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Comparison of Two Distributed Fuzzy Logic Controllers for Flexible-Link Manipulators

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COMPARISON OF TWO DISTRIBUTED FUZZY LOGIC CONTROLLERS FOR FLEXIBLE-LINK MANIPULATORS

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Control methodology:

- Fuzzy Logic Control (FLC) presents a computationally efficient and robust alternative to conventional controllers
- This paper uses FLC in the single-link flexible manipulator.

Structure design:

- The first structure is based on the expert observation of the system;
- The second structure uses the importance information of the inputs to the system output.

Learning ability:

Both approaches are tuned using one of nonlinear programming methods, Simplex.

Comparison:

The paper compares the two FLC structures with a linear quadratic regulator method to prove the effectiveness of FLCs.

Literature survey:

1 Lin, Y. and T. Lee, "An Investigation of fuzzy logic control of flexible robots," Robotica, vol. 11, pp. 363-371, 1993.

2 Arciniegas, J., A. Eltimsahy, and K. Cios, "Fuzzy inference and the control of flexible robotic manipulators," IEEE International Symposium on Intelligent Control, pp. 250-254. 1993.

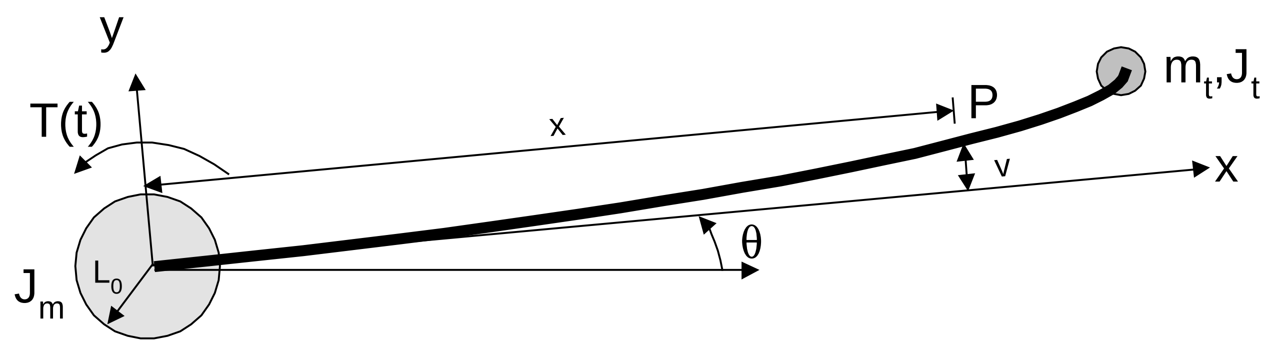
3 Moudgal, V. G., W. A. Kwong, K. M. Passino, and S. Yurkovich, "Fuzzy learning control for a flexible-link robot," IEEE Transactions on Fuzzy Systems, vol. 3, pp. 199-210, 1995.

4 Yoo, B., S. Jeong, and W. Ham, "Hybrid control of flexible manipulator based on fuzzy relations," IEEE International Conference on Fuzzy Systems, pp. 817-823, 1996.

5 Kubica, E. and Wang, D., "A two-stage fuzzy logic controller for a flexible single link robot," International Journal of Robotics and Automation, Vol. 14, No., pp. 9-14, 1999.

6 Trabia, M., 1998, "Tuning of Distributed Fuzzy Logic Controller for a Flexible-Link Robot," Vibration and Noise Control, DE-Vol. 97, ASME, New York, pp. 57-66.

