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Structural analysis and design of a flexible three-link hydraulically-actuated robotic arm

Tarek Mitri Bannoura
University of Nevada, Las Vegas

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Structural analysis and design of a flexible three-link hydraulically-actuated robotic arm

Bannoura, Tarek Mitri, M.S.

University of Nevada, Las Vegas, 1988
STRUCTURAL ANALYSIS AND DESIGN OF A FLEXIBLE THREE–LINK HYDRAULICALLY ACTUATED ROBOTIC ARM

by

Tarek Mitri Bannoura

A thesis submitted in partial fulfillment of the requirements for the degree of

Masters of Science in Civil Engineering in

Structural Engineering

Department of CE/ME Engineering
University of Nevada, Las Vegas
December, 1988
The thesis of Mr. Tarek Mitri Bannoura for the degree of masters in Civil Engineering is approved.

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University of Nevada, Las Vegas
December, 1988
The structural design of a flexible three-link hydraulically activated robotic mechanism is presented. Static and quasi dynamic, three-dimensional analysis of the robotic mechanism is shown. The analysis assumes loading on the links in the form of generalized force and moment vectors. Force and deflection equations are derived for the robotic mechanism and the finite element analysis is used to model its dynamic behavior and to study the fundamental 3-D bending and twisting frequencies of the arm as it reaches various positions inside the workspace. Using beam-column theory and finite element method, the design of a flexible three-link robotic mechanism is shown. The arm has 2.8m (110.24 in.) reach, is driven by two hydraulic actuators and is attached to a stiff rotating 1m (39.37 in.) long vertical shaft manipulated by a heavy hydraulic base. The links are hollow 7.62 cm (3 in.) square grade 46 steel tubes, 0.635 cm (0.25 in.) thick. The flexibility of the arm is set so that the total deflection of the arm is limited to 2%-3% of its maximum reach; and its first two fundamental frequencies are less than 6 Hz. The arm is capable of carrying a load equal or greater to its own weight.
To My Wonderful Parents
Mitri And Afifeh
And Wife
Linda
ACKNOWLEDGMENTS

I would like to express my warm thanks to my advisor and major professor Dr. Samaan G. Ladkany for his continuous help and direction throughout this research project. I would also like to thank Dr. Richard V. Wyman for his advise and support to me ever since I started my studies at UNLV.

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Tarek Mitri Bannoura
NOTATIONS

The following list contains the most frequently used symbols in this thesis. Also the symbols are defined more clearly in the thesis where they appear.

ANGLES

$\phi_1$ is the angle of member No.1 measured counter clockwise from horizontal
$\phi_2$ is the angle of member No.2 measured counter clockwise from horizontal
$\phi_3$ is the angle of member No.3 measured counter clockwise from horizontal
$\phi_4$ is the angle of the hydraulic actuator "ab" measured counter clockwise from horizontal
$\phi_5$ is the angle measured counter clockwise from horizontal to the tail of link No. 2
$\phi_6$ is the angle measured counter clockwise from horizontal to hydraulic actuator "de"

REACTIONS

$R^s_{4x}$, $R^d_{4x}$ are the applied static and dynamic force at the tip of the end effector in the global x–axis
$R^s_{4y}$, $R^d_{4y}$ are the applied static and dynamic force at the tip of the end effector in the global y–axis
$R^s_{4z}$, $R^d_{4z}$ are the applied static and dynamic force at the tip of the end effector in the global z–axis
$M^s_{4x}$, $M^d_{4x}$ are the applied static and dynamic moment at the tip of the end–effector in the global x–axis.
$M^s_{4y}$, $M^d_{4y}$ are the applied static and dynamic moment at the tip of the
end-effector in the global y-axis.

\( M^s_{4z}, M^d_{4z} \) are the applied static and dynamic moment at the tip of the end-effector in the global z-axis.

\( R^s_{3x}, R^d_{3x} \) are the static and dynamic force reaction at joint No.3 in the global x-axis

\( R^s_{3y}, R^d_{3y} \) are the static and dynamic force reaction at joint No.3 in the global y-axis

\( R^s_{3z}, R^d_{3z} \) are the static and dynamic force reaction at joint No.3 in the global z-axis

\( M^s_{3x}, M^d_{3x} \) are the static and dynamic moment reaction at joint No.3 in the global x-axis.

\( M^s_{3y}, M^d_{3y} \) are the static and dynamic moment reaction at joint No.3 in the global y-axis.

\( M^s_{3z}, M^d_{3z} \) are the static and dynamic moment reaction at joint No.3 in the global z-axis.

\( R^s_{2x}, R^d_{2x} \) are the static and dynamic force reaction at joint No.2 in the global x-axis.

\( R^s_{2y}, R^d_{2y} \) are the static and dynamic force reaction at joint No.2 in the global y-axis.

\( R^s_{2z}, R^d_{2z} \) are the static and dynamic force reaction at joint No.2 in the global z-axis.

\( M^s_{2x}, M^d_{2x} \) are the static and dynamic moment reaction at joint No.2 in the global x-axis.

\( M^s_{2y}, M^d_{2y} \) are the static and dynamic moment reaction at joint No.2 in the global y-axis.
the global y–axis.

\( F^s_{de}, F^d_{de} \) are the static and dynamic force in hydraulic actuator "de".

\( R^s_{1x}, R^d_{1x} \) are the static and dynamic force reaction at joint No.1 in the global x–axis.

\( R^s_{1y}, R^d_{1y} \) are the static and dynamic force reaction at joint No.1 in the global y–axis.

\( R^s_{1z}, R^d_{1z} \) are the static and dynamic force reaction at joint No.1 in the global z–axis.

\( M^s_{1x}, M^d_{1x} \) are the static and dynamic moment reaction at joint No.1 in the global x–axis.

\( M^s_{1y}, M^d_{1y} \) are the static and dynamic moment reaction at joint No.1 in the global y–axis.

\( F^s_{ab}, F^d_{ab} \) are the static and dynamic force in hydraulic actuator "ab".

\( R^s_{0x}, R^d_{0x} \) are the static and dynamic force reaction at joint No.0 in the global x–axis.

\( R^s_{0y}, R^d_{0y} \) are the static and dynamic force reaction at joint No.0 in the global y–axis.

\( R^s_{0z}, R^d_{0z} \) are the static and dynamic force reaction at joint No.0 in the global z–axis.

\( M^s_{0x}, M^d_{0x} \) are the static and dynamic moment reaction at joint No.0 in the global x–axis.

\( M^s_{0y}, M^d_{0y} \) are the static and dynamic moment reaction at joint No.0 in the global y–axis.
are the static and dynamic moment reaction at joint No.0 in the global z-axis.

are the total applied forces at the tip of the end-effector in the X, Y, Z axis respectively.

are the total applied moments at the tip of the end-effector in the X, Y, Z axis respectively.

are the total reaction force at joint No.3 in the X, Y, Z axis respectively.

are the total reaction moments at joint No.3 in the global X, Y, Z axis respectively.

are the total reaction force at joint No.2 in the X, Y, Z axis respectively.

are the total moment reaction at joint No.2 in the X, Y axis respectively.

is the total force in the hydraulic actuator "de".

are the total reaction force at joint No.1 in the X, Y, Z axis respectively.

are the total moment reaction at joint No.1 in the X, Y axis respectively.

is the total force in the hydraulic actuator "ab".

are the total reaction force at joint No.0 in the X, Y, Z axis respectively.

are the total reaction moments at the base of the robotic arm in the X, Y, Z axis respectively.
APPLIED FORCES

$Q_{g0x}$ is the dynamic global component of the applied force at the center of gravity in the $x$-axis of link No.0.

$Q_{g0y}$ is the dynamic global component of the applied force at the center of gravity in the $y$-axis of link No.0.

$Q_{g0z}$ is the dynamic global component of the applied force at the center of gravity in the $z$-axis of link No.0.

$Q_{g1x}$ is the dynamic global component of the applied force at the center of gravity in the $x$-axis of link No.1.

$Q_{g1y}$ is the dynamic global component of the applied force at the center of gravity in the $y$-axis of link No.1.

$Q_{g1z}$ is the dynamic global component of the applied force at the center of gravity in the $z$-axis of link No.1.

$Q_{g2x}$ is the dynamic global component of the applied force at the center of gravity in the $x$-axis of link No.2.

$Q_{g2y}$ is the dynamic global component of the applied force at the center of gravity in the $y$-axis of link No.2.

$Q_{g2z}$ is the dynamic global component of the applied force at the center of gravity in the $z$-axis of link No.2.

$Q_{g3x}$ is the dynamic global component of the applied force at the center of gravity in the $x$-axis of link No.3.

$Q_{g3y}$ is the dynamic global component of the applied force at the center of gravity in the $y$-axis of link No.3.

$Q_{g3z}$ is the dynamic global component of the applied force at the center of gravity in the $z$-axis of link No.3.
gravity in the z-axis of link No.3.

LOAD COMPONENTS IN LOCAL COORDINATES

$q_{1y}^0, q_{2y}^0$ are the distributed forces on link No. 1 & 2, in the local y-axis, due to the torques applied at their centers of gravity.

$q_{1z}^0, q_{2z}^0$ are the distributed forces on link No. 1 & 2, in the local z-axis due to the torques applied at their centers of gravity.

$q_{1y}^l, q_{2y}^l$ are the distributed forces on link No. 1 & 2, in the local y-axis, due to the forces applied at their centers of gravity.

$q_{1z}^l, q_{2z}^l$ are the distributed forces on link No. 1 & 2, in the local z-axis, due to the forces applied at their centers of gravity.

$R_{2x}^l, R_{3x}^l$ are the local reaction force in the x-axis of members No. 2 & 3.

$R_{2y}^l, R_{3y}^l$ are the local force reaction in the y-axis of members No. 2 & 3.

$M_{2y}^l, M_{3y}^l$ are the local moment reactions in the y-axis of members No.2 & 3

MATERIAL PROPERTIES

$j_1$ is the polar moment of inertia of link No.1.

$j_2$ is the polar moment of inertia of link No.2.

$I_{1y}, I_{2y}$ are the bending moment of inertia for member No.1 & 2 around the y-axis

$I_{1z}, I_{2z}$ are the bending moment of inertia for member No.1 & 2 around the z-axis

$E_1, E_2$ are the Young's modulus of elasticity for member No.1 & 2.

$A_1, A_2$ are the cross-sectional areas for link No.1 & 2.
Aab, Ade are the cross-sectional areas of actuators "ab" & "de".

Err1, Err2 are errors accounted for by the controllers.

**DISTANCES**

\[ P_{01} \] is distance between joint No.0 and joint No.1

\[ P_{12} \] is the distance between joint No.1 and joint No.2.

\[ P_{34} \] is the distance between joint No.3 and the tip of the end-effector.

\[ P_{1b} \] is the distance between joint No.1 and point "b".

\[ P_{2e} \] is the distance between joint No.2 and point "e".

\[ P_{0g0} \] is the local distance between joint No.0 and center of gravity of link No.0.

\[ P_{1g1} \] is the local distance between joint No.1 and center of gravity of link No.1.

\[ P_{2g2} \] is the local distance between joint No.2 and center of gravity of link No.2.

\[ P_{3g3} \] is the local distance between joint No.3 and center of gravity of link No.3.

**LOCAL DISPLACEMENTS**

\[ V1, V2 \] are the local deflection of members No.1 & 2 in y-axis.

\[ \theta_{z1}, \theta_{z2} \] are the local slopes, for member No.1 & 2, of the deflected links in the x-z coordinate.

\[ W_{i} \] are the local deflection of members No.1 & 2 in z-axis.

\[ \theta_{y1}, \theta_{y2} \] are the local slopes, for member No.1 & 2, of the deflected links in the x-y coordinate.

\[ \theta_{x1}, \theta_{x2} \] are the local twists of members No.1 & 2.

\[ U_{x1}, U_{x2} \] are the local extensions of members No.1 & 2.

**GLOBAL DISPLACEMENTS**

ROTANG2X is the rotational angle of joint No.2 in the Global x-axis.

ROTANG2Y is the rotational angle of joint No.2 in the Global y-axis.
ROTANG2Z is the rotational angle of joint No. 2 in the Global z-axis.

DEFL2x is the total deflection in the global x-axis.

DEFL2Y is the total deflection in the global y-axis.

DEFL2Z is the total deflection in the global z-axis.

ROTANGX is the rotational angle of joint No. 3 in the global x-axis.

ROTANGY is the rotational angle of joint No. 3 in the global y-axis.

ROTANGZ is the rotational angle of joint No. 3 in the global z-axis.

TOTDEFX is the Total deflections of joints No. 3 in the global x-axis.

TOTDEFY is the Total deflections of joints No. 3 in the global y-axis.

TOTDEFZ is the Total deflections of joints No. 3 in the global z-axis.
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CHAPTER ONE

INTRODUCTION

This work is done under grant to the University of Nevada, Las Vegas (UNLV) by the Army Research Office (ARO). The objective is to design a lightweight three linkage mobile elastic robotic arm capable of carrying a payload greater or equal to its own weight. This research project will expand existing robot kinematics and dynamic theory to include the influence of elasticity and damping on robot arm dynamics and oscillations. The analytical model will serve as a basis for the design and testing of the lightweight robotic arm. The modeling effort will include, a mathematical model and a development of an integrated computer graphics and on-line simulation package that can provide realistic animated simulation of the robot motion, including elastic deformation and oscillation. The control of the multi-link elastic manipulator utilizes optical deflections and slope sensors in each elastic link of the manipulator along with encoder output at each
joint to decouple the dynamic interaction between elastic links and cancel non-linear terms in the dynamic equations of for independent linear position servo at each joint. An optical deflection/slope sensor is used to provide feedback information of link deflection and slope for modifying joint angles in order to compensate the end effector position error due to the flexibility of the robot links.

Lightweight robot arms promise to open up new applications especially for mobile robots, or for the use in space applications; consequently, the performance of the robotic systems and their speed will improve dramatically with advances in flexible robot technology.

This thesis will concentrate on the structural analysis and design of the flexible elastic arms to study their behavior due to applied static and dynamic forces, and to determine the fundamental mode shapes and frequencies of the structure in order to control its behavior under operating conditions.

Generalized force, moment, and deflection equations in 3-D space are derived for the hydraulically actuated robotic structure. The solutions to these equations are in closed form which allow fast numerical determination of their values. Forces, moments, and deflection are calculated due to various types of static and quasi-dynamic loading for real-time control purposes. Since the robotic arms are hydraulically actuated, they could easily be attached to the hydraulic systems used on trucks, earth moving equipments or in aerospace structures. The lightweight flexible robotic arm could have various types of applications such as the handling of hazardous substances.

Rigid robots are usually designed as heavy and robust mechanisms that perform precise tasks. The weight ratios between the rigid robotic arms and their
load capacities usually exceeds ten to one, as in the case of the Puma arm which utilizes a stiff structure that avoids problems associated with the appreciable flexibility under load, as well as an array of oscillatory mode forms which are characteristic of light, flexible robots.

Elastic distortion and dynamic oscillations inate to the behavior of flexible lightweight robotic mechanisms, make their design and real time control difficult. Nevertheless, the design of elastic robot manipulator arm is expanding due to their inherent speed, ease of transportation and higher efficiency \[\text{Book (1984), Geradin and Robert (1984), Hestings and Book (1986), Nicosia et al. (1986), Singh and Schy (1986 a,b), Wang (1987)}\]. While most industrial robots use electric stepper motors at the joints, the flexible robotic arm design presented in this thesis uses hydraulic actuators which allow the robotic arm to handle large payloads equal to or greater than its own weight.

The analysis presented here applies the beam—column theory and the combined axial and bending loading interaction equations of steel structures for the design of the robotic linkages out of square tubular steel, AISC (1980). Elastic displacements of linkages in 3—D space are derived from the elastic beam bending theories, Timoshenko (1959), in the local coordinates of the linkages, while the deflected position of the end effector, relative to the undeformed shape of the robot, is found in the global coordinate system using coordinate transformations. The analysis assumes that the position, acceleration and the velocity vectors at the end effector and at the centers of gravity of all linkages are determined for the rigid body motion of the linkages by the controller at any given time and position inside the work space of the robot. The final deflected positions of the joints and
the end effector are found by the superposition of the rigid body displacements and
the elastic displacements. The fundamental mode shapes and frequencies of the
structure at any given position is determined in 3-D by finite element using
GIFTS (1985).
CHAPTER TWO

BACKGROUND

The term robot comes from a Czechoslovakian word "Robotnik" meaning worker. It was introduced in 1922 by Karl Capek in his play "Rossum's Universal Robots". In the 1930's, the term "android" which comes from andros that means human was introduced by the science fiction writers to refer to an artificially constructed robot that had the characteristics of human, (Derek, Kelly 1986).

Industrial robots are defined by the British Robot Association as, "A reprogrammable device designed to both manipulate and transport parts, tools, or specialized manufacturing implements through variable programmable motions for the performance of specific manufacturing tasks", (Simon, 1986).

Presently, manipulators are playing an important role in industry. They can perform some manipulative tasks fast and accurate. The use of manipulators results in enhanced product quality, greater direct and indirect labor productivity,
and increased worker safety while simultaneously requiring less floor space than a non-automated operation.

The use of manipulators in industrial applications is expanding at a rapid growth rate in all industrial countries especially in the United States and Japan. Table (2.1) shows the projection of robots in industrial use, Derek, Kelly (1986)

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</tr>
<tr>
<td>1995</td>
<td>1,000,000</td>
<td>10,000,000</td>
</tr>
<tr>
<td>2000</td>
<td>2,000,000</td>
<td>20,000,000</td>
</tr>
</tbody>
</table>

Table (2.1) Projection of robots in industry (Derek, Kelly)

Considerable research activity is being conducted in response to the demand and need for high performance robotic systems which operate at high speed while providing superior performance characteristics. The evolution of high performance robotic systems has been impeded by the lack of standardized methodology for designing light-weight robotic arms. The need for such methodology has been recognized by the academic and industrial communities.

The current generation of robots are designed with robust members that have substantial weight, large moment of inertia, and high structural stiffness.
Lightweight manipulator arm will enhance the quality of robots, while on the other hand, they add a high degree of complexity to the control and design aspects of the robot.

Flexible robotic arms exhibit many advantages over rigid arms; they are lighter, consume less power, are more maneuverable, require smaller actuators and are more mobile. However, production industries require that manipulators must function within a reasonable accuracy in response of the arm’s end-point to the joint control system input commands, this makes the design and control of flexible lightweight robots complex. Several researchers have suggested that the finite element method is the right analytical tool to model light-weight robots accurately in order to study the structural behavior of the mechanical linkages and to obtain their dynamic response, Usoro, Nadira, and Mahil(1986), Gerardin, Robert, and Bernardin(1984).

New methodologies are being considered that study the fabrication of different materials to be used in the design of light-weight robots to achieve the very high structural stiffness usually found in rigid robots. This philosophy promises the construction of light-weight robots with higher end-point accuracy, faster response times, shorter settling times upon completion of a maneuver, and lower power consumption, Sung and Thompson (1987). Therefore a model of symmetric laminated hollow-box-section beam was developed. The use of composite material to build light-weight robots is based on the proposition that the optimal performance of a robot with a specific kinematic chain and controller, operating under specific kinematic constraints and payload, is governed by the material
properties of the structural elements and their geometrical configuration, Sung and Thompson (1987).

Non–linear flexibility studies for manipulators were done by Naganathan and Soni (1986) where Galerkin’s Technique was used to render the kinematic and kinetic relations for differential segment in an integral form suitable for a finite element scheme to solve these equations. Three link flexible robot using tubular aluminum with outside diameter of 0.1m (3.94in.) and an inside diameter of 0.094m (3.7in.) was studied. A finite element model was developed to determine the non–linear effects of link flexibilities, when a manipulator undergoes gross motions at the joints. The model allows for the nonlinear coupling effects between the the nonlinear gross motions of the manipulator links and their elastic deformations. Another finite element model for two links was developed and studied by Usoro, Nadira, Mahil (1986) where the two links have a length of 1m (39.37in.), the same moment of inertia of 5x10^{-3}m^{4}, and the same material constant (E) of 2.0x10^{11}N/m^{2}. The model was tested once with and without payload at the tip. A Lagrangian approach to the dynamic model was used. Each link of the manipulator is treated as an assemblage of a finite number of elements for each of which kinetic and potential energies are derived. These elemental kinetic and potential energies are then suitably combined to derive the dynamic model for the system.

A flexible single link manipulator arm model was developed by Hastings and Book (1986), the resultant model was compared to an experimental 1.22m long manipulator. The method employed to generate the model utilizes a separable formulation of assumed modes to represent the transverse displacement due to
bending. Langragian dynamic equations were organized into a state space model suitable for use in linear control system design procedures. The material used for the model was aluminum 6061–T6 with a rectangular cross–section of 0.0191m (0.75in.) by 0.0048m (0.188in.) and 1.22m (48 in.) long, Hastings and book (1986).
CHAPTER THREE

STRUCTURAL ANALYSIS OF THE ROBOT

3.1 Basic Assumption For The Structural Analysis.

The determination of the elastic deformations of the segments and the final deflected position of the end effector, at a given position and loading conditions, requires correct expressions for the variations of the moment and shears across the segments which in turn require the correct determination of the weight of the structure, the loads carried by the end effector and the dynamic inertia forces.

Some basic assumptions used in this analysis are:

1. The links are slender prismatic cantilever beams.
2. Materials are homogeneous isotropic.
3. Deformations are within the small deflection theory of beam–column structures.

4. Stresses in the flexible linkages are within the elastic range.

5. The hydraulic actuators are attached to the rigid joints that connect the links but not to the links themselves.

6. The mass of the structure and its payload are subject to 3–D translational and rotational acceleration vectors.

7. The flexible links are attached to a stiff column which acts as the first link of the mechanism; which in turn is rigidly attached to a hydraulic base that provides fixed end conditions at the bottom of the column.

Figure(3.11) shows a schematic of a 3–link robotic mechanism attached to a rotating rigid base and manipulated by two hydraulic actuators acting in the plane of the arms. The third actuator is horizontally placed within the base and is used to rotate the arms.

3.2 Loading Conditions

Static and quasi dynamic loads are assumed to apply on the three link robotic structure. Static loads include the weight of the links, the hydraulic actuators, and the end–effector loads; That is three axial forces and three moment couples. Dynamic loadings are due to linear and angular acceleration vectors at the
center of gravity of the elastic links and the end effector. Therefore, three axial forces and three moment couples are assumed to act at the centers of gravity of the links and at the payload. These loading conditions are used in deriving the reactions at the joints of the links and in the hydraulic actuators.

3.2.1 The end effector refer to figure(3.4)

In the 3-D analysis of the robot, six degrees of freedom are assumed to exist at the tip of the end effector, three translational and three rotational vectors. Consequently, six different loading conditions can be applied to the end effector while the robot is operating, three axial forces and three moment couples.

3.2.2 Static weights of the arms, actuators and joint (figure 3.1—3.4)

Static loading includes the weight of the links, the actuators plus the static loads and couples applied to the end effector. These loading conditions are considered in the analysis of the robotic structure. Figure 3.5 shows the free body diagram of the structure with the applied static loads. Equivalent vectors representing the weights of the various links are shown to apply to the centers of gravity of the links.

3.2.3 Quasi—dynamic loading conditions

The quasi dynamic loading conditions affecting the robotic arm are derived
using the results of a complete kinematics analysis of the 3—link mechanism which
predicts the displacement and rotational accelerations $\ddot{a}_{xi}, \ddot{\theta}_{xi}$ of the structure.
For a given position of the end effector, the angles of linkages and the
displacement speeds of the hydraulic actuators, the kinematic analysis predicts the
velocities and accelerations at the end effector and the centers of gravity of the
linkages, Trabia (1988). Refer to figures (3.6—3.10).

Thus the forces and moment applied to the robotic mechanism are derived
as follows:

$$F_i = m_i \ddot{a}_i$$  \hspace{1cm} (3.1)

$$M_k = J_k \dddot{\theta}_k$$  \hspace{1cm} (3.2)

Where:

$m_i$ are masses.
$\ddot{a}_i$ are accelerations vectors.
$M_k$ are torques vectors.
$J_k$ are mass moment of inertias of the linkages and the payload.
$\dddot{\theta}_k$ are angular acceleration vectors.

From equation (3.1—3.2) expressions for the forces at the joints, and in the
hydraulic actuators are obtained for any 3—D position of the arm and for any set of
applied static and dynamic inertia loads.
3.3 Structural Analysis Of The Robot Due To Static Loading Conditions

The general static equations for the reactions at each joint and for the forces in the hydraulic actuators are derived and presented. The static analysis considers the weights of the links, joints, hydraulic actuators plus the forces and torques at the end effector due to applied loads, see figure (3.1–3.5).

The determination of the general static equations for the robotic arm are derived using the six vector equations:

\[ \Sigma F_i = \mathbf{0} \quad (3.3) \]
\[ \Sigma \mathbf{r}_i \times F_i + \Sigma M_j = \mathbf{0} \quad (3.4) \]

Where:
\[ \mathbf{r}_i \] are positions vectors.
\[ F_i \] are force vectors.
\[ M_j \] are moment vectors.

