Investigating the Effects of Addition with Regrouping Strategy Instruction Among Elementary Students with Learning Disabilities

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INVESTIGATING THE EFFECTS OF ADDITION WITH REGROUPING STRATEGY INSTRUCTION AMONG ELEMENTARY STUDENTS WITH LEARNING DISABILITIES

by

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A dissertation submitted in partial fulfillment of the requirements for the

Doctor of Philosophy in Special Education

Department of Special Education

College of Education

The Graduate College

University of Nevada, Las Vegas
August 2011
THE GRADUATE COLLEGE

We recommend the dissertation prepared under our supervision by

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entitled

Investigating the Effects of Addition with Regrouping Strategy Instruction Among Elementary Students with Learning Disabilities

be accepted in partial fulfillment of the requirements for the degree of

Doctorate of Philosophy in Special Education
Department of Special Education

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August 2011
ABSTRACT

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by

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Many students, specifically those with learning disabilities, struggle to master foundational computation skills such as addition with regrouping. With this in mind, the purpose of this research was to examine the effects of strategy instruction that involved the use of the concrete-representational-abstract teaching sequence on the addition with regrouping computation and word problem-solving skills of students with learning disabilities. This study involved the use of a multiple probe across participants design with two replications. The participants included nine second through sixth graders who had been identified as having a learning disability and were demonstrating mathematics difficulties. There were three females (i.e., one White third grader, one Hispanic fourth grader, and one Hispanic sixth grader) and six males (i.e., two White second graders, two Hispanic third graders, one White third grader, and one Hispanic fifth grader). The participants received 20 lessons (Miller, Kaffar, & Mercer, 2011) that involved the use of strategy instruction and the concrete-representational-abstract teaching sequence to teach addition with regrouping to students with learning disabilities. The instructional method used in these lessons involved the combination of the concrete-representational-abstract sequence and the use of two mathematics strategies (i.e., RENAME and FAST RENAME). The results revealed that students with learning disabilities improved their
abilities to solve addition with regrouping computation and word problems after receiving strategy instruction that involved the use of the concrete-representational-abstract teaching sequence. Additionally, most participants were able to maintain and generalize their abilities to solve addition with regrouping computation and word problems two weeks after receiving the intervention.
ACKNOWLEDGMENTS

I owe the deepest gratitude to those individuals whose assistance and assurances made this dissertation possible. First, I would like to thank the exceptional members of my committee, Susan Miller, Tom Pierce, Kyle Higgins, and Dick Tandy. Each one brought a new way to look at things to the table and encouraged me to think beyond the obvious. A special thank you to Susan Miller who quietly leads her students through her own extreme level of professionalism and leadership. I have learned so much about being a teacher and a professional through watching you. You have taught me to be quite and listen, a gift for which I will be eternally grateful. One day, I hope I am as able to lead through example as you are.

To my parents, brothers, and grandparents, I owe a lifetime of happy memories and the strong belief that it is better to try and fail than never to have tried at all. Thank you for being an integral part of both my childhood and my adulthood. Mom, thank you for encouraging me to start and finish this journey. Without your belief in my ability to climb this mountain, I would never have finished. Thank you for always believing in me and always encouraging me.

To my husband and children I owe my life’s happiness and fulfillment. Thank you, Jay, for always standing quietly by and encouraging me to follow my educational path. You have taken care of our family without complaint and you have never failed to make me laugh when I needed it most. I am grateful to have you as my husband and friend. To my son, Braedon, who has grown into a fine young man. You have never complained when I was gone to class at night or tried from being up late studying the night before. You make me smile every day. I am proud of the person you have become. To my
daughter, Emma, who brings joy to my life. You make be believe in miracles, and you inspire me to be a better person than I am. I love you all.

Finally, I want to thank God for His love and guidance in my life. While I often miss the mark, I desire to walk in your footsteps and touch those around me in positive ways.
# TABLE OF CONTENTS

ABSTRACT ........................................................................................................ iii

ACKNOWLEDGEMENTS .................................................................................... v

LIST OF TABLES ............................................................................................... x

LIST OF FIGURES ............................................................................................. xi

CHAPTER 1 INTRODUCTION ............................................................................ 1
  Historical Review of Mathematics Education Within the United States ......... 1
  Statement of the Problem ............................................................................. 15
  Purpose of the Study and Related Research Questions ............................. 18
  Significance of the Study ............................................................................. 19
  Limitations of the Study ............................................................................. 22
  Definition of Terms ..................................................................................... 22
  Summary ..................................................................................................... 27

CHAPTER 2 REVIEW OF LITERATURE .............................................................. 29
  Literature Review Procedures .................................................................... 29
  Selection Criteria ...................................................................................... 30
  Summary and Analysis of Studies Related to Mathematics Strategy Instruction
  for Students with Learning Disabilities ..................................................... 30
  Summary and Analysis of Studies Related to the Concrete-Representational-Abstract
  Teaching Sequence for Students with Learning Disabilities .................. 48
  Review of Literature Summary .................................................................. 59

CHAPTER 3 METHODOLOGY ........................................................................ 62
  Participants ............................................................................................... 63
  Setting ...................................................................................................... 65
  Instrumentation ....................................................................................... 65
  Materials and Equipment ......................................................................... 75
  Design ...................................................................................................... 76
  Procedures .............................................................................................. 79
  Interscorer Reliability ............................................................................. 85
  Fidelity of Treatment .............................................................................. 86
  Treatment of Data Related to Research Questions .................................... 86

CHAPTER 4 DATA ANALYSIS ......................................................................... 90
  Research Questions and Related Findings ................................................. 90
  Interscorer Reliability ........................................................................... 122
  Fidelity of Treatment ............................................................................. 124
  Summary of Findings ............................................................................. 126

CHAPTER 5 SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS ....... 129
Discussion of Results ................................................................. 130
Conclusions ........................................................................ 139
Practical Implications .......................................................... 140
Recommendations for Further Study ...................................... 141
APPENDIX A    LETTER FOR USE OF COPYRIGHTED MATERIAL .... 143
APPENDIX B    LETTER FOR USE OF COPYRIGHTED MATERIAL .... 144
APPENDIX C    LETTER FOR USE OF COPYRIGHTED MATERIAL .... 145
APPENDIX D    PARENT CONSENT FORM .................................... 146
APPENDIX E    STUDENT ASSENT FORM .................................... 149
APPENDIX F    ADDITION WITH REGROUPING PRETEST ............... 150
APPENDIX G    ADDITION WITH REGROUPING POSTTEST .......... 151
APPENDIX H    ADDITION WITH REGROUPING MAINTENANCE TEST .... 152
APPENDIX I    ADDITION WITH REGROUPING MINUTE ............... 153
APPENDIX J    ADDITION WITH REGROUPING WORD PROBLEM
PRETEST .................................................................................. 154
APPENDIX K    ADDITION WITH REGROUPING WORD PROBLEM
POSTTEST .............................................................................. 155
APPENDIX L    ADDITION WITH REGROUPING WORD PROBLEM
MAINTENANCE TEST ............................................................ 156
APPENDIX M    CONCEPTUAL UNDERSTANDING PRETEST ........... 157
APPENDIX N    CONCEPTUAL UNDERSTANDING POSTTEST ........ 158
APPENDIX O    CONCEPTUAL UNDERSTANDING PRETEST SCORING
PROTOCOL .............................................................................. 159
APPENDIX P    CONCEPTUAL UNDERSTANDING POSTTEST SCORING
PROTOCOL .............................................................................. 161
APPENDIX Q  BASELINE PROBE A ................................................................. 163
APPENDIX R  BASELINE PROBE B ................................................................. 164
APPENDIX S  BASELINE PROBE C ................................................................. 165
APPENDIX T  INTERVENTION PROBE EXAMPLE ........................................... 166
APPENDIX U  MAINTENANCE PROBE ............................................................. 168
APPENDIX V  GENERALIZATION PROBE .......................................................... 169
APPENDIX W  ADDITION WITH REGROUPING SATISFACTION QUESTIONNAIRE ................................................................. 170
APPENDIX X  INTERVENTION SCRIPTED LESSON SAMPLE ......................... 171
APPENDIX Y  PLACE VALUE MAT EXAMPLE .................................................... 172
APPENDIX Z  RENAME MNEMONIC DEVICE ..................................................... 173
APPENDIX AA  FAST RENAME MNEMONIC DEVICE ....................................... 174
APPENDIX BB  IMPLEMENTATION SCHEDULE ............................................... 175
APPENDIX CC  ADDITION WITH REGROUPING INTERVENTION LESSON MATRIX ................................................................. 186
APPENDIX DD  FIDELITY OF TREATMENT CHECKLIST .................................... 192
APPENDIX EE  SUMMARY OF DATA BY PARTICIPANT .................................... 193
REFERENCES .................................................................................................... 194
VITA ..................................................................................................................... 207
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 1</td>
<td>Participant Demographic Data</td>
<td>64</td>
</tr>
<tr>
<td>Table 2</td>
<td>Triad One: Baseline and Intervention Probe Scores</td>
<td>94</td>
</tr>
<tr>
<td>Table 3</td>
<td>Triad Two: Baseline and Intervention Probe Scores</td>
<td>95</td>
</tr>
<tr>
<td>Table 4</td>
<td>Triad Three: Baseline and Intervention Scores</td>
<td>95</td>
</tr>
<tr>
<td>Table 5</td>
<td>Triad One: Addition with Regrouping and Addition with Regrouping Word Problem Pretest and Posttest Scores</td>
<td>98</td>
</tr>
<tr>
<td>Table 6</td>
<td>Triad Two: Addition with Regrouping and Addition with Regrouping Word Problem Pretest and Posttest Scores</td>
<td>99</td>
</tr>
<tr>
<td>Table 7</td>
<td>Triad Three: Addition with Regrouping and Addition with Regrouping Word Problem Pretest and Posttest Scores</td>
<td>99</td>
</tr>
<tr>
<td>Table 8</td>
<td>Triad One: Conceptual Understanding Pretest and Posttest Scores</td>
<td>100</td>
</tr>
<tr>
<td>Table 9</td>
<td>Triad Two: Conceptual Understanding Pretest and Posttest Scores</td>
<td>101</td>
</tr>
<tr>
<td>Table 10</td>
<td>Triad Three: Conceptual Understanding Pretest and Posttest Scores</td>
<td>102</td>
</tr>
<tr>
<td>Table 11</td>
<td>Triad One: Intervention and Maintenance Probe Scores</td>
<td>114</td>
</tr>
<tr>
<td>Table 12</td>
<td>Triad Two: Intervention and Maintenance Probe Scores</td>
<td>115</td>
</tr>
<tr>
<td>Table 13</td>
<td>Triad Three: Intervention and Maintenance Probe Scores</td>
<td>115</td>
</tr>
<tr>
<td>Table 14</td>
<td>Triad One: Addition with Regrouping and Addition with Regrouping Word Problem Post- and Maintenance Test Scores</td>
<td>117</td>
</tr>
<tr>
<td>Table 15</td>
<td>Triad Two: Addition with Regrouping and Addition with Regrouping Word Problem Post- and Maintenance Test Scores</td>
<td>118</td>
</tr>
<tr>
<td>Table 16</td>
<td>Triad Three: Addition with Regrouping and Addition with Regrouping Word Problem Post- and Maintenance Test Scores</td>
<td>118</td>
</tr>
<tr>
<td>Table 17</td>
<td>Triad One: Intervention and Generalization Probe Scores</td>
<td>120</td>
</tr>
<tr>
<td>Table 18</td>
<td>Triad Two: Intervention and Generalization Probe Scores</td>
<td>120</td>
</tr>
<tr>
<td>Table 19</td>
<td>Triad Three: Intervention and Generalization Probe Scores</td>
<td>121</td>
</tr>
<tr>
<td>Table 20</td>
<td>Satisfaction Rating for Students with Learning Disabilities</td>
<td>123</td>
</tr>
<tr>
<td>Table 21</td>
<td>Addition with Regrouping Satisfaction Questionnaire Frequency Count and Mean Scores</td>
<td>124</td>
</tr>
<tr>
<td>Table 22</td>
<td>Interscorer Reliability</td>
<td>125</td>
</tr>
<tr>
<td>Table 23</td>
<td>Fidelity of Treatment</td>
<td>126</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Figure 1  Data Set for Triad One.................................................................91
Figure 2  Data Set for Triad Two ...............................................................92
Figure 3  Data Set for Triad Three .............................................................93
Figure 4  Triad One: Addition with Regrouping Minute Pretest, Posttest, and Maintenance Test Digits Correct in One Minute...........................................103
Figure 5  Triad One: Addition with Regrouping Minute Pretest, Posttest, and Maintenance Test Error Digits in One Minute .................................................103
Figure 6  Triad Two: Addition with Regrouping Minute Pretest, Posttest, and Maintenance Test Digits Correct in One Minute ..............................................104
Figure 7  Triad Two: Addition with Regrouping Minute Pretest, Posttest, and Maintenance Test Error Digits in One Minute .................................................105
Figure 8  Triad Three: Addition with Regrouping Minute Pretest, Posttest, and Maintenance Test Digits Correct in One Minute .............................................106
Figure 9  Triad Three: Addition with Regrouping Minute Pretest, Posttest, and Maintenance Test Error Digits in One Minute .................................................106
Figure 10 Participant 1: Addition with Regrouping Minute Celeration Rate ..........108
Figure 11 Participant 2: Addition with Regrouping Minute Celeration Rate ..........109
Figure 12 Participant 3: Addition with Regrouping Minute Celeration Rate ..........110
Figure 13 Participant 4: Addition with Regrouping Minute Celeration Rate ..........110
Figure 14 Participant 5: Addition with Regrouping Minute Celeration Rate ..........111
Figure 15 Participant 6: Addition with Regrouping Minute Celeration Rate ..........111
Figure 16 Participant 7: Addition with Regrouping Minute Celeration Rate ..........112
Figure 17 Participant 8: Addition with Regrouping Minute Celeration Rate ..........112
Figure 18 Participant 9: Addition with Regrouping Minute Celeration Rate ..........113
CHAPTER 1

INTRODUCTION

With the expectation of reading literacy, there is no subject that stirs controversy and is as hotly debated as mathematics education (Lepage & Sackett, 2002). Mathematics education has been one of the three core components of education in the United States since the mid-19th century (Boutwell, 2001). Traditionally known as the three Rs of education, the inclusion of reading, writing, and arithmetic in a school’s curriculum has remained unchallenged, but political, historical, and social events have radically influenced education, especially mathematics education in the United States (Boutwell, 2001).

Mathematics researchers and educators have been persistent in their mission to improve the quality of mathematics teaching and learning since the early 1700s. Numerous divergent philosophies related to the best ways to teach mathematics have emerged throughout the long history of mathematics education. This quest to improve the mathematics performance of students in the United States is likely to continue for the foreseeable future.

**Historical Review of Mathematics Education Within the United States**

**1700-1900: The Beginning of Mathematics Education**

Initially, formal education in the colonial states did not include mathematics (Furr, 1996). Education focused primarily on preparing privileged college-bound students in the classics and on teaching them literacy skills (Furr, 1996). The town schools located in the Northeast originally included mathematics in the curriculum, but religious leaders thought more traditional subjects such as religion and literacy should receive greater
emphasis. Some of these leaders viewed mathematics as a non-academic subject (Furr, 1996). However, larger towns and cities with small industry and business interests needed mechanical mathematics skills taught in schools (Willoughby, 1967). Benjamin Franklin’s influence in promoting a more utilitarian education encouraged the inclusion of arithmetic in school curricula because of its real world applications and intrinsic value (Furr, 1996).

The first mathematics professor was hired at Harvard in 1726, and soon the prestigious university began requiring competence in mathematics as a prerequisite for college acceptance (Willoughby, 1967). In response, arithmetic began to be taught in most secondary schools. It is very interesting to note that the order in which various topics in mathematics are taught in today’s secondary schools is the same order in which Harvard began requiring such disciplines for entrance: arithmetic, algebra (1820), geometry (1844), and later advanced topics (Furr, 1996).

From 1800 through 1860, several states opened public schools, also known as common schools, which increased the number of students learning mathematics (National Museum of American History [NMAH], 2002). As the number of citizens trained in mathematics was limited, there were few individuals with the capabilities to teach mathematics (Furr, 1996). Mathematics was initially taught as a series of topics to learn through rote memorization (Bidwell & Clason, 1970). Sometimes referred to as the rule method in which rules for a particular type of problem were modeled, students memorized the example and were drilled on the acquired knowledge (Bidwell & Clason, 1970). The focus of instruction was on memorization and students rarely understood the concepts or operations (Furr, 1996). Considered a very difficult subject to master,
mathematics was rarely introduced to boys before the age of 12. Girls were never taught mathematics and relied solely on number sense gathered from real life experiences (Furr, 1996).

Johann Pestalozzi introduced the idea that learners would understand mathematics better if the skills were connected to concrete objects and tangible images (NMAH, 2002). Pestalozzi’s ideas were coupled with the availability of textbooks that had become less expensive and offered more reliable content (NMAH, 2002). Warren Colburn’s textbook was first introduced in the United States in 1821 (Furr, 1996) and was considered one of the most influential mathematics textbooks ever published (Bidwell & Clason, 1970).

Colburn’s program used the discovery of concepts of numbers and operations as a basis to teach children, even children as young as five, mathematics (Furr, 1996). The discovery method was contrary to previous instruction that introduced abstract concepts followed by practice with problems (Furr, 1996). The conflicts between the two distinct schools of thought still exist (Furr, 1996). According to Furr, a great deal of the history of mathematics is, in large part, the continuing struggle to determine if conceptual understanding is necessary for abstract understanding of mathematical concepts (1996).

Beginning in the 1870s, many Americans studied overseas, especially in Germany, and they uncovered new mathematical doctrines (NMAH, 2002). Eager to share the new knowledge, scholars in the field of mathematics obtained equipment specifically associated with the advances in the field in order to assist the rising number of high school students in understanding mathematics (NMAH, 2002). Around the same time, the relatively new field of psychological research began focusing on limiting the teaching of
mathematics to immediately useful topics (Furr, 1996). Research in cognitive development, headed by Hall in the 1880s, promoted teaching mathematics with manipulative devices and practical experience to enhance student learning and motivation (Furr, 1996). Hall recommended deferring introduction in mathematics until later years (Furr, 1996). Hall’s suggestions paralleled the post-World War I anti-intellectual sentiment in which the role of mathematics as one of the core curriculum subjects was questioned (Furr, 1996).

**1900-1950: Beginning of Contemporary Mathematics Education**

In the early 20th century, William Kilpatrick shared many common views with Hall, mainly that content, including mathematics, should be taught to students based on the direct practical value that it held, or if students independently wanted to learn the content (Klein, 2003). Kilpatrick, deemed by many as the nation’s most significant educational leader of the 20th century, majored in mathematics and eventually joined the faculty at Teachers College in 1911 (Klein, 2003). Kilpatrick did not consider mathematics an academic subject and encouraged limiting mathematical content to simple utilitarian concepts (Klein, 2003). Kilpatrick believed that geometry, algebra, and other advanced topics should not be taught in primary or secondary education (Klein, 2003). In fact, Kilpatrick claimed that the thinking required for mathematics was detrimental to ordinary living and believed advanced mathematics courses were offered to too many students (Klein, 2003).

Snedden, a former Commissioner of Education for the state of Massachusetts and professor at Teachers College, agreed with Kilpatrick’s view regarding limiting access to algebra and geometry (Klein, 2003). Snedden asserted that algebra was essentially
useless to over 90% of the population (Klein, 2003). Since Kilpatrick and Snedden were prominent professors at Teachers College, their opinions regarding mathematics education were shared with more than 35,000 future educators (Klein, 2003).

In 1915, Kilpatrick was approached by the National Education Association’s Commission on the Reorganization of Secondary Education and accepted the challenge to chair a committee charged with investigating the problems of teaching mathematics in secondary institutions (Klein, 2003). The committee’s report called *The Problem of Mathematics in Secondary Education* published in 1920 and authored by Kilpatrick stated that the selection of appropriate content should not be determined by tradition, but by merit. The report further stated that mathematics, especially algebra and geometry, had no merit, and thus no logical place within the mathematics curriculum (Klein, 2003). Not surprisingly, mathematicians objected to Kilpatrick’s attack on mathematics and tried to block the publication of the report (Klein, 2003).

*The Problem of Mathematics in Secondary Education* triggered vigorous opposition from mathematicians and members of the Mathematics Association of America (MAA) (Klein, 2003). The same year, the National Council of Teachers of Mathematics (NCTM) was formed in response to anti-mathematics opinion (Willoughby, 1967). In 1923, the MAA, with support from the newly formed NCTM, responded with the *Report of the National Committee on Mathematics Requirements* (1923), which suggested a new mathematics curriculum based on psychology research in education, mathematics education in other countries, and successful school mathematics programs (Furr, 1996; Klein, 2003). The improved 6-3-3 curriculum included an explanation for the importance of the subject and provided a framework for a variety of junior and senior high school
curricula (Furr, 1996). The 1923 report was provided an extremely comprehensive view on the topic of school mathematics while stressing the importance of mathematics, specifically algebra, to every knowledgeable citizen (Klein, 2003). Although the 1923 report had some impact on mathematics education, Kilpatrick’s report had a stronger influence until the 1940s (Klein, 2003).

In the 1940s, the army discovered that draftees knew so little about basic mathematics that the military had to begin teaching basic arithmetic so soldiers could perform simple gunnery maintenance and bookkeeping (Stotsky, 2000). As a matter of national defense, especially during World War II, the U.S. government’s interest in mathematics education increased (Furr, 1996). By the mid 1940s, the need for soldiers to have more advanced mathematical skills was highlighted with the development of radar, navigation, operations analysis, cryptology, rockets, and atomic weapons demanded that more mathematical (Stotsky, 2000).

The 1950s saw a decline in the number of students enrolled both in general mathematics courses and in advanced high school mathematics courses (Walmsley, 2003). Algebra enrollment dropped over 30% from 1909 until 1955 (Klein, 2003). In the 1950s, at the beginning of the Cold War and the Sputnik era, the popular progressive education of the early 1900s lost prominence (Walmsley, 2003).


In the early 1950s, the rumor of reform began to circulate among educators and mathematicians (Stotsky, 2000). In 1957, however, the nation began actively looking for reform when on October 4, 1957 the USSR launched the satellite Sputnik into space. This is believed by many to be the onset of the space race between the United States and
the USSR (Walmsley, 2003). Wooten expressed the consequence of Sputnik as raising questions and doubts about the mathematics programs in the United States (Walmsley, 2003).

Public outcry and panic caused mathematics education to be placed at the forefront of the educational debate (Walmsley, 2003). The federal government increased spending on mathematics education and development (Walmsley, 2003). The budget of the National Science Foundation (NSF), established in 1950, increased from $15,000,000 before Sputnik to nearly 10 times that amount after Sputnik’s launch (Walmsley, 2003).

During this unsettling time, the public became aware of the New Math movement and was skeptical (Amit & Fried, 2002). Although much different from mathematics education of the past, New Math was embraced as a solution to the mathematics crisis (Walmsley, 2003). The new curriculum, referred to as New Math, quickly became extremely controversial (NMAH, 2002).

The uniqueness of mathematics education during the New Math era is credited to the active involvement of mathematicians (Furr, 1996). Hoping to train students in higher mathematics before their entrance in college, mathematicians concentrated on the importance of mathematics education for all ages (Furr, 1996). The mathematicians’ involvement, a job market requiring increased technical knowledge, and research proving that children were capable of learning quite advanced topics at much younger ages spurred the New Math era (Furr, 1996).

In the New Math era, educators, mathematicians, and psychologists worked together to revisit various methods of teaching mathematics (NMAH, 2002). Reformers agreed that a more abstract approach to arithmetic and algebra that included more sophisticated
mathematical ideas would provide greater benefits than teaching approaches of the past (NMAH, 2002). The New Math movement introduced new topics, but the emphasis was on new teaching techniques (Walmsley, 2003). While many reformers associated the New Math movement with radical reform, the changes made amounted to shifts in emphasis (Furr, 1996). Furr (1996) identified five categorical emphasis changes in the New Math movement. First, topics were rearranged in a more logical sequence. Second, advanced mathematical ideas were presented to students at a much earlier age. Third, superfluous topics were removed in order to create time to cover new subject matters. Next, set theory (i.e., the mathematical science of the infinite) was introduced in the classroom as a unifying theme. Finally, Furr explained the New Math movement placed more emphasis on formal logic, applications, and manipulative devices for introduction of analytical-based instruction. Barlage believed the only new portion of the New Math consisted of contemporary topics presented by specialized teachers who were trained through workshops, conferences, college courses, and in-service days (Furr, 1996).

New Math was on its way to extinction by the early 1970s (Klein, 2003). Klein (2003) identified two main reasons for the failure of the New Math movement. The main reason was the underlying belief that all students could be taught more advanced mathematics at an earlier age and with less time. The second explanation was the unrealistic conviction that teachers could be trained in New Math in a short time period. Klein asserted, while the New Math movement set forth true attempts at reform, the movement as a whole failed when students did not experience the anticipated educational gains.
1970-1990: Back to Basics

The Back to Basics ideals, predominant from 1970 to 1990, took mathematics education to where it was in the 1950s (Jones, Langrall, Thornton, & Nisbet, 2002). The student-centered progressive approach leading mathematics education in the 1960s shifted to a more traditional approach in the 1970s (Boutwell, 2001). The experts agreed that the progressive New Math initiative had caused rising school dropout rates, increases in school violence, and declining standardized test scores (Boutwell, 2001). A Back to Basics philosophy ensued (Boutwell, 2001).

In 1972, the federal government created the National Institute of Education with the intent to improve education through conducting research (Stotsky, 2000). The Back to Basics movement continued when the National Council of Mathematics Teachers (NCTM) published An Agenda for Action in 1980 (Stotsky, 2000). The brief report urged educators to adopt problem solving as the main focus of mathematics education (Stotsky, 2000). Still without contributions from mathematicians, the report was similar to reports published in the 1950s other than the mention of the importance of computers and calculators (Stotsky, 2000).

The decade following the 1980 publication by the NCTM of An Agenda for Action created numerous committees charged with satisfying the specifics of mathematics education (Stotsky, 2000). The work of the committees culminated in 1989 with the release of the first of three documents, the Curriculum and Evaluation Standards for School Mathematics (Stotsky, 2000). Often referred to as the NCTM Standards, the first volume spurred the National Science Foundation to financially support a similar study,
which resulted in each state compiling a similar set of standards. Many states already had standards in place, but with the 1980s federal legislation, many states adopted the NCTM Standards or adapted a shorter version of the Standards to meet the particular needs of the individual state (Walmsley, 2003).

In addition to the NCTM agenda, a landmark document entitled *A Nation at Risk* (1983) emerged and created the backdrop for the encroachment of national standards on state standards (Klein, 2003). Specifically, in August 1981, Secretary of Education T. H. Bell formed the National Commission on Excellence in Education (National Commission on Excellence in Education [NCEE], 1983). The committee was given 18 months to make a report to the nation regarding the quality of education in the United States (NCEE, 1983). The report referred to mathematics 10 times and made recommendations regarding the status of mathematics education (NCEE, 1983). First, the report recommended that students be obligated to take at least three years of high school mathematics as a requirement for graduation (NCEE, 1983). Next, the report suggested that high school mathematics would enable students to (a) understand geometric and algebraic concepts; (b) understand elementary probability and statistics; (c) apply mathematics in everyday situations; and (d) estimate, approximate, measure, and test the accuracy of their calculations. In addition to the traditional sequence of studies available for college-bound students, the report indicated a need for, new, equally demanding mathematics curricula for those who do not plan to continue their formal education immediately (NCEE, 1983). The report encouraged the work of professional groups to continue to update and make available innovative curricula. Finally, the shortage of mathematics teachers was addressed. This report was titled *A Nation at Risk* and is
credited with maintaining interest among the American public related to mathematics reform.

The combined effect of the development of the NCTM Standards initiated in 1980 and ultimately published in 1989 coupled with the Nation at Risk report published in 1983 was an educational environment positioned for change (Boutwell, 2001). In 1989, President Bush hosted an Educational Summit for the governors of all 50 states (Boutwell, 2001). The goal of the summit was to establish a set of national educational goals and to reallocate educational policy responsibilities among the federal, state, and local governments (Boutwell, 2001; Walmsley, 2003).

1990-Present: National Goals and Standards

As a result of the 1989 Educational Summit, President Bush announced AMERICA 2000: An Education Strategy. This strategy represented a long-range plan for school communities in the United States to meet six national goals and create a high standard for elementary and secondary education in the United States (Odland, 1993; United States Department of Education, 1991). Six educational goals were identified in the act, and two of the six goals specifically mentioned mathematics education (Odland, 1993). Goal three required students leaving grades 4, 8, and 12 to be proficient in “English, mathematics, science, history and geography; and every school in America will ensure that all students learn to use their minds well, so they may be prepared for responsible citizenship, further learning and productive employment in our modern economy” (Odland, 1993 p. 32). Goal four compelled U.S. students to be ranked first in the world in mathematics and science achievement (Odland, 1993). Support for these six national goals continued even with the election of a new United States president (i.e., Bill Clinton)
and new members of the United States congress. While President Clinton was in office, two new goals related to professional development and parent involvement were added to the original six national education goals (National Education Goals Panel, 1997). The two earlier goals related to mathematics learning remained intact.

Following President Clinton’s terms as president, the subsequently elected president (i.e. George W. Bush) provided leadership related to the enactment of the No Child Left Behind Act of 2001 (NCLB). George W. Bush’s policy, NCLB, was the most ambitious federal guideline for education in decades (Finn & Hess, 2004). In signing the NCLB Act of 2001, George W. Bush (2002) confidently affirmed that all students now have a better chance to learn, excel, and live out their dreams because expectation are higher and it is believed that every child can learn.

With a strong focus on teacher preparation programs, NCLB has directly impacted mathematics instruction (Riddle, 2003). According to NCLB, all public school teachers who teach core academic subjects will be highly qualified (Riddle, 2003). In his report to Congress, Secretary of Education Rod Paige remarked that institutes of higher education and formal teacher training programs were failing to produce the types of highly qualified teachers that the No Child Left Behind Act demands (United States Department of Education, 2002). The report declared that states must revamp teacher preparation and certification requirements because the current academic standards were too low (Cochran-Smith, 2002). Preparing highly qualified teachers of mathematics is necessary to meet the mandates proposed by NCLB (Bybee & Stage, 2005).

Accountability and testing are not new ideas in education (Bybee & Stage, 2005), but NCLB has given testing and accountability renewed importance (Charp, 2003).
Connecting student achievement through high-stakes testing as a measure of effective instructional practices is daunting, yet it is viewed by many as necessary (Bybee & Stage, 2005). Charp believes that many innovative approaches to learning have been ignored as teachers teach to the test in response to the pressures of standardized testing (2003). The dilemma widens with mathematics education because not only do students need to know the operational facts but must also be accomplished problem solvers (Horn, 2004).

The National Council of Teachers of Mathematics (NCTM) led the professional mathematics associations in providing updated standards to assist school district personnel with the implementation of quality mathematics education programs (Anhalt, Ward, & Vinson, 2004). The organization published *Principles and Standards for School Mathematics* in 2000 (Anhalt et. al, 2004). The principles, in conjunction with local and state standards, offered a solid framework for current mathematics education strategies (Anhalt et. al, 2004).

*Principles and Standards for School Mathematics* provided six overall themes referred to as principles. The six principles are equity, curriculum, teaching, learning assessment, and technology (NCTM, 2000). Along with the six principles, the NCTM (2000) document offered numerous standards described as follows:

The Standards for school mathematics describe the mathematical understanding, knowledge, and skills that students should acquire from prekindergarten through grade 12. Each Standard consists of two to four specific goals that apply across all the grades. For the five Content Standards, each goal encompasses as many as seven specific expectations for the four grade bands considered in Principles and Standards: prekindergarten through grade 2, grades 3–
5, grades 6–8, and grades 9–12. For each of the five Process Standards, the goals are described through examples that demonstrate what the Standard should look like in a grade band and what the teacher’s role should be in achieving the Standard. Although each of these Standards applies to all grades, the relative emphasis on particular Standards will vary across the grade bands. (p. 1).

