5-2015

Roulette: More than just a Chance

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ROULETTE: MORE THAN JUST CHANCE

By

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Honors Thesis submitted in partial fulfillment

for the designation of Department Honors

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May, 2015

ABSTRACT

Data were generated using a physical roulette wheel to test whether an association exists between initial conditions (the pocket from which the ball is released) and output (the pocket where the ball lands). I have generated data to determine whether there exists statistical significance in distributions of adjoined pockets. Using the statistical software *Excel* for data tabulation and mapping and using *R* for statistical computations, I determined a possible method for cheating. I also established an association does in fact exist between initial conditions and output. The existence of this association diminishes roulette as a game of "pure" chance. The results indicate there is an opportunity to change the odds in favor of the common gambler.

ACKNOWLEDGEMENTS

At this point, I would like to personally thank those who have helped this project grow and blossom into what it is today. Without their help, this project would have never come to fruition. These people have supported me from the beginning and guided me along my journey. I would like to thank my committee members: Dr. Andrew Hanson, Dr. Ashok Singh, and Dr. Rohan Dalpatadu. I would like to give special thanks to my thesis advisor, Dr. Dalpatadu, for all the advice and support over the past two years. I am most humbled and thankful that these fine men oversaw my efforts throughout this project.

I would also like to thank those who have helped me with the technical aspects of this project. First, I would like to thank those at the Stan Fulton Building – International Gaming Institute for their time and help with the project: Dr. Bo Bernhard and Daniel Sahl. Without them, I would never have been able to collect my data nor would I have had the insight to explore additional topics outside of the statistical realm. Finally, I would like to thank those who have given me the support and inspiration to work hard and stay focused. I would like to thank my parents: Robert and Ghadir McCauley. I also thank a good personal friend for helping overcome the monotony during the data collection stage and revising the wheel mapping: Jacob Poffinbarger.

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INTRODUCTION

The game of roulette has been a casino staple for the last two and half centuries. Its invention has typically been falsely accredited to the famous mathematician: Blaise Pascal. Critics of the time were convinced that Pascal had sold his soul to the devil, because the summation of all the numbers on a roulette wheel sum to 666: the number of the beast in Judeo-Christian epistemology. Pascal, however, did not invent the roulette wheel. Pascal is credited with the invention because he had constructed a wheel that was meant to explore perpetual motion (Small). There is no universally accepted inventor for the roulette wheel; however, there is no debate that the appearance of the wheel originated in France: roulette was introduced to Parisian casinos in the mid-eighteenth century.

Today, roulette is highly popular and widely used in casinos all over the world. According to Barth Holland, the creation of the roulette wheel has, "… meant fortune or ruin to thousands of people" (Holland, 8). Roulette wheels are placed on a flat table and accompanied by an adjacent betting table. The roulette dealer, known as the croupier, allows the gamblers to place their bets. The croupier spins the wheel, which spins for a relatively long period with a slow deceleration time. The croupier then spins the ball in the opposite rotational direction that the wheel was spun. After the ball is spun, eventually it slows down and lands in a pocket. After the pocket is determined, the croupier takes all bets lost for the house while paying out all winning bets.

Roulette is a game of pure chance, meaning, statistically, all events are random, and all outcomes are independent (i.e. outputs do not depend on inputs). The probability of any given number arising as an output is constant. Because of this, the probability of the ball landing on 24 is the same as landing on 00 or any other number. This point is illustrated by Jörg Bewersdorff who states, "Roulette is a pure game of chance, whose odds are in many aspects symmetric: it makes no difference whatsoever whether one bets on 17, 25, or 32" (Bewersdorff, 27). The outcomes are random, thus the ability to win is entirely based upon pure chance. However, if knowledge of initial conditions, such as the entrance position (the pocket the ball is released above), associates with the exit position (the pocket the ball lands), then the randomness of roulette wheels needs to be questioned.

Roulette is a wheel-based gambling game that has two different variants: American roulette and European roulette. There are 37 numbers on a European roulette wheel containing the inclusive set of whole numbers from 0 to 36. There are 38 numbers in American roulette containing the same set of numbers in European roulette wheel plus an additional pocket: 00 (double zero). For an American roulette wheel, if we let a discrete, random variable X denote the number of the pocket the ball falls into (exit position), then the $P(X = x) = 1/38$[1] = $0.026315789\ldots \approx 2.63\%$. Therefore, if one were to bet on the exit position being 7 for 100 spins, then 7 should be the outcome approximately 3 times[2].

