Inversion of input/output map, sliding mode and nonlinear flight control system design

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Inversion of input/output map, sliding mode and nonlinear flight control system design

Romano, James J., M.S.

University of Nevada, Las Vegas, 1989
INVERSION OF INPUT/OUTPUT MAP, SLIDING

MODE AND NON-LINEAR FLIGHT

CONTROL SYSTEM DESIGN

by

James J. Romano

A thesis submitted in partial fulfillment of the requirements for the degree of

Master of Science

in

Electrical Engineering

Department of Computer Science and Electrical Engineering
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May, 1989
ABSTRACT

In this thesis we study the control of multivariable nonlinear systems in the presence of uncertainty. The control of uncertain systems is an interesting and important problem, since an exact mathematical representation of a physical system is practically impossible to obtain. We present the review of two approaches for control system design. These are: (i) Non-Linear Inverse Dynamics and (ii) Variable Structure Control. The inverse control design is based on inversion of an input-output map of the nonlinear systems and the variable structure control system includes essentially a discontinuous controller.

These two schemes of design are applied to a realistic aircraft flight model. The equations of motion of an aircraft are highly nonlinear and the design of a control system for large roll-coupled maneuvers is not a simple task. Analytical derivations of control laws are presented in this thesis for the maneuver of the aircraft using inversion and variable structure control techniques. A short theoretical treatment of each technique is developed and their applications to flight control design are presented.

Several cases of flight conditions are simulated and the resulting data is analyzed and compared. The simulation results are presented to show that, while both
techniques can control the system, the Variable Structure Control has a greater ability to control the model with less error and reduced sensitivity to perturbations.
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Chapter 1

Introduction

The growing requirement for realistic representation of present day complex systems has necessitated the development of nonlinear control theory. Nonlinear modeling allows the more realistic representation of the system throughout the entire spectrum of interest and most importantly, is able to provide a realistic representation in the presence of uncertainty. Unfortunately, the nonlinear models of systems have normally not been used on an universal basis for control system design due to the complexity of the computations and the previous lack of adequate control laws. It has been common to linearize the model of the system at a narrow range of parameters and to provide different models or look up tables for the wider range of the system's environment. The original limited analytical methods for nonlinear systems like the describing function and phase portraits for second-order systems have been supplanted by a number of results of greater power and flexibility. These include input output stability theory, i.e. Non-Linear Inverse Dynamics (NID), and advanced Lyapunov based methods such as Variable Structure Control
(VSC). The advancement of the computational devices in terms of speed, memory, and precision have allowed more flexible use of these nonlinear techniques.

It is generally very difficult to control a multi-variable system due to the fact that every input controls more than one output and every output is controlled by more than one input. Often it is desired that decoupled control is designed into the closed loop system. The multi-variable system is said to be decoupled if each input controls only one output and each output is controlled by no more than one input. The difficulty of decoupling is exponentially increased when nonlinear systems are involved. In many cases of nonlinear systems it is not possible to directly decouple the system outputs and inputs.

The method of Non-Linear Dynamics necessitates the development of a control law of intermediate variables. This will allow decoupled control of the outputs through an intermediate step. The first phase of the NID technique is to develop the derivatives of the appropriate control outputs. The desired output element equations are differentiated a sufficient number of times until a term containing one or more elements of the original control input appears. The second phase concerns the development of the control law. The control law is developed containing the intermediate control vector with each control output dependent upon only one of these intermediate control
elements. This is the integrator decoupled form. The final phase is to negatively feed back the intermediate variables to allow control of the output variables. The control law is developed with these intermediate variables as the new external control point.

The method of Variable Structure Control (VSC) is a high speed switching feedback control technique which has been determined to be an effective and robust method of controlling nonlinear systems. The control gains of this technique switch according to the location with reference to some chosen switching surface. There are two main phases to the VSC technique, a sliding mode and a reaching mode. The sliding mode consists of some chosen switching surface about which it is desired that the system maintain a narrow bound. The state trajectory is allowed to 'slide' along this surface but not to depart from the surface for any region away from some infinitesimal tolerance about the surface. During this sliding phase the motion of the surface is insensitive to parameter variations and disturbances of the system [32]. In the reaching phase the state trajectory begins from any arbitrary initial condition and is forced to the switching surface. If the state trajectory is 'above' the surface the state velocity vector will 'force' the state trajectory 'down' to the surface and if the state trajectory goes 'below' the surface, the velocity vector will 'force' the state trajectory 'up' to the switching surface. The velocity vector will change dependent upon where in the
target area the state trajectory is located in relation to the switching surface. The actual chosen velocity vectors must not abruptly switch at the switching surface or system perturbations and instabilities may result. Some continuous function must be inserted that allows a smooth transition from one vector to another.

Flight control systems have long been designed utilizing the theory of piecewise linear models due to the relative simplicity of linear system theory. The actual mathematical representation of aircraft models, however, is based upon nonlinear dynamics. For minor maneuvers or development of the model using a limited range of flight parameters the linearization is an adequate model. However, over the entire flight spectrum or for rapid or large maneuvers nonlinear dynamics must be considered for an adequate description of the system. In maneuvers of any angle of attack most linearized models are inadequate to describe any but the most basic or limited maneuvers. The need for the implementation of nonlinear equations lies in the basis that the fundamental dynamical equations required to adequately describe flight are nonlinear in the inertia terms and in the kinematical variables. Additionally, the external forces, especially the aerodynamic ones (i.e. wind), may contain inherent nonlinearities. Also, some current high performance aircraft have control and stability difficulties over some portion of their flight envelopes. These difficulties arise from highly nonlinear aerodynamic
and propulsion characteristics, from undesirable coupling between axes, and from the extreme range of flight conditions. The cliche that the real world is ultimately nonlinear is nowhere more evident than in the investigation of flight. As mentioned at the beginning of this paragraph, although the equations of motion of an aircraft are highly nonlinear, airplane stability and control analysis have, in the past and in present day practical aircraft design and modeling, been largely limited to linear dynamic system theory. A practical application of nonlinear dynamics would aid in more realistically simulating the actual operation of the aircraft.

Several methods of developing control methods for nonlinear models have been developed. In this paper the two major techniques introduced above will be identified, described, and compared utilizing a standard nonlinear swept wing fighter aircraft model. The system model that provides the basis for technique portrayal in this thesis is the seven state variable model (constant speed) that has often been used to simulate airplane maneuvers.

A mathematical model of a physical system contains a set of equations that is a true1 image of the physical systems with the assumptions and approximation contained in

---

1. True is a relative term that depends upon the requirements of the system. No model is absolutely an exact duplicate of the physical system, the degree of accuracy will depend on the end need.
the model. The set of \( n \) variables that defines the state of the system is the state vector, and the corresponding \( n \)-dimensional space is the state space. Some or all of the state variables are selected, either directly or indirectly, as outputs. In addition to these chosen state variables, named outputs, there is usually associated with a system a second set of variables called inputs. These inputs are variables outside the province of the system. An output of one system may be the input to another, or to itself in certain cases. The state variables are unique functions of the non-autonomous inputs and of the initial conditions of the system [35].

Realistic control outputs and inputs were chosen for control system design. The pilot of a fighter aircraft most directly would like to set a yaw angle (\( \beta \)), a bank angle (\( \phi \)), and a pitch angle (\( \theta \)) so these were taken as the controlled outputs. For control inputs, the aileron control, \( \delta a \), the elevator control, \( \delta e \), and the rudder control, \( \delta r \), are used.

This development portion of this thesis differs in several areas from previous works on the various subjects. The control of the outputs of \( \beta \), \( \phi \), and \( \theta \) has not been considered in previous papers [25,27,28,32,34]. Previous works have applied nonlinear VSC theory only to linear helicopter models [25,27] or to \( \alpha \), \( \beta \), and \( \phi \) control [32,34]. The outputs \( \beta \), \( \phi \), \( \theta \) are the final values that are directly
considered by the pilot of the aircraft when he wishes to
perform maneuvers. The choice of \( \theta \) instead of \( \alpha \) seems to be
more appropriate from the pilot's point of view. The inverse
control law in this thesis differs from previous works [36]
in that integral feedback is applied. Although Stengel has
applied inverse control techniques to flight control system
his control law differs from the one posed here. The
derivation of the control law is fully developed in this
thesis with respect to the three controlled outputs \( \beta, \phi, \theta \).

Both the NID and the VSC technique were applied to the
nonlinear aircraft model referenced above. Each technique
was demonstrated using two cases of final values
\((\beta=0^\circ, \phi=45^\circ, \theta=75^\circ \text{ and } \beta=0^\circ, \phi=75^\circ, \theta=45^\circ)\)
in three different model conditions. These conditions are a nominal case, an
initial error case, and a robust case where the model was
developed under one flight regime set of parameters and
flown under a second set of flight parameters. An
interesting development occurred in this model simulation.
Commanded high final angles demonstrated an instability in
the model in all three axes. Further investigation
demonstrated that when angles above 90° were required for
the \( \phi \) and \( \theta \) final angles, the \( B \) matrix became singular and
the model became unstable. It must be emphasized that this
is not a limitation of inverse control law. The aircraft
model is poorly defined for \( \theta = \pi/2 \) by this choice of Euler
sequence of rotations. Previously model simulations [32]
have demonstrated adequate high \( \phi \) angle utilizing angle of
attack, $\alpha$, as one of the control variables. In this instance, however, pitch angle, $\theta$, attained a maximum of only 30°. Therefore, for this simulation all final angles were kept to a maximum of no more than 75°.

The NID technique handled the nominal case with little or no problem. All parameters were within desired maximum, final values were reached, inputs were realistic and the system maintained stability. The non-zero initial conditions case repeated the success of the nominal case. It appeared that the initial errors did not cause any great difficulty for the controller. The robust case caused some difficulty for the NID technique. The final values, rates, and control inputs were within adequate values but the errors appear to maintain a constant or slightly increasing oscillatory profile. This oscillatory curve is outside of the desired limits for some of the control outputs. A further experimentation was conducted by inserting a sinusoidal perturbation in the $\alpha$ model equation. In this case the inability of the NID technique to handle the perturbation was accentuated by the increasing sinusoidal error noted in the $\phi$ and $\theta$ axes.

The VSC technique handled the nominal case with little problem. All parameters were within desired maximums, final values were reached, and inputs were realistic. The control motions seem to be sensitive to initial errors and the initial errors had to be reduced to bring the control inputs
within the desired maximums. There were no other adverse effects from the initial errors in that rates were within limits and the error results were well within desired limits. This sensitivity is not a detriment in that the model could be constructed to start at an initial position of zero error in all cases, negating the sensitivity to initial errors. The VSC technique handled the robust case with little problem. A slight perturbation was noted in the $\theta$ angle when approaching the final value of 45°, although this perturbation did not appear to affect the rest of the simulation. Final values, rates, errors, and control inputs were well within desired limits. As described, in the VSC technique a 'buffer' must be used between the positive and negative sliding states. If this buffer is not used a 'bang-bang' effect results. One simulation was run to show this 'bang-bang' effect. Without the transition the control inputs were extremely out of limits and sharply oscillated back and forth. An additional case was run in the same manner as the NID technique above. The same sinusoidal perturbation of the $\alpha$ model equation as was used in the NID technique was inserted. Despite this perturbation, the error rate for all three axes reduced to a near zero steady state.

A direct comparison of NID and VSC control techniques has not been previously considered in works on either subject. The comparison of perturbation effects between the two techniques is especially relevant and unique to this
thesis. The ability to handle nonlinear systems is well documented for both techniques, however the investigation of perturbation effects in this thesis presents a direct substantiation of the merits of the VSC technique over the NID technique. Analysis of the data indicates both the NID and VSC techniques appear adequate to handle this limited case of nonlinear system modeling.

The Non Linear Inverse Dynamics Technique is a proven technique for controlling specific cases of nonlinear systems. This technique does not have a great ability to maintain accurate control under widely varying parameters and under any perturbations of the system. The method may or may not work for a specific case or may be affected by slight changes in coefficients. This was demonstrated by the oscillatory error noted in the sinusoidal experimentation. The Variable Structure Control was a more robust technique in all cases. The theory indicates that it should allow for wide changes in system operation and should be relatively impervious to system perturbations and changes. The robust case is the most demanding case for the two techniques and it that case it appeared that the VSC technique can more accurately follow the model output. The one additional simulation was attempted to accentuate any differences between the two techniques. As stated previously, a sinusoidal input was added to the state equation in both model simulations and similar parameters were run. The results of this simulation show that the VSC
technique more faithfully followed the actual model perturbations and steady state value while the NID technique maintained a greater error rate that appeared to be increasing.

Several further areas of interest were indicated by this research. The model itself needs further study in order to adequately demonstrate high angle performance. Limited study determined $\theta$ is obviously dependent upon the $\alpha$ control and must be taken into account for any simulation. Previous studies have not investigated high pitch ($\theta$) angle and investigation of the $\beta/\alpha$ control should also focus upon the high angle regime. In the high angle regime the effects of nonlinearity will be more evident. The alpha sinusoidal input is not an accurate representation of external forces on the aircraft model such as wind. Investigation of the effect of random wind gusts at the previously mentioned high angles should be accomplished to better determine the relative merits of the Non Linear Inverse Dynamics and Variable Structure Control techniques. A thorough investigation of these effect should adequately define this model and amply compare the techniques. Furthermore, sensor and actuator dynamics and noise should be included in any advanced study.
Chapter 2

Inversion of Input-Output Map
and Control System Design

2.1 Introduction

The main font of the theory of nonlinear inverse
dynamics is based in nonlinear decoupling mechanics. It is
generally very difficult to control a multi-variable system
due to the phenomenon that every input controls more than
one output and that every output is controlled by more than
one input. Therefore some compensator must be introduced so
that the multi-variable system becomes decoupled in the
sense that every input controls only one output and every
output is controlled by only one input. Consequently, a
decoupled system can be considered as consisting of a set of
independent single variable systems. Several authors have
aptly described the decoupling theory in relation to
nonlinear theory [1-8]. In nonlinear multi-variable theory
the desired outputs are able to be controlled by some
combination of the original input variables in the same
basic manner as in a standard linear control system. As in
any realistic multi-variable system every input controls
more than one output and every output is controlled by more than one input. However, in the nonlinear decoupling theory (as in multi-variable linear state feedback), one must select a control law that insures that each element of the output is independent from all but one element of the input variables. This is not possible to do in most cases with the original input variables and a control law of intermediate variables must be developed. This procedure, called nonlinear inverse dynamics (NID) has been well documented by [2,8].

This technique offers the potential for providing a much higher level of performance representation throughout the entire flight regime over the competing designs developed using linearizing assumptions for narrow regimes of flight parameters.

The nonlinear dynamic controller more accurately represents the involved forces and moments that arise in response to large state and control perturbations [8]. The NID control laws also will allow the desired state variables to be directly controlled (although not directly by the original input variables). State variables of interest to the investigator can be decoupled. In most cases, the bank angle, pitch angle, and yaw angle are of direct interest to the pilot of an aircraft. This NID system has the desirable property of decoupling the rolling, directional, and pitch responses. In the following sections
the decoupling of roll, pitch, and sideslip in rapid nonlinear airplane movements is derived.

2.2 Theory Development

In order to effectively develop a useful system one must first determine the composition of this modeled system. In the case of inverse dynamics it is required to have the system of interest in the following recognizable form [8].

\[ x = A(x) + B(x)u \]  \[ y = Cx \]

where \( A(x) \) is a \((n \times 1)\) vector, \( B(x) \) is an \((n \times m)\) matrix, and \( Cx \) is an \((m \times n)\) matrix. This is the desired form, however, the general form of any nonlinear system is of a different format.

\[ \dot{x}' = f(x', u') \]  \[ y = Cx' \]

where \( x' \) is an \((n \times 1)\) vector, \( u' \) is an \((m \times 1)\) vector, \( y \) is an \((1 \times 1)\) vector and \( C \) is an \((1 \times n)\) matrix. A transformation is required to convert equations 2.3 and 2.4 to the more readily utilized 2.1 and 2.2 system. The most commonly discussed method to accomplish this transformation is to develop the derivatives of the appropriate control
inputs, the inverse dynamics, and then insert them within the original system dynamics.

These inverse dynamics are developed by differentiating the selected elements of $y$, the desired output elements, a sufficient number of times until a term containing one or more elements of $u$, the original input, appears. Since only $m$ outputs can be independently controlled with $m$ inputs the dimension of the selected $y$ output must be equal to the dimension of the input $u$. This control law must be developed in the format below [2]

$$u = F(x) + G(x)w$$ \[2.5\]

where $w$ is the intermediate control vector and each element of $y$ is dependent upon only one and only one element of $w$.

Stated in general terms, the differentiation operator required for this transformation can be written as [8]

$$L_A^k(x) = \left[ \frac{\partial}{\partial x} L_A^{k-1}(x) \right] A(x)$$ \[2.6\]

where

$$L_A^0(x) = x$$
Inserting this operator into the \( y \) vector elements \( j \) of equation 2.4 and performing the differentiation operations we have (\( C_i \) is the \( i \)th row of \( C \))

\[
\dot{y}_j = C_j \dot{x} = C_j A(x) + C_j B(x)u = C_j L_{A^1}(x) \tag{2.7}
\]

\[
\ddot{y}_j = C_j \ddot{x} = C_j \left[ \frac{\partial}{\partial x} L_{A^1}(x) \right] A(x) + \left[ \frac{\partial}{\partial x} L_{A^1}(x) \right] B(x)u \tag{2.8}
\]

\[
= C_j L_{A^2}(x)
\]

\[
\dddot{y}_j = C_j \dddot{x} = C_j \left[ \frac{\partial}{\partial x} L_{A^2}(x) \right] A(x) + \left[ \frac{\partial}{\partial x} L_{A^2}(x) \right] B(x)u \tag{2.9}
\]

\[
= C_j L_{A^3}(x)
\]

\[
\dddot{y}_j = C_j \dddot{x} = C_j \left[ \frac{\partial}{\partial x} L_{A^3}(x) \right] A(x) + \left[ \frac{\partial}{\partial x} L_{A^3}(x) \right] B(x)u
\]

where \( d_j \) is the order of the differentiation with elements of \( u \) in each element of \( y_j \).

