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SLIDING MODE FOR USER EQUILIBRIUM DYNAMIC TRAFFIC ROUTING CONTROL

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ABSTRACT

This paper presents a solution to the user equilibrium Dynamic Traffic Routing (DTR) problem for a point diversion case using feedback control methodology. The sliding mode control technique which is a robust control methodology applicable to nonlinear systems in canonical form is employed to solve the user equilibrium DTR problem. The canonical form for this problem is obtained by using feedback linearization technique, and the uncertainties of the system are countered by using sliding mode principle. Simulation results show promising results.

1. INTRODUCTION

Real-time control of point diversion is an important topic as part of an overall incident management process. The advent of Intelligent Transportation Systems (ITS) has made clear the need for real time control for diversion. Researchers have used expert systems [1, 2], mathematical programming and more recently feedback control techniques [4, 5] for solving this real-time diversion problem. The method of "rolling horizon" which uses finite horizon nonlinear optimization technique at different sampling times to achieve feedback configuration has also been tried as one of the solution approaches [3, 6, 7]. Recently some feedback control methodologies have been designed to address the Dynamic Traffic Assignment and DTR problem. [10-13]. The feedback linearization technique works on exact cancellation [12]. This paper enhances that work by using sliding mode control, which takes care of a class of uncertainties in the system.

2. SYSTEM DYNAMICS

Many researchers have studied and designed optimal open loop controllers utilizing space and time discretized models of traffic flow [8, 9, 14]. In order to utilize the various linear and nonlinear [15 - 16] control techniques available for lumped parameter systems, the distributed parameter model of the original hydrodynamic model of traffic flow is space discretized [14].

Consider the following traffic model..

$$\frac{d}{dt}\rho_i = \frac{1}{\delta_i}[q_i(t) - q_{i+1}(t) + r_i(t) - s_i(t)], \quad i = 1,2,\ldots n \qquad (1)$$

$$q(t) = \rho(t)\,v(t) \qquad (2)$$

$$v = v_f\left(1 - \frac{\rho}{\rho_{max}}\right) \qquad (3)$$

Here, $r_i(t)$ and $s_i(t)$ terms indicate the on-ramp and off-ramp flows, $\rho(t)$ is the density of the traffic as a function of x, and time t, and $q(t)$ is the flow at given x, and t, v_f is the free flow speed, and ρ_{max} is the jam density. Equation (1) and the output equations (4) give the mathematical model for a highway, which can be represented in a standard nonlinear state space form for control design purposes.

$$y_j = g_j(\rho_1, \rho_2, \ldots, \rho_n), \quad j = 1,2,\ldots,p, \qquad (4)$$

The standard state space form is

$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{f}[\mathbf{x}(t), \mathbf{u}(t)],$$
$$\mathbf{y}(t) = \mathbf{g}[\mathbf{x}(t), \mathbf{u}(t)],$$
$$\mathbf{x}(0) = \mathbf{x}_0, \qquad (5)$$

where $\mathbf{x} = [\rho_1, \rho_2, \ldots, \rho_n]^T$ and $\mathbf{u}(t) = q_0(t)$.

3. FEEDBACK CONTROL FOR THE TRAFFIC

In the discretized traffic flow model, the freeway is divided into sections with aggregate traffic densities. Sensors are used to measure variables such as densities, traffic flow and traffic average speeds in these sections, which can be used by

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the feedback controller to give appropriate commands to actuators like VMS, HAR, etc.

3.1 User Equilibrium Formulation of the DTR Problem We present here a DTR formulation for the two alternate routes problem. The two routes are divided into n_1, and n_2 sections respectively. For simplicity, we are considering static velocity relationship, and ignoring the effect of downstream flow. Hence, the model used is

$$\frac{d}{dt}\rho_{i,j} = \frac{1}{\delta_i}[q_{i,j-1}(t) - q_{i,j}(t)], \tag{6}$$

$$(i,j) = ((1,1),(1,2),\ldots,(1,n_1),(2,1),(2,2),\ldots,(2,n_2))$$

with relationships (2) and (3). The control input is given by

$$\beta(t)U(t) = q_{1,0}(t), \quad 0 \le \beta \le 1, \tag{7}$$

$$(1-\beta(t))U(t) = q_{2,0}(t)$$

The flow U(t) is measured as a function of time, and the splitting rate $\beta(t)$ is the control input. The output measurement could be the full state vector, i.e., vector of flows of all the sections, or a subset of that. The control problem can be stated as: find $\beta_o(t)$, the optimal $\beta(t)$, which minimizes