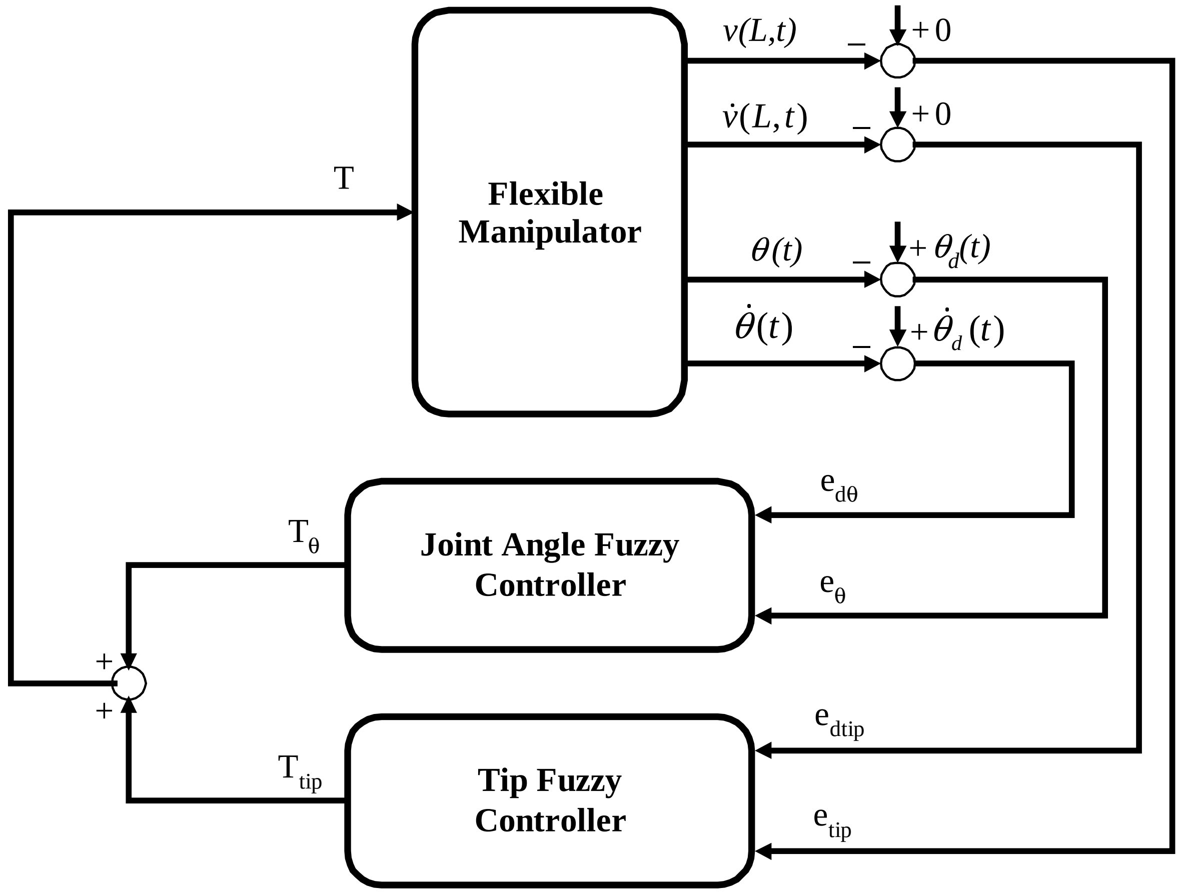
1 **Schematic of a Flexible Manipulator**



- Modeling techniques: Finite element approach and Lagrangian
- Input variable: *T*, the motor torque.
- Output variables: joint angle, $\theta(t)$, joint angular velocity, $\dot{\theta}(t)$, displacement of the tip point, $v_n(t)$and velocity of tip point, $\dot{v}_n(t)$.

2 **PD-like Distributed Fuzzy Logic Controller**

(1) Structure of PD-like distributed fuzzy logic controller



(2) Rule of PD-like fuzzy logic controller

Table I Rules for the Joint Angle Fuzzy logic controller

Displacement Error ⇒ Velocity Error ⇓	N	Z	P
N	N	N	Z
Z	N	Z	P
P	Z	P	P

Table II Rules for the Tip Fuzzy logic controller

Displacement Error ⇒ Velocity Error ⇓	N	Z	P
N	P	P	Z
Z	P	Z	N
P	Z	N	N

4 Importance-based Distributed Fuzzy Logic Controller

(1) Taylor Series Expansion for the system $y = f(x_1, x_2, \cdots, x_n)$

- Normalizing to the form of such that, $[x_1, x_2, \cdots, x_n, y]^T \in [0,1]^n$
- Collect p sample data $[x_{j,1}, x_{j,2}, \cdots, x_{j,n}, y_j]^T \quad \forall j = 1, \cdots p,$
- *Taylor Series Expansion* on a fixed point $[\chi_1, \chi_2, \cdots, \chi_n]^T$:

$$y_j = f(\chi_1, \chi_2, \cdots \chi_n) + \sum_{i=1}^{n} \left.\frac{\partial f}{\partial x_i}\right|_{x_i = \chi_i} (x_{j,i} - \chi_i) + r_j$$

$$y_k = f(\chi_1, \chi_2, \cdots \chi_n) + \sum_{i=1}^{n} \left.\frac{\partial f}{\partial x_i}\right|_{x_i = \chi_i} (x_{k,i} - \chi_i) + r_k$$

- Linear approximation: subtracting the above two equations

$$y_j - y_k = \sum_{i=1}^{n} b_i (x_{j,i} - x_{k,i}) \qquad \text{where } b_i = \left.\frac{\partial f}{\partial x_i}\right|_{x_i = \chi_i}$$

- Importance information: each b_i represents the ratio of the variance of the output variable y with the variance of each input variable x_i.