This differentiation will be continued until terms for each element of \( y \) emerge containing one or more \( u \) elements. Let, for each \( x \),

\[
C_j \left[ \frac{\partial}{\partial x} L_{A^{d_j-1}}(x) \right] B(x) \neq 0 \tag{2.10}
\]

where \( d_j \) is the order of the differentiation with elements of \( u \) in each element of \( y_j \).

To simplify the notation for the further development of the transformation we define the following substitutions
\[ A_j^z(x) = C_j \left[ L_{A,j}(x) \right] \]  \[ B_j^z(x) = C_j \left[ \frac{\partial}{\partial x} L_{A,j-1}(x) \right] B(x) \]

Define

\[ A^z = \begin{bmatrix} A^z_1(x) \\ \vdots \\ A^z_m(x) \end{bmatrix} \quad B^z = \begin{bmatrix} B^z_1(x) \\ \vdots \\ B^z_m(x) \end{bmatrix} \]

Utilizing these substitutions in the final derivation of \( y \) we arrive at

\[ y_d = A^z(x) + B^z(x)u \quad [2.12] \]

where \( y_d = \begin{bmatrix} y_1(x_1), \ldots, y_m(x_m) \end{bmatrix} \)

A sufficient condition for the existence of an inverse system model to the original system is that \( B^z \) must be non-singular [34]. If \( B^z \) is non-singular throughout the state space of interest then the decoupled control law may be developed as follows. \( y_d \) must be set equal to \( w \) which is called the integrator decoupled form. \( w \) is now required to be the new external control input and this provides us with the following output equations

\[ y_d = w = A^z(x) + B^z(x)u \quad [2.13] \]

\[ B^z(x)u = w - A^z(x) \quad [2.14] \]

\[ u = B^{-1}(x) \left[ w - A^z(x) \right] \]

The desired system equations must be in the form stated in 2.1 and 2.2.

\[ \dot{x} = A(x) + B(x)u \quad [2.1] \]

\[ y = C(x) \quad [2.2] \]
Combining the control law from equation 2.14 and utilizing simple mathematical substitution gives

\[ \dot{x} = A(x) + B(x)B^{-1}(x)[w - A^T(x)] \]
\[ u = B^{-1}(x)[w - A^T(x)] \]

Define

\[ F(x) = B^{-1}(x)A^T(x) \quad [2.15] \]
\[ G(x) = B^{-1}(x) \quad [2.16] \]

Then

\[ \dot{x} = A(x) + B(x)B^{-1}(x)w - B(x)B^{-1}(x)A^T(x) \]
\[ u = -B^{-1}(x)A^T(x) + B^{-1}(x)w \]

The end results are the system equations and control law desired

\[ \dot{x} = [A(x) - B(x)F(x)] + B(x)G(x)w \quad [2.17] \]
\[ u = -F(x) + G(x)w \quad [2.18] \]

with \( w \) as the new external control point.

This gives the output equation as

\[ y_1^{(d)} = w_i \]
\[ W = [w_1, ..., w_m]^T \]
Figure 2.1: Non-linear inverse dynamic control system

The closed loop system \([2.17, 2.18]\) is now decoupled. The system is directly controlled by the intermediate control inputs \(w\). It is interesting to note that the output responses for \(y_i\) are described by independent linear differential equations. There are several control system design techniques (such as pole placement, optimal control, frequency domain analysis, etc) well known for linear techniques. One can use any one of these schemes to control each of the selected output variables \([38-40]\).
Chapter 3

Description of Variable Structure Control

3.1 Introduction

Variable Structure Control is a high speed switching feedback control technique providing an effective and robust means of controlling nonlinear plants. The origination of this technique appears to lie in the 'bang-bang' control theory and is an outgrowth of the relay control theory. This technique has been called Variable Structure Control because the control gains (and thus the closed loop system dynamics) switch according to the state location with reference to a surface in the state space [20]. The control is designed to force the motion of the system towards this desired surface and once intercepting the surface, the trajectory is confined to this surface. In order to maintain the state on this surface the control must be a very high speed switching law. This technique has become practical due to the advancements in rapid computer technology including large scale, inexpensive memories and the development of high-speed low noise switching circuitry
and several authors have described aspects of techniques development [9-22]. There are two phases to the Variable Structure Control: (1) the Reaching Phase and (2) the Sliding Phase.

The Reaching Phase is the portion of the control law in which the trajectory begins from any arbitrary (within limits\(^2\)) initial condition and moves towards the discontinuity surface (switching surface)[21]. No system will be able to control in an infinitely large state space, so the state space must be limited to some region surrounding the switching surface. As long as this region is limited, stability can be guaranteed. As noted in Figure 3.1 the reaching phase requires the state velocity vector to be directed towards the switching surface from every portion of the target area. This state velocity vector has the dual purpose of initially forcing the state trajectory to the switching surface and, once there, of maintaining the state trajectory on this surface.

The Sliding Phase is the portion of the motion in which the state trajectory or describing point cannot ideally stray from the switching surface. Once on the discontinuity surface, the describing point evidently cannot move along any trajectory away from that surface over any

---

2. No technique will be able to handle an infinite distance or removal from the desired state. The more robust a technique the better able it is to handle a greater perturbation.
period. Actually, due to imperfections in the actuator such as time delay, a motion always starts that returns the describing point to the switching surface [20]. During this sliding phase the motion of the surface is insensitive to parameter variations and disturbances of the system [21]. The desired surface is named the switching surface since the control law must have one gain when the state trajectory is 'above' the surface and another gain when the state trajectory is 'below' the surface. In theory all succeeding motion is limited to this surface. In practical applications the switching delays or hysteresis cause a 'chattering' effect and it is required to limit the state trajectory to a very minute and transitory volume surrounding the surface.

![Figure 3.1: VSC Phases](image)

3.2 Theory Development

In this section we will consider a general system that is nonlinear in the state vector and linear in the control vector and that can be decoupled by state variable
feedback. We will demonstrate a discontinuous control law which will perform trajectory tracking in the closed loop system. A general description of the nonlinear system is

\[
x(t) = A(t,x) + \Delta A(t,x) + (B(t,x) + \Delta B(t,x))u(t)
\]

\[
y(t) = C(t,x)
\]

where

\[
C(t,x) = (c_1(t,x),...,c_m(t,x))^T
\]

\[
y = [y_1,...,y_m]^T
\]

where the state vector \(x(t)\) is a \((n \times 1)\) vector, the control vector \(u(t)\) is a \((m \times 1)\) vector, \(y(t)\) is a \((m \times 1)\) vector, and \(C\) is an \((m \times n)\) vector. \(A, B,\) and \(C\) are analytic functions of \(x\). In general they are time varying due to presence of nonlinearities in the original system. The functions \(\Delta A\) and \(\Delta B\) are continuously differentiable with respect to \(x\) and \(t\). The nominal system is obtained by setting \(\Delta A = 0\) and \(\Delta B = 0\) in the above equations.

As previously mentioned, the systems considered here are those which can be decoupled by state variable feedback in the nominal case (\(\Delta A\) and \(\Delta B\) are zero). For deriving the control law the derivatives of \(y_i(t)\) must be computed in the same manner as the Non-Linear Inverse Dynamic case indicated. This will provide the final derivation of \(y\) in the same format as equation 2.12 [34].
\[
\dot{y}_i = \frac{\partial c_i}{\partial t} + \frac{\partial c_i}{\partial x} \left[ A(x,t) + \Delta A(x,t) + [B(x,t) + \Delta B(x,t)]u(t) \right]
\]

\[
y_i^{(d)} = A^{*}(x,t) + \Delta A^{*}(x,t) + [B^{*}(x,t) + \Delta B^{*}(x,t)]u(t)
\]

where \( A^* \) and \( B^* \) are similarly defined as in 2.11 and

\[
L_{A^i} c_i(x,t) = \left[ \frac{\partial}{\partial t} c_i(x,t) \right] + \left[ \frac{\partial}{\partial x} c_i(x,t) \right] A(x,t)
\]

\[
L_{A^i} = L_{A^{(i-1)}}
\]

d_i \text{ when } \left[ \frac{\partial}{\partial x} L_{A^{(d^i-1)}} c_i(x,t) \right] B(x,t) \neq 0 \text{ for each } x,t

give

\[
\Delta A^{*} = \left[ \frac{\partial}{\partial x} L_{A^{(d^i-1)}} c_i(x,t) \right] \Delta A(x,t)
\]

\[
\Delta B^{*} = \left[ \frac{\partial}{\partial x} L_{A^{(d^i-1)}} c_i(x,t) \right] \Delta B(x,t)
\]

Every entry of the control \( u(t) \) has the form

\[
u_i(t,x) = \begin{cases} u_i^+(t,x) \text{ with } s_i(x) > 0 \\ u_i^-(t,x) \text{ with } s_i(x) < 0 \end{cases} \quad i = 1, \ldots, m
\]

where \( s_i(x) = 0 \) is the ith switching surface of the switching surface
\[ S(x) = [s_1(x), \ldots, s_m(x)]^T = 0 \quad [3.5] \]

The switching surface is a \((n - m)\) dimensional manifold determined by the intersection of switching surfaces \(s_i(x) = 0\). These switching surfaces are designed so as to have stability when the state trajectory remains on this switching surface (sliding mode). This is the first phase of VSC design in the construction of a switching surface so that the system remaining on the surface demonstrates the desired behavior. Two techniques available for determination of the existence of the switching surface are the method of equivalent control [20] and the method of Filippov [22]. The method utilized here is the Filippov technique, which uses the theory of Lyapunov for derivation of the control law. This method will be utilized in the following chapter to develop the control law. The design requires the construction of feedback gains which will drive the state trajectory during the reaching phase to the switching surface and maintain on it thereafter.

The state trajectory to be tracked can be represented by [32]
\[ y_r(t) = (y_{r1}(t), \ldots, y_{rm}(t))^T \quad [3.6] \]
and a vector is defined
\[ z = (\tilde{y}_1, \ldots, \tilde{y}_1^{(d^1-1)}; \tilde{y}_m, \ldots, \tilde{y}_m^{(d^m-1)})^T \quad [3.7] \]
where the tracking error \( \tilde{y} \) is denoted by
\[ \bar{y}_i = (y_1 - y_{r1}, \ldots, y_m - y_{rm})^T \]  \hspace{1cm} [3.8]

\[ \bar{y}_i^{(j)} = \frac{d^j \bar{y}_i}{dt^j} \]

The theory, [32], that will be applied later in this thesis to the model simulation requires the choice of the switching surface as a stationary hyperplane

\[ S(z, z_s) = Gz + G_0z_s = 0 \]  \hspace{1cm} [3.9]

where \( G_0 = \text{diag}(g_{10}, \ldots, g_{m0}) \), \( G = \text{diag}(G_1, \ldots, G_m) \)

\[ G_1 = (g_{11}, \ldots, g_{1u}, 1), \quad g_{1u}, 1 = 1 \]  \hspace{1cm} [3.10]

When the trajectory of the state space is in the vicinity of 3.9 it is said to be in sliding mode. The matrices \( G_0 \), and \( G \) are chosen so that the state trajectory \( z(t) \) is asymptotically stable about the origin during the sliding phase of the control law. We have assumed a switching surface of \( S(z, z_s) = 0 \) and will also utilize integral feedback of the form

\[ \dot{z_s} = Lz = \bar{y} \]  \hspace{1cm} [3.11]
\[ \bar{y} = y_1 - y_{r1} \]

To develop the switching surface we must differentiate \( S \) and substitute equation 3.11 into the result.
The above mathematical equations, when combined, give the following switching surface (\( \bar{y} = Lz \)), \( i = 1, \ldots, m \)

\[
\bar{y}_1 \bar{y}_1^{(d_1-1)} + g_1 \bar{y}_1^{(d_1-1)} + \ldots + g_1 \bar{y} = 0
\]  

[3.13]

The coefficients \( g_{i,j} \) are chosen such that the system is asymptotically stable and the state trajectory \( z(t) \) approaches 0 as time approaches \( \infty \) after the state trajectory has initially intercepted the switching surface.

The controller that causes the state trajectory to move toward the switching surface has been chosen as a Lyapunov function of the form [32]

\[
W = \sum_{i=1}^{m} |s_i|
\]  

[3.14]

where \( S = (s_1, \ldots, s_m)^T \), and this controller is chosen so that the derivative of \( W \) is less than or equal to some negative \( \varepsilon \) less than zero whenever \( S \neq 0 \).

\[
\dot{W} \leq -\varepsilon < 0
\]  

[3.15]

The gradient of \( W \) is not defined when \( S = 0 \) and

\[
\dot{W}(S(t)) = \bar{y}^T \bar{S}
\]  

[3.16]

for all \( \bar{y} \) belonging to the set \( \partial W \), the generalized gradient of \( W \) [10].
The derivative of $S$ is given by

$$
\dot{S} = (GE + G_o L)z + A^a(t, x) + (B^a(t, x) \\
+ \Delta B^a(t, x))u(t) + \Delta A^a(t, x) - Y
$$

where

$$
E = \text{diag}(E_i) \quad i = 1, \ldots, m
$$

and

$$
E_i = \begin{bmatrix}
0 & 1 & 0 & \ldots & 0 \\
0 & 0 & 1 & \ldots & 0 \\
0 & 0 & 0 & \ldots & 1 \\
0 & 0 & 0 & \ldots & 0
\end{bmatrix}
$$

and

$$
Y(t) = (y_r^{d_1}(t), \ldots, y_r^{d_m}(t))^T
$$

To make $W$ negative, one must choose $u(t)$ of the form

$$
u(t) = (B^a(t, x))^{-1}[-A^a(t, x) + Y - (GE + G_o L) \\
- k(\text{sgn}(S))]
$$

or

$$
u(t) = F(x, z, y, t) - kB^a(t, x)\text{sgn}(S)
$$

where $\text{sgn}(S) = (\text{sgn}(s_1), \ldots, \text{sgn}(s_m))^T$ and

$$
\text{sgn}(s_i) = \begin{cases}
1, & s_i > 0 \\
0, & s_i = 0 \\
-1, & s_i < 0
\end{cases}
$$

substituting 3.21 into 3.17 gives us

$$
\dot{S} = -k(\text{sgn}(S)) + \Delta A^a(t, x) + \Delta B^a(t, x)
$$
for \( S \neq 0 \) and no uncertainty about the \( A \) and \( B \) matrixes (\( \Delta A^* = 0 \), and \( \Delta B^* = 0 \)).

\[
\dot{S} = -k \, \text{sgn}(S) \tag{3.24}
\]

The above calculations demonstrate the theory of the controller 'forcing' the state trajectory to the desired switching surface and once the state trajectory attains this switching surface the controller maintains the state trajectory on the surface to within acceptable limits in the nominal case. For cases including uncertainty it is necessary to restrict the uncertain function to insure

\[
\dot{W}(t) < 0 \text{ for } S \neq 0 \tag{3.25}
\]

\( k \) must be chosen to insure that 3.25 is followed. It must be assumed that there are functions \( \tau_0, \tau_1(t,x) \) and \( \tau_2(t,x) \) such that

\[
\begin{align*}
|| \Delta A^*(t,x) + \Delta B^*(t,x)F(t,Y,x) || & \leq \tau_1(t,x) \tag{3.26} \\
|| \Delta B^*(t,x)(B^*(t,x))^{-1} || & \leq \tau_2(t,x) < \tau_0 < 1
\end{align*}
\]

\( k \) is then chosen as

\[
k \geq (1 - \tau_1(t,x))^{-1}(\varepsilon + \tau_2(t,Y,z,x)), \quad \varepsilon > 0 \tag{3.27}
\]
Therefore $S$ converges to 0 in finite time and remains 0 afterwards. Thus $z(t) \to 0$ as $t \to \infty$, which implies that $y(t) \to y_r(t)$ as $t \to \infty$. This is indicated by, for all $S \neq 0$ and almost all $t \cap (0,\infty)$

$$\dot{W}(t) \leq -\varepsilon$$  \hspace{1cm} [3.28]

The trajectory is confined to the surface $S(z, z_s) = 0$ after a finite period of time in spite of the uncertainty.
Chapter 4
Development of Aircraft Model

There have been several investigations of nonlinear theory in relation to aircraft flight [23-31, 36-37]. There are several roll-coupled and stability problems that must be addressed for realistic simulation of flight. Flight characteristics change at high angle and maneuvers require cross coupling of control inputs for accurate representation. Not all nonlinearities inherent in flight flight will be addressed here. For our comparison a standard nonlinear model without effects such as wind gusts, high angle, or slow speed will be used.

The aircraft model that will be used in this thesis to demonstrate the two nonlinear techniques is taken from [33]. The following expansion is a review of this model that can be utilized to demonstrate a nonlinear control technique that is valid over the entire flight envelope for this system.

The non linear equations for the standard aircraft model equating to our general system [2.1, 2.2] are taken from the seven state variable model that is commonly used to
simulate aircraft maneuvers. The example used contains all the nonlinear rotational coupling terms, but only a relatively few simple nonlinear aerodynamic effects [34]. The model is sufficiently sophisticated to provide an adequate example for this discussion on the NID and VSC techniques.