Betting varies slightly between the two different roulette variants. These two variants, despite the different number of pockets, share common bets divided between two types: inside and outside bets. Inside bets include: betting on one number (single/straight bets), betting on two adjoining numbers on the betting table (split bets), betting on 3 numbers in the same row of the betting table (row/street bets), betting on 4 adjacent numbers on the betting table (corner/square bets), and betting on 6 adjacent numbers on two rows (double street/six line bets). Outside bets include: betting on the range of numbers from 1-18 (low bets), betting on the range from 19-36 (high bets), betting that red or black will appear (red or black bets), betting that an even or odd

[1] $P(X = x)$ is the probability that the discrete, random variable $X = x$, where $x = \{0, 00, 1, 2, \ldots, 36\}$.
[2] The amount of times 7 appears is 0.0263 / spin. 100 spins implies that 7 will be the exit position 2.63 times, which is approximately 3 times in 100 spins.

number will appear (even or odd bets), betting on the range of numbers [1,12], [13,24], [25,36][3] (dozen bets), and betting on numbers in one of the three columns on the betting table (column bets) (Ethier, 462).

Roulette is intrinsically fair, with a slight advantage given to the house (the casinos). This advantage is known as the house advantage. Peter Olofsson, author of *Probabilities: The Little Numbers That Rule Our Lives,* describes the house advantage as, "the expected percentage that the house gains" (Olofsson, 180). Olofsson calculates the house advantage as "simply the difference between the probability that you win and the probability that you lose" (Olofsson, 180). Olofsson calculates the house advantage as about 5% (in American roulette). For example, on red and black bets, the only way the better can lose is if either the opposite color appears or the ball lands in the 0 or 00 pocket. There are 18 red, 18 black and 2 green slots (the 0 and 00) on a roulette wheel. According to Olofsson, betting on red would give us a winning probability of 18/38, while the probability of losing is 20/38. Therefore, the player has a disadvantage compared to the house calculated by the probability of landing on a red number (18/28) minus the probability of landing on a non-red number (20/38) i.e. (18/38 – 20/38) = -(2/38) = -(1/19) = -0.0526315789 ≈ -5%. The house advantage, also known as the house edge, is the average profit the house takes in on a given bet. For European roulette, the house advantage is much smaller because the advantage would be calculated as: the probability of red coming up (18/37) minus the probability of landing on black or 0 (19/37) which equals approximately a 2.7% advantage for the house.

It would be reasonable to assume that a bet on a single number (single/straight bets) that the house has a considerable advantage. However, payouts for single/straight bets are typically 35:1 (sometimes 36:1); therefore, the house advantage remains similar to other bets. The house

[3] [ ] indicates a range of whole numbers inclusively

advantage is calculated by taking the probability of winning, multiplied by the payout, and subtracting the probability of losing multiplied by the amount bet. According to Robert Hannum and Anthony Cabot, the expected value[4] (EV) is calculated by EV = (+$35)(1/38) + (-1)(37/38) = -$0.0526 (Hannum, 86). The product of (+$35)*(1/38) describes the probability of the ball falling into the pocket bet on multiplied by the payout, while (-$1)(37/38) represents the dollar originally bet multiplied by the chance of losing the bet. This results in a negative expected value of return for the better. Thus, the house advantage is the same amongst single/straight bets as it is with red or black bets. In fact, the house advantage remains constant between all American roulette bets, except for when betting on five numbers (betting on 0, 00, 1, 2, 3) where the advantage is approximately 7.8% in favor of the house (Ethier, 474). However, if initial conditions predict the outcome of the roulette wheel, the player could alter the house advantage. The ability to predict outcomes through the use of initial conditions disallows roulette to be classified as a game of pure chance.

There have been cases throughout roulette's history in which players have tried to overturn the probabilities the game dictates: that is to say, players attempted to topple the casinos through roulette. Holland describes one lucky player who was able to make about $300,000 in four days in 1873 from roulette. Joseph Jaggers had his assistants go into the casino at Monte Carlo and note all the numbers that landed on six different wheels. He then took the numbers, looking for random patterns. Holland explains, "Five of the six roulette wheels in operation were perfectly normal. The sixth, however, had nine numbers that came up far more often than chance would suggest" (Holland, 9-10). Jaggers's win over the casino was not due to statistical analysis: he was lucky. The fact that these numbers came up more often than not was not due to physics,

[4] Expected value is the amount expected on a given bet.

but instead to a physical abnormality: there was an apparent scratch in the wheel that led to these nine numbers coming up more frequently.