Although criticized for not providing specific mastery requirements, claims outlined in NCLB were that the standards would provide a vision for current mathematics education (Clements, Sarama, & Dibiase, 2003). The overarching vision supported the goals outlined by the federal NCLB Act (Mabry, 2004). The federal government’s interest in improving school performance was grounded in wanting to maintain the nation’s worldwide competitive edge (Stimson, 2003). It is believed that many educators and parents have little knowledge about the education in other countries other than the idea that the United States is not number one in mathematics and science (Stewart & Kagan, 2005). The elaborate NCLB document showed the desire to lessen the perceived gap between the United States and other industrialized nations (Anhalt et al., 2004).

Throughout the standards-based movement, variations of standards and guidelines have directly or indirectly impacted the mathematics instruction provided to students. Whether focusing on federal, state, or local school district standards, the goal of standardizing education has permeated the educational system. The Common Core State Standards initiative, the newest initiative, continues the standards-based education reform pattern. Announced in 2009, the Common Core State Standards, sponsored by The National Governors Association Center for Best Practices (NGACBP) and The Council of Chief State School Officers (CCSSO), proposed to provide a clear and consistent
understanding of what students are expected to learn (NGACBP & CCSSO, 2010). In June, 2010, the Common Core State Standards were published with the backing and input from educators and researchers from 48 states, the District of Columbia, Puerto Rico, and the Virgin Islands. The agenda associated with this initiative is to establish and implement a set of common standards within language arts and mathematics that are cohesive and consistent across grade levels and across states. These comprehensive core standards, founded within educational best practices and scholarly research, attempt to provide instructional guidelines for all teachers, administrators, and parents with the anticipation that this common focus will enhance and improve the mathematics performance of the nation’s students. The Common Core State Standards function on the premise that it is the ultimate goal for all American children to graduate from high school ready for college, career pathways, and success in a global economy (NGACBP & CCSSO, 2010).

**Statement of Problem**

After a century of debate about what constitutes effective mathematics instruction, the effectiveness of current mathematics instruction within the United States is still questionable. Resolving the disparity between what the educational community considers quality teaching practices and the instruction students actually receive is daunting. Professional discussions related to the measurement of instructional effectiveness while appropriately and effectively providing mathematics instruction adds to the challenges that current educators and researchers must face. Unfortunately, the result related to the various mathematics reform endeavors still reveals a country

Mathematics Performance of General Population

As an industrialized nation, the need for mathematics literacy and performance ability is necessary for our continued success. According to The Final Report of the National Mathematics Advisory Panel (National Mathematics Advisory Panel (NMAP), 2008), mathematics performance of the nation’s students is daunting. It is reported that more than 60% of eighth grade students scored below mathematics proficient levels and more than 75% of twelfth grade students scored below the mathematics proficient level (United States Department of Education, 2008). Murnane and Levy (2005) suggest that about 40% of the nation’s high school age students do not possess the mathematical skills necessary to adequately function within today’s entry level manual labor employment. Likewise, the National Advisory Educational Panel (NAEP) (2007) reported that four-year and community colleges have had drastic increases in the number of remedial mathematics courses they offer.

Mathematics Performance of Students with Learning Disabilities

Just as the mathematical performance of the general population is of great concern, so is the performance level of students with learning disabilities. During the 2008-2009 school years, only 9% of eighth-grade students identified as having learning disabilities performed at or above the proficient level on mathematics standardized assessments (National Center for Educational Statistics, 2010). Moreover, estimates reveal that between 5% and 13.8% of the school population have mathematics learning disabilities (Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005; Geary, 2004) and that a
substantial number of these students have limited understanding specifically related to
numbers and operations (Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008).
Historically, mathematics instruction for at-risk students and students with learning
disabilities has not received the same level of consideration from the research community
and policy makers as the field of reading.

A recent review of the ERIC literature base (Gersten, Clark, & Mazzocco, 2007)
revealed, that from 1996 to 2005, there was over 80% more research conducted on the
topic of reading for students with learning disabilities than was conducted on the topic of
mathematics for students with learning disabilities. While this is a dramatic improvement
over the prior decade where there was over 93% more research focused on reading, the
disproportionate level of consideration can have drastic effects for students with learning
disabilities. While a relatively small segment of the general population has mathematics
learning disabilities, these disabilities have the potential to result in higher levels of
school dropout, delinquency, and lifelong underachievement (Dunn, Chambers, &
Rabren, 2004).

Given the importance of mathematics learning and the poor mathematics performance
of many students, research is needed to identify appropriate and improved interventions.
This is especially important for elementary students with learning disabilities in
mathematics that struggle with basic operational concepts and skills (e.g., addition).
Students who fail to master this foundational operation are likely to struggle with
subsequent mathematics concepts, primarily due to the hierarchical nature of
mathematics instruction (i.e., knowledge of new skills is dependent on mastery of
pervious skills).
Purpose of the Study and Related Research Questions

The purpose of this study was to investigate the effects of strategy instruction that involved the use of the concrete-representational-abstract teaching sequence to teach addition with regrouping to students with learning disabilities. The following research questions addressed this purpose.

Research Question 1: Do students with learning disabilities improve their ability to solve addition with regrouping computation and word problems after receiving strategy instruction that involves concrete-representational-abstract sequencing?

Research Question 2: Do students with learning disabilities improve their conceptual understanding related to addition with regrouping after receiving strategy instruction that involves concrete-representational-abstract sequencing?

Research Question 3: Do students with learning disabilities increase their fluency related to addition with regrouping after receiving strategy instruction that involves concrete-representational-abstract sequencing?

Research Question 4: Do students with learning disabilities maintain their ability to solve addition with regrouping computation and word problems after receiving strategy instruction that involves concrete-representational-abstract sequencing?

Research Question 5: Do students with learning disabilities generalize their ability to solve addition with regrouping computation and word problems after receiving strategy instruction that involves concrete-representational-abstract sequencing?

Research Question 6: Do students with learning disabilities report high levels of satisfaction related to strategy instruction that involves concrete-representational-abstract sequencing for learning addition with regrouping skills?
Significance of the Study

Providing effective mathematics instruction during the early formative years has been shown to minimize mathematics difficulties for students with and without disabilities (Fuson, Smith, & LoCicero, 1997; Jordan, Kaplan, Locuniak, & Ramineni, 2007; Bryant, Bryant, Gersen, et. al., 2008). One skill that is imperative for students to master early is computation (Gersten, Jordan, & Flojo, 2005). Recently, the Center on Instruction conducted a meta-analysis titled *The Historical and Contemporary Perspectives on Mathematical Learning Disabilities* (2007) on the topic of teaching mathematics to students with learning disabilities. The meta-analysis identified effective instructional practices that have been shown to be effective in increasing the mathematics performance of students with learning disabilities. These practices included the use of explicit instruction and strategy instruction (Gersten, et al., 2008). Additionally, it has been shown that the use of strategy instruction and concrete-representational-abstract sequenced instruction is effective for teaching mathematics computation skills to students with learning disabilities (Carnine, 1997). Strategy instruction involves teaching a series of steps for student to follow (Carnine, 1997). Explicit instruction involves teaching new material in highly structured, small steps based on student performance data (Miller & Hudson, 2007). Advanced organizers, demonstrations, guided practice, independent practice, and maintenance probes are usually a part of explicit instruction (Miller & Hudson, 2007). The concrete-representational-abstract sequence involves the use of three distinct teaching phases with students showing mastery at each phase prior to moving to the next (Miller & Hudson, 2007). The first phase involves that use of three-dimensional manipulative devices, while the second phase involves the use of two-dimensional
pictures to represent the problem (Miller & Hudson, 2007). Lastly, the third phase focuses on the abstract level of understanding (i.e., problems solved without using manipulative devices or pictures) (Miller & Hudson, 2007).

The current literature base dealing with explicit instruction, strategy instruction, and concrete-representational-abstract sequencing tends to focus on basic math facts rather than more advanced computational skills (Harris, Miller, & Mercer, 1995; Kroesbergen & Van Luit, 2003; Mercer & Miller, 1992; Montague, 2008; Swanson & Hoskyn, 2001). Because basic computational skills are the building blocks of mathematical comprehension and the ability to navigate more difficult mathematical material, it is vital that students be fluent with both basic math facts and more advanced computational skills (Boerst & Schielack, 2003). The NCTM defines computational fluency as “having efficient and accurate methods for computing” (NCTM, 2000, p 152). To be efficient, students must perform calculations at a rate appropriate for a given skill level. To be accurate, students must perform calculations correctly. Therefore, to be computationally fluent, students must correctly answer mathematics problems at an identified level of difficulty within a given time period. Computational fluency is so important that the NCTM lists “the ability to compute fluently” (2000, p. 152) as a number and operation standard for kindergarten through eighth grade. The NCTM standards assert that by the eighth grade, students should be able to fluently apply mental computation to whole and rational numbers. By the ninth grade, computational fluency is assumed and is thus no longer listed as a standard. The National Research Council (2001) suggested that poor computational fluency may interfere with mathematical comprehension. Computational fluency is a complex process involving the basic building blocks of mathematics (Boerst
& Schielack, 2003), and without this ability students’ acquisition of higher order mathematics skills is severely impeded (Johnson & Layng, 1992). Furthermore, computational fluency aids in the ability to problem solve by allowing students to use generalizable methods while monitoring, organizing, and navigating within these methods without getting lost (Calhoon, Emerson, Flores, & Houchins, 2007).

Although research has been conducted related to the use of concrete-representational-abstract sequencing for teaching basic math facts (Harris, Miller, & Mercer, 1995; Miller, Harris, Strawser, Jones, & Mercer, 1998; Morin & Miller, 1998), place value (Peterson, Mercer, & O’Shea, 1988), fractions (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Jordan, Miller, & Mercer, 1999), and algebra (Maccini & Hughes, 2000; Witzel, Mercer, & Miller, 2003) to students with learning disabilities, studies related to advanced addition (i.e. 2- and 3-digit problems that require regrouping) appear to be absent from the literature. Bryant, Bryant, and Hammill (2000) conducted a study that resulted in the identification of 29 specific mathematics behaviors associated with learning disabilities in mathematics and then asked learning disability teachers to rank order the skills based on the frequency their students displayed the behaviors. The statement about having difficulty with word problems was ranked first and the statement about having difficulty with multi-step problems was ranked second. The statement about students making borrowing (i.e. regrouping) errors was ranked seventh (Bryant et. al, 2000). Thus word problems, multi-step problems, and regrouping were clearly identified as great concern from both the mathematics literature and teachers of students with mathematics disabilities. More recently, addition with regrouping was identified as one of the early building blocks to mathematical comprehension that students with learning disabilities
struggle to master (Murphy, Mazzocco, Hanich, & Early, 2007). It appears that little progress has been made in this area of the mathematics curriculum. Thus, this study adds important information related to teaching addition with regrouping to students with mathematics learning disabilities. This study also contributes to the apparently non-existent literature specifically related to applying evidence-based practices (e.g. explicit instruction, strategy instruction, concrete-representation-abstract sequencing) to the teaching and learning of addition with regrouping multi-step computation and related word problems skills.

**Limitations of the Study**

There are two limitations related to this study. First, all participants within this study were identified as having a learning disability. As such, the findings are not generalizable to other dissimilar populations of students. Second, the selection of participants was based on a convenience sampling. The participants were second-, third-, fourth-, fifth-, and sixth-grade students who all attended a charter school within the Southwestern United States. Again, generalization to students in other grades and schools is limited.

**Definition of Terms**

The following definitions and terms are applicable to this study.

**Addend**

Addends are the numbers within a mathematical equation which are being added (Stein, Kinder, Silbert, & Carnine, 2006).
Advanced Organizer

An advanced organizer is the information introduced at the beginning of a lesson in which previously learned information is reviewed, the current lesson objectives are explained and a connection is made with previously learned information, and justification for learning the objectives and its relationship to students’ lives is made (Hudson & Miller, 2006).

Base Ten Blocks

Base Ten Blocks are mathematics manipulative devices used to assist conceptual understanding of the base-ten number system. The base ten blocks are 3-dimensional blocks in three different shapes. Individual cubes represent units of one. Rectangular rods, which equal the length of ten cubes joined together, represent tens. Square tiles equal to ten rods joined together represent hundreds. Finally, a large cube equal to ten square tiles represents thousands (Fuson & Briars, 1990).

Basic Facts

Basic facts are the arithmetic operations of addition, subtraction, multiplication, and division that include single-digit numbers (0-9). There are 390 basic facts (i.e., 100 addition, 100 subtraction, 100 multiplication, and 90 division facts). Basic fact equations consist of three single-digit numbers (i.e., $2 + 2 = 4; 8 - 1 = 7; 3 \times 3 = 9; 8 \div 4 = 2$) or two single-digit numbers and one double-digit number (i.e. $7 + 5 = 12; 10 - 4 = 6; 5 \times 5 = 25; 49 \div 7 = 9$) (Stein, et. al, 2006).

Charter school

A charter school is a nonsectarian public school of choice that is publically funded and open to all students with no admission testing or screening. Each charter school has a
Charter, or performance contract, detailing its program, goals, and methods of assessment. Charter schools operate with increased autonomy in exchange for accountability. They are accountable for both academic results and fiscal practices to several groups: authorizer that grants the charter, the parents who choose to send their children, and the public that funds them (Brouillette, 2003).

**Cognitive Strategies**

Cognitive strategies involve the use of step-by-step mental procedures to solve a problem or complete a task. Cognitive strategies provide structure for learning when a task can be completed through a series of steps. Cognitive strategies serve to support the learner while internal procedures that enable complex task performance are developed (Harris & Pressley, 1991).

**Concrete-Representational-Abstract Teaching Sequence**

The concrete-representational-abstract teaching sequence is an instructional method that sequentially introduces a mathematics concept using (a) concrete three-dimensional manipulative devices, (b) two-dimensional representational drawings, and (c) abstract representations of mathematical concepts often times in the form of a number sentence (Hudson & Miller, 2006).

**Conceptual Knowledge**

Conceptual knowledge is an individual’s representation of the major concepts within a system that involves understanding concepts and recognizing their application within various situations (Robinson & Dube, 2009).
Curriculum-based Assessment (CBA)

Curriculum-based assessments are an approach to assessment that uses direct observation and recording of a student’s performance in the school curriculum as a basis for obtaining information to make instructional decisions (Mercer, 1997).

Declarative Knowledge

Declarative knowledge is the information commonly thought of as facts that are automatically retrieved to answer questions and solve problems (Mercer, 1997).

Explicit Instruction

Explicit instruction is a highly-structured teacher-driven instructional method that is used to present new skills in small steps. This instructional approach relies on student progress to determine instructional pace and promotes student understanding through direct, clearly-defined teaching of concepts and skills (Hudson & Miller, 2006).

Fluency

Fluency is the act of being able to recall information with automaticity: having instant, efficient, and accurate recognition of information (e.g., recalling computation facts) (Calhoon, Emerson, Flores, & Houchins, 2007).

Focused Curriculum-based Assessment

Focused curriculum-based assessments are measurement tools designed to assess a narrow span of skills (Hudson & Miller, 2006).

Guided Practice

Guided practice is the lesson section in which students practice new mathematics skills with mindful teacher guidance. As students attain independence with the new skill, teacher guidance is gradually withdrawn (Hudson & Miller, 2006).
**Independent Practice**

Independent practice is the lesson section in which students independently practice new mathematics skills without teacher support (Hudson & Miller, 2006).

**Learning Disability**

A Learning Disability is defined as “A disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, which may manifest itself in the imperfect ability to listen, think, speak, read, write, spell, or do mathematical calculations” (20 U.S.C. §1401 [30]) (IDEA, 2004).

**Place Value**

Place value is the value of a digit determined by its position in a number (Fuson & Briars, 1990).

**Procedural Knowledge**

Procedural knowledge is the knowledge of the steps required to carry out activities and perform tasks (Mercer, 1997).

**Regrouping**

Regrouping is the action necessary to solve an addition problem when an exchange of base groups is required (Robinson & Dube, 2009).

**Strategy Instruction**

Strategy instruction as a plan that specifies the sequence of needed actions as well as incorporating critical guidelines and rules related to making effective decisions during a problem solving process (Ellis & Lenz, 1996).
Sum

The sum is the amount that is obtained as the result of adding numbers (Stein, et. al, 2006).

Word Problems

Word problems are mathematical exercises expressed as a hypothetical situation explained in words (Baroody & Dowker, 2003).

Summary

The purpose and process of mathematics education has been heavily debated over the last 100 years. Curriculum, a common theme, has remained near the center of the debate. This debate has repeatedly resonated with key professionals in the field of mathematics and has ultimately manifested itself in political reform. A number of mathematics-related agendas (i.e., progressive curriculum, New Math, NCTM standards, Common Core Standards) have influenced how mathematics is taught and learned. The current calls for reform seem to remain steadfast in adopting, revising, and condensing national mathematics standards (NCTM, 1989, 2000, 2006).

As the mathematics debates continue, it seems that a definitive answer related to the most effective mathematics curriculum still remains in the distant future. Despite the debate and unsettled nature of mathematics instruction, one easily agreed upon issue is that basic computation must be addressed effectively. Unfortunately, students in the United States continue to perform below acceptable expectations (NAEP, 2007; National Mathematics Advisory Panel, 2008). Students with learning disabilities are of particular concern. The last decade has seen a drastic increase of students with learning disabilities in mathematics and simultaneous agreement regarding the importance of basic
computation skills for this population of students (Bryant, Bryant, Kethley et al., 2008). Perhaps this is why there has been a primary focus within mathematics literature related to basic math fact instruction for students with disabilities (Garnett, 1992; Gersten et al., 2008; Miller et al., 1998; Montague, 2008; Montague & Brooks, 1993). While there is a solid base of literature related to basic math facts, there is limited research on other basic computation skills, such as multi-digit addition, that is equally important in terms of further progress to higher order mathematics skills. The intent of this study was to contribute information regarding the effectiveness of strategy instruction and the concrete-representational-abstract sequence when teaching addition with regrouping to students with learning disabilities. The results of this study have direct and immediate practical implications for classroom teachers of mathematics.

Details related to this study are discussed in the subsequent chapters. Chapter 2 includes a review of literature relevant to this study. Chapter 3 includes a discussion of the methodology used in this study. The results of the study and a discussion of their implications are reported in chapters 4 and 5.
CHAPTER 2

REVIEW OF LITERATURE

This chapter serves two purposes. The first is to summarize and examine existing professional literature related to mathematics strategy instruction for students with learning disabilities. The second purpose is to summarize and examine existing professional literature related to the concrete-representational-abstract teaching sequence. To understand best practices for teaching mathematical concepts such as addition with regrouping for students with learning disabilities, knowledge in the two above stated areas is necessary. First, this chapter includes a discussion of the literature review procedures and the selection criteria used for experimental studies related to mathematics strategy instruction for students with learning disabilities. Second, this chapter includes the review of studies related to mathematics strategy instruction for students with learning disabilities. Third, this chapter includes the review of studies related to the concrete-representational-abstract teaching sequence. Finally, a summary and synthesis of the research related to mathematics strategy instruction for students with learning disabilities and the concrete-representational-abstract teaching sequence is provided.

Literature Review Procedures

This review includes studies located through a comprehensive search of the following databases: Academic Search Premier, Elton B. Stephens Company (EBSCO), Educational Resources Information Center (ERIC), and Digital Dissertations. The following descriptors were used: learning disabilities, disabilities, mathematics learning disabilities, strategy instruction, and concrete-representational-abstract teaching.
sequence. Additionally, an ancestral search through the reference lists of obtained articles was conducted.

**Selection Criteria Used for Studies Included within this Review**

Specific criteria were used to identify appropriate studies to include in this review of literature. These criteria were: (a) publication between 1975 and 2011, (b) purpose of the study was to examine the effects of mathematics strategies, (c) participants were elementary or middle school students, and (d) at least part of the study results involved the mathematics performance of students with learning disabilities. Additionally, studies were excluded from this review of literature if they (a) were published prior to 1975, (b) failed to involve an investigation related to the effects of mathematics strategies, (c) included participants that did not attend either elementary or middle school, or (d) were designed to explore only the mathematics performance of students without learning disabilities or the performance of students with learning disabilities in subjects other then mathematics.

**Summary and Analysis of Studies Related to Mathematics Strategy Instruction for Students with Learning Disabilities**

Finding instructional techniques and curricula that promote independence and success for students with learning disabilities has been an ongoing quest for special educators for many years (Ellis, 1990). The articles included within this review of literature imply that students with learning disabilities are able to experience success and independence within a wide range of mathematical constructs when adherence to specific instructional procedures is followed. Ellis and Lenz (1996) define strategy instruction as a plan that
specifies the sequence of needed actions as well as incorporating critical guidelines and rules related to making effective decisions during a problem solving process.

**Diagram-Related Strategy Instruction**

Schema-based strategy instruction involves instruction that uses visual representations or drawings to assist individuals in solving mathematical problems (Griffin & Jitendra, 2009). Schema-based strategy instruction assists individuals’ ability to develop conceptual understanding, declarative knowledge, and procedural knowledge.

Jitendra and Hoff (1996) conducted a multiple probe-across-participants study to assess the effects of a schema-based direct instruction strategy on word-problem-solving performance. The three participants were enrolled in grades 3 and 4 and ranged in age from 8 years 10 months to 10 years 10 months. All participants were Caucasian and attended a private elementary school. This private elementary school focused on educating students with learning disabilities.

Baseline included each participant taking three probes assessing three problem types (i.e. change problems, group problems, and compare problems) during three concurrent sessions. The next phase of intervention included participants receiving instruction on how to identify and represent problem schemata. This phase concluded with another probe. The next phase of intervention had participants participate in staggered schema-based direct strategy instruction that used scripted lessons. When the first student reached a criterion level of 100% correct for two consecutive days, another probe was administered to the remaining participants and the second participant began intervention. Likewise, once the second participant reached a criterion level of 100% correct for two consecutive days, another probe was administered to the final participant and the final
participant began intervention. At the conclusion of the intervention participants again were administered another probe. The study concluded with a maintenance probe given between two to three weeks after the final probe was given for each participant.

Fidelity of intervention implementation checks took place during 20% of the problem schemata and intervention training sessions. Likewise, interscorer reliability checks were completed on 20% of the probes to ensure accurate scoring. Visual analysis of data was used to determine intervention effectiveness.

The results of this study indicated a significant level of increase between baseline probes and all other administered probes for all three participants. Likewise, the maintenance probe indicated a high level of skill maintenance. The researchers suggested that further research be conducted to determine the extent to which participants that learn schema-based instruction would be able to generalize these skills into typical math classrooms. They further recommended that using a larger population of participants would be beneficial in determining if other students could benefit from this form of instruction. Additionally, they recommended further investigation into whether instructional effectiveness was related to the use of schema-based diagrams or whether effectiveness was a result of fostering conceptual understating.

Jitendra, Griffin, McGoey, Gardill, Bhar, and Riley (1998) conducted a study comparing the effects of a schema-based instructional strategy and a traditional basal strategy to teach basic addition and subtraction word problems to students with learning disabilities and students without disabilities who were identified at being at-risk for failure in mathematics. The 34 second- through fifth-grade participants were enrolled in four public school classrooms located in the southeastern United States. Ten participants
included in the treatment group were identified as having learning disabilities. The comparison group included 24 typically achieving third-grade students.

This study had two phases. The first phase involved investigation of the effects of the schematic strategy and the traditional basil strategy on use of basic addition and subtraction word problems, while the second phase involved investigation of the maintenance and generalization related to the two instructional strategies. The study began with pretesting which included a variety of word problems that required participants to add and subtract basic numbers. Participants were instructed to read and solve the pretest problems to the best of their ability. Next, instructional lessons began. The 17 to 20 scripted instructional lessons were 45-minutes each and conducted as small groups of three to six participants. The participants receiving the schema-based strategy instruction were taught three schema diagrams to aid the word problem-solving process: (a) change the story situation, (b) group the story situation, and (c) compare the story situation drawings. Each schema-diagram was taught individually and participants were given time to practice identifying the schema-diagram, to draw the schema-based diagram, and to review problems using the schema-based diagram. Participants receiving traditional basil instruction strategy were taught to solve addition and subtraction word problems using the assigned school textbook. Posttests were administered to all participants at the completion of the intervention lessons. One day later, a generalization assessment was given as well.

Participants were assessed a second time one to two weeks after the posttest to investigate the maintenance of the strategies. Additionally, a strategy questionnaire was completed by each participant at the end of the study. An analysis of variance used on
the pretest indicated no significant difference between the schema-based group and the traditional basil group (F (1, 23) = 0.29, p = 0.59). A significant difference was found between the schema-based group and the traditional basil group on both the posttest and maintenance test as indicated by an Analysis of Covariance (i.e., for schema-based group 77% and 81% correct and for the traditional basil group 65% and 64% correct). Both groups demonstrated increases in their ability to solve word problems (i.e., schema-based group increased 26% and traditional basil group increased 16%); however, participants from the schema-based group demonstrated the greatest effect. The schema-based group performed at rates comparable to the comparison third-grade students without disabilities. On the maintenance test, the schema-based group scored a mean of 81%, the comparison group scored a mean of 82%, and the traditional basil group scored a mean of 64%. The results of this study indicated that the use of schema-based instructional methods assists students with learning disabilities to perform similarly to students without learning disabilities when solving addition and subtraction word problems.

Jitendra, Hoff, and Beck (1999) conducted a replication study on the earlier research (Jitendra, Griffin, McGoey, Gardill, Bhat, & Riley, 1998) investigating the effects of schema-based instructional strategies and the generalization from one-step addition and subtraction word problems to two-step word problems. The four participants within this study ranged in age from 12 to 14 years old and all attended a middle school located in the northeastern United States. The four participants had been identified as having a learning disability. A comparison group of 21 typically achieving middle school students was used during testing only. The intervention took place in a special education resource room during a 45-minute period.
A multiple baseline across subjects and across behaviors design was used to investigate the effects of the schema-based instructional strategy while teaching mathematical word problem-solving abilities. This study included the following phases: (a) baseline, (b) two instructional levels (schema-based instruction on one-step and two-step word problems), (c) posttests, (d) setting and behavior generalization, and (e) maintenance. Intervention included instruction on the procedures of solving one-step addition and subtraction word problems while using the schema-based instructional strategy. Once participants reached 90% criterion on two consecutive days, they received instruction on solving two-step addition and subtraction word problems with the use of the schema-based instructional strategy. A total of three schema-based diagrams were used: (a) change diagram, (b) group diagram, and (c) compare diagram.

The results of the study indicated an increase in word problem solving abilities after the intervention of schema-based instruction was taught. The researchers found that the participants increased their abilities to solve one-step word problems by 26% after receiving the one-step word problems with schema-based instructional strategy. Likewise, the participants increased their ability to solve two-step word problems by 71% after receiving instruction on solving two-step word problems with the schema-based instructional strategy when comparing the pre- and post-test scores. Generalization and maintenance test score means increased 39% when compared to the baseline data.

The data from this study indicated that the schema-based instructional strategy was effective when teaching middle school-aged students with learning disabilities how to solve one- and two-step addition and subtraction word problems. Additionally, the questionnaire interviews indicated that the participants found the schema-based diagrams...
to be useful for planning and solving mathematic word problems. The researchers stated that future research should be conducted to examine whether the participants would generalize their skill to new mathematics word problems (i.e., three- or four-step word problems). The study followed the preset procedures with precision and routinely checked procedure usage through observations, fidelity checklists, and reliability procedures; all of which added to the study’s strengths.

Xin, Jitendra, and Deatline-Buchman (2005) conducted a study to compare the effects of a schema-based instructional method and a traditional textbook instructional method. The schema-based instructional method, which had been used in an earlier study conducted by Jitendra and Hoff (1996), consisted of two steps: (a) identify the type of problem, and (b) determine the structure of the problem to be used in a schematic diagram. The second approach involved the use of a traditional strategy that was adapted from a commercial mathematics textbook. The traditional strategy consisted of four steps: (a) read to understand, (b) develop a plan, (c) solve, and (d) look back.

Twenty-two individuals attending a middle school in the northeast region of the United States who had academic difficulties (i.e., 18 who were identified as having a learning disability, three who were identified as being at-risk for mathematics failure, and one identified as having an emotional disturbance) participated in this study. Participants’ ability to acquire, maintain, and generalize mathematics problem solving skills were measured using word problem assessments. The results indicated that participants who were taught with the schema-based strategy instruction performed significantly better than participants who were taught with the traditional strategy instruction. All three measures (i.e., posttests, maintenance tests, and generalization
tests) showed statistically significant differences between groups. The traditional strategy did not involve the use of diagrams and appeared to lack the specificity that students with learning disabilities need when solving challenging mathematics problems.

Jitendra, Griffin, Haria, Leh, Adams, and Kaduvettoor (2007) conducted a study to assess the effects of schema-based instruction as opposed to multiple strategy instruction. The 88 participants (male=49; female=39) were in third grade. Just under 10% of the participants had an identified learning disability. All participants attended the same elementary school in a northeastern urban school district. Mathematical problem solving and computational pre- and posttests were administered to all participants. Additionally, all participants completed the Pennsylvania System of School Assessment Mathematics (PSSAM) test as a posttest measure of participants’ progress on current state mathematical standards. The participants were placed in six instructional groups with three groups receiving schema-based instruction (SBI) that included schematic diagrams designed to promote mathematical problem solving. The additional three groups served as a comparison group and received general strategy instruction (GSI) that included instruction in the use of objects, drawing a diagram, writing a number sentence, and using data from a graph. Both the SBI and GSI groups were taught how to solve a word problem under their respective conditions using scripted lessons for 25 minutes a day five days a week.

A one-way between subjects analysis of covariance (ANCOVA) was applied to posttest scores. The results indicated a significant difference between the two instruction groups in regards to mathematical word problem solving. The SBI group showed greater
gains in word problem solving on the posttest and the PSSAM than their GSI counterparts.

The researchers concluded that schema-based instruction resulted in significant improvement for a group of third grade participants who were solving mathematical word problems; however, this research could be extended in multiple ways. The participant sample was not reflective of current variances within a typical general education classroom. There was a small sample size of students with learning disabilities and a lack of participants who represented those with specific mathematical learning disabilities. The statistical finding among this subgroup of participants differed from the larger group outcomes. There were no statistical differences between the SBI group and the GST group when looking at only the performance of the posttest of those students with learning disabilities. Thus, in this study students with learning disabilities seemed to benefit from both the use of schema-based diagrams and the use of objects more than traditional diagrams, writing number sentences, and using graphs.

Fuchs, Fuchs, Prentice, Hamlett, Finelli, and Courey (2004) conducted a study to investigate the effects of schema-based instruction in promoting mathematical problem solving while also examining schema-based instruction as a mechanism in the development of mathematical problem solving. This study also examined the added value of guided sorting practice on scheme development and problem solving skills. The participating 24 female third-grade teachers from six southeastern urban schools were divided into three groups. Each group comprised of approximately 122 third-grade students, focused on a different intervention: (a) a schema-based instruction group, (b) a schema-based instruction plus sorting practice group, and (c) a comparison group.
group which included teacher-designed and implemented instruction on the four problem types. Three weeks prior to intervention, each group was administered a pretest. The intervention phase lasted 16 weeks and involved whole class instruction conducted inside their math classroom and focused on the individual groups’ intervention method (i.e., schema-based instruction, schema-based instruction plus sorting practice, and teacher-designed and implemented instruction on the four problem types). Upon the completion of the intervention phase, each group was administered a posttest that included mathematical problem solving and schema development. This study applied a two-factor model analysis of variance (ANOVA) to measure the effects of the interventions. The between-teacher variable was the condition while the within-teacher variable was the initial participant status.

The results indicated that the two schema-based instruction groups performance was greater than that of the teacher-designed instruction group on both problem solving and schema development. The researchers included general problem solving strategies (i.e. lining up numbers from the test to perform math operations; checking computation; and labeling work with words, monetary sign, and mathematical symbols) within each of the three intervention groups, and thus were able to isolate the effects of schema-based instruction from more general problem-solving strategies.

The results of this study found the use of schema-based instruction to be effective in teaching mathematical problem solving to third-grade students with and without learning disabilities, but there were no significant differences between the two types of schema-based instruction (with and without sorting activities). Future research
examining the difference between these two types of schema-based instruction among students with and without learning disabilities is needed.

