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Pushkin Kachroo
University of Nevada, Las Vegas, pushkin@unlv.edu

Masayoshi Tomizuka
University of California at Berkeley

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Pushkin Kachroo and *Masayoshi Tomizuka*
Department of Mechanical Engineering
University of California at Berkeley
Berkeley, California 94720

Abstract

Sliding mode control is a standard approach to tackle the parametric and modeling uncertainities of a non-linear system. However, the robust control obtained by using sliding mode has a price, which is the high frequency chattering encountered during the digital implementation of the control. The idea of introducing a boundary layer around the switching surface and approximating a continuous control inside it has been extended for different kinds of systems. The systems should be analyzed to ascertain which continuous law is appropriate within the boundary layer. In this paper the effect of various continuous control approximations within the boundary layer to chattering and error convergence in different systems is studied.

1. Introduction

Sliding mode control originated in Soviet Union some thirty years ago. It was mostly studied for the control of linear systems as a variable structure formulation (e.g [1] and [4]). The major drawback in the classical sliding mode control is high chattering across the switching surface and consequently a high control action.

Chattering is most of the times undesirable because it can excite the unmodeled high frequency dynamics of the system. It is also undesirable if the control variable is expensive, in which cases a high control activity should be avoided.

One approach to reduce chattering is to introduce a boundary layer around the switching surface and use a continuous control within the boundary layer, keeping the boundary layer attractive outside (see [2] and [3]). In the method proposed by Slotine, et al., ϕ, the boundary layer thickness is made varying in order to take advantage of the system bandwidth. In order to utilize the bandwidth of the system it is not necessary to vary ϕ. The same precision is obtained by using an alternate method proposed. An example is given of a system where using the sat function does not work and hence a different continuous approximation law is proposed. This shows that systems must be classified to facilitate the use of various continuous control laws within the boundary.

2. Background

Let a nonlinear system be defined as

$$x^n = g(\mathbf{x},t) + c(\mathbf{x},t)u \quad (1)$$

$\mathbf{x}$ is the state vector, u is the control input and x is the output state. The other states in the state vector are the higher order derivatives of x up to the (n-1)st order. Here, g and c are generally nonlinear functions of time and the states.

Sliding mode control is basically a robust feedback linearization where the linearization is obtained by introducing a time varying surface s(t) as

$$s = \left(\frac{d}{dt} + b\right)^{n-1}\tilde{x} \quad (2)$$

where b is a constant, taken to be the bandwidth of the system, $\tilde{x} = x - x_d$, where x_d is the desired state.

Keeping s within a value π is equivalent to keeping the ith derivative of the state within $(2b)^i\xi$, where $\xi = \Pi/b^{n-1}$. Condition

$$\frac{1}{2}\frac{d}{dt}(s^2) \le -\psi|s|,\ y > 0 \quad (3)$$

forces the trajectories to point towards s = 0 when |s| > 0.

Consider a second order system

$$\ddot{x} = g + u \quad (4)$$

where g is generally non-linear and/or time varying and u is the control input x being the state to be controlled to follow a desired trajectory x_d. Also, $\hat{g}$ is the estimate of g so that $|\hat{g} - g| \le G$, then defining

$$s = \left(\frac{d}{dt} + b\right)^{n-1}\tilde{x} \quad (5)$$

$$\hat{u} = -\hat{g} + \ddot{x}_d - b\dot{\tilde{x}} \quad (6)$$

and taking

$$u = \hat{u} - k\,\mathrm{sgn}(s) \quad (7)$$

and

$$k = G + \psi \quad (8)$$

ensures the equation(3) condition.

Consider the system

$$\ddot{x} = g + cu \quad (9)$$

where c is bounded as $0 \le c_{min} \le c \le c_{max}$. We can take estimate of c as

$$\hat{c} = \sqrt{c_{min}c_{max}} \quad (10)$$

Define

$$a = \sqrt{c_{max}/c_{min}} \quad (11)$$

Taking

$$u = \hat{c}^{-1}[\hat{u} - k\ \mathrm{sgn}(s)] \quad (12)$$

and

$$k \ge a(G + \psi) + (\alpha - 1)|\hat{u}| \quad (13)$$

ensures the equation(3) condition.