Predicting roulette outcomes has been attempted in contemporary times as well. A group of physicists, called chaos scientists, calculated the rotation of the wheel, the speed of the ball, the friction, and various other factors to determine an equation that would predict the exit position for a given entrance position (Small). The scientists were able to determine which half of the wheel the ball would land, given its entrance position about 52% of the time. Using the European wheels, these scientists raised the advantage of roulette betting from 2.7% in favor of the house to approximately 18% in their own favor. Since then, casinos all over the world switched their roulette wheels so that the slots are deeper, thus negating the original calculations. However, a statistical analysis of the final location of the ball compared to where the ball enters the wheel has not been done.

My aim for this thesis was to determine whether an association exists between the entrance position of a roulette ball (input) and the exit position of the ball (the output). I hypothesized that a positive association between the input and the output of a roulette wheel exists leading me to believe that roulette is not a game of pure chance. The game is not entirely random because a specified output, or range of outputs is possible, given the input. Consequently, knowledge of this association may lead to an increase in the probability of a ball landing in a given exit position, which in turn gives the statistical advantage to the gambler.

METHODOLOGY

I used a roulette wheel at the Stan Fulton Gaming Institute, located at the University of Nevada, Las Vegas, in order to generate data to test my hypothesis. The wheel used was an older wheel that had some defects. These defects did not hinder the spin of the wheel nor did it hinder the spin of the ball along the wheel. Most of the defects were scratches and dings centered on the frame of the wheel that only helped to show the wheel's age. There was one prominent defect that I had noticed before I started the project, which would ultimately lead to a disturbance in the outputs. This defect occurred on the wheel face itself.

I reconstructed the conditions necessarily found in typical casino play. The first condition was the speed at which the wheel was spun. After observing roughly thirty roulette tables and croupiers from a variety of casinos, I determined that the wheel is roughly spun on average 2 - 3 seconds per revolution. To replicate this speed, I calculated speeds on the test wheel with a timer to ensure that the speed of the test wheel matched the speed of the wheels used in casinos. The wheel was spun between 2 and 3 seconds per revolution. The ball was always released over the 0 pocket. I did this to ensure a constant entrance position for the ball in order to determine if associations exist between where the ball is released and where it ends up.

The last condition concerned the number of times the ball spins around the wheel before dropping into the wheel. After observing a few croupiers, I determined that the ball spins roughly 18-24 revolutions before hitting a spoke and falling into the wheel. I also noticed that the ball spins for approximately 20 seconds before dropping. For my experiment, I spun the ball so that it had 18-24 revolutions before falling onto the wheel, and I confirmed, with a timer, that the ball spun for approximately 20 seconds. In order to practice, I went to the Stan Fulton Gaming Institute a few times prior to launching the experiment in order to practice spinning the ball. I rejected any ball spin that did not meet any of the prior conditions and would re-spin the ball. For this experiment, I assumed that the method to "put the ball in play" is constant amongst all croupiers. Although, this assumption is hardly valid, I believe the greatest error will originate from the placement of the ball onto the wheel. This assumption reduces the errors when determining the initial entrance position of the ball on the wheel. The wheel was placed on a flat surface to eliminate errors from outside forces and to remain consistent with casino play.

In order to have statistical confidence, I repeated the process of entering a ball into play 1000[5] times to reach a 95% level of significance and a 1% margin of error. From these 1000 iterations, I extrapolated two variables: the number of rotations the ball makes around the wheel and the final pocket in which the ball landed. I noted the number of rotations the ball made around the wheel in order to make sure I met the conditions I previously set out. I then used Excel as a way to collect and store the data. From there, I reordered the number of hits of each pocket such that they fit the pattern of the American roulette wheel. Afterwards, I did a visual mapping of the numbers in order to view any significant pockets. Finally, I set up a null hypothesis as well as an alternative hypothesis and conducted a one-sample proportion z-test in order to either reject or fail to reject my null hypothesis based on results generated from the statistical software: *R*.

RESULTS

After compiling all the data, I made a few observations. First, the frequency of the end pockets, as shown in Figure 1, are skewed from having a consistent individual pocket of the same likelihood of hitting to having irregular pockets of “strong” and “weak” numbers, where “strong” numbers indicate that the numbers hit more frequently than others, and conversely, that “weak” numbers refer to those that did not hit as often as they probabilistically should have.