Van Garderen (2007) conducted a study to investigate the use of diagrams to solve one- and two-step mathematical problems by students with learning disabilities. A multiple-probe-across-participants design was used in this study. The three eighth-grade participants had all been identified as having a learning disability. The participants were instructed in how to use a diagram strategy to solve mathematical problems. The four research questions of this study were: (a) can participants with learning disabilities improve their ability to generate diagrams to represent mathematical problems, (b) can those participants improve their problem performance while incorporating the diagram strategy, (c) will the participants generalize the skills to authentic, real-world problems, and (d) how will the participants evaluate the effectiveness of the strategy? This study included four phases: (a) baseline, (b) intervention, (c) generalization, and (d) maintenance. The diagram strategy’s effectiveness was measured using a pre- and posttest.

The results of this study indicated that all participants showed improved mathematical problem-solving performance. While the researchers found inconsistencies between participants’ performances, they stressed the importance of supporting any diagram-based instructional method with several lessons to teach the participants about what a diagram is and how it can be used to assist in solving problems. Additionally, the researchers stated that researchers and educators should start instruction with an emphasis on the use and conceptualization of diagrams and how to generate diagrams.
This review of literature substantiates that the use of schema-based diagram strategies and more traditional diagram strategies help students with learning disabilities improve their ability to solve mathematical problems. Only four studies among this literature involved elementary students. Of these four studies involving elementary students, none of them involved the use of diagram-based strategies to teach addition with regrouping skills.

**Cognitive Strategy Instruction**

Cognitive strategy instruction is comprised of written cues or prompts that assist in the mathematical solving process. Usually, cognitive strategies include a mnemonic device that guides students through the process or steps necessary to solve a mathematical problem while assisting with the development of procedural knowledge.

Case, Harris, and Graham (1992) conducted a study to investigate the effect of a self-regulated strategy developed to improve mathematical word problem abilities in students with learning disabilities. Investigation of the effects of a cognitive strategy to assist with solving simple addition and subtraction word problems was the overall purpose of this study. The four participants were enrolled in fifth- and sixth-grade and had been identified as having a learning disability. All of the participants attended a school in a large metropolitan area in the northeastern United States and received their mathematics instruction in a self-contained special education classroom. While each participant scored 80% or higher on a computation test measuring basic addition and subtraction ability, the participants scored between 40% and 70% on the baseline word problem test. The intervention phase was five weeks long and conducted by two undergraduate students enrolled in a special education program. Each participant received intervention
lessons in a one-on-one setting. Each lesson was approximately 35-minutes in length and was administered two or three times per week.

Overall, 25 probes with seven addition problems and seven subtraction problems were administered. Each probe contained six different types of word problems (i.e., addition-joining, addition-combining, subtraction-separate, subtraction-comparison, subtraction-joining missing addend, and subtraction-combining). The self-regulated strategy involved the use of cognitive strategies in which the participant took on an active collaborator role that included scaffolding and Socratic dialogue. The self-regulation also involved self-assessment, self-recording, and self-instruction.

Because the instructional sessions were criterion based, the participants could not progress through the lessons until mastery in the current lesson was obtained. Each intervention lesson followed the same procedures which included (a) conferencing, (b) discussion of the problem-solving strategy, (c) modeling the strategy and self-instruction, (d) mastery of strategy steps, (e) collaborative practice of the strategy and self-instruction, (f) independent practice, and (g) generalization and maintenance components.

Three components of data collection were used (a) word problems that were scored in two categories: number of correctly written equations and number of correctly written equations with correct answers, (b) strategy usage, and (c) social validation in which the students and their special education teachers provided perspectives about the intervention through interviews.

This study involved the use of a multiple-baseline-across-subject and across two behaviors design. The mean baseline score for problems being written correctly followed by the correct answer was 56%. Immediately after learning the strategy, the participants’
mean score for writing and correctly solving addition problems was 95%. Participants’ mean score on writing and correctly solving subtraction problems was 82%.

Additionally, the data indicated a successful effect during the generalization probes in which the participants’ mean score for writing and correctly solving mixed addition and subtraction problems was 88%. The researchers stated that while participants’ accuracy on solving word problems increased after the intervention, participants were still more likely to write the problem and circle words during the strategy than draw pictures. Both participants and their teachers reported high levels of satisfaction during social validity interviews.

Montague (1992) conducted a study that combined cognitive and metacognitive strategies to assist in solving mathematics word problems. This multiple baseline across subjects design included six participants ranging in age from 12 to 14 years. Each participant was enrolled in grades 6 through 8 and had been identified as having a learning disability.

Participants received intervention instruction during their regularly scheduled special education class period. The intervention phase of this study took place over two months at the end of the school year, while the generalization phase took place during two separate months the following school year.

Baseline data, collected prior to intervention, included test scores and timeframes for test completion for each participant. Intervention included the use of the following materials: (a) scripted lessons, (b) wall charts listing the seven cognitive strategy steps, a metacognitive strategy, and the combined seven strategy steps and metacognitive strategy, (c) strategy study cards, (d) 50 practice probes, and (e) graphs for recording
individual and group scores. Likewise, intervention included two treatments: (a) teaching of seven cognitive strategy steps, (b) teaching of the metacognitive strategy, and (c) practicing solving mathematical problems with missing components. Generalization included using learned strategy steps and problem solving in alternate settings.

The results of this study showed that cognitive and metacognitive strategies are effective for teaching students with learning disabilities how to solve mathematics word problems. While conducting generalization during a different school year was strength for the study, replicating the results within a more urban area may be difficult due to higher transiency rates among students.

Montague, Applegate, and Marquard (1993) conducted a study to investigate the effects of a cognitive strategy to assist students with learning disabilities in solving mathematical problems. Determining the effects of the strategy on the performance of middle school students with learning disabilities was the overall purpose of this study. The researchers stated an effective cognitive strategy was necessary because students with disabilities had not improved in mathematics problem solving as a result of typical classroom instruction.

The participants included 72 seventh to ninth graders identified as having learning disabilities from four schools in the southeastern part of the United States. This study involved three treatment conditions: (a) cognitive instruction only (COG), (b) metacognitive instruction (MET), and (c) a combined cognitive and metacognitive instruction (COG-MET). Random assignment was used to determine participant placement within each treatment condition. The mean age within each treatment was 14.5 years (COG), 14.3 years (MET), and 13.9 years (COG-MET). Of the 72
participants, 19 were female, and 53 were male. Among the participants, 35 identified as Anglo, seven identified as African American, and 30 identified as Hispanic.

This study had four phases: (a) a seven-day unit of instruction incorporating a 10-problem test of mathematical problems each day, (b) a five-day unit of instruction incorporating one of the condition groups (i.e., COG, MET, or COG-MET), (c) a posttest, and (d) a final maintenance test. This study used a repeated-measures design to measure the effectiveness of the treatment over time. This four-month study took place during the last semester of the academic school year.

The COG treatment consisted of direct instruction in the seven processes used in the cognitive strategy: (a) Read, (b) Paraphrase, (c) Visualize, (d) Hypothesize, (e) Estimate, (f) Compute, and (g) Check. The MET treatment consisted of only the metacognitive process of the cognitive strategy. The COG-MET treatment consisted of both the COG treatment and the MET treatment.

All 72 participants demonstrated an increase of score from pretest to posttest; however, only the COG treatment group demonstrated a statistically significant higher posttest score when compared to the pretest score. Likewise, the data revealed that all 72 participants scored significantly lower on the maintenance measure, administered five weeks after the posttest, compared to the posttest. The researchers concluded that students with learning disabilities (a) can benefit from strategy instruction for solving mathematical problems, (b) may become more confident about their ability to solve mathematics after mastering a strategy, and (c) may increase self-esteem and motivation to solve mathematics word problems in the future.
Ozaki, Williams, and McLaughlin (1996) assessed the effects of the Cover-Copy-Compare procedure on the percent of multiplication facts correctly completed by a sixth grade student with a learning disability using a multiple baseline across behaviors design. This study took place in a resource room. The participant was 11 years and 1 month old.

The pretesting included assessing the amount of prior declarative knowledge related to multiplication facts the participant had. The intervention phase included the participant receiving instruction in five steps of the Cover-Copy-Compare procedures. The steps were: (a) look at the completed math fact, (b) read the problem out loud and copy the answer, (c) cover the problem, (d) read the problem out loud and write it from memory, and (e) compare the answer to the original problem. The instruction included 18 sessions, each about 15 minutes per session 3 times a week.

A substantial level of increase between the participant’s baseline scores and the participant’s post-intervention probe scores was evident. While this study provides a basis for the use of strategy instruction for learning declarative multiplication knowledge, it allows for limited generalization because only a single participant was included within this research. To confirm the effectiveness of the Cover-Copy-Compare on student achievement, further research that includes more participants and an alternate means of improving declarative knowledge is required.

Naglieri and Johnson (2000) conducted a study to determine if instruction designed to facilitate planning, given by teachers to their class as a group, would have differential effects on the specific Planning, Attention, Simultaneous, Successive (PASS) cognitive characteristics of each child.
The participants included 19 (male = 16; female = 2) sixth to eighth graders who ranged from 12 to 14 years of age. While most of the participants had been identified as having a learning disability, several were identified as having mild intellectual impairments. All participants attended a public school in southern California that served rural and suburban communities with low to lower-middle class levels of socioeconomic status.

The Cognitive Assessment System (CAS) was administered to all participants and the results were used to place participants into the experimental group or one of four comparison groups. Participant placement was based on their ability levels related to the four fundamental processes for planning and successfully executing cognitive tasks. The intervention condition consisted of participants completing subtraction worksheets with and without regrouping and teachers identifying effective strategies the participants used to solve math problems. The results indicated that the participants who were identified as having low planning scores for the CAS measure demonstrated the greatest gains from baseline to intervention on the math worksheets. Researchers point out that this instruction does no use teacher scripts or rigidly formatted procedures that make the intervention easily replicated. Replication studies investigating the effects of the PASS cognitive instruction are needed, especially related to other students with various types of learning challenges.

This review of literature substantiates that the use of cognitive strategy instruction assists students with learning disabilities who struggle with solving mathematics problems. This review of literature reveals that students in elementary and middle school have shown positive gains in their procedural knowledge related to the ability to
solve mathematics problems, with the assistance of cognitive strategies alone. Specifically, the literature reveals that cognitive strategies have been used successfully to teach students to solve computation and word problems that involve basic addition and subtraction (i.e., single-digit) (Case, Harris, & Graham, 1992), multi-digit subtraction with and without regrouping (Naglieri & Johnson, 2000), and multiplication and division (Montague, 1992; Ozaki, Williams, & Mclaughlin, 1996). There were no studies found, however, related to the use of cognitive strategies to assist students with addition with regrouping. Moreover, there was a lack of research within the context of the cognitive strategies literature that also involved the use of other evidenced-based strategies designed to help students with learning disabilities develop conceptual knowledge.

**Summary and Analysis of Studies Related to the Concrete-Representational-Abstract Teaching Sequence for Students with Learning Disabilities**

The Concrete-Representational-Abstract Teaching Sequence (CRA) is a researched-based, scaffolding instructional sequence that promotes conceptual understanding, procedural knowledge, and declarative knowledge for students with learning disabilities (Ketterlin-Geller, Chard, & Fien, 2008). There are three distinct stages of CRA: (a) concrete, (b) representational, and (c) abstract. The concrete stage promotes conceptual understanding and procedural knowledge through the use of concrete, three-dimensional objects that are used to solve mathematics problems. The second stage, representational, involves the use of two-dimensional drawings or pictorial representations of previously used manipulative objects to promote conceptual understanding, procedural knowledge, and declarative knowledge. The final stage of CRA is the abstract stage. This stage involves the moving from manipulative devices or visual aids while independently
solving mathematical problems and primarily focuses on the development of declarative knowledge.

Harris, Miller, and Mercer (1995) conducted a study to investigate the effects of using strategy instruction and the concrete-representational-abstract teaching sequence to teach initial multiplication skills to students with disabilities in general education classrooms. This multiple baseline across classroom design with one replication took place at a public elementary school located in north-central Florida. The participants included 112 second graders in six second-grade general education classrooms with 12 being identified as having a learning disability and one being identified as having an emotional disability. The instruction was provided by the general education teachers within the six second-grade classrooms during the regularly scheduled mathematics period. Mercer and Miller’s Multiplication Facts 0 to 81 (1992), a scripted manual from the Strategic Math Series (Mercer & Miller, 1991-1994), was implemented as the intervention in the study. Prior to intervention, the six general education teachers participated in a two-hour training session that discussed the procedures.

This study used four measures: (a) a one-minute timed multiplication facts sheet, (b) multiplication pretest, (c) multiplication posttest, and (d) the daily learning sheet, which accompanied the 21 scripted lessons. Baseline, which covered several days, consisted of participants completing one-minute timed multiplication fact probes that measured the rate of computation on basic multiplication facts. The baseline data were scored through counting the number of correct and number of incorrect digits that each participant listed in one minute. Once a stable baseline trend was established, the participants were administered a pretest followed by the intervention lessons.
Participants identified as having a learning disability showed a mean increase of 52.2% from pretest to posttest. The average pretest scores among participants with learning disabilities ranged from 5% to 50%. The average posttest scores among participants with learning disabilities ranged from 60% to 100%. In addition to comparing pre- and posttest scores of the participants identified as having a learning disability, the researchers also compared the performance of participants with disabilities to their general education peers without identified disabilities. During baseline and pretest data collection phases, both groups began instruction at the same level. Median scores were the same for both groups on seven of the nine learning sheets that comprised the first 10 lessons (i.e., developing conceptual understanding of multiplication). The two groups began to differ when the instructional emphasis changed from conceptual understanding of multiplication fact computation to requiring participants to solve and create their own mathematics word problems. During this phase of instruction, participants with identified disabilities scored 10-20% lower on Intervention probes than their peers without disabilities. Likewise, the two groups differed at posttest with participants with disabilities scoring lower on the posttest measure than their peers without disabilities (i.e., participants with disabilities median posttest score was 80% and participants without disabilities median posttest score was 90%). These findings indicated that participants with learning disabilities were able to learn multiplication skills at acceptable levels (i.e., at least 80% accuracy on posttest) within a general education classroom.

Several implications for future practice were noted by the researchers: (a) effective teaching approaches (i.e., CRA) benefit students with and without disabilities
while teaching conceptual understanding of multiplication, (b) pretest data can assist teachers in developing appropriate instructional delivery, (c) mastery levels are critical for teachers to make data-based informed decisions related to planning instruction and delivering instructional feedback, and (d) students with disabilities can perform similar to their general education peers, while solving multiplication, given the instruction involves the use of appropriate curricular materials.

Maccini and Hughes (2000) conducted a study investigating the effectiveness of a problem-solving strategy that involved the use of the CRA teaching sequence to introduce algebra to students with learning disabilities. This study took place over a 168-day period and used a multiple-probe across subjects design to answer three questions: (a) can the participants learn the multi-stepped, self-instructional graduated instructional sequence, (b) will the participants improve their word problem-solving abilities after the intervention, and (c) will the participants generalize and maintain their skills when presented with novel mathematic word problems? All six secondary-aged participants had been identified as having a learning disability and attended a school in central Pennsylvania.

The first phase of the intervention involved a concrete application that used individual manipulation of physical objects to represent mathematical problems. The second phase in the instructional sequence involved a representational application where participants were taught to draw pictures to represent the previously used physical objects. The last phase of the intervention was the abstract application where the participants were taught to use mathematical symbols combined with written numbers to solve mathematical problems. Participants were also taught a first letter mnemonic, STAR, to assist in the
process for solving mathematics word problems. The STAR mnemonic device steps were (a) Search the word problem, (b) Translate the problem, (c) Answer the problem, and (d) Review the solution. Each of the CRA and strategy lessons included phases adapted from the Strategic Math Series (Mercer & Miller as cited in Maccini & Hughes, 2000): (a) advance organizer, (b) describe and model, (c) guided practice, (d) independent practice, (e) give a posttest, and (f) provide feedback.

The following data were collected for each participant: (a) correct problem solution, (b) answer percentage, and (c) strategy-use abilities. Points per component were provided using a holistic scoring guide and scale. An improvement in the percentage of strategy use with all participants (23% at baseline, 80% near-transfer generalization, 54% far-transfer generalization, and 69% for maintenance) was indicated by a visual analysis of multiple probe data and an analysis of the pretest and posttest results. Additionally, participants increased their accuracy on problem solving (addition baseline M = 33.38% to instructional M = 94.12%, subtraction baseline M = 26.88% to instructional M = 93%, multiplication baseline M = 13.88% to instructional M = 93%, and division baseline M = 10.04% to instructional M = 97%). Participants also demonstrated accuracy on problem solutions, ranging from 38.87% to 57.89% on the baseline measure to 89.4% to 100% on the instructional measures. Generalization measures showed similar results were obtained for percentage accuracy. The participants responded positively to the strategy and the teachers as determined by a Likert-scale questionnaire. Likewise, the results obtained from three open-ended questions were also positive indicating the social validity was very high.
Maccini and Ruhl (2000) extended the above research of using the CRA and STAR mnemonic strategy to investigate the effectiveness with solving algebraic subtraction problems. This study used a multiple probe design across subjects. The three eighth-grade participants ranged in age from 14 to 15 years old and each had been identified as having a learning disability. Additionally, each participant demonstrated deficits in subtraction skills and each participant received specialized education in the area of mathematics. The study was conducted in a public middle school located in central Pennsylvania.

The study used instructional procedures adapted from the Strategic Math Series (Mercer & Miller cited in Maccini & Ruhl, 2000). The participants increased their percent of strategy use and increased their operation abilities within algebraic problem solving. Again, a Likert-scale questionnaire was used to measure social validity. The participants within this study rated the strategy higher than the previous study with a mean satisfaction of 4.67, using a five-point scale (5 being the greatest satisfaction).

Butler, Miller, Crehan, Babbitt, and Pierce (2003) conducted a study to investigate the effects of fraction-related instruction using two instructional methods: (a) the concrete-representational-abstract instructional sequence CRA), and (b) the representational-abstract (R-A) instructional sequence. The purpose of this study was to compare the effects of the two instructional sequences. The 50 participants ranging in age from 11 to 15 were enrolled in grades, 6, 7, and 8 at a public middle school located in a large urban area of the southwestern United States. All 50 participants had been identified with mild-to moderate disabilities in mathematics and received mathematics instruction in a resource room setting. Twenty-six participants received the CRA instructional sequence
and 24 participants received the RA instructional sequence. Additionally, a comparison group of 65 students, without disabilities, enrolled in eighth-grade were administered the post measure to determine what typical students without disabilities know about fractions at the end of eighth-grade. This study used five subtests (three subtests from the Brigance Comprehensive Inventory of Basic Skills-Revised and two subtests designed by the researchers). These instruments were used as pretests and posttests and measured the participants’ knowledge of fraction skills. Each participant was also administered an attitude questionnaire to assess the participants’ attitude towards mathematics instruction.

This study used ten scripted lessons that included (a) an advance organizer, (b) a teacher demonstration, (c) guided practice, (d) independent practice, (e) problem-solving practice (i.e., word problems), and (f) feedback. A learning sheet accompanied each lesson and contained problems for guided practice, independent practice, and problem-solving practice. Four investigator-designed cue cards also were used. The concrete materials included (a) fraction circles, (b) dried beans, and (c) fraction squares made of paper. Two special education teachers who were trained in teaching the scripted lessons taught the four math classes in which the participants were enrolled (i.e. two classes were taught the CRA sequence and two classes were taught the R-A sequence). Both the CRA and R-A lessons lasted 45 minutes and followed the same lesson format (i.e., advanced organizer, teacher demonstration, guided practice, problem-solving practice, and feedback routines). Additionally, notes were given to participants in both groups to assist with lesson understanding.

Lessons 1 through 3 evidenced the key difference between the two treatment groups. The group receiving the CRA instructional sequence received three lessons that focused
on conceptual development using concrete manipulative devices and three lessons that involved the use of representational devices. The group receiving the R-A instructional sequence received six lessons that involved representational drawings and no concrete manipulative devices. The remaining lessons were the same for each group.

The results of this study indicated that both treatment groups improved from pre- to posttest. Each subtest indicated that participants in the CRA treatment group had overall higher mean scores than did the participants in the R-A treatment group. The researchers stated that the participants in both the CRA and R-A treatment groups performed as well as the comparison group. The data revealed similar performance between the CRA and R-A groups on the attitude questionnaire. The researchers suggested that future studies be designed to examine the use of concrete level instruction for a longer period of time.

The detailed descriptions of the settings and procedures that provide sufficient detail for replication, the use of scripted intervention lessons to strengthen internal validity, and the social validation of a cost effective intervention designed to teach a skill that must be taught to all students in public education were all strengths of this study. However, the study could have been strengthened by including pretest data on the comparison group. The researchers indicated that a majority of the participants involved within this study had identified learning disabilities so caution should be taken when generalizing the results to other dissimilar populations.

Witzel, Mercer, and Miller (2003) conducted a study to measure the effects of concrete-representational-abstract teaching sequence on middle school students’ with learning disabilities ability to solve complex algebraic equations. This study involved 12 sixth- and seventh-grade classrooms, 358 sixth- and seventh-grade participants, and 10
teachers and took place in a southeastern United States urban county. The researchers identified 34 participants with learning disabilities and matched them with 34 participants with similar characteristics. Sets of participants were assigned to two different treatment groups: (a) equivalent algebra lessons using the CRA teaching sequence and (b) traditional algebra instruction. The instruction in both treatment groups included the following: (a) introduction of skill, (b) skill modeling, (c) guided practice, and (d) independent practice. The CRA treatment group received instruction at the concrete, representational, and abstract levels, while the traditional instruction treatment group received instruction at the abstract level only.

Repeated measures of analysis of variance were performed on two levels of instruction (i.e., CRA vs. abstract) and three levels of occasions (i.e., pretest, posttest, and maintenance). The study’s results indicated that both treatment groups improved from pretest to posttest, but the CRA treatment group demonstrated a larger gain in performance than the traditional instruction group. The researchers noted that the pre-, post-, and maintenance measures had not been fully evaluated, and thus did not address all of the participants gains especially those gain related to conceptual understating.

Scheuermann, Deshler, and Schumaker (2009) conducted a study designed to explore the CRA instructional sequence through explicit instruction while solving word problems. The study’s purpose was to investigate the effectiveness of this approach in both general education and special education settings. The 20 participants ranged in age from 11 to 14. All participants had been identified as having a learning disability and scored in the lower 25th percentile on a standardized mathematics assessment. The study was conducted in a charter school that specialized in teaching students with learning
disabilities. The procedures for the study included the use of an Explicit Inquiry Routine (EIR) that combines validated mathematics practices from general education (i.e., inquiry and dialogue) and mathematics practices from special education (i.e., explicit instruction). This study involved the use of a multiple-probe-across-students design. The participants received intervention during a daily 55-minute mathematics lesson using the direct-teaching approach. Each participant was administered a follow-up worksheet at the end of each lesson; a score of 75% represented mastery. The pre- and posttest and maintenance probe data indicated that all subjects made significant growth after the intervention was provided. The participants’ ability to generalize the skills taught during the intervention was measured and the data indicated participants made significant growth in a Far-Generalization Test. The researchers concluded that students with mathematics learning disabilities can increase their knowledge of mathematical concepts using direct instruction and the CRA instructional sequence.

Flores (1992) conducted a study to investigate the effect of the CRA sequence on the computational performance of students with learning disabilities. Specifically, the effects of the CRA sequence to assist with fluency in computing subtraction with regrouping problems was studied. The six participants were enrolled in third grade and had been identified as having a learning disability or identified at risk for failure in mathematics. All of the participants attended a school in a rural district outside of a major city in the southwestern United States and received their mathematics instruction in the general education classroom. The researcher used a multiple probe across groups design. Baseline for all participants regarding subtraction with regrouping in the tens place was considered stable when three consecutive probes varied no more than 5% from the
average rate. Once baseline was stable, the first participant began the intervention condition while the other participants remained in the baseline condition. Once the first participant achieved a criterion for 20 digits correct on a 2-minute probe on three consecutive trials, the first participant moved into a 4-week maintenance condition during which no instruction or practice was provided. The second and third participants began the intervention condition when the first participant met criterion. When the second and third participants met the above stated criterion, they moved into the maintenance condition and the three remaining participants began the intervention condition. Each participant received the maintenance measure four weeks after he or she met criterion.

The CRA instruction provided during the intervention condition contained three concrete lessons, three representational lessons, one lesson teaching the DRAW strategy, three abstract lessons, and fluency lessons (the number of fluency lessons was determined by how long it took for criterion to be met). The six participants reached criterion in 10 to 15 lessons. Five of the six participants maintained performance at or above the criterion level after four weeks of no instruction or practice. The one participant who did not maintain performance demonstrated a six digit decrease in performance, however, his maintenance score was 14 digits more than his baseline mean score.

The results of this study indicated that the use of the CRA sequence improved subtraction with regrouping fluency of students with learning disabilities. Likewise, students taught subtraction with regrouping with the use of the CRA sequence maintained their performance ability four weeks after the end of the intervention condition. Some limitations of this study as stated by the researcher included the instruction taking place outside of the classroom and the lack of a comparison group. The researcher suggested
that further research be done to (a) measure how long the performance gains could be maintained, (b) measure the ability of performance to be generalized into a classroom setting, and (c) measure the problem-solving abilities of students who receive instruction involving the CRA sequence.

Based on this review of the literature, the concrete-representational-abstract teaching sequence appears to be an effective instructional strategy for teaching mathematical computation and word problems involving initial place value, fractions, basic multiplication facts, and algebra. Likewise, the CRA teaching sequence was shown to be an effective instructional strategy for educating diverse populations (i.e., students with and without identified learning disabilities). However, what appears to be missing from the literature is research in the use of a CRA teaching sequence for teaching addition with regrouping computation and problem solving.

**Review of Literature Summary**

Over the past several decades, researchers and educators have investigated various interventions to assist students with learning disabilities in the curricular area of mathematics. Based on this review of literature, several evidence-based practices emerged that assist this population of students as they progress through the mathematics curricula.

A variety of studies (Jitendra & Hoff, 1996; Jitendra, et al., 1998; Jitendra, Hoff, & Beck, 1999; Xin, Jintendra, & Deatline-Bachman, 2005; Jitendra, et al., 2007; Van Garderen, 2007) revealed that students with learning disabilities benefit from the use of diagrams to solve word problems. A majority of these studies (Jitendra & Hoff, 1996; Jitendra, et al., 1998; Jitendra, Hoff, & Beck, 1999; Xin, Jintendra, & Deatline-Bachman,
2005; Jitendra, et al., 2007; Fuchs et al., 2004) were designed to investigate the use of schema-based diagrams, while only one was designed to investigate the use of more traditional diagrams. Results from these studies consistently support the use of diagrams to solve word problems that involve basic addition and subtraction, two-step addition and subtraction, and multiplication.

Several studies (Case, Harris, & Graham, 1992; Montague, 1992; Montague, Applegate, & Marquard, 1993; Ozaki, Williams, & McLaughlin, 1996) revealed that cognitive strategies help improve the mathematics performance of students with learning disabilities. Again, the emphasis in this body of literature was on helping students use systematic procedural steps to solve word problems. The word problems used in these studies required students to use self-regulation strategies to monitor their own performance, mnemonic devices, and memorized procedures related to the problem-solving steps to find the problem solutions after setting up the problem successfully. Results from these studies support the use of cognitive strategies for addition, subtraction, and multiplication.

The final area of literature that emerged in this review was the use of the concrete-representational-abstract instructional sequence for teaching students with learning disabilities a variety of mathematics skills (i.e., initial place value; subtraction with regrouping; multiplication; fractions; algebraic subtraction problems, word problems, and complex equations). It is interesting to note that the CRA sequence requires the integration of diagrams as students progress through the representational aspect of the sequence. The representational diagrams used in the CRA sequence typically mirror the manipulative devices used during the concrete aspect of the sequence. The body of
literature related to the CRA sequence typically involved the use of (a) an advanced organizer, (b) modeling, (c) guided practice, and (d) independent practice. In a few of the studies (Harris, Miller, & Mercer, 1995; Maccini & Hughes, 2000; Maccini & Rahl, 2000), the researchers mentioned the integration of a cognitive strategy to assist with the transition from representational to abstract lessons. Results from this body of literature indicated that the use of the concrete-representational-abstract sequence improves the mathematics performance of students with learning disabilities.

It is interesting to note that no studies were identified that involved the investigation of these evidence-based practices when teaching addition with regrouping to students with learning disabilities. Advanced addition skills that require regrouping are considered foundational computation skills that should be mastered during the elementary grades in school. Unfortunately, many students with learning disabilities struggle with this aspect of the curriculum (Bryant, Bryant, Gersen, et al., 2008). The abstractness of regrouping and students’ limited conceptual understanding related to place value skills associated with the regrouping process contribute to poor performance in this challenging component of the mathematics curriculum.

The gap in the literature related to addition with regrouping skills is puzzling, but clearly additional research is needed to investigate the effectiveness of strategy instruction integrated with the CRA teaching sequence for students with learning disabilities. Research that incorporates these research-based practices to teach addition with regrouping skills is needed to determine if success can be replicated when focusing on this particular skill.
CHAPTER 3

METHODOLOGY

The purpose of this study was to investigate the effect of concrete-representational-abstract sequencing within strategy instruction while teaching addition with regrouping to students with learning disabilities. To address this purpose, the following research questions were answered.

Research Question 1: Do students with learning disabilities improve their ability to solve addition with regrouping computation and word problems after receiving strategy instruction that involves concrete-representational-abstract sequencing?

Research Question 2: Do students with learning disabilities improve their conceptual understanding related to addition with regrouping after receiving strategy instruction that involves concrete-representational-abstract sequencing?

Research Question 3: Do students with learning disabilities increase their fluency related to addition with regrouping after receiving strategy instruction that involves concrete-representational-abstract sequencing?

Research Question 4: Do students with learning disabilities maintain their ability to solve addition with regrouping computation and word problems after receiving strategy instruction that involves concrete-representational-abstract sequencing?

Research Question 5: Do students with learning disabilities generalize their ability to solve addition with regrouping computation and word problems after receiving strategy instruction that involves concrete-representational-abstract sequencing?
Research Question 6: Do students with learning disabilities report high levels of satisfaction related to strategy instruction that involves concrete-representational-abstract sequencing for learning addition with regrouping skills?

This chapter addresses the methodology used within this study and includes a discussion on the following topics: (a) research questions, (b) participants, (c) settings, (d) instrumentation, (e) materials and equipment, (f) design, (g) procedures, (h) interscorer reliability, (i) fidelity of treatment, and (j) treatment of data.

**Participants**

A total of nine elementary-aged students with learning disabilities participated in this study. The participants ranged in age from 7 years 7 month to 11 years 7 months. Of the nine participants, 6 were male and 3 were female. The following ethnicities were represented in this sample: Hispanic (n=5), White (n=3), Black (n=1). All nine participants demonstrated need for addition with regrouping instruction. See Table 1 for a summary of participant demographic data.

**Participant Pool**

A convenience sample was used to select the participants. The participant pool consisted of students enrolled at one publically-funded charter school.