Free body diagrams showing the forces and moments acting on each link and the structure, as a whole, are shown in figures (4.3–4.8). In the analysis the following symbols are used:

Superscript "s" refers to static forces and "d" refers to dynamic forces

"i" (i=0,3) refers to a joint number,"gk" refers to the center of gravity of the link (gk=1,3) and "k" is a general point number in the structure.

\[ W_i \] is the weight of link "i".
\( R_{ix}, R_{iy}, R_{iz} \) are the global components of the force reactions at joint \( i \) (\( i=0,3 \)).
\( M_{ix}, M_{iy}, M_{iz} \) are the global components of the moment reaction at (\( i=0,3 \)).
\( Q_{gix}, Q_{giy}, Q_{giz} \) are the global components of the applied forces at the centers of gravity of the members
\( T_{gix}, T_{giy}, T_{giz} \) are the global components of the applied torques at the centers of gravity of the members.
\( P_{ik} \) is the local distance between points "i" and "k".
\( P_{igk} \) is the local distance between points "i" and center of gravity of link "k".
\( F_{ab}, F_{de} \) are the forces in actuators "ab" and "de".
\( \phi_l, (l=0,6) \) are the angles measured counterclockwise from the horizontal to the center lines of the links, the end effector and the actuators.

3.3.1 Static derivation for the whole structure (figure 3.5)

\[ \Sigma F = \Sigma F_x i + \Sigma F_y j + \Sigma F_z k \]

\[ \Sigma F = (-R_{0x} + R_{4x}) i + (-R_{0y} - W - W - W - W + R_{4y}) j + \]

\[ (-R_{0z} + R_{4z}) k = 0 \]
\[ \sum \mathbf{r}_i \times F_i + \sum M_j = \]

\[ \left[ (P_{12} \sin \phi_1, P_{23} \sin \phi_2, P_{34} \sin \phi_3) \mathbf{R} \right] \hat{i} - \]

\[ \left[ (P_{12} \cos \phi_1, P_{23} \cos \phi_2, P_{34} \cos \phi_3) \mathbf{R} \right] \hat{j} - \]

\[ \left[ -(P_{12} \cos \phi_1) \mathbf{W} - (P_{23} \cos \phi_2 + P_{34} \cos \phi_3) \mathbf{W} \right] \hat{k} - \]

\[ (P_{12} \cos \phi_1, P_{23} \cos \phi_2, P_{34} \cos \phi_3) \mathbf{W} \hat{l} + \]

\[ (P_{12} \sin \phi_1, P_{23} \sin \phi_2, P_{34} \sin \phi_3) \mathbf{R} \hat{x} + \]

\[ (P_{12} \cos \phi_1, P_{23} \cos \phi_2, P_{34} \cos \phi_3) \mathbf{R} \hat{y} \]

\[ M_{0x} \hat{i} - M_{0y} \hat{j} - M_{0z} \hat{k} + M_{4x} \hat{i} + M_{4y} \hat{j} + M_{4z} \hat{k} = 0 \]

**3.3.2 Static derivation for link No.1, refer to figure(3.2)**

\[ \Sigma \mathbf{F} = \Sigma F_{x} \hat{i} + \Sigma F_{y} \hat{j} + \Sigma F_{z} \hat{k} \]

\[ \Sigma \mathbf{F} = (\mathbf{F}_{1x} \mathbf{R}_{de} \mathbf{cos} \phi_{5} \mathbf{F}_{ab} \cos \phi_{4} \mathbf{R}_{2x}) \hat{i} + \]

\[ (\mathbf{F}_{1y} \mathbf{R}_{de} \mathbf{sin} \phi_{5} \mathbf{F}_{ab} \sin \phi_{4} \mathbf{R}_{2x}) \hat{j} + \]

\[ (\mathbf{F}_{1z} \mathbf{R}_{1z} \hat{k} = 0 \]
\[ \sum r_i \times F_i + \sum M_j = \]
\[
\left[ \left( P_{12} \sin \phi_{12} \right) R_{2z} \right] i + \left[ \left( P_{12} \sin \phi_{12} \right) R_{2x} \right] j + \\
\left[ \left( P_{12} \cos \phi_{12} \right) R_{2y} - \left( P_{12} \sin \phi_{12} \right) R_{2x} \right] k - \\
\left( P_{1d} \sin \phi_{de} \right) F_{de} + \left( P_{1d} \cos \phi_{de} \right) F_{de} - \\
\left( P_{1g} \cos \phi_{de} \right) W_{de} + \left( P_{1g} \sin \phi_{de} \right) F_{de} \cos \phi_{de} - \\
\left( P_{1b} \cos \phi_{de} \right) F_{de} \sin \phi_{de} \]
\< k - \\
M_{ix} - M_{iy} + M_{ix} + M_{iy} = 0 \\
\]

3.3.3 Static derivation for link No.2, refer to figure(3.3)

\[ \sum F = \sum F_x i + \sum F_y j + \sum F_z k \\
\]
\[ \sum F = \left( -R_{2x} - F_{de} \cos \phi_{de} + R_{3y} \right) i + \\
\left( -R_{2y} - F_{de} \sin \phi_{de} + R_{3y} - W_2 \right) j + \\
\left( -R_{2z} + R_{3y} \right) k = 0 \]
\[ \Sigma \hat{r}_i \times \hat{F}_i + \Sigma M_j = \\
\left[ (P_{34} \sin \phi_{34}) R_{34} \right] \hat{i} + \left[ -(P_{34} \cos \phi_{34}) R_{34} \right] \hat{j} + \\
\left[ -(P_{34} \cos \phi_{34}) W_3 + (P_{34} \cos \phi_{34}) R_{34} \right] \hat{k} - \\
(P_{34} \sin \phi_{34}) R_{34} \hat{i} + \left[ -(P_{34} \cos \phi_{34}) R_{34} \right] \hat{j} + \\
\left[ -(P_{34} \cos \phi_{34}) W_3 + (P_{34} \cos \phi_{34}) R_{34} \right] \hat{k} = 0 \]

3.3.4 Static derivation for link No.3, refer to figure(3.4)

\[ \Sigma F = \Sigma F_x \hat{i} + \Sigma F_y \hat{j} + \Sigma F_z \hat{k} \]

\[ \Sigma F = (-R_{33} + R_{33}) \hat{i} + (-R_{3y} + R_{3y} - W_3 Y_{3y}) \hat{j} + \\
(-R_{3z} + R_{3z}) \hat{k} = 0 \]

\[ \Sigma \hat{r}_i \times \hat{F}_i + \Sigma M_j = \\
\left[ (P_{34} \sin \phi_{34}) R_{34} \right] \hat{i} + \left[ -(P_{34} \cos \phi_{34}) R_{34} \right] \hat{j} + \\
\left[ -(P_{34} \cos \phi_{34}) W_3 + (P_{34} \cos \phi_{34}) R_{34} \right] \hat{k} - \\
(P_{34} \sin \phi_{34}) R_{34} \hat{i} + \left[ -(P_{34} \cos \phi_{34}) R_{34} \right] \hat{j} + \\
\left[ -(P_{34} \cos \phi_{34}) W_3 + (P_{34} \cos \phi_{34}) R_{34} \right] \hat{k} = 0 \]
Derivation continued

\[
(P_{34} \sin \phi)R_3 \hat{k} - M_{3x} \hat{i} - M_{3y} \hat{j} - M_{3z} \hat{k} +
\]

\[
M_{4x} \hat{i} + M_{4y} \hat{j} + M_{4z} \hat{k} = 0
\]

3.4 Static And Dynamic Force At The Joints And The Hydraulic Actuators

By solving the simultaneous equations for the links and checking with the extra six equations for the whole structure, the following expressions for the reactions at the joints and forces in the hydraulic actuators are derived:

3.4.1 Base reactions due to static loading conditions, Refer to figure(3.1)

\[
R_{0x}^S = R_{4x}^S
\]

(3.5)

\[
R_{0y}^S = -W_{0} - W_{1} - W_{2} - W_{3} + R_{4y}^S
\]

(3.6)

\[
R_{0z}^S = R_{4z}^S
\]

(3.7)

\[
M_{0x}^S = (P_{01} + P_{12} \sin \phi + P_{23} \sin \phi + P_{34} \sin \phi)R_{4z}^S + M_{4z}^S
\]

(3.8)

\[
M_{0y}^S = -(P_{12} \cos \phi + P_{23} \cos \phi + P_{34} \cos \phi)R_{4z}^S + M_{4y}^S
\]

(3.9)
\[ M_{oz}^S = -P_{ig1} \cos \phi \cdot W - (P_{i2} \cos \phi + P_{2g2} \cos \phi)W - \\
(P_{i2} \cos \phi + P_{23} \cos \phi + P_{3g3} \cos \phi)W + \\
(P_{i2} \cos \phi + P_{23} \cos \phi + P_{34} \cos \phi)R_{12}^S - \\
(P_{i2} \sin \phi + P_{23} \sin \phi + P_{34} \sin \phi)R_{12}^S + M_{oz}^S \] (3.10)

3.4.2 Reaction at joint No.1 due to static loading conditions

refer to figure(3.2)

\[ F_{ab}^S = (P_{l1} \cos \phi \cdot F_{de}^S \sin \phi - P_{l1} \sin \phi \cdot F_{de}^S \cos \phi - \\
P_{l1} \cos \phi \cdot W + P_{l1} \cos \phi \cdot R_{ab}^S - P_{l1} \sin \phi \cdot R_{ab}^S) / \\
(P_{l1} \cos \phi \sin \phi - P_{l1} \sin \phi \cos \phi) \] (3.11)

\[ R_{1x}^S = F_{de}^S \cos \phi - F_{de}^S \cos \phi + R_{1x} \] (3.12)

\[ R_{1y}^S = F_{de}^S \sin \phi - W - F_{de}^S \sin \phi + R_{1y} \] (3.13)
Reactions at joint No.2 due to static loading conditions refer to figure (3.3)

\[
F_{de} = \left( -P_{23} \cos \phi \cdot R_{23} + P_{3y} \sin \phi \cdot R_{3y} + P_{2x} \cos \phi \cdot W_{2x} - M_{2z} \right) / \left( P_{2e} \sin \phi \cos \phi - P_{2e} \cos \phi \sin \phi \right) 
\]  
(3.18)

\[
R_{2x} = -F_{de} \cos \phi + R_{3x} 
\]  
(3.19)

\[
R_{2y} = -W_{2y} + R_{3y} - F_{de} \sin \phi 
\]  
(3.20)

\[
R_{2z} = R_{3z} 
\]  
(3.21)
\[ M_{2x} = P \sin \phi \cdot R_{2x} + M \]  
\[ (3.22) \]

\[ M_{2y} = -P \cos \phi \cdot R_{2y} + M \]  
\[ (3.23) \]

\[ M_{2z} = 0.0 \]  
\[ (3.24) \]

3.4.4 Reaction at joint No.3 due to static loading conditions

refer to figure(3.4).

\[ R_{3x} = R_{4x} \]  
\[ (3.25) \]

\[ R_{3y} = -W_{3y} + R_{4y} \]  
\[ (3.26) \]

\[ R_{3z} = R_{4z} \]  
\[ (3.27) \]

\[ M_{3x} = P \sin \phi \cdot R_{3x} + M \]  
\[ (3.28) \]

\[ M_{3y} = -P \cos \phi \cdot R_{3y} + M \]  
\[ (3.29) \]

\[ M_{3z} = P \cos \phi \cdot R_{3z} - P \sin \phi \cdot R_{3z} - P \cos \phi \cdot W_{3z} + M \]  
\[ (3.30) \]
3.5 Structural Analysis Of The Robot Due To Dynamic Loading Condition

The general dynamic equations for the reactions at each joint and forces in the hydraulic actuators are derived and presented. The quasi dynamic loadings on the robotic arm is derived using the results of a complete kinematics analysis Trabia (1988) of the 3–link mechanism which predicts the displacement and rotational accelerations of the structure for a given position of the end effector, the angles of linkages and displacement speeds of the hydraulic actuators.

3.5.1 Dynamic derivation for the whole structure, refer to figure(3.10)

\[ \Sigma F = \Sigma F^x \hat{i} + \Sigma F^y \hat{j} + \Sigma F^z \hat{k} \]

\[ \Sigma F = (-R_{0x} + Q_{g0x} + Q_{g1x} + Q_{g2x} + Q_{g3x}) \hat{i} + \]

\[ (-R_{0y} + Q_{g0y} + Q_{g1y} + Q_{g2y} + Q_{g3y}) \hat{j} + \]

\[ (-R_{0z} + Q_{g0z} + Q_{g1z} + Q_{g2z} + Q_{g3z}) \hat{k} = 0 \]

\[ \Sigma \bar{r}_i \times F_i + \Sigma M_j = \]

\[ \left[ (P_{0g} X Q_{g0z}) + (P_{01} + P_{1g} \sin \phi) X Q_{g1z} + \right. \]
Derivation continued

\[
\begin{align*}
(P_{01} + P_{12} \sin \phi + P_{23} \sin \phi)X_{Q_{g2z}} + \\
(P_{01} + P_{12} \sin \phi + P_{23} \sin \phi + P_{3g3} \sin \phi)X_{Q_{g3z}} \hat{i} + \\
\left[ -(P_{lg1} X_{Q_{g1x}}) + (P_{lg1} \cos \phi + P_{2g2} \cos \phi) \right] \hat{j} + \\
\left[ -(P_{0g0} Q_{g0x}) + (P_{lg1} \cos \phi Q_{lg1y}) - (P_{0l1} + P_{lg1} \sin \phi) \right] \hat{k} + \\
(P_{12} \cos \phi + P_{23} \cos \phi + P_{3g3} \cos \phi)Q_{g3y} - \\
(P_{01} + P_{12} \sin \phi + P_{23} \sin \phi + P_{3g3} \sin \phi)Q_{g3x} \hat{k} - \\
-M_{0x} \hat{i} - M_{0y} \hat{j} - M_{0z} \hat{k} + T_{g0x} \hat{i} + T_{g0y} \hat{j} + T_{g0z} \hat{k} + \\
+ T_{g1x} \hat{i} + T_{g1y} \hat{j} + T_{g1z} \hat{k} + T_{g2x} \hat{i} + T_{g2y} \hat{j} + T_{g2z} \hat{k} + \\
+ T_{g3x} \hat{i} + T_{g3y} \hat{j} + T_{g3z} \hat{k} = 0
\end{align*}
\]
3.5.2 Dynamic Derivation for link No.1, refer to figure (3.7)

\[ \Sigma F = \Sigma F_x \hat{i} + \Sigma F_y \hat{j} + \Sigma F_z \hat{k} \]

\[ \Sigma F = (R_{lx} + F_{de} \cos \phi + Q_{gtx} - F_{de} \cos \phi + R_{2x}) \hat{i} + \]

\[ (R_{ly} + F_{de} \cos \phi + Q_{gyy} - F_{de} \cos \phi + R_{2y}) \hat{j} + \]

\[ (-R_{lz} + Q_{glz} + R_{2z}) \hat{k} = 0 \]

\[ \Sigma \vec{r}_i \times F_i + \Sigma M_j = \]

\[ \left[ (P_{g11} \sin \phi)Q_{gtx} + (P_{g12} \sin \phi)R_{2x} \right] \hat{i} + \left[ -(P_{g11} \cos \phi)Q_{glz} + \right. \]

\[ (P_{g12} \cos \phi)R_{2x} \hat{j} + \left[ (P_{g11} \cos \phi)(F_{de} \sin \phi) - \right. \]

\[ (P_{g12} \sin \phi)(F_{de} \cos \phi) + (P_{g11} \cos \phi)Q_{gly} - \]

\[ (P_{g11} \sin \phi)Q_{glz} - (P_{g12} \cos \phi)(F_{de} \sin \phi) + \]

\[ (P_{g12} \sin \phi)(F_{de} \cos \phi) + (P_{g12} \cos \phi)R_{2y} - \]

\[ (P_{g12} \cos \phi)R_{2x} \hat{k} - M_{lx} - M_{ly} + M_{2x} + M_{2y} + \]

\[ T_{g1x} \hat{i} + T_{g1y} \hat{j} + T_{g1z} \hat{k} = 0 \]
3.5.3 Dynamic derivation for link No.2, refer to figure (3.8)

\[ \Sigma F = \Sigma F_x \hat{i} + \Sigma F_y \hat{j} + \Sigma F_z \hat{k} \]

\[ \Sigma F = (-R_{2x} + Q_{g2x} - F_{de \cos \phi_5} + R_{3x}) \hat{i} + \]

\[ (-R_{2y} + Q_{g2y} - F_{de \sin \phi_5} + R_{3y}) \hat{j} + \]

\[ (-R_{2z} + Q_{g2z} + R_{3y}) \hat{k} = 0 \]

\[ \Sigma \hat{r}_i \times F_i + \Sigma M_j = \]

\[ \left[ (P_{g2} \sin \phi_2')Q_{g2x} + (P_{g2} \sin \phi_2')R_{3x} \right] \hat{i} + \]

\[ \left[ (P_{g2} \cos \phi_2')Q_{g2x} - (P_{g2} \cos \phi_2')R_{3x} \right] \hat{j} + \]

\[ \left[ (P_{g2} \cos \phi_2')Q_{g2x} - (P_{g2} \sin \phi_2')Q_{g2x} \right] \hat{k} \]

\[ (P_{2g} \cos \phi_2')R_{2y} - (P_{2g} \sin \phi_2')R_{2y} - \]

\[ (P_{2e} \cos \phi_2')(F_{de \sin \phi_5}) + (P_{2e} \sin \phi_2')(F_{de \cos \phi_5}) \hat{k} \]

\[ -M_{2x} -M_{2y} + M_{3x} + M_{3y} + M_{3z} + T_{g2x} \hat{i} + T_{g2y} \hat{j} + T_{g2z} \hat{k} = 0 \]
3.5.4 Dynamic derivation for link No.3, refer to figure (3.9)

\[ \Sigma F = \Sigma F_i + \Sigma F_j + \Sigma F_k \]

\[ \Sigma F = (-R_{3x} + Q_{g3x})i + (-R_{3y} + Q_{g3y})j + \]

\[ (-R_{3z} + Q_{g3z})k = 0 \]

\[ \Sigma \bar{r}_i \times F_j + \Sigma M_j = \]

\[ \left[ (P_{3g3} \sin \phi_3)Q_{g3x} \right] i - \left[ (P_{3g3} \cos \phi_3)Q_{g3y} \right] j + \]

\[ \left[ (P_{3g3} \cos \phi_3)Q_{g3y} - (P_{3g3} \sin \phi_3)Q_{g3x} \right] k \]

\[ -M_{3x} -M_{3y} -M_{3z} + T_{g3x} \hat{i} + T_{g3y} \hat{j} + T_{g3z} \hat{k} = 0 \]

3.6 Dynamic Forces At The Joints And The Hydraulic Actuators

By solving the simultaneous equations and by checking with the six equations for the whole structure, for accuracy, the following expressions for the reactions at the joints and forces in the hydraulic actuators are derived:
3.6.1 Base reactions due to dynamic loading conditions

refer to figure(3.6)

\[ R_{0x}^d = Q_{0x}^d + Q_{g1x}^d + Q_{g2x}^d + Q_{g3x}^d \]  
(3.31)

\[ R_{0y}^d = Q_{0y}^d + Q_{g1y}^d + Q_{g2y}^d + Q_{g3y}^d \]  
(3.32)

\[ R_{0z}^d = Q_{0z}^d + Q_{g1z}^d + Q_{g2z}^d + Q_{g3z}^d \]  
(3.33)

\[ M_{0x}^d = P_{0g0}^d * Q_{0x}^d + (P_{01} + P_{2g2} \sin \phi) Q_{g1z}^d + \]

\[ (P_{01} + P_{2g2} \sin \phi + P_{2g2} \sin \phi) Q_{g2z}^d + \]

\[ (P_{01} + P_{2g2} \sin \phi + P_{2g2} \sin \phi) Q_{g3z}^d + \]

\[ T_{0x}^d + T_{g1x}^d + T_{g2x}^d + T_{g3x}^d \]  
(3.34)

\[ M_{0y}^d = -P_{1g1}^d * Q_{g1z}^d - (P_{12} \cos \phi + P_{2g2} \cos \phi) Q_{g2z}^d - \]

\[ (P_{12} \cos \phi + P_{2g2} \cos \phi + P_{3g3} \cos \phi) Q_{g3z}^d + T_{0y}^d + T_{g1y}^d + \]

\[ T_{g2y}^d + T_{g3y}^d \]  
(3.35)

\[ M_{0z}^d = -P_{0z}^d * Q_{0x}^d + P_{0g1}^d * Q_{g1y}^d - (P_{01} + P_{2g1} \sin \phi) Q_{g1x}^d + \]
Equation (3.36) continued

\[
(P_{12} \cos \phi + P_{2g2} \cos \phi)Q^d_{g2y} - (P_{12} + P_{2g2} \sin \phi + P_{g2} \sin \phi)Q^d_{g2x} + \]

\[
(P_{12} \cos \phi + P_{23} \cos \phi + P_{3g3} \cos \phi)Q^d_{g3y} - \]

\[
(P_{01} + P_{23} \sin \phi + P_{3g3} \sin \phi + P_{g3} \sin \phi)Q^d_{g3x} + \]

\[
T^d_{g0z} + T^d_{g1z} + T^d_{g2z} + T^d_{g3z} \quad (3.36)\]

3.6.2 Reactions at joint No.1 and actuator "ab" due to dynamic loading conditions

refer to figure(3.7)

\[
F_{ab} = (P_{1d} \cos \phi * F_{de} - P_{1d} \sin \phi * F_{de} + P_{5d} \cos \phi * Q^d_{l1g1} - P_{5d} \sin \phi * Q^d_{l1g1})/ \]

\[
(P_{zb} \sin \phi \cos \phi - P_{zb} \sin \phi \cos \phi) \quad (3.37)\]

\[
R_{1x} = F_{de} \cos \phi + Q_{g1x} - F_{ab} \cos \phi + R_{2x} \quad (3.38)\]

\[
R_{1y} = F_{de} \sin \phi + Q_{g1y} - F_{ab} \sin \phi + R_{2y} \quad (3.39)\]
3.6.3 Reactions at joint No.2 and actuator "de" due to dynamic loading conditions refer to figure(3.8)

\[
\begin{align*}
R_{1z} &= Q_{g1z} + R_{2z} \tag{3.40} \\
M_{1x} &= P_{lg1} \sin \phi * Q_{g1z} + P_{lg1} \sin \phi * R_{1z} + T + M_{g1x} + M_{2x} \tag{3.41} \\
M_{1y} &= -P_{lg1} \cos \phi * Q_{g1z} - P_{lg1} \cos \phi * R_{1z} + T + M_{g1y} + M_{2y} \tag{3.42}
\end{align*}
\]

\[
\begin{align*}
R_{2x} &= Q_{g2x} - F_{de} \cos \phi + R_{3x} \tag{3.43} \\
R_{2y} &= Q_{g2y} - F_{de} \cos \phi + R_{3y} \tag{3.44} \\
R_{2z} &= Q_{g2z} + R_{3z} \tag{3.45} \\
M_{2x} &= P_{2g2} \cos \phi * Q_{g2z} + P_{2g2} \sin \phi * R_{2z} + T + M_{g2x} + M_{3x} \tag{3.46} \\
M_{2y} &= -P_{2g2} \cos \phi * Q_{g2z} - P_{2g2} \cos \phi * R_{2z} + T + M_{g2y} + M_{3y} \tag{3.47} \\
F_{de} &= (P_{2g2} \cos \phi * Q_{g2y} - P_{2g2} \sin \phi * Q_{g2x} + P_{2g2} \cos \phi * R_{3y} - \ldots
\end{align*}
\]
Equation (3.48) continued

\[ P_{23} \sin \phi \sin j - R_{3x} + T_{23} + M_{3z} \]

\[ (P_{2e} \cos \phi - P_{2e} \sin \phi \cos \phi) \] (3.48)

3.6.4 Reactions at joint No.3 due to dynamic loading conditions

refer to figure (3.9)

The dynamic loads on link No.3, the end effector, are applied to its center of gravity.

\[ R_{3x} = Q_{g3x} \] (3.49)

\[ R_{3y} = Q_{g3y} \] (3.50)

\[ R_{3z} = Q_{g3z} \] (3.51)

\[ M_{3x} = P_{3g3} \sin \phi \sin j - Q_{g3x} + T_{g3x} \] (3.52)

\[ M_{3y} = P_{3g3} \cos \phi \sin j - Q_{g3y} + T_{g3y} \] (3.53)

\[ M_{3z} = P_{3g3} \cos \phi \sin j - P_{3g3} \sin \phi \cos \phi + T_{g3z} \] (3.54)
3.7 Total Force And Moment Reactions

The total moment and force reactions are the sum of the expressions for the static and dynamic loading conditions. The final equations for the reactions at each joint and the actuators are presented below.