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[5] Where n, denotes the number of spins, $\approx \frac{1.96^2(\rho)(1-\rho)}{d^2}$; $\rho \approx 1/38 = 0.263$, and d denotes the half-length of the confidence interval, d = 0.01 Thus, n $\approx \frac{1.96^2(.0263)(0.9737)}{0.01^2}$ = 983.768837 spins. I rounded up and spun the wheel 1000 times.

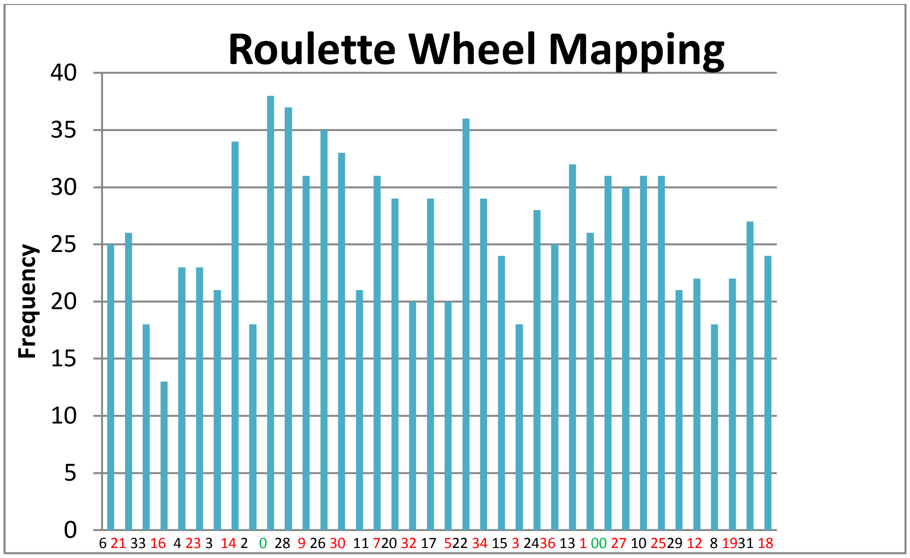

Figure 1: Roulette Wheel Mapping

Figure 1 depicts the distribution of the final pockets in the same pattern layout as would appear on a roulette wheel. Figure 1 further illustrates the peaks and valleys of an irregular distribution. In order to test for individual pockets of distributions, I must test the variance between the pockets and the statistical average. The distributions will be determined as grouping of four or more pockets above or below the average number of hits. The resulting distributions are labeled both above and below the average red line in Figure 2.

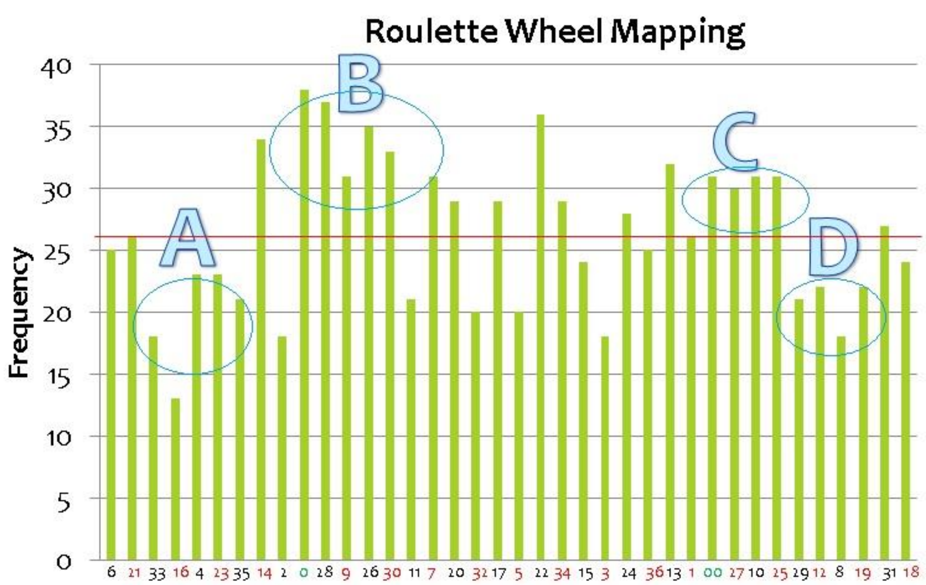

Figure 2 Roulette Wheel Mapping-Advanced

Based on Figure 2 I can see that a particular distribution of pockets appeared more frequently than should statistically happen. In fact, there are two distinct distributions of pockets that hit more frequently (B and C) than the norm and two distributions that hit much less frequently (A and D) than the norm. The average number of outcomes for each pocket is statistically 26.3[6], modeled by the red line in Figure 2. Overall, all spins met the criteria set out in the methods section: the wheel spun between 2 and 3 seconds per revolution, each ball was released over the 0 pocket, and each ball traveled between 18 and 24 revolutions before dropping into the wheel. The average number of revolutions the ball traveled was 19.972 per spin.