The general form of a nonlinear system can be stated as

\[
\dot{x} = A(x) + B(x) u \quad [4.1]
\]

\[
y = C(x)
\]

as mentioned in our previous section's discussion on NID techniques. The actual seven state equations for the aircraft model to be used in this simulation are

\[
\begin{align*}
\dot{p} &= L_{qq} q + L_{rr} r + (L_{qg} q + L_{r} r) \Delta \alpha + L_{q} p - I_{q} q r + L_{r} \Delta \delta a \\quad + L_{r} \delta r \\
\dot{q} &= M_{a} \Delta \alpha + M_{q} q + I_{q} p r - M_{q} p + M_{q}(g/v)(\cos \theta \cos \phi - \cos \theta) \\
&\quad + M_{q} \Delta \delta e \\
\dot{r} &= N_{q} q + N_{r} r + N_{p} p + N_{p} \Delta \alpha - I_{p} p q - N_{r} q + N_{r} \Delta \delta a + N_{r} \Delta \delta r \\
\dot{\alpha} &= q - p \delta + Z_{e} \Delta \alpha + (g/v)(\cos \theta \cos \phi - \cos \phi) + Z_{e} \Delta \delta e \\
\dot{\beta} &= Y_{q} \beta + p(\sin \alpha + \Delta \alpha) - r \cos \alpha + (g/v)(\cos \theta \sin \phi) + Y_{q} \Delta \delta a \\
\dot{\phi} &= p + q \tan \phi \sin \phi + r \tan \phi \cos \phi \\
\dot{\theta} &= q \cos \phi - r \sin \theta 
\end{align*}
\]

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\[ y = C(x) \]

where \( x \) is equal to all seven output variables (\( p, q, r, \alpha, \beta, \varphi, \theta \)) and \( C \) is a vector (filter) allowing only the three desired controlled variables \( \beta, \varphi, \text{ and } \theta \) to pass.

This model ignores all external forces such as wind, turbulence, and also assumes that velocity remains constant during all maneuvers. While this is not realistic in that velocity will change during any large aircraft maneuver, the model is sufficiently accurate for demonstration of the NID technique. There are three control inputs: \( \delta a \), the aileron input for maneuvers around the longitudinal axis, \( \delta e \), the elevator input for maneuvers around the lateral axis, and \( \delta r \), the rudder input for maneuvers around the vertical axis. Since there are three control inputs there must be no more than three controlled outputs as indicated by our requirement that the dimension of selected output must be equal to the dimension of the input.

The pilot of the standard fighter jet aircraft is normally concerned with the ability to change aircraft pitch and to roll the aircraft. The use of the forward/aft position of the stick changes the pitch \( \theta \) (climb angle) of the aircraft or indirectly the angle of attack \( \alpha \), the left/right position of the stick changes the bank angle \( \varphi \).

---

3. angle of attack \( \alpha \) is directly affected by velocity, where \( \alpha = \tan^{-1} \frac{w}{u} \), \( w \) being the vertical component of velocity and \( u \) being the longitudinal component of velocity.
of the aircraft, and the rudder provides control over the yaw $\beta$ (sideslip angle), of the aircraft. The pilot more directly sets a pitch angle than angle of attack so for the three controlled outputs this model will use $\beta$, $\phi$, and $\theta$.

![Aircraft Axis](image)

Figure 4.1: Aircraft Axis

Many additional factors could be inserted for more accurate representation of aircraft flight parameters. Outside forces such as wing gusts and turbulence are not considered. Fuel consumption will shift center of gravity and change flight parameters during flight. Large altitude differences as in takeoff and landing will also change aircraft performance parameters. Additionally, configuration changes such as spoilers, flaps, and speedbrakes will also significantly change flight characteristics. These are not required for this discussion of techniques, but would be required to apply these technique to an actual aircraft flight model.
Chapter 5

Development of the NID Model

5.1 Introduction

The initial problem is to apply the NID techniques to the aircraft model described previously. The desired command variable set must be selected. In the previous chapter we have selected

\[
Y = \begin{bmatrix}
\beta \\
\phi \\
\theta
\end{bmatrix}
\] [5.1]

The aircraft model must be decoupled to allow the formulation of the control law. Once decoupled the selected intermediate control variables must be negatively feed back to control the model.

5.2 Problem Formulation

To simplify the nonlinear equations of motion of the aircraft one term will be substituted for all terms that are
not affected by any of the inputs (δa, δe, or δr) in the equations.

\[
p = f_p + L_{a\delta} \delta a + L_{r\delta} \delta r
\]

\[
q = f_a + M_{\delta e} \delta e
\]

\[
r = f_r + N_{\delta a} \delta a + N_{\delta r} \delta r
\]

\[
a = f_a + Z_{\delta e} \delta e
\]

\[
b = f_{\delta} + Y_{\delta a} \delta a
\]

\[
s = f_s
\]

\[
\theta = f_{\theta}
\]

Two of the desired output equations have no control inputs and according to the described NID technique, all three desired output equations must be differentiated as demonstrated in equation [2.6] in the previous section. The controlled output variables have been picked as β, φ, and θ and they have been rewritten in block form.

\[
Y = \begin{bmatrix}
\beta \\
\phi \\
\theta
\end{bmatrix}
\]

[5.1]
This output matrix must be differentiated until each element has one or more of the control variables included in the equation.

\[
\dot{Y} = \begin{bmatrix} \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} f_\theta \\ f_\phi \end{bmatrix} \quad \ddot{Y} = \begin{bmatrix} \partial/\partial t f_\theta \\ \partial/\partial t f_\phi \end{bmatrix}
\]

The required level of differentiation is performed on the three output equations to obtain the necessary control inputs.

Performing the differentiation on the \( \dot{\phi} \) term.

\[
\dot{\phi} = (Y_\phi + p(\sin\alpha_0 + \Delta \alpha) - r \cos\alpha_0 + (g/v)(\cos\theta \sin\phi) + Y_{\delta \alpha} \delta \alpha)
\]

\[
\partial/\partial \dot{\phi} = \ddot{\phi} = Y_\phi \dot{\phi} + p(\sin\alpha_0 + \alpha - \alpha_0) + \dot{p} \partial - r \cos\alpha_0
\]

\[
+ (g/v)(\cos\theta \cos\phi \dot{\phi})
\]

\[
- (g/v)(\sin\theta \sin\phi \dot{\phi})
\]

\[
\ddot{\phi} = Y_\phi [f_\phi + Y_{\delta \alpha} \delta \alpha] + [f_r + \Lambda_\delta \delta \alpha + \Lambda_\delta \delta r](\sin\alpha_0 + \Delta \alpha)
\]

\[
+ p(f_\alpha + Z_\delta \delta \epsilon) - (f_r + N_\delta \delta \alpha + N_\delta \delta r)(\cos\alpha_0)
\]

\[
+ (g/v)(\cos\theta \cos\phi f_\phi) - (g/v)(\sin\theta \sin\phi f_\phi)
\]

---

4. The input \( Y_{\delta \alpha} \delta \alpha \) in this equation has not been entered at this point for simplicity although it rightly should be part of this equation. However in the first differentiation it will not add any elements (\( \partial/\partial t Y_{\delta \alpha} \delta \alpha = 0 \)).
The differentiated $\theta$ term must be separated into the A and B elements. The B term is further separated in the individual control input elements.

\[ a_1 = Y_b[f_b + Y_b \delta a] + f_p(\sin \alpha + \Delta \alpha) + p \delta \alpha - f_r(\cos \alpha) \]
\[ + \left( \frac{g}{v} \right)(\cos \alpha \cos \phi \delta\alpha) - \left( \frac{g}{v} \right)(\sin \alpha \sin \phi \delta \alpha) \]
\[ b_1 = \left[ (Y_b + Y_b \delta a) + (\sin \alpha + \Delta \alpha) L_{\delta a} - N_{\delta a} \cos \alpha \right] \delta a, \]
\[ \left[ p z_{\delta a} \right] \delta e, \]
\[ \left[ L_{\delta r} (\sin \alpha + \Delta \alpha) - N_{\delta r} \cos \alpha \right] \delta r \]

Performing the differentiation on the $\phi$ term

\[ \dot{\phi} = p + q \tan \theta \sin \phi + r \tan \theta \cos \phi \]

\[ \frac{\partial}{\partial t} \dot{\theta} = \ddot{\theta} = \dot{\phi} = p + q \tan \theta \sin \phi + q \sec^2 \theta \sin \phi \dot{\theta} + q \tan \theta \cos \phi \dot{\theta} \]
\[ + r \tan \theta \cos \phi + r \sec^2 \theta \cos \phi \dot{\theta} - r \tan \theta \sin \phi \]
\[ \ddot{\theta} = (f_p + L_{\delta \alpha} \delta a + L_{\delta r} \delta r) + (f_r + M_{\delta e} \delta e) \tan \theta \sin \phi \]
\[ + q \sec^2 \theta \sin \phi \delta \theta + q \tan \theta \cos \phi \delta \theta + (f_r + N_{\delta \alpha} \delta a + N_{\delta r} \delta r) \tan \theta \cos \phi \]
\[ + r \sec^2 \theta \cos \phi \delta \theta - r \tan \theta \sin \phi \delta \theta \]

The differentiated $\phi$ term must be separated into the A and B elements. The B term is further separated in the individual control input elements.

\[ a_2 = f_p + f_q \tan \theta \sin \phi + q \sec^2 \theta \sin \phi \delta \theta + q \tan \theta \cos \phi \delta \theta \]

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+ \upsilon_{r} \tan \theta \cos \phi + r \sec \theta \cos \phi \delta \theta - r \tan \theta \sin \phi \delta \phi

\begin{align*}
b_2 &= \begin{bmatrix} L_a + N_a \tan \theta \cos \phi \delta \alpha, \\
M_a \tan \theta \sin \phi \delta \epsilon, \\
L_r + N_r \tan \theta \cos \phi \delta r \end{bmatrix} \\
\text{Performing the differentiation on the } \theta \text{ term}
\end{align*}

\begin{align*}
\dot{\theta} &= q \cos \phi - r \sin \phi \\
\therefore \dot{\theta} &= q \cos \phi - q \sin \phi \delta \phi - r \sin \phi - r \cos \phi \delta \phi
\end{align*}

\begin{align*}
\ddot{\theta} &= (f_\alpha + M_\alpha \delta e) \cos \phi - q \sin \phi \delta \phi \\
&\quad - (f_r + N_\alpha \delta \alpha + N_r \delta r) \sin \phi - r \cos \phi \delta \phi
\end{align*}

The differentiated \( \theta \) term must be separated into the A and B elements. The B term is further separated in the individual control input elements.

\begin{align*}
a_3 &= f_\alpha \cos \phi - q \sin \phi \delta \phi - f_r \sin \phi - r \cos \phi \delta \phi \quad [5.9] \\
b_3 &= \begin{bmatrix} N_\alpha \sin \phi \delta \alpha, \\
M_\alpha \cos \phi \delta \epsilon, \\
N_r \sin \phi \delta r \end{bmatrix}
\end{align*}
The equation [2.10] is now satisfied at the second differentiation of the output equations. Placing the results in the form of equation [2.11] provides us with the following format.

\[ \ddot{Y} = A^* + B^*u \]  \hspace{1cm} [5.10]

where

\[ \ddot{Y} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} u = \begin{bmatrix} \ddot{\beta} \\ \ddot{\phi} \\ \ddot{\theta} \end{bmatrix} \]  \hspace{1cm} [5.11]

Next substitute the intermediate input variable \( w \) for \( \ddot{Y} \)

\[ w = \ddot{Y} \Longleftrightarrow \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} \ddot{\beta} \\ \ddot{\phi} \\ \ddot{\theta} \end{bmatrix} \]  \hspace{1cm} [5.12]

To conclude this phase of the development, the inverse dynamics control law can now be written in the described in the beginning of the previous section.

\[ u = B^{*-1} \begin{bmatrix} -A^* + w \end{bmatrix} \]  \hspace{1cm} [5.13]
5.3 Derivation of Control Law

The next step in the aircraft model progress is to develop the feedback control model. At this point standard linear feedback methods can be used. Three term control, or Proportional (P) Integral (I) Derivative (D) control appears to be one of the best methods for feedback in this system. In the PID controller the error signal is multiplied by some constant k to yield a signal which is negatively fed back to the original process. The dynamical behavior is altered by varying the magnitude of this constant. In the optimum range of this case there will always be a steady state error.\(^5\) Introducing an integral action will change the system from a type 0 to a type 1 and provide 0 or very small steady state error. The error signal is integrated within the controller and even a very small error eventually produces a corrective signal of sufficient amplitude to correct the error. The PI controller, while settling to 0 steady state error can have a large overshoot prior to settling.\(^6\) The derivative of the PID controller action acts as an anticipating device of the required zero steady state point, and begins to damp any overshoot so as to smoothly

\(^5\) In either extreme of this proportional case with too small a constant the system will behave sluggishly and with too great a constant the system will behave in an unstable manner.

\(^6\) The more quickly the system is required to reach the steady state zero point, the greater the overshoot will be.
settle to the desired zero error steady state within the required time, with little or no overshoot. [39].

As indicated in Figure 5.1 the standard PID controller selected to be used in the example aircraft model inner loop is described by the equation

\[ \frac{P_s s + P_1 + P_0}{s} \]  \[ 5.14 \]

feedback control. In addition provision for some sort of control in the outside loop must be made to be able to externally control this model. As indicated in Figure 5.2 control input and derivatives of the control input must be provided in order to adequately control this model. For simplicity, the control input of the outer loop will be patterned after the inner loop aircraft model.
Figure 5.2: Simplified Aircraft Model with Control

\[ W = -P_2(\dot{Y} - \dot{Y_c}) - P_1(Y - Y_c) - P_0 \int(Y - Y_c) + \ddot{Y_c} \quad [5.15] \]

where the output \( Y \) and control output \( Y_c \) contain the following elements.

\[ Y = \begin{bmatrix} \beta \\ \phi \\ \theta \end{bmatrix} \quad \quad Y_c = \begin{bmatrix} \beta_c \\ \phi_c \\ \theta_c \end{bmatrix} \]
substituting $\ddot{Y}$ for $W$ in equation [3.15]

\[
(\ddot{Y} - \dot{Y}_c) + P_2(\ddot{Y} - \dot{Y}_c) + P_1(Y - Y_c) + P_0 \int (Y - Y_c) = 0
\]  \[5.17\]

taking the first derivative of this equation for ease of
the following mathematical manipulation and substituting $E$
for $Y - Y_c$

\[
\ddot{E} + P_2 \ddot{E} + P_1 \dot{E} + P_0 E = 0
\]  \[5.18\]

and solving for the constants in the transformed
characteristic equation

\[
s^3 + P_2 s^2 + P_1 s + P_0 = 0
\]  \[5.19\]

\[
(s + \lambda_e)(s^2 + 2\gamma \omega_n e + \omega_n e^2) = 0
\]

\[
P_0 = \lambda_e \omega_n e^2
\]

\[
P_1 = 2\gamma \omega_n e \lambda_e + \omega_n e^2
\]

\[
P_2 = \lambda_e = 2\gamma \omega_n e
\]

The command generator is a third order system where

\[
(s^3 + G_2 s^2 + G_1 s + G_0) Y_c = G_0 Y^* \]  \[5.20\]

and

\[
G_0 = \lambda_e \omega_n e^2
\]

\[
G_1 = 2\gamma \omega_n e \lambda_e + \omega_n e^2
\]

\[
G_2 = \lambda_e = 2\gamma \omega_n e
\]
It is desired to have the poles of the model in the same plane as indicated in Figure 5.3 and for this depicted case the variables must be set equal as indicated in the following equation.

\[ w_a = \frac{\lambda}{\gamma} \]  \[ 5.21 \]

*with \( \gamma = .707 \) this is actually 1.007\( \times \lambda \)

**Figure 5.3: Poles of the Model**

Values must be chosen for the aircraft model and for the outer loop controller. In both cases the damping ratio \( \gamma \) will be selected as .707. A damping ratio of .707 in an underdamped second-order system has been found to be satisfactory over a number of years in positioning systems. Response is more accurate than with critical damping and overshoot is negligible. \( w_a \) is the damped natural frequency of the characteristic polynomial. Although the characteristic equation is of a higher order and the terms \( \gamma \) and \( w_a \) do not have exactly the same meaning, the expression

\[ \text{The } (s^2 + 2\gamma w_n + w_n^2) \text{ portion of the characteristic polynomial.} \]
can still apply for dominant roots. All three roots of the third order polynomial can be placed the same distance from the imaginary axis by use of equation [3.21] as indicated in Figure 5.2. Since all three of these roots are equidistant from the imaginary axis the resulting response is a exponentially (from the real root) decreasing oscillatory (from the complex conjugate pair) term. With proper placement of these roots the system will have the desired settling time and overshoot.

For the total model the terms of the controller are of more importance than the terms of the inner loop aircraft model. \( \lambda \) will be selected as 9 (equating \( \omega_n \) to 12.7298) but, within limits, any variation would not significantly affect the outcome. The controller \( \kappa_c \) will be set to 2.5 (equating \( \omega_n \) to 3.5361). This controller should provide a quicker response to force the model to the condition desired.

---

8. The closer to the imaginary axis the less stable, or the more quicker the response to an input.
Chapter 6

Development of the VSC Model

6.1 Introduction

The Variable Structure Control method must be applied to the aircraft model discussed in the preceding chapter. Much of the decoupling accomplished to apply the Non-Linear Inverse Dynamics technique must also be accomplished in order to apply the VSC technique. Once the model has been decoupled then a sliding mode must be chosen that meets the robustness requirements. Once controlled in the sliding mode, the feedback system must be insensitive to certain parameter variations and disturbances. This sliding mode must be maintained by the control law whose feedback gain coefficients switch on hypersurfaces defined in the state space. The control law, $u$, is chosen so that trajectories are attained near the intersection of the hypersurfaces [25].

The design process is composed of two major steps. The sliding surface is chosen so that the system has the desired properties in the sliding mode and the control law,
u, is chosen to guarantee reaching and existence of the sliding mode over the feasible part of the state space.