3. Methodology

To remove chattering, a thin boundary of thickness ϕ is defined around the switching surface s,

so that $B(t) = \{x, |s| < \phi\}$. (14)

To gurantee that all the trajectories outside the boundary layer are attracted towards the boundary the following condition should be satisfied.

$$\frac{1}{2}\frac{d}{dt}s^2 \le (\dot{\phi} - \psi)|s| \quad (15)$$

For the system of the form of equation(4) we can define

$$\bar{k}(x) = k(x) - \dot{\phi} \quad (16)$$

so that the control law becomes (see [2] for details)

$$u = \hat{u} - \bar{k}(x)\mathrm{sat}(s/\phi) \quad (17)$$

Near the boundary approximate dynamics can be approximated as

$$\dot{s} = -k(x_d)s/\phi + (-\Delta g(x_d) + o(\xi)) \quad (18)$$

where

$$\Delta g = \hat{g} - g \quad (19)$$

A first order filter of bandwidth b can be obtained by taking

$$k(x_d)/\phi = b \quad (20)$$

so that the variation of Φ with time can evaluated from

$$\dot{\phi} = -b\phi + k(x_d). \quad (21)$$

Fig.1 shows the filter, where D is the laplace operator d/dt.

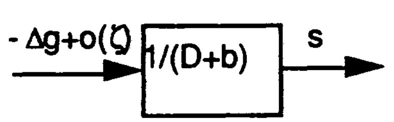

Figure 1

The same filter can be obtained by using a constant width boundary so that

$$\dot{\phi} = 0 \quad (22)$$

so that equation(3) is satisfied instead of equation(15) and

$$\bar{k}(x) = k(x) \quad (23)$$

Take control law as

$$u = \hat{u} - k(x)\ \mathrm{push}(as/\phi) \quad (24)$$

where

$$\mathrm{push}(as/\Phi) = as/\Phi \quad \text{for } |s| < \Phi \quad (25)$$

$$\mathrm{push}(as/\Phi) = \mathrm{sgn}(s) \quad \text{otherwise} \quad (26)$$

so that within the boundary we get

$$\dot{s} = -k(x_d)as/\phi + (-\Delta g(x_d) + o(\xi)) \quad (27)$$

and to utilize bandwidth we need only to design for the value of a for a fixed ϕ.

Taking

$$a = \frac{b\phi}{k(x_d)} \quad (28)$$

gives the same filter as shown in Figure 4.

Now, for the system given by equation(9) and the values used as given by equations(10) and (11), we can still use a fixed ϕ to obtain a filter by taking

$$\frac{k(x_d)a}{\phi}(c/\hat{c})_{max} = b \quad (29)$$

so that

$$a = \frac{b\phi}{\alpha k(x_d)} \quad (30)$$

Implementation of this scheme is simpler because ϕ is constant and the design of a is straightforward.

Now if the input $-\Delta g(x_d) + o(\xi)$ to the first order filter is a step input, then s doesn't go to zero but has a steady state error. Similarly if the input term is a ramp then s keeps increasing and if sat function with a varying boundary is being used the boundary might also keep increasing. When a fixed boundary width is used, s increases until it hits the boundary layer and once it is out, it is forced back inwards bacause of the attractiveness of the boundary layer causing chattering on the boundary as shown in Fig.2. If the magnitude of k is not large, then the s trajectory will stabilize either on the boundary or chatter on the lower boundary as when using the push function or might stabilize within the boundary for instance while using the sat function with a fixed boundary, for in that case the dynamics of the filter are not linear time invariant inside the boundary.