**Participant Selection**

The following criteria were used to determine participant eligibility for this study. Participants must have: (a) identified as having a specific learning disability, (b) been enrolled within second, third, fourth, fifth, or sixth grade at the participating charter school, and (c) scored 80% or less on the Addition with Regrouping Pretest (Miller, Kaffar, & Mercer, 2011). Permission for use was granted (see Appendices A-C). Parent
Table 1

**Participant Demographic Data**

<table>
<thead>
<tr>
<th>Participant</th>
<th>Age</th>
<th>Gender</th>
<th>Grade</th>
<th>Ethnicity</th>
<th>LD Identification Area</th>
<th>Mathematics Achievement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 1</td>
<td>11.3</td>
<td>Male</td>
<td>5th</td>
<td>Hispanic</td>
<td>SLD (M, R, W)</td>
<td>WJ III – 71</td>
</tr>
<tr>
<td>Participant 2</td>
<td>11.7</td>
<td>Female</td>
<td>6th</td>
<td>Hispanic</td>
<td>SLD (M, R, W)</td>
<td>WJ III - 65</td>
</tr>
<tr>
<td>Participant 3</td>
<td>9.11</td>
<td>Male</td>
<td>4th</td>
<td>Black</td>
<td>SLD (M, R)</td>
<td>WJ III - 75</td>
</tr>
<tr>
<td>Participant 4</td>
<td>8.8</td>
<td>Female</td>
<td>3rd</td>
<td>White</td>
<td>SLD (M, R, W)</td>
<td>WJ III - 76</td>
</tr>
<tr>
<td>Participant 5</td>
<td>8.8</td>
<td>Male</td>
<td>3rd</td>
<td>Hispanic</td>
<td>SLD (M, R, W)</td>
<td>WJ III - 83</td>
</tr>
<tr>
<td>Participant 6</td>
<td>9.7</td>
<td>Female</td>
<td>4th</td>
<td>Hispanic</td>
<td>SLD (M, R, W)</td>
<td>WJ III - 74</td>
</tr>
<tr>
<td>Participant 7</td>
<td>7.7</td>
<td>Male</td>
<td>2nd</td>
<td>White</td>
<td>SLD (M, R)</td>
<td>KTEA - 72</td>
</tr>
<tr>
<td>Participant 8</td>
<td>9.4</td>
<td>Male</td>
<td>3rd</td>
<td>Hispanic</td>
<td>SLD (M, R)</td>
<td>KTEA - 68</td>
</tr>
<tr>
<td>Participant 9</td>
<td>8.5</td>
<td>Male</td>
<td>2nd</td>
<td>White</td>
<td>SLD (M, R, W)</td>
<td>WJ III - 71</td>
</tr>
</tbody>
</table>

*Note. M = Math; R = Reading; W = Written Expression; WJ III = Woodcock-Johnson Test of Achievement III; KTEA = Kaufman Test of Educational Achievement 2nd Ed.*

informed consent and participant assent also had to be provided to be selected (see Appendices D & E).

**Triad Formations**

Triads were formatted taking into consideration the grade in which the participants were enrolled and the classroom setting from which participants were being removed for intervention. This resulted in one triad of fourth to sixth graders, one triad of third and fourth graders, and one triad of second and third graders.
Setting

This study took place within a K-12 charter school located within a metropolitan city in the Southwestern United States that covers approximately 8,091 square miles and serves approximately 308,500 students. This publicly funded charter school is open to any student living within the school district regardless of school district designated school zoning. This charter school is located within one of the largest school districts in the United States. The school employs 29 grade level teachers, four teacher specialists (i.e., physical education teachers, music teachers, art teachers) two special education teachers, two special education paraprofessionals, and three general education paraprofessionals. This charter school is a full inclusion school that provides a majority of its special education services within the general education classroom. The population of this school consists of approximately 750 students. The percentage rate of students identified as having disabilities is 9%. The following is a breakdown of the school population’s demographic information: (a) 49% of student population is female and 51% is male, (b) 2% of the population is of Asian/Pacific Islander descent, (c) 20% of the population is Black, (e) 14% of the population is White, (f) 55% of the population is Hispanic, (g) 9% of the population is multiracial, and (h) 38% of the population is eligible for free or reduced lunch.

Instrumentation

There were seven curriculum-based assessments, two conceptual understanding tests, baseline probes, intervention probes, and a participant satisfaction questionnaire used in this study. Details related to these instruments are provided in this section.
Pre- and Posttests

The first curriculum-based assessment (CBA), the Addition with Regrouping Pretest (Miller, Kaffar, & Mercer, 2011), had 20 problems that required regrouping to solve (see Appendix F). Out of the 20 problems, ten problems contained two two-digit addends and ten problems contained two three-digit addends. Of the problems with three-digit addends, seven required a single regrouping and three required two regroupings to solve the problem. This CBA was designed to measure the participants’ ability to correctly solve addition with regrouping problems without time restrictions. Thus, the CBA was considered an untimed-focused curriculum based assessment (Hudson & Miller, 2006).

The second CBA, the Addition with Regrouping Posttest (Miller, Kaffar, & Mercer, 2011) contained identical problems as those found within the Addition with Regrouping Pretest to control for problem difficulty level (see Appendix G). The problems on the Addition with Regrouping Posttest were presented in a different order than those on the pretest. Differing the order of problems on the pre- and posttest helped reduce the likelihood of practice effect on these measures. Permission for use was granted (see Appendices A-C).

The third CBA, the Addition with Regrouping Maintenance Test contained identical problems as those found within the Addition with Regrouping Pre- and Posttest to control for problem difficulty level (see Appendix H). The problems on the Addition with Regrouping Maintenance Test were presented in a different order than those on the pretest. Differing the order of problems on the pretest and maintenance test helped reduce the likelihood of practice effect on these measures.
The fourth CBA, the Addition with Regrouping Minute (Miller, Kaffar, & Mercer, 2011), had 16 addition with regrouping problems (see Appendix I). Permission for use was granted (see Appendices A-C). Of the 16 problems, 8 problems contained two-digit addends, while the remaining 8 problems contained three-digit addends. Six of the three-digit addend problems required participants to regroup only once, while two of the three-digit addend problems required participants to regroup twice to correctly solve the problem. The participants were given one minute to complete this CBA. This CBA was designed to measure the participants’ ability to correctly solve addition with regrouping problems in a fluent manner. Thus, this CBA was considered a timed-focused curriculum-based assessment (Hudson & Miller, 2006). The Addition with Regrouping Minute was administered as a pre- and posttest, as part of lessons 11 through 20 as a practice test, and as a maintenance. However, to reduce the likelihood of practice effect on this measure, participants began each completion of it on the problem after the last problem completed on the subsequent attempt.

The fifth CBA, the Addition with Regrouping Word Problem Pretest, consisted of ten world problems that required regrouping to correctly solve (see Appendix J). In an effort to measure the participants’ problem-solving ability and not his or her reading ability, this CBA was read aloud to each participant. Within this CBA, five problems contained two-digit addends. One of these problems contained extraneous information. Participants were expected to ignore the unnecessary information and use two-digit numbers to correctly solve the problem. Additionally, this CBA contained five problems consisting of three-digit addends. All of the three-digit addend problems required only a single regrouping within the problem. As this CBA was intended to measure the participants’
ability to correctly solve addition with regrouping word problems with no time restriction, it was considered an untimed-focused curriculum-based assessment (Hudson & Miller, 2006).

The sixth CBA, the Addition with Regrouping Word Problem Posttest contained identical problems as those found within the Addition with Regrouping Word Problem Pretest to control for problem difficulty level (see Appendix K). The problems on the Addition with Regrouping Word Problem Posttest were presented in reverse order from the items on the pretest. Differing the order of problem presentation on the pre- and posttest helped reduce the likelihood of practice effect on these measures. Consistent with the Addition with Regrouping Word Problem Pretest, the problems were read aloud to the participants.

The last CBA, the Addition with Regrouping Word Problem Maintenance Test contained identical problems as those found within the Addition with Regrouping Word Problem Pretest and Posttest to control for problem difficulty level (see Appendix L). The problems on the Addition with Regrouping Word Problem maintenance Test were presented in reverse order from the items on the pretest. Differing the order of problem presentation on the pretest and maintenance test helped reduce the likelihood of practice effect on these measures. Consistent with the Addition with Regrouping Word Problem Pretest and Posttest, the problems were read aloud to the participants.

**Conceptual Understanding Pre- and Posttest**

Additionally, two conceptual understanding tests related to addition with regrouping were administered. The first, the Conceptual Understanding Pretest, contained six addition with regrouping problems (see Appendix M). The first three problems, asked
the participant to show how he or she would solve an addition with regrouping problem using base ten blocks. The participant was prompted to explain what he or she is doing with the base ten blocks as the problem was solved. One of these problems required the participant to solve a two-digit addend addition with regrouping problem, while the other two problems required the participant to solve three-digit addend addition with regrouping problems that involved regrouping a single time. The last three problems required the participant to show how to solve the addition with regrouping problems without the use of base ten blocks. Again, the participant was prompted to explain how he or she was solving the problems, but without the use of the base ten blocks. One of these problems required the participant to solve a two-digit addend addition with regrouping problem, while the other two problems required the participant to solve three-digit addend addition problems that involved regrouping a single time.

Finally, the Conceptual Understanding Posttest was administered (see Appendix N). This test was administered using the same procedures used for the Conceptual Understanding Pretest. Using base ten blocks, the participant solved and explained the solving process for three addition with regrouping problems. The problems were the same as those on the Conceptual Understanding Pretest to control for problem difficulty level. The problems were, however, arranged in a different order to prevent the likelihood of practice effect.

During both conceptual understanding tests, the student investigator scored the participants’ actions based on scoring conditions listed on the Conceptual Understanding Pretest and Posttest Scoring Protocols (see Appendices O and P). The Conceptual Understanding Pretest Protocol contained 34 conditions. During the first three problems
of the pretest, each participant was asked to use base ten blocks to explain how he or she was solving the problems. Problem one included seven conditions: (a) participant represents the first number accurately, (b) the participant represents the second number accurately, (c) participant adds ones correctly, (d) participant adds tens correctly, (e) participant states need to regroup tens, (f) participant regroups tens correctly, and (g) participant adds hundreds correctly. Problem two included six conditions: (a) participant represents the first number accurately, (b) the participant represents the second number accurately, (c) participant adds ones correctly, (d) participant states need to regroup ones, (e) participant regroups ones accurately, and (f) participant adds tens accurately. Problem three included seven conditions: (a) participant represents the first number accurately, (b) participant represents the second number accurately; (c) participant adds ones correctly, (d) participant states need to regroup ones, (e) participant regroups one accurately, (f) participant adds tens accurately, and (g) participant adds hundreds accurately.

The final three problems of the Conceptual Understanding Pretest required the participants to explain how he or she was solving the problems without using base ten blocks. Problem four contained five conditions: (a) participants adds ones correctly, (b) participant states need to regroup ones, (c) participant regroups ones accurately, (d) participant adds tens accurately, and (e) participant adds hundreds accurately. Problem five contained five conditions: (a) participant adds ones correctly, (b) participant adds tens correctly, (c) participant states need to regroup tens, (d) participant regroups tens accurately, and (e) participant adds hundreds accurately. Finally, problem six contained
The Conceptual Understanding Posttest Scoring Protocol contained 38 conditions. Just as in the Conceptual Understanding Pretest, during the first three problems of the posttest, each participant was asked to use base ten blocks to explain how he or she was solving the problems. Problem one included six conditions: (a) participant represents first number accurately, (b) participant represents second number accurately, (c) participant adds ones correctly, (d) participant states need to regroup ones, (e) participant regroups ones accurately, and (f) participant adds tens accurately. Problem two included seven conditions: (a) participant represents first number accurately, (b) participant represents second number accurately, (c) participant adds ones correctly, (d) participant states need to regroup ones, (e) participant regroups ones accurately, (f) participant adds tens accurately, and (g) participant adds hundreds accurately. Problem three contained nine conditions: (a) participant represents first number accurately, (b) participant represents second number accurately, (c) participant adds ones correctly, (d) participant states need to regroup ones, (e) participant regroups ones accurately, (f) participant adds tens correctly, (g) participant states need to regroup tens, (h) participant regroups tens correctly, and (i) participant adds hundreds correctly.

The final three problems of the posttest required each participant to explain how he or she was solving the problems without using base ten blocks. Problem four contained four conditions: (a) participant adds ones correctly, (b) participant states need to regroup ones, (c) participant regroups ones accurately, and (d) participant adds tens accurately. Problem five consisted of five conditions: (a) participant adds ones correctly, (b)
participant states need to regroup ones, (c) participant regroups ones accurately, (d) participant adds tens accurately, and (e) participant adds hundreds accurately. Finally, problem six contained seven conditions: (a) participant adds ones correctly, (b) participant states need to regroup ones, (c) participant regroups ones accurately, (d) participant adds ones correctly, (e) participant states need to regroup tens, (f) participant regroups tens correctly, and (g) participant adds hundreds correctly. The Conceptual Understanding Pre- and Posttests were untimed.

Baseline Probes

Once pretesting was complete, participants completed a minimum of three Baseline Probes (See appendices Q-S). Each probe contained ten problems. Eight of these problems were addition with regrouping problems, while two of these problems were addition with regrouping word problems. Of the eight addition with regrouping problems, four were two-digit problems and four of the problems were three-digit problems. Of the three-digit problems, three required a single regrouping and one required two regrouping to correctly solve the problem. There was one two-digit word problem and one three-digit word problem. The three-digit word problem required a single regrouping to correctly solve. The word problems on each probe were read to the participants to prevent the degree to which low reading ability may interfere with mathematics problem solving. These probes were designed to help determine the efficacy of the intervention (Barlow & Hersen, 1984). Additionally, these probes did not contain time restrictions.
**Intervention Probes**

Participants were required to complete a Learning Sheet as part of each addition with regrouping lesson (see example in Appendix T). Permission for use was granted (see Appendices A-C). These Learning Sheets contained three problems that were used during the describe and model stage of the lesson, three problems that were used during the guided practice stage of the lesson (the last two guided practice problems were solved without discussion of the correct answer), six problems that were used during the independent practice stage of the lesson, and two problems that were used during the problem-solving practice stage of the lesson. The final 10 problems on each Learning Sheet were used as Intervention Probes to measure participants’ ongoing progress during the study per the parameters of a multiple probe across participants design (Barlow & Herson, 1984).

**Maintenance Probe**

The Maintenance Probe was used to measure the participants’ continued ability to solve addition with regrouping problems (see Appendix U). The Maintenance Probe contained ten problems. Eight were addition with regrouping problems, while two of the problems were addition with regrouping word problems. Of the eight addition with regrouping problems, four were two-digit problems and four of the problems were three-digit problems. Of the three-digit problems, three required a single regrouping and one required two regroupings to correctly solve the problem. There was one two-digit word problem and one three-digit word problem. The three-digit word problems required a single regrouping to correctly solve. The word problems on the Maintenance Probe were read to the participants to prevent the degree to which low reading ability may interfere
with mathematics problem solving. These problems were designed to measure participants’ retention of addition with regrouping skills. Also, this probe did not contain time constraints.

**Generalization Probe**

The Generalization Probe was used to measure participants’ ability to generalize addition with regrouping ability to an alternate setting (see Appendix V). The Generalization Probe contained eight addition with regrouping problems, while two of the problems were addition with regrouping word problems. Of the eight addition with regrouping problems, four were two-digit problems and four of the problems were three-digit problems. Of the three-digit problems, three required a single regrouping and one required two regroupings to correctly solve the problem. There was one two-digit word problem and one three-digit word problem. The three-digit word problems required a single regrouping to correctly solve. The word problems on the Generalization Probe were read to the participants to prevent the degree to which low reading ability may interfere with mathematics problem solving. Also, this probe did not contain time constraints.

**Addition with Regrouping Satisfaction Questionnaire**

The Addition with Regrouping Satisfaction Questionnaire was used to measure the participants’ level of satisfaction with the various addition with regrouping intervention lessons (see Appendix W). The questionnaire contained eight questions and included a four-point Likert scale with 4 being strongly agree and 1 being strongly disagree. Participants were provided verbal instructions prior to completing the questionnaire and the questionnaire statements were read aloud to the participants.
Materials and Equipment

Addition with Regrouping Lessons

The Addition with Regrouping Lessons (Miller, Kaffar, & Mercer, 2011) contained materials lists, goals to be addressed during each lesson, and sample lesson presentation scripts to ensure that each lesson presentation involved systematic, explicit instruction (see an example in Appendix X). The pedagogically sound instruction included advance organizers and multiple stages of instruction that included describing, modeling, guided practice, independent practice, and problem solving. Permission for use was granted (see Appendices A-C).

Base Ten Blocks

Base ten blocks were used as a manipulative device to assist participants’ conceptual understanding of the addition with regrouping process. The base ten blocks were 3-dimensional plastic blocks in three different shapes. Individual cubes represented units of one. Rectangular rods that equal the length of ten cubes joined together represented tens. Finally, square tiles equal to ten rods joined together represented the hundreds.

Place Value Mat

A sheet of 8 ½ inch by 24 inch construction paper was be used to construct place value mats for each participant (see example in Appendix Y). The place value mat was divided into three columns. The right column was titled Ones, the middle column was titled Tens, and the left column was titled Hundreds. The place value mat was used during the initial five lessons as a means to assist in the development of conceptual understanding related to addition with regrouping.
Design

This study involved the use of a multiple probe across participants design with two replications (Barlow & Hersen, 1984; Zirpoli, 2008). There were four design conditions: baseline, intervention, maintenance, and generalization. There were three triads with two serving as replications.

Baseline Condition

Upon the completion of pretesting, the multiple probe study began. All participants received concurrent Baseline Probes (see Appendices Q-S). The baseline condition included collection of data to establish the participants’ pre-instructional skills related to addition with regrouping. As soon as one student from each triad demonstrated baseline stability, the intervention condition began for those three participants. These participants were considered Participant 1, Participant 4, and Participant 7 and represented the first participant in each of the three triads. The remaining participants continued to receive baseline probes on a weekly basis until the first three participants reached mastery level performance on the first three lessons. When the mastery level was reached on the first three lessons, a baseline probe was administered to the remaining participants. As all remaining participants demonstrated stability in baseline trends, the intervention condition began with an additional three participants. These participants were considered Participant 2, Participant 5, and Participant 8 and represented the second participant in each of the three triads. The remaining three participants received an additional Baseline Probe once Participants 2, 5, and 8 reached a mastery level performance on the first three lessons. Because the final three participants demonstrated stability in baseline trends, the intervention condition for these three participants began. These participants were
considered Participant 3, Participant 6, and Participant 9 and represented the third participant in each of the three triads.

**Intervention Condition**

Participants 1, 4, and 7 began initial instruction of intervention lessons at the same time. The participants received scripted lessons that follow explicit instruction pedagogy including (a) an advanced organizer, (b) a describe and model instructional stage, (c) a guided practice instructional stage, (d) an independent practice instructional stage, and (e) a problem solving instructional stage. To ensure the accuracy of material presentation, the student investigator followed Power Point slides and notes based on the scripted lessons when presenting the different lessons to each participant. The program lessons also followed the concrete-representational-abstract instructional process. Of the 20 intervention lessons, five focused on concrete methodology, three focused on representational methodology, two focused on the teaching and mastery of mnemonic devices, five focused on abstract methodology, and the remaining five focused on building advanced word problem and fluency skills. Current best practices reported within CRA literature was used to determine the number and types of intervention lessons that were used.

Lessons focusing on concrete methodology were designed to facilitate the conceptual understanding of addition with regrouping. Base ten blocks were used to provide hands-on experiences that correlate to the verbal descriptions of what took place when adding with regrouping. The use of these three-dimensional objects allowed participants to understand and develop mental images of the mathematics concept (i.e. trading ones to form a ten; trading tens to form a hundred).
The representational methodology lessons focused on moving the participants’ use of addition with regrouping from a three-dimensional understanding to a two-dimensional understanding. Instead of using base-ten blocks, visual depictions were used to assist with the solving of addition with regrouping problems.

The lessons focusing on a mnemonic device consisted of participants learning the mnemonics RENAME and FAST RENAME (see Appendices Z and AA). The purpose of teaching these mnemonics was to aid the participants in remembering and using the steps required to solve addition with regrouping computation and word problems independently. The abstract methodology lessons focused on removing visual supports the participants had previously used in solving addition with regrouping problems.

The lessons focusing on word problems and fluency allowed participants to practice addition with regrouping skills within word problems and to focus on increasing the rate at which they were able to solve addition with regrouping computation problems. The scaffolding of instruction within the concrete-representational-abstract process supported participants’ movement from a level of understanding that required tangible objects to a more abstract understanding of this new mathematics concept (Hudson & Miller, 2006).

The parameters of a multiple probe design required ongoing probes of participant performance as part of the Intervention Condition. Specifically, the percentage scores of Participants 1, 4, and 7 were graphed to monitor their individual success with the new skill. When Participants 1, 4, and 7 achieved 80% correct on the first three Intervention Probes, all remaining participants received an additional baseline probe prior to Participants 2, 5, and 8 beginning the intervention lessons. Because Participants 2, 5, and 8 demonstrated stability in baseline trends, they began the intervention condition.
Likewise, when participants 2, 5, and 8 achieved 80% correct on the first three Intervention Probes, the remaining participants received an additional baseline probe. Because Participants 3, 6, and 9 demonstrated stability in baseline trends, they began the intervention condition.

**Maintenance Condition**

A Maintenance Probe was administered seven days after the intervention condition concluded for each participant (see Appendix U). For each participant, the seven days included five typical days of attendance in school and two weekend days. Maintenance scores were used to measure participants’ retention of addition with regrouping skills.

**Generalization Condition**

A generalization probe was administered fourteen days after the intervention condition concluded for each participant (see Appendix V). For each participant, the fourteen days included ten typical days of attendance in school and four weekend days. The generalization probe was administered by a teacher within the general education classroom. The participants’ ability to generalize addition with regrouping skills to an alternate setting was measured using percentage scores on this probe.

**Procedures**

This study consisted of six phases: (a) study preparation, (b) pretest and baseline, (c) mathematics intervention lesson implementation, (d) post-assessments, (e) maintenance, and (f) generalization.

**Phase 1: Study Preparation**

The study preparation phase included two activities: (a) participant selection and (b) obtaining permission for participation. The following criteria was used to determine
participant eligibility for this study: (a) identified as having a specific learning disability, (b) been enrolled within second, third, fourth, fifth, or sixth grade at the participating charter school, and (c) scored 80% or less on the Addition with Regrouping Pretest (Miller, Kaffar, & Mercer, 2011).

Permission for study implementation was obtained from the Institutional Review Board at the sponsoring university and the administrator at the participating charter school. Both the approved letters of parent consent and participant assent (see Appendices D and E) were placed within sealed envelopes and sent home with potential participants (i.e., students enrolled in second through sixth grade with learning disabilities in mathematics) by the special education facilitator at the site school. After review, the potential participants and their parents returned the forms to the special education facilitator. Ten potential participants returned signed parental consent and student assent forms. These ten potential participants were given the Addition with Regrouping Pretest (Miller, Kaffar, & Mercer, 2011). Nine of the ten potential participants earned a score of 80% or less on the pretest and were thus considered eligible to participate within this study.

Phase 2: Pretest and Baseline

The student investigator administered three CBAs and one conceptual understanding test to each participant at the school site. First, the Addition with Regrouping Pretest and the Addition with Regrouping Word Problem Pretest were administered (see Appendices F and J). As both of these pretests were untimed-focused curriculum-based assessments, participants were given as much time as necessary for completion of the assessments.
Percentage scores were calculated to measure how accurately participants were able to solve addition with regrouping problems.

Second, the Addition with Regrouping Minute was administered to all participants (see Appendix I). Participants were given one minute to solve as many problems as possible in this timed-focused curriculum-based assessments. Participants’ addition with regrouping skills were measured by determining the number of correct and incorrect digits recorded during the Addition with Regrouping Minute.

Third, the Conceptual Understanding Pretest was administered (see Appendix M). If participants meet the stated conditions on the Conceptual Understanding Pretest Scoring Protocol (e.g. participant adds ones accurately, participant states need to regroup tens) points were awarded on the Conceptual Understanding Pretest Scoring Protocol (see Appendix O).

Following the administration of these curriculum-based pretests and the conceptual understanding pretest, the baseline condition, (i.e., administration of the Baseline Probes), began following the parameters for multiple probe across participants designs (see Appendices Q-S). Baseline Probes were given to all participants over a minimum of three sessions until stability was established.

**Phase 3: Mathematics Intervention Lesson Implementation**

Upon establishment of baseline stability, the addition with regrouping intervention lessons began according to the implementation schedule (Appendix BB). The lessons used explicit teaching principles and the concrete-representational-abstract process within scripted lessons. The series of 20 lessons were used to gradually teach participants the
skills necessary to solve complex addition with regrouping computation and word
problems (for a summary of lesson goals see Appendix CC).

First, each lesson began with an advance organizer that (a) reviewed previously
learned skills, (b) presented the lesson objective in a manner that directly related to prior
knowledge, and (c) provided relevance for why participants were learning the new
concept or skill, thus enhancing participant motivation to participate (Hudson & Miller,
2011). Next, the describe and model stage of the lesson was implemented. This stage
included three items. First, the instructor modeled what participants were expected to do
in order to solve the problem. Participants were exposed to the metacognitive process
through instructor think-alouds while problem solving. Second, the instructor maintained
participant attention and engagement by seating participants within two feet of the
instructor and using verbal cues. Third, participant comprehension was monitored
through the use of questioning and feedback.

During the guided practice stage of the lessons, the instructor gradually encouraged
participants to take more responsibility while working toward independent problem
solving. The instructor provided various levels of support during guided practice to
ensure participant success. Gradually, assistance was removed so that participants were
supported while working toward independence. The instructor simultaneously asked
both factual and process type questions to monitor participant performance with the new
concept or skill.

The independent practice stage of the lessons required participants to independently
solve addition with regrouping problems with instructor supports removed. This gave
participants an opportunity to show their current levels of performance with the concept or skill.

Both guided practice and independent practice stages of each lesson involved performance feedback. Specifically, a feedback routine was provided that included: (a) helping the participant plot his or her score on a progress chart, (b) providing one specific positive statement about the participant’s work, (c) identifying one area for improvement, (d) demonstrating how to solve missed problems using think-aloud methodology, (e) asking participants to complete one similar problem, and (f) closing the feedback session by stating positive expectations related to future performance on similar problems.

**Phase 4: Posttests**

Three curriculum-based assessments, the Addition with Regrouping Posttest, the Addition with Regrouping Minute, and the Addition with Regrouping Word Problem Posttest, were given to participants at the conclusion of the addition with regrouping intervention lessons (see Appendices G, I, and K). Additionally, the Conceptual Understanding Posttest was administered as a posttest measure of participants’ conceptual understanding of addition with regrouping skills (see Appendix N).

The Addition with Regrouping Posttest and the Addition with Regrouping Word Problem Posttest were untimed-focused curriculum-based assessments, allowing participants as much time as necessary for completion. Participants’ abilities to solve addition with regrouping problems were measured using percentage scores.

For the Addition with Regrouping Minute Posttest, the number of correct and incorrect digits participants provided within one minute was determined to measure participants’
fluency in solving addition with regrouping problems. Participants were given one minute to complete this posttest.

The Conceptual Understanding Posttest also was administered to each participant. If participants met the stated criteria on the Conceptual Understanding Posttest Scoring Protocol, points were awarded (see Appendix P).

The Addition with Regrouping Satisfaction Questionnaire was used to measure social validity of the study (see Appendix W). The questionnaire measured the participants’ satisfaction levels related to the various individual program components as well as their satisfaction with the program as a whole. The questionnaire contained nine questions and was based on a four-point Likert scale with 4 being strongly agree and 1 being strongly disagree. Participants were provided verbal instructions prior to completing the questionnaire and the questionnaire statements were read aloud to the participants.

During individual meetings, each participant was shown his or her test results. Each participant was also shown the level of improvement he or she demonstrated by comparing pre- and posttest results.

**Phase 5: Maintenance**

Three CBA maintenance tests, Addition with Regrouping Maintenance Test, Addition with Regrouping Word Problem Maintenance Test, and the Addition with Regrouping Minute- and the Maintenance Probe were administered seven days after the initial Posttests were administered (see Appendices H, I, L, and U). Results from the maintenance posttests and Maintenance Probes were shared with participants to demonstrate progress with addition with regrouping skills.
Phase 6: Generalization

To measure the participants’ ability to generalize his or her addition with regrouping skills in alternate settings, a Generalization Probe was administered (see Appendix V). Within the general education classroom, participants completed a teacher given addition with regrouping worksheet that included both addition with regrouping problems and addition with regrouping word problems. Participants’ ability to generalize addition with regrouping skills was measured using a percentage score on this worksheet.

Interscorer Reliability

The student investigator scored each participant’s pre-, post-, and maintenance tests – the Addition with Regrouping Pre-, Post-, and Maintenance Tests, the Addition with Regrouping Minute, the Addition with Regrouping Word Problem Pre-, Post-, and Maintenance Tests, and the Maintenance Probe. Likewise, the student investigator scored the Conceptual Understanding Pre- and Posttests. Similarly, the student investigator scored all of the Baseline Probes, the Intervention Probes, and the Generalization Probes. The research assistant scored 20% of each of these measures (i.e. pre-, post-, and maintenance tests; Baseline Probes, Intervention Probes, Maintenance Probes, and Generalization Probes) to determine interscorer reliability. The probes were randomly selected across types. The primary scorer was the student investigator and the secondary scorer was the research assistant. When both the student investigator and the research assistant recorded the same score for an answer, an agreement was counted. Reliability levels were determined using the formula agreements ÷ (agreements + disagreements) x 100 (Barlow & Hersen, 1984).
Fidelity of Treatment

The research assistant observed 100% of the recorded addition with regrouping lessons. She watched videos of each lesson and completed fidelity of treatment checklists (see Appendix DD). The primary investigator observed 25% of randomly selected lessons. She watched videos of the selected lessons and completed fidelity of treatment checklists. To determine interobserver agreement, the formula agreements ÷ (agreements + disagreements) x 100 was used.

Treatment of Data Related to Visual Analysis

To measure the effects of the addition with regrouping intervention lessons, visual analysis of the participants’ Baseline Probes, Intervention Probes, Maintenance Probes, and Generalization Probes was used. Individual participant performance was graphed using multiple probe design specifications (Barlow & Hersen, 1984; Zirpoli, 2008). Visual inspection of the level, trend, and variability of participant performance data was used to identify the effectiveness of the intervention lessons. Level change was determined using the mean scores of the dependent variable (Barlow & Hersen, 1984). If the performance level of the dependent variable (Intervention Probes) increased when compared to Baseline Probes, then the intervention lessons were considered successful. Visual inspection of the data that revealed consistency within the rate of behavior in either an upward, downward, or stable manner was used to assess trend (Barlow & Hersen, 1984). If there was an acceptable increase in the trends line’s stability or slope, the intervention lessons were considered successful. The consistency of data points around the mean performance inspected to evaluate variability (Barlow & Hersen, 1984). The intervention was considered successful when little variability was shown, thereby
indicating consistent performance along with a change in level and trend. To address issues of external validity and increase the confidence that performance ability within addition with regrouping skills was due to the intervention lessons, two replications were conducted with six additional participants.

**Treatment of Data Related to Research Questions**

**Research Question 1**

Do students with learning disabilities improve their ability to solve addition with regrouping computation and word problems after receiving strategy instruction that involves concrete-representational-abstract sequencing? This question was answered using two data sets: the on-going monitoring probes (i.e. Baseline Probes and Intervention Probes) and two of the curriculum-based pre- and posttest measures (Addition with Regrouping and Addition with Regrouping Word Problem). Baseline Probe scores were compared to Intervention Probe scores taking into consideration level, trend, and variability. To analyze the data obtained from the ongoing Intervention Probes, the percentage of non-overlapping data (PND) (i.e., nonparametric approach to determining treatment effects in single subject design studies) was calculated by (a) identifying the highest Baseline Probe among all participants, (b) identifying the number of treatment probes from all nine participants that were greater than the highest Baseline Probe by the total number of treatment probes and multiplying by 100 to determine the PND. To provide supplemental information related to this research question, curriculum-based pretest percentage scores were compared to the curriculum-based posttest percentage scores.
**Research Question 2**

Do students with learning disabilities improve their conceptual understanding related to addition with regrouping after receiving strategy instruction that involves concrete-representational-abstract sequencing? This question was answered using descriptive data from the Conceptual Understanding Pre- and Posttests. Points earned on the Conceptual Understanding Scoring Protocols were translated to percentage scores. The pretest percentage score of each participant was compared to the respective posttest percentage score.

**Research Question 3**

Do students with learning disabilities increase their fluency related to addition with regrouping after receiving strategy instruction that involves concrete-representational-abstract sequencing? This question was answered using the Addition with Regrouping Minute scores related to the number of correct and error digits on these minutes were compared. Additionally, the celebration rate for each participant was calculated.