(2) Least Square Error Algorithm:

- randomly choosing a subset q sample data pairs in the form of, $[x_{j,1}-x_{k,1},\cdots,x_{j,n}-x_{k,n},y_j-y_k]^T$ from the original data set such that q is greater than n.
- Rewriting in a matrix form:

$$\Delta Y = \Delta X B$$

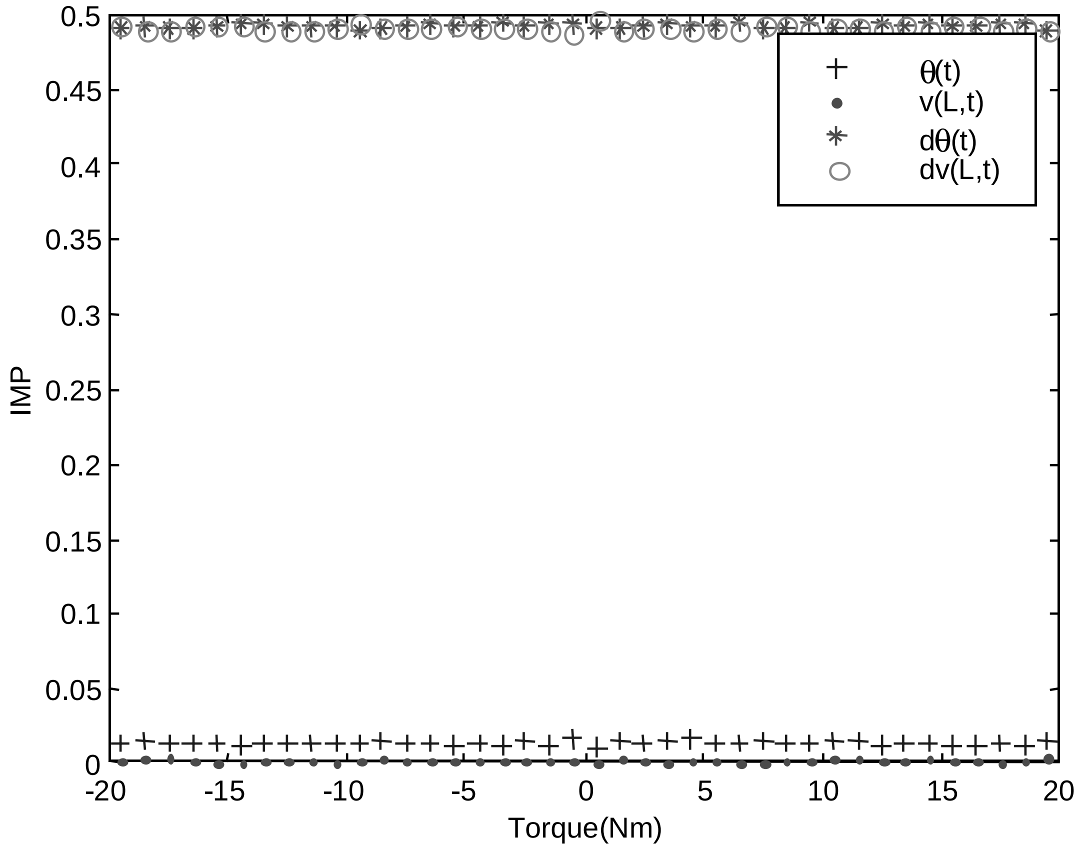
- using the pseudo-inverse formula: $B^* = (\Delta X^T \Delta X)^{-1} \Delta X^T \Delta Y$
- If $\Delta X^T \Delta X$ is a singular matrix, let the i^{th} row vector of matrix ΔX be Δx_i^T and the i^{th} element of ΔY be Δy_i^T, B can be calculated by the following sequential formulations:

$$B_{i+1} = B_i + D_{i+1}\Delta x_{i+1}(\Delta y_{i+1}^T - \Delta x_{i+1}^T B_i) \qquad (8)$$