6.2 Problem Formulation

The control inputs are selected as described in the preceding section where

\[ Y = \begin{bmatrix} \dot{\beta} \\ \dot{\phi} \\ \dot{\theta} \end{bmatrix} \]  \hspace{1cm} [6.1]

the same differentiation as accomplished in the previous NID chapter is required to decouple the desired output variables.

\[ \ddot{Y} = \begin{bmatrix} \ddot{\beta} \\ \ddot{\phi} \\ \ddot{\theta} \end{bmatrix} = A^2 + B^2 u \]  \hspace{1cm} [6.2]

in this case we will set

\[ z_s = E \hspace{1cm} \text{where} \hspace{1cm} E = Y - Y_c = \begin{bmatrix} \beta - \beta_c \\ \phi - \phi_c \\ \theta - \theta_c \end{bmatrix} \]  \hspace{1cm} [6.3]

and as previously discussed in Chapter 5 we will develop the inverse control law
\[ u = B^*^{-1} \left( -A^* + w \right) \]  \hspace{1cm} [6.4] 

\[ \ddot{y} = w \]

\[ w = P_2 \dot{E} + P_1 E + P_0 \int E \, dt \]

\[ u = B^{* -1} \left[ -A^* + P_2 \dot{E} + P_1 E + P_0 z_s \right] \]

### 6.2.1 Sliding Mode

In the Variable Structure Control law we need to choose a sliding surface \( S \). For the aircraft model, we choose the switching surface as

\[ S = b_0 E + \dot{E} + g_0 z_s = 0 \]  \hspace{1cm} [6.5] 

This is graphically depicted in Figure 6.1.

\[ \begin{align*} 
S &= 0 \\
\text{Figure 6.1: Sliding Surface} 
\end{align*} \]

During the sliding phase \( S = 0 \). Therefore, differentiating \( S \) gives

\[ \dot{S} = b_0 \dot{E} + \ddot{E} + g_0 E = 0 \]  \hspace{1cm} [6.6] 

- 49 -
since $z_s = E$.

In terms of the characteristic equation, one has

$$s^2 + b_o s + g_o = 0 \quad [6.7]$$

where

$$g_o = \omega_{ae}^2 \text{ and } b_o = 2\gamma \omega_{ae} \quad [6.8]$$

And this provides us with a sliding surface that happens to be quite similar to our development of the Non-Linear Inverse Dynamics model (and incidentally command generator). If $\omega_{ae} > 0$ and $\gamma > 0$ then the system is asymptotically stable about the origin and $E(t)$ approaches 0 as $t$ approaches $\infty$, whenever the trajectory lies on the surface $S = 0$ [32].

### 6.2.2 Reaching Mode

The reaching phase requires that in all regions above and below the desired hyperspace the derivative of the state space force the trajectory towards the switching surface. Figure 6.2 indicates the differing $S$ and derivative of $S$ needed to meet this requirement.
If
\[ S = b_0 E + \dot{E} + g_0 z_s \]  \hspace{1cm} [6.5]
then
\[ \dot{S} = b_0 \dot{E} + \ddot{E} + g_0 E \]  \hspace{1cm} [6.9]

Substituting (6.2) in 6.9 gives
\[ \dot{S} = b_0 \dot{E} - \ddot{y}_r + g_0 E + A^* + B^* u \]  \hspace{1cm} [6.10]

In order to have the system trajectory always tend towards the switching surface the combination of the state space and the state space derivative must be a negative value. This is determined by the following with derivative of \( V \) the control law.
\[ V = S^2 \]  \hspace{1cm} [6.11]
\[ V = S^T S \]

\[ \dot{V} = 2SS < 0 \]

Since we are utilizing three control outputs we take the Lyapunov function in the following form

\[ V = |s_1| + |s_2| + |s_3| \quad [6.12] \]

\[ = \begin{bmatrix} sgn_{s_1}, sgn_{s_2}, sgn_{s_3} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} \]

\[ = (sgnS)^T S \]

where

\[ (sgnS) = \begin{bmatrix} sgn_{s_1} \\ sgn_{s_2} \\ sgn_{s_3} \end{bmatrix} \quad sgn_{s_1} = \begin{cases} 1, s_1 > 0 \\ 0, s_1 = 0 \\ -1, s_1 < 0 \end{cases}, \ldots \]

Taking the derivative of (6.12), gives

\[ \dot{V} = (sgnS)^T \dot{S} \quad [6.14] \]

\[ = (sgnS)^T \left[ b_o \dot{E} - \dot{Y}_r + g_o E + A^2 + B^T u \right] \]

In accordance with the theory discussed in the previous chapter the control input \( u \) is of the following form

\[ u = B^{*-1} \left[ -b_o \dot{E} + \dot{Y}_r - g_o E - A^2 - ksgnS \right] \quad [6.15] \]
\[ F = -kB_{-1} \text{sgn} S \]

Substituting (6.15) into (6.14), gives

\[ \dot{V} = (\text{sgn} S)^T \begin{bmatrix} -k \text{sgn} S \end{bmatrix} \]

\[ \dot{V} = -k \left[ (\text{sgn} s_1)^2 + (\text{sgn} s_2)^2 + (\text{sgn} s_3)^2 \right] \]

so if \( S \geq 0 \), one has

\[ \dot{V} \leq -k \]  

This is all predicated upon no uncertainty. With bounded uncertainty, the above variable can be made negative provided that \( k \) is chosen sufficiently large and thus stability can be maintained in face of the uncertainty. Since we do not have available adequate information concerning the uncertainty of the model in lieu of a mathematical outcome we used a digital simulation and observation of the results. A \( k = 10 \) was sufficient for adequate model simulation and was used in each Variable Structure Control model run.

6.2.3 Chattering

An abrupt transition from a positive to negative value will cause a chattering problem with the model. A device must be implemented that allows for some smooth transition. Figure 6.3 demonstrates the transition phases of the switching surface and linear slope that would allow for a
quick yet not abrupt transition. For this VSC model simulation an $\varepsilon$ of .1 was implemented.

Figure 6.3: Chattering Avoidance
Chapter 7

Simulation of NID Aircraft Model

7.1 Model Descriptions

The desired aircraft model simulations were done using a FORTRAN program. The desired output variables were selected as β (yaw), ϕ (bank), and θ (pitch) to relate to direct pilot inputs as discussed in the previous sections. Two flight conditions were chosen to be compared. These parameters included a nominal case at one flight condition, a robust case where the model was developed using one set of flight regime coefficients and run inserting a second set of flight regime coefficients, and a last simulation with initial conditions other than zero. For each of these three conditions two flight final values were chosen. These flight regime coefficients are documented in Table 7.1. The first parameter set included the final desired conditions of β = 0°, ϕ = 75°, and θ = 45°. The second parameter set included

9. The initial condition case is an indication of the ability of the system to compensate for initial error. A more exacting method of providing for initial conditions would be to change the axis chosen for the simulation to coincide with the desired initial conditions.
the final desired conditions of $\beta = 0'$, $\phi = 45'$, and $\theta = 75'$. Specific parameters are delineated in Table 7.0.

<table>
<thead>
<tr>
<th>CASE</th>
<th>INITIAL COND</th>
<th>FINAL COND</th>
<th>FLIGHT COND</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta'$</td>
<td>$\phi'$</td>
<td>$\theta'$</td>
</tr>
<tr>
<td>Nominal</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Nominal</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>IC</td>
<td>.5</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>IC</td>
<td>.5</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Robust</td>
<td>.5</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Robust</td>
<td>.5</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 7.1: Model simulation parameters

7.2 Data Description

Each case has a full set of data in the form of graphs enclosed in appendix A. The data includes a time tagged plot of each output parameter sideslip angle, bank angle, and pitch angle ($\beta$, $\phi$, $\theta$) from 0 to 3.5 seconds, a time tagged plot of roll, pitch, and yaw rates ($p$, $q$, $r$) from 0 to 3.5 seconds, a time tagged plot of the aileron input $\delta a$ (marked as $u(1)$), the elevator input $\delta e$ (marked as $u(2)$), and the rudder input $\delta r$ (marked as $u(3)$) from 0 to 3.5 seconds, and finally the error between the controller and the aircraft model for the parameters $\beta$, $\phi$, and $\theta$. In the majority of cases, the parameters came to rest at a steady
state or appeared to be approaching a steady state. Those that did not reach a steady state in the 3.5 seconds are indicated on Table 7.2 and 7.3.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>NOMINAL MAX</th>
<th>MIN</th>
<th>SS</th>
<th>INITIAL COND MAX</th>
<th>MIN</th>
<th>SS</th>
<th>ROBUST MAX</th>
<th>MIN</th>
<th>SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
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<td>0</td>
<td>.5</td>
<td>-.1</td>
<td>0</td>
<td>.5</td>
<td>-.2</td>
<td>0</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>45</td>
<td>0</td>
<td>45</td>
<td>45</td>
<td>-.1</td>
<td>45</td>
<td>45</td>
<td>-.2</td>
<td>45</td>
</tr>
<tr>
<td>$\theta$</td>
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<td>75</td>
<td>75</td>
<td>-.1</td>
<td>75</td>
<td>75</td>
<td>0</td>
<td>75</td>
</tr>
<tr>
<td>$p$</td>
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<td>11</td>
<td>.1</td>
<td>25</td>
<td>-15</td>
<td>-1</td>
<td>25</td>
<td>-16</td>
<td>-.6</td>
</tr>
<tr>
<td>$q$</td>
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<td>0</td>
<td>60</td>
<td>-23</td>
<td>0</td>
<td>56</td>
<td>-22</td>
<td>-.1</td>
</tr>
<tr>
<td>$r$</td>
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<td>-5</td>
<td>0</td>
<td>8</td>
<td>-5</td>
<td>0</td>
<td>8</td>
<td>-7</td>
<td>.3</td>
</tr>
<tr>
<td>$u(1)$</td>
<td>1</td>
<td>-2</td>
<td>.1</td>
<td>18</td>
<td>-5</td>
<td>.1</td>
<td>14</td>
<td>-5</td>
<td>0</td>
</tr>
<tr>
<td>$u(2)$</td>
<td>-2</td>
<td>-26</td>
<td>1*</td>
<td>34</td>
<td>-25</td>
<td>-4*</td>
<td>31</td>
<td>-25</td>
<td>-4*</td>
</tr>
<tr>
<td>$u(3)$</td>
<td>6</td>
<td>-5</td>
<td>0</td>
<td>6</td>
<td>-23</td>
<td>0</td>
<td>6</td>
<td>-19</td>
<td>0</td>
</tr>
<tr>
<td>error1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>.5</td>
<td>-.1</td>
<td>0</td>
<td>.5</td>
<td>-.2</td>
<td>0</td>
</tr>
<tr>
<td>error2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>-.5</td>
<td>0</td>
<td>2</td>
<td>-1</td>
<td>.3</td>
</tr>
<tr>
<td>error3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>-.9</td>
<td>0</td>
<td>3</td>
<td>-8</td>
<td>.1*</td>
</tr>
</tbody>
</table>

* Elements had not decreased to steady state at this time, however they appeared to be approaching a reference point.

Table 7.2: Case 1 Final Conditions $\beta=0^\circ$ $\varphi=45^\circ$ $\theta=75^\circ$

Tables 7.2 and 7.3 also list the maximum and minimum for each parameter in a format so as to easily compare between the differing cases. From this table, several items of interest can quickly be deciphered as will be indicated in the following sections.
In order to determine the usefulness of the model in examining the NID technique, several criteria must be established which will be used to judge the NID technique results when implemented upon this non-linear model.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>NOMINAL MAX MIN SS</th>
<th>INITIAL COND MAX MIN SS</th>
<th>ROBUST MAX MIN SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0 0 0</td>
<td>.5 -.1 0</td>
<td>.5 -.2 0</td>
</tr>
<tr>
<td>$\phi$</td>
<td>75 0 75</td>
<td>75 .2 75</td>
<td>75 .1 75</td>
</tr>
<tr>
<td>$\theta$</td>
<td>45 0 45</td>
<td>45 -.4 45</td>
<td>45 -.3 45</td>
</tr>
<tr>
<td>$p$</td>
<td>47 -5 -3</td>
<td>47 -14 -3</td>
<td>47 -15 -.4*</td>
</tr>
<tr>
<td>$q$</td>
<td>51 0 3</td>
<td>50 -23 3</td>
<td>4 -24 0</td>
</tr>
<tr>
<td>$r$</td>
<td>14 -.7 .8*</td>
<td>13 -.8 .8*</td>
<td>12 -2 1</td>
</tr>
<tr>
<td>$u(1)$</td>
<td>1 -4 .4</td>
<td>18 -6 .4</td>
<td>13 -6 0</td>
</tr>
<tr>
<td>$u(2)$</td>
<td>0 -23 -6*</td>
<td>34 -23 -6*</td>
<td>30 -20 -5*</td>
</tr>
<tr>
<td>$u(3)$</td>
<td>4 -8 .1</td>
<td>3 -23 .1</td>
<td>6 -19 0</td>
</tr>
<tr>
<td>error1</td>
<td>0 0 0</td>
<td>.5 -.1 0</td>
<td>.5 -.2 0</td>
</tr>
<tr>
<td>error2</td>
<td>0 0 0</td>
<td>2 -.6 0</td>
<td>2 -.7 .1*</td>
</tr>
<tr>
<td>error3</td>
<td>0 0 0</td>
<td>3 -.9 0</td>
<td>3 -.8 0</td>
</tr>
</tbody>
</table>

* Elements had not decreased to steady state at this time, however they appeared to be approaching a reference point.

Table 7.3: Case 2 Final Conditions $\beta=0^\circ$, $\phi=75^\circ$, $\theta=45^\circ$
Criteria established.

- Output approaches final desired value smoothly with little overshoot
- Roll rate less than maximum of 100° per second and settle to small steady state value
- Pitch rate less than maximum of 100° per second and settle to small steady state value
- Maximum aileron deflection of 30°
- Maximum elevator deflection of 30°
- Maximum rudder deflection of 30°
- Error in all cases less than .1° at final steady state

Review of Tables 7.2, 7.3, and the graphs in appendix A indicate the model simulation is adequate in demonstrating the NID technique.

7.3 Nominal Cases

As expected, the nominal cases [Figures A1.1 - A1.8] more closely follow the ideal representation of the model results. These nominal cases have no initial conditions (or conditions are 0°) and the only differences between the two cases in the final values of ϕ and θ. In the first case ϕ = 45° and θ=75°, and in the second case ϕ=75° and θ=45°.

7.3.1 β, ϕ, θ Results

By inspection of the graphs [Figures A1.1, A1.5] of both final cases, it appears that all criteria have been met. β maintains 0° as far as can be read on the graphs and
by actual inspection of the data the $\theta$ angle does maintain 0° conditions. The graphs indicate an interesting phenomena concerning the $\phi$ and $\theta$ angles. The path of the final value appears to depend upon the final value itself not upon the parameter. The 75° path is the same for $\phi$ and $\theta$ while the 45° path is also the same for $\phi$ and $\theta$. I would expect that, in order to accomplish this, the control inputs would have to be widely different due to the differing stability regimes for the two axes. This will be further discussed in the following sections. It is noticed that the higher angle of the case does take approximately .25 seconds longer to reach a steady state value than the lower angle. The $\phi$ and $\theta$ angles do start at 0° and do smoothly increase to the final value.

7.3.2 p, q, r Results

Again inspection of the graphs [Figures A1.2, A1.6] indicate that all criteria have been met. The maximum and minimum values are well within the range required and all parameters settle to a small steady state value. The roll rate $p$ follows the perceived path in that an initial rate is set in the positive direction and then as the final value is approached the rate must be reversed to slow the approach down. Finally the rate approaches the small steady state point as the final value is approached. It is interesting to note that the final steady state value of the higher angle is a -3°. This indicates that the model has a
tendency to continue roll and a negative input must be maintained to have a steady final value. The pitch rate shows an interesting phenomena in that the maximum rate at both angles is approximately the same even though the angles are widely different. A possible explanation is that as the bank is further increased the pitch rate is also required to maintain the final aircraft position. Also a positive rate of 3° is required to be maintained at the lower pitch angle with the higher bank angle. The yaw rate follows the standard positive rate to reversal to 0 steady state in the same manner. It is interesting to note that a higher yaw rate \( r \), is required when the higher final value \( \phi \) is implemented.

7.3.3 Error Results

The errors [Figures A1.3, A1.7] of all three parameters \( (\beta, \varphi, \theta) \) are well within the desired ranges. the maximum of any error is no more than \( .0014 \). It is interesting to note that the error1 \( (\beta) \) is of a order of magnitude greater than the other two errors. Since it is still such a small value this is not significant for our simulations.

7.3.4 \( \delta a \delta e \delta r \) Results

The control inputs [Figures A1.4, A1.8] again are well within our desired ranges. The aileron input \( \delta a \) is the smallest maximum value, ranging from 3 to 5 degrees absolute maximum in the two cases. The low inputs required are
indicative of the inherent instability in the roll portion of the model. This is also the experience of the actual pilot of a jet aircraft in that most aircraft are more able to quickly roll than change rate in any other axis. The elevator control input \( \delta e \) is of greater maximum value and, as in the same manner, as the pitch rate \( q \), does not have as wide a variance (when normalized) between the two cases as does the aileron input \( \delta a \). Again this may be due to the requirement of elevator input for coupling with the roll rate. The peak of the \( \delta e \) input does come later than the peak of the \( \delta a \) input while the \( \phi \) angle is approaching the final value. The rudder input \( \delta r \), also follows the expected curve as the aircraft model rolls, and pitches. It is interesting to note that \( \delta r \) has a greater maximum value with the higher bank angle than with the higher pitch angle. This indicates that sideslip is more highly coupled with roll than pitch.

7.3.5 Conclusion of Nominal Case

For the nominal case the NID technique appears to have provided an excellent non-linear technique for handling this aircraft model. All parameters were within desired maximum, final values were reached, and inputs were realistic. The model handled in the same manner as a real aircraft with the increased responsiveness in the roll compared to the pitch. The coupling required during a roll with the yaw was also
evident. Therefore, for the nominal case it appears that the NID is a success.

7.4 Initial Conditions Cases

The initial condition model [Figures A2.1 - A2.8] are more exactly termed initial error models. This case demonstrates how rapidly the model adjusts for errors. At time 0 one would expect the greatest errors and the highest rates of correction. As in the nominal case, the first case \( \phi = 45^\circ \) and \( \theta = 75^\circ \), and in the second case \( \phi = 75^\circ \) and \( \theta = 45^\circ \).