**Research Question 4**

Do students with learning disabilities maintain their ability to solve addition with regrouping computation and word problems after receiving strategy instruction that involves concrete-representational-abstract sequencing? This question was answered using four posttest curriculum-based assessments – Addition with Regrouping Post- and Maintenance Tests and the Addition with Regrouping Word Problem Post- and Maintenance Tests- and the Maintenance Probe. All were administered seven days after the instruction condition. Performance on the respective post- and maintenance test
scores was compared. Likewise, performance on the Maintenance Probe was compared to mean Intervention Probe scores.

**Research Question 5**

Do students with learning disabilities generalize their ability to solve addition with regrouping computation and word problems after receiving strategy instruction that involves concrete-representational-abstract sequencing? This question was answered using a teacher given classroom-based addition with regrouping worksheet. Percentage scores were calculated for each participant and compared to posttest and maintenance scores.

**Research Question 6**

Do students with learning disabilities report high levels of satisfaction related to strategy instruction that involves concrete-representational-abstract sequencing for learning addition with regrouping skills? This question was answered using the Addition with Regrouping Satisfaction Questionnaire. Response frequencies and related percentage scores were reported for each statement on the questionnaire.
CHAPTER 4
DATA ANALYSIS

The purpose of this study was to investigate the effects of strategy instruction that involves the use of the concrete-representational-abstract sequence to teach addition with regrouping to students with learning disabilities. Data were collected to answer six research questions related to the participants’ ability to acquire, maintain, and generalize knowledge related to solving addition problems that require regrouping. Additionally, participant’s satisfaction levels were assessed in relation to learning through the concrete-representational-abstract sequence. This chapter begins with a sequential presentation of results related to each of the six research questions. Next, interscorer reliability and fidelity of treatment data are provided. The chapter concludes with a summary of the results obtained in this study.

Research Questions and Related Findings

Research Question 1

Do students with learning disabilities improve their ability to solve addition with regrouping computation and word problems after receiving strategy instruction that involves concrete-representational-abstract sequencing?

Two data sets (i.e., ongoing probes and pre-posttests) were used to determine whether the computation and word problem performance of students with learning disabilities improved after receiving strategy instruction that involved the concrete-representational-abstract sequence. The first data set consisted of the Baseline Probes and the Intervention Probes that were collected throughout baseline and intervention conditions (see Figures 1, 2, and 3). Visual analysis was used to analyze these data.
Figure 1

Data Set for Triad One
Figure 2

Data Set for Triad Two

Baseline | Intervention | Maintenance | Generalization

Sessions

![Graphs showing data set for Triad Two](image)
Figure 3

Data Set for Triad Three
(i.e., level, trend, and variability) per the parameters of the multiple probe design.

Additionally, the percentage of non-overlapping data (PND) was calculated to determine the magnitude of the treatment effects.

Visual inspection of Figures 1, 2, and 3 reveals that all nine participants demonstrated improvement in performance level upon the initiation of the strategy instruction that involved the use of concrete-representational-abstract sequence. The mean Baseline Probe scores for the participants in triad one ranged from 5.77% to 65% (M=23.33%, SD = 31.14). The mean Intervention Probe scores for the participants in triad one ranged from 89.52% to 95.5% (M= 92.74%, SD =9.44). This represents a mean percentage point improvement of 68.78 for triad one. See Table 2 for a summary of individual Baseline and Intervention Probe percentage scores for triad one. The mean Baseline Probe scores for the participants in triad two ranged from 0% to 67.5% (M=22.5%, SD =33.34). The mean Intervention Probe scores for the participants in triad two ranged from 92.38% to 96% (M= 94.43%, SD =8.47). This represents a mean

Table 2

<table>
<thead>
<tr>
<th>Participants</th>
<th>Baseline Probes M / SD</th>
<th>Intervention Probes M / SD</th>
<th>Percentage Point Increase from Baseline to Intervention Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 1</td>
<td>6.67 / 5.77</td>
<td>89.52 / 12.03</td>
<td>82.85</td>
</tr>
<tr>
<td>Participant 2</td>
<td>65.00 / 5.77</td>
<td>93.00 / 7.33</td>
<td>28.00</td>
</tr>
<tr>
<td>Participant 3</td>
<td>0.00 / 0.00</td>
<td>95.50 /7.59</td>
<td>95.50</td>
</tr>
</tbody>
</table>
percentage point improvement of 71.96 for triad two. See Table 3 for a summary of individual Baseline and Intervention Probe percentage scores for triad two. The mean Baseline Probe scores for the participants in triad three ranged from 0% to 12% (M=5.00%, SD = 6.74). The mean Intervention Probe scores for the participants in triad three ranged from 85.91% to 97% (M= 91.29%, SD =10). This represents a mean percentage point improvement of 87.47 for triad three. See Table 4 for a summary of individual Baseline and Intervention Probe percentage scores for triad three.

Table 3

*Triad Two: Baseline and Intervention Probe Scores*

<table>
<thead>
<tr>
<th></th>
<th>Baseline Probes</th>
<th>Intervention Probes</th>
<th>Percentage Point Increase from Baseline to Intervention Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M / SD</td>
<td>M / SD</td>
<td></td>
</tr>
<tr>
<td>Participant 4</td>
<td>0.00 / 0.00</td>
<td>92.38 / 11.36</td>
<td>92.38</td>
</tr>
<tr>
<td>Participant 5</td>
<td>67.5 / 5.00</td>
<td>96.00 / 5.98</td>
<td>28.50</td>
</tr>
<tr>
<td>Participant 6</td>
<td>0.00 / 0.00</td>
<td>95.00 / 6.88</td>
<td>95.00</td>
</tr>
</tbody>
</table>

Table 4

*Triad Three: Baseline and Intervention Probe Scores*

<table>
<thead>
<tr>
<th></th>
<th>Baseline Probes</th>
<th>Intervention Probes</th>
<th>Percentage Point Increase from Baseline to Intervention Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M / SD</td>
<td>M / SD</td>
<td></td>
</tr>
<tr>
<td>Participant 7</td>
<td>0.00 / 0.00</td>
<td>85.91 / 12.21</td>
<td>85.91</td>
</tr>
<tr>
<td>Participant 8</td>
<td>0.00 / 0.00</td>
<td>91.50 / 7.45</td>
<td>91.50</td>
</tr>
<tr>
<td>Participant 9</td>
<td>12.00 / 4.47</td>
<td>97.00 / 5.71</td>
<td>85.00</td>
</tr>
</tbody>
</table>
With regard to trend, all nine participants demonstrated relatively stable baseline performance and moderately high performance after the initiation of the strategy instruction that involved the use of the concrete-representational sequence. Participants 1, 4, and 7 demonstrated very slight ascending trends during the intervention condition while all other participants demonstrated stable, but high performance trends during the intervention condition. With regard to variability, Participant 1 demonstrated baseline variability that ranged from 0% to 10%, which meant a difference of only one problem. Participants 2 and 5 demonstrated baseline variability that ranged from 60% to 70%, which again translates to a difference of one problem. Participants 3, 4, 6, 7, and 8 demonstrated no baseline variability. Their performance scores remained at 0% throughout the baseline condition. Likewise, Participant 9 demonstrated no baseline variability. His performance scores remained at 10% throughout the baseline condition. More variability was demonstrated during the intervention condition. Participants 1 and 7 demonstrated intervention variability that ranged from 50% to 100% which meant a difference of 5 problems. Participant 4 demonstrated intervention variability that ranged from 60% to 100% which meant a difference of 4 problems. Participants 2, 3, 5, 6, 8, and 9 demonstrated intervention variability that ranged from 80% to 100% which meant a difference of 2 problems. Thus, intervention variability for all nine participants ranged from 50% to 100%, which translates to a difference of five problems. See appendix EE for a summary of data by participant.

Calculation of the PND involved: (a) identifying the highest Baseline Probe among all participants (i.e., 70%, Participants 5 and 7), (b) identifying the number of treatment probes from all participants that were greater than the highest Baseline Probe
(i.e., 180), and (c) dividing the number of treatment probes greater than the highest Baseline Probe by the total number of treatment probes (i.e., 184), and multiplying by 100. Thus, the PND for these nine participants was 97.83%, which represents a very large effect size (Mathur, Kavale, Quinn, Forness, & Rutherford, 1998).

The second data set used to assess the performance of students with learning disabilities consisted of the Addition with Regrouping and the Addition with Regrouping Word Problem Pretest scores and the Addition with Regrouping and the Addition with Regrouping Word Problem Posttest scores. All nine participants increased their scores from pre- to posttest. The Addition with Regrouping Pretest scores for the participants in triad one ranged from 0% to 75% (M= 43.33%, SD = 38.84). All three participants in triad one scored 100% on the Addition with Regrouping Posttest. This represents a mean percentage point improvement of 56.68 for triad one. The Addition with Regrouping Word Problem Pretest scores for the participants in triad one ranged from 0% to 70% (M= 23.33%, SD = 40.41). All three participants in triad one scored 100% on the Addition with Regrouping Word Problem Posttest. This represents a mean percentage point improvement of 76.67 for triad one. See Table 5 for a summary of individual pretest and posttest scores for triad one. The Addition with Regrouping Pretest scores for the participants in triad two ranged from 0% to 75% (M= 25.00%, SD = 43.30). The Addition with Regrouping Posttest scores for the participants in triad two ranged from 95% to 100% (M = 98.33%, SD = 2.89). This represents a mean percentage point improvement of 73.33 for triad two. The Addition with Regrouping Word Problem Pretest scores for the participants in triad two ranged from 0% to 80% (M= 26.67%, SD =
46.19). The Addition with Regrouping Word Problem Posttest scores for the participants in triad two ranged from 90% to 100% (M = 93.33%, SD = 5.77). This represents a mean percentage point improvement of 66.66 for triad two. See Table 6 for a summary of individual pretest and posttest scores for triad two. The Addition with Regrouping Pretest scores for the participants in triad three ranged from 0% to 10% (M= 5.00%, SD = 5.00). The Addition with Regrouping Posttest scores for the participants in triad three ranged from 90% to 95% (M = 91.67%, SD = 2.89). This represents a mean percentage point improvement of 86.67 for triad three. The Addition with Regrouping Word Problem Pretest scores for all participants in triad three were 0%. The Addition with Regrouping Word Problem Posttest scores for the participants in triad three ranged from 90% to 100% (M = 96.67%, SD = 5.77). This represents a mean percentage point improvement of 96.67 for triad three. See Table 7 for a summary of individual pretest and posttest scores for triad three.

Table 5

**Triad One: Addition with Regrouping and Addition with Regrouping Word Problem**

**Pretest and Posttest Scores**

<table>
<thead>
<tr>
<th>Participants</th>
<th>Pretests</th>
<th>Posttests</th>
<th>Percentage Point Increase from Pretest to Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Addition with Regrouping / Addition with Regrouping Word Problem</td>
<td>Addition with Regrouping / Addition with Regrouping Word Problem</td>
<td></td>
</tr>
<tr>
<td>Participant 1</td>
<td>55 / 0</td>
<td>100 / 100</td>
<td>45 / 100</td>
</tr>
<tr>
<td>Participant 2</td>
<td>75 / 70</td>
<td>100 / 100</td>
<td>25 / 30</td>
</tr>
<tr>
<td>Participant 3</td>
<td>0 / 0</td>
<td>100 / 100</td>
<td>100 / 100</td>
</tr>
</tbody>
</table>
Table 6

**Triad Two: Addition with Regrouping and Addition with Regrouping Word Problem**

**Pretest and Posttest Scores**

<table>
<thead>
<tr>
<th>Participants</th>
<th>Pretests</th>
<th>Posttests</th>
<th>Percentage Point Increase from Pretest to Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Addition with Regrouping / Addition with Regrouping Word Problem</td>
<td>Addition with Regrouping / Addition with Regrouping Word Problem</td>
<td></td>
</tr>
<tr>
<td>Participant 4</td>
<td>0 / 0</td>
<td>100 / 90</td>
<td>100 / 90</td>
</tr>
<tr>
<td>Participant 5</td>
<td>75 / 80</td>
<td>100 / 100</td>
<td>25 / 20</td>
</tr>
<tr>
<td>Participant 6</td>
<td>0 / 0</td>
<td>95 / 90</td>
<td>95 / 90</td>
</tr>
</tbody>
</table>

Table 7

**Triad Three: Addition with Regrouping and Addition with Regrouping Word Problem**

**Pretest and Posttest Scores**

<table>
<thead>
<tr>
<th>Participants</th>
<th>Pretests</th>
<th>Posttests</th>
<th>Percentage Point Increase from Pretest to Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Addition with Regrouping / Addition with Regrouping Word Problem</td>
<td>Addition with Regrouping / Addition with Regrouping Word Problem</td>
<td></td>
</tr>
<tr>
<td>Participant 7</td>
<td>0 / 0</td>
<td>90 / 90</td>
<td>90 / 90</td>
</tr>
<tr>
<td>Participant 8</td>
<td>5 / 0</td>
<td>95 / 100</td>
<td>90 / 100</td>
</tr>
<tr>
<td>Participant 9</td>
<td>10 / 0</td>
<td>90 / 100</td>
<td>80 / 100</td>
</tr>
</tbody>
</table>
Research Question 2

Do students with learning disabilities improve their conceptual understanding related to addition with regrouping after receiving strategy instruction that involves concrete-representational-abstract sequencing? One data set, the Conceptual Understanding Pretest and Posttest, was used to determine whether the conceptual understanding of students with learning disabilities improved after receiving strategy instruction that involved the concrete-representational-abstract sequence. Points earned on the Conceptual Understanding Scoring Protocols were translated to percentage scores; these protocol scores were compared to determine whether conceptual understanding related to regrouping improved.

All nine participants demonstrated an increase in their conceptual understanding from pre- to posttest. The Conceptual Understanding Pretest scores for the participants in triad one ranged from 38% (13/34) to 76% (26/34) (M= 52.67%, SD = 20.43). The Conceptual Understanding Posttest scores for the participants in triad one ranged from 91% (31/34) to 100% (34/34) (M= 96%, SD = 4.58). This represents a mean percentage point improvement of 43.33 for triad one. See Table 8 for a summary of individual pretest and posttest scores for triad one. The Conceptual Understanding Pretest scores

<table>
<thead>
<tr>
<th>Participant</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Percentage Point Increase from Pretest to Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 1</td>
<td>38 (13/34)</td>
<td>97 (33/34)</td>
<td>59</td>
</tr>
<tr>
<td>Participant 2</td>
<td>76 (26/34)</td>
<td>100 (34/34)</td>
<td>24</td>
</tr>
<tr>
<td>Participant 3</td>
<td>44 (15/34)</td>
<td>91 (31/34)</td>
<td>47</td>
</tr>
</tbody>
</table>
for the participants in triad two ranged from 35% (12/34) to 56% (19/34) (M= 46%, SD = 10.54). All three participants in triad two scored a 100% (34/34) on the Conceptual Understanding Posttest. This represents a mean percentage point improvement of 54.00 for triad two. See Table 9 for a summary of individual pretest and posttest scores for Table 9

**Triad Two: Conceptual Understanding Pretest and Posttest Scores**

<table>
<thead>
<tr>
<th>Participants</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Percentage Point Increase from Pretest to Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 4</td>
<td>35 (12/34)</td>
<td>100 (34/34)</td>
<td>65</td>
</tr>
<tr>
<td>Participant 5</td>
<td>56 (19/34)</td>
<td>100 (34/34)</td>
<td>44</td>
</tr>
<tr>
<td>Participant 6</td>
<td>47 (16/34)</td>
<td>100 (34/34)</td>
<td>53</td>
</tr>
</tbody>
</table>

triad two. The Conceptual Understanding Pretest scores for the participants in triad three ranged from 0% (0/34) to 88% (30/34) (M= 39.00%, SD = 44.84). The Conceptual Understanding Posttest scores for the participants in triad three ranged from 74% (25/34) to 100% (34/34) (M= 91.33%, SD = 15.01). This represents a mean percentage point improvement of 52.33 for triad three. See Table 10 for a summary of individual pretest and posttest scores for triad three.

**Research Question 3**

Do students with learning disabilities increase their fluency related to addition with regrouping after receiving strategy instruction that involves concrete-representational-abstract sequencing?

The Addition with Regrouping Minute scores were used to determine
Table 10

<table>
<thead>
<tr>
<th>Participants</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Percentage Point Increase from Pretest to Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 7</td>
<td>0 (0/34)</td>
<td>74 (25/34)</td>
<td>74</td>
</tr>
<tr>
<td>Participant 8</td>
<td>29 (10/34)</td>
<td>100 (34/34)</td>
<td>71</td>
</tr>
<tr>
<td>Participant 9</td>
<td>88 (30/34)</td>
<td>100 (34/34)</td>
<td>12</td>
</tr>
</tbody>
</table>

whether the addition with regrouping fluency of students with learning disabilities improved during the intervention condition and whether fluency was maintained after receiving strategy instruction that involved the concrete-representational-abstract sequence. The Addition with Regrouping Minute scores related to the number of correct and error digits for each triad of participants were compared. The pretest mean scores for number of correct and error digits per minute for the participants in triad one ranged from 2 to 12 (correct) (M=7.00, SD = 5.00) and 9 to 16 (error) (M= 11.67, SD = 3.79). The posttest mean scores for the number of correct and error digits in one minute for the participants in triad one ranged from 17 to 29 (correct) (M = 24.00; SD = 6.24) and 0 to 1 (error) (M = 0.33; SD = 0.58). The maintenance test mean scores for the number of correct and error digits per minute for the participants in triad one ranged from 14 to 21 (correct) (M = 16.67; SD = 3.79) and 0 to 1 (error) (M = 0.33; SD = 0.58). This represents a mean score increase of 17 correct digits and a mean score decrease of 11.34 error digits pre- to posttest for triad one. Additionally, this represents a mean score decrease of 7.33 correct digits and no change in the number of error digits from posttest to maintenance test for triad one. See Figures 4 and 5 for a summary of individual pre-,
post- and maintenance test raw scores for correct digits and error digits in one minute for
the participants in triad one. The pretest mean scores for number of correct and error

Figure 4

*Triad One: Addition with Regrouping Minute Pretest, Posttest, and Maintenance Test*

**Digits Correct in One Minute**

![Graph](image1)

Figure 5

*Triad One: Addition with Regrouping Minute Pretest, Posttest, and Maintenance Test*

**Error Digits in One Minute**

![Graph](image2)
digits per minute for the participants in triad two ranged from 1 to 16 (correct) (M= 6.67, SD = 8.14) and 5 to 15 (error) (M= 10.00, SD = 5.00). The posttest mean scores for the number of correct and error digits in one minute for the participants in triad two ranged from 14 to 25 (correct) (M = 18.00; SD = 6.08) and 0 to 3 (error) (M = 1.00; SD = 1.73). The maintenance test mean scores for the number of correct and error digits per minute for the participants in triad two ranged from 18 to 25 (correct) (M = 22.33; SD = 3.79) and 0 to 2 (error) (M = 1.33; SD = 1.15). This represents a mean score increase of 11.33 correct digits and a mean score decrease of 9 error digits from pre- to posttest for triad two. Additionally, this represents a mean score increase of 4.33 digits correct and a mean score increase of 0.33 error digits per minute from posttest to maintenance test for triad two. See Figures 6 and 7 for a summary of individual pre-, post-, and maintenance test raw scores for correct digits and error digits in one minute for the participants in triad two.

Figure 6

*Triad Two: Addition with Regrouping Minute Pretest, Posttest, and Maintenance Test*

*Digits Correct in One Minute*
two. The pretest mean scores for number of correct and error digits per minute for the participants in triad three ranged from 0 to 4 (correct) \( (M = 2.33, \ SD = 2.08) \) and 0 to 10 (error) \( (M = 5.67, \ SD = 5.13) \). The posttest mean scores for the number of correct and error digits in one minute for the participants in triad three ranged from 9 to 13 (correct) \( (M = 11.00; \ SD = 2.00) \) and 0 to 1 (error) \( (M = 0.33; \ SD = 0.58) \). The maintenance test mean scores for the number of correct and error digits per minute for the participants in triad three ranged from 10 to 13 (correct) \( (M = 11.00; \ SD = 1.73) \) and 0 to 1 (error) \( (M = 0.50; \ SD = 0.58) \). This represents a mean score increase of 8.67 correct digits and a mean score decrease of 5.34 error digits from pre- to posttest for triad three. Additionally, this represents no change in the mean score for correct digits and a slight increase of 0.17 error digits per minute from posttest to maintenance test for triad three. See Figures 8 and 9 for a summary of individual pre-, post-, and maintenance test raw scores for digits correct and error digits in one minute for the participants in triad three.
Additionally, the Addition with Regrouping Minute was used to determine whether the addition with regrouping fluency of students with learning disabilities improved after
receiving strategy instruction that involved the concrete-representational-abstract sequence. The Addition with Regrouping Minute was administered during sessions 11 through 20 of the intervention condition. These data were used to determine the celeration rate of individual participants’ fluency performance. The celeration rate, which measures the extent or magnitude of learning over time, was calculated by (a) identifying two week’s (i.e., ten sessions) worth of Addition with Regrouping Minute scores for each participant, (b) identifying the median score for each week, and (c) dividing the larger median score by the smaller median score (Evans, Evans, & Mercer as cited in Miller, 2009). If the first week’s median score is less than the second week’s median score, then the participant’s fluency rate is increasing. Likewise, if the first week’s median score is larger than the second week’s median score, then the participant’s fluency rate is decreasing. All nine participants demonstrated increases in fluency as determined by individual celeration rates. Participant 1 demonstrated a celeration rate of 1.29 which translates into a weekly improvement of 4 digits correct per minute. See Figure 10 for a description of individual session digits correct and error digits for Participant 1. Participant 2 demonstrated a celeration rate of 1.19 which translates into a weekly improvement of 4 digits correct per minute. See Figure 11 for a description of individual session digits correct and error digits for Participant 2. Participant 3 demonstrated a celeration rate of 1.55 which translates into a weekly improvement of 6 digits correct per minute. See Figure 12 for a description of individual session digits correct and error digits for Participant 3. Participant 4 demonstrated a celeration rate of 1.12 which translates into a weekly improvement of 2 digits correct per minute. See Figure 13 for a description of individual session digits correct and error digits for
Participant 4. Participant 5 demonstrated a celeration rate of 1.40 which translates into a weekly improvement of 6 digits correct per minute. See Figure 14 for a description of individual session digits correct and error digits for Participant 5. Participant 6 demonstrated a celeration rate of 1.41 which translates into a weekly improvement of 3 digits correct per minute. See Figure 15 for a description of individual session digits correct and error digits for Participant 6. Participant 7 demonstrated a celeration rate of 4.5 which translates into a weekly improvement of 7 digits correct per minute. See Figure 16 for a description of individual session digits correct and error digits for Participant 7. Participant 8 demonstrated a celeration rate of 1.14 which translates into a weekly improvement of 1 digit correct per minute. See Figure 17 for a description of individual session digits correct and error digits for Participant 8. Participant 9
Participant 2: Addition with Regrouping Minute Celeration Rate

demonstrated a celeration rate of 1.50 which translates into a weekly improvement of 3
digits correct per minute. See Figure 18 for a description of individual session digits
correct and error digits for Participant 9.

As determined by the calculation of individual celebration rates based on the Addition
with Regrouping Minute and by the comparison of pre-, post-, and maintenance tests of
the Addition with Regrouping Minute, all nine participants demonstrated increases in
fluency rates during the intervention condition. Likewise, three participants (i.e.,
Participants 4, 7, and 8) demonstrated the ability to maintain posttest intervention fluency
rates seven days after the conclusion of the intervention condition. Two participants (i.e.,
Participants 5 and 6) demonstrated an increase in fluency rates seven days after the
conclusion of the intervention condition. Finally, while four participants (i.e., Participants
1, 2, 3, and 9) demonstrated decreases in fluency rates seven days after the conclusion of
the intervention treatment, their maintenance test fluency rates were still considerably higher than their pretest fluency rates.

Figure 12

**Participant 3: Addition with Regrouping Minute Celeration Rate**

![Graph for Participant 3]

Figure 13

**Participant 4: Addition with Regrouping Minute Celeration Rate**

![Graph for Participant 4]
Figure 14

*Participant 5: Addition with Regrouping Minute Celeration Rate*

Celeration Rate: 
\[ \frac{21}{15} = 1.40 \]

Digits Correct 
Median=15

Digits Correct 
Median=21

Figure 15

*Participant 6: Addition with Regrouping Minute Celeration Rate*

Celeration Rate: 
\[ \frac{10}{7} = 1.41 \]

Digits Correct 
Median=10

Digits Correct 
Median=7

Sessions
Figure 16

Participant 7: Addition with Regrouping Minute Celeration Rate

![Graph showing celeration rate and digits correct for Participant 7.]

Figure 17

Participant 8: Addition with Regrouping Minute Celeration Rate

![Graph showing celeration rate and digits correct for Participant 8.]

Celeration Rate:
9 ÷ 2 = 4.5

Digits Correct
Median = 2

Celeration Rate:
8 ÷ 7 = 1.14

Digits Correct
Median = 7
Research Question 4

Do students with learning disabilities maintain their ability to solve addition with regrouping computation and word problems after receiving strategy instruction that involves concrete-representational-abstract sequencing?

Two data sets (i.e., ongoing probes and posttests-maintenance tests) were used to determine whether the computation and word problem performance of students with learning disabilities was maintained after receiving strategy instruction that involved the use of the concrete-representational-abstract sequence. The first data set consisted of the Intervention Probes that were collected throughout the intervention condition and the Maintenance Probe (see Figures 1-3) that were administered one week after the conclusion of the intervention phase. The mean scores for each triad on these Intervention Probe were compared to their respective maintenance probe mean scores.
The Intervention Probe scores for the participants in triad one ranged from 50% to 100% (M= 92.74%, SD = 3.44) (see Figure 1). The Maintenance Probe scores for the participants in triad one ranged from 90% to 100% (M= 93.33%, SD = 5.77) (see Figure 1). Thus, the participants in triad one maintained and actually increased (+0.59%) their performance after one week of no intervention instruction. See Table 11 for a summary of individual participant probe scores for triad one. The Intervention Probe score for the participants in triad two ranged from 60% to 100% (M= 94.43%, SD = 8.47) (see Figure 2). The Maintenance Probe scores for the participants in triad two ranged from 90% to 100% (M= 96.67%, SD = 5.77) (see Figure 2). Thus, the participants in triad two maintained and actually increased (+2.24%) their performance after one week of no intervention instruction. See Table 12 for a summary of individual participant probe scores for triad two. The Intervention Probe score for the participants in triad three ranged from 50% to 100% (M= 91.29%, SD = 10.00) (see Figure 3). The Maintenance Probe scores for the participants in triad three ranged from 80% to 100% (M= 90.00%, SD = 10.00) (see Figure 3). Thus the participants in triad three demonstrated a slight

Table 11

**Triad One: Intervention and Maintenance Probe Scores**

<table>
<thead>
<tr>
<th>Participants</th>
<th>Intervention Probes M / SD</th>
<th>Maintenance Probe</th>
<th>Percentage Point Increase/Decrease from Intervention to Maintenance Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 1</td>
<td>89.52 / 12.03</td>
<td>100.00</td>
<td>+10.48</td>
</tr>
<tr>
<td>Participant 2</td>
<td>93.00 / 7.33</td>
<td>90.00</td>
<td>-3.00</td>
</tr>
<tr>
<td>Participant 3</td>
<td>95.50 / 7.59</td>
<td>90.00</td>
<td>-5.50</td>
</tr>
</tbody>
</table>
Table 12

*Triad Two: Intervention and Maintenance Probe Scores*

<table>
<thead>
<tr>
<th>Participants</th>
<th>Intervention Probes M / SD</th>
<th>Maintenance Probe</th>
<th>Percentage Point Increase/Decrease from Intervention to Maintenance Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 4</td>
<td>92.38 / 11.36</td>
<td>100.00</td>
<td>+7.62</td>
</tr>
<tr>
<td>Participant 5</td>
<td>96.00 / 5.98</td>
<td>100.00</td>
<td>+4.00</td>
</tr>
<tr>
<td>Participant 6</td>
<td>95.00 / 6.88</td>
<td>90.00</td>
<td>-5.00</td>
</tr>
</tbody>
</table>

Triad Three: Intervention and Maintenance Probe Scores

<table>
<thead>
<tr>
<th>Participants</th>
<th>Intervention Probes M / SD</th>
<th>Maintenance Probe</th>
<th>Percentage Point Increase/Decrease from Intervention to Maintenance Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 7</td>
<td>85.91 / 12.21</td>
<td>80.00</td>
<td>-5.91</td>
</tr>
<tr>
<td>Participant 8</td>
<td>91.50 / 7.45</td>
<td>90.00</td>
<td>-1.50</td>
</tr>
<tr>
<td>Participant 9</td>
<td>97.00 / 5.71</td>
<td>100.00</td>
<td>+3.00</td>
</tr>
</tbody>
</table>

Visual analysis of Figures 1, 2, and 3 reveals that all nine participants maintained their performance at mastery level (i.e., 80% or higher) one week after the completion of the intervention condition. A total of four participants (i.e., Participants 1, 4, 5 and 9) scored...
100%, four participants (i.e., Participants 2, 3, 6, and 8) scored 90%, and one participant (i.e., Participant 7) scored 80%.

The second data set used to determine whether the computation and word problem solving performance of students with learning disabilities was maintained after receiving strategy instruction that involved the use of the concrete-representational-abstract sequence consisted of the scores from the Addition with Regrouping and the Addition with Regrouping Word Problem Posttests which were administered at the conclusion of the intervention condition and the scores from the Addition with Regrouping and the Addition with Regrouping Word problem Maintenance Tests that were administered during the maintenance condition seven days later. All three participants in triad one scored 100% on both the Addition with Regrouping Post- and Maintenance Tests. This demonstrates maintenance of addition with regrouping ability for triad one. All three participants in triad one scored 100% on the Addition with Regrouping Word Problem Posttest. The scores for these three participants on the Addition with Regrouping Word Problem Maintenance Test ranged from 90% to 100% (M = 96.67%, SD = 5.77). This represents a mean percentage point decrease of 3.33 for triad one. See Table 14 for a summary of individual posttest and maintenance test scores for triad one. The Addition with Regrouping Posttest scores for the participants in triad two ranged from 95% to 100% (M = 98.33%, SD = 2.89). The Addition with Regrouping Maintenance Test scores for the participants of triad two range from 95% to 100% (M = 96.67%, SD = 2.89). This represents a mean percentage point decrease of 1.66 for triad two. The scores on the Addition with Regrouping Word Problem Posttest for the participants of triad two ranged from 90% to 100% (M = 93.33%, SD = 5.77). The scores for the participants in
Table 14

**Triad One: Addition with Regrouping and Addition with Regrouping Word Problem**

*Post- and Maintenance Test Score*

<table>
<thead>
<tr>
<th>Participants</th>
<th>Posttests</th>
<th>Maintenance Tests</th>
<th>Percentage Point Increase/Decrease from Posttest to Maintenance Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Addition with Regrouping / Addition with Regrouping Word Problem</td>
<td>Addition with Regrouping / Addition with Regrouping Word Problem</td>
<td></td>
</tr>
<tr>
<td>Participant 1</td>
<td>100 / 100</td>
<td>100 / 90</td>
<td>0 / -10</td>
</tr>
<tr>
<td>Participant 2</td>
<td>100 / 100</td>
<td>100 / 100</td>
<td>0 / 0</td>
</tr>
<tr>
<td>Participant 3</td>
<td>100 / 100</td>
<td>100 / 100</td>
<td>0 / 0</td>
</tr>
</tbody>
</table>

Triad two on the Addition with Regrouping Word Problem Maintenance Test ranged from 90% to 100% (M = 96.67%, SD = 5.77). This represents a mean percentage point increase of 3.33 for triad one. See Table 15 for a summary of individual posttest and maintenance test scores for triad two. The Addition with Regrouping Posttest scores for the participants in triad three ranged from 90% to 95% (M = 91.67%, SD = 2.89). The Addition with Regrouping Maintenance Test scores for the participants of triad three range from 80% to 90% (M = 86.67%, SD = 5.77). This represents a mean percentage point decrease of 5.00 for triad three. The scores on the Addition with Regrouping Word Problem Posttest for the participants of triad three ranged from 90% to 100% (M = 96.67%, SD = 5.77). The scores for the participants in triad three on the Addition with Regrouping Word Problem Maintenance Test ranged from 90% to 100% (M = 96.67%, SD = 5.77). This demonstrates maintenance of addition with regrouping ability for triad three. See Table 16 for a summary of individual posttest and maintenance test scores for...
triad three.