3.7.1 Total base reactions

\[
\begin{align*}
R_{0x} &= R_x^S + Q_{4x}^d + Q_{g0x}^d + Q_{g1x}^d + Q_{g2x}^d + Q_{g3x}^d \\
R_{0y} &= -W_0 - W_{1} - W_{2} - W_{3} + R_y^S + Q_{4y}^d + Q_{g0y}^d + Q_{g1y}^d + Q_{g2y}^d + Q_{g3y}^d \\
R_{0z} &= R_z^S + Q_{4z}^d + Q_{g0z}^d + Q_{g1z}^d + Q_{g2z}^d + Q_{g3z}^d \\
M_{0x} &= (P_{01} + P_{12} \sin \phi + P_{23} \sin \phi + P_{34} \sin \phi) R_x^S + M_x^S + P_{0g0} * Q_{g0x}^d + \\
&\quad (P_{01} + P_{1g1} \sin \phi) Q_{g1z}^d + \\
&\quad (P_{01} + P_{12} \sin \phi + P_{2g2} \sin \phi) Q_{g2z}^d + \\
&\quad (P_{01} + P_{12} \sin \phi + P_{23} \sin \phi + P_{3g3} \sin \phi) Q_{g3z}^d + 
\end{align*}
\]
Equation (3.58) continued

\[ T^d \begin{array}{c} g_{1x} \\ g_{1x} \\ g_{2x} \\ g_{3x} \end{array} + T^d \begin{array}{c} g_{2x} \\ g_{2x} \\ g_{3x} \\ g_{3x} \end{array} + T^d \begin{array}{c} g_{2x} \\ g_{2x} \\ g_{3x} \\ g_{3x} \end{array} + T^d \begin{array}{c} g_{3x} \\ g_{3x} \\ g_{3x} \end{array} \]

\[ M_{0y} = -(P_{12} \cos \phi + P_{23} \cos \phi + P_{34} \cos \phi)R^s_{g_{1y}} + M^s - P_{1g_1} \cos \phi \cdot Q^d \begin{array}{c} g_{1z} \\ g_{1z} \\ g_{2z} \\ g_{2z} \end{array} - (P_{12} \cos \phi + P_{23} \cos \phi)Q^d \begin{array}{c} g_{2z} \\ g_{2z} \end{array} - (P_{12} \cos \phi + P_{23} \cos \phi + P_{34} \cos \phi)Q^d \begin{array}{c} g_{3z} \\ g_{3z} \end{array} + T^d \begin{array}{c} g_{0y} \\ g_{0y} \end{array} + T^d \begin{array}{c} g_{0z} \\ g_{0z} \end{array} \]

\[ T^d \begin{array}{c} g_{1y} \\ g_{1y} \\ g_{2y} \\ g_{2y} \end{array} + T^d \begin{array}{c} g_{2y} \\ g_{2y} \end{array} + T^d \begin{array}{c} g_{2y} \\ g_{2y} \end{array} + T^d \begin{array}{c} g_{3y} \\ g_{3y} \end{array} \]

\[ M_{0x} = -P_{1g_1} \cos \phi \cdot W - (P_{12} \cos \phi + P_{23} \cos \phi)W - (P_{12} \cos \phi + P_{23} \cos \phi + P_{34} \cos \phi)W + \]

\[ (P_{12} \cos \phi + P_{23} \cos \phi + P_{34} \cos \phi)W + \]

\[ (P_{12} \cos \phi + P_{23} \cos \phi + P_{34} \cos \phi)R^s \begin{array}{c} g_{1y} \\ g_{1y} \\ g_{2y} \\ g_{2y} \end{array} - (P_{12} \sin \phi + P_{23} \sin \phi + P_{34} \sin \phi)R^s \begin{array}{c} g_{4y} \\ g_{4y} \\ g_{4z} \\ g_{4z} \end{array} + M^s - P_{0g_1} \sin \phi \cdot Q^d \begin{array}{c} g_{0x} \\ g_{0x} \\ g_{1x} \\ g_{1x} \end{array} - (P_{01} \sin \phi + P_{1g_1} \sin \phi)Q^d + \]

\[ (P_{12} \cos \phi + P_{23} \cos \phi)Q^d \begin{array}{c} g_{2y} \\ g_{2y} \end{array} - (P_{12} \sin \phi + P_{23} \sin \phi + P_{2g_2} \sin \phi)Q^d \begin{array}{c} g_{2y} \\ g_{2y} \end{array} + \]

\[ (P_{12} \cos \phi + P_{23} \cos \phi + P_{34} \cos \phi)Q^d \begin{array}{c} g_{3y} \\ g_{3y} \end{array} - \]

\[ (P_{12} \cos \phi + P_{23} \cos \phi + P_{34} \cos \phi)Q^d \begin{array}{c} g_{3y} \\ g_{3y} \end{array} - \]
Equation (3.60) continued

\[ (P_{01} + P_{12} \sin \phi + P_{23} \sin \phi + P_{3g3} \sin \phi) Q^d + \]

\[ T^d_{g0z} + T^d_{g1z} + T^d_{g2z} + T^d_{g3z} \]  

(3.60)

3.7.2 Total reactions at joint No.1 and actuator "ab"

\[ F_{ab} = (P_{1d} \cos \phi * F_{1de} ^s - P_{1de} \sin \phi * F_{1de} ^s - \]

\[ P_{1d} \cos \phi * W_{lg1} ^s + P_{1d} \cos \phi * R_{l1d} ^s - P_{1d} \sin \phi * R_{l1d} ^s) )/ \]

\[ (P_{l1b} \cos \phi * \sin \phi - P_{l1b} \sin \phi * \cos \phi) + (P_{l1b} \cos \phi * F_{l1de} ^s - \]

\[ P_{l1d} \sin \phi * F_{l1de} ^s + P_{l1d} \cos \phi - \]

\[ P_{l1g1} \sin \phi * Q_{l1g1} ^d + P_{l1g1} \cos \phi * R_{l1g1} ^d - P_{l1g1} \sin \phi * R_{l1g1} ^d + T_{l1g1} ^d) )/ \]

\[ (P_{l1b} \cos \phi * \sin \phi - P_{l1b} \sin \phi * \cos \phi) \]  

(3.61)

\[ R_{ix} = F_{de} ^s \cos \phi - F_{de} ^s \cos \phi + R_{2x} ^s + F_{de} ^s \cos \phi + \]

\[ Q_{gx} ^d - F_{ab} ^d \cos \phi + R_{2x} ^d \]  

(3.62)
\[ R_{ly} = F \sin \phi - W - F \sin \phi + R_{ly} \]
\[ F \sin \phi + Q - F \sin \phi + R_{ly} \]
\[ R_{lz} = R_{lz} + Q + R_{lz} \]
\[ M_{lx} = P \sin \phi * R_{lz} + M + P \sin \phi * Q + \]
\[ M_{lx} = -P \sin \phi * R_{lz} + M - P \cos \phi * Q - \]
\[ M_{lz} = 0.0 \]

3.7.3 Total reactions at No.2 and actuator "de"

\[ F_{de} = (P \cos \phi * R_{2y} - P \sin \phi * R_{2y} - P \cos \phi + M) / \]
Equation (3.68) continued

\[
(P_{\phi} \cos \phi \sin \phi - P_{\phi} \sin \phi \cos \phi) + (P_{\phi} \cos \phi \sin \phi) = 0
\]

\[
P_{\phi} \sin \phi \cos \phi - P_{\phi} \sin \phi \cos \phi
\]

\[
R_{x} = -F_{\phi} \cos \phi + R_{x} + Q_{x} - F_{\phi} \cos \phi + R_{x}
\]

\[
R_{y} = -W_{\phi} + R_{y} - F_{\phi} \sin \phi + Q_{y} - F_{\phi} \cos \phi + R_{y}
\]

\[
R_{z} = R_{z} + Q_{z} + R_{z}
\]

\[
M = P_{\phi} \sin \phi \cos \phi + M_{z} + P_{\phi} \cos \phi \sin \phi
\]

\[
M = -P_{\phi} \cos \phi \sin \phi + M_{z} + P_{\phi} \cos \phi \sin \phi
\]
Equation (3.73) continued

\[ P \cos \phi \cdot R + T + M \] (3.73)

\[ M_{2z} = 0.0 \] (3.74)

3.7.4 Total reaction forces at joint No.3

Static loading to link No.3, the end effector, is assumed to apply to the tip while dynamic loading is assumed to apply to the center of gravity. If static and dynamic loadings are to coordinate then distances \( P_{34} \) in static equations and \( P_{3g3} \) are to the same values, \( P_{34} = P_{3g3} \).

\[ R_{3x} = R_{4x} + Q \] (3.75)

\[ R_{3y} = -W_{34} + R_{4y} + Q_{3g3} \] (3.76)

\[ R_{3z} = R_{4z} + Q_{3g3} \] (3.79)

\[ M_{3x} = P_{34} \sin \phi \cdot R_{4z} + M_{4x} + P_{3g3} \sin \phi \cdot Q_{3g3} + T_{g3x} \] (3.80)

\[ M_{3y} = -P_{34} \cos \phi \cdot R_{4z} + M_{4y} - P_{3g3} \cos \phi \cdot Q_{3g3} + T_{g3y} \] (3.81)
\[ M_{3z} = P \cos \phi_3 R_s - P \sin \phi_4 R_s - P \cos \phi_4 W_s + M_{4z} + \]

\[ P_3 \cos \phi_3 Q_{gy} - P_3 \sin \phi_3 Q_{gy} + T_{gz} \]  

(3.82)
CHAPTER FOUR

DISPLACEMENT ANALYSIS OF THE ROBOTIC STRUCTURE

4.1 Elastic Displacement Equations Of The Linkages In The Local $X_i - Y_i$ Plane

Elastic displacements of linkages will first be derived in the local coordinates of the structure. Total displacements of the end effector will be shown later in this chapter. The linkages are treated as slender flexible elastic beams. The differential equation for the beam bending with axial compression or tension, Timoshenko (1959), in the local coordinates of the beam is:

\[
\frac{d^4 W_i}{dx_i^4} - \frac{S_i}{E_i I_i} \frac{d^2 W_i}{dx_i^2} - \frac{d^2 M_i(x_i)}{dx_i^2} = \frac{q_i(x_i)}{E_i I_i} \quad (4.1)
\]
$W_i = W_i(x_i)$ is the transverse displacement normal to the local axis of the beam.

$M(x_i)$ is the moment due to applied transverse loading when $S_i = 0$

$S_i$ is the axial force in the beam, $S_i$ is positive for tensile forces and negative for compressive forces

(i) is the link beam number.

The general solution for equation (4.1) is:

$$W_i = W_{ih} + W_{ip}$$

(4.2)

Where:

$W_{ih}$ is the homogeneous solution

$W_{ip}$ is the particular solution due to the loading $q_1(x_i)$.

The characteristic homogeneous solutions are:

For $S_i > 0$ tensile

$$W_h = A \cosh \frac{2 \mu_i x_i}{L_i} + A_2 \sinh \frac{2 \mu_i x_i}{L_i}$$

(4.3)

For $S_i < 0$ compression
\[
W_h = B_1 \cos \left( \frac{2 \mu \ x_i}{L_i} \right) + B_2 \sin \left( \frac{2 \mu \ x_i}{L_i} \right) \quad (4.4)
\]

Where:

\[
\mu_i^2 = \frac{S_i \ L_i^2}{4 \ E_i \ I_i} \quad (4.5)
\]

\(x_i\) is the local axis of the beam

\(I_i, E_i\) are its moment of inertia and modulus of elasticity respectively. The particular solution \(W_{ip}\) depends on the loading \(q_i(x_i)\).

Figures (4.1–4.2) show the distributed and concentrated end loads on the cantilever link \(i\), where, \(L_i\) is the length of the link and \(q_{i y}, q_{i z}, q_{i y}^0, q_{i z}^0\) are defined as the following:

\[
q_{iz1}^0 = \frac{6}{L_i^2} \left( T_{g_{ly}^i} \cos \phi_i - T_{g_{ly}^i} \sin \phi_i \right) \quad (4.6)
\]

\[
q_{iz2}^0 = \frac{6}{L_i^2} \left( T_{g_{2y}^i} \cos \phi_2 - T_{g_{2y}^i} \sin \phi_2 \right) \quad (4.7)
\]

\[
q_{ly1}^0 = \frac{6 \ T_{g_{1z}^i}}{L_i^2} \quad (4.8)
\]
\[ q_{2y2}^0 = \frac{6Tg^2z}{L_2^2} \quad (4.9) \]

\[ q_{1y1} = \frac{Q_{g1y \cos \phi_1} - Q_{g1x \sin \phi_1}}{L_1} - \frac{W_1}{L_1} \cos \phi_1 \quad (4.10) \]

\[ q_{2y2} = \frac{Q_{g2y \cos \phi_2} - Q_{g2x \sin \phi_2}}{L_2} - \frac{W_2}{L_2} \cos \phi_2 \quad (4.11) \]

\[ q_{1z1} = \frac{Q_{gzi}}{L_1} \quad (4.12) \]

\[ q_{2z2} = \frac{Q_{gzi}}{L_2} \quad (4.13) \]

Where:

\( q_{ix}, q_{iy}, q_{iz} \) are the equivalent distributed forces per unit distance on the link due to forces and torques applied at the centers of gravity of the links.

If the effect of the axial force \( S_i \) on bending is neglected, \( S_i = 0 \), then the deflection equations in the local coordinates for the cantilever beams under the applied static and dynamic forces are shown in equations (4.14–4.17). Where:

\( V_i \) is the beam displacement in the local direction of \( y_i \).

\( W_i \) is the beam displacement in the local direction of \( z_i \).

\( \theta_{yi} = -\frac{dW_i}{dx_i} \), \( \theta_{zi} = \frac{dV_i}{dz_i} \) are the slopes of the deflected beams in
the local coordinates \( x_i - y_i \) and \( x_i - z_i \).

\[
V_i = \frac{q_i y_i}{24 E_i I_{zi}} \left[ x_i^4 - 4L_i x_i^3 + 6L_i^2 x_i^2 \right] + \\
\frac{q_i^0 y_i}{120 E_i I_{zi}} \left[ 2x_i^5 - 5L_i x_i^4 + 12L_i^3 x_i^2 \right] + \frac{R_i (i + 1)}{6 E_i I_{zi}} y_i (3L_i x_i^2 - x_i^3) + \\
\frac{M_i (i + 1) x_i^2}{2 E_i I_{zi}} \tag{4.14}
\]

\[
\theta_{zi} = \frac{q_i y_i}{24 E_i I_{zi}} \left[ 4x_i^3 - 12L_i x_i^2 + 12L_i x_i \right] + \\
\frac{q_i^0 y_i}{120 E_i I_{zi}} \left[ 10x_i^4 - 20L_i x_i^3 + 12L_i^3 x_i \right] + \\
\frac{LR_i (i + 1) y_i (6L_i x_i - 3x_i^2)}{6 E_i I_{zi}} + \frac{M_i (i + 1) x_i^3}{E_i I_{zi}} x_i \tag{4.15}
\]

\[
W_i = \frac{q_i z_i}{24 E_i I_{yi}} \left[ x_i^4 - 4L_i x_i^3 + 6L_i^2 x_i^2 \right] + \\
\frac{q_i^0 z_i}{120 E_i I_{yi}} \left[ 2x_i^5 - 5L_i x_i^4 + 12L_i^3 x_i^2 \right] + \\
\frac{R_i (i + 1)}{6 E_i I_{yi}} z_i (3L_i x_i^2 - x_i^3) + \\
\frac{M_i (i + 1) x_i^2}{2 E_i I_{yi}} \tag{4.16}
\]
Equation (4.16) continued

\[
\frac{q^0_i z_i}{120 E_i I_y L_i} \left[ 2x_i^5 - 5L_i x_i^4 + 12L_i^2 x_i^2 \right] +
\]

\[
\frac{R((i+1)z_i)(3L_i x_i^2 - x_i^3)}{6 E_i I_y L_i} - \frac{M_l}{2 E_i I_y L_i} x_i^2
\]  \hspace{1cm} (4.16)

\[
\frac{q^0_i y_i}{24 E_i I_y L_i} \left[ 4x_i^3 - 12L_i x_i^2 + 12L_i^2 x_i \right] +
\]

\[
\theta_y = -\frac{q^0_i y_i}{24 E_i I_y L_i} \left[ 10x_i^4 - 20L_i x_i^3 + 24L_i^2 x_i \right] +
\]

\[
\frac{R((i+1)z_i)(6L_i x_i^2 - 3x_i^3)}{6 E_i I_y L_i} - \frac{M_l}{E_i I_y L_i} x_i
\]  \hspace{1cm} (4.17)

Where:

- \(E_i, I_i\) are the Young’s modulus of elasticity and the moment of inertia.
- \(M_l\) is the moment reaction in the local \(y_i\)-axis.
- \(R_l\) is the force reaction in the local \(y_i\)-axis.
- \(M_{l(i+1)i} = M_{l(i+1)i} \sin \phi_i + M_{l(i+1)i} \cos \phi_i\)
- \(R_{(i+1)i} = R_{(i+1)i} \cos \phi_i - R_{(i+1)i} \sin \phi_i\)
4.2 Torsional Twist And Axial Extension

The angle of twist due to distributed and applied components of the dynamic torques in the direction of the local axis of the beams and due to applied torques at the joints produce torsional rotations $\theta_{x_1}$ as:

$$\theta_{x_1} = \frac{T}{L_1 J_1 G_1} \left( \frac{g_1 x \cos \phi_1 + T g_1 y \sin \phi_1}{L_1 J_1 G_1} \right) + \frac{M_2 x \cos \phi_1 + M_2 y \sin \phi_1}{L_1 J_1 G_1} (x_1)$$

(4.18)

$$\theta_{x_2} = \frac{T}{L_2 J_2 G_2} \left( \frac{g_2 x \cos \phi_2 + T g_2 y \sin \phi_2}{L_2 J_2 G_2} \right) + \frac{M_3 x \cos \phi_2 + M_3 y \sin \phi_2}{L_2 J_2 G_2} (x_2)$$

(4.19)

Where:

$L_{12} = L_1 - L_2$ which is the distance between joint No. 1 & 2.

$L_{23} = L_2 - L_3$ which is the distance between joint No. 2 & 3.
\[ J_i \] is the polar moment of inertia.

\[ G_i \] is the shear modulus of elasticity.

The extensions along the local axis \((x_i)\) of the beams due distributed and applied dynamic forces in the direction of the axis \((x_i)\) and due to the applied force reaction at the joint produce extensional displacement \((u_{x_i})\) as:

\[
u_{x1} = \frac{Q_{g1x} \cos \phi_1 + Q_{g1y} \sin \phi_1}{L_1 A_1 E_1} (L_1 x_1 - \frac{x_1}{2}) + \]

\[
\frac{R_{2x} \cos \phi_1 + R_{2y} \sin \phi_1}{L_1 A_1 E_1} (x_1)
\]

\[
u_{x2} = \frac{Q_{g2x} \cos \phi_2 + Q_{g2y} \sin \phi_2}{L_2 A_2 E_2} (L_2 x_2 - \frac{x_2}{2}) + \]

\[
\frac{R_{3x} \cos \phi_2 + R_{3y} \sin \phi_2}{L_2 A_2 E_2} (x_2)
\]

4.3 Total Global Displacements Of The Robotic Arm

Refer to figures(4.3–4.8)

The elastic displacements and rotations were derived in the local coordinates at the tip of each link of the robotic structure. These local
displacements and rotations were used in determining the total global displacements at the tip of joint No.2 & 3. The total global rotations and displacements of the joints are determined by the local translational and rotational displacements. The effect of the virtual elongations on the global displacements of joint No.3 were absorbed in the hydraulic actuators "ab" and "de" due to their local strain and to the changes in the volumetric flow rates entering the chamber of the hydraulic actuators.

4.3.1 Global displacements and rotations of joint No.2

4.3.1.1 Joint 2, plane of the robot X–Y

The total global displacements and rotations at joint No.2 are due the reaction forces at the joint, small change in the angle \( \phi_1 \) of link No.1 which is due to slight movement in the hydraulic actuator "ab", and the extension of the link due to its axial loading.

\[
\text{DEFL2X} = P \cos \phi_1 \left( P \cos \phi_1 - P \cos \phi_1 d \phi_1 \right) + \]
\[
\left[ P \cos \phi_1 \left( P \cos \phi_1 - V \sin \phi_1 \right) \right] + U_1 \cos \phi_1
\]

(4.22)
DEFL2Y = \[ P_{12} \sin \phi_1 - (P_{12} \sin \phi_1 + P_{12} \cdot d \phi_1 \cos \phi_1) + \]
\[ P_{12} \sin \phi_1 - (P_{12} \sin \phi_1 + V_1 \cos \phi_1) + U_{1x} \sin \phi_1 \]  \hspace{1cm} (4.23)

\[ \text{ROTANG2X} = (-\theta_1 \sin \phi_1 + \theta_1 \cos \phi_1) \]  \hspace{1cm} (4.24)

\[ \text{ROTANG2Y} = (\theta_1 \cos \phi_1 + \theta_1 \sin \phi_1) \]  \hspace{1cm} (4.25)

4.3.1.2 Joint 2, out of plane displacements and rotations

DEFL2Z = W_1 \hspace{1cm} (4.26)

\[ \text{ROTANG2Z} = \theta_2 \]  \hspace{1cm} (4.27)

4.3.3 Total displacements and rotations of joint No.3

The total deflected position of joint No.3 due to loading is found relative to the frame of the undeflected robot when no loads are applied. Superposition of the deflection components and displacements due to slopes of the deflected curves is achieved by transforming the components of displacements equations into the global coordinates. The total global displacements at the tip of joint No.3 is the addition of the displacement of joint No.2 & 3 plus the components of the global extension of link No.1 & 2. In addition, small extension or contraction of the
hydraulic actuators, or virtual changes in the actuator length due to volumetric changes in the oil flow rates will cause a virtual change in $\phi_1$ and $\phi_2$ which in turn will cause deflection at joints No.2 & 3. The total global displacements in the X–Y–Z axis are presented in the equations below:

4.3.3.1 Joint 3, plane of the robot X–Y

\[
\text{TOTDEF}_{X} = \left[ (P_{12} \cos\phi_1 + P_{23} \cos\phi_2) - (P_{12} \cos\phi_1 + P_{23} \cos\phi_2) - \\
(V_1 \sin\phi_1) - (V_2 \sin\phi_2) \right] + \left[ (P_{12} \cos\phi_1 + P_{23} \cos\phi_2) - \\
(P_{12} \cos\phi_1 + P_{23} \cos\phi_2) - (P_{12} * \Delta\phi_1 \sin\phi_1) - \\
(P_{23} * \Delta\phi_2 \sin\phi_2) - (P_{23} * \Delta\phi_2 \sin(\phi_2 + \Delta\phi_1)) \right] + \\
\left[ U_1 \cos\phi_1 + U_2 \cos\phi_2 \right]
\]

\[
\text{TOTDEF}_{Y} = \left[ (P_{12} \sin\phi_1 + P_{23} \sin\phi_2) - (P_{12} \sin\phi_1 + P_{23} \sin\phi_2) - \\
(V_1 \cos\phi_1) + (V_2 \cos\phi_2) \right] + \left[ (P_{12} \sin\phi_1 + P_{23} \sin\phi_2) - \\
\right]
\]
Equation (4.29) continued

\[
\left( P_{12} \sin \phi_1 + P_{23} \sin \phi_2 \right) - \left( P_{12} \right) - \\
\left( P_{23} \right) - \\
\left( P_{23} \cos \phi_2 - (P_{23} \cos(\phi_2 + \phi_1)) \right) + \\
\left[ U_{1x} \sin \phi_1 + U_{2x} \sin \phi_2 \right]
\]

\[ (4.29) \]

\[
\text{ROTANGX}= \left[ \left( - \theta \sin \phi + \theta \cos \phi \right) - \left( \theta \sin \phi \right) + \right.
\]
\[
\left. \left( \theta \cos \phi \right) \right]
\]

\[ (4.30) \]

\[
\text{ROTANGY}= \left[ \left( \theta \cos \phi + \theta \sin \phi \right) - \left( \theta \cos \phi \right) + \right.
\]
\[
\left. \left( \theta \sin \phi \right) \right]
\]

\[ (4.31) \]

4.3.3.2 Joint 3, out of plane displacements and rotations

\[
\text{TOTDEFZ}= \left[ \left( \theta \cos(\phi_2 - \phi_1) \right) + \right.
\]
\[
\left. \left( \theta \sin(\phi_2 - \phi_1) \right) \right]
\]

\[ (4.32) \]

\[
\text{ROTANGZ}= \theta
\]

\[ (4.33) \]
4.4 Virtual Displacements Of The Hydraulic Actuators

The virtual displacements of the hydraulic actuators are due to the changes in the volumetric flow rates entering the chambers of the hydraulic actuators. Similarly the axial forces in the actuators lead to small changes in their lengths. These small displacements will lead to a virtual change in angles \( \phi_1 \) and \( \phi_2 \) which results in larger displacements at joints No.2 & 3. The derivation of the expressions for the global displacements due to the virtual change of angles \( \phi_1 \) and \( \phi_2 \) are presented below:

Figure (4.7) the triangular part of the robot containing actuator "ab".