7.4.1 \( \beta, \phi, \theta \) Results

By inspection of the graphs [Figures A2.1, A2.5] it appears that all criteria have been met. \( \theta \) maintains less than .5' as indicated by direct inspection of the data. Both \( \phi \) and \( \theta \) are initially at the programmed initial conditions of 2' and 3'. There is some minor overshoot to a negative value (less than -.4) in the controller's attempt to correct the model, but within .25 seconds of start of run the model is positive and increasing to the final values. Unlike the nominal case the curves of \( \phi \) and \( \theta \) do not appear to be interchangeable for the same final angles although they are close probably due to the differing initial values. Both \( \phi \) and \( \theta \) do, however, smoothly increase to the final value and have no visible overshoot.
7.4.2 \( p, q, r \) Results

Inspection of the graphs [Figures A2.2, A2.6] indicate that all criteria have been met. Maximum and minimum values are well within the stated range and all parameters do settle to a steady state value. There is a definite difference between the two cases, however, in that the steady state for the \( \theta = 45^\circ \) case is within a \( \pm 1^\circ \) range, while the steady state for the \( \theta = 75^\circ \) case is within a \( \pm 3^\circ \) range. This would indicate again that the roll has a greater effect on the model than the pitch. The roll rate \( p \), also increases for the higher \( \theta \) angle. The pitch rate \( q \) also increases for the higher \( \theta \) angle, however, it does no increase to the same extent as does the roll rate. Both rates become negative at the first to compensate for the initial errors, but quickly return to the positive proportion of the graph. The roll rate demonstrates a smooth curve for both cases of the model. The pitch rate indicates a 'crook' in the positive slope, but it is surprisingly more evident in the case where \( \theta = 45^\circ \). Other than this 'crook' the curves are standard and appeared to be smooth and regular.

7.4.3 Error Results

The errors [Figures A2.3, A2.7] of all three parameters (\( \beta, \phi, \theta \)) are well within the desired ranges after .5 seconds. Initially the error is the initial condition presented (\( \beta = .5^\circ, \phi = 2^\circ, \theta = 3^\circ \)), rapidly decreases to 0,
overshoots to less than \(|-1'|\) and returns to 0. Error \(\approx 0.6\) seconds. This indicates that the model is still highly accurate within a very short period of time.

### 7.4.4 \(\delta a, \delta e, \delta r\) Results

The control inputs [Figures A2.4, A2.8] have an increasing amount of work to do to provide desired results. The actual curves are almost identical for the two cases. The aileron input, \(\delta a\), is an initial high input of approximately 18° for both cases. \(\delta a\) then rapidly decreases to a negative number \(\approx -5°\), maintains this negative position slightly longer for the \(\theta = 75°\) case, returns to 0° and then has a slight decreasing oscillation about the final value of less than 0.4°. Since the aileron is primary responsible for the roll rate, it is realistic for the control input to maintain a negative position longer for the higher angle. The elevator input, \(\delta e\), has an initial rate of \(\approx 35°\) due to the initial condition, but it rapidly drives to a negative number. This angle appears to show no difference in maximum rate for the different angles, but there appears to be an attempted reversal at \(\approx 0.2\) seconds. Although this attempted reversal stays negative for both cases, for the \(\theta=75°\) case this reversal is much less pronounced. The rudder input, \(\delta r\), has an initial input of \(\approx -24°\) and rapidly drives to a steady state conditions. It appears to be an underdampened case, and oscillates a few cycles prior to its final value. For the \(\theta=45°\) this
oscillation tends to favor the positive portion of the graph and for the \( \phi = 75^\circ \) this oscillation tends to favor the negative portion of the graph but, in either case, it does not appear to be significant.

7.4.5 Conclusion

In the initial conditions case it appears that the NID technique again provided an excellent technique for handling this aircraft model. All parameters were reached within the desired maximum and minimum, final values were reached, and inputs were realistic. Errors do not cause any great difficulty for the NID controller. It does appear that the higher roll angle has more of an effect on the model than the higher pitch angle.

7.5 Robust Cases

The robust cases [Figures A3.1 - A3.8] are an experiment to test the robustness of the model. If the model is sufficiently robust, the developed model at one flight regime would adequately perform when run under the conditions of the second flight regime. As in all other cases, the first case is \( \phi = 45^\circ \) and \( \theta = 75^\circ \), and the second case is \( \phi = 75^\circ \) and \( \theta = 45^\circ \).
7.5.1 $\beta$, $\phi$, $\theta$ Results

As in all other cases, the desired criteria have been met [Figures A3.1, A3.5]. $\beta$ maintains less than .5 as indicated by direct inspection of the data. Both $\phi$ and $\theta$ smoothly rise to the desired final values. For this robust case, the initial conditions were also used. The same positioning is evident as in the initial conditions case where the angles are initially at the .5', 2', and 3' conditions, rapidly decrease to 0', and then begin the smoothly curve to the final values. The curves for the higher and lower angles again follow the same path for both $\phi$ and $\theta$.

7.5.2 $p, q, r$ Results

The criteria for the roll, pitch, and yaw rates are easily met in this model [Figures A3.2, A3.6]. The roll rate, $p$, again appears to have a higher rate when $\phi=75'$ than when $\phi=45'$. The curves are wider and indicate a more constant input is required. When $\phi=75'$, $q$ is maintained at it's peak for a longer period of time than in the other case. In both cases the yaw rate rises to a small positive due to the initial error, decreases to 0, then increases again into a slight oscillatory curve to a steady state of 0. In the case of $\phi=75'$ the $r$ curve is favoring the positive side of the graph whereas in the case of $\phi=45'$ the $r$ curve appears to split equally about the 0 reference line.
7.5.3 Error Results

In these cases [Figures A3.3, A3.7] it appears that there is a greater oscillatory nature to all error curves. As noted previously, all errors are at a maximum at the initiation of the run due to the initial conditions. All curves start at the positive point of the initial condition, decrease rapidly to 0, and then begin a oscillatory curve. It is interesting to note that the case of $\phi=75^\circ \theta=45^\circ$ appears to much more quickly decrease to a smaller oscillatory level. The errors of the $\phi=45^\circ \theta=75^\circ$ maintain a more negative value and oscillate a greater extent. At the end of the run it appears that the $\phi$ error will be oscillating for a greater time.

7.5.4 $\delta a \delta e \delta r$ Results

There is a greater variance of control inputs [Figures A3.4, A3.8] in order to adequately control these model cases. The aileron $\delta a$, and elevator $\delta e$ inputs have an initially high positive rate due to the initial conditions. The $\delta a$ input decreases to below 0 and then to a 0 steady state rate in within $\approx 4$ seconds. In the $\phi=75^\circ$ case the curve has a slightly greater tendency to maintain a negative value than the other case. The elevator input $\delta e$, rapidly decreases from a high positive initial rate to a negative rate and stays negative for the length of the simulation although the indicators are that the curve is decreasing to a small steady state. For $\theta=75^\circ$ the input is more negative.
although it appears to rapidly follow the other case curve. The rudder input os again an oscillatory input with an initial input of approximately -19°. It does decrease oscillatory to 0. The value of δr also appears to stay more negative for the $\psi=75^\circ$ case than for the $\psi=45^\circ$ case.

7.5.5 Conclusion

The robust case would appear to be the hardest case for the technique to adequately handle. Even so, the technique appears to handle all of our criteria with aplomb. All final values were reached in adequate time, and inputs were still realistic. Errors, although more oscillatory than previously encountered, are still well within our criteria. Obviously, the technique is a success.

7.6 Overall Results

In all cases the final results were indicative of what was desired in that they all reached the desired final value (in most cases 0 or at defined final state). All actual maximum and minimum values were well within the requirements.

7.6.1 $\beta$, $\phi$, $\theta$ Results

In all cases [Figures A1.1, A1.5, A2.1, A2.5, A3.1, A3.5] the desired $\beta$, $\phi$, and $\theta$ angles were attained in approximately 2 seconds and were maintained at the final
desired condition for the duration of the simulation run. No reverses or overshoot in the initiation of the simulations are noted even in the worst case of robust. The graphs also indicate that all of these parameters approached the final conditions in a smooth manner with no overshoot. The desired outcome for \( \beta \) was to maintain the 0° during the runs, or for \( \beta \) to quickly decrease to 0° during the manner with no overshoot. In the majority of cases the desired \( \beta \) result was actually accomplished. The desired outcome for \( \phi \) was to smoothly approach the final condition in a critically dampened manner. In all cases the final condition was exactly as desired with little variance in time. The desired outcome for \( \theta \) was also to smoothly approach the final value in a critically damped manner. As in the case of \( \phi \), the outcome of \( \theta \) was exactly as desired.

There are specific differences in all three parameters, which are described in more detail in the individual sections following this section.

### 7.6.2 \( p, q, r \) Results

The roll, pitch, and yaw rates \( p, q, \) and \( r \) all fell within the desired maximum and minimum and did settle to a steady state in the time allotted [Figures A1.2, A1.6, A2.2, A2.6, A3.2, A3.6]. These rates (degree/second) did indicate some oscillatory characteristics that depended upon the specific model simulation. The roll rate \( p \), reacted exactly as anticipated. The maximum rate was higher or lower,
dependent upon the degree of the final angle required. For a final angle of 75° p was over twice the p for a final angle of 45°. The shape of all p curves was initially a positive rate to get to the bank angle and then a reversal to stop the roll rate and stabilize the bank angle at its desired final value. P then decreased to a small steady state value as the final bank angle was approached. An interesting overall feature is the continually high (relatively) maximum rate of the pitch rate q. While the roll rate p, would be noticeably larger or smaller dependent upon the final angle desired, the change for the q rate with change in final pitch angle desired was no larger than 10 degrees/second over all model simulations. This is an indication the greater stability in the vertical axis as opposed to the longitudinal, and gravity possibly has a large part to play in this. The yaw rate r appears similar in all cases, with the exception of the slight positive rise initially for the robust and initial condition cases.

7.6.3 Error Results

Again, in all cases, the error was negligible enough to be considered zero [Figures A1.3, A1.7, A2.3, A2.7, A3.3, A3.7]. The exception to this was only in the initial time of the robust and initial condition cases were the model was not started at a zero reference. The controller adequately followed the model in all cases.
7.6.4 $\delta a$, $\delta e$, $\delta r$ Results

The control inputs $\delta a$, $\delta e$, and $\delta r$ in all cases fell well within the maximum and minimum limits and did decrease to a zero or near zero steady state [Figures A1.4, A1.8, A2.4, A2.8, A3.4, A3.8]. The elevator input $\delta e$ was exercised the largest amount in the negative direction and appeared to have the hardest time reaching the steady state point.

7.7 High Angle Anomalies

Several other model conditions were experimented with in this simulation. It was found that angles above 90° caused a run away of the simulation. Further investigation revealed that the B matrix would become singular in these cases and therefore the technique was no longer valid.\textsuperscript{10}

Several attempts to isolate the actual culprit met with failure until it was discovered that the angle of attack $\alpha$, was becoming extremely large as these cases began to run away. Once the angle of attack was controlled the system was controllable, although control inputs $\delta a$ and $\delta e$ were too large for our requirements. Research into previous control outputs used in simulation of this system demonstrates that while angle of attack instead of pitch was a controlled

\textsuperscript{10} Note the discussion in the section on development of this theory.
output, pitch would not rise above approximately 30°. This indicates that \( \alpha \) should be one of the controlled variables for high angles, however this limits the pitch angle that can be attained. Further investigation should be accomplished in the use of \( \alpha \), \( \theta \), and the most important of the two remaining variables, \( \phi \). However, by limiting ourselves to less than 90° in our cases, the NID controller proves itself to be a more than adequate technique for control of a non-linear system.

Table 7.4: Flight Parameters

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Flight Cond 1</th>
<th>Flight Cond 2</th>
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<tbody>
<tr>
<td>( I_1 )</td>
<td>.727</td>
<td>.727</td>
</tr>
<tr>
<td>( I_2 )</td>
<td>-.949</td>
<td>-.949</td>
</tr>
<tr>
<td>( I_3 )</td>
<td>.716</td>
<td>.716</td>
</tr>
<tr>
<td>( L_p )</td>
<td>-3.933</td>
<td>-5.786</td>
</tr>
<tr>
<td>( L_q )</td>
<td>.107</td>
<td>.108</td>
</tr>
<tr>
<td>( L_r )</td>
<td>.126</td>
<td>.221</td>
</tr>
<tr>
<td>( L_r \alpha )</td>
<td>8.39</td>
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</tr>
<tr>
<td>( L_\phi )</td>
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<td>( L_\phi \alpha )</td>
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<td>-543.8</td>
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<tr>
<td>( L_\phi \omega )</td>
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</tr>
<tr>
<td>( L_{\phi r} )</td>
<td>63.5</td>
<td>64.6</td>
</tr>
<tr>
<td>( M_q )</td>
<td>-814</td>
<td>-1.168</td>
</tr>
<tr>
<td>( M_\alpha )</td>
<td>-23.18</td>
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</tr>
<tr>
<td>( M_\phi )</td>
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<td>-.173</td>
</tr>
<tr>
<td>( M_{\phi \alpha} )</td>
<td>-28.37</td>
<td>-31.64</td>
</tr>
<tr>
<td>( N_q )</td>
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<td>.013</td>
</tr>
<tr>
<td>( N_\alpha )</td>
<td>.223</td>
<td>.222</td>
</tr>
<tr>
<td>( N_\phi )</td>
<td>-.235</td>
<td>-.377</td>
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<tr>
<td>( N_{\phi \alpha} )</td>
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<td>-1.583</td>
</tr>
<tr>
<td>( N_{\phi \omega} )</td>
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<td>8.88</td>
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<tr>
<td>( N_{\phi r} )</td>
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<tr>
<td>( N_{\phi r} )</td>
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<td>2.459</td>
</tr>
<tr>
<td>( Y_\phi )</td>
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<td>-.280</td>
</tr>
<tr>
<td>( Y_{\phi \alpha} )</td>
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<td>.0119</td>
</tr>
<tr>
<td>( Z_\alpha )</td>
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<td>-1.746</td>
</tr>
<tr>
<td>( Z_{\phi \alpha} )</td>
<td>.168</td>
<td>-.224</td>
</tr>
<tr>
<td>( g/v )</td>
<td>.0345</td>
<td>.0412</td>
</tr>
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</table>
Chapter 8

Simulation of VSC Aircraft Model

8.1 Model Descriptions

The nonlinear aircraft model simulations from the section concerning Non-Linear Inverse Dynamics were altered so as to follow the development of the preceding chapter describing the Variable Structure Control modeling. The desired output variables were again taken as $\phi$ (yaw), $\psi$ (bank), and $\theta$ (pitch) which relate to the direct pilot inputs as previously discussed. Additionally, taking the same output variables allows an exact comparison of the Non-Linear Inverse Dynamics and the Variable Structure Control. As previously, these parameters include a nominal case at one flight condition, an initial error simulation (although with smaller initial error than the NID case), and a robust case where the model was developed utilizing one set of flight regime coefficients and run inserting a second set of flight regime conditions. For each of these three conditions, the Variable Structure Control appears to accept less of an initial error than the NID and therefore the robust conditions do not include an initial error in the same.
flight conditions two flight final values were chosen. These flight regime coefficients are the same as were used in the NID simulations and are documented in Table 7.4 in the preceding chapter. The first parameter set included the final desired conditions of $\beta = 0^\circ$, $\phi = 75^\circ$, and $\theta = 45^\circ$. The second parameter set included the final desired conditions of $\beta = 0^\circ$, $\phi = 45^\circ$, and $\theta = 75^\circ$. Specific parameters are delineated in Table 8.1. Additionally, a single case where the $\epsilon$ parameter was set to 0 was simulated. The flight final value of $\beta = 0^\circ$, $\phi = 75^\circ$, and $\theta = 75^\circ$ was chosen for this single case. This demonstrates the abrupt reversal in the sliding case with the expectations that the result will show severe overshoots in the control parameters when trying to approach the final conditions. All other cases had $\epsilon = .1$ set for the model simulations.

The model simulations gave similar results in most of the flight conditions. It appears that final values were more of a variable for the results than flight conditions as will be specified in the following sections.

manner as the NID cases. Since all simulations should be initiated from a zero error condition this was not considered to be a major factor or difference in the model simulation.
### Table 8.1: VSC Model simulation parameters

<table>
<thead>
<tr>
<th>CASE</th>
<th>INITIAL COND</th>
<th>FINAL COND</th>
<th>FLIGHT COND</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta', \theta', \phi'$</td>
<td>$\beta', \theta', \phi'$</td>
<td>develop</td>
</tr>
<tr>
<td>Nominal</td>
<td>0 0 0</td>
<td>0 45 75</td>
<td>1</td>
</tr>
<tr>
<td>Nominal</td>
<td>0 0 0</td>
<td>0 75 45</td>
<td>1</td>
</tr>
<tr>
<td>IC</td>
<td>.05 .25 .25</td>
<td>0 45 75</td>
<td>1</td>
</tr>
<tr>
<td>IC</td>
<td>.05 .25 .25</td>
<td>0 75 45</td>
<td>1</td>
</tr>
<tr>
<td>Robust</td>
<td>0 0 0</td>
<td>0 45 75</td>
<td>1</td>
</tr>
<tr>
<td>Robust</td>
<td>0 0 0</td>
<td>0 75 45</td>
<td>1</td>
</tr>
</tbody>
</table>

### 8.2 Data Description

Each case has a full set of data in the form of graphs enclosed in appendix A. The data includes a time tagged plot of each output parameter. These are sideslip angle, bank angle, and pitch angle ($\beta, \phi, \theta$) from 0 to 3.5 seconds, a time tagged plot of roll, pitch, and yaw rates ($p, q, r$) from 0 to 3.5 seconds, a time tagged plot of the aileron input $\delta a$ (marked as $u(1)$), the elevator input $\delta e$ (marked as $u(2)$), and the rudder input $\delta r$ (marked as $u(3)$) from 0 to 3.5 seconds, and finally the error between the controller and the aircraft model for the parameters $\beta, \phi, \theta$. In the majority of cases, the parameters came to rest at a steady state or appeared to be approaching a steady state.
Those that did not reach a steady state in the 3.5 seconds are indicated on Table 8.2 and 8.3.

Tables 8.2 and 8.3 also list the maximum and minimum for each parameter in a format so as to easily compare between the differing cases. From this table, several items of interest can quickly be deciphered as will be indicated in the following sections.