Table 15

*Triad Two: Addition with Regrouping and Addition with Regrouping Word Problem*

*Post- and Maintenance Test Score*

<table>
<thead>
<tr>
<th>Participants</th>
<th>Posttests</th>
<th>Maintenance Tests</th>
<th>Percentage Point Increase/Decrease from Posttest to Maintenance Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Addition with Regrouping / Addition with Regrouping Word Problem</td>
<td>Addition with Regrouping / Addition with Regrouping Word Problem</td>
<td></td>
</tr>
<tr>
<td>Participant 4</td>
<td>100 / 90</td>
<td>100 / 100</td>
<td>0 / +10</td>
</tr>
<tr>
<td>Participant 5</td>
<td>100 / 100</td>
<td>95 / 100</td>
<td>-5 / 0</td>
</tr>
<tr>
<td>Participant 6</td>
<td>95 / 90</td>
<td>95 / 90</td>
<td>0 / 0</td>
</tr>
</tbody>
</table>

Table 16

*Triad Three: Addition with Regrouping and Addition with Regrouping Word Problem*

*Post- and Maintenance Test Score*

<table>
<thead>
<tr>
<th>Participants</th>
<th>Posttests</th>
<th>Maintenance Test</th>
<th>Percentage Point Increase/Decrease from Posttest to Maintenance Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Addition with Regrouping / Addition with Regrouping Word Problem</td>
<td>Addition with Regrouping / Addition with Regrouping Word Problem</td>
<td></td>
</tr>
<tr>
<td>Participant 7</td>
<td>90 / 90</td>
<td>80 / 90</td>
<td>-10 / 0</td>
</tr>
<tr>
<td>Participant 8</td>
<td>95 / 100</td>
<td>90 / 100</td>
<td>-5 / 0</td>
</tr>
<tr>
<td>Participant 9</td>
<td>90 / 100</td>
<td>90 / 100</td>
<td>0 / 0</td>
</tr>
</tbody>
</table>

118
Research Question 5

Do students with learning disabilities generalize their ability to solve addition with regrouping computation and word problems after receiving strategy instruction that involves concrete-representational-abstract sequencing?

One data set (i.e., ongoing probes) was used to determine whether the computation and word problem solving performance of students with learning disabilities was generalized after receiving strategy instruction that involved the use of the concrete-representational-abstract sequence. The data set consisted of the Intervention Probes that were collected throughout the intervention condition and the Generalization Probe that was administered two weeks after the conclusion of the intervention phase. The mean scores for each triad on the Intervention Probes were compared to their respective Generalization Probe mean scores. The Intervention Probes score for the participants in triad one ranged from 50% to 100% (M = 92.74%, SD = 3.44) (see Figure 1). All three participants in triad one scored 90% on the Generalization Probe. Thus, the participants in triad one demonstrated a 2.74% decline in performance from the intervention condition to the generalization condition. See Table 17 for a summary of individual intervention mean scores and generalization probes scores for triad one. The Intervention Probe scores for the participants in triad two ranged from 60% to 100% (M = 94.43%, SD = 8.47) (see Figure 2). The Generalization Probe scores for the participants in triad two ranged from 90% to 100% (M = 93.33%, SD = 5.77) (see Figure 2). Thus, the participants in triad two demonstrated a 1.10% decline in performance from intervention condition to generalization condition. See Table 18 for a summary of individual
Table 17

**Triad One: Intervention and Generalization Probe Scores**

<table>
<thead>
<tr>
<th>Participants</th>
<th>Intervention Probes M / SD</th>
<th>Generalization Probe</th>
<th>Percentage Point Increase/Decrease from Intervention to Generalization Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 1</td>
<td>89.52 / 12.03</td>
<td>90.00</td>
<td>+0.48</td>
</tr>
<tr>
<td>Participant 2</td>
<td>93.00 / 7.33</td>
<td>90.00</td>
<td>-3.00</td>
</tr>
<tr>
<td>Participant 3</td>
<td>95.50 / 7.59</td>
<td>90.00</td>
<td>-5.50</td>
</tr>
</tbody>
</table>

Table 18

**Triad Two: Intervention and Generalization Probe Scores**

<table>
<thead>
<tr>
<th>Participants</th>
<th>Intervention Probes M / SD</th>
<th>Generalization Probe</th>
<th>Percentage Point Increase/Decrease from Intervention to Generalization Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 4</td>
<td>92.38 / 11.36</td>
<td>90.00</td>
<td>-2.36</td>
</tr>
<tr>
<td>Participant 5</td>
<td>96.00 / 5.98</td>
<td>100.00</td>
<td>+4.00</td>
</tr>
<tr>
<td>Participant 6</td>
<td>95.00 / 6.88</td>
<td>90.00</td>
<td>-5.00</td>
</tr>
</tbody>
</table>

intervention mean scores and generalization probe scores for triad two. The Intervention Probe score for the participants in triad three ranged from 50% to 100% (M= 91.29%, SD = 10.00) (see Figure 3). The Generalization Probe scores for the participants in triad three ranged from 80% to 100% (M= 90.00%, SD = 10.00) (see Figure 3). Thus, the participants in triad three demonstrated a 1.29% decline in performance from intervention condition to generalization condition. See Table 19 for a summary of individual
intervention mean scores and maintenance probe scores triad three. Overall, six participants demonstrated slight declines (i.e., -1.50% to -5.91%) in performance from the intervention condition to

Table 19

*Triad Three: Intervention and Generalization Probe Scores*

<table>
<thead>
<tr>
<th>Participants</th>
<th>Intervention Probes M / SD</th>
<th>Generalization Probe</th>
<th>Percentage Point Increase/Decrease from Intervention to Generalization Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 7</td>
<td>85.91 / 12.21</td>
<td>80.00</td>
<td>-5.91</td>
</tr>
<tr>
<td>Participant 8</td>
<td>91.50 / 7.45</td>
<td>90.00</td>
<td>-1.5</td>
</tr>
<tr>
<td>Participant 9</td>
<td>97.00 / 5.71</td>
<td>100.00</td>
<td>+3.00</td>
</tr>
</tbody>
</table>

the generalization condition which translates into less than one problem difference.

Three participants demonstrated slight increases (i.e., 0.48% to 4.00%) in performance from intervention condition to generalization condition which translates into less than one problem difference.

**Research Question 6**

Do students with learning disabilities report high levels of satisfaction related to strategy instruction that involves concrete-representational-abstract sequencing for learning addition with regrouping skills? The Addition with Regrouping Satisfaction Questionnaire was administered to each participant immediately after completing the intervention condition. The purpose of the Addition with Regrouping Satisfaction Questionnaire was to assess the satisfaction levels of each participant with regard to the
instruction they received. The eight statement Addition with Regrouping Satisfaction Questionnaire was designed using a four-point Likert scale: (a) circling the numeral 1 indicated that the participant strongly disagreed with the statement, (b) circling the numeral 2 indicated that the participant disagreed with the statement, (c) circling the numeral 3 indicated that the participant agreed with the statement, and (d) circling the numeral 4 indicated that the participant strongly agree with the statement.

On statement 1 (i.e., The base ten blocks helped me with addition), Participant 1 stated that he agreed, while the other eight participants stated that they strongly agreed with statement 1. Participant 5 stated that he agreed with statement 2 (i.e., Drawings helped me with addition), while the other eight participants stated that they strongly agreed with statement 2. For statements 3 though 8, all participants stated that they strongly agreed. See Table 2 for a summary of each participants’ responses. Overall, the nine participants indicated a high level of satisfaction with the strategy instruction that included the concrete-representational-abstract sequence (i.e., M = 99.5%; SD 0.05). See Table 21 for a summary of response frequencies and the mean scores for each statement.

**Interscorer Reliability**

The student investigator scored each participant’s pre-, post-, and maintenance tests (i.e., the Addition with Regrouping Pre-, Post-, and Maintenance Tests, the Addition with Regrouping Minute, the Addition with Regrouping Word Problem Pre-, Post-, and Maintenance Tests). Likewise, the student investigator scored the Conceptual Understanding Pre- and Posttests. Similarly, the student investigator scored all of the Baseline Probes, the Intervention Probes, the Maintenance Probes, and the
### Table 20

**Satisfaction Rating for Students with Learning Disabilities**

<table>
<thead>
<tr>
<th>Statements</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The base ten blocks helped me with addition.</td>
<td>P1</td>
<td>P2-P9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Drawings helped me with addition.</td>
<td></td>
<td>P1-P4 &amp; P5-P6-P9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. The REMANE strategy helped me with addition.</td>
<td></td>
<td>P1-P9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. The Addition Minute helped me get faster at addition.</td>
<td></td>
<td>P1-P9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. The FAST RENAME strategy helped me with word problems.</td>
<td></td>
<td>P1-P9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. The PIG Game helped me with addition.</td>
<td></td>
<td>P1-P9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. This program helped me with addition.</td>
<td></td>
<td>P1-P9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Overall, I like this Addition Program.</td>
<td></td>
<td>P1-P9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note: P1 = Participant 1; P2 = Participant 2; P3 = Participant 3; P4 = Participant 5; P6 = Participant 6; P7 = Participant 7; P8 = Participant 8; P9 = Participant 9

Generalization Probes. The research assistant scored 20% of each separate measure to determine interscorer reliability. The instruments were randomly selected across types. The primary scorer was the student investigator and the secondary scorer was the research assistant. When both the student investigator and the research assistant recorded the same score for an answer, an agreement was counted. Reliability levels were determined using the formula agreements ÷ (agreements + disagreements) x 100 (Barlow & Hersen, 1984). The percentage of agreement for Addition with Regrouping Pre-, Post-, and Maintenance Tests; the Addition with Regrouping Minute; the Addition with Regrouping Word Problem Pre-, Post-, and Maintenance Tests; and the Maintenance
Probe; Conceptual Understanding Pre- and Posttests; the Baseline Probes; and the Generalization Probes was 100%. The percentage of agreement for Intervention Probes was 99.75%, having identified one disagreement out of 400 Intervention Probe questions.

Table 22 provides the data from the interscorer reliability checks.

Table 21

*Addition with Regrouping Satisfaction Questionnaire Frequency Count and Mean Scores*

<table>
<thead>
<tr>
<th>Statements</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The base ten blocks helped me with addition.</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>8</td>
<td>3.89</td>
</tr>
<tr>
<td>2. Drawings helped me with addition.</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>8</td>
<td>3.89</td>
</tr>
<tr>
<td>3. The REMANE strategy helped me with addition.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9</td>
<td>4.0</td>
</tr>
<tr>
<td>4. The Addition Minute helped me get faster at addition.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9</td>
<td>4.0</td>
</tr>
<tr>
<td>5. The FAST RENAME strategy helped me with word problems.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9</td>
<td>4.0</td>
</tr>
<tr>
<td>6. The PIG Game helped me with addition.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9</td>
<td>4.0</td>
</tr>
<tr>
<td>7. This program helped me with addition.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9</td>
<td>4.0</td>
</tr>
<tr>
<td>8. Overall, I like this Addition Program.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9</td>
<td>4.0</td>
</tr>
</tbody>
</table>

*Note: 1 = Strongly Disagree; 2 = Disagree; 3 = Agree; 4 = Strongly Agree*

**Fidelity of Treatment**

The student investigator used a digital video camera to record each lesson. To determine interobserver agreement related to fidelity of treatment, the research assistant observed 100% of all recorded lessons and the principal investigator observed 25% of randomly selected lessons. Both the research assistant and the primary investigator completed fidelity of treatment checklists (see Appendix DD) for each lesson viewed.
Table 22

*Interscorer Reliability*

<table>
<thead>
<tr>
<th>Measures</th>
<th>Total Agreements</th>
<th>Total Agreements + Disagreements</th>
<th>Percentage of Agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition with</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regrouping Pretest</td>
<td>40</td>
<td>40</td>
<td>100</td>
</tr>
<tr>
<td>Addition with</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regrouping Word Problem Pretest</td>
<td>20</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>Addition with</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regrouping Minute Problem Pretest</td>
<td>72</td>
<td>72</td>
<td>100</td>
</tr>
<tr>
<td>Conceptual Understanding Pretest</td>
<td>68</td>
<td>68</td>
<td>100</td>
</tr>
<tr>
<td>Baseline Probes</td>
<td>90</td>
<td>90</td>
<td>100</td>
</tr>
<tr>
<td>Intervention Probes</td>
<td>399</td>
<td>400</td>
<td>99.75</td>
</tr>
<tr>
<td>Addition with</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regrouping Posttest</td>
<td>40</td>
<td>40</td>
<td>100</td>
</tr>
<tr>
<td>Addition with</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regrouping Word Problem Posttest</td>
<td>40</td>
<td>40</td>
<td>100</td>
</tr>
<tr>
<td>Conceptual Understanding Posttest</td>
<td>68</td>
<td>68</td>
<td>100</td>
</tr>
<tr>
<td>Addition with</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regrouping Maintenance Test</td>
<td>20</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>Addition with</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regrouping Word Problem Maintenance Test</td>
<td>20</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>Maintenance Probe</td>
<td>20</td>
<td>20</td>
<td>100</td>
</tr>
</tbody>
</table>
To determine interobserver agreement, the formula agreements ÷ (agreements + disagreements) x 100 was used (Barlow & Hersen, 1984). The percent of agreement related to the fidelity of treatment was 100% (see Table 23).

Table 23

<table>
<thead>
<tr>
<th>Fidelity of Treatment</th>
<th>Measure</th>
<th>Total Agreements</th>
<th>Total Agreements + Disagreements</th>
<th>Percentage of Agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fidelity of Treatment</td>
<td>75</td>
<td>75</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

**Summary of Findings**

All nine participants increased their ability to solve addition with regrouping computation and word problems after receiving strategy instruction that involved the use of the concrete-representational-abstract sequence. This improvement was evident through both ongoing probes (i.e., Intervention Probes) and through the pre- and posttest measures (i.e., Addition with Regrouping Pre- and Posttests; Addition with Regrouping Word Problem Pre- and Posttests). Likewise, all nine participants increased their conceptual understating of addition with regrouping from pre- to posttest after receiving strategy instruction that involved the use of the concrete-representational-abstract sequence. This improvement was evident through the Conceptual Understanding Pre- and Posttests.
All nine participants increased their addition with regrouping problem-solving fluency rate from pre- to posttest after receiving strategy instruction that involved the use of the concrete-representational-abstract sequence. This improvement was evident through the Addition with Regrouping Minutes. Additionally, three participants maintained their posttest intervention fluency rates seven days after the conclusion of the intervention condition, while two participants demonstrated an increase in fluency rates seven days after the conclusion of the intervention condition. Four participants demonstrated declines in fluency rates seven days after the conclusion of the intervention condition; however, their maintenance measure fluency rates were still considerably higher than their pretest measure rates.

As evident through the Addition with Regrouping Posttest and Maintenance Test, four participants maintained their ability to solve addition with regrouping computation problems seven days after the posttest, whereas three participants showed a one problem increase from posttest to maintenance. As evident through the Addition with Regrouping Word Problem Posttest and Maintenance Test, seven participants maintained their ability to solve addition with regrouping word problems seven days after the posttest. One participant demonstrated slight declines (i.e., losses of one problem) in his ability to solve addition with regrouping word problems after seven days. However, one participant demonstrated a slight increase in his ability to solve addition with regrouping word problems (i.e., one problem) after seven days.

Three participants demonstrated the ability to generalize their addition with regrouping computation and word problem solving ability into a new setting after receiving strategy instruction that involved the use of the concrete-representational-
abstract sequence by increasing their posttest scores fourteen days after the conclusion of the intervention condition. The remaining six participants demonstrated the ability to generalize their addition with regrouping computation and word problem solving skills by showing only slight declines (i.e., less than one problem) after fourteen days.

Finally, the data from the Addition with Regrouping Satisfaction Questionnaire indicated that all nine of the participants’ responses for all eight statements were either Agree or Strongly Agree. This suggests that the use of strategy instruction that involves the use of the concrete-representational-abstract sequence is a socially valid method of teaching addition with regrouping skills to individuals with learning disabilities.
CHAPTER 5
SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

The purpose and process of mathematics education has been heavily debated over the last 100 years. Curriculum, a common theme, has remained near the center of the debate. This debate has repeatedly resonated with key professionals in the field of mathematics and has ultimately manifested itself in political reform. A number of mathematics-related agendas (i.e., progressive curriculum, New Math, NCTM standards, Common Core Standards) have influenced how mathematics is taught and learned. The current calls for reform seem to remain steadfast in adopting, revising, and condensing national mathematics standards (NCTM, 1989, 2000, 2006). As the mathematics debates continue, it seems that a definitive answer related to the most effective mathematics curriculum still remains in the distant future. Despite the debate and unsettled nature of mathematics instruction, one easily agreed upon issue is that basic computation must be addressed effectively. Unfortunately, students in the United States continue to perform below acceptable expectations (NAEP, 2007; National Mathematics Advisory Panel, 2008). Students with learning disabilities are of particular concern. The last decade has seen a drastic increase of students with learning disabilities in mathematics and simultaneous agreement regarding the importance of basic computation skills for this population of students (Bryant, Bryant, Kethley, et al., 2008). Perhaps this is why there has been a primary focus within mathematics literature related to basic math fact instruction for students with disabilities (Garnett, 1992; Gersten et al., 2008; Miller et al., 1998; Montague, 2008; Montague & Brooks, 1993). While there is a solid base of literature related to basic math facts, there is limited research on other basic computation
skills, such as multi-digit addition, that is equally important in terms of further progress to higher order mathematics skills.

The current study was designed to investigate the effect of concrete-representational-abstract sequencing within strategy instruction while teaching addition with regrouping to students with learning disabilities. This chapter includes (a) a sequential discussion related to the results associated with the six research questions, (b) a list of conclusions based on the results obtained, (c) a discussion of practical implications obtained from the current research, and (d) a list of recommendations for future research.

**Discussion of Results**

The purpose of this study was to investigate the effects of concrete-representational-abstract sequencing within strategy instruction while teaching addition with regrouping to students with learning disabilities. A sequential discussion of results related to each research question is provided.

**Research Question 1**

Do students with learning disabilities improve their ability to solve addition with regrouping computation and word problems after receiving strategy instruction that involves concrete-representational-abstract sequencing?

There were two data sets (i.e., ongoing probes and pre-posttests) used to determine whether the computation and word problem performance of students with learning disabilities improved after receiving strategy instruction that involved the concrete-representational-abstract sequence. Both data sets revealed that all nine participants improved their ability to solve addition with regrouping computation and word problems after receiving strategy instruction that involved the concrete-representational-abstract
sequence. Additionally, all nine participants reached mastery on each intervention lesson. It is interesting to note that even the participants with very low baseline and pretest scores were able to meet mastery criteria after receiving the intervention.

Of the nine participants completing 20 intervention lessons each, only three participants (i.e., Participants 1, 4, and 9) had to repeat intervention lessons due to not meeting criteria (i.e., 80% or higher on the Intervention Probe). Participants 1 and 4 made errors on Lessons 2 related to improperly manipulating the base ten blocks. This was the first lesson with base ten blocks that required the participant to physically move blocks from one place value column to another. Based on the difficulties Participant 1 and Participant 4 displayed related to this new concept, it appeared that both participants needed additional instruction and practice in this area to be successful. Thus, Lesson 2 was repeated and both participants achieved the required mastery score. Participant 7 also had difficulties with Lesson 2, but his errors resulted from difficulty related to one-to-one correspondence while counting the base ten blocks within the ones column. This resulted in inaccurate sums for problems in Lesson 2. It appeared that additional practice with this skill (i.e., repeating the lesson) improved his ability to accurately count the manipulative devices because he was able to reach lesson mastery on the second attempt. Participant 4 also had difficulty with Lesson 10. She made errors in adding the crutch numbers when solving the problems in this lesson. This was the first lesson that required participants to solve addition with regrouping problems without the aid of a manipulative device or a drawing. Participant 4 reached lesson mastery when the lesson was repeated. Thus, when teaching individuals with learning disabilities to solve addition with regrouping computation and word problems, it appears that additional time, instruction,
and practice may be necessary for some students to reach mastery. According to the findings in this study, the need for additional instruction and practice may emerge when students are first learning to use manipulative devices to represent regrouping from ones to tens, when students have difficulty with prerequisite skills such as one-to-one correspondence, or when students are transitioning from representational to abstract level of instruction in the CRA sequence.

The findings related to research question 1 concur with previous research (i.e., Harris, Miller, & Mercer, 1995; Manccini & Hughes, 2000; Witzel, Mercer, & Miller, 2003) related to the use of the CRA sequence when teaching students with learning disabilities. These previous studies revealed that students with learning disabilities are able to acquire multiplication, and initial algebra skills using the CRA sequence. The findings of the current study also concur with previous research (i.e., Jitendra, Griffin, McGoey, Gardill, Bhar, & Riley, 1998; Jitendra, Hoff, & Beck, 1999; Jitendra & Hoss, 1996; Van Garderen, 2007) related to the use of diagram-related strategy instruction while working with students with learning disabilities. It is important to note, however, that in these previous studies, diagrams were used to help students in solving word problems, whereas, in this current study diagrams were used to help students with both computation and word problems. Additionally, the findings of this study also concur with previous research on cognitive strategies (i.e., Case, Harris, & Graham, 1992; Montague, 1992; Montague, Applegate, & Marquard, 1993). However, this study extends previous research by combining strategy instruction with the CRA sequence to teach addition with regrouping to students with learning disabilities; whereas previous research was limited to
investigating the use of strategy instruction with the CRA sequence to teach basic math fact skills and algebra skills.

**Research Question 2**

Do students with learning disabilities improve their conceptual understanding related to addition with regrouping after receiving strategy instruction that involves concrete-representational-abstract sequencing?

One data set, the Conceptual Understanding Pretest and Posttest, was used to determine whether the conceptual understanding of students with learning disabilities improved after receiving strategy instruction that involved the concrete-representational-abstract sequence. All nine participants demonstrated an increase in their conceptual understanding from pre- to posttest. It is interesting to note that most of the errors participants made on the posttest were errors in verbally expressing how they were solving problems, not in inability to actually solve the problems themselves. It is quite typical for students with learning disabilities to also have deficits in language (e.g., expressive language). Thus, errors related to verbally explaining how they were solving the problem were not surprising. In spite of these errors, participants’ improvement related to explaining why they were trading ones for a ten was impressive. Clearly, the addition with regrouping lessons helped the students learn the steps needed to solve the problems correctly, but also helped them develop conceptual understanding related to why the steps were necessary. This type of information is particularly important as students progress to more advanced mathematics skills.

The findings related to research question 2 extends the previous research (i.e., Harris, Miller, & Mercer, 1995; Manccini & Hughes, 2000; Butler, Miller, Crehan, Babbitt, &...
Pierce, 2003) related to strategy instruction that involves the use of the CRA sequence through the direct measurement of conceptual understanding. Previous research in this area measured skill acquisition without assessing conceptual understanding through student talk-alouds. This study extends previous research by applying evidence-based practices (i.e., CRA and strategy instruction) to a new skill area (i.e., addition with regrouping) and provides a foundation for a new area of mathematics research (i.e., direct measurement of conceptual understanding).

**Research Question 3**

Do students with learning disabilities increase their fluency related to addition with regrouping after receiving strategy instruction that involves concrete-representational-abstract sequencing?

One data set (i.e., Addition with Regrouping Minute) were used to determine whether the addition with regrouping fluency of students with learning disabilities improved after receiving strategy instruction that involved the concrete-representational-abstract sequence. The data related to research question 3 revealed that all nine participants demonstrated increases in fluency rates during the intervention condition. Likewise, three participants (i.e., Participants 4, 7, and 8) demonstrated the ability to maintain posttest intervention fluency rates seven days after the conclusion of the intervention condition. Two participants (i.e., Participants 5 and 6) demonstrated an increase in fluency rates seven days after the conclusion of the intervention condition. Additionally, while four participants (i.e., Participants 1, 2, 3, and 9) demonstrated declines in fluency rates seven days after the conclusion of the intervention treatment, their maintenance fluency rates were still considerably higher than their pretest fluency rates.
The findings of research question 3 concur with previous researchers (i.e., Harris, Miller, & Mercer, 1995; Flores, 2009) who found that the use of strategy instruction that involves the use of the CRA sequence increases students’ with learning disabilities fluency in computation. The current study extends the literature in that it examined fluency rates related to addition with regrouping which had not been previously examined.

**Research Question 4**

Do students with learning disabilities maintain their ability to solve addition with regrouping computation and word problems after receiving strategy instruction that involves concrete-representational-abstract sequencing?

Two data sets (i.e., ongoing probes and posttests-maintenance tests) were used to determine whether the computation and word problem solving performance of students with learning disabilities was maintained after receiving strategy instruction that involved the use of the concrete-representational-abstract sequence. Four participants (i.e., Participants 1, 4, 5 and 9) increased in maintenance probe score compared to their mean Intervention Probe score. It is interesting to note that these same four participants received that same maintenance probe score (i.e., 100%) as they received on their last Intervention Probe. With regard to the pre-posttest data set, the same four participants also increased or maintained their posttest scores seven days later. Three participants (i.e., Participants 2, 3, and 6) scored exactly the same at maintenance as their mean Intervention Probe score. These same participants did, however, show slight declines based on the posttest to maintenance measure (i.e., each showed less than a one problem decline). Two participants (i.e., Participants 7 and 8) showed slight declines from the
mean Intervention Probe to maintenance and declines from posttest to maintenance (i.e., showed less than a one problem increase). Participant 7 showed the largest decline during the maintenance condition (i.e., he decreased by 5.91% from mean Intervention Probe to maintenance and 10% from posttest to maintenance). It is interesting to note that he began the intervention condition working much slower than the other participants, but began to increase his speed and accuracy during the middle of the intervention condition. He finished the intervention condition strong (i.e. he scored 100% on three of the final five Intervention Probes). However, during the maintenance condition, he appeared distracted and inattentive (i.e., he spent several minutes staring at a wall even after the student investigator attempted to verbally redirect his attention; he took two 45 minute sessions to complete the maintenance condition as compared to only one 45 minute session to complete the posttesting; he began answering one problem, became distracted, and then after redirection began answering a different problem). It was revealed that he had undergone a change in medication during the seven days between the end of the intervention condition and the maintenance condition. It is possible that this change in medication and the possible associated behaviors of being distracted and inattentive may have impacted his performance ability during the maintenance condition.

The findings related to research question 4 concur with previous research (i.e., Maccini & Hughes, 2000; Witzel, Mercer, & Miller, 2003; Scheuermann, Deshler, & Schumaker, 2009; Flores, 2009) who found that strategy instruction that involved the use of the CRA sequence could promote maintenance of a variety of mathematical skills. Again, the current study extends the maintenance literature related to solving mathematical problems to a new skill area (i.e., addition with regrouping problems).
Research Question 5

Do students with learning disabilities generalize their ability to solve addition with regrouping computation and word problems after receiving strategy instruction that involves concrete-representational-abstract sequencing?

One data set (i.e., ongoing probes) was used to determine whether the computation and word problem solving performance of students with learning disabilities was generalized after receiving strategy instruction that involved the use of the concrete-representational-abstract sequence. The data revealed that all nine participants scored at mastery level on the Generalization Probe. Six participants demonstrated slight declines in performance from intervention condition to generalization condition that translated into less than one problem difference. Three participants demonstrated slight increases in performance from intervention condition to generalization condition that translated into less than one problem difference. It is interesting to note that two participants (i.e., Participants 2 and 6) actually scored higher on the Generalization Probe than they did on the Maintenance Probe. Additionally, six participants (i.e., Participants 3, 4, 5, 7, 8, and 9) scored the same on both the Generalization and Maintenance Probe. Finally, only Participant 1 scored lower on the Generalization Probe than he scored on the Maintenance Probe.

The findings related to research question 5 concur with previous researchers (i.e., Jitendra at el., 1998; Jitendra, Hoff, & Beck, 1999; Xin, Jitendra, & Deatline-Backman, 2005) who indicated positive results in student ability related to the ability to generalize mathematical problem-solving skills with the use of diagrams. Likewise, the findings concur with researchers (i.e., Maccini & Hughes, 2000; Schuermann, Deshler, &
Schumaker, 2009) who indicated that students who receive strategy instruction combined with CRA sequence experience generalization success related to their ability to solve mathematical problems. The current study extends the literature related to solving mathematical problems to a new skill area (i.e., addition with regrouping problems).

**Research Question 6**

Do students with learning disabilities report high levels of satisfaction related to strategy instruction that involves concrete-representational-abstract sequencing for learning addition with regrouping skills?

The Addition with Regrouping Satisfaction Questionnaire was used to assess the satisfaction levels of each participant with regard to the instruction they received. The data revealed that all nine participants indicated a high level of satisfaction with the strategy instruction that included the CRA sequence that they received. It is interesting to note that after completing the Addition with Regrouping Satisfaction Questionnaire, two participants (i.e., Participants 2 and 6) separately asked the student investigator what math skill they were going to learn with her next. When they were told that the student investigator would not be returning, they both asked if she could talk to their teacher about teaching the same way that she did. Participant 2 said that she thought she could learn anything related to math if the teachers would break things down like the instruction she had just completed. Additionally, Participants 4 and 9 indicated they wished they had other tricks (i.e., mnemonic devices) to help them with other school-related tasks.

The findings related to research question 6 concur with previous researchers (i.e., Jitendra & Hoff, 1996) who reported students who received instruction that involved the use of diagrams for teaching mathematical skills reported high levels of satisfaction and
enjoyment. Likewise, the findings concur with Maccini and Ruhl (2000) that reported students who received instruction that included the CRA sequence indicated high levels of satisfaction with the instructional components.

Conclusions

Based on the results obtained in this research, the investigator’s conclusions include:

1. Strategy instruction that involves the use of the concrete-representational-abstract sequence improves the addition with regrouping computation and word problem-solving ability of students with learning disabilities.

2. Students with learning disabilities who receive strategy instruction that involves the use of the concrete-representational-abstract sequence demonstrate increases in conceptual understanding related to addition with regrouping.

3. Students with learning disabilities who receive strategy instruction that involves the use of the concrete-representational-abstract sequence increase the rate with which they solve addition with regrouping computation problems.

4. Some students with learning disabilities who receive strategy instruction that involves the use of the concrete-representational-abstract sequence need additional review and practice to maintain their ability to solve addition with regrouping computation and word problems.

5. Students with learning disabilities who receive strategy instruction that involves the use of the concrete-representational-abstract sequence are able to generalize their ability while solving addition with regrouping computation and word problems in a different setting with a different teacher.
6. Students with learning disabilities who receive strategy instruction that involves the use of the concrete-representational-abstract sequence report high levels of satisfaction with the strategy instruction and the components of the concrete-representational-abstract sequence.

**Practical Implications**

Several practical implications emerged from this study. First, the student investigator noted that the time allotment necessary for the concrete phase of the CRA sequence is about 45 minutes per session. Allowing participants the time necessary to construct their conceptual understanding though the use of manipulative devices takes more time than working within the representational and abstract phases. Clearly, there is additional management involved in lessons that require the use of manipulative devices (i.e., organizing materials, distributing materials, establishing rules for working with manipulative devices). While the student investigator noted that the concrete lessons took longer than the other lessons, it was dually noted that the time spent constructing conceptual understanding was invaluable and the process for solving addition with regrouping problems became easier and more efficient over time due to participants mastering the important concepts in earlier lessons.

A second practical implication that emerged from this study relates to the success of systematic instructional approaches when teaching students with learning disabilities addition with regrouping. The use of the CRA sequence that includes explicit instruction components (i.e., advanced organizer, modeling, guided practice, independent practice, problem-solving) established an organized and productive learning environment. Additionally, the participants seemed to enjoy the consistency of each lesson’s format
and noticed when the format changed. Lessons 19 and 20 had no modeling or guided practice. The structure within each lesson seemed to set participants up for success with the extensive teacher support at the beginning of each lesson that was gradually withdrawn as the lesson progressed. While the participants in this study had all experienced previous failure with addition with regrouping, there were no serious behavior issues during lesson implementation. Overall, participants seemed to enjoy the lessons and their success with the lesson’s content.