INTERPRETATIONS

In order to properly explore the results, one must first take a closer inspection of the wheel itself. Figure 3 displays the wheel that I had used for this experiment in the Stan Fulton Gaming Institute.

Figure 3: American Roulette Wheel

[6] 1000/38 =26.31578947 (the total number of spins / the total number of pockets).

The wheel is a standard American roulette wheel with 38 pockets. The wheel is an older model that had been previously used in casino play. Today, however, it is not fit to be used in casinos. There are various scratches along the wood frame itself; these scratches do not affect the ball's outcome whatsoever, since the frame is not a part of the ball's path. The treading in the wheel itself where the ball was released functioned as it should have. For the most part, all the pockets were identical, save for a few, which I address later.

As previously stated, there are two distinct distributions of pockets that hit more frequently than the norm. After analyzing these two distributions, I determined that the reasons these distributions were hit more often differ. Within the first distribution (distribution B) of pockets, 5 numbers (0, 28, 9, 26, 30) hit much more frequently than the norm. These numbers hit 174 times out of the 1000 spins. The mean for 5 numbers out of 1000 to hit is only 131.5[7]. Setting up a one-sample proportion z-test[8], with the null hypothesis (H_0): p = 0.1315 and alternate hypothesis (H_1): p > 0.1315, with an α of 0.05, I calculated my test statistic (z_o) to be 3.98[9]. Figure 4 displays the output these values yield analyzed using the statistical software *R*.

```
> prop.test(174,1000,p=.1315, alt="greater", correct=FALSE)

        1-sample proportions test without continuity correction

data:  174 out of 1000, null probability 0.1315
X-squared = 15.8155, df = 1, p-value = 3.491e-05
alternative hypothesis: true p is greater than 0.1315
95 percent confidence interval:
 0.1551673 1.0000000
sample estimates:
    p
0.174
```

Figure 4: 5 pocket high hit *R* results

---

7 The mean for 5 numbers is the average number of hits * 5 = 26.3*5 = 131.5.

8 In statistics, we reject H_0 if our calculated p-value is less than our α, and fail to reject H_0 if the p-value is greater than α.

9 One-sample proportion z-test: $z_o = \frac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}}$, where $\hat{p} = 0.174$, $p_o = 0.1315$, and n = 1000.

Given this p value, which is less than the alpha of 0.05, I reject the H_0 at a 95% level of significance, and conclude that the probability of the distribution of these 5 pockets is greater than 0.1315. The p-value is extremely low at 0.00003491, which I interpret as something other than the statistical nature of the wheel in play. At the start of the study, I noticed that there were two pockets not like the rest. These two pockets were the 0 and 2 pockets respectively. The plating above the two pockets where the numbers lie is textured differently than the rest. It appears as if someone spilled glue and did not clean it up. The texture is much rougher than over the other pockets. The difference in pockets can easily be seen in Figure 5.

Figure 5: Wheel Physical Deformities

I interpret the higher frequency of hits in this distribution to be the result of the irregularity of the 0 and 2 pockets. My interpretation is that when the ball lands on the wheel and before it drops into a pocket, once it spins around the 0 and 2, the ball slows down and falls preferentially into the next few pockets. This produced an extraordinary amount of hits above the average. Interestingly enough, the 2 pocket had one of the fewest hits (n=18) when compared to the other pockets, which is well below the average. The ball would hit these two pockets, travel a little further down the wheel and almost immediately drop into a pocket.

This finding introduces an aspect to this thesis that I had not originally thought: the idea of cheating roulette. To preface this notion, I would like to explain that these trials were not entirely random. My aim was to take a roulette wheel spinning at constant speed, with a ball released at the same place that traveled roughly the same distance every time in order to view any statistical significance in the outcomes. All the spins in this experiment are fixed, and thus are extremely biased. One could say my methods set up a basis for cheating the wheel under the present conditions. If one were to then take that application and apply an outside factor in order to skew the results, one would have ample opportunities for cheating.