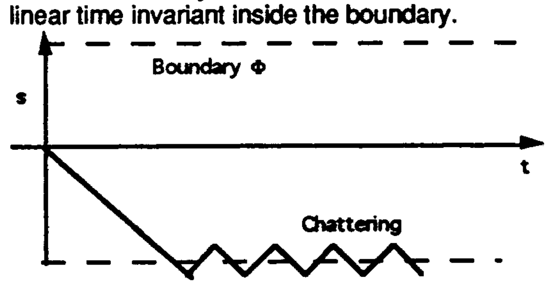

Figure 2

To overcome the step input of the filter of Fig.1 we need to introduce an integral action within the boundary layer i.e when the Laplace tranform of $-\Delta g(x_d) + o(\xi)$ is m/D, we modify the control to

$$u = \hat{u} - k(x)\ \mathrm{ipush}(a,p,s/\phi) \qquad (31)$$

where

$$\mathrm{ipush}(a,p,s/\phi) = as/\phi + \frac{p}{\phi}\int_0^t s\,dt \quad \text{for } |s| < \phi$$

$$\mathrm{ipush}(a,p,s/\Phi) = \mathrm{sgn}(s) \quad \text{otherwise}$$

The dynamics obtained by this in the boundary can be approximated as before as

$$\dot{s} = -\bar{k}(x_d)as/\phi - \bar{k}(x_d)\frac{p}{\phi}\int_0^t s\,dt + (-\Delta g(x_d) + o(\xi))$$

(32)

To design for the values of a and p we have a choice to make. We can either have a fixed boundary and take

$$a = 2b\phi/\bar{k}(x_d) \qquad (33)$$

and p = b/2. Notice that equation(23) is valid here. Alternately, we can also have a varying boundary by taking a = 1,

$\bar{k}(x_d)/\phi = 2b$ and p = b/2 to get

$$\dot{\phi} + 2b\phi = \bar{k}(x_d) \qquad (34)$$

The filter is the same for the constant and the time varying F and is shown in the Fig.3.

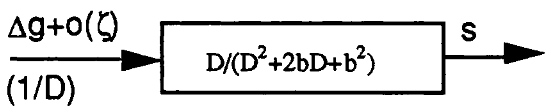

Figure 3

Extending the argument in the same vein, if the filter input has a term of the form m/D^n, we introduce an integral of order n and for the sake of this paper classify it as a system of form-n. For instance, a system of form-3 would have $\Delta f = -2.5x^2$ and $x_d = 0.23t$ and so in the boundary the control law should have integrals upto the third order.

Similarily, for the system of equation (9), the equation for $\dot{s}$ obtained by using the control law of equation (12) is

$$\dot{s} = (g - c\hat{c}^{-1}\hat{g}) + (1 - c\hat{c}^{-1})(-\ddot{x}_d + \lambda\dot{\tilde{x}}) - c\hat{c}^{-1}k\ \mathrm{sgn}(s)$$

(35)

which can be written as

$$\dot{s} = -c\hat{c}^{-1}k\ \mathrm{sgn}(s) - i(x) \qquad (36)$$

where

$$i(x) = -(g - c\hat{c}^{-1}\hat{g}) - (1 - c\hat{c}^{-1})(-\ddot{x}_d + \lambda\dot{\tilde{x}}) \qquad (37)$$

Here also because x is close to x_d, the input to the filter, similar to the one explained before is $-i(x_d)$ and therefore its form should be checked.

4. Simulation

Consider the system given by equation(4) with the parameters given by

$$g = -2.00;\ \hat{g} = -1.0; G = 1.01;\ \psi = 0.1; \quad b = 20;\ x_d = \sin(\Pi t/2) \qquad (38)$$

Notice that this system is of form-1. To reduce chattering, when a time varying boundary is used and the control law of equation (17) is applied, a steady state error is obtained in the value of s and the output error as shown in Fig.4.

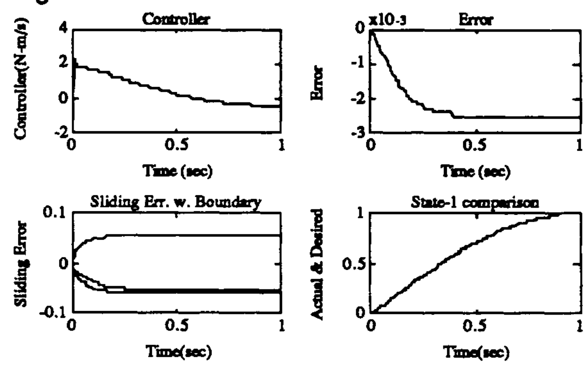

Figure 4

Using the same controller as described above on the same system with parameters given by

$$g = -2.00t;\ \hat{g} = -1.0t; G = 1.01t;\ \psi = 0.1; \quad b = 20;\ x_d = \sin(\Pi t/2) \qquad (39)$$

an increasing s is obtained. This system is of the form-2. Here the boundary is also increasing linearly so that s never comes out of the boundary, which in this case proves to be destabilizing.