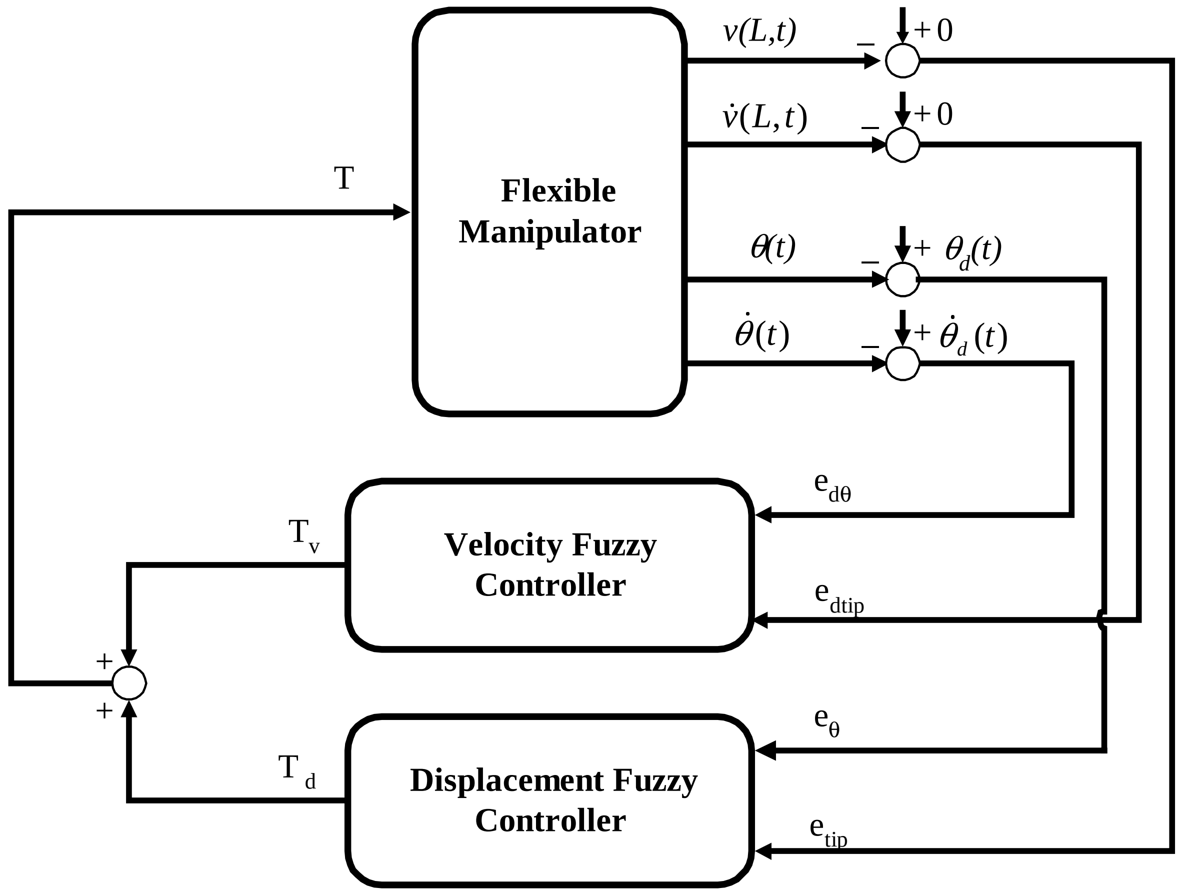
$$D_{i+1} = D_i - \frac{D_i u_{i+1} u_{i+1}^T D_i}{1 + u_{i+1}^T D_i u_{i+1}} \quad i = 0,\cdots,q-1 \qquad (9)$$

- the *degree of importance* of x_i $IMP(x_i)$ is $IMP(x_i) = |b_i| \Big/ \sum_{j=1}^{n} |b_j|$

(3) Analysis the Importance of Variables for Flexible Manipulator



(4) The structure of importance-based distributed fuzzy logic controller



(5) The rules of importance-based distributed fuzzy logic controller

Table III Rules for the Velocity Fuzzy logic controller

Tip Velocity Error ⇒ Joint Velocity Error ⇓	N	Z	P
N	N	N	Z
Z	N	Z	Z
P	Z	P	P

Table IV Rules for the Displacement Fuzzy logic controller

Tip Displacement Error ⇒ Joint Displacement Error ⇓	N	Z	P
N	P	P	Z
Z	P	Z	N
P	Z	N	N

5 Tuning of the distributed fuzzy logic controllers using nonlinear programming

(1) Reason for tuning the fuzzy logic controller:

A good estimate of the location and shape of each input/output variable may be unavailable or can be only obtained by operating the system extensively.

(2) Tuning algorithms:

Simplex Method of Nelder and Mead

(3) Performance index:

$$PI = \sum_{i=1}^{nt}\left(e_{\theta_i}{}^2 + e_{d\theta_i}{}^2 + \left(\frac{e_{tip_i}}{L}\right)^2 + \left(\frac{e_{dtip_i}}{L}\right)^2 \right) + \sum_{i=2}^{nt}\left(\left(e_{d\theta_i} - e_{d\theta_{i-1}}\right)^2 + \left(\frac{e_{dtip_i}}{L} - \frac{e_{dtip_{i-1}}}{L}\right)^2 \right)$$

(4) Optimization Parameters to tune:

Choose three membership functions for each input/output. Then there are three parameters to tune for each variable:

Table V Parameters Describing the Membership Functions of a Fuzzy Variable

Membership Function Parameters ⇒ Membership Function ⇓	c	σ
N	$-c_P$	σ_P
Z	0	σ_Z
P	c_P	σ_P

6 Tuning the PD-like distributed fuzzy logic controller

(1) Physical parameters and mechanical properties of the simulated flexible manipulator

Parameter	**Value**
Link length, L	1.0 m
Density, ρ	0.1 kg/m
Bending stiffness, EI	2.0 Nm^2
Moment of inertia of the hub, J_m	0.05 Kgm^2
Radius of the hub, L_0	0.01 m
Tip mass, m_t	1.0 kg
Tip mass moment of inertia, J_t	10^{-5} Kgm^2

(2) Number of nodes: Eight elements of equal length are used to model the beam.