Currently, according to a floor manager, the issue that most casinos concern themselves with is the imbalance of their roulette wheels. An imbalance in the wheel would allow for a distribution of pockets or several individual pockets to hit more frequently than should statistically occur. A gambler who notices this imbalance in the wheel has the opportunity to have a significant advantage when placing bets. My results indicate that there is an opportunity for an alternative problem that may be prevalent in casinos today. This second method involves a combined cheating from the croupier and a partner that makes the bets. Although this is pure speculation, a croupier can apply a thin, transparent substance to create a roughness over a certain patch of the wheel in order for the ball to drop in the subsequent pockets. Then, the partner can approach the wheel and place bets on the pockets after the tampered pockets.

The next three distributions of pockets do not appear to be affected by any physical deformities in the wheel. The first of these distributions I would like to discuss is the pocket of numbers that appear just before the major distribution (distribution A) above. This distribution is a collection of pockets with well below average hits. There are 5 numbers in this distribution (33, 16, 4, 23, 35) that sum up to 98 total hits in the 1000 spins. As seen previously, the mean number of hits for a collection of 5 pockets is 131.5. Setting up another hypothesis test at $\alpha = 0.05$ with H_0: $p = 0.1315$ and H_1: $p < 0.1315$, we note our z_o to be -3.13. Figure 6 displays the results of the previous data when run through *R*.

```
> prop.test(98,1000,p=.1315, alt="less", correct=FALSE)

        1-sample proportions test without continuity correction

data:  98 out of 1000, null probability 0.1315
X-squared = 9.8264, df = 1, p-value = 0.0008601
alternative hypothesis: true p is less than 0.1315
95 percent confidence interval:
 0.0000000 0.1145666
sample estimates:
    p
0.098
```

Figure 6: 5 pocket low hit *R* results

Because the p-value (at 0.0008601) is less than the α of 0.05, I reject H_0 at a 95% level of significance and conclude that for this distribution the probability of a ball landing in the one of the pockets is less than the statistical mean. From these results, I conclude that if the ball were to be spun in the same manner as my experiment, this distribution of pockets would be best avoided when betting.

Similarly, the next distribution (distribution D) of pockets I would like to discuss hit much less than the norm. This distribution occurred directly after the second high hit distribution, and interestingly enough is only 4 pockets away from the first distribution of pockets with low hits. There are four numbers in this distribution (29, 12, 8, 19) and the sum of hits was 83. The mean for total hits among 4 pockets is 105.2. Thus, setting up a hypothesis test with α of 0.05, H_0: $p = 0.1052$, H_1: $p < 0.1052$, I calculate a z_o of -2.29. Figure 7 displays the results of the test when run through *R*.

```
> prop.test(83,1000,p=.1052, alt="less", correct=FALSE)

        1-sample proportions test without continuity correction

data:  83 out of 1000, null probability 0.1052
X-squared = 5.2356, df = 1, p-value = 0.01106
alternative hypothesis: true p is less than 0.1052
95 percent confidence interval:
 0.00000000 0.09849987
sample estimates:
    p
0.083
```

Figure 7: 4 pocket low hit *R* results

With a p-value of 0.01106, which is less than α of 0.05, I concluded with a 95% level of significance, that this particular distribution of pockets hit less than the statistical mean. Thus, it would be unwise to bet on this group of numbers if the ball is released from the 0 pocket.

The final distribution (distribution C) is similar to the first in that these pockets hit much more than the statistical mean. The hits are not as prevalent as in the first distribution, but they are most definitely well above the average. This final pocket consists of the numbers 00, 27, 10, and 25. These four numbers hit 123 times collectively out of the 1000 spins. With the mean of 4 numbers hitting at 105.2, I set up a hypothesis test with an α of 0.05, H_0: p = 0.1052, H_1: p > 0.1052, I calculate a z_o of 1.83. When run through *R*, the results are tabulated in Figure 8.

```
> prop.test(123,1000,p=.1052, alt="greater", correct=FALSE)

        1-sample proportions test without continuity correction

data:  123 out of 1000, null probability 0.1052
X-squared = 3.3659, df = 1, p-value = 0.03328
alternative hypothesis: true p is greater than 0.1052
95 percent confidence interval:
 0.1069264 1.0000000
sample estimates:
    p
0.123
```

Figure 8: 4 pocket high hit *R* results

Noting a p-value of 0.03328, I reject H_0 at a 95% level of significance and conclude that these numbers hit more often than the statistical mean.