In order to determine the usefulness of the model in examining the VSC technique, several criteria must be established which will be used to judge the VSC technique results when implemented upon this non-linear model. In the interest of comparison these criteria are the same as the criteria for the NID examination. These criteria are repeated below.
PARAMETER NOMINAL MAX MIN SS INITIAL COND MAX MIN SS ROBUST MAX MIN SS

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>NOMINAL MAX MIN SS</th>
<th>INITIAL COND MAX MIN SS</th>
<th>ROBUST MAX MIN SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>β</td>
<td>0 0 0</td>
<td>.05 0 0</td>
<td>0 0 0</td>
</tr>
<tr>
<td>φ</td>
<td>45 0 45</td>
<td>45 .25 45</td>
<td>45 0 45</td>
</tr>
<tr>
<td>θ</td>
<td>75 0 75</td>
<td>75 .25 75</td>
<td>75 0 75</td>
</tr>
<tr>
<td>p</td>
<td>35 -13 -1.6</td>
<td>35 -13 -1.6</td>
<td>35 -17 -1.1</td>
</tr>
<tr>
<td>q</td>
<td>86 .2 .4</td>
<td>86 .2 .4</td>
<td>82 0 .15*</td>
</tr>
<tr>
<td>u(1)</td>
<td>4 -6 .16</td>
<td>18 -4 .16</td>
<td>4 -10 .1</td>
</tr>
<tr>
<td>u(2)</td>
<td>0 -30 -1.9*</td>
<td>23 -30 -1.6*</td>
<td>0 -30 -2.1*</td>
</tr>
<tr>
<td>u(3)</td>
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<td>11 -20 0</td>
<td>12 -10 0</td>
</tr>
<tr>
<td>error1</td>
<td>0 0 0</td>
<td>.05 0 0</td>
<td>0 0 0</td>
</tr>
<tr>
<td>error2</td>
<td>0 0 0</td>
<td>.25 0 0</td>
<td>0 -.1 0</td>
</tr>
<tr>
<td>error3</td>
<td>0 0 0</td>
<td>.25 0 0</td>
<td>.2 -.5 0</td>
</tr>
</tbody>
</table>

* Elements had not decreased to steady state at this time, however they appeared to be approaching a reference point.

Table 8.2: Case 1 Final Conditions β=0° φ=45° θ=75°

Criteria established.
- Output approaches final desired value smoothly with little overshoot
- Roll rate less than maximum of 100° per second and settle to small steady state value
- Pitch rate less than maximum of 100° per second and settle to small steady state value
- Maximum aileron deflection of 30°
- Maximum elevator deflection of 30°
- Maximum rudder deflection of 30°
- Error in all cases less than .1' at final steady state

```
<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>NOMINAL MAX MIN SS</th>
<th>INITIAL COND MAX MIN SS</th>
<th>ROBUST MAX MIN SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0 0 0</td>
<td>.5 -1 0</td>
<td>0 0 0</td>
</tr>
<tr>
<td>( \phi )</td>
<td>75 0 75</td>
<td>75 .2 75</td>
<td>75 0 75</td>
</tr>
<tr>
<td>( \theta )</td>
<td>45 0 45</td>
<td>45 -4 45</td>
<td>45 0 45</td>
</tr>
<tr>
<td>( p )</td>
<td>47 -5 -3</td>
<td>47 -14 -3</td>
<td>68 -5 -3.7</td>
</tr>
<tr>
<td>( q )</td>
<td>51 0 3</td>
<td>50 -23 3</td>
<td>75 2 3.7</td>
</tr>
<tr>
<td>( r )</td>
<td>14 -7 .8*</td>
<td>13 -8 .8*</td>
<td>21 .2 .9*</td>
</tr>
<tr>
<td>( u(1) )</td>
<td>1 -4 .4</td>
<td>18 -6 .4</td>
<td>5 -7 .38</td>
</tr>
<tr>
<td>( u(2) )</td>
<td>0 -23 -6*</td>
<td>34 -23 -6*</td>
<td>0 -27 -4*</td>
</tr>
<tr>
<td>( u(3) )</td>
<td>4 -8 .1</td>
<td>3 -23 .1</td>
<td>7 -16 .12</td>
</tr>
<tr>
<td>error1</td>
<td>0 0 0</td>
<td>.5 -1 0</td>
<td>0 0 0</td>
</tr>
<tr>
<td>error2</td>
<td>0 0 0</td>
<td>2 -.6 0</td>
<td>0 0 0</td>
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<tr>
<td>error3</td>
<td>0 0 0</td>
<td>3 -.9 0</td>
<td>0 0 0</td>
</tr>
</tbody>
</table>

* Elements had not decreased to steady state at this time, however they appeared to be approaching a reference point.
```

Table 8.3: Case 2 Final Conditions \( \beta=0' \) \( \phi=75' \) \( \theta=45' \)

Review of Tables 8.2, 8.3, and the graphs in appendix A indicate the model simulation is adequate in demonstrating the VSC technique with some small variances. It does appear that the VSC technique applies over a wider set of conditions than does the NID technique.
8.3 Nominal Cases

As expected, the nominal cases [Figures A5.1 - A5.8] more closely follow the ideal representation of the model results. These nominal cases have no initial conditions (or conditions are 0°) and the only differences between the two cases in the final values of \( \phi \) and \( \theta \). In the first case \( \phi=45° \) and \( \theta=75° \), and in the second case \( \phi=75° \) and \( \theta=45° \).

8.3.1 \( \beta \), \( \phi \), \( \theta \) Results

By inspection of the graphs of both final cases [Figures A5.1, A5.5], it does appear that all criteria have been met. \( \beta \) maintains 0° as far as can be read from the graphs and by actual inspection of the raw data the \( \beta \) angle is determined to maintain 0° within acceptable limits. The paths of the \( \phi \) and \( \theta \) angles also depict an interesting phenomena. The path of the final value appears to depend upon the final value itself and the differences between the \( \theta \) and \( \phi \) angles does not manifest itself. The 75° path is the same for \( \phi \) and \( \theta \) while the 45° path is also the same for \( \phi \) and \( \theta \). I would expect that, in order to accomplish this, the control inputs would have to be widely different due to the differing stability regimes for the two axes. I also expect that a more exacting aircraft model would demonstrate some measurable difference between the differing axes. In either case the higher angle does take a slightly longer time (approximately .25 seconds) to reach a steady state.
value than the lower angle. Both $\phi$ and $\theta$ angles do start at 0° and do smoothly increase to the final value.

8.3.2 $p, q, r$ Results

Again inspection of the graphs [Figures A5.2, A5.6] indicate that all criteria have been met. The maximum and minimum values are well within the range required and all parameters settle to a small steady state value. The roll rate $p$ does follow the expected path in that an initial rate is set, peaks, and then is decreased as the final value of $\phi$ is approached. The roll rate must be decreased to a negative value in order to stop the roll and attain the final $\phi$ value. It appears that some small negative value of $p$ is required to maintain the final aircraft position. This would indicate that the model would have a very slight tendency to roll in this final position and has to have a control input to prevent the roll continuation. The graphs easily show that the higher $\phi$ angle desired requires the higher maximum roll rate and a lesser roll rate in the negative direction. The higher bank angle also requires a more negative roll rate steady state value to maintain the desired aircraft position. The maximum pitch rate $q$ does not have as great a difference between the two different angles as does the roll rate $p$ possibly due to the greater stability in the $y$ axis. The final steady state value is noticeably higher for the pitch rate for $\theta = 45^\circ$. This may be due to the higher bank angle desired. A higher positive maximum yaw rate $r$ is also
required for the higher roll rate although the rate in the negative direction is of a lesser absolute magnitude.

8.3.3 Error Results

The errors of all three parameters ($\beta, \phi, \theta$) are well within the desired ranges [Figures A5.3, A5.7]. The absolute maximum of any error is no more than .005. It is interesting to note that the error in $\phi$ is of an order of magnitude greater than the other two errors. Since it is still such a small value this is not significant for our simulations.

8.3.4 $\delta a$ $\delta e$ $\delta r$ Results

The control inputs again are well within our desired ranges [Figures A5.4, A5.8]. The aileron input, $\delta a$, is the smallest maximum value, ranging from an absolute variance of 5 to 10 degrees. The lower $\phi$ angle has an anomaly in that while the control input is symmetrical about the $\theta$ axis for the higher $\phi$ angle, the 45° $\phi$ angle model has an additional reversal in the original input direction. This may be caused by the coupling with the higher pitch angle/pitch rate. Both inputs are still smaller in the absolute that the other two inputs. This does follow actual aircraft flying experience in that the rolling of an aircraft is normally the quicker rate change with less input than the other two axes. The elevator control input, $\delta e$, shows little difference between the two angles which follows from the
similarity of the pitch rates. It is interesting to note, however, that the higher pitch angle does require a slightly more quick onset of the control input and also a slightly higher absolute maximum, peaking .25 seconds earlier than the lower angle. The control input for the higher angle also appears to decrease toward 0 at a slightly faster rate. In both angle cases the peak of the $\delta e$ input comes well after the peak of the $\delta a$ input. The rudder input, $\delta r$, also follows the expected curve as the aircraft rolls and pitches. While the absolute difference between the maximum and minimum of $\delta r$ for both angle models is approximately the same, the negative peak of the $\phi = 45^\circ$ model is significantly more negative and the positive peak significantly less positive than the $\phi = 75^\circ$ model.

8.3.5 Conclusion of Nominal Case

For the nominal case the VSC technique appears to have provided an excellent non-linear technique for handling this aircraft model. All parameters were within desired maximum, final values were reached, and inputs were realistic. The model handled in the same manner as a real aircraft with the increased responsiveness in the roll compared to the pitch. The coupling required during a roll with the yaw was also evident. Therefore, for the nominal case it appears that the VSC technique is a success.
8.4 Initial Conditions Cases

The initial condition model [Figures A6.1 - A6.8] are more exactly termed initial error models. This case demonstrates how rapidly the model adjusts for errors. At time 0 one would expect the greatest errors and the highest rates of correction. As in the nominal case, the first case \( \phi = 45^\circ \) and \( \theta = 75^\circ \), and in the second case \( \phi = 75^\circ \) and \( \theta = 45^\circ \). The VSC technique appears to be more sensitive to initial errors than the NID technique and requires much higher initial control inputs to overcome the errors. In order to maintain the control inputs in the desired bounds for control inputs the initial errors had to be taken no greater than \( \beta = .05^\circ \), \( \phi = .25^\circ \), and \( \theta = .25^\circ \).

8.4.1 \( \beta, \phi, \theta \) Results

By inspection of the graphs [Figures A6.1, A6.5] it appears that all criteria have been met. \( \beta \) maintains less than \( .05^\circ \) (the initial error) as indicated by direct comparison of the raw data. Both \( \phi \) and \( \theta \) are initially at the programmed initial error of \( .25^\circ \) and smoothly approach the final values with no overshoot or undershoot. Like the nominal case the curves of \( \phi \) and \( \theta \) appear to be interchangeable for the same final angles. Both \( \phi \) and \( \theta \) do, however smoothly increase to the final value and have no visible overshoot.
8.4.2 \( p, q, r \) Results

Inspection of the graphs [Figures A6.2, A6.6] indicate that all criteria have been met. Maximum and minimum values are well within the stated range and all parameters do settle to a steady state value. There is a definite difference between the two cases, however, in that the steady state for the \( \phi=45^\circ \) case is in a range of \(-1.6^\circ + 0.4^\circ\) while the steady state for the \( \phi=75^\circ \) case is within a \( \pm 3^\circ \) range. This indicates that the roll coupling has a greater effect on the model than does the pitch. The roll rate, \( p \), increases for the higher bank angle. The pitch rate, \( q \), also increases for the higher pitch angle but not the same order of magnitude as does the roll rate. In both cases the pitch rate is higher than the roll rate. The yaw rate, \( r \), is greater for the higher bank angle demonstrating the higher coupling between the two axes. All three rates increase to a positive maximum and then \( p \) and \( r \) decrease to a negative peak and then settle to a steady state near zero. The rate \( q \) never decreases past zero and actually maintains a small positive steady state. In the case of \( \phi=75^\circ \) the final steady state for all three rates is of a greater absolute than of the \( \phi=45^\circ \) case. This would indicate a greater dominance of the variables in the \( x \) axis.

8.4.3 Error Results

The errors of all three parameters (\( \beta, \phi, \theta \)) are well within the desired ranges after .3 seconds [Figures A6.3,
Initially the error is the initial condition presented ($\beta=.05^\circ$, $\phi=.25^\circ$, $\theta=.25^\circ$), rapidly decreases to zero, and then perturbates very slightly about the zero axis. This perturbation is not significant but appears to be in the $\phi$ angle.

8.4.4 $\delta a$ $\delta e$ $\delta r$ Results

The control inputs have an increasing amount of work to do to provide desired results [Figures A6.4, A6.8]. The actual curves demonstrate some differences between the two cases. The aileron input, $\delta a$, is an initial positive rate of approximately $18^\circ$ for both cases that quickly decreases below zero and perturbates about the zero axis to a very small steady state value. While for the higher $\phi$ angle case the control input changes from positive to negative to positive to near zero, in the lower $\phi$ angle the control input makes an additional relatively large foray into the negative portion of the graph prior to reaching a very small positive steady state value. It appears that the higher $\theta$ angle shows a more prominent cross-coupling. The elevator input, $\delta e$, is very similar for the two cases. The higher $\theta$ case does require a slightly more negative control input and the peak negative come slightly later than the lower angle case. Although the time of simulation did not allow for the settling of the $\delta e$ to a steady state, in both cases it does appear that they are approaching zero. The rudder input, $\delta r$, follows the same profile in both cases with an initial
value of a -20 to a peak still in the negative area back to a valley in the negative area to finally a peak in the positive area to a steady state close to zero. The case of the higher \( \theta \) angle shifts these peaks and valleys in a more positive direction although the absolute differences seem to be the same. In every case the control inputs are well within the desired bounds.

8.4.5 Conclusion

In the initial conditions case it appears that the VSC technique again provided an excellent technique for handling this aircraft model. All parameters were reached within the desired maximum and minimum, final values were reached, and inputs were realistic. The model does appear to be sensitive to initial errors. This is not a detriment in that the model could be constructed to start at an initial position of zero error in all cases, negating the sensitivity to initial errors. It does appear that the higher roll rate has more of an effect on the model than the higher pitch rate with the exception of the additional control input perturbations for the higher pitch rate.

8.5 Robust Cases

The robust cases [Figures A7.1 - A7.8] are an experiment to test the robustness of the model. If the model is sufficiently robust, the developed model at one
flight regime would adequately perform when run under the conditions of the second flight regime. As in all other cases, the first case is $\phi=45^\circ$ and $\theta=75^\circ$, and the second case is $\phi=75^\circ$ and $\theta=45^\circ$. Due to the sensitivity of the VSC technique to initial error in this simulation, the robust case is initiated with zero error in all three angles.

### 8.5.1 $\beta$, $\phi$, $\theta$ Results

As in all other cases, the desired criteria have been met [Figures A7.1, A7.5]. $\beta$ maintains much less than $.5^\circ$ as indicated by direct inspection of the data. $\theta$ rises smoothly to the final value in both angle cases, however there appears to be a slight perturbation in the lower ($45^\circ$) $\phi$ angle. This does not appear to be of any great significance since the perturbation is slight and the final value is of the desired magnitude.

### 8.5.2 $p,q,r$ Results

The criteria for the roll, pitch, and yaw rates are easily met in this model [Figures A7.2, A7.6]. The roll rate, $p$, again appears to have a higher rate when $\phi=75^\circ$ than when $\phi=45^\circ$. Unlike the nominal case the pitch rate, $q$, has a significantly higher maximum for the higher $\theta$ angle than for the lower $\theta$ angle. The yaw rate, $r$, has a higher positive maximum for the higher $\phi$ angle and the maximum negative value is much less for the same $\phi$ angle. All three
rates decrease to a very small steady state value in both angle cases.

8.5.3 Error Results

The error rates are well within the established requirements, and with a maximum of no more than .7, $\theta$ is the worst case of all three error measurements [Figures A7.3, A7.7]. However, in these cases it appears that there is a greater oscillatory nature to the $\theta$ error curves in both angle cases. The $\beta$ error is almost negligible in both cases, with a very slight dip in the $\phi=45^\circ$ case. There is negligible $\phi$ error in the $\phi=75^\circ$ case, however there is a noticeable dip in the $\phi$ error for the $\phi=45^\circ$ case. This dip is at the same point that the perturbation in the $\phi$ angle shows in the first graph of this series. For both angles there is a positive error peak for $\theta$ at the same time although the lower $\theta$ angle case has a higher positive error. Additionally, for both angles there is a negative valley at the same time. This valley is larger for the higher $\theta$ angle. The final steady state value of the $\phi=45^\circ$ case is a much higher positive value that of the $\phi=75^\circ$. The errors in all cases do decrease to a small steady state value and are well within established limits.

8.5.4 $\delta a$ $\delta e$ $\delta r$ Results

All control inputs are well within the established criteria [Figures A7.4, A7.8]. The $\delta a$ control followed the
expected curve for the $\phi=75^\circ$, with an initial negative input and then a reversal to positive with a gradual decrease until zero steady state is reached. The lower $\phi$ case has an interesting reversal back into the negative area prior to decreasing to a zero steady state value. The $\delta e$ input closely compares in both cases. The input increases to a negative value and then decrease towards the zero line although the time of simulations does not allow for determination of a final steady state value. Additional extended time testing did indicate that the control input did decrease to a stable, small, steady state value. The rudder control input, $\delta r$, appeared to have the same absolute magnitude between the maximum positive and negative values in both angle cases. The $\phi=45^\circ$ did have the greater positive maximum value and the smaller negative maximum value. However, in both cases, the control input did decrease to a zero steady state value.

8.5.5 Conclusion

The robust case should be the hardest case for the technique to adequately handle. Even so, the technique appears to handle all of our criteria with little or no problems. All final values were reached and all perturbations were within adequate limits. There is a perturbation in the $\phi$ angle when approaching the final value of $45^\circ$ which follows through the negative reversal noted in the $\phi$ error and control input reversal for that $\phi$ axis.
This perturbation is not serious in that it does not appear to severely affect the results of the simulation. This technique is obviously successful in this robust case.