**Recommendations for Further Study**

Recommendations for further study emerged from the results obtained in this study. Included among these recommendations are the following:

1. Research should be conducted to compare strategy instruction that involves the use of the CRA sequence to teach addition with regrouping with another addition intervention. It may be that another addition intervention is even more effective in teaching addition with regrouping to students with learning disabilities.

2. As this study was conducted in a small group setting outside of the general education classroom, more research should be conducted to investigate the effects of strategy instruction that involves the use of the CRA sequence within a general education setting. Changes to the current teaching sequence may be needed to accommodate for the larger instructional groups (e.g., utilizing differentiated instruction and flexible grouping).

3. Research should be conducted to investigate the use of strategy instruction that involves the CRA sequence being taught by the students’ general education teachers. The lessons in the current study were presented by a student investigator with prior
experience in implementing the CRA sequence. The outcomes may be different if instruction is provided from a teacher without this precious experience.

4. Research should be done to investigate how long treatment effects of strategy instruction that involves the CRA sequence are maintained. Administering additional maintenance measures (i.e., four weeks, eight weeks, twelve weeks after posttest) may provide new information and assist in determining possible changes in the strategy instruction to increase maintenance skill levels.

5. Research should be conducted to investigate the effects of strategy instruction that involves the CRA sequence with different populations (i.e., students with attention-deficit/hyperactivity disorder, intellectual disabilities, or gifts and talents; or students without disabilities but, at-risk for school failure). The required number of lessons at each level may need to be different when teaching strategy instruction that involves the CRA sequence to diverse student populations.
APPENDIX A

PERMISSION LETTER FOR USE OF COPYRIGHTED MATERIAL

Christi J. Carmack
6584 McKinley Summit Court
Las Vegas, NV 89110

May 21, 2011

Dr. Ceci Mercer
6607 NW 50th Lane
Gainesville, FL 32653

Dear Dr. Mercer:

I am completing a doctoral dissertation at the University of Nevada Las Vegas entitled *Investigating the Effects of Addition with Regrouping Strategy Instruction Among Elementary Students with Learning Disabilities*. I would like your permission to reprint excerpts for the Strategic Math Series: Addition with Regrouping (Miller, Kaffar, & Mercer, 2011) within my dissertation. The excerpts to be reproduced include: (a) Lesson 4 Teacher Instructions, (b) Learning Sheet 4, (c) Addition with Regrouping Pre- and Posttest, (d) Addition with Regrouping Minute, and (e) Addition Review Minute.

The requested permission extends to any future revisions and editions of my dissertation, including non-exclusive world rights in all languages, and to the prospective publication of my dissertation by ProQuest through its UMI® Dissertation Publishing business. ProQuest may produce and sell copies of my dissertation on demand and may make my dissertation available for free Internet download at my request. These rights will in no way restrict republication of the material in any other form by you or by others authorized by you. Your signing of this lesson will also confirm that you own the copyright to the above described material.

If these arrangements meet with your approval, please sign this letter where indicated below and return it to me in the enclosed return envelope. Thank you for your assistance.

Sincerely,

Christi J. Carmack
Doctoral Candidate
University of Nevada, Las Vegas

PERMISSION GRANTED FOR THE USE REQUESTED ABOVE:

[Signature: Dr. Ceci Mercer]  5-27-11
APPENDIX B

PERMISSION LETTER FOR USE OF COPYRIGHTED MATERIAL

May 21, 2011

Dr. Susan P. Miller
Department of Special Education and Early Childhood
College of Education: University of Nevada Las Vegas
4505 Maryland Parkway, Box 43014
Las Vegas, NV 89154-4301

Dear Dr. Miller:

I am completing a doctoral dissertation at the University of Nevada Las Vegas entitled *Investigating the Effects of Addition with Regrouping Strategy Instruction Among Elementary Students with Learning Disabilities*. I would like your permission to reprint excerpts for the Strategic Math Series: Addition with Regrouping (Miller, Kuffar, & Mercer, 2011) within my dissertation. The excerpts to be reproduced include: (a) Lesson 4 Teacher Instructions, (b) Learning Sheet 4, (c) Addition with Regrouping Pre- and Posttest, (d) Addition with Regrouping Minute, and (e) Addition Review Minute.

The requested permission extends to any future revisions and editions of my dissertation, including non-exclusive world rights in all languages, and to the prospective publication of my dissertation by ProQuest through its UMI® Dissertation Publishing business. ProQuest may produce and sell copies of my dissertation on demand and may make my dissertation available for free internet download at my request. These rights will in no way restrict republication of the material in any other form by you or by others authorized by you. Your signing of this lesson will also confirm that you own the copyright to the above described material.

If these arrangements meet with your approval, please sign this letter where indicated below and return it to me in the enclosed return envelope. Thank you for your assistance.

Sincerely,

Christi J. Carmack
Doctoral Candidate
University of Nevada, Las Vegas

PERMISSION GRANTED FOR THE USE REQUESTED ABOVE:

[Signature: Dr. Susan P. Miller] [5-26-11]
APPENDIX C

PERMISSION LETTER FOR USE OF COPYRIGHTED MATERIAL

Christi J. Carnack
6584 McKinley Summit Court
Las Vegas, NV 89110

May 21, 2011

Dr. Bradley Kaffar
6726 Trumpeter Court
St Cloud, MN 56303

Dear Dr. Kaffar:

I am completing a doctoral dissertation at the University of Nevada Las Vegas entitled *Investigating the Effects of Addition with Regrouping Strategy Instruction Among Elementary Students with Learning Disabilities*. I would like your permission to reprint excerpts for the Strategic Math Series: Addition with Regrouping (Miller, Kaffar, & Mercer, 2011) within my dissertation. The excerpts to be reproduced include: (a) Lesson 4 Teacher Instructions, (b) Learning Sheet 4, (c) Addition with Regrouping Pre- and Posttest, (d) Addition with Regrouping Minute, and (e) Addition Review Minute.

The requested permission extends to any future revisions and editions of my dissertation, including non-exclusive world rights in all languages, and to the prospective publication of my dissertation by ProQuest through its UMI® Dissertation Publishing business. ProQuest may produce and sell copies of my dissertation on demand and may make my dissertation available for free internet download at my request. These rights will in no way restrict republication of the material in any other form by you or by others authorized by you. Your signing of this lesson will also confirm that you own the copyright to the above described material.

If these arrangements meet with your approval, please sign this letter where indicated below and return it to me in the enclosed return envelope. Thank you for your assistance.

Sincerely,

Christi J. Carnack
Doctoral Candidate
University of Nevada, Las Vegas

PERMISSION GRANTED FOR THE USE REQUESTED ABOVE:

[Signature of Dr. Bradley Kaffar]

Date: 6-3-11
APPENDIX D

PARENT CONSENT FORM

Department of Special Education

TITLE OF STUDY: Investigating the Effects of Addition with Regrouping Strategy Instruction among Elementary Students with Learning Disabilities

INVESTIGATORS: Susan P. Miller, PhD and Christi J. Carmack

CONTACT PHONE NUMBER: 466-8555 (Mrs. Carmack) or 895-1108 (Dr. Miller)

Purpose of the Study

Your child is invited to participate in a research study. The purpose of this study is to investigate the effectiveness of strategy instruction designed to help students solve addition with regrouping problems.

Participants

Your child is being asked to participate in this study because he/she has an identified learning disability and needs assistance to improve skills in solving addition with regrouping problems.

Procedures

If you give permission for your child to participate in this study, your child will be asked to do the following: (a) take five short pretests (approximately 40 minutes total) that involve solving addition problems and related word problems that require regrouping, (b) participate in 20 40-minute math lessons that will take place during school hours Monday – Friday for 4 weeks, (c) take five short posttests immediately following the conclusion of the math lessons that involve solving addition problems and related word problems (approximately 40 minutes total), (d) complete a short satisfaction questionnaire (approximately 10 minutes), and (e) complete a 20 regrouping problems two weeks after instruction ends. The mathematics instruction consists of daily lessons focusing on addition problems that require regrouping to solve. Each lesson includes the following components: (a) introduction to the lesson to ensure that students understand why the lesson is important; (b) teacher demonstration of how to solve three problems using the same materials the students will use in the lesson (e.g., math manipulative devices, drawings, and/or numbers); (c) practice that involves students and teacher solving three problems together; (d) practice that involves students solving six problems on their own; and (e) practice that involves the teacher asking questions that guide students through the solving of two word problems (the problems will be read aloud to the students). Two of the problems that your child solves with teacher support, the six problems that your child solves on his or her own, and the two word problems will be used to monitor your child’s progress throughout this program. If he or she has difficulty with any of the lessons, the teacher will provide additional feedback to help. Your child will also be taught two strategies (i.e., RENAME and FAST RENAME) to help with remembering the steps involved in solving problems that require regrouping. RENAME stands for Read the problem, Examine the ones.

Approved by the UNLV IRB. Protocol 1012-3663M
Received: 02-16-11 Approved: 02-16-11 Expiration: 02-15-12

Parent’s Initials __________

Page 1 of 3
TITLE OF STUDY: Investigating the Effects of Addition with Regrouping Strategy Instruction among Elementary Students with Learning Disabilities

columns, Note ones in the ones column, Address the tens column, Mark tens in tens column, and Examine and note hundreds. FAST RENAME, a strategy for word problems, stands for Find what you’re solving for, Ask yourself, “What are the parts of the problem?” Set up the numbers, Tie down the sign, Read the problem, Examine the ones columns, Note ones in the ones column, Address the tens column, Mark tens in tens column, and Examine and note hundreds. The lessons are designed to keep students actively involved, and they also include instructional games to help with motivation and mastery of regrouping skills. At the end of the instructional lessons, your child will be asked to complete a satisfaction questionnaire to see if he/she thought the lessons helped improve regrouping skills and to determine whether he/she enjoyed the lessons.

Additionally, the lessons will be video recorded to ensure lesson quality and effective teaching. The video recorder will be focused on the face of the investigator as she teaches the lesson, so it is mainly the back of your child’s head that will be recorded. Only the investigators will review the video recordings of the lessons.

Benefits of Participation

There may be direct benefits to your child as a participant in this study. Allowing the investigator to analyze your child’s mathematics performance will provide more information to better address your child’s educational needs. Additionally, your child may improve his/her ability to add with regrouping.

Risks of Participation

There are risks involved in all research studies. This study includes only minimal risks. It is possible that your child will only see minimal improvements to his/her mathematics problem solving skills.

Cost/Compensation

There will be no financial cost to you for your child to participate within this study. There will be no compensation.

Contact Information

If you have any questions or concerns about the study, you may contact Christi Carmack at 702-466-8555 or Susan Miller at 702-895-1108. For questions regarding the rights of research subjects or to discuss any complaints or comments regarding the manner in which the study is being conducted you may contact the UNLV Office for the Protection of Research Subjects at 702-895-2794.

Voluntary Participation

Your child’s participation in this study is voluntary. Your child may refuse to participate in this study or in any part of this study. Your child may withdraw at any time without prejudice to your relations with the university or Innovations International Charter School of Nevada. You or your child are encouraged to ask questions about this study at the beginning or any time during the research study.

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Received: 02-16-11 Approved: 02-16-11 Expiration: 02-15-12

Parent’s Initials ____________

Page 2 of 3
TITLE OF STUDY: Investigating the Effects of Addition with Regrouping Strategy Instruction among Elementary Students with Learning Disabilities

Confidentiality

All information gathered in this study will be kept confidential. No reference will be made in written or oral materials that could link your child to this study. All records will be stored in a locked facility at UNLV for 3 years after completion of the study. After the storage time, the information gathered will be destroyed and computer files erased.

CONSENT FOR STUDY PARTICIPATION

I have read the above information and agree to allow my child participate in this study. I am at least 18 years of age. A copy of this form has been given to me.

________________________________________________________________________________________
Signature of Parent __________________________ Date __________________________

________________________________________________________________________________________
Parent Name (Please Print) __________________________ Child’s Name (Please Print) __________________________

PARENT CONSENT FOR VIDEO RECORDING

I agree to allow my child to be video recorded for the purposes of this research study.

________________________________________________________________________________________
Signature of Parent __________________________ Date __________________________

________________________________________________________________________________________
Parent Name (Please Print) __________________________ Child’s Name (Please Print) __________________________

Approved by the UNLV IRB. Protocol 1012-3663M
Received: 02-16-11 Approved: 02-16-11 Expiration: 02-15-12

Parent’s Initials ____________
APPENDIX E

STUDENT ASSENT FORM

ASSENT TO PARTICIPATE IN RESEARCH

Investigating the Effects of Addition with Regrouping Strategy Instruction among Elementary Students with Learning Disabilities

1. My name is Mrs. Christi Carmack.
2. I am asking you to be in a project that will help us become better math teachers.
3. If you agree to be in the project, you will participate in 20 math lessons. I will teach the lessons to you during school time Monday – Friday for 4 weeks.
4. If you agree to be in the project, you will take five short tests before and after I teach the lessons. You will also take a short test two weeks after the lessons are over. These tests will tell us how much you learned.
5. If you agree to be in the project, you will also answer a short questionnaire about the lessons. The questions will be about whether you think the lessons helped you in math and whether you liked the lessons.
6. If you agree to be in the project, you will meet with me every day for 40 minutes. During our meetings, I will show you how to solve math problems. I will teach you some easy ways to remember the steps to solve math problems, and I will help you learn to solve math problems by yourself. During each lesson, you will complete a short math worksheet. This worksheet will tell us how much you learn each day.
7. It is possible that you will miss some of the math problems and that could make you feel bad, but I think you will get most of the problems correct.
8. If you agree to participate in this project, you may become better at math and we will learn how to better teach you math.
9. All of the math lessons will be video recorded so that my advisor and I can make sure that the lessons were taught correctly.
10. Please talk to your parents before you decide to participate in this project. I will also ask your parents to give their permission for you to take part in this project. But even if your parents say ‘yes’ you can say ‘no.’
11. If you don’t want to participate, you don’t have to. No one will be mad if you don’t want to do this. If you say ‘yes’ now and change your mind later, that’s OK. You can stop working with me any time you want.
12. You can ask any questions that you have about this project. You can call me at 702-466-8555 or my university advisor, Susan Miller at 702 895-1108. If I don’t answer your questions or you do not want to talk to me about your questions, you or your parent can call the UNLV Office for the Protection of Research Subjects at 702-895-2794.
13. Signing your name on the line below means that you agree to be in this project. You and your parents will be given a copy of this form after you have signed it.

__________________________________________  __________________________
Print your name                                           Date

Sign your name

Approved by the UNLV IRB. Protocol 1012-3663M
Received: 02-16-11 Approved: 02-16-11 Expiration: 02-15-12
Addition with Regrouping Pretest

\[
\begin{array}{cccc}
55 & 342 & 36 & 106 \\
+ 27 & + 464 & + 47 & + 225 \\
64 & 483 & 58 & 358 \\
+ 18 & + 119 & + 26 & + 161 \\
144 & 36 & 55 & 105 \\
+ 277 & + 16 & + 27 & + 217 \\
45 & 355 & 103 & 24 \\
+ 26 & + 188 & + 118 & + 18 \\
34 & 264 & 45 & 254 \\
+ 17 & + 572 & + 17 & + 228 \\
\end{array}
\]
### ADDITION WITH REGROUPING POSTTEST

**Addition with Regrouping Posttest**

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## APPENDIX H

### ADDITION WITH REGROUPING MAINTENANCE TEST

**Addition with Regrouping Maintenance Test**

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APPENDIX I

ADDITION WITH REGROUPING MINUTE

Addition with Regrouping Minute

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+ 17 & + 118 & + 19 & + 139 \\
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452 & 37 & 554 & 32 \\
+ 163 & + 37 & + 258 & + 29 \\
\hline
45 & 362 & 34 & 506 \\
+ 26 & + 362 & + 19 & + 347 \\
\hline
224 & 48 & 468 & 26 \\
+ 237 & + 35 & + 268 & + 46 \\
\end{array}
\]
**APPENDIX J**

**ADDITION WITH REGROUPING WORD PROBLEM PRETEST**

**Addition with Regrouping Word Problem Pretest**

1. Bill had 231 baseball cards. He bought 183 more. How many baseball cards does Bill have now?

2. Pat has 114 cans. Sue has 128 cans. How many cans do they have in all?

3. Mary has 45 pretzels. Bill has 10 potato chips. Betty has 18 pretzels. How many pretzels are there altogether?

4. Amy has 29 points. Ryan has 62 points. How many points do Amy and Ryan have altogether?

5. Jan knows 146 songs. Lee knows 271 songs. How many songs do they know in all?

6. Harry saw 324 cars. Sam saw 126 cars. How many cars did they see altogether?

7. Joe has 13 books. Ann has 18 books. How many books do they have altogether?

8. Joe saw 114 dogs at one pet store and 107 dogs at another pet store. How many dogs did Joe see altogether?

9. Sam rode 16 miles on Monday and 17 miles on Tuesday. How many miles did he ride in all?

10. Mary had 13 pens and Jose had 28 pens. How many pens did they have altogether?
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<td>1.</td>
<td>Mary had 13 pens and Jose had 28 pens. How many pens did they have altogether?</td>
</tr>
<tr>
<td>2.</td>
<td>Sam rode 16 miles on Monday and 17 miles on Tuesday. How many miles did he ride in all?</td>
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<tr>
<td>3.</td>
<td>Joe saw 114 dogs at one pet store and 107 dogs at another pet store. How many dogs did Joe see altogether?</td>
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<td>4.</td>
<td>Joe has 13 books. Ann has 18 books. How many books do they have altogether?</td>
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<td>6.</td>
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<td>7.</td>
<td>Amy has 29 points. Ryan has 62 points. How many points do Amy and Ryan have altogether?</td>
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<td>Mary has 45 pretzels. Bill has 10 potato chips. Betty has 18 pretzels. How many pretzels are there altogether?</td>
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<tr>
<td>9.</td>
<td>Pat has 114 cans. Sue has 128 cans. How many cans do they have in all?</td>
</tr>
<tr>
<td>10.</td>
<td>Bill had 231 baseball cards. He bought 183 more. How many baseball cards does Bill have now?</td>
</tr>
</tbody>
</table>
### Addition with Regrouping Word Problem Maintenance Test

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Mary had 13 pens and Jose had 28 pens. How many pens did they have altogether?</td>
<td></td>
</tr>
<tr>
<td>2. Sam rode 16 miles on Monday and 17 miles on Tuesday. How many miles did he ride in all?</td>
<td></td>
</tr>
<tr>
<td>3. Joe saw 114 dogs at one pet store and 107 dogs at another pet store. How many dogs did Joe see altogether?</td>
<td></td>
</tr>
<tr>
<td>4. Joe has 13 books. Ann has 18 books. How many books do they have altogether?</td>
<td></td>
</tr>
<tr>
<td>5. Harry saw 324 cars. Sam saw 126 cars. How many cars did they see altogether?</td>
<td></td>
</tr>
<tr>
<td>6. Jan knows 146 songs. Lee knows 271 songs. How many songs do they know in all?</td>
<td></td>
</tr>
<tr>
<td>7. Amy has 29 points. Ryan has 62 points. How many points do Amy and Ryan have altogether?</td>
<td></td>
</tr>
<tr>
<td>8. Mary has 45 pretzels. Bill has 10 potato chips. Betty has 18 pretzels. How many pretzels are there altogether?</td>
<td></td>
</tr>
<tr>
<td>9. Pat has 114 cans. Sue has 128 cans. How many cans do they have in all?</td>
<td></td>
</tr>
<tr>
<td>10. Bill had 231 baseball cards. He bought 183 more. How many baseball cards does Bill have now?</td>
<td></td>
</tr>
</tbody>
</table>
# Conceptual Understanding Pretest

1. Show me how to solve the following problems using these base ten blocks. As you solve the problems, tell me what you are doing to solve the problem.

<table>
<thead>
<tr>
<th>131</th>
<th>25</th>
<th>124</th>
</tr>
</thead>
<tbody>
<tr>
<td>+183</td>
<td>+16</td>
<td>+117</td>
</tr>
</tbody>
</table>

2. Show me how to solve the following problems without using the base ten blocks. As you solve the problems, tell me what you are doing to solve the problem.

<table>
<thead>
<tr>
<th>124</th>
<th>141</th>
<th>17</th>
</tr>
</thead>
<tbody>
<tr>
<td>+128</td>
<td>+175</td>
<td>+25</td>
</tr>
</tbody>
</table>
1. Show me how to solve the following problems using these base ten blocks. As you solve the problems, tell me what you are doing to solve the problem.

<table>
<thead>
<tr>
<th>25</th>
<th>124</th>
<th>131</th>
</tr>
</thead>
<tbody>
<tr>
<td>+16</td>
<td>+117</td>
<td>+183</td>
</tr>
</tbody>
</table>

2. Show me how to solve the following problems without using the base ten blocks. As you solve the problems, tell me what you are doing to solve the problem.

<table>
<thead>
<tr>
<th>17</th>
<th>124</th>
<th>141</th>
</tr>
</thead>
<tbody>
<tr>
<td>+25</td>
<td>+128</td>
<td>+175</td>
</tr>
</tbody>
</table>
### Conceptual Understanding Pretest Scoring Protocol

1. **Problem One:**
   - Participant represents first number accurately
   - Participant represents second number accurately
   - Participant adds ones correctly
   - Participant adds tens correctly
   - Participant states need to regroup tens
   - Participant regroups tens correctly
   - Participant adds hundreds correctly

2. **Problem Two:**
   - Participant represents first number accurately
   - Participant represents second number accurately
   - Participant adds ones correctly
   - Participant states need to regroup ones
   - Participant regroups ones accurately
   - Participant adds tens accurately

3. **Problem Three:**
   - Participant represents first number accurately
   - Participant represents second number accurately
   - Participant adds ones correctly
   - Participant states need to regroup ones
   - Participant regroups ones accurately
   - Participant adds tens accurately
   - Participant adds hundreds accurately

4. **Problem Four:**
   - Participant adds ones correctly
   - Participant states need to regroup ones
Participant regroups ones accurately  ____
Participant adds tens accurately  ____
Participant adds hundreds accurately  ____

5. Problem Five:
Participant adds ones correctly  ____
Participant adds tens correctly  ____
Participant states need to regroup tens  ____
Participant regroups tens accurately  ____
Participant adds hundreds accurately  ____

6. Problem Six:
Participant adds ones correctly  ____
Participant states need to regroup ones  ____
Participant regroups ones accurately  ____
Participant adds tens correctly  ____
APPENDIX P

CONCEPTUAL UNDERSTANDING POSTTEST SCORING PROTOCOL

Conceptual Understanding Posttest Scoring Protocol

1. Problem One:
   - Participant represents first number accurately ______
   - Participant represents second number accurately ______
   - Participant adds ones correctly ______
   - Participant states need to regroup ones ______
   - Participant regroups ones accurately ______
   - Participant adds tens accurately ______

2. Problem Two:
   - Participant represents first number accurately ______
   - Participant represents second number accurately ______
   - Participant adds ones correctly ______
   - Participant states need to regroup ones ______
   - Participant regroups ones accurately ______
   - Participant adds tens accurately ______
   - Participant adds hundreds accurately ______

3. Problem Three:
   - Participant represents first number accurately ______
   - Participant represents second number accurately ______
   - Participant adds ones correctly ______
   - Participant states need to regroup ones ______
   - Participant regroups ones accurately ______
   - Participant adds ones correctly ______
   - Participant states need to regroup tens ______
   - Participant regroups tens correctly ______
   - Participant adds hundreds correctly ______
| Problem Four: | Participant adds ones correctly | ______ |
|              | Participant states need to regroup ones | ______ |
|              | Participant regroups ones accurately | ______ |
|              | Participant adds tens accurately | ______ |
| Problem Five: | Participant adds ones correctly | ______ |
|              | Participant states need to regroup ones | ______ |
|              | Participant regroups ones accurately | ______ |
|              | Participant adds tens accurately | ______ |
|              | Participant adds hundreds accurately | ______ |
| Problem Six: | Participant adds ones correctly | ______ |
|              | Participant states need to regroup ones | ______ |
|              | Participant regroups ones accurately | ______ |
|              | Participant adds ones correctly | ______ |
|              | Participant states need to regroup tens | ______ |
|              | Participant regroups tens correctly | ______ |
|              | Participant adds hundreds correctly | ______ |
### Baseline Probe A

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<table>
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<td>765</td>
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<td></td>
<td>+29</td>
<td>+72</td>
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<tr>
<td>2)</td>
<td>409</td>
<td>454</td>
</tr>
<tr>
<td></td>
<td>+349</td>
<td>+315</td>
</tr>
<tr>
<td>3)</td>
<td>45</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>+667</td>
<td>+352</td>
</tr>
<tr>
<td>4)</td>
<td>54</td>
<td>37</td>
</tr>
</tbody>
</table>

9) Sylvia, Tiffany, and Nadia collect stickers. Sylvia has 58 butterfly stickers. Tiffany has 37 butterfly stickers. Nadia has 11 lady bug stickers. How many butterfly stickers do they have all together?

10) Marcus and Alex collected cans to recycle. Marcus collected 175 cans. Alex collected 251 cans. How many cans did they collect in all?
Baseline Probe B

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
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<tr>
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<td>+548</td>
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<td>4)</td>
<td>37</td>
<td>5)</td>
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<td></td>
<td>+46</td>
<td></td>
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<tr>
<td>7)</td>
<td>156</td>
<td>8)</td>
</tr>
<tr>
<td></td>
<td>+352</td>
<td></td>
</tr>
</tbody>
</table>

9) Nicole and Tyra shared a bag of potato chips. Nicole ate 26 potato chips. Tyra ate 15 potato chips. How many potato chips did they eat all together?

10) Tonya and Sam built a block tower. Tonya stacked 149 blocks. Sam stacked 236 blocks. How many blocks total did they stack?
## Baseline Probe C

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>1)</td>
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<td>2)</td>
<td>25</td>
<td>+47</td>
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<tr>
<td>3)</td>
<td>206</td>
<td>+487</td>
</tr>
<tr>
<td>4)</td>
<td>38</td>
<td>+26</td>
</tr>
<tr>
<td>5)</td>
<td>126</td>
<td>+458</td>
</tr>
<tr>
<td>6)</td>
<td>74</td>
<td>+27</td>
</tr>
<tr>
<td>7)</td>
<td>416</td>
<td>+145</td>
</tr>
<tr>
<td>8)</td>
<td>58</td>
<td>+27</td>
</tr>
</tbody>
</table>

9) Barbara and Melina played outside after school on Monday. Barbara played outside for 57 minutes. Melina played outside for 46 minutes. How many minutes did they play outside all together?

10) William, Max, and Jose sold cookie dough for a school fundraiser. William sold 154 containers of chocolate chip cookie dough. Max sold 237 containers of chocolate chip cookie dough. Jose sold 176 containers of peanut butter cookie dough. How many containers of chocolate chip cookie dough did the boys sell all together?
**Learning Sheet 4**

### Review Problems

<table>
<thead>
<tr>
<th></th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>324</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>245</td>
<td></td>
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<td></td>
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</tbody>
</table>

### Describe and Model

<table>
<thead>
<tr>
<th></th>
<th>1) 135</th>
<th>2) 143</th>
<th>3) 245</th>
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</thead>
<tbody>
<tr>
<td></td>
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<td>192</td>
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</tbody>
</table>

### Guided Practice

<table>
<thead>
<tr>
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<th>4) 138</th>
<th>5) 326</th>
<th>6) 126</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>127</td>
<td>135</td>
<td>329</td>
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</table>
Independent Practice

<p>| | | |</p>
<table>
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</thead>
<tbody>
<tr>
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<td>+ 317</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<tr>
<td>8)</td>
<td>124</td>
<td>+ 193</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9)</td>
<td>486</td>
<td>+ 140</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10)</td>
<td>326</td>
<td>+ 428</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11)</td>
<td>234</td>
<td>+ 182</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12)</td>
<td>253</td>
<td>+ 264</td>
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</tbody>
</table>

Problem-Solving Practice

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>13)</td>
<td>Kim has 237 stickers. Bob has 119 stickers. How many stickers do they have in all?</td>
</tr>
<tr>
<td>14)</td>
<td>There are 182 pages in Juan’s book. There are 154 pages in Sara’s book. How many pages are there in all?</td>
</tr>
</tbody>
</table>
## Maintenance Probe

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
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<td>+</td>
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<td>14</td>
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<td>2)</td>
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<td></td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>3)</td>
<td>37</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>4)</td>
<td>19</td>
<td>34</td>
</tr>
<tr>
<td>5)</td>
<td>172</td>
<td>764</td>
</tr>
<tr>
<td>6)</td>
<td>488</td>
<td>266</td>
</tr>
<tr>
<td>7)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8)</td>
<td>Jose had 237 paper clips and Walter had 126 paper clips. How many paper clips did they have altogether?</td>
<td></td>
</tr>
<tr>
<td>9)</td>
<td>Martha listened to 19 songs on Monday and 27 songs on Tuesday. How many songs did she listen to in all?</td>
<td></td>
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</tbody>
</table>
APPENDIX V
GENERALIZATION PROBE

NAME:

<table>
<thead>
<tr>
<th>54</th>
<th>45</th>
<th>33</th>
<th>29</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ 37</td>
<td>+ 26</td>
<td>+ 57</td>
<td>+ 12</td>
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</table>

<table>
<thead>
<tr>
<th>409</th>
<th>667</th>
<th>146</th>
<th>349</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ 454</td>
<td>+ 250</td>
<td>+ 265</td>
<td>+ 315</td>
</tr>
</tbody>
</table>

Jeorge and Phil collect baseball cards. Jeorge has 53 baseball cards. Phil has 39 baseball cards. Danny has 11 birthday cards. How many baseball cards do Jeorge and Phil have in all?

Tyra and Anna buy a bag of candy. Tyra buys a bag with 346 jelly beans in it. Anna buys a bag with 326 jelly beans in it. How many jelly beans do Tyra and Anna have all together?
## APPENDIX W

### ADDITION WITH REGROUPING SATISFACTION QUESTIONNAIRE

#### Addition with Regrouping Satisfaction Questionnaire

<table>
<thead>
<tr>
<th></th>
<th>Strongly Agree</th>
<th>Agree</th>
<th>Disagree</th>
<th>Strongly Disagree</th>
</tr>
</thead>
<tbody>
<tr>
<td>I liked using base ten blocks helped with addition.</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>I liked using drawings with addition.</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>I liked using the RENAME strategy with addition.</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>I liked using the Addition Minute to get faster at addition.</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>I liked using the FAST RENAME Strategy with word problems.</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>I liked playing the PIG Games with addition problems.</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>This program helped me become better at addition.</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Overall, I liked this Addition Program.</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
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</tbody>
</table>
Lesson 4
Introduce the Concrete Method of 3-Digit Addition With Regrouping
From Ones to Tens or Tens to Hundreds

GOALS
☐ To review place value concept related to the identification of hundreds, tens, and ones.
☐ To promote the students’ ability to use objects to solve 3-digit addition problems that require regrouping
☐ To promote the students’ ability to use concrete objects to solve word problems that require 3-digit addition with regrouping.

MATERIALS
☐ Whiteboard and marker
☐ Concrete objects (i.e., base ten blocks) including at least 7 hundreds, 14 tens, and 16 ones for each student
☐ Learning Sheet 4
☐ Place Value Mat: Hundreds, Tens, and Ones, one per student
☐ Addition With Regrouping Progress Charts
☐ Overhead projector and screen (optional)
☐ Overhead transparency of Learning Sheet 4 (optional)
☐ Overhead transparency of Place Value Mat: Hundreds, Tens and Ones (optional)

GIVE AN ADVANCE ORGANIZER
1. Tell the students what they will be doing and why.