Since there are four pockets of distributions that vary from the standard uniformity of chance, the fairness of the wheel must be taken into account. To test this factor, I set a new null hypothesis Ho: the wheel is fair, and a new alternative hypothesis H_1: the wheel is not fair. The fairness of a roulette wheel implies that the probability distribution of X, the number on which the ball comes to rest, is uniform over the 38 pockets, i.e. P(X=x) = 1/38 = 0.026316. This probability distribution yields the expected frequencies E_i for each number in 1000 trials, which is the statistical average number of hits: 1000(1/38) = 26.3158. The observed test statistic (X^2_{OBS}) of a chi-square test of goodness of fit is 53.06[10]. The null distribution of the chi-square test statist is chi-square with degrees of freedom (df) given by the number of pockets – 1 = 38-1 = 37. The p-value for this goodness of fit test is then calculated from the following equation: $P(\chi^2_{37} > \chi^2_{OBS}) = P(\chi^2_{37} > 53.06) = 0.042$. Since the p-value is less than the α of 0.05, I reject the null hypothesis and conclude that the roulette wheel used in this experiment is not fair. The calculations for the chi-square test statistic are shown in Figure 9 and the chart for observed versus expected data is shown in Figure 10.

Category	O_i	E_i	Chi-square	Category	O_i	E_i	Chi-square
0	38	26.32	5.19	19	22	26.32	0.71
1	26	26.32	0.00	20	29	26.32	0.27
2	18	26.32	2.63	21	26	26.32	0.00
3	18	26.32	2.63	22	36	26.32	3.56
4	23	26.32	0.42	23	23	26.32	0.42
5	20	26.32	1.52	24	28	26.32	0.11
6	25	26.32	0.07	25	31	26.32	0.83
7	31	26.32	0.83	26	35	26.32	2.87
8	18	26.32	2.63	27	30	26.32	0.52
9	31	26.32	0.83	28	37	26.32	4.34
10	31	26.32	0.83	29	21	26.32	1.07

[10] $X^2_{OBS} = \sum_{i=1}^{38} \frac{(O_i - E_i)^2}{E_i}$, where $O_i = observed\ frequency\ of\ number, i = 0, 00, 1, 2, \ldots, 36$, and $E_i = expected\ frequency\ of\ number\ i\ assuming\ uniform\ distribution\ of\ numbers = 1000 * \frac{1}{38} = 26.32.$

11	21	26.32	1.07	30	33	26.32	1.70
12	22	26.32	0.71	31	27	26.32	0.02
13	32	26.32	1.23	32	20	26.32	1.52
14	34	26.32	2.24	33	18	26.32	2.63
15	24	26.32	0.20	34	29	26.32	0.27
16	13	26.32	6.74	35	21	26.32	1.07
17	29	26.32	0.27	36	25	26.32	0.07
18	24	26.32	0.20	00	31	26.32	0.83