(3) Simulation trajectory:

- The initial angle of the manipulator is equal to zero
- The desired final angle is equal to one radian at 1 second.
- The desired joint angle motion is a bang-bang acceleration profile.
- The frequency of sampling is hundred samples per second.

(4) Choose the initial Response for the Initial Guess of PD-like fuzzy logic controller.

Variable	Parameter	Value
E_θ	C_P (rad.)	0.8
	σ_P	0.24
	σ_Z	0.12
$E_{d\theta}$	C_P (rad./s)	5.0
	σ_P	1.5
	σ_Z	0.75
T_θ	C_P (Nm)	10.0
	σ_P	3
	σ_Z	1.5
e_{tip}	C_P (m)	0.3
	σ_P	0.09
	σ_Z	0.045
e_{dtip}	C_P (m/s)	5.0
	σ_P	1.5
	σ_Z	0.75
T_{tip}	C_P (Nm)	10.0
	σ_P	3
	σ_Z	1.5

(5) Response of the initial guess of PD-like fuzzy logic controller

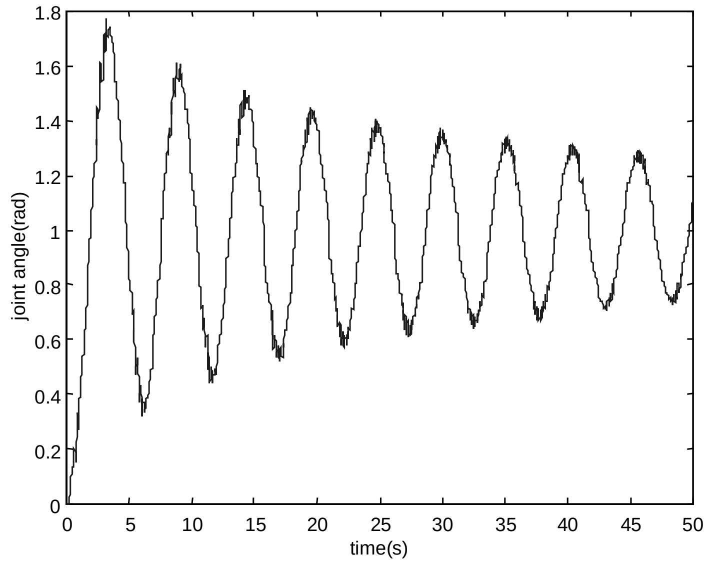


Figure 5 Joint Angle Response

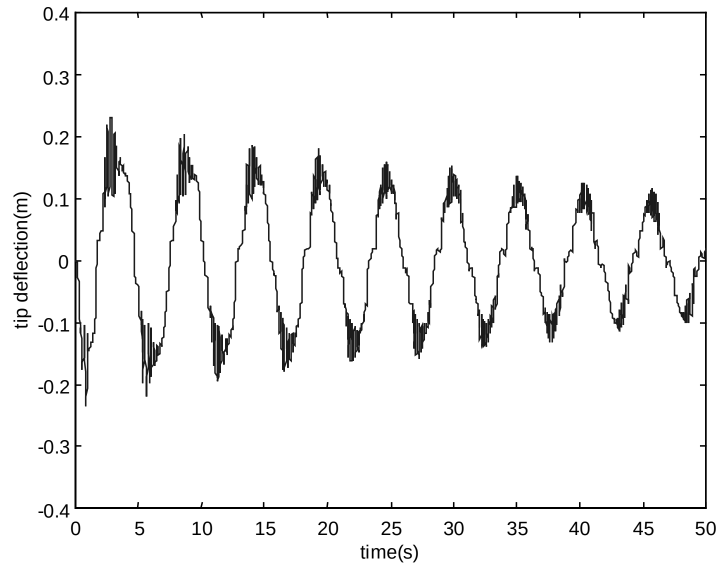


Figure 6 Tip Point Displacement

- The controller, based on these membership functions, needs a long time to stabilize. Its performance is clearly unacceptable.