\( C_l = P_{hb} \), represents the length of hydraulic actuator "ab", and \( a_1 = P_{h1}, b_1 = P_{1b} \) are known distances on the robotic arm, \( \beta \) is a known angle. Therefore the length \( C_l \) is:

\[
C_l^2 = a_1^2 + b_1^2 - 2(a_1)(b_1) \cos \beta
\]

(4.34)

Using the law of sine will get the following expression for the angle between the base and the hydraulic actuator "ab":

\[
\text{Ang}_2 = \sin^{-1}(\frac{a_1 \sin \beta}{C_l})
\]

By differentiating equation (4.34) with respect to \( \beta \) the following expression for \( d\beta \) is obtained:
Where:

\[ d\beta = d\phi_1 \]
\[ \beta = 90^\circ + \phi_1 \]

\[ d\beta = \frac{C_1}{(a_1)(b_1) \sin \beta} \ dC_1 \]

If the error in measuring the length of the actuator due to virtual changes in the volumetric flow rate is defined as \( Err_1 \), then the virtual change in the hydraulic actuator length, \( dC_1 \), due to its own axial force and \( Err_1 \):

\[ dC_1 = \frac{F_{ab} C_1}{A_{ab} E_{ab}} + Err_1 \]

\( dC_2 \) is derived similarly as shown in figure (4.8) as:

\[ dC_2 = \frac{F_{de} C_2}{A_{de} E_{de}} + Err_2 \]
CHAPTER FIVE

STRUCTURAL DESIGN OF A MODEL ROBOT

5.1 Description of the Lab Model

A 1/3 scale laboratory model of the robotic mechanism discussed in this paper has been constructed out of tubular high strength steel. The three link robot arm is supported and powered by a PRAB hydraulic base. The first link is vertical and has a length of 1.06m (41.73in.) is made of a heavy steel pipe section having a diameter of 0.28m (11.02in.)and thickness of 0.05m (1.97in),it is rigidly attached to the base.

The second and third links have a combined reach of 2.8m (110.24in.) are driven by two hydraulic actuators attached to the vertical shaft. The second link has a length of 1.7m (66.93 in) and is manipulated by the first hydraulic actuators
that is attached to the vertical link. The third link has a length of 0.7m (27.56in.) and a rigidly attached inclined back section of 0.267m (10.5in.) long, it is manipulated by the second hydraulic actuator, which is connected to the top of the vertical shaft and the curved end of the third link. The end effector of the robotic arm will be manipulated by an electric stepper motor or a small hydraulic actuator and will have a length of approximately 0.15m (5.91in.).

5.2 DESIGN CRITERIA

The following criteria have been used in the design of the robotic links:

1. Forces in the hydraulic actuators are not to exceed a preset maximum limit.
2. Maximum elastic deflection of the end effector under worst loading conditions is not to exceed 2% of the total length of the extended robot arm.
3. The first fundamental frequencies of the unloaded robot both in its own plane and transverse to its own plane to be close to each other and under 6 HZ.
4. The robot is to be light-weight and to carry load at least equal to its own weight.

The preset conditions for the design of the robot are essential for the purpose of real time control techniques being considered by the robotics research group at the
University of Nevada, Las Vegas. Using the above design criteria high strength Grade 46 steel was selected for the design of the model. The lengths of the second and third links were chosen on the basis of a preselected workspace for the end effector.

By using the forces of the robot structure from various cases and a maximum deflection of about 2% of the total extended length of the robot arms equations 4.1–4.5 were used to obtain initial guesses for the moment of inertias and the areas of the links assuming $E_i=30 \times 10^6$ psi for steel. The maximum values for the forces, and moments along with the initial guesses of the areas and the moment of inertias of the links were also substituted in the combined bending and axial load interaction equations of steel structures (AISC 1980), equations (5.1–5.3) which are:

\[
\frac{f_a}{F_a} + \frac{C_{m} \times f_{b_x}}{F_{b_x}} + \frac{C_{n} \times f_{b_y}}{F_{b_y}} \leq 1.0 \quad \text{Eqn.(5.1)}
\]

\[
\frac{f_a}{0.6F_y} + \frac{f_{b_x}}{F_{b_x}} + \frac{f_{b_y}}{F_{b_y}} \leq 1.0 \quad \text{Eqn.(5.2)}
\]

\[
\frac{f_a}{F_a} + \frac{f_{b_x}}{F_{b_x}} + \frac{f_{b_y}}{F_{b_y}} \leq 1.0 \quad \text{Eqn.(5.3)}
\]

where:

$C_m$ is the coefficient applied to bending term in interaction formula for
prismatic members and dependent upon column curvature caused by applied moments.

- $F_a$ is the axial compressive stress permitted in a prismatic member in the absence of bending moment.
- $F_b$ is the bending stress permitted in a prismatic member in the absence of axial force.
- $F_e$ is the Euler stress for a prismatic member divided by factor of safety.
- $F_y$ is the specified minimum yield stress of the type of steel being used.
- $f_a$ is the computed axial stress.
- $f_b$ is the computed bending stress.

Several design cycles are necessary to select structurally sound members. To satisfy the third criteria regarding the fundamental frequencies of the robot finite element analysis was performed by Dr. S.G. Ladkany and M.S. Rouas, members of the ARO research group, to study the dynamic behavior of the structure. Modifications on the moment of inertias of the members, that did not violate the conditions set by equations (4.1–4.5) and (5.1–5.3), were made and a finite element dynamic analysis was performed for every iterative cycle of the design until satisfactory results were achieved. The finite element program GIFTS was used in the analysis.

The cross-sectional area selected for the design of links 1 and 2 is a square tubing of Grade 45 steel, having outer dimensions of 1 1/2 x 1 1/2 inch and inner dimensions of 1x1 inches (3.81x3.81cm and 2.54x2.54cm respectively).
The first three fundamental frequencies of the robot are shown in figures (5.1–5.4). Mode 1 at a frequency of 5.388 Hz. represents out of plane flapping vibrations. Mode 2 at a frequency of 5.919 Hz. represents in plane flapping vibrations. Mode 3 at frequency of 31.09 represents second order in plane flapping vibrations.

When link 2 is in a vertical position with link 1 in a horizontal position, mode 1 at frequency of 5.67 represents an out of plane twisting vibration as seen in figure (5.4).

Using a payload of a 100 lb and Young's modulus of $30 \times 10^6$ psi and a maximum deflection of 2%, a first design value of the moment of inertia was obtained. Few iterations were required before selecting the final section for the robotic arm. Sample calculations is listed below to illustrate the required calculations for the steel section of the links.

**First Design Iteration:**

The total length of the links $(L)=82.5$ in., therefore, the maximum deflection is equal to $(2\% \times 82.5)=1.65$ inch. Using the previous informations, the moment inertia of the section could be obtained from the following equation:

$$\text{Deflection} = \frac{P L^3}{3 E I}$$

$$1.65 = \frac{100 \times (82.5)^3}{3 \times 30 \times 10^6 I}$$
\[ I = 0.378 \text{ in}^4 \]

Thus, try a section of 1.5x1.5 inch and thickness of 0.25 inch with \((I = 0.388 \text{ in}^4)\).

Having obtained a cross sectional area of the links, the section is to be checked against the dynamic loading and to stay within 6 Hz.

The weights of the hydraulic actuators and the links is calculated to be 37 lb from which, the mass is calculated to be \(37/386.4 \text{ (lb S}^2/\text{ in)}\).

\[ W = (K/m)^{0.5} \]

Where:

- \(W\) is the angular velocity.
- \(K\) is the stiffness of the beam.
- \(m\) is the mass of the links and the actuators.

\[ K = \frac{3 E I}{L^3} \]

\[ K = 20.73 \text{ lb/in} \]
\[ W = 14.80 \text{ rad/S} \]

The frequency can be calculated as the following:

\[ f = \frac{W}{2 \pi} = 2.35 \text{ Hz} < 6 \text{ Hz}. \]
Hence, the criteria were achieved and the cross-sectional area of 1.5x1.5x0.25 is substituted in equations (5.1–5.3) as shown later in this thesis.

The maximum bending deflection is obtained when links No. 1 & 2 are in their most extended positions, that is with $\phi_1 = \phi_2 = 0.0$. Therefore, a force of 100 lb is applied in the negative y-axis, the reactions at joints No.1 & 2 are calculated and listed below:

<table>
<thead>
<tr>
<th>$R_{2x}$</th>
<th>$R_{2y}$</th>
<th>$R_{2z}$</th>
<th>$M_{2x}$</th>
<th>$M_{2y}$</th>
<th>$R_{1x}$</th>
<th>$R_{1y}$</th>
<th>$R_{1z}$</th>
<th>$M_{1x}$</th>
<th>$M_{1y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>808</td>
<td>–151</td>
<td>0.0</td>
<td>0.0</td>
<td>356</td>
<td>893</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>

The total global displacement at joint No.3 is calculated as follows:

In plane deflection

Deflection in y-axis = \[ \frac{P_3}{3EI_2} \frac{L^3}{2} + \frac{P_2}{3EI_1} \frac{L^3}{1} \]

Where:

- $P_3$ is the reaction force at joint No.3 in the y-axis and equal to 100 lb.
- $P_2$ is the reaction force at joint No.2 in the y-axis and equal 151 lb.
- $I_1 = I_2 = 0.388 \text{ in}^4$.
- $E_1 = E_2 = 30 \times 10^6 \text{ psi}$. 
Deflection in y-axis = 0.7645 in.

The deflections of the two links are added separately due to the existence of the z-axis hinge which does not allow for moment transfer between the two members.

Out-of-plane Deflection

The deflection in the out-of-plane is calculated as if the two members are welded together because the connection allows for moment transfer between the members. If a force of 100 lb is applied in the direction of the z-axis, the following deflection is obtained:

\[
\text{Deflection in z-axis} = \frac{P_3}{3E_2} \left( \frac{L_1 + L_2}{I_2} \right)^3
\]

Deflection in z-axis = 1.608 in.

The adequacy of the selected cross-section is now tested by using equations (5.1–5.3). When a force of 100 lb. is applied in the negative y-axis and another force of 50 lb. is applied in the negative z-axis at the tip of joint No.3, the following reactions at joints No.1 & 2 are obtained:
Member No.2

The axial force \( P \) along member No.2 is calculated as the following

\[
P = 266 \cos(44) + 364 \sin(44) = 444.2 \text{ psi}
\]

The allowable computed axial stress \( f_a \) is:

\[
f_a = \frac{P}{A} = \frac{444.2}{2.25 \text{ in}^2} = 197.4 \text{ psi}
\]

The axial compressive stress permitted in the member \( F_a \) is obtained from the AISC—steel manual table (3—50) after calculating the slenderness ratio which is

\[
\frac{K \cdot L}{r}
\]

Where:

- \( K \) is the effective length factor for the member which is \( 2.1 \).
- \( L \) is the length of the member which is \( 28 \text{ in} \).
- \( r \) is the radius of gyration which equals \( (I/A)^{0.5} \).
- \( A \) is the cross-sectional area of the member which is \( 2.25 \text{ in}^2 \).
- \( I \) is the moment of inertia of the member which is \( 0.388 \text{ in}^4 \).
\[ \frac{KL}{r} = 140 \]

Using the slenderness ratio we find \( F_a = 7.62 \text{ KSI} \)

AISC section 1.6.1 if \( \left( \frac{f_a}{F_a} \right) \leq 0.15 \) use equation (5.3)

\[
\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0
\]

Since the member is rectangular \( F_{bx} = F_{by} \)

\[
F_{bx} = 0.66 \times F = 0.66 \times 45 = 29.7 \text{ KSI}
\]

The moment in around the x–axis which is the z–axis in the AISC manual is calculated at the cut–off point before the hinge which is:

\[
M_x = 100(28 \cos 44) = 2014 \text{ lb in}
\]

\[
f_{bx} = \frac{M_x}{I} \times C = \frac{2014 (0.75)}{0.388} = 3893 \text{ psi}
\]

\[
f_{by} = \frac{M_y}{I} \times C = \frac{1389 (0.75)}{0.388} = 2685.9 \text{ psi}
\]
0.24 \leq 1.0 \quad \text{design is Ok.}

**Member No.1**

The maximum axial force \( P \) along member No.1 is calculated as the following:

\[
P = 308 \cos(37) + 1206 \sin(37) = 972 \text{ psi}
\]

The allowable computed axial stress \( f_a \) is:

\[
f_a = \frac{P}{A} = \frac{972}{2.25 \text{ in}^2} = 432 \text{ psi}
\]

The axial compressive stress permitted in the member \( F_a \) is obtained from the AISC–steel manual table (3–50) after calculating the slenderness ratio which is:

\[
\frac{KL}{r}
\]

Where:

- \( K \) is the effective length factor for the member which is (2.1).
- \( L \) is the length of the member starting from the end of the thick 6″ sleeve which is (48.5 in)
- \( r \) is the radius of gyration which equal to \((I/A)^{0.5}\)
A is the cross-sectional area of the member which is (2.25) in².

I is the moment of inertia of the member which (0.388) in⁴

\[ \frac{KL}{r} = 245.3 \]

If slenderness ratio > 200 use the following equation:

\[ F_a = \frac{12 \pi^2 E}{23 (KL/r)^2} \]

Where:

E is the young's modulus of elasticity which is (30 x 10⁶ psi).

\[ F_a = 2573 \text{ psi} \]

AISC section 1.6.1 \( (f_a/F_a) \geq 1.0 \) use equation (5.1–5.2)

Where:

\[ C_a = 0.85 \] for compressions members subject to joint translation.

\[ F'_{ex} = F'_{ey} = \frac{12 \pi^2 E}{23 (KL/r)^{0.5}} = 2573 \text{ psi} \]

Since the member is rectangular \( F_{bx} = F_{by} \)

\[ F_{by} = 0.66 F_y = 0.66(45) = 29.7 \text{ KSI} \]
The moment in around the x-axis which is the z-axis in the AISC manual is calculated at the cut-off point before the hinge which is:

\[ M_x = 364(54.5\cos37) - 265(265\sin37) = 7152 \text{ lb in} \]

\[ f_{bx} = \frac{M_x}{I} C = \frac{7152}{0.388} (0.75) = 13825 \text{ psi} \]

\[ f_{by} = \frac{M_y}{I} C = \frac{3029}{0.388} (0.75) = 5855 \text{ psi} \]

\[ \frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{1 - \frac{f_a}{F_{ex}}} + \frac{C_{my} f_{by}}{1 - \frac{f_a}{F_{ey}}} \leq 1.0 \]

\[ \frac{f_a}{0.6 F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \]

\[ \frac{0.432}{2.04} + \frac{0.85}{29.7 (1 - 0.432 / 2.57)} + \frac{0.85 \times 5.85}{29 (1 - 0.432 / 2.57)} \leq 1.0 \]

\[ 0.85 \leq 1.0 \]
\[
\frac{0.432}{27} + \frac{13.83}{29.70} + \frac{5.9}{29.7} = 0.68 < 1.0 \text{ design is ok.}
\]

Since, the above equations were satisfied, the 1.5x1.5x0.25 inch section is ok.

5.3 The Finite Element Model

The finite element analysis was performed by other members of the ARO robotic research group. It will be discussed briefly in this thesis since its results were used in the design of the robot.

The vibration of the robot was performed using the finite element program GIFTS, the Graphical Interactive Finite Element Total System software. The type of elements used are 3-dimensional, beam/column elements, with each node in the structure having six degrees of freedom, three translations and three rotational. The base of the robot is assumed to be perfectly rigid. The nodes connecting the actuators, are treated as two dimensional pins, in order to take into account the fact that the actuators do not carry any moment; The actuators are modeled as rod elements.

The model contained a total of 33 nodes and 31 element with 192 unknowns. Figures (5.1–5.4) shows the vibration mode shapes and corresponding natural frequencies. Note that in the figures both in-plane and out-of-plane vibrations are shown.
Several computer runs were performed inorder to observe the effect of concentrated masses carried by the end effector, on the vibration response of the robot. Similarly dynamic finite element runs were made for the robot at varios positions within its workspace.
CHAPTER SIX

CONCLUSION

The force and displacement analysis of a 3-link lightweight elastic robot has been presented. The design of a 1/3 scale laboratory model which satisfies stringent criteria of loading, deflection and dynamic response is achieved using iterative analysis. The finite element method was used to determine the dynamic response of the robot and to compare the deflections under loading with those obtained by the analytical techniques.

A fortran computer program was written to calculate the force and moment reactions and the displacements at joints No. 2 and 3 due to the static and dynamic forces applied to the end effector and to the centers of gravity of the links. The program accepts as input linear and rotational accelerations at the center of gravity of the links and at the tip of the robot. The model has been built and is being tested at the Civil and Mechanical Engineering Laboratory at UNLV.
Future research will involve testing in the laboratory of the constructed robot and a comparison between actual and predicted structural behavior under normal operating conditions in order that fully effective control algorithms be derived.
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APPENDIX—A

FIGURES CORRESPONDS TO CHAPERS THREE, FOUR AND FIVE
Figure (3.1) Free Body Diagram of Member No. 0 Showing the Applied Static Loads and Reactions.
Figure (3.2) Free Body Diagram of Member No. 1 showing the applied static loads and reactions.
Figure (3.3) Free Body Diagram of Member No. 2
Showing the Applied Static Loads and Reactions
Figure (3.4) Free Body Diagram of the End Effector (Member No. 3) showing the applied static loads and reductions.
Figure (3.5) Free Body Diagram of the Whole Structure Showing the Applied Static Loads and Reactions.
Figure (3.6) Free Body Diagram of Member No. 0 Showing the Applied Dynamic Loads and Reactions.
Figure (3.7) Free Body Diagram of Member No. 1 Showing the Applied Dynamic Loads and Reactions.
**Figure (3.8)** Free Body Diagram of Member No. 2 Showing the Applied Dynamic Loads and Reactions.
Figure (3.9) Free Body Diagram of the End Effector (Member No. 3) showing the Applied Dynamic Loads and Reactions.
Figure (3.10) Free Body Diagram of the Whole Structure Showing the Applied Dynamic Loads and Reactions.
Figure (3.11) Schematic of a Three-Link Robotic Mechanism.
Figure (4.1) Equivalent Distributed Dynamic Loads and Cantilever Tip Loads on Link "i" in the Local $x_i$-$z_i$ Plane.
Figure (4.2) Equivalent Distributed Dynamic Loads and Cantilever Tip Loads on Link "I" in Local $x_i-y_i$ Plane.
Figure (4.3) **Local Rotational Angles That Affect The Global Z-Displacements.**

\[
\begin{align*}
\theta_{1y1x2} &= \theta_{1y1} \sin(\phi_2 - \phi_1) \\
\theta_{1y1y2} &= \theta_{1y1} \cos(\phi_2 - \phi_1) \\
\theta_{1x1y2} &= -\theta_{1x1} \sin(\phi_2 - \phi_1) \\
\theta_{1x1x2} &= \theta_{1x1} \cos(\phi_2 - \phi_1)
\end{align*}
\]
Figure (4.4) Global Displacements of Links No. 1 and 2 in the Z-Direction.
Figure (4.5) Global Displacements of Links No. 1 and 2.
Figure (4.6) Global Rigid Body Displacements of Links No. 1 and 2 Due to Virtual Changes of Angles $\phi_1$ and $\phi_2$. 
\[ \beta = 90 + \Phi_1 \]

\[ C_1^2 = a_1^2 + b_1^2 - 2a_1b_1 \cos \beta \]

\[ \text{Ang}2 = \sin^{-1}\left( \frac{a_1 \sin \beta}{C_1} \right) \]

\[ d\beta = \frac{C_1}{ab \sin \beta} \, dC_1 \]

\[ dC_1 = \frac{F_{ab} C_1}{A_1 E_1} \]

**Figure (4.7) Triangular Part of the Robot Containing Actuator "ab".**
\[ C_2 = a_2^2 + b_2^2 - 2a_2b_2 \cos \alpha \]

\[ \text{Ang1} = \sin \left( \frac{a_2 \sin \alpha}{C_2} \right) \]

\[ d \alpha = \frac{C_2}{a_2 b_2 \sin \alpha} \quad dc_2 \]

\[ d C_2 = \frac{F_{de} c_2}{A_2 E_2} \]

\[ d \alpha = d \phi_2 \]

**Figure (4.8)** Triangular part of the robot containing actuator "de".
Figure (5.1) First Mode (5.38 CPS). Out-of-Plane Vibrations, with the Second and Third Links of the Robot at Their Maximum Elevation.
Figure (5.2) Second Mode (5.92 CPS). In-Plane Vibrations, With the Second and the Third Links of the Robot at Their Maximum Elevation.
Figure (5.3) Third Mode (3.1 CPS). Out-of-Plane Second Order Vibrations, with the Second and Third Links of the Robot at Their Maximum Elevation.
Figure (5.4) First Mode (5.67 CPS). Out-of-Plane Vibrations, With the Third Link of The Robot in a Vertical Position.
APPENDIX—B

THE COMPUTER PROGRAM
COMPUTER PROGRAM NOTATIONS

The following list contains the most frequently used symbols in the computer program.

ANGLES

fe1 is the angle of member No.1 measured counter clockwise from horizontal
fe2 is the angle of member No.2 measured counter clockwise from horizontal
fe3 is the angle of member No.3 measured counter clockwise from horizontal
fe4 is the angle of the hydraulic actuator "ab" measured counter clockwise from horizontal
fe5 is the angle measured counter clockwise from horizontal to the tail of link No. 2
fe6 is the angle measured counter clockwise from horizontal to hydraulic actuator "de"

REACTIONS

R4xs,R4xd are the applied static and dynamic force at the tip of the end effector in the global x-axis
are the applied static and dynamic force at the tip of the end effector in the global y-axis

are the applied static and dynamic force at the tip of the end effector in the global z-axis

are the applied static and dynamic moment at the tip of the end-effector in the global x-axis.

are the applied static and dynamic moment at the tip of the end-effector in the global y-axis.

are the applied static and dynamic moment at the tip of the end-effector in the global z-axis.

are is the static and dynamic force reaction at joint No.3 in the global x-axis

are the static and dynamic force reaction at joint No.3 in the global y-axis

are the static and dynamic force reaction at joint No.3 in the global z-axis.

are the static and dynamic moment reaction at joint No.3 in the global x-axis.

are the static and dynamic moment reaction at joint No.3 in the global y-axis.

are the static and dynamic moment reaction at joint No.3 in the global z-axis.

are the static and dynamic force reaction at joint No.2 in the global x-axis.
R2ys, R2yd are the static and dynamic force reaction at joint No.2 in the global y–axis.

R2zs, R2zd are the static and dynamic force reaction at joint No.2 in the global z–axis.

M2xs, M2xd are the static and dynamic moment reaction at joint No.2 in the global x–axis.

M2ys, M2yd are the static and dynamic moment reaction at joint No.2 in the global y–axis.

Fdes, Fded are the static and dynamic force in hydraulic actuator "de".

R1xs, R1xd are the static and dynamic force reaction at joint No.1 in the global x–axis.

R1ys, R1yd are the static and dynamic force reaction at joint No.1 in the global y–axis.

R1zs, R1zd are the static and dynamic force reaction at joint No.1 in the global z–axis.

M1xs, M1xd are the static and dynamic moment reaction at joint No.1 in the global x–axis.

M1ys, M1yd are the static and dynamic moment reaction at joint No.1 in the global y–axis.

Fabs, Fabd are the static and dynamic force in hydraulic actuator "ab".

R0xs, R0xd are the static and dynamic force reaction at joint No.0 in the global x–axis.

R0ys, R0yd are the static and dynamic force reaction at joint No.0 in the global y–axis.
R0zs,R0zd are the static and dynamic force reaction at joint No.0 in the global z-axis.

M0xs,M0xd are the static and dynamic moment reaction at joint No.0 in the global x-axis.

M0ys,M0yd are the static and dynamic moment reaction at joint No.0 in the global y-axis.

M0zs,M0zd are the static and dynamic moment reaction at joint No.0 in the global z-axis.

R4X,R4Y,R4Z are the total applied forces at the tip of the end-effector in the X, Y, Z axis respectively.

M4X,M4Y,M4Z are the total applied moments at the tip of the end-effector in the X, Y, Z axis respectively.

R3x,R3y,R3z are the total reaction force at joint No.3 in the X, Y, Z axis respectively.

M3x,M3y,M3z are the total reaction moments at joint No.3 in the global X, Y, Z axis respectively.

R2x,R2y,R2z are the total reaction force at joint No.2 in the X, Y, Z axis respectively.

M2x,M2y are the total moment reaction at joint No.2 in the X, Y axis respectively.

Fde is the total force in the hydraulic actuator "de".

R1x,R1y,R1z are the total reaction force at joint No.1 in the X, Y, Z axis respectively.

M1x,M1y are the total moment reaction at joint No.1 in the X, Y axis respectively.
respectively.

 fabricated

 respectively.

 $F_{ab}$ is the total force in the hydraulic actuator "ab".

 $R_{0X}, R_{0Y}, R_{0Z}$ are the total reaction force at joint No.0 in the X, Y, Z axis respectively.

 $M_{0X}, M_{0Y}, M_{0Z}$ are the total reaction moments at the base of the robotic arm in the X, Y, Z axis respectively.

 **APPLIED FORCES**

 $dQ_{g0x}$ is the dynamic global component of the applied force at the center of gravity in the x-axis of link No.0.

 $dQ_{g0y}$ is the dynamic global component of the applied force at the center of gravity in the y-axis of link No.0.

 $dQ_{g0z}$ is the dynamic global component of the applied force at the center of gravity in the z-axis of link No.0.

 $dQ_{g1x}$ is the dynamic global component of the applied force at the center of gravity in the x-axis of link No.1.

 $dQ_{g1y}$ is the dynamic global component of the applied force at the center of gravity in the y-axis of link No.1.

 $dQ_{g1z}$ is the dynamic global component of the applied force at the center of gravity in the z-axis of link No.1.

 $dQ_{g2x}$ is the dynamic global component of the applied force at the center of gravity in the x-axis of link No.2.