Sample dialogue:
During our last lesson, we practiced addition with regrouping using _____ (name objects used in first lesson). Remember? (Wait for response.) You did a good job representing the first number in the problem, representing the second number in the problem. You also did a great job remembering that when addition involves two-digit numbers, you Is add the numbers in the ones column first and then you add the numbers in the tens column second. After you added the ones, you realized that you had enough ones to trade for a ten. Ir word for trade is regroup. You regrouped ones to form a ten. Does everyone remember doing that? (Wait for response.) After you regrouped ones to a ten, you added the numbers in the ones column on your Learning Sheet and also added the numbers in the tens column on your Learning Sheet. Let’s do a problem together to be sure we remember all the steps. (Write 24+27 in vertical format on the board and call on students to identify the steps for representing the problem with base ten blocks and solving the problem).

Today we’re going to practice addition using base ten blocks. We’ll use them to do problems similar to the ones we did yesterday except the problems we do today are going to be hundreds. What are the problems going to be? (Elicit response, “hundreds.”) By the end of today’s lesson, you’ll be able to add problems such as 123+218, 134+128, and 191+226. (Write problems on whiteboard in vertical format as you say them.)
Before we get started with problems that involve adding hundreds, we need to review some important information about ones, tens, and hundreds. This review will help you succeed in this lesson, so pay close attention.

2. Review place value concept related to hundreds, tens, and ones.

Give base ten blocks and one blank copy of Learning Sheet 4 to each student. Also, give each student a place value mat that includes a ones, tens, and hundreds column.

Let’s look at base ten blocks to be sure we understand what each one represents. This is a one block. (Hold up a one block.) What is it? Yes, a one block. This is a ten block. (Hold up ten block.) What is it? Yes, a ten block. Who remembers why this is called a ten block? That’s correct because there are ten ones in a ten block. (If students seem confused, have them place ten one blocks in a row next to a ten block to illustrate they are the same length and same quantity.) This is a hundred block. (Hold up a hundred block.) What is it? Yes, a hundred block. Who remembers why this is called a hundred block? (Elicit response such as, “There are 100 one blocks in the hundred block.) How many ten blocks are there in a hundred block? (Elicit response, “ten.”) Yes, there are 100 ones in a hundred block (hold up hundred block) and there are 10 tens in a hundred block. (If students seem confused, have them place 10 ten blocks on top of or in a row next to the hundred block to illustrate they are the same quantity.)

Look at the first review problem at the top of your Learning Sheet. I see the number 324. I also see a blank place value chart next to the problem. I’m going to use my base ten blocks to represent the number 324. First, I’ll use my hundred blocks and count 3 because I see “3” in the hundreds column of my number. (Count aloud, “1, 2, 3,” as you individually pick up three hundred blocks and place them on the place value mat.) So, we have 100, 200, 300. (Point to each hundred block as you say this.)

Next, I need to represent the tens. To do this, I’m going to use my ten blocks and count two tens because I see “2” in the tens column of my number. (Count aloud, “1, 2,” as you individually pick up two ten blocks and place them on the place value mat.) So, we have 10, 20. (Point to each ten as you say this.)

Next, I need to represent the ones. To do this, I’m going to use my one blocks and count four ones because I see “4” in the ones column of my number. (Count aloud, “1, 2, 3, 4,” as you individually pick up four one blocks and place them on the place value mat.) So, we see there are 3 hundreds, 2 tens, and 4 ones in the number 324. Let’s write these numbers in the correct columns on our Learning Sheets. (Check to be sure student write numbers in the correct column.)

3. Use the same process to demonstrate the second review problem on the Learning Sheet. Guide students through the process of counting hundreds, tens, and ones to represent the number 245 and then have them write the number of hundreds, tens, and ones in the appropriate column of the place value chart on the Learning Sheet.

DESCRIBE AND MODEL

1. Ensure that students have place value mat cleared, base ten blocks pushed to the side of the work space, and their Learning Sheets in front of them.

2. Demonstrate how to compute Problem 1.
Sample dialogue:
I'm now going to show you how to do these problems. To start out, I want you to watch me and leave your base ten blocks alone. You'll get a chance to use your blocks in just a few minutes.

Look at Problem 1 on your Learning Sheet. It says, “135 plus 216 equals how many?” To begin, I'm going to look at the first number, 135, and represent that number using my blocks. (Point to the 1 in 135 and say “one hundred” aloud. Put one hundred block in the hundreds column on your place value mat. Point to the “3” in “135” and count three tens aloud. Put the three tens on your place value mat in the tens column in full view of the students. Point to the “5” in “135” and count five one blocks aloud. Put the five ones on your place value mat in the ones column in full view of the students.)

So I have one hundred, three tens and five ones or 135. According to the problem, I need to add 216. I will represent 216 with my blocks. (Point to the “2” in “216” and count two hundred blocks aloud. Put the two hundred blocks in the hundred’s column on your place value mat. Point to the “1” in “216” and count one ten block aloud. Put the one ten on your place value mat in the tens column in full view of the students. Point to the “6” in “216” and count six one blocks aloud. Put the six one blocks on your place value mat in the ones column in full view of the students.) When addition involves two- or three-digit numbers, you always add the numbers in the ones column first. What do you do when addition involves two- or three-digit numbers? (Elicit response, “You always add the numbers in the ones column first.”). To do this, I look at the ones column and see that I must add 5 and 6. (Point to the “5” and “6” in the problem and also point to the 5 ones objects and the 6 ones objects on the place value mat.) I'll count the ones now to see how many I have in all (count the ones objects ending with eleven).

I have 11 ones. Do I have enough to trade for a ten block? (Elicit the response, “Yes.”). So I’ll do that now (count ten ones, trade for a ten block, place the ten block in the tens column.) I traded ten ones for one ten and put my ten in the tens column. So, 5 plus 6 equals 11; one ten and one one. On my problem, I record the 1 in the ones column, and the one ten in the tens column. (Write “1” in the ones column answer space and write 1 above the “3” in “135” to represent the crutch number.) Everyone write “1” in the ones column on your learning sheets and write “1” in the tens column above the “3”. (Check to see that students have written the correct numbers in the correct columns.)

Next, I look at the tens column and see that I must add 1 ten plus 3 tens plus 1 ten. (Point to the “1” crutch number, the “3” in “135”, and the “1” in “216” and also point to the 1 ten, 3 tens and 1 ten on the place value mat.). I’ll count the tens now to see how many I have in all (count the ten blocks ending with five).

I have five tens. Do I have enough tens to trade for a hundred block? (Elicit the response, “no.”) How many tens would I need to trade for a hundred block? (Elicit the response, “ten.”) That’s correct I would need ten. Because I only have five tens, I need to write “5” in the tens column on my Learning Sheet. Everyone write “5” in the tens column on your learning sheets. (Check to see that students have written the correct answer in the correct column.)

Next, I look at the hundreds column and see that I must add 1 hundred plus 2 hundreds. (Point to the “1” in “135” and the “2” in “216” and also point to the one hundred block and the two hundred blocks on the place value mat.) I’ll count the hundreds now to see how many I have in all (count the hundred blocks ending in three.) I have three hundreds, so I need to write “3” in the hundreds column on my Learning Sheet. Everyone write “3” in the hundreds column on your learning sheets. (Check to see that students have written the correct answer in the correct
3. Demonstrate how to compute Problem 2.

Sample dialogue:

Look at Problem 2 on your Learning Sheet. It says, “143 plus 192 equals how many?” To begin, I’m going to look at the first number, 143, and represent that number using my blocks. (Point to the “1” in “143” and say “one hundred” aloud. Put one hundred block in the hundreds column on your place value mat. Point to the “4” in “143” and count four tens aloud. Put the four tens on your place value mat in the tens column in full view of the students. Point to the “3” in “143” and count three one blocks aloud. Put the three ones on your place value mat in the ones column in full view of the students.)

So I have one hundred, four tens and three ones or 143. According to the problem, I need to add 192. I will represent 192 with my blocks. (Point to the “1” in “192” and count one hundred block aloud. Put the one hundred block in the hundred’s column on your place value mat. Point to the “9” in “192” and count nine ten blocks aloud. Put the nine ten blocks on your place value mat in the tens column in full view of the students. Point to the “2” in “192” and count two one blocks aloud. Put the two one blocks on your place value mat in the ones column in full view of the students.) When addition involves two- or three-digit numbers, you always add the numbers in the ones column first. What do you do when addition involves two- or three-digit numbers? (Elicit response, “You always add the numbers in the ones column first.”). To do this, I look at the ones column and see that I must add 3 and 2. (Point to the “3” and “2” in the problem and also point to the 3 one blocks and the 2 one blocks on the place value mat.) I’ll count the ones now to see how many I have in all (count the ones objects ending with five).

I have 5 ones. Do I have enough to trade for a ten block? (Elicit the response, “No.”). So I’ll go ahead and write “5” in the ones column on my Learning Sheet. Everyone write “5” in the ones column on your learning sheets. (Check to see that students have written the correct number in the correct column.)

Next, I look at the tens column and see that I must add 4 tens plus 9 tens. (Point to the “4” in 143 and the “9” in “192” and also point to the 4 ten blocks and the 9 ten blocks on the place value mat.). I’ll count the tens now to see how many I have in all (count the ten blocks ending with thirteen).

I have thirteen tens. Do I have enough tens to trade for a hundred block? (Elicit the response, “yes.”) So I’ll do that now (count 10 tens, trade for a hundred block, place the hundred block in the hundreds column on the place value mat.) I traded ten tens for one hundred block and put my hundred in the hundreds column. So, 4 tens plus 9 tens equals 13 tens and 13 tens equals one hundred and three tens. On my problem, I record the 3 tens in the tens column, and the one hundred in the hundreds column. (Write “3” in the tens column answer space and write 1 above the “1” in “143” to represent the crutch number.) Everyone write “3” in the tens column on your learning sheets and write “1” in the hundreds column above the “1” in “143.” (Check to see that students have written the correct numbers in the correct columns.

Next, I look at the hundreds column and see that I must add 1 hundred plus 1 hundred plus 1 hundred. (Point to the “1” crutch number, the “1” in “143” and the “1” in “192” and also point to the one hundred block, the one hundred block, and the one hundred block in the hundreds column on the place value mat.) I’ll count the hundreds now to see how many I have in all (count the hundred blocks ending in three.) I have three hundreds, so I need to write “3” in the
hundreds column on my Learning Sheet. Everyone write “3” in the hundreds column on your learning sheets. (Check to see that students have written the correct answer in the correct column.)

4. Instruct the students to solve Problem 3 with you.

Sample dialogue:
Now take your blocks and let’s do Problem 3 together. This problem says, “245 plus “372” is how many?” The first number, 245, tells us what number to represent on our place value mat. Let’s represent 245 with our blocks. (Point to the 245 on the Learning Sheet and then count two hundreds and place in the hundreds column on the place value mat, count four tens and place in the tens column on the place value mat and then count five ones and place in the ones column). (Check to see that each student has represented 245 accurately on his place value mat.)

The second number, 372, tells us how many to add. Let’s represent 372 with our blocks. (Point to the 372 on the Learning Sheet and then count 3 hundred blocks and place in the hundreds column on the place value mat, count 7 ten blocks and place in the tens column on the place value mat and then count 2 one blocks and place in the ones column. Check to see that each student has represented 372 accurately on his place value mat.)

When addition involves two- or three-digit numbers, you always add the numbers in the ones column first. What do you do when addition involves two- or three-digit numbers?...Yes, you always add the numbers in the ones column first. To do this, we look at the ones column and see that we must add 5 and 2. (Point to the “5” and “2” in the problem and also point to the 5 one blocks and the 2 one blocks on the place value mat.) Let’s count the ones now to see how many we have in all (count the ones objects ending with seven).

We have 7 ones. Do we have enough to trade for a ten block? (Elicit the response, “no.”). So let’s record the 7 in the ones column on our Learning Sheet. (Check to be sure students write “7” in the ones column of the problem answer space.)

Next, we look at the tens column and see that we must add 4 tens plus 7 tens. (Point to the “4” in “245” and the “7” in “372” and also point to the 4 ten blocks and 7 ten blocks on the place value mat.). Let’s count the tens now to see how many we have in all (Count the ten blocks ending with eleven.) Do we have enough tens to trade for a hundred block? (Elicit the response, “yes.”) So let’s do that now (count ten tens, trade for a hundred block, place the hundred block in the hundreds column on the place value mat.) We traded ten tens for one hundred and put our hundred in the hundreds column. So, 4 tens plus 7 tens equals 11 tens and 11 tens equals one hundred and one ten. Everyone write “1” in the tens column on your learning sheets and write “1” in the hundreds column above the “2” in “245.” (Check to see that students have written the correct numbers in the correct columns.

Let’s count the hundreds now to see how many we have in all (count the hundred blocks ending in six.) We have six hundreds, so let’s write “6” in the hundreds column on our Learning Sheets. (Check to see that students have written the correct answer in the correct column.)

So, we see that 245 plus 372 equals 617.

CONDUCT GUIDED PRACTICE

1. Guide students through Problem 4. Do not demonstrate the process unless further
demonstrations appear necessary.

*Sample dialogue:*
Now it is your turn to do Problem 4. Look at your Learning Sheet. Find the first number and represent that number on your place value mat. How many hundreds did you put on the mat? (Elicit the response, “1.”) How many tens did you put on the mat? (Elicit the response, “3.”) Good. How many ones did you put on the mat? (Elicit the response, “8”) Good.

Now look at the second number in the problem and represent that number on your place value mat. How many hundreds did you put on the mat? (Elicit the response, “1.”) How many tens did you put on the mat? (Elicit the response, “2.”) How many ones did you put on the mat? (Elicit the response, “7.”) Good. That’s correct.

When addition involves two- or three-digit numbers, which numbers do we add first?...Yes the numbers in the ones column. Do that now. How many ones do we have?...yes, we have 15 ones. So what do we need to do with these ones?...That’s correct we trade for a ten block. Do that now. (Check to be sure students make the trade and note the trade accurately).

What do we do now? (Elicit the response, “we record the 5 ones in the ones column on our Learning Sheets.”) What else do we record? (Elicit the response, the one ten.) (Check to be sure students do this accurately.)

What do we do now? (Elicit response, “We add the numbers in the tens column.”). Do that now. Don’t forget to trade for a hundred IF you have enough tens. Write the number of tens in the tens column once you know how many there are.

What do we do next? (Elicit response, “Add the numbers in the hundreds column.”) Good do that now and write the number of hundreds in the hundreds column.

What is 138 plus 127? (Elicit the response, “265.” Praise students for a good job.)

2. Guide the students through Problem 5 using the same procedure that you followed with Problem 4. Do not ask for the answer, however, as this is the first problem to be scored on the Learning Sheet.

3. Instruct the students to solve Problem 6 by themselves. Tell them to use their objects to solve the problem, but do not guide them through the process. Provide prompts and assistance only if needed. Again, do not ask for the answer, as this is the second problem to be scored on the Learning Sheet.

**CONDUCT INDEPENDENT PRACTICE**

1. **Instruct the students to solve Problems 7-12 independently.**

*Sample dialogue:*
Now please do Problems 7-12 on your Learning Sheet. For each problem, remember to use your base ten blocks. When you finish Problem 12, stop and put down your pencil. What are you going to do when you finish?

2. **Repeat the directions if needed.**

3. **Tell the students to begin.**

4. **Circulate and monitor work while students solve problems.** Provide assistance with the
CONDUCT PROBLEM-SOLVING PRACTICE

1. Give instructions for Problems 13 and 14.

Sample dialogue:
Now look at Problem 13. Let’s read this problem together. Kim has 237 stickers. Bob has 119 stickers. How many stickers do they have in all? Good reading. To determine what the first number in our problem is going to be, we need to think about the number of stickers Kim has. How many stickers does Kim have? (Elicit the response “237” and then praise the student). So, write “237 stickers” on your learning sheet. How many stickers does Bob have? (Elicit the response “119” and praise.) So, write “119 stickers” under “237 stickers” on your learning sheet. Then draw your equals line under “119.” Now use your blocks and place value mat to solve this problem.

2. Instruct the students to solve Problem 14.

Sample dialogue:
Now I’d like you to solve Problem 14. Read the problem. Let me know if you need help reading any of the words. Then, write the problem and use your blocks to solve the problem. When you have finished, raise your hand and I’ll collect your paper.

3. Collect all papers when students indicate that they are finished.

PROVIDE FEEDBACK

(For more specific information about any of these steps, please refer to Provide Feedback.)

1. Score the last 10 problems of each student’s Learning Sheet (Problems 5-14) for correct and incorrect responses; determine the total percentage of correct responses.

2. Individually meet with each student; help the student plot his score on his Addition Progress Chart. Begin by making at least one specific, positive statement about the student’s work. Compare the student’s score to the mastery goal line, noting any progress.

3. Specify incorrect responses and corresponding error patterns if they exist. Explain where errors have occurred. Try to make these statements without using the word “you.”

4. Show the student how to perform the task. For at least one problem missed, show the student how to compute the problem correctly by manipulating the objects used in the lesson. While demonstrating how to correctly compute a problem, verbalize how the student should “think” or talk to himself the next time he encounters a similar fact.

5. Ask the student to practice the application. Using a different problem, ask the student to show you how he will proceed in the future. Check to see that the student correctly manipulates objects to solve the problem.

6. Close the feedback session. Make a positive statement about the student’s performance in the feedback process and your expectations for the future.

Mastery: If the student scores 80% or better on his Learning Sheet, tell him that he is ready for Lesson 5. If he scores less than 80% however, explain that he needs to repeat this lesson until he obtains a score of 80% or better.
### Place Value Mat

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>Read the Problem.</td>
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<td>---</td>
<td>-------------------</td>
<td></td>
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<tr>
<td>E</td>
<td>Examine the one column; 10 or more, go next door.</td>
<td></td>
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<tr>
<td>N</td>
<td>Note the ones in the ones column.</td>
<td></td>
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<tr>
<td>A</td>
<td>Address the tens column; 10 or more, go next door.</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>Mark the tens in the tens column.</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>Examine and note the hundreds; Exit with a quick check.</td>
<td></td>
</tr>
</tbody>
</table>

(Adapted from Miller, Kaffar, & Mercer, 2011)
APPENDIX AA
FAST RENAME MNEMONIC DEVICE

<table>
<thead>
<tr>
<th>F</th>
<th>Find what you’re solving for.</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>Ask yourself, “What are the parts of the problem?”</td>
</tr>
<tr>
<td>S</td>
<td>Set up the numbers.</td>
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<td>T</td>
<td>Tie down the sign.</td>
</tr>
<tr>
<td>R</td>
<td>Read the Problem.</td>
</tr>
<tr>
<td>E</td>
<td>Examine the one column; 10 or more, go next door.</td>
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<tr>
<td>N</td>
<td>Note the ones in the ones column.</td>
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<tr>
<td>A</td>
<td>Address the tens column; 10 or more, go next door.</td>
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<tr>
<td>M</td>
<td>Mark the tens in the tens column.</td>
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<tr>
<td>E</td>
<td>Examine and note the hundreds; Exit with a quick check.</td>
</tr>
</tbody>
</table>

(Adapted from Miller, Kaffar, & Mercer, 2011)
## Implementation Schedule

<table>
<thead>
<tr>
<th>Estimated Sessions</th>
<th>Participants</th>
<th>Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session 1</td>
<td>Participants 1, 2, 3, 4, 5, 6, 7, 8, &amp; 9</td>
<td>Administer Pretests (Conceptual Understanding Pretest, Addition Pretest, Word Problem Pretest, Addition with Regrouping Minute)</td>
</tr>
<tr>
<td>Session 2</td>
<td>Participants 1, 2, 3, 4, 5, 6, 7, 8, &amp; 9</td>
<td>Baseline Probe A</td>
</tr>
<tr>
<td>Session 3</td>
<td>Participants 1, 2, 3, 4, 5, 6, 7, 8, &amp; 9</td>
<td>Baseline Probe B</td>
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<tr>
<td>Session 4</td>
<td>Participants 1, 2, 3, 4, 5, 6, 7, 8, &amp; 9</td>
<td>Baseline Probe C</td>
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<tr>
<td>Session 5</td>
<td>Participants 1, 4, &amp; 7</td>
<td>Lesson 1 (assuming baseline stability)</td>
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<tr>
<td>Session 6</td>
<td>Participants 1, 4, &amp; 7</td>
<td>Lesson 2</td>
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<tr>
<td>Session 7</td>
<td>Participants 1, 4, &amp; 7</td>
<td>Lesson 3</td>
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<tr>
<td>Session 8</td>
<td>Participants 1, 4, &amp; 7</td>
<td>Lesson 4</td>
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<tr>
<td></td>
<td>Participants 2, 5, &amp; 8</td>
<td>Baseline Probe A.2 (assuming Participants 1, 4, &amp; 7 met criterion of 80% on Lessons 1, 2, &amp; 3) and Lesson 1 (assuming baseline stability)</td>
</tr>
<tr>
<td></td>
<td>Participants 3, 6, &amp; 9</td>
<td>Baseline Probe A.2 (assuming Participants 1, 4, &amp; 7 met criterion of 80% on Lessons 1, 2, &amp; 3)</td>
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<tr>
<td>Session 9</td>
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<tr>
<td>Session</td>
<td>Participants</td>
<td>Baseline Probe B.2 (assuming Participants 2, 5, &amp; 8 met criterion of 80% on Lessons 1, 2, &amp; 3) and Lesson 1 (assuming baseline stability)</td>
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<td>Session 30</td>
<td>Participants 1, 4, &amp; 7</td>
<td>Administer Maintenance Probe &amp; Maintenance Measures (Addition Posttest, Word Problem Posttest, Addition With Regrouping Minute)</td>
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<tr>
<td>Participants 3, 6, &amp; 9</td>
<td>Lesson 20</td>
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<tr>
<td>Session 31</td>
<td>Participants 3, 6, &amp; 9</td>
<td>Administer Posttest (Conceptual Understanding Posttest, Addition Posttest, Word Problem Posttest, Addition With Regrouping Minute) &amp; Satisfaction Survey</td>
</tr>
<tr>
<td>Session 32</td>
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<td></td>
</tr>
<tr>
<td>Session 33</td>
<td>Participants 2, 5, &amp; 8</td>
<td>Administer Maintenance Probe &amp; Maintenance Measures (Addition Posttest, Word Problem Posttest, Addition With Regrouping Minute)</td>
</tr>
<tr>
<td>Session 34</td>
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<td></td>
</tr>
<tr>
<td>Session 35</td>
<td>Participants 1, 4, &amp; 7</td>
<td>General Education Teacher administers Generalization Probe within general education classroom</td>
</tr>
<tr>
<td>Session 36</td>
<td>Participants 3, 6, &amp; 9</td>
<td>Administer Maintenance Probe &amp; Maintenance Measures (Addition Posttest, Word Problem Posttest,</td>
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<tr>
<td>Session</td>
<td>Participants</td>
<td>General Education Teacher administers Generalization Probe within general education classroom</td>
</tr>
<tr>
<td>----------</td>
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<td>------------------------------------------------------------------------------------------------</td>
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<tr>
<td>Session 37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Session 38</td>
<td>Participants 2, 5, &amp; 8</td>
<td></td>
</tr>
<tr>
<td>Session 39</td>
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<tr>
<td>Session 40</td>
<td></td>
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</tr>
<tr>
<td>Session 41</td>
<td>Participants 3, 6, &amp; 9</td>
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</tr>
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</table>
## APPENDIX CC

### ADDITION WITH REGROUPING INTERVENTION LESSON MATRIX

#### Addition with Regrouping Intervention Lesson Matrix

<table>
<thead>
<tr>
<th>Lesson</th>
<th>CRA Sequence Level</th>
<th>Lesson Focus</th>
<th>Lesson Goals</th>
</tr>
</thead>
</table>
| 1      | Concrete           | To review the concrete method of multi-digit addition without regrouping | • To review the concept of 2-digit addition without regrouping using concrete objects  
• To promote students’ ability to solve addition word problems that require 2-digit addition without regrouping |
| 2      | Concrete           | To introduce the concrete method of 2-digit addition with regrouping from ones to tens | • To review place value concept related to the identification of tens and ones  
• To promote the students’ ability to use concrete objects to solve 2-digit addition problems that require regrouping  
• To promote the students’ ability to use concrete objects to solve addition word problems that require 2-digit addition with regrouping |
| 3      | Concrete           | To continue the concrete method of 2-digit addition with regrouping from ones to tens | • To promote the students’ ability to use concrete objects to solve 2-digit addition problems that require regrouping  
• To promote the students’ ability to use concrete objects to solve addition word problems that require 2-digit addition with regrouping |
<p>|        |                    | To introduce the concrete method of 3-digit addition with | • To review place value concept related to the identification of hundreds, tens, and ones |</p>
<table>
<thead>
<tr>
<th></th>
<th>Concrete</th>
<th>Representational</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>regrouping from ones to tens</td>
<td>To complete the concrete method of 2-digit or 3-digit addition with regrouping from ones to tens or tens to hundreds</td>
</tr>
<tr>
<td>5</td>
<td>To promote the students’ ability to use objects to solve 3-digit addition problems that require regrouping</td>
<td>To introduce the concept of 2-digit addition with regrouping using pictures of objects</td>
</tr>
<tr>
<td>6</td>
<td>To promote the students’ ability to use concrete objects to solve word problems that require 3-digit addition with regrouping</td>
<td>To promote the students’ ability to solve 2-digit addition word problems that require regrouping using pictures of objects</td>
</tr>
<tr>
<td>7</td>
<td>To promote the students’ ability to use objects to solve 3-digit addition problems that require regrouping</td>
<td>To promote the students’ ability to use pictures of objects to solve 3-digit addition word problems that require regrouping</td>
</tr>
<tr>
<td>8</td>
<td>To promote the students’ ability to use pictures of objects to solve 2-digit, 3-digit, or 3-digit plus 2-digit addition problems that require regrouping</td>
<td>To promote the students’ ability to use pictures of objects to solve 2-digit addition word problems that require regrouping</td>
</tr>
</tbody>
</table>
|   |   | ones to tens or tens to hundreds | regrouping  
|   |   |   |   । To promote the students’ ability to use pictures of objects to solve 2-digit or 3-digit addition word problems that require regrouping  
|   |   |   |   । To introduce the students to “RENAME,” a learning strategy than can be used to solve addition problems without using concrete objects or drawings  
|   |   |   |   । To ensure that students can name each step of the “RENAME” Strategy  
| 9 | Mnemonic Device | To introduce the “RENAME” strategy |   । To introduce the students to “RENAME,” a learning strategy than can be used to solve addition problems without using concrete objects or drawings  
|   |   |   |   । To ensure that students can name each step of the “RENAME” Strategy  
| 10 | Abstract | To introduce the abstract method of 2-digit addition with regrouping from ones to tens |   । To provide the students with practice in using the “RENAME” Strategy when they solve 2-digit addition problems with regrouping  
|   |   |   |   । To promote the students’ ability to solve 2-addition word problems that require regrouping  
| 11 | Abstract | To introduce the abstract method of 3-digit addition with regrouping from ones to tens or tens to hundreds |   । To provide the students with practice in using the “RENAME” Strategy when they solve 3-digit addition problems with regrouping  
|   |   |   |   । To promote the students’ ability to solve 3-digit addition word problems that require regrouping  
|   |   |   |   । To increase students’ speed when solving addition with regrouping problems  
| 12 | Abstract | To continue the abstract method of 2- or – digit addition with regrouping from ones to tens to hundreds (includes |   । To provide the students with practice in using the “RENAME” Strategy when they solve 2-digit, 3-digit, or 3-digit plus 2-digit addition problems with regrouping  

<p>| | | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
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</thead>
</table>
| 13 | Abstract | 3-digit plus 2-digit problems | • To promote the students’ ability to solve 2-digit, 3-digit, or 3-digit plus 2-digit addition word problems that require regrouping  
• To increase students’ speed when solving addition with regrouping problems  
To introduce the abstract method of 3-digit with regrouping from ones to tens and tens to hundreds (regrouping twice)  
To provide the students with practice in using the “RENAME” Strategy when they solve 3-digit addition problems with regrouping of ones and tens  
To promote the students’ ability to solve 3-digit addition word problems that require regrouping of ones and tens  
To increase students’ speed when solving addition with regrouping problems  |
| 14 | Abstract | To introduce the abstract method of 3-digit addition with zeros and regrouping from ones to tens or tens to hundreds | • To provide the students with practice in using the “RENAME” Strategy when they solve 3-digit addition problems with regrouping of ones and tens and include zeros  
• To promote the students’ ability to solve 3-digit addition word problems that require regrouping of ones and tens and include zeros  
• To increase students’ speed when solving addition with regrouping problems  |
| 15 | Mnemonic Device | To introduce the “FAST RENAME” strategy for solving word problems | • To introduce the students to the “FAST RENAME” Strategy  
• To provide the students with further practice in solving addition with regrouping problems  
• To increase the students’ speed when solving addition with regrouping problems  |
<table>
<thead>
<tr>
<th>No.</th>
<th>Topic</th>
<th>Description</th>
<th>Objectives</th>
</tr>
</thead>
</table>
| 16  | Word Problems/Fluency | To extend the application of the “FAST RENAME” strategy to word problems containing extraneous information | • To introduce the students to the concept of solving addition word problems containing extraneous information  
• To provide the students with further practice in solving addition problems using the “RENAME” and “FAST RENAME” strategies  
• To increase the students speed when solving addition with regrouping problems |
| 17  | Word Problems/Fluency | To extend the application of the “FAST RENAME” strategy to word problems with and without extraneous information | • To provide the students with discrimination practice related to solving addition word problems with and without extraneous information  
• To provide the students with further practice in solving addition problems using the “RENAME” and “FAST RENAME” strategies  
• To increase the students’ speed when solving addition with regrouping problems |
| 18  | Word Problems/Fluency | To complete the application of the “FAST RENAME” strategy to word problems with and without extraneous information | • To provide the students with discrimination practice related to solving addition word problems with and without extraneous information  
• To provide the students with further practice in solving addition problems using the “RENAME” and “FAST RENAME” strategies  
• To increase the students’ speed when solving addition with regrouping problems |
| 19  | Word Problems/Fluency | To introduce the concept of student-originated word problems                  | • To provide the students with further practice in solving addition with regrouping problems using the “RENAME” and “FAST RENAME” strategies  
• To provide the students with practice in making up and
<table>
<thead>
<tr>
<th>20</th>
<th>Fluency</th>
<th>To provide practice to fluency</th>
<th>To promote the students’ ability to solve addition with regrouping problems with accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>To increase the students’ speed at solving addition with regrouping problems</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>To provide the students with practice in creating and solving addition with regrouping word problems</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>To provide the students with review practice in discriminating addition problems that require regrouping from addition problems that do not require regrouping</td>
</tr>
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</table>
APPENDIX DD
FIDELITY OF TREATMENT CHECKLIST

Fidelity of Treatment Checklist

Lesson: Group:

For each lesson component included within the lesson, place a check mark in the corresponding box.

<table>
<thead>
<tr>
<th>Advanced Organizer (lessons 1-20)</th>
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<tr>
<td>Describe &amp; Model Stage of Instruction (lessons 1-18)</td>
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<tr>
<td>Guided Practice Stage of Instruction (lessons 1-18)</td>
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<tr>
<td>Independent Practice Stage of Instruction (lessons 1-20)</td>
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<tr>
<td>Problem Solving Stage of Instruction (lessons 1-20)</td>
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<tr>
<td>Fluency Stage of Instruction (lessons 16-20)</td>
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APPENDIX EE

SUMMARY OF DATA BY PARTICIPANT

Summary of Data by Participant

<table>
<thead>
<tr>
<th>Participant</th>
<th>Baseline Probe Mean Score</th>
<th>Intervention Probe Mean Score</th>
<th>Maintenance Probe Score</th>
<th>Generalization Probe Score</th>
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REFERENCES


Bybee, R. W., & Stage, E. (2005, Winter). No country left behind: International comparisons of student achievement tell U.S. educators where they must focus their efforts to create the schools the country needs. *Issues in Science and Technology, 21*, 69-75.


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Assessment Accommodation Options for Classroom Teachers of Students with
Disabilities presented at Conference of Council for Exceptional Children on April 5, 2008

Dissertation Title: Investigating the Effects of Addition with Regrouping Strategy
Instruction Among Elementary Students with Learning Disabilities

Dissertation Examination Committee:
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Committee Member, Kyle Higgins, Ph. D.
Committee Member, Dick Tandy, Ph.D.