(6) Response for PD-like fuzzy logic controller after Simplex tuning

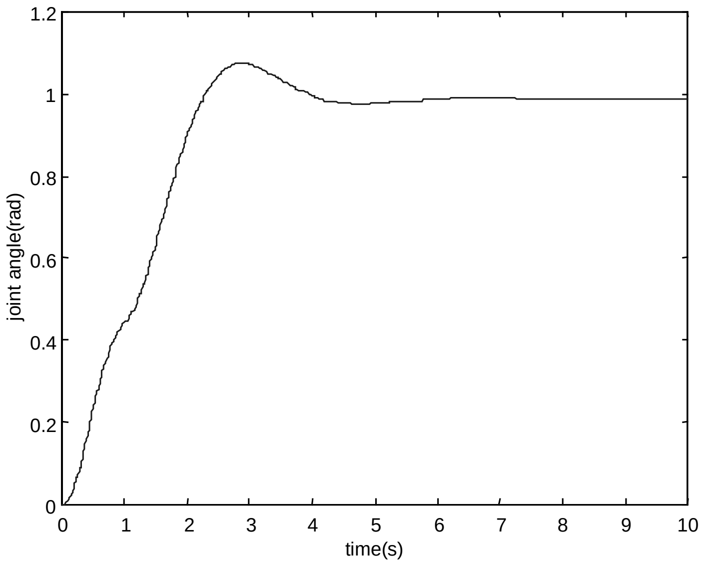


Figure 4 Joint Angle Response

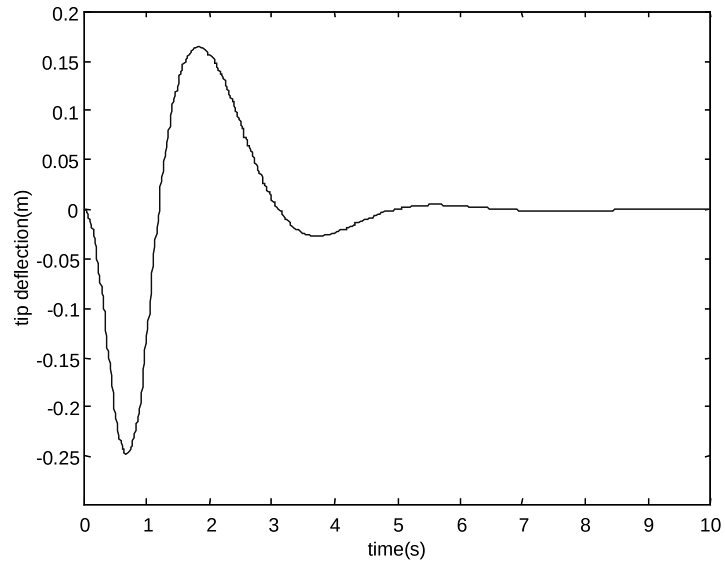


Figure 5 Tip Point Displacement Response

- The algorithm reaches a performance index value of 1.429.
- The performance of the tuned controller shows a marked improvement.
- The tuned controller reaches a steady state at less than seven seconds.

7 Tuning the importance-based distributed fuzzy logic controller

(1) Choose the initial Response for the Initial Guess of Importance-based fuzzy logic controller:

- Results of using the same initial values in PD-like fuzzy logic controller: extremely unacceptable magnitudes of the errors and high frequency responses.
- Choose of initial values for the output of less important fuzzy logic controller: T_d is scaled down by a factor of twenty-five to indicate that the *Displacement Controller* is less important than the *Velocity Controller*.
- All other variables have members similar to that for PD-like fuzzy logic controller.