 $dQ_{g2y}$ is the dynamic global component of the applied force at the center of
gravity in the y-axis of link No. 2.

\[ dQg2z \]
is the dynamic global component of the applied force at the center of gravity in the z-axis of link No. 2.

\[ dQg3x \]
is the dynamic global component of the applied force at the center of gravity in the x-axis of link No. 3.

\[ dQg3y \]
is the dynamic global component of the applied force at the center of gravity in the y-axis of link No. 3.

\[ dQg3z \]
is the dynamic global component of the applied force at the center of gravity in the z-axis of link No. 3.

**LOAD COMPONENTS IN LOCAL COORDINATES**

\[ q01y, q02y \]
are the distributed forces on link No. 1 & 2, in the local y-axis, due to the torques applied at their centers of gravity.

\[ q01z, q02z \]
are the distributed forces on link No. 1 & 2, in the local z-axis due to the torques applied at their centers of gravity.

\[ q1y, q2y \]
are the distributed forces on link No. 1 & 2, in the local y-axis, due to the forces applied at their centers of gravity.

\[ q1z, q2z \]
are the distributed forces on link No. 1 & 2, in the local z-axis, due to the forces applied at their centers of gravity.

\[ LR2x, LR3x \]
are the local reaction force in the x-axis of members No. 2 & 3.

\[ LR2y, LR3y \]
are the local force reaction in the y-axis of members No. 2 & 3.
LM2y, LM3y are the local moment reactions in the y-axis of members No. 2 and 3.

LdQg1x, LdQg2x are the local applied dynamic forces at the centers of gravity of member No. 1 & 2 in the x-axis.

**DYNAMIC ACCELERATIONS**

- **a0x** is the acceleration at the center of gravity of member No. 0 in the global x-axis.
- **a0y** is the acceleration of member No. 0 in the global y-axis.
- **a0z** is the acceleration of member No. 0 in the global z-axis.
- **a1x** is the acceleration of member No. 1 in the global x-axis.
- **a1y** is the acceleration of member No. 1 in the global y-axis.
- **a1z** is the acceleration of member No. 1 in the global z-axis.
- **a2x** is the acceleration of member No. 2 in the global x-axis.
- **a2y** is the acceleration of member No. 2 in the global y-axis.
- **a2z** is the acceleration of member No. 2 in the global z-axis.
- **a3x** is the acceleration of member No. 3 in the global x-axis.
- **a3y** is the acceleration of member No. 3 in the global y-axis.
- **a3z** is the acceleration of member No. 3 in the global z-axis.
- **th0x** is the angular acceleration vector of link No. 0 in the x-axis.
- **th0y** is the angular acceleration vector of link No. 0 in the y-axis.
- **th0z** is the angular acceleration vector of link No. 0 in the z-axis.
- **th1x** is the angular acceleration vector of link No. 1 in the x-axis.
th1y is the angular acceleration vector of link No.1 in the y-axis.
th1z is the angular acceleration vector of link No.1 in the z-axis.
th2x is the angular acceleration vector of link No.2 in the x-axis.
th2y is the angular acceleration vector of link No.2 in the y-axis.
th2z is the angular acceleration vector of link No.2 in the z-axis.
th3x is the angular acceleration vector of link No.3 in the x-axis.
th3y is the angular acceleration vector of link No.3 in the y-axis.
th3z is the angular acceleration vector of link No.3 in the z-axis.

MATERIAL PROPERTIES

xM0 is the mass of link No. 0.
xM1 is the mass of link No.1.
xM2 is the mass of link No.2.
xM3 is the mass of link No.3.
j0x is the mass moment of inertia of link No.0 around the x-axis.
j0y is the mass moment of inertia of link No.0 around the y-axis.
j0z is the mass moment of inertia of link No.0 around the z-axis.
j1x is the mass moment of inertia of link No.1 around the x-axis.
j1y is the mass moment of inertia of link No.1 around the y-axis.
j1z is the mass moment of inertia of link No.1 around the z-axis.
j2x is the mass moment of inertia of link No.2 around the x-axis.
j2y is the mass moment of inertia of link No.2 around the y-axis.
j2z is the mass moment of inertia of link No.2 around the z-axis.
\[ j_3x \] is the mass moment of inertia of link No.3 around the \( x \)-axis.
\[ j_3y \] is the mass moment of inertia of link No.3 around the \( y \)-axis.
\[ j_3z \] is the mass moment of inertia of link No.3 around the \( z \)-axis.

\[ I_{1y}, I_{2y} \] are the bending moment of inertia for member No.1 & 2 around the \( y \)-axis.
\[ I_{1z}, I_{2z} \] are the bending moment of inertia for member No.1 & 2 around the \( z \)-axis.

\[ E_1, E_2, E_3 \] are the Young's modulus of elasticity for member No.1 & 2.

\[ A_1, A_2 \] are the cross-sectional areas for link No.1 & 2 and the Hydraulic actuators.

\[ A_{ab}, A_{de} \] are the cross-sectional areas of actuators "ab" & "de".

\[ \text{ERR1, ERR2} \] are errors accounted for by the controllers.

**DISTANCES**

\[ P_{01} \] is distance between joint No.0 and joint No.1.
\[ P_{12} \] is the distance between joint No.1 and joint No.2.
\[ P_{34} \] is the distance between joint No.3 and the tip of the end-effector.
\[ P_{1b} \] is the distance between joint No.1 and point "b".
\[ P_{2e} \] is the distance between joint No.2 and point "e".
\[ P_{0g0} \] is the local distance between joint No.0 and center of gravity of link No.0.
\[ P_{1g1} \] is the local distance between joint No.1 and center of gravity of link No.1.
P2g2 is the local distance between joint No.2 and center of gravity of link No.2.

P3g3 is the local distance between joint No.3 and center of gravity of link No.3.

LOCAL DISPLACEMENTS

V1,V2 are the local deflection of members No.1 & 2. in y-axis.
thtaz1,thtaz2 are the local slopes, for member No.1 & 2, of the deflected links in the x–z coordinate.
xW1,xW2 are the local deflection of members No.1 & 2 in z-axis.
thtay1,thtay2 are the local slopes, for member No.1 & 2, of the deflected links in the x–y coordinate.
thtax1,thtax2 are the local twists of members No.1 & 2.
U1x,U2x are the local extensions of members No.1 & 2.

GLOBAL DISPLACEMENTS

ROtx,ROTY are the global components of the local extensions of the members.

Dfy2X,Dfy3X are the global deflections of joints No.2 & 3 in the X–axis.

Dfy2Y,Dfy3Y are the global deflections of joints No.2 & 3 in the Y–axis.

Dfy2Z,Dfy3Z are the global deflections of joints No.2 & 3 in the Z–axis.

thta2X,thta3X are the global rotational angles at joints No.2 & 3 in the
$\theta_2^Y, \theta_3^Y$ are the global rotational angles at joints No. 2 & 3 in the Y-axis.

$\theta_2^Z, \theta_3^Z$ are the global rotational angles at joints No. 2 & 3 in the Z-axis.
FORTRAN PROGRAM FOR THE DETERMINATION OF FORCE, AND DISPLACEMENTS IN A
THREE-LINK ROBOTIC MECHANISM

c $ THIS PROGRAM WAS WRITTEN BY TAREK MITRI BANNOURA FOR THE MASTERS $
$ OF SCIENCE DEGREE IN Civil ENGINEERING $

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realxB I0,I1,I2,I3,H2XS,H2YS,N1XS,M1YS,H0XS
+H0Y S,M0Z S,M 3XS,H 3Y S,M 3Z S,M 4X S,M 4Y S,H 4Z S
+ijOx, jOy,jOz,M0x,MOy, MOz,M1x,M1y, M2x, M2y
+j1x, j1y, j1z,j2x,j2y,j2z, j3x, j3y, j3z
+M3x,M3y,M3z,M0xd,M0yd,M0zd,M1xd,M1yd
+M2xd,M2yd,M3xd,M3yd,M3zd,L1,L2
+jLR2y,LR2x,LM2y,LR3y,LR3x,LR3z,LM3y
+H1y,H1z,H2y,H2z,LM2y, LM2z
+j3x1,j3y1,j3z1,j3x2

open(unit=1, file=’lindal.dat’, status=’old’)
open(unit=6, file=’tarek.out’, status=’new’)
Read(1,*)P01,Plb,P12,P23,P34,P2e
Read(1,*)P0g0,P1g1,P2g2,P3g3,P1d
Read(1,*)fe1,fe2,fe3,Gama
Read(1,*)w0,w1,w2,w3
Read(1,*)R4xS,R4yS,R4zS,M4xS,M4yS,M4zS
Read(1,*)x0x,x1x,x2x,x3
Read(1,*)a0x,a0y,a0z,a1x,a1y,a1z
Read(1,*)a2x,a2y,a2z,a3x,a3y,a3z
Read(1,*)j0x,j0y,j0z,j1x,j1y,j1z
Read(1,*)j2x,j2y,j2z,j3x,j3y,j3z
Read(1,*)tth0x,tth0y,tth0z
Read(1,*)tth1x,tth1y,tth1z,tth2x,tth2y,tth2z
Read(1,*)tth3x,tth3y,tth3z
Read(1,*)E1,E2,E3,I1y,I1z,I2y,I2z
Read(1,*)A1,A2,Aab,Ade,ERR1,ERR2
Read(1,*)xJ1,b1,xJ2,b2

c $ This fortran program was written to calculate the complicated $
$ 3-D forces, and moments in the links of the robot and, forces $
$ in the hydraulic actuators, global deflections, and global $
$ rotational angles at joint No.1, 2 & 3. This program can also $
$ be slightly modified to calculate the deflection at the tip of $
$ the end-effector when decision is taken on what kind of end $
$ effector will be used in the model. $


fe1=fe1/57.29578
fe2=fe2/57.29578

Redefine the angles in radians
fe3 = fe3 / 57.29578

c Gamma is the angle between link No. 2 and its tail

Gamma = Gamma / 57.29578
beta = fe1 + (90 / 57.29578)
pi = 180 / 57.29578
alfa = pi - gamma + fe1 - fe2
Pai = 23.15
C1 = (Pai**2 + Pib**2) - (2 * Pai * Pib * cos(beta)) ** 0.5
C2 = ((P2e**2 + P12**2) - (2 * P2e * P12 * cos(alfa))) ** 0.5
dci = ((Fab * C1) / (Aab * E3)) + ERR1
dc2 = ((Fde * C2) / (Adet * E3)) + ERR2
dfe = (ci / (Pai * Pib * sin(beta))) * dc1
dfe2 = (c2 / (P2e * P12 * sin(alfa))) * dc2

Angl is the angle between actuator de and link 1.
Angl = Asin((P2e * sin(Alfa)) / C2)
Fe5 = fe1 + Angl

Ang2 is the angle between the base and actuator ab
Ang2 = Asin((Plb * sin(beta)) / Cl)
Fe4 = (pi / 2.) - Ang2
Fe6 = Fe1 * Gamma

R3s is the static reaction at joint No. 3 in Global x-axis
R3xS = R4xS
R3yS = -w2 + R4yS
R3zS = R4zS
M3xS = P34*(sin(fe3)) * R4zS + H4xS
M3yS = -P34*(cos(fe3)) * R4zS + H4yS
M3zS = P34*(cos(fe3)) * R3yS - (P34*sin(fe3)) * R4xS
+ (-P3g3*(cos(fe3)) * M3) + M4zS

aaa = P23*(cos(fe2)) * M3yS
bbb = P23*(sin(fe2)) * R3zS
ccc = P2g2*(cos(fe2)) * M2 - M32S
ddd = P2e*(cos(fe6)) * (sin(fe5))
eee = P2e*(sin(fe6)) * (cos(fe5))
FdeS = (-aaa + bbb + ccc) / (-ddd + eee)
R2zS = FdeS * (cos(fe5)) + R3zS
R2yS = W2 + R3yS - FdeS * (sin(fe5))
R2zS = R3zS
M2xS = P23*(sin(fe2)) * R3zS + M3zS
M2yS = -P23*(cos(fe2)) * R3zS + M3yS

Static calculations for hydraulic actuator ab and reactions

c at joint No. 1

caaa = Pld*cos(fe1) * FdeS*sin(fe5)
bbb = Pld*sin(fe1) * FdeS*cos(fe5)
ccc = Plg1*(cos(fe1)) * M1
ddd = P12*(cos(fe1)) * R2yS - P12*(sin(fe1)) * R2zS
\[
\begin{align*}
\text{eee} &= \text{Pib}(\cos(fel))\sin(fe4)) \\
\text{xff} &= \text{Pib}(\sin(fel))\cos(fe4)) \\
\text{FabS} &= (\text{aaa}-\text{bbb}-\text{ccc}+\text{ddd})/(\text{eee} \times \text{xff}) \\
\text{R1xS} &= \text{FabS}(\cos(fe4)) + \text{R2xS} \\
\text{R1yS} &= \text{W1} - \text{FabS}(\sin(fe4)) + \text{R2yS} \\
\text{R1zS} &= \text{R2zS} \\
\text{M1xS} &= \text{P12}(\sin(fel))\text{R2zS} + \text{M2zS} \\
\text{M1yS} &= \text{P12}(\cos(fel))\text{R2zS} + \text{M2yS} \\
\text{RlxS} &= -\text{FabS}(\cos(fe4)) + \text{R2xS} \\
\text{RlySM} &= -\text{FabS}(\sin(fe4)) + \text{R2yS} \\
\text{RlzS} &= \text{R2zS} \\
\text{HlxS} &= \text{P12}(\sin(fel))\text{R2zS} + \text{H2xS} \\
\text{HlyS} &= -(\text{P12}\cos(fel))\text{R2zS} + \text{H2yS} \\
\text{HlzS} &= -(\text{P12}\cos(fel))\text{R2zS} + \text{H2zS} \\
\end{align*}
\]

\text{Reaction calculation at the base}
\[
\begin{align*}
\text{R0xS} &= \text{R4xS} \\
\text{R0yS} &= \text{W0} - \text{W1} - \text{W2} - \text{W3} + \text{R4yS} \\
\text{R0zS} &= \text{R4zS} \\
\text{M0xS} &= (\text{P01} + \text{P12}\sin(fel)) + \text{P23}\sin(fe2) + \text{P34}\sin(fe3))\text{R4zS} + \text{M4zS} \\
\text{M0yS} &= -(\text{P12}\cos(fel)) + \text{P23}\cos(fe2) + \text{P34}\cos(fe3))\text{R4zS} + \text{M4yS} \\
\text{M0zS} &= -(\text{P12}\cos(fel))\text{W1} + (\text{P12}\sin(fel)) + \text{P23}\cos(fe2))\text{W2} + (\text{P12}\cos(fel)) + \text{P23}\cos(fe2)\text{W3} + (\text{P12}\sin(fel)) + \text{P23}\sin(fe2)\text{W4} \\
\text{H0xS} &= \text{P12}(\sin(fel))\text{R4zS} + \text{H4xS} \\
\text{H0yS} &= -(\text{P12}\cos(fel))\text{R4zS} + \text{H4yS} \\
\text{H0zS} &= -(\text{P12}\cos(fel))\text{R4zS} + \text{H4zS} \\
\end{align*}
\]

\text{This part of the program will be utilized to calculate the dynamic reaction forces and moments at the joints and in the hydraulic actuators (ab, de).}

\text{These are the dynamic forces at the centers of gravity of each member}
\[
\begin{align*}
\text{dQg0x} &= \text{x0}\times\text{a0x} \\
\text{dQg0y} &= \text{x0}\times\text{a0y} \\
\text{dQg0z} &= \text{x0}\times\text{a0z} \\
\text{dQg1x} &= \text{x1}\times\text{a1x} \\
\text{dQg1y} &= \text{x1}\times\text{a1y} \\
\text{dQg1z} &= \text{x1}\times\text{a1z} \\
\text{dQg2x} &= \text{x2}\times\text{a2x} \\
\text{dQg2y} &= \text{x2}\times\text{a2y} \\
\text{dQg2z} &= \text{x2}\times\text{a2z} \\
\text{dQg3x} &= \text{x3}\times\text{a3x} \\
\text{dQg3y} &= \text{x3}\times\text{a3y} \\
\text{dQg3z} &= \text{x3}\times\text{a3z} \\
\end{align*}
\]

\text{These are the dynamic torques at the centers of gravity of each member}
\[
\begin{align*}
\text{dTg0x} &= \text{j0}\times\text{th0x} \\
\text{dTg0y} &= \text{j0}\times\text{th0y} \\
\text{dTg0z} &= \text{j0}\times\text{th0z} \\
\text{dTg1x} &= \text{j1}\times\text{th1x} \\
\text{dTg1y} &= \text{j1}\times\text{th1y} \\
\text{dTg1z} &= \text{j1}\times\text{th1z} \\
\text{dTg2x} &= \text{j2}\times\text{th2x} \\
\text{dTg2y} &= \text{j2}\times\text{th2y} \\
\end{align*}
\]
\[ dTg_2 = j_2 \times th_2 \]
\[ dTg_3 = j_3 \times th_3 \]
\[ dTg_3 = j_3 \times th_3 \]
\[ R_3x = d_0g_3x \]
\[ R_3y = d_0g_3y \]
\[ R_3z = d_0g_3z \]
\[ M_3x = P_3g_3x \times (\sin(f_3)) \times d_0g_3z + dTg_3x \]
\[ M_3y = -P_3g_3x \times (\cos(f_3)) \times d_0g_3x + dTg_3y \]
\[ M_3z = P_3g_3x \times (\cos(f_3)) \times d_0g_3y - P_3g_3x \times (\sin(f_3)) \times d_0g_3x + dTg_3z \]

\[ F_{ded} \text{ is the force in actuator } de \text{ due to dynamic loading condition} \]

\[ a_{aa} = P_2g_2 \times \cos(f_2) \times d_0g_2y \]
\[ b_{bb} = P_2g_2 \times \sin(f_2) \times d_0g_2x \]
\[ c_{cc} = P_23 \times (\cos(f_2)) \times R_3yd \]
\[ d_{dd} = P_23 \times (\sin(f_2)) \times R_3xd \]
\[ x_{xx} = P_2e \times (\cos(f_5)) \times \sin(f_5) \]
\[ e_{ee} = P_2e \times (\sin(f_6)) \times \cos(f_5) \]
\[ F_{ded} = \left( \frac{a_{aa} \times b_{bb} + c_{cc}}{x_{xx} - e_{ee}} \right) \]

\[ R_{2xd}, R_{2yd}, R_{2zd} \text{ are the reaction force at joint No.2 in the global x-axis due to dynamic loading condition} \]
\[ R_{2xd} = -F_{ded} \times (\cos(f_5)) + R_3xd + d_0g_2x \]
\[ R_{2yd} = d_0g_2y + R_3yd - F_{ded} \times (\sin(f_5)) \]
\[ R_{2zd} = R_3zd + d_0g_2z \]

\[ M_{2xd}, M_{2yd}, M_{2zd} \text{ are the reaction moment at joint No.2 in the global x-axis due to dynamic loading condition} \]
\[ M_{2xd} = P_2g_2 \times (\sin(f_2)) \times d_0g_2z + P_23 \times (\sin(f_2)) \times R_3zd + dTg_2x + M_3x \]
\[ M_{2yd} = P_2g_2 \times (\cos(f_2)) \times d_0g_2y - P_23 \times (\cos(f_2)) \times R_3zd + dTg_2y + M_3y \]

\[ F_{abd} \text{ is the force in actuator } ab \text{ due to dynamic loading condition} \]
\[ a_{aa} = P_1d \times \cos(f_1) \times F_{ded} \times (\sin(f_6)) - P_1d \times \sin(f_6) \times \cos(f_6) \]
\[ b_{bb} = P_1d \times \cos(f_1) \times d_0g_1y - P_1d \times \sin(f_1) \times d_0g_1x \]
\[ c_{cc} = P_12 \times (\cos(f_1)) \times R_2yd - P_12 \times (\sin(f_1)) \times R_2xd + dTg_1z \]
\[ d_{dd} = P_1b \times (\cos(f_1)) \times (\sin(f_4)) \]
\[ e_{ee} = P_1b \times (\sin(f_4)) \times (\cos(f_4)) \]
\[ F_{abd} = \left( a_{aa} + b_{bb} + c_{cc} \right) / (d_{dd} - e_{ee}) \]

\[ R_{1xd}, R_{1yd}, R_{1zd} \text{ are the reaction force at joint No.1 in the global x-axis due to dynamic loading condition} \]
\[ R_{1xd} = -F_{abd} \times (\cos(f_4)) + R_2xd + d_0g_1x \]
\[ R_{1yd} = d_0g_1y - F_{abd} \times (\sin(f_4)) + R_2yd \]
\[ R_{1zd} = R_2zd + d_0g_1z \]

\[ M_{1xd}, M_{1yd} \text{ are the reaction moment at joint No.1 in the global x-axis due to dynamic loading condition} \]
\[ M_{1x} = P_1 g_1 (\sin(f_1)) dQ_{g1z} + P_2 g_2 (\sin(f_1)) dQ_{g2z} + dT_{g1x} + M_{2x} \]
\[ M_{1y} = -P_1 g_1 (\cos(f_1)) dQ_{g1z} - P_2 g_2 (\cos(f_2) + P_3 g_3 (\cos(f_3)) dQ_{g3z} \]
\[ R_{0x} = dQ_{g0x} + dQ_{g1x} + dQ_{g2x} + dQ_{g3x} \]
\[ R_{0y} = dQ_{g0y} + dQ_{g1y} + dQ_{g2y} + dQ_{g3y} \]
\[ R_{0z} = dQ_{g0z} + dQ_{g1z} + dQ_{g2z} + dQ_{g3z} \]

**Reaction calculation at the base**

**R0x, R0y, R0z** are the reactions force at joint No.1 in the global x-axis due to dynamic loading condition.

**HQxd, HQyd, HQzd** are the reactions moment at joint No.1 in the global x-axis due to dynamic loading condition.

**This part of the program will be utilized to calculate the total reactions (Statics and Dynamics at the joints and in hydraulic actuators (AB, DE).**
R3y = R3ys + R3yd
R3z = R3zs + R3zd
M3x = M3xs + M3xd
M3y = M3ys + M3yd
M3z = M3zs + M3zd
write(6, 111)

111 format('//, 'table 1 moment and force input at the gripper ')
   write(6, 112)

112 format('___________________________________________________________________')
   write(6, 113)

113 format('//, x, 'R4x', 10x, 'R4y', 6x, 'R4z', 10x, 'M4x', 8x, 'M4y', 8x, 'M4z')
   write(6, 150)

114 format('___________________________________________________________________')
   write(6, 115)

115 format('___________________________________________________________________')
   write(6, 116)

116 format('___________________________________________________________________')
   write(6, 188)

117 format('x, x, 'f11', 5x, 'f12', 5x, 'f13', 5x, 'f14', 5x, 'f15', 5x, '+P01', 5x, 'P11', 5x, 'P12', 5x, 'P13', 5x, 'P14', 5x)
   write(6, 118)

118 format('___________________________________________________________________')
   write(6, 119)

119 format('___________________________________________________________________')
   write(6, 120)

120 format('___________________________________________________________________')
   write(6, 121)

121 format('___________________________________________________________________')
   write(6, 122)

122 format('___________________________________________________________________')
   write(6, 123)

123 format('___________________________________________________________________')
   write(6, 124)

124 format('___________________________________________________________________')
   write(6, 300)