\section*{8.6 \(\epsilon = 0\)}

As described in the model development section in the VSC technique a 'buffer' must be used between the +1 and -1 sliding states [Figures A9.1 - A9.4]. If this buffer is not used a 'bang-bang' effect results. This instantaneous reversal could cause a number of problems and one example simulation \(\beta=0^\circ, \phi=45^\circ, \theta=75^\circ\) was run to evaluate any problems.

\subsection*{8.6.1 \(\beta, \phi, \theta\) Results}

All parameters followed the nominal case graphs [Figure A9.1]. The initial angles were all at zero and smoothly climbed to the desired steady state value. These output values had no noticeable complication from the instantaneous reversal.

\subsection*{8.6.2 \(p, q, r\) Results}

The initial curves [Figure A9.2] of these parameters follow quite closely the nominal case with an increase in the positive direction to the maximum rates noted in the nominal case and then the reversal in the negative direction. However, as all three rates approach the nominal
steady state final value they all demonstrate a perturbation of a oscillatory nature. It even appears that the roll rate, $p$, has an increasing oscillatory modulation that may drive the parameter into instability.

8.6.3 Error Results

While the error rates [Figure A9.3] are still within acceptable limits during the time of this simulation, it still indicates a 10 times greater error maximum than the nominal rate. Additionally, the $\phi$ and $\theta$ error do not appear to be decreasing to a zero steady state value. It does appear that the $\theta$ error may be slowly increasing to a greater negative error rate.

8.6.4 $\delta a$, $\delta e$, $\delta r$ Results

All three of the control inputs [Figure A9.4] are extremely out of the established limits. The abrupt reversals cause large overshoots with a maximum of $+450^\circ$ to $-380^\circ$ in the $\delta a$ control input and even $\delta r$ with the smallest variance still had a $+50^\circ$ to $-50^\circ$ swing.

8.6.5 Conclusion

The use of $\epsilon=0$ does not provide for an adequate simulation. The wide swings of the control inputs indicate the inability of this technique to accomplish the instantaneous reversals. The rest of the simulations were run with an $\epsilon = .1$. 

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8.7 Overall Results

In all cases the final results were indicative of what was desired in that they all reached the desired final value (in most cases 0 or at defined final state). All actual maximum and minimum values were well within the requirements and no values demonstrated a tendency for oscillatory or unstable behavior other than a slight perturbation in the $\phi$ axis under a robust condition.

8.7.1 $\beta$, $\phi$, $\theta$ Results

In all cases the desired $\beta$, $\phi$, and $\theta$ angles [Figures A5.1, A5.5, A6.1, A6.5, A7.1, A7.5] were attained in approximately 2 seconds and were maintained at the final desired condition for the duration of the simulation run. No reverses or overshoot in the initiation of the simulations are noted even in the worst case of robust. The graphs also indicate that most of these parameters approached the final conditions in a smooth manner with no overshoot. The desired outcome for $\beta$ was to maintain the 0° during the runs, or for $\beta$ to quickly decrease to 0° during the manner with no overshoot. In all cases the desired $\beta$ result was actually accomplished. The desired outcome for $\phi$ was to smoothly approach the final condition in a critically dampened manner. In most cases the final condition was exactly as desired with little variance in time. The robust condition with $\phi=45°$ did show a slight
perturbation just prior to reaching the final value. This perturbation followed through all portions of that axis's graphed conditions. The desired outcome for $\theta$ was also to smoothly approach the final value in a critically damped manner. As in the case of $\phi$, the outcome of $\theta$ was exactly as desired.

8.7.2 $p, q, r$ Results

The roll, pitch, and yaw rates $p, q,$ and $r$ all fell within the desired maximum and minimum and did settle to a steady state in the time allotted [Figures A5.2, A5.6, A6.2, A6.6, A7.2, A7.6]. These rates (degree/second) did indicate some oscillatory characteristics that depended upon the specific model simulation. In almost all cases the different cases of nominal, initial condition, and robust made little difference in the rates graphed. The only exception appears to be the $\phi=45^\circ$ case where the pitch rate, $q$, actually had a decreased positive maximum over the initial condition and the nominal cases for that angle. The roll rate, $p$, reacted exactly as anticipated. The maximum rate was higher or lower, dependent upon the degree of the final angle required. For the higher angle $\phi=75^\circ$, $p$ was over twice the maximum of $p$ for the lower angle $\phi=45^\circ$. The shape of the $p$ curve was generally the same for all cases. $p$ would increase to a positive maximum and then reverse to a small negative maximum prior to settling a very small steady state value. The cases of the higher pitch angle,
\( \theta = 75^\circ \), did have a greater negative value during the reversal than did the lower pitch angle \( \theta = 45^\circ \). The yaw rate, \( r \), followed the same general path as did the \( q \) curve with the exception that the positive and negative maximum values were an order smaller.

### 8.7.3 Error Results

The maximum error in all cases was \( .7^\circ \) and all steady state conditions were less than \( .08^\circ \) error [Figures A5.3, A5.7, A6.3, A6.7, A7.3, A7.7]. With the removal of the \( \theta \) error in the \( \theta = 45^\circ \) robust case, all errors were less than \( .001^\circ \). This demonstrates the adequate ability of the controller to follow the model although it would be more satisfactory to have the error in all cases to be less than \( .001^\circ \) final steady state.

### 8.7.4 \( \delta a, \delta e, \delta r \) Results

The control inputs \( \delta a, \delta e, \) and \( \delta r \) in all cases fell well within the maximum and minimum limits and did decrease to a zero or near zero steady state [Figures A5.4, A5.8, A6.4, A6.8, A7.4, A7.8]. The elevator input \( \delta e \) was exercised the largest amount in the negative direction and appeared to have the hardest time reaching the steady state point. The differing cases of nominal, initial condition, and robust appeared to have little difference on the magnitude and curve of the inputs. The different angle cases appeared to have more of an effect on the curve although the magnitudes
were approximately the same in all cases. The noticeable
difference was in the $\delta a$ input which had an additional
reversal in all $\varphi=45^\circ$ cases. Obviously initial error cases
causd a large control input at the initiation of the
simulation, however they very shortly followed the other
cases's curves.

8.7.5 Conclusion

The VSC technique provides creditable results
throughout all flight conditions and values. The differing
cases of nominal, initial condition, and robust do not
appear to severely change the parameters and inputs of the
graphed conditions. It appears that the VSC technique, with
some sensitivities to error, more evenly represents the
model simulation over a range of conditions. Several
variates of the models were run with differing gains,
control $\lambda$, and $\epsilon$. Within limits these parameters did not
significantly change the graphed results. This is an
additional indication of the stability of this technique in
the model simulation.

8.8 High Angle Anomalies

Several other model conditions were experimented with
in this simulation. It was found that angles approaching
90° and above caused large errors in the simulation.
Further investigation revealed that the B matrix would
become singular or close singular in these cases and therefore the technique was no longer valid. From previous failures of the NID model it was determined that angle of attack \( \alpha \), was becoming extremely large as these errors grew. This indicates that \( \alpha \) must be considered in this technique also prior to any large angle maneuvers.
Chapter 9

Comparison of NID and VSC Simulations

9.1 Introduction

The Non-Linear Inverse Dynamics Technique is a proven technique for controlling specific cases of non-linear systems. This technique does not have a great ability to maintain accurate control under widely varying parameters and under any perturbations of the system. The method may or may not work for a specific case or may be affected by slight changes in coefficients. The Variable Structure Control should be a more robust technique in all cases. It should allow for wide changes in system operation and should be relatively impervious to system perturbations and changes. The following sections will evaluate the two techniques under all simulation conditions and determine the actual advantage of either technique.
9.2 Nominal Cases

It appears that both techniques were able to handle the nominal cases [Figures A1.1 - A1.8, A5.1 - A5.8]. These nominal cases would be ideal conditions with no initial error and a narrow range of flight conditions.

9.2.1 $\beta$, $\varphi$, $\theta$ Results

There appears to do no difference in the two technique as far as the actual final conditions shows. Both techniques reach the same final condition in all cases with no overshoot and maintain the desired final condition during the model simulation. There is a noticeable difference in the speed at which the final value is obtained. The Variable Structure Control technique appears to reach 90% of the final values almost .5 seconds sooner than the Non-Linear Inverse Dynamics.

9.2.2 $p, q, r$ Results

The roll, pitch, and yaw rates in both techniques are well within the desired maximum and minimum limits. Both techniques also settle to a small steady state value. This final value appears to depend upon the angle of the final value as opposed to the technique used, with the higher $\varphi$ angle case requiring the larger steady state rates. The maximum rates for the VSC case is significant higher than for the NID case which would follow from the faster approach
to the final values discussed in the previous section. This difference is as much as 20°/sec at the peak rate. Additionally, the peak rates are reached sooner in the VSC technique while the NID technique peaks later and maintains a positive rate for a longer period of time.

9.2.3 Error Results

Both cases indicate very small amounts of error. The θ error is the only measurable error in both techniques. The peak absolute error for the NID technique appears to be on the order of .0015° while the peak absolute error for the VSC technique is slightly higher at .0045°. There does appear to be some difference in the time to settle this error. The NID technique required over 2 seconds to settle to within 10° of the steady state value while the VSC technique required no more than 1.5 seconds. This is not significant due to the small amount of error.

9.2.4 δa δe δr Results

The control inputs in both techniques were well within established limits and within a few degrees of each other. The VSC technique did require slightly larger control inputs and also approached the final steady state value slightly more rapidly than did the NID technique. The VSC technique had more movement in the δa control input to the point of actually reversing additionally into the negative area of the chart.
9.2.5 Conclusion

Both techniques were sufficient for the nominal cases. The VSC technique was slightly faster and required moderately higher control inputs. Either technique probably could have been varied to emulate the opposite results. A slow down of the VSC technique would have decreased the input control and reverse a speed up of the NID technique would have increased the control inputs.

9.3 Initial Condition Cases

The initial condition cases cannot be directly compared due to the differing initial error used [Figures A2.1 - A2.8, A6.1 - A6.8]. The VSC technique was more sensitive to the initial error and required much larger control inputs to complete the simulation. In order to reduce the initial control inputs to acceptable maximums the VSC initial error were greatly reduced. The two techniques may be compared from the time the initial errors are negated.

9.3.1 $\beta$, $\phi$, $\theta$ Results

The NID technique had the same slower approach to the final values over the VSC technique that was noted in the nominal case. Once adjusted for the differing initial conditions, it appears that the times differences between the two reaching the final values are the same also.
9.3.2 p, q, r Results

Both techniques maintain well within the desired limits for all values. Compensating for the initial error differences, it still appears that the NID technique required smaller rates and a longer settling time. The NID technique appears to have a more difficult time recovering from the initial error at least for the q rate. Once the q rate recovers to zero and began its positive climb, it develops a 'crook'. This is an indication of a less than desirable pitch rate. One test was run with the VSC technique utilizing the same initial errors and the VSC technique provided a smooth curve in all parts of the graph.

9.3.3 Error Results

The errors of both cases are well within the desired levels after the initial errors are overcome. The NID technique appears to overshoot more readily than the VSC technique although both techniques have negligible error within .5 seconds. One test was run of the VSC technique with the higher initial errors and this case also demonstrated little or no overshoot after correcting for the initial error.

9.3.4 8a, 8e, 8r Results

Both techniques are well within the desired maximum limits for the control inputs. As mentioned earlier, the VSC technique required less initial error due to the high
control inputs required to overcome that initial error. There appears to be slightly more control input required for the VSC case even with the lesser initial error.

9.3.5 Conclusion

Both techniques are still acceptable for use in this model simulation even with the initial error. There appears to be a disadvantage in that the VSC technique cannot handle as high as initial error as can the NID technique. Since model simulation should be initiated at a zero error condition this is not a significant factor. The 'crook' noted in the pitch rate of the NID technique, while not a serious disadvantage is an indication of the NID system faltering ability to handle wide ranging conditions accurately.

9.4 Robust Cases

The robust cases should be the most difficult cases for either technique to handle [Figures A3.1 - A3.8, A7.1-A7.8]. The NID case was simulated using the same initial errors as the initial condition simulations. The VSC technique was simulated with no initial errors. Both techniques can be compared once the initial errors are compensated.
9.4.1 $\beta$, $\phi$, $\theta$ Results

Both techniques satisfactorily reached the final values desired. There are no overshoots or major perturbations in any of the cases. The $\phi = 45^\circ$ case for the VSC technique does show a very slight perturbation in the $\phi$ angle approach to the final value but it is not of any large order of magnitude.

9.4.2 $p,q,r$ Results

The NID technique is now appearing to have more of a problem with the robust case than does the VSC technique. Both techniques are still within the maximum limits for the rates, but the NID case is now showing a 'crook' in both the roll and the pitch rates. As before, the VSC technique results in higher maximum rates than the NID technique, however they are still well within the desired levels. Additionally, the NID pitch rate has to be maintained for an extended period of time, almost 3 seconds. Both techniques do settle to a near zero steady state prior to the end of the simulations, although the VSC technique settles approximately one second sooner than the NID technique.

9.4.3 Error Results

There are major differences in the error results for both techniques. The initial errors caused by the initial conditions of the NID can be disregarded, however, the $\phi$ and $\theta$ errors do not settle to a small steady state. For the case
of $\gamma = 45^\circ$, $\theta = 75^\circ$ these errors appear to be following a small sinusoidal curve and are not within the desired minimums by the completion of the simulation. For the NID case of $\gamma = 75^\circ$, $\theta = 45^\circ$ these errors are smaller but still appear sinusoidal and outside of the desired limits. The VSC technique appears to better follow the model. Maximum error is still small (0.68°) and all errors settle to a much lesser value than the NID case. The largest state value for the VSC case appears to be 0.08° for the $\theta$ angle in the $\gamma = 75^\circ$, $\theta = 45^\circ$ case.

9.4.4 $d_a$, $d_e$, $d_r$ Results

For both techniques all control inputs were well within the desired limits. It does appear that the VSC technique again required a slightly higher control input in all cases than did the NID technique. Although of a lesser maximum, the control inputs for the NID technique had to be maintained for a much longer time duration than did the VSC technique. It did appear that all inputs did settle to a small steady state case.

9.5 Sinusoidal Input

A last simulation was attempted to evaluate any differences between the two techniques. A sinusoidal input was added to the $\alpha$ state equation in both model simulations and the same parameters were run (Figures A4.1 - A4.4, A8.1
In this case the robust case with initial errors the same for both techniques was used. There were differences in all parameters. The NID techniques indicated a smooth approach to the final values while the VSC technique demonstrated small perturbations in all three (\( \beta, \phi, \theta \)) angles. The p,q,r rates were again smoother for the NID technique although they were at a greater rates for a longer period of time. The rates for the VSC technique all showed moderate perturbations due to the sinusoidal input. There were major differences in the errors between the two techniques. Although the VSC technique had some moderate (+1° to -2°) perturbations, all three errors settled to a near zero steady state value in a short period of time. The errors for the NID technique appeared to maintain the sinusoidal rate and the \( \theta \) error was developing an increasing sinusoidal rate at the conclusion of the simulation. The VSC technique had a large variance in the control inputs \( \delta a \), moderate variance in \( \delta r \), and a decreasing sinusoidal in \( \delta e \). The \( \delta a \) and \( \delta r \) control inputs decrease to a near zero steady state value while the \( \delta e \) control input appears to maintain a sinusoidal value about the zero axis. The NID technique appears to maintain a negative constant sinusoidal for \( \delta e \) while the other control inputs decrease to a small steady state value.
9.6 Conclusion

This study has demonstrated that both techniques are adequate in the nominal cases and are also adequate for narrow specifications or parameters. Given the proper limitations, either technique appears quite capable of handling the nonlinear model described in this thesis. In the robust cases the results also do show that the VSC technique is able to more exactly control the aircraft model. The technique results show a reduced error for the VSC technique over the NID technique in the more stringent cases. The sinusoidal inputs to the $a$ equations also show that the NID technique is less capable of handling a class of perturbations about the exact parameters than the VSC technique. In the comparisons given in the preceding paragraphs, the Variable Structure Control technique emerges a clear winner for this nonlinear model.
Chapter 10

Conclusion

An exploratory study of certain nonlinear techniques was discussed in this thesis. The two control laws using Non Linear Inverse Dynamics and Variable Structure Control were derived. The NID technique necessitated the development of a control law of intermediate variables than have been decoupled. These intermediate variables were then negatively fed back into the closed loop system. The control law was developed with these intermediate variables as the new external control point. The VSC technique required the selection of a sliding mode which we wished the state trajectory to maintain. A reaching mode was developed that forced the trajectory to this sliding mode and maintained the trajectory within desired bounds around the sliding mode. These techniques were applied to a standard seven state nonlinear aircraft model. The results of each technique were individually compared to a desired bounded area of system parameters. These comparisons were made on the final values of $\beta$ (yaw angle), $\phi$ (bank angle), $\theta$ (pitch angle), $p$ (roll rate), $q$ (pitch rate), $r$ (yaw rate), $\beta$ error, $\phi$ error, $\theta$ error, $\delta a$ (ailerón control input), $\delta e$ (
elevator control input), and \( \delta r \) (rudder control input). Once each comparison made between the technique and the bounded parameters a second comparison between the values of the two techniques was made.