$$J(\beta) = \int_0^{t_f} [\sum_{i=1}^{m} \chi(\rho_i) - \sum_{m+1}^{m+p} \chi(\rho_j)]^2 dt \tag{8}$$

where $\chi(.,.)$ is the travel time function and t_f is the final time. The controller should provide

$$Lt_{t\to\infty}[\sum_{i=1}^{m} \chi(\rho_i) - \sum_{m+1}^{m+p} \chi(\rho_j)] \to 0 \tag{9}$$

and some transient behavior characteristics like some specified settling time, percent overshoot, etc.

For the generalized case with n alternate route problem we have:

<u>Problem</u>: Find β_o^i, i=1,2,...,n, which minimize

$$J(\beta_0^i, i=1,2,..n) = \int_0^{t_f} [\sum_{k,p} \{\sum_{i=1}^{n_k} \chi(\rho_{i,k}) - \sum_{j=1}^{n_p} \chi(\rho_{j,p})\}^2 dt \tag{10}$$

(k=1,2,...,n, p=1,2,...,n, and the summations are taken over total number of combinations of n and p, and not permutations so that (k,p)=(1,2) is considered the same as (k,p)=(2,1), and hence only one of these two will be in the summation)), or which guarantee

$$Lt_{t\to\infty}\mathbf{e} \to \mathbf{0}, \tag{11}$$

$$\text{where } \mathbf{e} = [\{\sum_{i=1}^{n_1} \chi(\rho_{i,1}) - \sum_{j=1}^{n_2} \chi(\rho_{j,2})\},\ldots,\{\sum_{i=1}^{n_k} \chi(\rho_{i,k}) - \sum_{j=1}^{n_p} \chi(\rho_{j,p})\},\ldots]$$

with some transient behavior characteristics like some specified settling time, percent overshoot, etc. for the system

$$\frac{d}{dt}\rho_{i,j} = \frac{1}{\delta_i}[q_{i,j-1}(t) - q_{i,j}(t)], \tag{12}$$

$$(i,j) = ((1,1),\ldots,(1,n_1),(2,1),\ldots,(2,n_2),\ldots,(n,1),\ldots,(n,n_n))$$

with given full and partial state observation, and input constraints

$$\sum_{i=1}^{n} q_{i,0}(t) = U(t) \text{ and } \sum_{i=1}^{n} \beta^i = 1. \tag{13}$$

4. FEEDBACK LINEARIZATION

Feedback linearizaton is an appropriate technique for developing feedback controllers for nonlinear systems similar to the DTR model described above. The feedback linearization technique is applicable to an input affine square multiple input multiple output (MIMO), system. The details on exact nonlinear decoupling technique (feedback linearization) can be found in [19-22], and is briefly summarized here for the DTR application. Let us consider the following square MIMO system:

$$\dot{x}(t) = \mathbf{f}(\mathbf{x}) + \sum_{i=1}^{p} g_i(\mathbf{x})u_i \tag{14}$$

$$y_j = h_j(\mathbf{x}) \quad j = 1,2,\ldots,p$$

This can be written in a compact form as

$$\Sigma: \begin{array}{c} \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} \\ \mathbf{y} = \mathbf{h}(\mathbf{x}) \end{array} \tag{15}$$

where, $\mathbf{x} \in R^n, \mathbf{f}(\mathbf{x}): R^n \to R^n$, $\mathbf{g}(\mathbf{x}): R^p \to R^n$, $\mathbf{u} \in R^p$, and $\mathbf{y} \in R^p$. The vector fields of $\mathbf{f}(\mathbf{x})$ and $\mathbf{g}(\mathbf{x})$ are analytic functions.

It is assumed that for the system Σ, each output y_j has a defined relative degree γ_j. The concept of relative degree implies that if the output is differentiated with respect to time γ_j times, then the control input appears in the equation. This can be succinctly represented using Lie derivatives. Definition of a Lie derivative is given below, after which the definition of relative degree in terms of Lie derivatives is stated.

Definition (Lie Derivative): Lie derivative of a smooth scalar function $h: R^n \to R$ with respect to a smooth vector field $\mathbf{f}: R^n \to R^n$ is given by $L_f h = \frac{\partial h}{\partial \mathbf{x}}\mathbf{f}$. Here, $L_f h$ denotes the Lie derivative of order zero. Higher order Lie derivatives are given by $L_f^i h = L_f(L_f^{i-1}h)$.