300 format('___________________________________________________________________')
write(6,301)
301 format('')
    write(6,302)
302 format('/a4x,'/xm0',/7x,'xm1',/7x,'xm2',/7x,'xm3')
    write(6,303)
303 format('')
    write(6,304) xm0,xm1,xm2,xm3
304 format('/a12(2x,f7.4)')
    WRITE(6,125)
125 format(/'table 5 global linear accelerations of the center of g')
    write(6,126)
126 format('')
    write(6,127)
127 format(/'2x,'/a0x',/5x,'a0y',/5x,'a0z',/5x,'a1x',/5x,'a1y',/5x,'a1z'
    +/5x,'a2x',/5x,'a2y',/5x,'a2z',/5x,'a3x',/5x,'a3y',/5x,'a3z')
    write(6,128)
128 format('')
    write(6,129) a0x,a0y,a0z,a1x,a1y,a1z,a2x,a2y,a2z,a3x,a3y,a3z
129 format(/'6.1,2x,6.1,2x,6.1,2x,6.1,2x,6.1,2x,6.1,2x,6.1,2x,6.1,2x,6.1,2x,6.1,2x')
    write(6,130)
130 format(/'table 6 mass moment of inertia the four members')
    write(6,131)
131 format('')
    write(6,132)
132 format(/'2x,'/j0x',/6x,'j0y',/6x,'j0z',/5x,'j1x',/5x,'j1y',/5x,'j1z'
    +/5x,'j2x',/5x,'j2y',/4x,'j2z',/4x,'j3x',/4x,'j3y',/4x,'j3z')
    write(6,133)
133 format('')
    write(6,134) j0x,j0y,j0z,j1x,j1y,j1z,j2x,j2y,j2z,j3x,j3y,j3z
134 format(/'5.2,3x,5.2,3x,5.2,3x,5.2,3x,5.2,3x,5.2,3x,5.2,3x,5.2,3x')
    write(6,135)
135 format(/'table 7 rotational accelerations of members 0, 1, 2 &')
    write(6,136)
136 format('')
    write(6,137)
137 format(/'15x,'/th0x',/7x,'th0y',/7x,'th0z',/7x,'th1x',/7x,'th1y',/7x,'th1z')
write(6,138)
138 format("
+__________________________"
)
write(6,139)th0x,th0y,th0z,th1x,th1y,th1z
139 format(10x, f6.1, 5x, f6.1, 5x, f6.1, 5x, f6.1, 5x, f6.1, 5x, f6.1, 5x)
write(6,500)
500 format(","table 7a rotational accelerations continued.")
write(6,144)
write(6,501)
501 format(15x,'th2x','7x','th2y','7x','th2z','7x','th3x','7x','th3y','7x','th3z'
)
write(6,144)
write(6,502)th2x,th2y,th2z,th3x,th3y,th3z
502 format(10x,f6.1,6x,f6.1,6x,f6.1,6x,f6.1,6x,f6.1,6x)
write(6,143)
143 format(","table 8 polar moment of inertia and shear modulus of
+elasticity and weights of links 0,1,2,3")
write(6,144)
144 format("
+____________________________________________________"
)
write(6,145)
145 format(12x,'xj1','6x','xj2','6x','x61','6x','x62','6x','w0','5x','w1','5x',
+','w3','5x','w3'
)
write(6,146)
146 format("
+____________________________________________________"
)
write(6,147)xj1,xj2,x61,x62,w0,w1,w2,w3
147 format(6x,8(4x,f5.2))
write(6,148)
148 format(","table 9 applied forces at the centers of gravity of
each link")
write(6,149)
149 format("
+____________________________________________________"
)
write(6,150)
150 format(7,4x,'dQg0x','3x','dQg0y','3x','dQg0z','3x','dQg1x','3x','dQg1y'
+,'3x','dQg1z','3x','dQg2x','3x','dQg2y','3x','dQg2z','3x','dQg3x','3x','dQg3y',
+,'3x','dQg3z'
)
write(6,151)
151 format("
+____________________________________________________"
)
write(6,152)dQg0x,dQg0y,dQg0z,dQg1x,dQg1y,dQg1z,dQg2x,dQg2y,dQg2z,
dQg3x,dQg3y,dQg3z
152 format(15x,'f6.2,2x,f6.2,2x,f6.2,2x,f6.2,2x,f6.2,2x,f6.2,2x,f6.2,2x,f6.2,2x,f6.2,2x,f6.2,2x,f6.2,2x,f6.2,2x')
write(6,153)
153 format(","table 10 applied torques at the centers of gravity of
each link")
This part of the program is to calculate the deflection due to static and dynamic forces, and due to small angles change dfel and dfe2 in the actuators which will cause the arm to move in a rigid body motion.

write(6,11)
11 format('Table 1 shows the force and moment reactions at the joints')
write(6,12)
12 format('R0x, R0y, R0z, R1x, R1y, R1z, R2x, R2y, R2z, R3x, R3y, R3z')
write(6,30)
30 format('Fab, R1x, R1y, R1z, M1x, M1y')
write(6,40)
40 format('R2x, R2y, R2z, M2x, M2y')
write(6,50)
50 format('R3x, R3y, R3z, M3x, M3y, M3z')
write(6,60)
write(6,70)
write(6,80)
80 format('L1=PI12')
\[ L_2 = P_{23} \]
\[ q_{01x} = (6.0 / (L_{1} L_{2})) (d_t g_{1x} \cos(f_{1}) - d_t g_{1y} \sin(f_{1})) \]
\[ q_{02x} = (6.0 / (L_{2} L_{2})) (d_t g_{2x} \cos(f_{2}) - d_t g_{2y} \sin(f_{2})) \]
\[ q_{01y} = (6.0 / (L_{1} L_{2} L_{1} L_{2})) \]
\[ q_{02y} = (6.0 / (L_{2} L_{2} L_{1} L_{2})) \]
\[ q_{1y} = ((d_t g_{1x} \cos(f_{1}) - d_t g_{1y} \sin(f_{1}))) / (L_{1} / L_{1}) - ((W_{1} / L_{1}) \cos(f_{1})) \]
\[ q_{2y} = ((d_t g_{2x} \cos(f_{2}) - d_t g_{2y} \sin(f_{2}))) / (L_{2} / L_{2}) - ((W_{2} / L_{2}) \cos(f_{2})) \]
\[ q_{1z} = (d_t g_{1z} / L_{1}) \]
\[ q_{2z} = (d_t g_{2z} / L_{2}) \]

- LR_{2y}, LM_{2y} are forces and moments in the local y-axis respectively.
- L_{0} are forces and moments in the local y-axis.
- LR_{3y}, LM_{3y} are forces and moments in the local y-axis for joint No. 3.

Calculations of the maximum total deflections in the local x-y plane for each of the links:

\[ \text{Deflection calculations in the local x-y plane for member No.1} \]
\[ a_{1} = (q_{1y} / (2 \times E_{1} \times L_{1} \times L_{1} \times L_{1})) \times (3.0 \times E_{1} \times L_{1} \times L_{1} \times L_{1}) \]
\[ b_{1} = (q_{01y} / (120.0 \times E_{1} \times L_{1} \times L_{1} \times L_{1})) \]
\[ c_{1} = (L_{R_{2}x} / (L_{1} L_{1} \times L_{1} L_{1} \times L_{1} \times L_{1})) \]
\[ V_{1} = a_{1} + b_{1} + c_{1} \]
\[ a_{1} = (q_{1y} / (2 \times E_{1} \times L_{1} \times L_{1} \times L_{1})) \times (4.0 \times (L_{1} \times L_{1} \times L_{1} \times L_{1})) \]
\[ b_{1} = (q_{01y} / (120.0 \times E_{1} \times L_{1} \times L_{1} \times L_{1})) \]
\[ c_{1} = (L_{R_{2}x} / (L_{1} \times L_{1} \times L_{1} \times L_{1})) \]
\[ V_{1} = a_{1} + b_{1} + c_{1} \]

- \( \theta_{1z} \) is the rotation angle in the local z-axis.

\[ \theta_{1z} = a_{1} + b_{1} + c_{1} \]

Calculations of the maximum total deflections in the local x-y plane for member No.2:

\[ a_{2} = (q_{2y} / (L_{2} \times L_{2} \times L_{2} \times L_{2})) \]
\[ b_{2} = (9.0 \times (L_{2} \times L_{2} \times L_{2} \times L_{2})) \times (q_{02y} / (120.0 \times E_{2} \times L_{2} \times L_{2} \times L_{2})) \]
\[ c_{2} = (L_{R_{3}y} / (L_{2} \times L_{2} \times L_{2} \times L_{2})) \times (2.0 \times (L_{2} \times L_{2} \times L_{2} \times L_{2})) \]
\[ d_{2} = (M_{3} / (L_{2} \times L_{2} \times L_{2} \times L_{2})) \]
\[ V_{2} = a_{2} + b_{2} + c_{2} \]
\[ = (q_{2y} / (L_{2} \times L_{2} \times L_{2} \times L_{2})) \times (4.0 \times (L_{2} \times L_{2} \times L_{2} \times L_{2})) \]
\[ b_{2} = (9.0 \times (L_{2} \times L_{2} \times L_{2} \times L_{2})) \times (q_{02y} / (120.0 \times E_{2} \times L_{2} \times L_{2} \times L_{2})) \]
\[ c_{2} = (L_{R_{3}y} / (L_{2} \times L_{2} \times L_{2} \times L_{2})) \times (3.0 \times (L_{2} \times L_{2} \times L_{2} \times L_{2})) \]
\[ x_{2} = (M_{3} / (L_{2} \times L_{2} \times L_{2} \times L_{2})) \]

- \( \theta_{2z} \) is the local rotation angle in the z-axis.

\[ \theta_{2z} = a_{2} + b_{2} + c_{2} + d_{2} \]

Calculations of the maximum total deflections in the local x-z plane for member No.1:

\[ a_{3} = (q_{3y} / (L_{3} \times L_{3} \times L_{3} \times L_{3})) \]
\[ b_{3} = (9.0 \times (L_{3} \times L_{3} \times L_{3} \times L_{3})) \times (q_{03y} / (120.0 \times E_{3} \times L_{3} \times L_{3} \times L_{3})) \]
\[ c_{3} = (L_{R_{4}y} / (L_{3} \times L_{3} \times L_{3} \times L_{3})) \times (2.0 \times (L_{3} \times L_{3} \times L_{3} \times L_{3})) \]
\[ d_{3} = (M_{4} / (L_{3} \times L_{3} \times L_{3} \times L_{3})) \]
\[ V_{3} = a_{3} + b_{3} + c_{3} \]
\[ = (q_{3y} / (L_{3} \times L_{3} \times L_{3} \times L_{3})) \times (4.0 \times (L_{3} \times L_{3} \times L_{3} \times L_{3})) \]
\[ b_{3} = (9.0 \times (L_{3} \times L_{3} \times L_{3} \times L_{3})) \times (q_{03y} / (120.0 \times E_{3} \times L_{3} \times L_{3} \times L_{3})) \]
\[ c_{3} = (L_{R_{4}y} / (L_{3} \times L_{3} \times L_{3} \times L_{3})) \times (3.0 \times (L_{3} \times L_{3} \times L_{3} \times L_{3})) \]
\[ x_{3} = (M_{4} / (L_{3} \times L_{3} \times L_{3} \times L_{3})) \]

- \( \theta_{3z} \) is the local rotation angle in the z-axis.

\[ \theta_{3z} = a_{3} + b_{3} + c_{3} + d_{3} \]
\[ a_{a3} = \frac{q_{1z}}{(2.4 \cdot \text{EI}_{1y})} \times (3 \cdot \text{L} \cdot \text{I} \cdot \text{I} \cdot 3) \]
\[ b_{b3} = \frac{q_{01z}}{(12.0 \cdot \text{EI}_{1y})} \times (9 \cdot \text{L} \cdot \text{I} \cdot \text{I} \cdot 3) \]
\[ c_{c3} = \frac{R_{2z}}{(6 \cdot \text{EI}_{1y})} \times (2 \cdot \text{L} \cdot \text{I} \cdot \text{I} \cdot 3) \]
\[ d_{d4} = \frac{L_{M2y}}{(2 \cdot \text{EI}_{1y})} \]
\[ x_{wl} = a_{a3} + b_{b3} + c_{c3} - d_{d4} \]
\[ x_{a3} = -\frac{q_{1z}}{(2.4 \cdot \text{EI}_{1y})} \times (4 \cdot \text{L} \cdot \text{I} \cdot \text{I} \cdot 3) \]
\[ x_{b3} = \frac{q_{01y}}{(12.0 \cdot \text{EI}_{1y})} \times (14 \cdot \text{L} \cdot \text{I} \cdot \text{I} \cdot 3) \]
\[ x_{c3} = \frac{R_{2z}}{(2 \cdot \text{EI}_{1y})} \times (2 \cdot \text{L} \cdot \text{I} \cdot \text{I} \cdot 3) \]
\[ x_{d3} = \frac{L_{M2y}}{(2 \cdot \text{EI}_{1y})} \]
\[ \theta_{tayl} = x_{a3} + x_{b3} \times x_{c3} - x_{d3} \]

\text{c} \ \theta_{tayl} \text{ is the local rotation angle in the y-axis}

\[ \text{Deflection calculations in the local x-z plane for member No. 2} \]
\[ a_{a4} = \frac{q_{2z}}{(2.4 \cdot \text{EI}_{2y})} \times (3 \cdot \text{L} \cdot \text{I} \cdot \text{I} \cdot 4) \]
\[ b_{b4} = \frac{q_{02z}}{(12.0 \cdot \text{EI}_{2y})} \times (9 \cdot \text{L} \cdot \text{I} \cdot \text{I} \cdot 4) \]
\[ c_{c4} = \frac{R_{3z}}{(6 \cdot \text{EI}_{2y})} \times (2 \cdot \text{L} \cdot \text{I} \cdot \text{I} \cdot 3) \]
\[ d_{d4} = \frac{L_{M3y}}{(2 \cdot \text{EI}_{2y})} \]
\[ x_{wl} = a_{a4} + b_{b4} + c_{c4} - d_{d4} \]
\[ x_{a4} = -\frac{q_{2z}}{(2.4 \cdot \text{EI}_{2y})} \times (4 \cdot \text{L} \cdot \text{I} \cdot \text{I} \cdot 3) \]
\[ x_{b4} = \frac{q_{02y}}{(12.0 \cdot \text{EI}_{2y})} \times (14 \cdot \text{L} \cdot \text{I} \cdot \text{I} \cdot 3) \]
\[ x_{c4} = \frac{R_{3z}}{(2.4 \cdot \text{EI}_{2y})} \times (2 \cdot \text{L} \cdot \text{I} \cdot \text{I} \cdot 3) \]
\[ x_{d4} = \frac{L_{M3y}}{(2 \cdot \text{EI}_{2y})} \]
\[ \theta_{tay2} = x_{a4} + x_{b4} + x_{c4} - x_{d4} \]

\text{c} \ \theta_{tay2} \text{ is the local rotation angle in the y-axis}

\[ \theta_{tay2} = x_{1l} + x_{2l} + x_{3l} - x_{4l} \]

\text{c Calculations of the deflections due to twist in the members}
\[ \theta_{tt} = (d_{Tg1x} \times \cos(f_{el}) + d_{Tg1y} \times \sin(f_{el})) \times 0.5 \text{SL1} \]
\[ \theta_{tt} = (d_{Tg1x} \times \cos(f_{el}) + d_{Tg1y} \times \sin(f_{el})) \times 0.5 \text{SL1} \]
\[ \theta_{tt} = (d_{Tg2x} \times \cos(f_{el}) + d_{Tg2y} \times \sin(f_{el})) \times 0.5 \text{SL2} \]
\[ \theta_{tt} = (d_{Tg2x} \times \cos(f_{el}) + d_{Tg2y} \times \sin(f_{el})) \times 0.5 \text{SL2} \]
\[ \theta_{tt} = (d_{Tg3x} \times \cos(f_{el}) + d_{Tg3y} \times \sin(f_{el})) \times 0.5 \text{SL3} \]
\[ \theta_{tt} = (d_{Tg3x} \times \cos(f_{el}) + d_{Tg3y} \times \sin(f_{el})) \times 0.5 \text{SL3} \]
\[ \theta_{tt} = (d_{Tg4x} \times \cos(f_{el}) + d_{Tg4y} \times \sin(f_{el})) \times 0.5 \text{SL4} \]
\[ \theta_{tt} = (d_{Tg4x} \times \cos(f_{el}) + d_{Tg4y} \times \sin(f_{el})) \times 0.5 \text{SL4} \]

\text{c Global rotational angles at joint No. 2}
\[ \theta_{taz2} = \theta_{tay1} \times \cos(f_{el}) + \theta_{tax1} \times \sin(f_{el}) \]
\[ \theta_{taz2} = \theta_{tay1} \times \cos(f_{el}) + \theta_{tax1} \times \sin(f_{el}) \]
\[ \theta_{taz2} = \theta_{tay1} \times \cos(f_{el}) + \theta_{tax1} \times \sin(f_{el}) \]

\text{c Global rotational angles at joint No. 3}
\[ \theta_{taz3} = \theta_{taz2} \]
\[ \theta_{taz3} = \theta_{taz2} \times \theta_{tay2} \times \cos(f_{2e}) + \theta_{tax2} \times \sin(f_{2e}) \]
\[ \theta_{taz3} = \theta_{taz2} \times \theta_{tay2} \times \cos(f_{2e}) + \theta_{tax2} \times \sin(f_{2e}) \]
\[ \theta_{taz3} = \theta_{taz2} \times \theta_{tay2} \times \cos(f_{2e}) + \theta_{tax2} \times \sin(f_{2e}) \]
\[ \theta_{taz3} = \theta_{taz2} \times \theta_{tay2} \times \cos(f_{2e}) + \theta_{tax2} \times \sin(f_{2e}) \]
\[ \theta_{taz3} = \theta_{taz2} \times \theta_{tay2} \times \cos(f_{2e}) + \theta_{tax2} \times \sin(f_{2e}) \]
\[ \theta_{taz3} = \theta_{taz2} \times \theta_{tay2} \times \cos(f_{2e}) + \theta_{tax2} \times \sin(f_{2e}) \]

\text{c the above portion only includes e1, e2, i1y, i2y, i2z, i2y and a1, a2 only}

\text{c Calculations for the extensions due to the effects of in plane deflections}
\[ a_{al} = (L_{Dg1x} / (A \times \text{EI}_{1l})) \times 0.5 \text{SL1} \]
\[ b_{bl} = (L_{Dg2x} / (A \times \text{EI}_{2l})) \]
\[ U_{xl} = a_{al} + b_{bl} \]
\[ a_{al} = (L_{Dg1x} / (A \times \text{EI}_{1l})) \times 0.5 \text{SL1} \]
\[ b_{bl} = (L_{Dg2x} / (A \times \text{EI}_{2l})) \]
\[ U_{xl} = a_{al} + b_{bl} \]
c Deflections in plane due to rigid body motion
c C1, C2 are the lengths of actuators "ab", "de" respectively
c

c joint 2 in the global coordinates before deflection
x_{j2x} = L1 \cos(fel)
y_{j2y} = L1 \sin(fel)

c joint 2 components in the global coordinates after deflection
x_{j2x} = L1 \sin(fel) + L2 \sin(fel)
y_{j2y} = L1 \cos(fel) + L2 \cos(fel)

c joint 2 in the global coordinates after deflection
x_{j2x} = x_{j2x} - L1 \sin(fel)
y_{j2y} = y_{j2y} + L1 \cos(fel)

f_{2x}, f_{2y} are the final coordinates of joint 2 after \( \Delta fel \) deflection
f_{2x} = x_{j2x} - x_{j2x}
f_{2y} = y_{j2y} + y_{j2y}

c joint 3 in the global coordinates before deflection
x_{j3x} = L1 \cos(fel) + L2 \cos(fel)
y_{j3y} = L1 \sin(fel) + L2 \sin(fel)

c joint 3 in the global coordinates due to rigid body motion
x_{j3x} = x_{j3x} - L1 \Delta fel \sin(fel)
y_{j3y} = y_{j3y} + L1 \Delta fel \cos(fel)

c joint 3 in the global coordinate after the effects of \( \Delta fel \)
x_{j3x} = x_{j3x} - L1 \Delta fel \sin(fel)
y_{j3y} = y_{j3y} + L1 \Delta fel \cos(fel)

c joint 3 coordinates before deflection
x_{j3x} = L1 \cos(fel) + L2 \cos(fel)
y_{j3y} = L1 \sin(fel) + L2 \sin(fel)

c joint 3 coordinates after deflection due to forces at joint 2
x_{j3x} = x_{j3x} - V1 \sin(fel) + L2 \cos(fel)
y_{j3y} = y_{j3y} + V1 \cos(fel) + L2 \sin(fel)

R_0TX = U_1x \cos(fel) + U_2z \cos(fel)
R_0TY = U_1x \sin(fel) + U_2z \sin(fel)

c joint 3 coordinates after deflections due to forces at joint 3
x_{j3x} = x_{j3x} - V1 \sin(fel) + L2 \cos(fel)
y_{j3y} = y_{j3y} + V1 \cos(fel) + L2 \sin(fel)

C deflection due to the forces moments on both members

c joint 2 coordinates before deflection
x_{j2x} = L1 \cos(fel)
y_{j2y} = L1 \sin(fel)

c joint 3 coordinates before deflection
x_{j3x} = L1 \cos(fel) + L2 \cos(fel)
y_{j3y} = L1 \sin(fel) + L2 \sin(fel)

c joint 3 coordinates after deflection due to forces at joint 2
x_{j3x} = x_{j3x} - V1 \sin(fel)
y_{j3y} = y_{j3y} + V1 \cos(fel)

R_0TX = U_1x \cos(fel) + U_2z \cos(fel)
R_0TY = U_1x \sin(fel) + U_2z \sin(fel)

C deflection due to the forces moments on both members

C coordinate of out of plane deflections due to forces
c j3zz=(thtaxl\cos(\theta2-\phi)l)*l2*w1+w2-(l2*t(thtaxl\sin(\theta2-\phi)))
c total Global deflections of joint No.2
Df12x=-(xj2x-f2x)-(j2xx-j2xi)+U1x*cos(f1)
Df12y=-(xj2y-f2y)-(j2yy-j2yi)+U1y*sin(f1)
Df12z=w1
aaa=(thtaxl\cos(\theta2-\phi)l)*l2
bbb=xH2
ccc=L2*(thtaxl\sin(\theta2-\phi))
Df12z=aaa+bbb+ccc
c calculating the final total deflection in global x-axis at joint 3
Df13x=-(j3xx-j3x2)-(xj3x-xj3x3)+Rotx
c calculating the final total deflection in global y-axis at joint 3
Df13y=-(j3yy-j3y2)-(xj3y-xj3y3)+roty
c calculating the final total deflection in global z-axis at joint 3
c Df13z=j3zz
write(6,66)
66 format(*,Table 2 shows the global deflections in the x-y-z c +coordinates*)
write(6,77)
77 format(*
+____________________*)
write(6,88)
88 format(*,2x,'joint',3x,'Deflection X',5x,'Deflection Y',5x,
+ 'Deflection Z')
joint=2
write(6,101) joint,Df12x,Df12y,Df12z
101 format(*,2x,11,4x,12.8,6x,12.8,4x,12.8)
joint=3
write(6,109) joint,Df13x,Df13y,Df13z
99 format(*,2x,11,4x,12.8,6x,12.8,4x,12.8)
write(6,105)
105. format(*,Table 3 shows the global rotational angles in the x-y-z +coordinates*)
write(6,102)
102 format(*
+____________________*)
write(6,107)
107 format(*,2x,'joint',3x,'Rotational X',5x,'Rotational Y',5x,
+ 'Rotational Z')
joint=2
write(6,103) joint,thta2x,thta2y,thta2z
103 format(*,2x,11,4x,12.8,6x,12.8,4x,12.8)
joint=3
write(6,104) joint,thta3x,thta3y,thta3z
104 format(*,2x,11,4x,12.8,6x,12.8,4x,12.8)
end
APPENDIX—C

THE COMPUTER OUTPUT
### Table 1: Moment and Force Input at the Gripper

<table>
<thead>
<tr>
<th>R4x</th>
<th>R4y</th>
<th>R4z</th>
<th>M4x</th>
<th>M4y</th>
<th>M4z</th>
</tr>
</thead>
<tbody>
<tr>
<td>100.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
</tr>
</tbody>
</table>

### Table 2: Angles of Link 1 & 2 and Their Section Properties and Lengths of Links

<table>
<thead>
<tr>
<th>f1</th>
<th>f2</th>
<th>l1y</th>
<th>l1z</th>
<th>l2y</th>
<th>l2z</th>
<th>P01</th>
<th>P12</th>
<th>P23</th>
<th>P34</th>
</tr>
</thead>
<tbody>
<tr>
<td>.5</td>
<td>-.3</td>
<td>.4</td>
<td>.4</td>
<td>.4</td>
<td>.4</td>
<td>43.9</td>
<td>54.5</td>
<td>28.0</td>
<td>.0</td>
</tr>
</tbody>
</table>

### Table 3: Material Constants and Areas of Members

<table>
<thead>
<tr>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>A1</th>
<th>A2</th>
<th>Aab</th>
<th>Ade</th>
</tr>
</thead>
<tbody>
<tr>
<td>30000000.00</td>
<td>30000000.00</td>
<td>30000000.00</td>
<td>2.250</td>
<td>2.250</td>
<td>1.480</td>
<td>.786</td>
</tr>
</tbody>
</table>

### Table 4: The Masses of Link 0, 1, 2 & 3

<table>
<thead>
<tr>
<th>x0</th>
<th>x1</th>
<th>x2</th>
<th>x3</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
</tr>
</tbody>
</table>

### Table 5: Global Linear Accelerations of the Center of Gravity of Each Link

<table>
<thead>
<tr>
<th>a0x</th>
<th>a0y</th>
<th>a0z</th>
<th>a1x</th>
<th>a1y</th>
<th>a1z</th>
<th>a2x</th>
<th>a2y</th>
<th>a2z</th>
<th>a3x</th>
<th>a3y</th>
<th>a3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
</tr>
<tr>
<td>$j_{0x}$</td>
<td>$j_{0y}$</td>
<td>$j_{0z}$</td>
<td>$j_{1x}$</td>
<td>$j_{1y}$</td>
<td>$j_{1z}$</td>
<td>$j_{2x}$</td>
<td>$j_{2y}$</td>
<td>$j_{2z}$</td>
<td>$j_{3x}$</td>
<td>$j_{3y}$</td>
<td>$j_{3z}$</td>
</tr>
<tr>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Table 7a Rotational accelerations continued.**

<table>
<thead>
<tr>
<th>$t_{0x}$</th>
<th>$t_{0y}$</th>
<th>$t_{0z}$</th>
<th>$t_{1x}$</th>
<th>$t_{1y}$</th>
<th>$t_{1z}$</th>
<th>$t_{2x}$</th>
<th>$t_{2y}$</th>
<th>$t_{2z}$</th>
<th>$t_{3x}$</th>
<th>$t_{3y}$</th>
<th>$t_{3z}$</th>
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<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

**Table 8 Polar moment of inertia and shear modulus of elasticity and weights of links 0, 1, 2, 3**

<table>
<thead>
<tr>
<th>$x_{j1}$</th>
<th>$x_{j2}$</th>
<th>$x_{61}$</th>
<th>$x_{62}$</th>
<th>$w_{0}$</th>
<th>$w_{1}$</th>
<th>$w_{3}$</th>
<th>$w_{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.78</td>
<td>0.78</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 9 Applied forces at the centers of gravity of each link