The aircraft model control outputs were selected to be \( \beta \), the sideslip or yaw angle, \( \phi \), the bank angle, and \( \theta \), the pitch angle. These parameters were chosen as they most directly represent the outputs desired by the pilot of an actual aircraft. The pilot perceives a final position to which he wishes to maneuver his aircraft. He estimates a combination of bank, pitch, and yaw that will allow him to place his aircraft in the desired position. The control inputs were selected as \( \delta a \), the aileron, \( \delta e \), the elevator, and \( \delta r \), the rudder as these are the major control elements for the three axes of the aircraft. Actual nonlinear effects such as wind were ignored for the purpose of this comparison. Investigation during the simulations demonstrated that the model was not sufficient for high angle maneuvers in any axis. It was ultimately determined that the \( B \) matrix for either control technique would become singular for any combination of angles over 90\(^\circ\) for the \( \phi \) and \( \theta \) angles. This singularity would cause instability in the feedback to the model and a 'run away' condition. A cursory investigation of the control of the \( \alpha \) final value at high angles of \( \phi \) and \( \theta \) appeared to limit the instability of the model although results were still outside of the desired parameters for the control inputs.
The Non Linear Inverse Dynamic Technique was proven to be an adequate method for control of the nonlinear model. The data indicated that all parameters of final values \((\beta, \phi, \theta)\), all rates \((p, q, r)\), and all control inputs \((\delta a, \delta e, \delta r)\) were within desired values for all cases. The errors \((\delta e r r o r, \phi e r r o r, \theta e r r o r)\) were all acceptable with the exception of the two robust cases. This most difficult case indicated the control technique was not sufficiently robust to faithfully control the model under all conditions. The errors did not exceed the desired maximums to a great extent during the period of simulations although it was assumed they would continue to increase during a longer simulation run.

The Variable Structure Control technique also proved itself to be an adequate technique for control of this nonlinear model. A weakness was noted in that the technique was sensitive to initial errors and they had to be reduced for adequate regulation of the model. With the reduced initial errors the control law was more than adequate for use. It was assumed that the sensitivity to the initial errors was of marginal importance since the model should be constructed to initially have zero error. The data indicated that all parameters of final values \((\beta, \phi, \theta)\), all rates \((p, q, r)\), all errors, \((\beta e r r o r, \phi e r r o r, \theta e r r o r)\) and all control inputs \((\delta a, \delta e, \delta r)\) were within desired values for all cases to include the most stringent robust case.
The comparison of the two techniques indicated superior results for the Variable Structure Control. The error comparison demonstrated that the error rate during the comparison for the nominal cases was slightly higher for the VSC technique than for the NSC technique; however, both cases were well within the maximum desired parameter. The comparison of the errors for the robust cases indicated that the VSC technique was clearly superior. The VSC case did have all errors settle to a near zero steady state while the NID case indicated a oscillatory or slightly increasing error in the $\varphi$ and $\theta$ angles. Since real world cases are more comparable to the robust case in this study, this indicates the VSC would be more applicable to an exacting model. To investigate the ability of each technique to handle some forms of perturbations a sinusoidal input was added to the $\alpha$ equation of each technique. The NID simulation resulted in an increasing oscillatory error while the VSC simulation again settled to a near zero steady state. This additional comparison of the sinusoidal input indicated that the VSC technique more capably handled perturbations, and was a more robust technique.

This study has left unanswered questions in several areas and a need for further investigation.

1. Further investigation of the aircraft model is warranted with a study into use of the $\alpha$ output as a control variable. Since the three control outputs documented in this
study are still desirable some investigation into one or more of these outputs dependent upon $\alpha$ is needed if the three original control inputs are maintained. Previous studies have been accomplished with $\theta$ as a dependent of $\alpha$, [2,28,32,34] however, they have only been investigated for low (< 30°) angles of $\theta$.

2. While the alpha sinusoidal perturbation utilized in this study is sufficient for demonstration of the robustness of the two techniques, a more exacting model of nonlinear effects such as wind gusts should be developed and investigated for a more critical appraisal of the accuracy of either technique in realistic aircraft modeling.

3. Although the simulations do indicate that errors are oscillatory or increasing for a number of the NID cases, the simulations should be further investigated to insure that the increase in errors actually occurs.

4. Once performance of the model has been improved the results of simulations should be compared to actual aircraft flight data to critically evaluate the model and nonlinear control technique.
References


Appendix A

Simulation Results
A.1 NID Nominal Cases
Non-Linear Inverse Dynamics
Nominal

![Graph showing Non-Linear Inverse Dynamics with parameters Beta=0, Phi=45, Theta=75.](image)
Non-Linear Inverse Dynamics
Nominal

FIGURE A1.2
Non-Linear Inverse Dynamics
Nominal

FIGURE A1.3
Non-Linear Inverse Dynamics
Nominal

FIGURE A1.5
Non-Linear Inverse Dynamics
Nominal

FIGURE A1.6
Non-Linear Inverse Dynamics
Nominal

![Graph showing error1, error2, and error3 with time axis from 0 to 4 seconds and degrees axis from -15 to 15 degrees.](image)

Beta=0 Phi=75 Theta=45

FIGURE A1.7
Non-Linear Inverse Dynamics
Nominal

FIGURE A1.8
A.2 NID Initial Condition Cases
Non-Linear Inverse Dynamics
Initial Condition

TIME, SEC
Beta=0 Phi=45 Theta=75
FIGURE A2.1
Non-Linear Inverse Dynamics
Initial Condition

Figure A2.2

Beta=0 Phi=45 Theta=75
Non-Liner Inverse Dynamics
Initial Condition

TIME, SEC
Beta=0 Phi=45 Theta=75

FIGURE A2.3
Non-Linear Inverse Dynamics

Initial Condition

FIGURE A2.4

Beta=0 Phi=45 Theta=75
Non-Linear Inverse Dynamics

Initial Condition

![Graph showing Non-Linear Inverse Dynamics]

**Figure A2.5**

**Equations**

\[ \phi, \theta, \beta \]

**Conditions**

- Beta = 0
- Phi = 75
- Theta = 45

**Degrees**

- 0 to 80 degrees

**Time, SEC**

- 0 to 4 seconds
Non-Linear Inverse Dynamics
Initial Condition

FIGURE A2.6
Non-Linear Inverse Dynamics
Initial Condition

FIGURE A2.7
Non-Linear Inverse Dynamics
Initial Condition

TIME, SEC

Beta=0 Phi=75 Theta=45

FIGURE A2.8
A.3 NID Robust Cases
Non-Linear Inverse Dynamics
Robust

TIME, SEC
Beta=0 Phi=45 Theta=75

FIGURE A3.1
Non-Linear Inverse Dynamics
Robust

![Graph showing DEGREES/SECOND vs TIME, SEC with curves labeled q, p, r.]

Beta=0 Phi=45 Theta=75

FIGURE A3.2
Non-Linear Inverse Dynamics
Robust

FIGURE A3.3
Non-Linear Inverse Condition
Robust

TIME, SEC
Beta=0 Phi=45 Theta=75

FIGURE A3.4
Non-Linear Inverse Dynamics
Robust

FIGURE A3.5
Non-Linear Inverse Dynamics
Robust

![Graph showing non-linear inverse dynamics with curves labeled p, q, and r.](image)

- Degrees/Second
- Time, Sec

Beta=0 Phi=75 Theta=45

FIGURE A3.6
Non-Linear Inverse Dynamics
Robust

FIGURE A3.7

Error 1, Error 2, and Error 3 vs. Time (sec)

Beta=0 Phi=75 Theta=45
Non-Linear Inverse Dynamics
Robust

FIGURE A3.8
A.4 NID Alpha Sinusoid
Variable State Control
Initial Condition

Figure A6.1
Beta=0 Phi=45 Theta=75

TIME, SEC

θ

ϕ

β
Variable State Control
Initial Condition

Beta=0 Phi=45 Theta=75

FIGURE A6.2
Variable State Control
Initial Condition

Beta=0 Phi=45 Theta=75

FIGURE A6.3
Variable State Control
Initial Condition

Beta=0 Phi=45 Theta=75

FIGURE A6.4
Variable State Control
Initial Condition

FIGURE A6.5
Variable State Control
Initial Condition

Beta=0 Phi=75 Theta=45

FIGURE A6.6
Variable State Control
Initial Condition

![Graph showing time in seconds on the x-axis and degrees on the y-axis, with three lines labeled error1, error2, and error3. The graph describes the behavior of errors over time, with Beta=0, Phi=75, and Theta=45.]

FIGURE A6.7
Variable State Control
Initial Condition

![Graph showing the response of u(1), u(2), and u(3) with time.]

Beta=0  Phi=75  Theta=45

FIGURE A6.8
A.7 VSC Robust Cases
Variable State Control
Robust

![Graph showing the variables θ, φ, and β over time, with labels for Beta=0, Phi=45, Theta=75.]

FIGURE A7.1
Variable State Control
Robust

Beta=0 Phi=45 Theta=75

FIGURE A7.2
Variable State Control

Robust

Error1

Error2

Error3

TIME, SEC

Beta=0 Phi=45 Theta=75

FIGURE A7.3
Variable State Control
Robust

FIGURE A7.4

Beta=0 Phi=45 Theta=75
Variable State Control
Robust

![Graph showing variables phi, theta, and beta over time with annotations]

Beta=0 Phi=75 Theta=45

FIGURE A7.5
Variable State Control
Robust

FIGURE A7.6

Beta=0 Phi=75 Theta=45
Variable State Control
Robust

FIGURE A7.7

TIME, SEC

DEGREES

error3

error1

error2

Beta=0 Phi=75 Theta=45
Variable State Control
Robust

FIGURE A7.8
A.8 VSC Alpha Sinusoid
Variable State Control
Robust, Alpha Sinusoid Input

FIGURE A8.1
Variable State Control
Robust, Alpha Sinusoid Input

DEGREES/SECOND

TIME, SEC

Beta=0 Phi=75 Theta=45

FIGURE A8.2
Variable State Control
Robust, Alpha Sinusoid Input

FIGURE A8.3

Time, SEC

Beta=0, Phi=75, Theta=45
Variable State Control
Robust, Alpha Sinusoid Input

FIGURE A8.4
A.9 VSC \( \text{Epsilon} = 0 \)
Variable State Control

\[ e = 0 \]

\[ \beta = 0 \quad \Phi = 45 \quad \Theta = 75 \]

Figure A9.1
Variable State Control

\( e = 0 \)

\[ \begin{align*}
q(t) \\
p(t) \\
r(t)
\end{align*} \]

\[ \begin{align*}
\text{DEGREES} \\
\text{TIME, SEC}
\end{align*} \]

\( \beta = 0 \quad \phi = 45 \quad \theta = 75 \)

FIGURE A9.2
Variable State Control

$e = 0$

**Figure A9.3**
Variable State Control

\( e = 0 \)

**Figure A9.4**

Graph showing the control variables over time with labels for\( u(1) \), \( u(2) \), and \( u(3) \) with specific values and time points indicated.

Parameters:
- \( \beta = 0 \)
- \( \Phi = 45 \)
- \( \Theta = 75 \)
Nominal Non-Linear Inverse Dynamics
Non-Linear Inverse Dynamics

FIGURE A1.2

Beta=0 Phi=45 Theta=75

Nominal
FIGURE A.3

BETA = 0 PHI = 45 THETA = 75

TIME, SEC

Nominal
Non-Linear Inverse Dynamics
FIGURE A1.4

Nominal
Non-Linear Inverse Dynamics

\[
\begin{align*}
\text{Beta} &= 0, \quad \text{Phi} = 45, \quad \text{Theta} = 75
\end{align*}
\]
Non-Linear Inverse Dynamics

Nominal
Figure A1.6

Beta=0 Phi=75  Theta=45

Time, SEC

Nominal

Non-Linear Inverse Dynamics
FIGURE A1.7

Beta=0  Phi=75  Theta=45

TIME SEC

Nominal
Non-Linear Inverse Dynamics

Error 2  Error 3
Non-Linear Inverse Dynamics

FIGURE A1.8
Beta=0 Phi=75 Theta=45

Nominal
A.2 NID Initial Condition Cases
FIGURE A2.1
Beta=0 Phi=45 Theta=75

TIME, SEC

Initial Condition
Non-linear Inverse Dynamics
Figure A2.2

Beta=0 Phi=45 Theta=75

Time: SEC

Initial Condition

Non-Linear Inverse Dynamics
Initial Condition
Non-Linear Inverse Dynamics

FIGURE A2.3
Beta=0 Phi=45 Theta=75
TIME, SEC

TIME SEC

Initial Condition
Non-Linear Inverse Dynamics
Non-Linear Inverse Dynamics

Initial Condition

Figure A2.4

Time, SEC

Beta=0 Phi=45 Theta=76

0
-50
-25
0
25
50

u(1)
u(2)
u(3)
Figure A2.5

Beta=0 Phi=75 Theta=45

Time, sec

Non-linear Inverse Dynamics

Initial Condition
FIGURE A2.6

Initial Condition

Non-Linear Inverse Dynamics
Figure A2.7

Beta=0 Phi=76 Theta=45

Time, sec

Initial Condition

Non-Linear Inverse Dynamics

Errors
Figure A.2.8

Beta=0 Phi=75 Theta=45

TIME
SEC

Initial Condition
Non-linear Inverse Dynamics
A.3 NID Robust Cases
Robust
Non-Linear Inverse Dynamics

FIGURE A3.1
Beta=0 PHI=45 Theta=75
TIME, SEC

Y

\theta

\phi

\theta
FIGURE A3.2

Beta=0  phi=45  Theta=75

TIME, SEC

Non-Linear Inverse Dynamics

Robust
Figure A3.3

Beta=0, Phi=45, Theta=76

Time, SEC

Robust Non-Linear Inverse Dynamics
Figure A3.4

Beta=0 Phi=45 Theta=75

Time, Sec

Robust
Non-Linear Inverse Condition
FIGURE A3.5
Beta=0 Phi=75 Theta=45

TIME SEC

Robust
Non-Linear Inverse Dynamics
Non-Linear Inverse Dynamics

Figure A3.6

Beta=0 Phi=75 Theta=45
FIGURE A3.7

Robust Non-Linear Inverse Dynamics

Beta = 0 PHI = 75 Theta = 45

TIME SEC

Error1
Error2
Error3

Degrees
Figure A3.8

Robust Non-Linear Inverse Dynamics
A.4 NID Alpha Sinusoid
Non-Linear Inverse Dynamics
Robust, Alpha Sinusoid Input

FIGURE A4.1

Beta=0 Phi=75 Theta=45
Non-Linear Inverse Dynamics
Robust, Alpha Sinusoid Input

FIGURE A4.2
Non-Linear Inverse Dynamics
Robust, Alpha Sinusoid Input

FIGURE A4.3

TIME, SEC
Beta=0 Phi=75 Theta=45
Non-Linear Inverse Dynamics
Robust, Alpha Sinusoid Input

FIGURE A4.4
A.5 VSC Nominal Cases
Variable State Control
Nominal

Figure A5.1

Degrees

Time, Sec

Beta=0 Phi=45 Theta=75
Variable State Control
Nominal

FIGURE A5.2
Variable State Control
Nominal

FIGURE 5.3

Beta=0 Phi=45 Theta=75
Variable State Control

Nominal

Figure A5.4

Beta=0 Phi=45 Theta=75
Variable State Control
Nominal

Figure A5.5
Variable State Control
Nominal

Figure A5.6
Variable State Control
Nominal

FIGURE A5.7
Variable State Control
Nominal

![Graph showing time in seconds on the x-axis and degrees on the y-axis with curves labeled u(1), u(2), u(3).]

Beta=0 Phi=75 Theta=45

FIGURE A5.8
A.6 VSC Initial Condition Cases
Variable State Control
Initial Condition

DEGREES

TIME, SEC

Beta=0 Phi=45 Theta=75

FIGURE A6.1
Variable State Control

Initial Condition

Figure A6.2

Beta = 0, Phi = 45, Theta = 75

TIME, SEC

0 1 2 3 4

100 80 60 40 20 0 -20
Variable State Control
Initial Condition

![Graph showing the behavior of errors over time with labels: error1, error2, error3. The graph indicates that the errors decrease over time.]

**FIGURE A6.3**

Beta=0  Phi=45  Theta=75
Variable State Control
Initial Condition

Figure A6.4
Variable State Control
Initial Condition

FIGURE A6.5
Variable State Control
Initial Condition

![Graph showing the time response of variables p, q, and r with initial conditions and time expressed in seconds. The graph includes annotations for Beta=0, Phi=75, and Theta=45.]

FIGURE A6.6
Variable State Control
Initial Condition

Beta=0 Phi=75 Theta=45

FIGURE A6.7
Variable State Control
Initial Condition

FIGURE A6.8
A.7 VSC Robust Cases
Variable State Control
Robust

Figure A7.2

Beta=0 Phi=45 Theta=75
Variable State Control
Robust

![Graph showing time response with error lines labeled error1, error2, and error3.](image)

**FIGURE A7.3**
Variable State Control
Robust

FIGURE A7.4
Variable State Control
Robust

Figure A7.5

- $\phi$
- $\theta$
- $\beta$

Time, SEC

Degrees

Beta=0 Phi=75 Theta=45
Variable State Control
Robust

DEGREES/SECOND

TIME, SEC

Beta=0 Phi=75 Theta=45

FIGURE A7.6
Variable State Control
Robust

![Graph showing error over time with labels error1, error2, and error3. The graph includes a timeline from 0 to 4 seconds and a vertical axis labeled DEGREES ranging from -0.2 to 0.8. The labels indicate Beta=0, Phi=75, Theta=45.]

FIGURE A7.7
Variable State Control
Robust

Beta=0 Phi=75 Theta=45

FIGURE A7.8
A.8 VSC Alpha Sinusoid
Variable State Control
Robust, Alpha Sinusoid Input

Figure A8.1

Beta=0  Phi=75  Theta=45
Variable State Control
Robust, Alpha Sinusoid Input

FIGURE A8.2
Variable State Control
Robust, Alpha Sinusoid Input

FIGURE A8.3

Beta=0 Phi=75 Theta=45
Variable State Control
Robust, Alpha Sinusoid Input

TIME, SEC
Beta=0 Phi=75 Theta=45
FIGURE A8.4
A.9 VSC Epsilon = 0
Variable State Control

\[ e = 0 \]

\[ \text{FIGURE A9.1} \]
Variable State Control

\[ e = 0 \]

\[ q, p, r \]

\[ \text{FIGURE A9.2} \]
Variable State Control

\[ e = 0 \]

**Figure A9.3**

- **Axes:**
  - Y-axis: Degrees
  - X-axis: Time, SEC

- **Graph Components:**
  - Error 1
  - Error 2
  - Error 3

- **Parameters:**
  - Beta = 0
  - Phi = 45
  - Theta = 75
Variable State Control

$e = 0$

FIGURE A9.4