Figure 9: Chi-Square Test Data

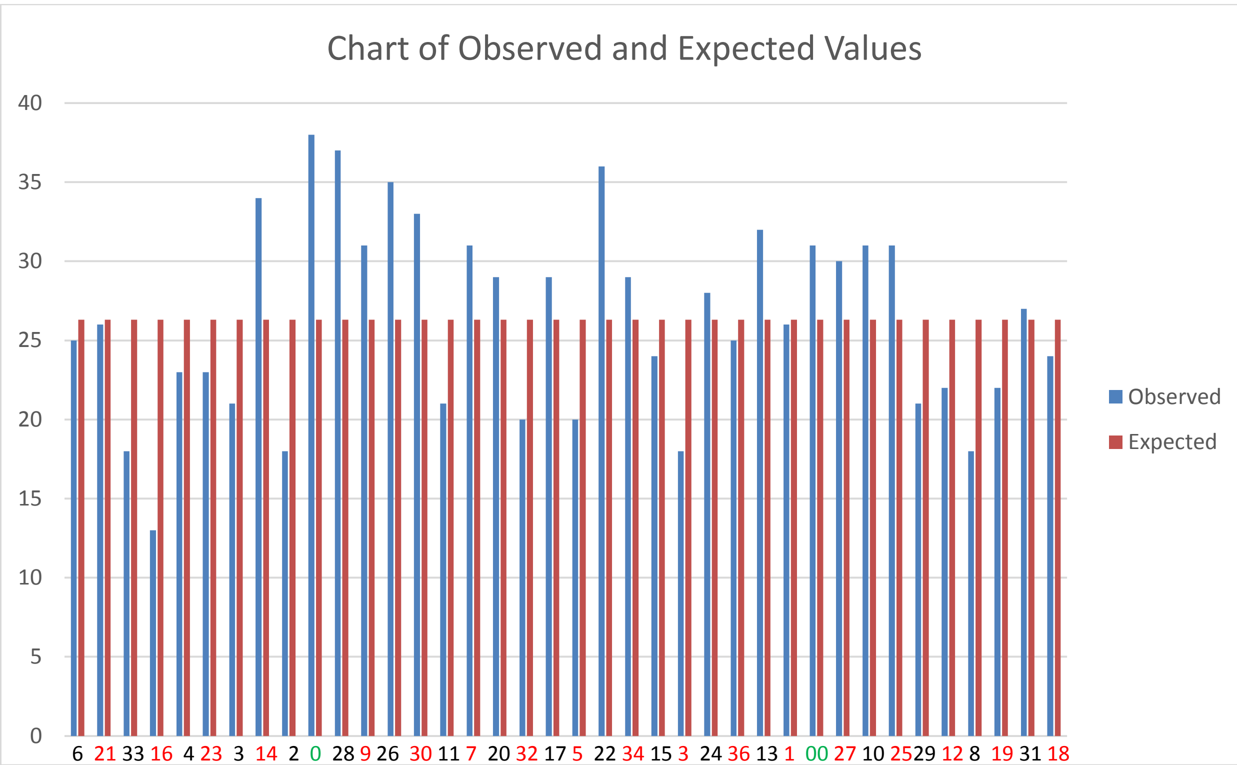

Figure 10: Chart of Observed and Expected Values

Limitations of the Study:

The roulette wheel turned out to be biased, and thus unfair, which caused a problem in my attempt to answer the main question of this study: can we accurately predict the pocket the ball lands in by using initial conditions such as were set in my methods section? The ball was dropped near the single 0 pocket, and the number of spins of the ball was controlled to have an average of about 20 rotations before landing in the wheel. The 95% confidence interval, computed using a one-sample t-method test, turned out to be (19.8904, 20.0536), which implies that the result would be very close to the 0 pocket if the roulette wheel was fair. I did see a lot of activity near the 0 pocket, which could affirm the previous statement; however, since the wheel is biased, I cannot say with statistical certainty that this activity was due to the nature of the wheel and not the deformities in the wheel itself.

Conclusion:

Based on the results, I reached a few conclusions. Firstly, the physical aspects of the wheel have a great impact on the outcome of the ball. Physical deformities affect the wheel and skew the data. I infer that if the wheel did not have those deformities portrayed in Figure 5, then there would have been a rightward shift for distribution B in Figure 2. The hits would have occurred later in the wheel, most likely closer to the unbiased distribution C in Figure 2. There would be a tendency for the ball to hit more often on the opposite half of the wheel from where it was originally spun. However, there is the possibility that regardless of the deformities, there would have been a distribution amongst those five numbers. To test either of these two hypotheses, more testing must be done on an undamaged wheel.

Secondly, unlike distribution B centered on the deformities, the other three distributions appear to have not been affected by the deformities. I mean to say that there did not appear to be any outside bias forcing these three distributions to occur as often. These three distributions, although not as statistically skewed as the biased distribution B, occur under the conditions presented in the experiment. These distributions came from the statistical nature in the wheel under the three conditions used instead of coming from a natural state of fair chance. I conclude there are two distributions (A and D) of numbers that did not hit as much. It would be best to avoid betting on these numbers when the ball is released from the 0 pocket on this wheel.

Distribution C has an unbiased inclination for numbers that hit higher than the mean when released from the 0 pocket. If this pattern exists among other wheels, I would take full advantage of this opportunity and bet on these numbers.

Finally, the advantage from these distributions slightly changes the outcomes. When placing bets on this wheel, it would be best to place bets on the numbers in distribution B and C. This experiment yielded data that I interpret, with statistical significance, that the wheel used was biased. The data can be applied only to the wheel that I used. If I were to replicate this experiment again, I would attempt to get possession of a handful of roulette wheels used in casinos. I would then spin each wheel 2000 times: the first 1000 in order to get a control spin and the second 1000 spins would abide by the conditions I had set up in the methods section. I can conclude from this experiment that there is an association between the entrance position of the ball and the final pocket. Roulette is not a game of fair chance when played on this particular wheel.

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