<table>
<thead>
<tr>
<th>$d_{G0x}$</th>
<th>$d_{G0y}$</th>
<th>$d_{G0z}$</th>
<th>$d_{G1x}$</th>
<th>$d_{G1y}$</th>
<th>$d_{G1z}$</th>
<th>$d_{G2x}$</th>
<th>$d_{G2y}$</th>
<th>$d_{G2z}$</th>
<th>$d_{G3x}$</th>
<th>$d_{G3y}$</th>
<th>$d_{G3z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Table 10 applied torques at the centers of gravity of each link

<table>
<thead>
<tr>
<th>$dT_{g0x}$</th>
<th>$dT_{g0y}$</th>
<th>$dT_{g0z}$</th>
<th>$dT_{glx}$</th>
<th>$dT_{gly}$</th>
<th>$dT_{glz}$</th>
<th>$dT_{g2x}$</th>
<th>$dT_{g2y}$</th>
<th>$dT_{g2z}$</th>
<th>$dT_{g3x}$</th>
<th>$dT_{g3y}$</th>
<th>$dT_{g3z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 1 shows the force and moment reactions at the joints

<table>
<thead>
<tr>
<th>$-R_{0x}$</th>
<th>$-R_{0y}$</th>
<th>$-R_{0z}$</th>
<th>$-M_{0x}$</th>
<th>$-M_{0y}$</th>
<th>$-M_{0z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-6161.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$F_{ab}$</th>
<th>$R_{1x}$</th>
<th>$R_{1y}$</th>
<th>$R_{1z}$</th>
<th>$M_{1x}$</th>
<th>$M_{1y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-299.52</td>
<td>262.23</td>
<td>359.50</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$F_{de}$</th>
<th>$R_{2x}$</th>
<th>$R_{2y}$</th>
<th>$R_{2z}$</th>
<th>$M_{2x}$</th>
<th>$M_{2y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-110.72</td>
<td>185.89</td>
<td>69.88</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$R_{3x}$</th>
<th>$R_{3y}$</th>
<th>$R_{3z}$</th>
<th>$M_{3x}$</th>
<th>$M_{3y}$</th>
<th>$M_{3z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 2 shows the global deflections in the x-y-z coordinates

<table>
<thead>
<tr>
<th>joint</th>
<th>Deflection I</th>
<th>Deflection Y</th>
<th>Deflection Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.07516479</td>
<td>-0.13018800</td>
<td>0.00000000</td>
</tr>
<tr>
<td>3</td>
<td>0.08251953</td>
<td>-0.10998340</td>
<td>0.00000000</td>
</tr>
</tbody>
</table>

Table 3 shows the global rotational angles in the x-y-z coordinates

<table>
<thead>
<tr>
<th>joint</th>
<th>Rotational I</th>
<th>Rotational Y</th>
<th>Rotational Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.00000000</td>
<td>0.00000000</td>
<td>-0.00413747</td>
</tr>
<tr>
<td>3</td>
<td>0.00000000</td>
<td>0.00000000</td>
<td>0.00115182</td>
</tr>
</tbody>
</table>
**Table 1: Moment and Force Input at the Gripper**

<table>
<thead>
<tr>
<th>R4x</th>
<th>R4y</th>
<th>R4z</th>
<th>M4x</th>
<th>M4y</th>
<th>M4z</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0</td>
<td>100.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
</tr>
</tbody>
</table>

**Table 2: Angles of Link 1 & 2 and Their Section Properties and Lengths of Links**

<table>
<thead>
<tr>
<th>fel</th>
<th>fe2</th>
<th>l1y</th>
<th>l1z</th>
<th>l2y</th>
<th>l2z</th>
<th>P01</th>
<th>P12</th>
<th>P23</th>
<th>P34</th>
</tr>
</thead>
<tbody>
<tr>
<td>.5</td>
<td>-.3</td>
<td>.4</td>
<td>.4</td>
<td>.4</td>
<td>.4</td>
<td>43.9</td>
<td>54.5</td>
<td>28.0</td>
<td>.0</td>
</tr>
</tbody>
</table>

**Table 3: Material Constants and Areas of Members**

<table>
<thead>
<tr>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>A1</th>
<th>A2</th>
<th>Aab</th>
<th>Ade</th>
</tr>
</thead>
<tbody>
<tr>
<td>30000000.00</td>
<td>30000000.00</td>
<td>30000000.00</td>
<td>2.250</td>
<td>2.250</td>
<td>1.480</td>
<td>.786</td>
</tr>
</tbody>
</table>

**Table 4: The Masses of Link 0, 1, 2 & 3**

<table>
<thead>
<tr>
<th>xM0</th>
<th>xM1</th>
<th>xM2</th>
<th>xM3</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
</tr>
</tbody>
</table>

**Table 5: Global Linear Accelerations of the Center of Gravity of Each Link**

<table>
<thead>
<tr>
<th>a0x</th>
<th>a0y</th>
<th>a0z</th>
<th>a1x</th>
<th>a1y</th>
<th>a1z</th>
<th>a2x</th>
<th>a2y</th>
<th>a2z</th>
<th>a3x</th>
<th>a3y</th>
<th>a3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
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</tr>
</tbody>
</table>
### Table 5: Mass Moment of Inertia of the Four Members

<table>
<thead>
<tr>
<th>j₀x</th>
<th>j₀y</th>
<th>j₀z</th>
<th>j₁x</th>
<th>j₁y</th>
<th>j₁z</th>
<th>j₂x</th>
<th>j₂y</th>
<th>j₂z</th>
<th>j₃x</th>
<th>j₃y</th>
<th>j₃z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

### Table 7: Rotational Accelerations of Members 0, 1, 2 & 3

<table>
<thead>
<tr>
<th>th₀x</th>
<th>th₀y</th>
<th>th₀z</th>
<th>th₁x</th>
<th>th₁y</th>
<th>th₁z</th>
<th>th₂x</th>
<th>th₂y</th>
<th>th₂z</th>
<th>th₃x</th>
<th>th₃y</th>
<th>th₃z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
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</tr>
</tbody>
</table>

### Table 7a: Rotational Accelerations Continued

<table>
<thead>
<tr>
<th>th₂x</th>
<th>th₂y</th>
<th>th₂z</th>
<th>th₃x</th>
<th>th₃y</th>
<th>th₃z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

### Table 8: Polar Moment of Inertia and Shear Modulus of Elasticity and Weights of Links 0, 1, 2, 3

<table>
<thead>
<tr>
<th>x₁</th>
<th>x₂</th>
<th>x₃</th>
<th>x₄</th>
<th>w₀</th>
<th>w₁</th>
<th>w₂</th>
<th>w₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.78</td>
<td>0.78</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

### Table 9: Applied Forces at the Centers of Gravity of Each Link

<table>
<thead>
<tr>
<th>dG₀x</th>
<th>dG₀y</th>
<th>dG₀z</th>
<th>dG₁x</th>
<th>dG₁y</th>
<th>dG₁z</th>
<th>dG₂x</th>
<th>dG₂y</th>
<th>dG₂z</th>
<th>dG₃x</th>
<th>dG₃y</th>
<th>dG₃z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Table 10 applied torques at the centers of gravity of each link

<table>
<thead>
<tr>
<th>(dTgO_x)</th>
<th>(dTgO_y)</th>
<th>(dTgO_z)</th>
<th>(dTgI_x)</th>
<th>(dTgI_y)</th>
<th>(dTgI_z)</th>
<th>(dTg2_x)</th>
<th>(dTg2_y)</th>
<th>(dTg2_z)</th>
<th>(dTg3_x)</th>
<th>(dTg3_y)</th>
<th>(dTg3_z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 1 shows the force and moment reactions at the joints

<table>
<thead>
<tr>
<th>-ROx</th>
<th>-ROy</th>
<th>-ROz</th>
<th>-MOx</th>
<th>-MOy</th>
<th>-MOz</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>100.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>7350.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fab</th>
<th>R1x</th>
<th>R1y</th>
<th>R1z</th>
<th>M1x</th>
<th>M1y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1245.80</td>
<td>-81.55</td>
<td>-912.56</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fde</th>
<th>R2x</th>
<th>R2y</th>
<th>R2z</th>
<th>M2x</th>
<th>M2y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-304.21</td>
<td>235.98</td>
<td>291.99</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>R3x</th>
<th>R3y</th>
<th>R3z</th>
<th>M3x</th>
<th>M3y</th>
<th>M3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>100.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 2 shows the global deflections in the x-y-z coordinates

<table>
<thead>
<tr>
<th>joint</th>
<th>Deflection X</th>
<th>Deflection Y</th>
<th>Deflection Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-.31263350</td>
<td>.54149440</td>
<td>.00000000</td>
</tr>
<tr>
<td>3</td>
<td>-.29242710</td>
<td>.59700390</td>
<td>.00000000</td>
</tr>
</tbody>
</table>

Table 3 shows the global rotational angles in the x-y-z coordinates

<table>
<thead>
<tr>
<th>joint</th>
<th>Rotational X</th>
<th>Rotational Y</th>
<th>Rotational Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>.00000000</td>
<td>.00000000</td>
<td>.01720912</td>
</tr>
<tr>
<td>3</td>
<td>.00000000</td>
<td>.00000000</td>
<td>.00316460</td>
</tr>
</tbody>
</table>
**Table 1** Moment and force input at the gripper

<table>
<thead>
<tr>
<th>R4x</th>
<th>R4y</th>
<th>R4z</th>
<th>M4x</th>
<th>M4y</th>
<th>M4z</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0</td>
<td>.0</td>
<td>100.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
</tr>
</tbody>
</table>

**Table 2** Angles of link 1 & 2 and their section properties and lengths of links

<table>
<thead>
<tr>
<th>fel</th>
<th>fe2</th>
<th>l1x</th>
<th>l1y</th>
<th>l2x</th>
<th>l2y</th>
<th>l2z</th>
<th>P01</th>
<th>P12</th>
<th>P23</th>
<th>P34</th>
</tr>
</thead>
<tbody>
<tr>
<td>.5</td>
<td>-.3</td>
<td>.4</td>
<td>.4</td>
<td>.4</td>
<td>.4</td>
<td>43.9</td>
<td>54.5</td>
<td>28.0</td>
<td>.0</td>
<td>.0</td>
</tr>
</tbody>
</table>

**Table 3** Material constants and areas of members

<table>
<thead>
<tr>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>A1</th>
<th>A2</th>
<th>Aab</th>
<th>Ade</th>
</tr>
</thead>
<tbody>
<tr>
<td>30000000.00</td>
<td>30000000.00</td>
<td>30000000.00</td>
<td>2.250</td>
<td>2.250</td>
<td>1.480</td>
<td>.765</td>
</tr>
</tbody>
</table>

**Table 4** The masses of link 0, 1, 2 & 3

<table>
<thead>
<tr>
<th>x0m</th>
<th>x1m</th>
<th>x2m</th>
<th>x3m</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
</tr>
</tbody>
</table>

**Table 5** Global linear accelerations of the center of gravity of each link

<table>
<thead>
<tr>
<th>a0x</th>
<th>a0y</th>
<th>a0z</th>
<th>a1x</th>
<th>a1y</th>
<th>a1z</th>
<th>a2x</th>
<th>a2y</th>
<th>a2z</th>
<th>a3x</th>
<th>a3y</th>
<th>a3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
</tr>
</tbody>
</table>
Table 6: Mass moment of inertia for the four members.

<table>
<thead>
<tr>
<th>j0x</th>
<th>j0y</th>
<th>j0z</th>
<th>j1x</th>
<th>j1y</th>
<th>j1z</th>
<th>j2x</th>
<th>j2y</th>
<th>j2z</th>
<th>j3x</th>
<th>j3y</th>
<th>j3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 7: Rotational accelerations of members 0, 1, 2, 3.

<table>
<thead>
<tr>
<th>th0x</th>
<th>th0y</th>
<th>th0z</th>
<th>th1x</th>
<th>th1y</th>
<th>th1z</th>
<th>th2x</th>
<th>th2y</th>
<th>th2z</th>
<th>th3x</th>
<th>th3y</th>
<th>th3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 7a: Rotational accelerations continued.

<table>
<thead>
<tr>
<th>th2x</th>
<th>th2y</th>
<th>th2z</th>
<th>th3x</th>
<th>th3y</th>
<th>th3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 8: Polar moment of inertia and shear modulus of elasticity and weights of links 0, 1, 2, 3.

| x1j | xj2 | xj3 | xj2 | xj1 | xj2 | xj1 | xj2 | xj1 | xj2 | xj1 | xj2 | xj1 | xj2 | xj1 | xj2 | xj1 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0.78| 0.78| 0.00| 0.00| 0.00| 0.00| 0.00| 0.00| 0.00| 0.00| 0.00| 0.00| 0.00| 0.00| 0.00| 0.00| 0.00|

Table 9: Applied forces at the centers of gravity of each link.

<table>
<thead>
<tr>
<th>dQg0x</th>
<th>dQg0y</th>
<th>dQg0z</th>
<th>dQglx</th>
<th>dQg1y</th>
<th>dQg1z</th>
<th>dQg2x</th>
<th>dQg2y</th>
<th>dQg2z</th>
<th>dQg3x</th>
<th>dQg3y</th>
<th>dQg3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Table 10 applied torques at the centers of gravity of each link

<table>
<thead>
<tr>
<th>dTg0x</th>
<th>dTg0y</th>
<th>dTg0z</th>
<th>dTg1x</th>
<th>dTg1y</th>
<th>dTg1z</th>
<th>dTg2x</th>
<th>dTg2y</th>
<th>dTg2z</th>
<th>dTg3x</th>
<th>dTg3y</th>
<th>dTg3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 1 shows the force and moment reactions at the joints

<table>
<thead>
<tr>
<th>R0x</th>
<th>R0y</th>
<th>R0z</th>
<th>M0x</th>
<th>M0y</th>
<th>M0z</th>
<th>Rlx</th>
<th>Rly</th>
<th>Rlz</th>
<th>M1x</th>
<th>M1y</th>
<th>M1z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>100.00</td>
<td>6151.00</td>
<td>-7350.00</td>
<td>0.00</td>
<td>100.00</td>
<td>1767.34</td>
<td>-7350.98</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fab Rlx Rly Rlz M1x M1y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>100.00</td>
<td>1767.34</td>
<td>-7350.98</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fde R2x R2y R2z M2x M2y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>100.00</td>
<td>-957.66</td>
<td>-2631.14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R3x R3y R3z M3x M3y M3z</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 shows the global deflections in the x-y-z coordinates

<table>
<thead>
<tr>
<th>joint</th>
<th>Deflection X</th>
<th>Deflection Y</th>
<th>Deflection Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.00000000</td>
<td>0.00000000</td>
<td>0.69320360</td>
</tr>
<tr>
<td>3</td>
<td>0.00000000</td>
<td>0.00000000</td>
<td>1.40653300</td>
</tr>
</tbody>
</table>

Table 3 shows the global rotational angles in the x-y-z coordinates

<table>
<thead>
<tr>
<th>joint</th>
<th>Rotational X</th>
<th>Rotational Y</th>
<th>Rotational Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-0.02146453</td>
<td>0.01297052</td>
<td>0.00000000</td>
</tr>
<tr>
<td>3</td>
<td>-0.02031271</td>
<td>0.01523522</td>
<td>0.00000000</td>
</tr>
</tbody>
</table>
### Table 1: Moment and Force Input at the Gripper

<table>
<thead>
<tr>
<th></th>
<th>R4x</th>
<th>R4y</th>
<th>R4z</th>
<th>M4x</th>
<th>M4y</th>
<th>M4z</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10000.0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 2: Angles of Link 1 & 2 and Their Section Properties and Lengths of Links

<table>
<thead>
<tr>
<th></th>
<th>f1</th>
<th>f2</th>
<th>l1x</th>
<th>l1y</th>
<th>l1z</th>
<th>l2x</th>
<th>l2y</th>
<th>l2z</th>
<th>P01</th>
<th>P12</th>
<th>P23</th>
<th>P34</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5</td>
<td>-0.3</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>43.3</td>
<td>54.5</td>
<td>28.0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 3: Material Constants and Areas of Members

<table>
<thead>
<tr>
<th></th>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>A1</th>
<th>A2</th>
<th>Aab</th>
<th>Ade</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30000000.00</td>
<td>30000000.00</td>
<td>30000000.00</td>
<td>2.250</td>
<td>2.250</td>
<td>1.480</td>
<td>0.786</td>
</tr>
</tbody>
</table>

### Table 4: The Masses of Link 0, 1, 2 & 3

<table>
<thead>
<tr>
<th></th>
<th>xM0</th>
<th>xM1</th>
<th>xM2</th>
<th>xM3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

### Table 5: Global Linear Accelerations of the Center of Gravity of Each Link

<table>
<thead>
<tr>
<th></th>
<th>a0x</th>
<th>a0y</th>
<th>a0z</th>
<th>a1x</th>
<th>a1y</th>
<th>a1z</th>
<th>a2x</th>
<th>a2y</th>
<th>a2z</th>
<th>a3x</th>
<th>a3y</th>
<th>a3z</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 6 Mass Moment of Inertia of the Four Members

<table>
<thead>
<tr>
<th>j₀ₓ</th>
<th>j₀ᵧ</th>
<th>j₀ᶻ</th>
<th>j₁ₓ</th>
<th>j₁ᵧ</th>
<th>j₁ᶻ</th>
<th>j₂ₓ</th>
<th>j₂ᵧ</th>
<th>j₂ᶻ</th>
<th>j₃ₓ</th>
<th>j₃ᵧ</th>
<th>j₃ᶻ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 7 Rotational Accelerations of Members 0, 1, 2 & 3

<table>
<thead>
<tr>
<th>th₀ₓ</th>
<th>th₀ᵧ</th>
<th>th₀ᶻ</th>
<th>th₁ₓ</th>
<th>th₁ᵧ</th>
<th>th₁ᶻ</th>
<th>th₂ₓ</th>
<th>th₂ᵧ</th>
<th>th₂ᶻ</th>
<th>th₃ₓ</th>
<th>th₃ᵧ</th>
<th>th₃ᶻ</th>
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</thead>
<tbody>
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</table>

Table 7a Rotational Accelerations Continued

<table>
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<th>th₃ᵧ</th>
<th>th₃ᶻ</th>
<th>th₄ₓ</th>
<th>th₄ᵧ</th>
<th>th₄ᶻ</th>
</tr>
</thead>
<tbody>
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</table>

Table 8 Polar Moment of Inertia and Shear Modulus of Elasticity and Weights of Links 0, 1, 2, 3

<table>
<thead>
<tr>
<th>x₀₁</th>
<th>x₀₂</th>
<th>x₁₁</th>
<th>x₁₂</th>
<th>w₀</th>
<th>w₁</th>
<th>w₂</th>
<th>w₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.78</td>
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<td>0.00</td>
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</tbody>
</table>

Table 9 Applied Forces at the Centers of Gravity of Each Link

<table>
<thead>
<tr>
<th>dₓ₀</th>
<th>dₓ₀ᵧ</th>
<th>dₓ₀ᶻ</th>
<th>dₓ₁</th>
<th>dₓ₁ᵧ</th>
<th>dₓ₁ᶻ</th>
<th>dₓ₂</th>
<th>dₓ₂ᵧ</th>
<th>dₓ₂ᶻ</th>
<th>dₓ₃</th>
<th>dₓ₃ᵧ</th>
<th>dₓ₃ᶻ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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</tr>
</tbody>
</table>
Table 10 applied torques at the centers of gravity of each link

<table>
<thead>
<tr>
<th>dTg0x</th>
<th>dTg0y</th>
<th>dTg0z</th>
<th>dTg1x</th>
<th>dTg1y</th>
<th>dTg1z</th>
<th>dTg2x</th>
<th>dTg2y</th>
<th>dTg2z</th>
<th>dTg3x</th>
<th>dTg3y</th>
<th>dTg3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
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<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 1 shows the force and moment reactions at the joints

<table>
<thead>
<tr>
<th>-R0x</th>
<th>-R0y</th>
<th>-R0z</th>
<th>-M0x</th>
<th>-M0y</th>
<th>-M0z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
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<td>0.00</td>
<td>10000.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fab</th>
<th>R1x</th>
<th>R1y</th>
<th>R1z</th>
<th>M1x</th>
<th>M1y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
<td>10000.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fde</th>
<th>R2x</th>
<th>R2y</th>
<th>R2z</th>
<th>M2x</th>
<th>M2y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>10000.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>R3x</th>
<th>R3y</th>
<th>R3z</th>
<th>M3x</th>
<th>M3y</th>
<th>M3z</th>
</tr>
</thead>
<tbody>
<tr>
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<td>10000.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 2 shows the global deflections in the x-y-z coordinates

<table>
<thead>
<tr>
<th>joint</th>
<th>Deflection X</th>
<th>Deflection Y</th>
<th>Deflection Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.00000000</td>
<td>0.00000000</td>
<td>0.53794030</td>
</tr>
<tr>
<td>3</td>
<td>0.00000000</td>
<td>0.00000000</td>
<td>-0.14305240</td>
</tr>
</tbody>
</table>

Table 3 shows the global rotational angles in the x-y-z coordinates

<table>
<thead>
<tr>
<th>joint</th>
<th>Rotational X</th>
<th>Rotational Y</th>
<th>Rotational Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.03218955</td>
<td>0.04551700</td>
<td>0.00000000</td>
</tr>
<tr>
<td>3</td>
<td>0.05592711</td>
<td>0.02822197</td>
<td>0.00000000</td>
</tr>
</tbody>
</table>
### Table 1: Moment and Force Input at the Gripper

<table>
<thead>
<tr>
<th>R4x</th>
<th>R4y</th>
<th>R4z</th>
<th>M4x</th>
<th>M4y</th>
<th>M4z</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>10000.0</td>
<td>.0</td>
</tr>
</tbody>
</table>

### Table 2: Angles of Link 1 & 2 and Their Section Properties and Lengths of Links

<table>
<thead>
<tr>
<th>fe1</th>
<th>fe2</th>
<th>I1y</th>
<th>I1z</th>
<th>I2y</th>
<th>I2z</th>
<th>P01</th>
<th>P12</th>
<th>P23</th>
<th>P34</th>
</tr>
</thead>
<tbody>
<tr>
<td>.5</td>
<td>-.3</td>
<td>.4</td>
<td>.4</td>
<td>.4</td>
<td>.4</td>
<td>43.9</td>
<td>54.5</td>
<td>28.0</td>
<td>.0</td>
</tr>
</tbody>
</table>

### Table 3: Material Constants and Areas of Members

<table>
<thead>
<tr>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>A1</th>
<th>A2</th>
<th>Aab</th>
<th>Adc</th>
</tr>
</thead>
<tbody>
<tr>
<td>30000000.00</td>
<td>30000000.00</td>
<td>30000000.00</td>
<td>2.250</td>
<td>2.250</td>
<td>1.480</td>
<td>.786</td>
</tr>
</tbody>
</table>

### Table 4: The Masses of Link 0, 1, 2 & 3

<table>
<thead>
<tr>
<th>x0m</th>
<th>x1m</th>
<th>x2m</th>
<th>x3m</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0000</td>
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</tbody>
</table>

### Table 5: Global Linear Accelerations of the Center of Gravity of Each Link

<table>
<thead>
<tr>
<th>a0x</th>
<th>a0y</th>
<th>a0z</th>
<th>a1x</th>
<th>a1y</th>
<th>a1z</th>
<th>a2x</th>
<th>a2y</th>
<th>a2z</th>
<th>a3x</th>
<th>a3y</th>
<th>a3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
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</table>
### Table 6: Mass Moment of Inertia of the Four Members

<table>
<thead>
<tr>
<th>$j_{0x}$</th>
<th>$j_{0y}$</th>
<th>$j_{0z}$</th>
<th>$j_{1x}$</th>
<th>$j_{1y}$</th>
<th>$j_{1z}$</th>
<th>$j_{2x}$</th>
<th>$j_{2y}$</th>
<th>$j_{2z}$</th>
<th>$j_{3x}$</th>
<th>$j_{3y}$</th>
<th>$j_{3z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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</tr>
</tbody>
</table>

### Table 7: Rotational Accelerations of Members 0, 1, 2 & 3

<table>
<thead>
<tr>
<th>$th_{0x}$</th>
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<th>$th_{0z}$</th>
<th>$th_{1x}$</th>
<th>$th_{1y}$</th>
<th>$th_{1z}$</th>
<th>$th_{2x}$</th>
<th>$th_{2y}$</th>
<th>$th_{2z}$</th>
<th>$th_{3x}$</th>
<th>$th_{3y}$</th>
<th>$th_{3z}$</th>
</tr>
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<tbody>
<tr>
<td>0.0</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
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<td>0.0</td>
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</table>

### Table 7a: Rotational Accelerations Continued

<table>
<thead>
<tr>
<th>$th_{2x}$</th>
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<th>$th_{2z}$</th>
<th>$th_{3x}$</th>
<th>$th_{3y}$</th>
<th>$th_{3z}$</th>
</tr>
</thead>
<tbody>
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</tr>
</tbody>
</table>

### Table 8: Polar Moment of Inertia and Shear Modulus of Elasticity and Weights of Links 0, 1, 2, 3

<table>
<thead>
<tr>
<th>$x_{j1}$</th>
<th>$x_{j2}$</th>
<th>$x_{11}$</th>
<th>$x_{12}$</th>
<th>$w_{0}$</th>
<th>$w_{1}$</th>
<th>$w_{3}$</th>
<th>$w_{3}$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