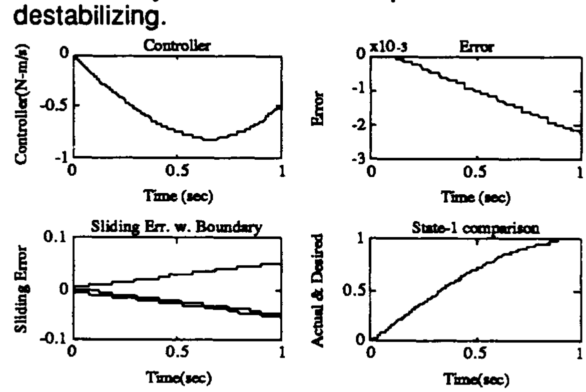

Figure 5

The chattering surface is shifted from 0 to the boundary, when the control law of eq. (24) is applied to the form-1 system of eq.(38). The boundary here is kept at 0.02.

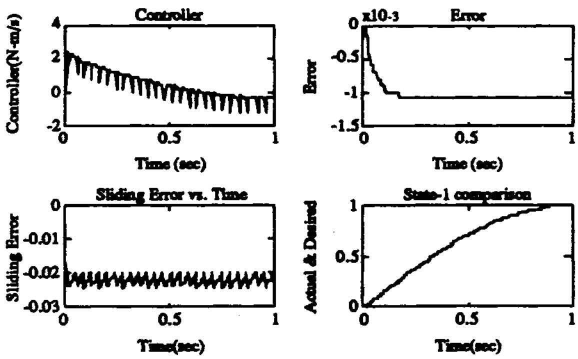

Figure 6

Finally the result of using the control law as proposed by equation (31) to form-1 system of equation (38) is shown below. This control law makes the output error as well as s go to zero.

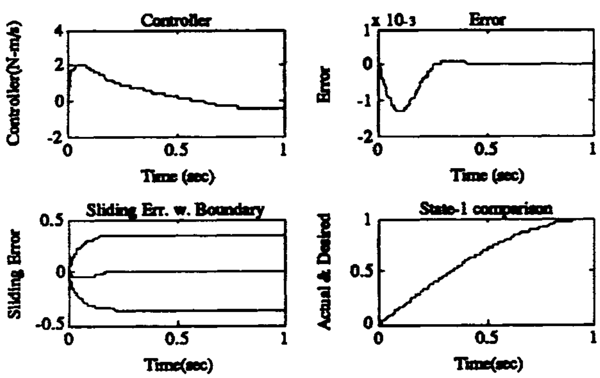

Figure 7

5. Conclusions

Various control laws are proposed within the boundary layer for different kinds of systems. Systems should be analyzed in order to decide which continuous control law should be used inside the boundary layer around the switching surface in order to reduce chattering and the same time drive the output error to zero. Specifically, for a system of the kind given by equation (4), function $\Delta g(d)$, where Δg is defined by equation (19) and d is the desired trajectory, and for a system given by equation (9), i(d) defined by equation (37) indicates the control law to be used. Finally the systems are classified as being of different forms, so that once the form of a system is identified the specific control for that form can be used.

References

[1] FILLIPOV, A. F., 1964, Differential equations with discontinuous right hand sides, *Mathematicheskii Sbornek*, 51, 1 (1960), in Russian. Translated in English, Am. *Math. Soc. Trans.*, 62, 199.

[2] SLOTINE, J.-J. E. and LI, WEIPING, 1991, *Applied Nonlinear Control*, Prentice Hall, New Jersey.

[3] SLOTINE, J.-J. E. and SASTRY S. S., 1983, Tracking Control of Nonlinear Systems Using Sliding Surfaces With Applications to Robot Manipulators, *Int. J. Control*, 39, 2.

[4] UTKIN, V. I., 1978, *Sliding Modes and Their Application to Variable Structure Systems*, MIR Publishers, Moscow.