(2) Response for the Initial Guess of Importance-base fuzzy logic controller

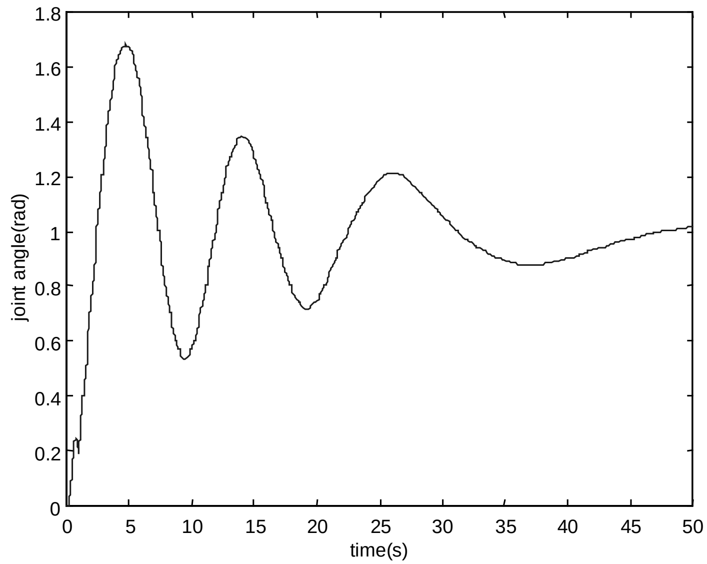


Figure 6 Joint Angle Response

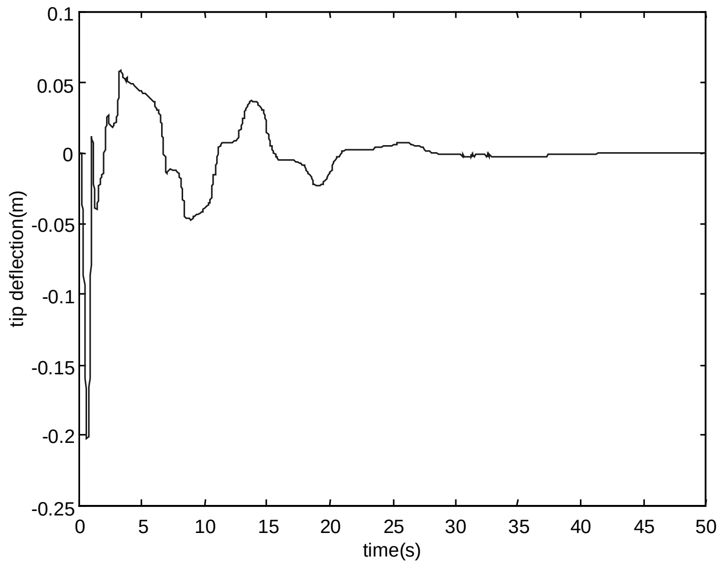


Figure 7 Tip Point Displacement

- The manipulator is under-powered since joint angles are not damped by the end of fifty seconds.
- As an initial guess, this controller is however better than the one used in the PD-Like distributed controller. This observation is generally correct for Importance-Based controllers.

(2) Response for Importance-base fuzzy logic controller after Simplex tuning

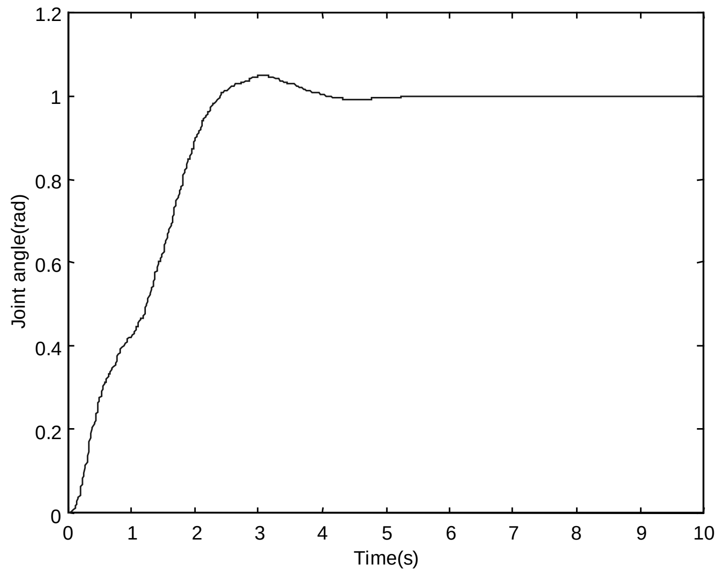


Figure 8 Joint Angle Response

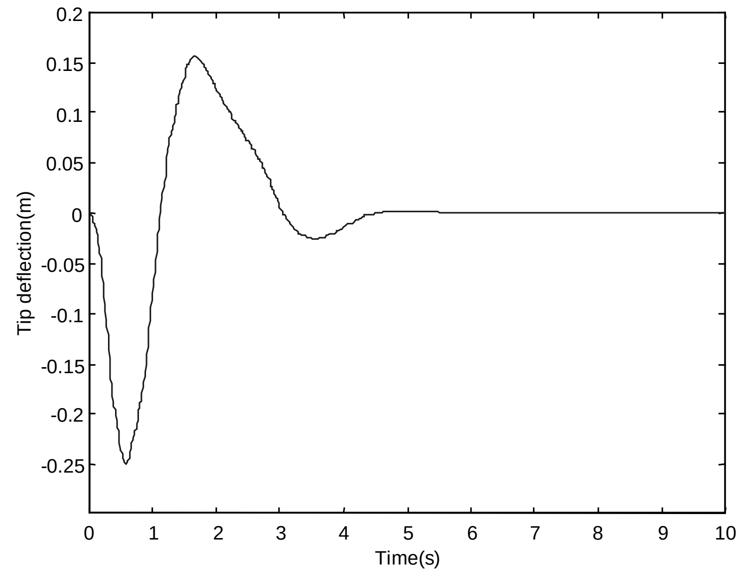


Figure 9 Tip Point Displacement Response

- The search converges to a point with a performance index of 1.483.
- The steady state is reached faster for the tuned Importance-Based controller that for the PD-Like controller with less overshoot.
- It also produces slightly less tip vibration as can be seen in its performance index.

8 Comparing to linear quadratic regulator

(1) Linear quadratic controller (LQR) theory

A LQR can be defined as finding the appropriate state feedback controller that minimizes the following cost functional:

$$J = \int_{t_0}^{t_f} (x^T Q x + u^T R u + 2x^T N u) dt$$

The above equation is subject to the state dynamic constraint,

$$\dot{x} = A x + Bu$$

The optimal control is obtained through feedback with a control law defined as,

$$u = -Kx$$

In this example identity matrices are used for both Q and R. N matrix is null. To properly compare the results to those of the fuzzy logic controllers, a different LQR is created for every time step.