### Table 9: Applied Forces at the Centers of Gravity of Each Link

<table>
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<tr>
<th>$d_{g0x}$</th>
<th>$d_{g0y}$</th>
<th>$d_{g0z}$</th>
<th>$d_{g1x}$</th>
<th>$d_{g1y}$</th>
<th>$d_{g1z}$</th>
<th>$d_{g2x}$</th>
<th>$d_{g2y}$</th>
<th>$d_{g2z}$</th>
<th>$d_{g3x}$</th>
<th>$d_{g3y}$</th>
<th>$d_{g3z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
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</table>
Table 10 applied torques at the centers of gravity of each link

<table>
<thead>
<tr>
<th>dTg0x</th>
<th>dTg0y</th>
<th>dTg0z</th>
<th>dTg1x</th>
<th>dTg1y</th>
<th>dTg1z</th>
<th>dTg2x</th>
<th>dTg2y</th>
<th>dTg2z</th>
<th>dTg3x</th>
<th>dTg3y</th>
<th>dTg3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 1 shows the force and moment reactions at the joints

<table>
<thead>
<tr>
<th>-R0x</th>
<th>-R0y</th>
<th>-R0z</th>
<th>-M0x</th>
<th>-M0y</th>
<th>-M0z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>10000.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>R1x</td>
<td>R1y</td>
<td>R1z</td>
<td>M1x</td>
<td>M1y</td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>10000.00</td>
<td></td>
</tr>
<tr>
<td>R2x</td>
<td>R2y</td>
<td>R2z</td>
<td>M2x</td>
<td>M2y</td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>10000.00</td>
<td></td>
</tr>
<tr>
<td>R3x</td>
<td>R3y</td>
<td>R3z</td>
<td>M3x</td>
<td>M3y</td>
<td>M3z</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>10000.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 2 shows the global deflections in the x-y-z coordinates

<table>
<thead>
<tr>
<th>joint</th>
<th>Deflection X</th>
<th>Deflection Y</th>
<th>Deflection Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.00000000</td>
<td>0.00000000</td>
<td>-1.10494500</td>
</tr>
<tr>
<td>3</td>
<td>0.00000000</td>
<td>0.00000000</td>
<td>-2.77887400</td>
</tr>
</tbody>
</table>

Table 3 shows the global rotational angles in the x-y-z coordinates

<table>
<thead>
<tr>
<th>joint</th>
<th>Rotational X</th>
<th>Rotational Y</th>
<th>Rotational Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-0.04551700</td>
<td>-0.02048432</td>
<td>0.00000000</td>
</tr>
<tr>
<td>3</td>
<td>-0.02822197</td>
<td>-0.03820803</td>
<td>0.00000000</td>
</tr>
</tbody>
</table>
### Table 1: Moment and Force Input at the Gripper

<table>
<thead>
<tr>
<th>R4x</th>
<th>R4y</th>
<th>R4z</th>
<th>H4x</th>
<th>H4y</th>
<th>H4z</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>10000.0</td>
</tr>
</tbody>
</table>

### Table 2: Angles of Link 1 & 2 and Their Section Properties and Lengths of Links

<table>
<thead>
<tr>
<th>f1</th>
<th>f2</th>
<th>I1y</th>
<th>I1z</th>
<th>I2y</th>
<th>I2z</th>
<th>P01</th>
<th>P12</th>
<th>P23</th>
<th>P34</th>
</tr>
</thead>
<tbody>
<tr>
<td>.5</td>
<td>-.3</td>
<td>.4</td>
<td>.4</td>
<td>.4</td>
<td>43.9</td>
<td>54.5</td>
<td>28.0</td>
<td>.0</td>
<td></td>
</tr>
</tbody>
</table>

### Table 3: Material Constants and Areas of Members

<table>
<thead>
<tr>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>A1</th>
<th>A2</th>
<th>Aab</th>
<th>Ade</th>
</tr>
</thead>
<tbody>
<tr>
<td>30000000.00</td>
<td>30000000.00</td>
<td>30000000.00</td>
<td>2.250</td>
<td>2.250</td>
<td>1.480</td>
<td>.786</td>
</tr>
</tbody>
</table>

### Table 4: The Masses of Link 0, 1, 2 & 3

<table>
<thead>
<tr>
<th>xM0</th>
<th>xM1</th>
<th>xM2</th>
<th>xM3</th>
</tr>
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<tbody>
<tr>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
</tr>
</tbody>
</table>

### Table 5: Global Linear Accelerations of the Center of Gravity of Each Link

<table>
<thead>
<tr>
<th>a0x</th>
<th>a0y</th>
<th>a0z</th>
<th>a1x</th>
<th>a1y</th>
<th>a1z</th>
<th>a2x</th>
<th>a2y</th>
<th>a2z</th>
<th>a3x</th>
<th>a3y</th>
<th>a3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>.0</td>
</tr>
</tbody>
</table>
### Table 6 Mass Moment of Inertia of the Four Members

<table>
<thead>
<tr>
<th>$j_{0x}$</th>
<th>$j_{0y}$</th>
<th>$j_{0z}$</th>
<th>$j_{1x}$</th>
<th>$j_{1y}$</th>
<th>$j_{1z}$</th>
<th>$j_{2x}$</th>
<th>$j_{2y}$</th>
<th>$j_{2z}$</th>
<th>$j_{3x}$</th>
<th>$j_{3y}$</th>
<th>$j_{3z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

### Table 7 Rotational Accelerations of Members 0, 1, 2 & 3

<table>
<thead>
<tr>
<th>$\theta_{0x}$</th>
<th>$\theta_{0y}$</th>
<th>$\theta_{0z}$</th>
<th>$\theta_{1x}$</th>
<th>$\theta_{1y}$</th>
<th>$\theta_{1z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

### Table 7a Rotational Accelerations Continued

<table>
<thead>
<tr>
<th>$\theta_{2x}$</th>
<th>$\theta_{2y}$</th>
<th>$\theta_{2z}$</th>
<th>$\theta_{3x}$</th>
<th>$\theta_{3y}$</th>
<th>$\theta_{3z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

### Table 8 Polar Moment of Inertia and Shear Modulus of Elasticity and Weights of Links 0,1,2,3

<table>
<thead>
<tr>
<th>$x_{j1}$</th>
<th>$x_{j2}$</th>
<th>$x_{61}$</th>
<th>$x_{62}$</th>
<th>$w_{0}$</th>
<th>$w_{1}$</th>
<th>$w_{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.78</td>
<td>0.78</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

### Table 9 Applied Forces at the Centers of Gravity of Each Link

<table>
<thead>
<tr>
<th>$d_{Qg0x}$</th>
<th>$d_{Qg0y}$</th>
<th>$d_{Qg0z}$</th>
<th>$d_{Qg1x}$</th>
<th>$d_{Qg1y}$</th>
<th>$d_{Qg1z}$</th>
<th>$d_{Qg2x}$</th>
<th>$d_{Qg2y}$</th>
<th>$d_{Qg2z}$</th>
<th>$d_{Qg3x}$</th>
<th>$d_{Qg3y}$</th>
<th>$d_{Qg3z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Table 10 applied torques at the centers of gravity of each link

<table>
<thead>
<tr>
<th>dTgOx</th>
<th>dTgOy</th>
<th>dTgOz</th>
<th>dTg1x</th>
<th>dTg1y</th>
<th>dTg1z</th>
<th>dTg2x</th>
<th>dTg2y</th>
<th>dTg2z</th>
<th>dTg3x</th>
<th>dTg3y</th>
<th>dTg3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 1 shows the force and moment reactions at the joints

<table>
<thead>
<tr>
<th>-R0x</th>
<th>-R0y</th>
<th>-R0z</th>
<th>-M0x</th>
<th>-M0y</th>
<th>-M0z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>10000.00</td>
</tr>
</tbody>
</table>

Fabric

<table>
<thead>
<tr>
<th>Fab</th>
<th>R1x</th>
<th>R1y</th>
<th>R1z</th>
<th>M1x</th>
<th>M1y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1694.74</td>
<td>464.90</td>
<td>-909.08</td>
<td>.00</td>
<td>.00</td>
<td>.00</td>
</tr>
</tbody>
</table>

Fde

<table>
<thead>
<tr>
<th>Fde</th>
<th>R2x</th>
<th>R2y</th>
<th>R2z</th>
<th>M2x</th>
<th>M2y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1156.20</td>
<td>896.86</td>
<td>729.69</td>
<td>.00</td>
<td>.00</td>
<td>.00</td>
</tr>
</tbody>
</table>

R3x

<table>
<thead>
<tr>
<th>R3x</th>
<th>R3y</th>
<th>R3z</th>
<th>M3x</th>
<th>M3y</th>
<th>M3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00</td>
<td>.00</td>
<td>.00</td>
<td>.00</td>
<td>.00</td>
<td>10000.00</td>
</tr>
</tbody>
</table>

Table 2 shows the global deflections in the x-y-z coordinates

<table>
<thead>
<tr>
<th>joint</th>
<th>Deflection X</th>
<th>Deflection Y</th>
<th>Deflection Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-0.42529300</td>
<td>0.73629500</td>
<td>0.00000000</td>
</tr>
<tr>
<td>3</td>
<td>-0.31011200</td>
<td>1.05308900</td>
<td>0.00000000</td>
</tr>
</tbody>
</table>

Table 3 shows the global rotational angles in the x-y-z coordinates

<table>
<thead>
<tr>
<th>joint</th>
<th>Rotational X</th>
<th>Rotational Y</th>
<th>Rotational Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.00000000</td>
<td>0.00000000</td>
<td>0.02341066</td>
</tr>
<tr>
<td>3</td>
<td>0.00000000</td>
<td>0.00000000</td>
<td>-0.02405498</td>
</tr>
</tbody>
</table>
### Table 1: Moment and Force Input at the Gripper

<table>
<thead>
<tr>
<th>R4x</th>
<th>R4y</th>
<th>R4z</th>
<th>M4x</th>
<th>M4y</th>
<th>M4z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

### Table 2: Angles of Link 1 & 2 and Their Section Properties and Lengths of Links

<table>
<thead>
<tr>
<th>fel</th>
<th>fe2</th>
<th>I1y</th>
<th>I1z</th>
<th>I2y</th>
<th>I2z</th>
<th>P01</th>
<th>P12</th>
<th>P23</th>
<th>P34</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>-0.3</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>43.9</td>
<td>54.5</td>
<td>28.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

### Table 3: Material Constants and Areas of Members

<table>
<thead>
<tr>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>A1</th>
<th>A2</th>
<th>Aab</th>
<th>Ade</th>
</tr>
</thead>
<tbody>
<tr>
<td>30000000.00</td>
<td>30000000.00</td>
<td>30000000.00</td>
<td>2.250</td>
<td>2.250</td>
<td>1.480</td>
<td>.785</td>
</tr>
</tbody>
</table>

### Table 4: The Masses of Link 0, 1, 2 & 3

<table>
<thead>
<tr>
<th>xm0</th>
<th>xm1</th>
<th>xm2</th>
<th>xm3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000</td>
<td>0.0450</td>
<td>0.0230</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

### Table 5: Global Linear Accelerations of the Center of Gravity of Each Link

<table>
<thead>
<tr>
<th>a0x</th>
<th>a0y</th>
<th>a0z</th>
<th>a1x</th>
<th>a1y</th>
<th>a1z</th>
<th>a2x</th>
<th>a2y</th>
<th>a2z</th>
<th>a3x</th>
<th>a3y</th>
<th>a3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>193.2</td>
<td>193.2</td>
<td>193.2</td>
<td>193.2</td>
<td>193.2</td>
<td>193.2</td>
<td>193.2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
Table 6: Mass moment of inertia of the four members

<table>
<thead>
<tr>
<th>j0x</th>
<th>j0y</th>
<th>j0z</th>
<th>j1x</th>
<th>j1y</th>
<th>j1z</th>
<th>j2x</th>
<th>j2y</th>
<th>j2z</th>
<th>j3x</th>
<th>j3y</th>
<th>j3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00</td>
<td>.00</td>
<td>.00</td>
<td>.01</td>
<td>11.14</td>
<td>11.14</td>
<td>.01</td>
<td>11.14</td>
<td>11.14</td>
<td>.00</td>
<td>.00</td>
<td>.00</td>
</tr>
</tbody>
</table>

Table 7: Rotational accelerations of members 0, 1, 2 & 3

<table>
<thead>
<tr>
<th>th0x</th>
<th>th0y</th>
<th>th0z</th>
<th>th1x</th>
<th>th1y</th>
<th>th1z</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00</td>
<td>.00</td>
<td>.00</td>
<td>4.7</td>
<td>4.7</td>
<td>4.7</td>
</tr>
</tbody>
</table>

Table 7a: Rotational accelerations continued

<table>
<thead>
<tr>
<th>th2x</th>
<th>th2y</th>
<th>th2z</th>
<th>th3x</th>
<th>th3y</th>
<th>th3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.7</td>
<td>4.7</td>
<td>4.7</td>
<td>.00</td>
<td>.00</td>
<td>.00</td>
</tr>
</tbody>
</table>

Table 8: Polar moment of inertia and shear modulus of elasticity and weights of links 0, 1, 2, 3

<table>
<thead>
<tr>
<th>xj1</th>
<th>xj2</th>
<th>xj3</th>
<th>xj4</th>
<th>u0</th>
<th>u1</th>
<th>u2</th>
<th>u3</th>
</tr>
</thead>
<tbody>
<tr>
<td>.78</td>
<td>.78</td>
<td>.00</td>
<td>.00</td>
<td>.00</td>
<td>.00</td>
<td>.00</td>
<td>.00</td>
</tr>
</tbody>
</table>

Table 9: Applied forces at the centers of gravity of each link

<table>
<thead>
<tr>
<th>dQg0x</th>
<th>dQg0y</th>
<th>dQg0z</th>
<th>dQg1x</th>
<th>dQg1y</th>
<th>dQg1z</th>
<th>dQg2x</th>
<th>dQg2y</th>
<th>dQg2z</th>
<th>dQg3x</th>
<th>dQg3y</th>
<th>dQg3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00</td>
<td>.00</td>
<td>.00</td>
<td>8.69</td>
<td>8.69</td>
<td>8.69</td>
<td>4.44</td>
<td>4.44</td>
<td>4.44</td>
<td>.00</td>
<td>.00</td>
<td>.00</td>
</tr>
</tbody>
</table>
Table 10 applied torques at the centers of gravity of each link

<table>
<thead>
<tr>
<th>( dT_{g0x} )</th>
<th>( dT_{g0y} )</th>
<th>( dT_{g0z} )</th>
<th>( dT_{g1x} )</th>
<th>( dT_{g1y} )</th>
<th>( dT_{g1z} )</th>
<th>( dT_{g2x} )</th>
<th>( dT_{g2y} )</th>
<th>( dT_{g2z} )</th>
<th>( dT_{g3x} )</th>
<th>( dT_{g3y} )</th>
<th>( dT_{g3z} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.05</td>
<td>52.36</td>
<td>52.36</td>
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<td>52.36</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 1 shows the force and moment reactions at the joints

<table>
<thead>
<tr>
<th>( R_{0x} )</th>
<th>( R_{0y} )</th>
<th>( R_{0z} )</th>
<th>( R_{1x} )</th>
<th>( R_{1y} )</th>
<th>( R_{1z} )</th>
<th>( M_{1x} )</th>
<th>( M_{1y} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.14</td>
<td>13.14</td>
<td>13.14</td>
<td>211.96</td>
<td>-411.93</td>
<td>-200.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( F_{ab} )</td>
<td>( R_{1x} )</td>
<td>( R_{1y} )</td>
<td>( R_{1z} )</td>
<td>( M_{1x} )</td>
<td>( M_{1y} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>73.52</td>
<td>6.07</td>
<td>-48.46</td>
<td>13.14</td>
<td>244.69</td>
<td>-411.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( F_{de} )</td>
<td>( R_{2x} )</td>
<td>( R_{2y} )</td>
<td>( R_{2z} )</td>
<td>( M_{2x} )</td>
<td>( M_{2y} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-15.05</td>
<td>16.12</td>
<td>13.94</td>
<td>4.44</td>
<td>-20.71</td>
<td>-4.68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R_{3x} )</td>
<td>( R_{3y} )</td>
<td>( R_{3z} )</td>
<td>( M_{3x} )</td>
<td>( M_{3y} )</td>
<td>( M_{3z} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 shows the global deflections in the x-y-z coordinates

<table>
<thead>
<tr>
<th>joint</th>
<th>Deflection X</th>
<th>Deflection Y</th>
<th>Deflection Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-0.01507568</td>
<td>0.02611542</td>
<td>0.03710918</td>
</tr>
<tr>
<td>3</td>
<td>-0.01407623</td>
<td>0.02898772</td>
<td>0.04719726</td>
</tr>
</tbody>
</table>

Table 3 shows the global rotational angles in the x-y-z coordinates

<table>
<thead>
<tr>
<th>joint</th>
<th>Rotational X</th>
<th>Rotational Y</th>
<th>Rotational Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-0.00020598</td>
<td>0.00027285</td>
<td>0.00081917</td>
</tr>
<tr>
<td>3</td>
<td>-0.00021813</td>
<td>0.00031801</td>
<td>-0.00015210</td>
</tr>
</tbody>
</table>
Table 1: Moment and force input at the gripper

<table>
<thead>
<tr>
<th>R4x</th>
<th>R4y</th>
<th>R4z</th>
<th>M4x</th>
<th>M4y</th>
<th>M4z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 2: Angles of link 1 & 2 and their section properties and lengths of links

<table>
<thead>
<tr>
<th>fe1</th>
<th>fe2</th>
<th>I1y</th>
<th>I1z</th>
<th>I2y</th>
<th>I2z</th>
<th>P01</th>
<th>P12</th>
<th>P23</th>
<th>P34</th>
</tr>
</thead>
<tbody>
<tr>
<td>.5</td>
<td>-.3</td>
<td>.4</td>
<td>.4</td>
<td>.4</td>
<td>.4</td>
<td>43.9</td>
<td>54.5</td>
<td>28.0</td>
<td>.0</td>
</tr>
</tbody>
</table>

Table 3: Material constants and areas of members

<table>
<thead>
<tr>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>A1</th>
<th>A2</th>
<th>Aab</th>
<th>Ade</th>
</tr>
</thead>
<tbody>
<tr>
<td>30000000.00</td>
<td>30000000.00</td>
<td>30000000.00</td>
<td>2.250</td>
<td>2.250</td>
<td>1.480</td>
<td>.786</td>
</tr>
</tbody>
</table>

Table 4: The masses of link 0, 1, 2 & 3

<table>
<thead>
<tr>
<th>xm0</th>
<th>xm1</th>
<th>xm2</th>
<th>xm3</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0000</td>
<td>.0450</td>
<td>.0230</td>
<td>.1300</td>
</tr>
</tbody>
</table>

Table 5: Global linear accelerations of the center of gravity of each link

<table>
<thead>
<tr>
<th>a0x</th>
<th>a0y</th>
<th>a0z</th>
<th>a1x</th>
<th>a1y</th>
<th>a1z</th>
<th>a2x</th>
<th>a2y</th>
<th>a2z</th>
<th>a3x</th>
<th>a3y</th>
<th>a3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0</td>
<td>.0</td>
<td>.0</td>
<td>193.2</td>
<td>193.2</td>
<td>193.2</td>
<td>193.2</td>
<td>193.2</td>
<td>193.2</td>
<td>193.2</td>
<td>193.2</td>
<td>193.2</td>
</tr>
</tbody>
</table>
Table 6: Mass moment of inertia of the four members

<table>
<thead>
<tr>
<th>j0x</th>
<th>j0y</th>
<th>j0z</th>
<th>j1x</th>
<th>j1y</th>
<th>j1z</th>
<th>j2x</th>
<th>j2y</th>
<th>j2z</th>
<th>j3x</th>
<th>j3y</th>
<th>j3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00</td>
<td>.00</td>
<td>.00</td>
<td>.01</td>
<td>.00</td>
<td>11.14</td>
<td>11.14</td>
<td>.01</td>
<td>11.14</td>
<td>11.14</td>
<td>10.00</td>
<td>10.00</td>
</tr>
</tbody>
</table>

Table 7: Rotational accelerations of members 0, 1, 2 & 3

<table>
<thead>
<tr>
<th>th0x</th>
<th>th0y</th>
<th>th0z</th>
<th>th1x</th>
<th>th1y</th>
<th>th1z</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00</td>
<td>.00</td>
<td>.00</td>
<td>4.7</td>
<td>4.7</td>
<td>4.7</td>
</tr>
</tbody>
</table>

Table 7a: Rotational accelerations continued.

<table>
<thead>
<tr>
<th>th2x</th>
<th>th2y</th>
<th>th2z</th>
<th>th3x</th>
<th>th3y</th>
<th>th3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.7</td>
<td>4.7</td>
<td>4.7</td>
<td>4.7</td>
<td>4.7</td>
<td>4.7</td>
</tr>
</tbody>
</table>

Table 8: Polar moment of inertia and shear modulus of elasticity and weights of links 0, 1, 2, 3

<table>
<thead>
<tr>
<th>xj1</th>
<th>xj2</th>
<th>x61</th>
<th>x62</th>
<th>w0</th>
<th>w1</th>
<th>w3</th>
<th>w3</th>
</tr>
</thead>
<tbody>
<tr>
<td>.78</td>
<td>.78</td>
<td>.00</td>
<td>.00</td>
<td>.00</td>
<td>.00</td>
<td>.00</td>
<td>.00</td>
</tr>
</tbody>
</table>

Table 9: Applied forces at the centers of gravity of each link

<table>
<thead>
<tr>
<th>dGg0x</th>
<th>dGg0y</th>
<th>dGg0z</th>
<th>dGglx</th>
<th>dGglz</th>
<th>dGg2x</th>
<th>dGg2y</th>
<th>dGg2z</th>
<th>dGg3x</th>
<th>dGg3y</th>
<th>dGg3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00</td>
<td>.00</td>
<td>.00</td>
<td>8.69</td>
<td>8.69</td>
<td>8.69</td>
<td>4.44</td>
<td>4.44</td>
<td>4.44</td>
<td>25.12</td>
<td>25.12</td>
</tr>
</tbody>
</table>
Table 10 applied torques at the centers of gravity of each link

<table>
<thead>
<tr>
<th>dTg0x</th>
<th>dTg0y</th>
<th>dTg0z</th>
<th>dTg1x</th>
<th>dTg1y</th>
<th>dTg1z</th>
<th>dTg2x</th>
<th>dTg2y</th>
<th>dTg2z</th>
<th>dTg3x</th>
<th>dTg3y</th>
<th>dTg3z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.05</td>
<td>52.36</td>
<td>52.36</td>
<td>0.05</td>
<td>52.36</td>
<td>52.36</td>
<td>47.00</td>
<td>47.00</td>
<td>47.00</td>
</tr>
</tbody>
</table>

Table 1 shows the force and moment reactions at the joints

<table>
<thead>
<tr>
<th>-R0x</th>
<th>-R0y</th>
<th>-R0z</th>
<th>-R0x</th>
<th>-R0y</th>
<th>-R0z</th>
</tr>
</thead>
<tbody>
<tr>
<td>38.25</td>
<td>38.25</td>
<td>38.25</td>
<td>2416.44</td>
<td>-2446.08</td>
<td>196.78</td>
</tr>
<tr>
<td>Fab</td>
<td>R1x</td>
<td>R1y</td>
<td>R1z</td>
<td>M1x</td>
<td>M1y</td>
</tr>
<tr>
<td>327.88</td>
<td>56.03</td>
<td>-196.34</td>
<td>38.25</td>
<td>735.58</td>
<td>-2262.69</td>
</tr>
<tr>
<td>Fde</td>
<td>R2x</td>
<td>R2y</td>
<td>R2z</td>
<td>M2x</td>
<td>M2y</td>
</tr>
<tr>
<td>-130.65</td>
<td>130.91</td>
<td>112.01</td>
<td>29.56</td>
<td>-214.24</td>
<td>-670.01</td>
</tr>
<tr>
<td>R3x</td>
<td>R3y</td>
<td>R3z</td>
<td>M3x</td>
<td>M3y</td>
<td>M3z</td>
</tr>
<tr>
<td>25.12</td>
<td>25.12</td>
<td>25.12</td>
<td>47.00</td>
<td>-4.49</td>
<td>98.49</td>
</tr>
</tbody>
</table>

Table 2 shows the global deflections in the x-y-z coordinates

<table>
<thead>
<tr>
<th>joint</th>
<th>Deflection X</th>
<th>Deflection Y</th>
<th>Deflection Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-0.07890701</td>
<td>0.13667300</td>
<td>0.21470840</td>
</tr>
<tr>
<td>3</td>
<td>-0.06985474</td>
<td>0.16155820</td>
<td>0.40106180</td>
</tr>
</tbody>
</table>

Table 3 shows the global rotational angles in the x-y-z coordinates

<table>
<thead>
<tr>
<th>joint</th>
<th>Rotational X</th>
<th>Rotational Y</th>
<th>Rotational Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-0.00546619</td>
<td>0.00352811</td>
<td>0.00433281</td>
</tr>
<tr>
<td>3</td>
<td>-0.00506967</td>
<td>0.00429428</td>
<td>0.00099930</td>
</tr>
</tbody>
</table>