) is about 25 times larger than the penetration depth. Consequently, a zero penetration depth or an infinite conductivity assumption is reasonable. Increases in plume temperature enhance the validity of the assumption. Second, the assumption regarding the constant density profile of the plume was chosen for simplicity. The rate of temporal and spatial changes requires a more detailed study. Even so, a low constant density was used in this section, assuming the plume density changes significantly within the 50-ns period during which the plume is being generated. Increasing or decreasing the plume density by an order of magnitude shifts the energy and power curves by an order of magnitude in the same direction. Consequently, one can easily deduce the source powers and energies required for other plume densities from the figures provided. Third, it is assumed that the substrate or target guiding the beam will not exceed temperatures needed to boil the target surface. Such a study is currently being examined. If the target surface did boil, it is conceivable that the pulsed electromagnetic source may provide a means to move some new bremsstrahlung target material in front of a forthcoming electron beam in a multi-pulsed, dynamic radiography machine.

6. CONCLUSION

A model was developed to examine the feasibility of using a surface wave to move a modeled perfectly conducting plasma plume slab a distance equal to twice the spot size of the electron beam used to generate the plume. Limitations in the model such as skin depth issues and modeled surface currents have been accounted for and discussed in the numerical simulations. The source energy and power are linearly related to the plume density. Care must be taken in interpreting the model when losses are neglected. Collisional losses dominate the effects due to the nearly free electron model as the frequency is decreased below a critical frequency. Consequently, the lossless nearly free electron model provides meaningless information at the lower frequencies. Near the electron plasma frequency, loss effects significantly alter the propagating characteristics of the source wave.
An optimal laser frequency exists where the laser power and energy needed to move the modeled plasma plume are a minimum. This minimum occurs at the frequency where the propagation losses of the surface wave and the height of the surface wave are relatively small. If the source frequency is increased from this point, propagation losses become significant because of skin depth effects and surface heating. If the source frequency is decreased from the energy minimum point, the height of the surface wave becomes larger. The energy is therefore distributed over a larger volume. Consequently, more source energy is required to exert the same amount of force on the slab as compared to the source at the optimal frequency.

The amount of energy required to move an initially drifting perfectly conducting modeled slab approaches a constant as the time duration for plasma removal becomes large. The dynamic nature of the plasma is responsible for this effect. As the initial velocity of the plume approaches zero and the time duration for removal approaches infinity, the source energy approaches zero.

REFERENCES


Simplistic plasma dust removal model