(2) The performance of the LQR controller

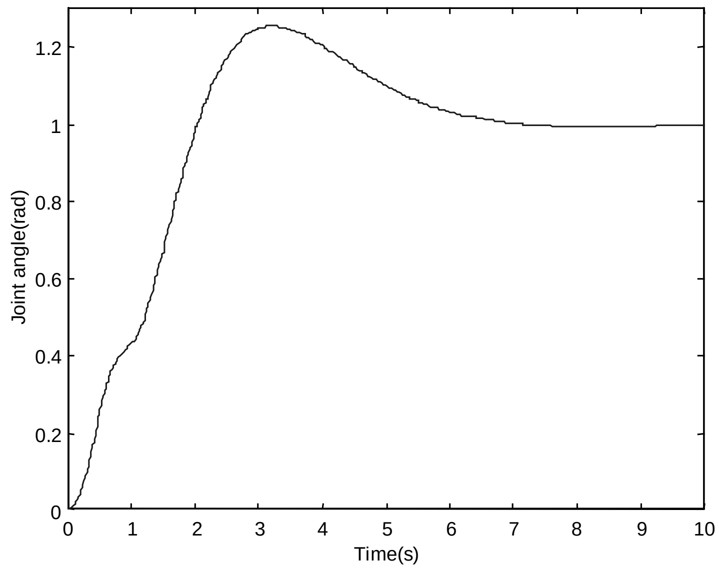


Figure 9 Joint Angle Response

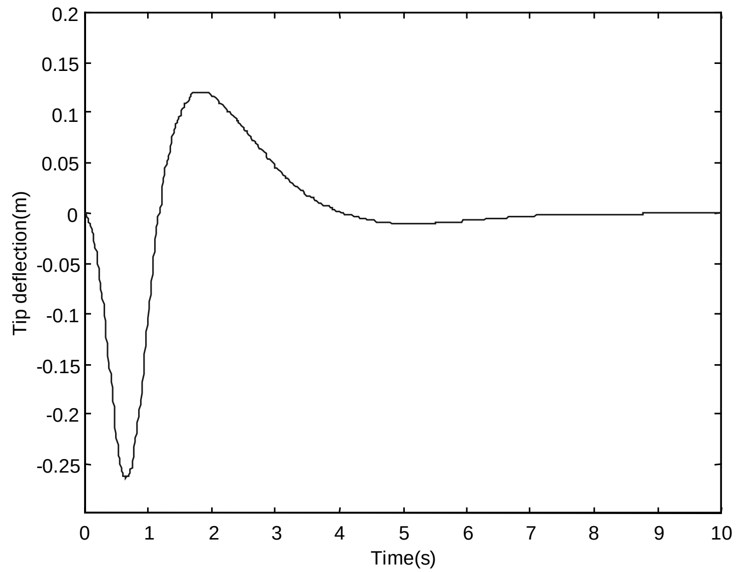


Figure 10 Tip Point Displacement Response

(3) Comparison with fuzzy logic controller:

- The manipulator with LQR exhibits similar behaviour to the two tuned fuzzy logic controllers.
- The joint angle in LQRs case has a larger error. Settling time is close to nine seconds, which is also larger than settling time for the two tuned fuzzy logic controllers.
- The range of the tip deflection is close to the results of the two previous examples.
- LQR controller is a full feedback controller. The error information on the eighteen variables (or more depended on the number of nodes to represent the flexible link) are needed to produce the feedback, which limit the possibilities of implementing this controller.

9 Conclusions:

(1) We explored two possible structures for the distributed fuzzy logic controller of a single-link flexible manipulator.

- *PD-Like* distributed fuzzy logic controller: *Joint Angle* and *Tip* fuzzy logic controllers.
- *Importance-Based* distributed fuzzy logic controller: *Velocity* controller and *Displacement* fuzzy logic controller.

(2) *Importance-Based* controller usually provides better performance based on an arbitrary initial guess.

(3) Tuning the locations and shapes of the membership functions using *Nelder and Mead Simplex* nonlinear programming technique.

(4) The algorithm is satisfactorily applied to both distributed controllers. Importance-Based distributed controller gives a slightly better performance in terms of better settling time, steady state error.

(5) Linear Quadratic Regulator (LQR) is used to compare with the above two distributed fuzzy logic controllers. Tuned fuzzy logic controllers are superior in terms of overshoot, settling time,

(6) The proposed structures of the distributed controllers will be extended to multiple-link flexible manipulators. The procedures presented in this paper can be applied to other systems that are difficult to characterize.