Definition (Relative Degree): The output y_j of the system Σ has a relative degree γ_j if, $\exists$ an integer, s.t.

$L_{g_i} L_f^{\ell} h(\mathbf{x}) \equiv 0 \quad \forall \ell < \gamma_j - 1, \forall 1 \le i \le p, \forall x \in U$, and $L_{g_i} L_f^{\gamma_j - 1} h(\mathbf{x}) \neq 0$. $U \subset R^n$ which is in a given neighborhood of the equilibrium point of the system $\sum$. The total relative degree of the system r, is defined to be the sum of the relative degrees of all the output variables, i.e., $r = \sum_{j=1}^{p} \gamma_j$. By successively taking the Lie derivatives of each of the output variables up to their respective relative degrees, we obtain

$$\begin{bmatrix} y_1^{\gamma_1} \\ y_2^{\gamma_2} \\ \cdot \\ \cdot \\ \cdot \\ y_1^{\gamma_p} \end{bmatrix} = \begin{bmatrix} L_f^{\gamma_1} h_1(\mathbf{x}) \\ L_f^{\gamma_2} h_2(\mathbf{x}) \\ \cdot \\ \cdot \\ \cdot \\ L_f^{\gamma_p} h_p(\mathbf{x}) \end{bmatrix} + \begin{bmatrix} L_{g_1} L_f^{\gamma_1 - 1} h_1(\mathbf{x}) & \dots & L_{g_p} L_f^{\gamma_1 - 1} h_1(\mathbf{x}) \\ L_{g_p} L_f^{\gamma_2 - 1} h_1(\mathbf{x}) & \dots & L_{g_p} L_f^{\gamma_2 - 1} h_1(\mathbf{x}) \\ & \cdot & \\ & \cdot & \\ & \cdot & \\ L_{g_1} L_f^{\gamma_p - 1} h_1(\mathbf{x}) & \dots & L_{g_p} L_f^{\gamma_p - 1} h_1(\mathbf{x}) \end{bmatrix} u \tag{16}$$

This can be written as

$$\mathbf{y}^{\gamma} = \mathbf{A}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{u} \tag{17}$$

where

$$\mathbf{y}^{\gamma} = \begin{bmatrix} y_1^{\gamma_1} & y_2^{\gamma_2} & \dots & y_1^{\gamma_p} \end{bmatrix}^T \tag{18}$$

$$\mathbf{A}(\mathbf{x}) = \begin{bmatrix} L_f^{\gamma_1} h_1(\mathbf{x}) & L_f^{\gamma_2} h_2(\mathbf{x}) & \dots & L_f^{\gamma_p} h_p(\mathbf{x}) \end{bmatrix}^T \tag{19}$$

$$\mathbf{B}(\mathbf{x}) = \begin{bmatrix} L_{g_1} L_f^{\gamma_1 - 1} h_1(\mathbf{x}) & \dots & L_{g_p} L_f^{\gamma_1 - 1} h_1(\mathbf{x}) \\ L_{g_p} L_f^{\gamma_2 - 1} h_1(\mathbf{x}) & \dots & L_{g_p} L_f^{\gamma_2 - 1} h_1(\mathbf{x}) \\ & \cdot & \\ & \cdot & \\ & \cdot & \\ L_{g_1} L_f^{\gamma_p - 1} h_1(\mathbf{x}) & \dots & L_{g_p} L_f^{\gamma_p - 1} h_1(\mathbf{x}) \end{bmatrix} \tag{20}$$

If the decoupling matrix $\mathbf{B}(\mathbf{x})$ is invertible, then we can use the feedback control law (21) to obtain the decoupled dynamics (22).

$$\mathbf{u} = (\mathbf{B}(\mathbf{x}))^{-1}[-\mathbf{A}(\mathbf{x}) + \mathbf{v}] \tag{21}$$

$$\mathbf{y}^{\gamma} = \mathbf{v} \tag{22}$$

where $\mathbf{v} = \begin{bmatrix} v_1 & v_2 & \dots & v_p \end{bmatrix}^T$

The vector $\mathbf{v}$ can be chosen to render the decoupled system (17) stable with desired transient behavior.

5. SLIDING MODE CONTROL

The point diversion problem, as will be illustrated in the following sections, has a special structure. The special structure is such that the matrix $\mathbf{B}(\mathbf{x})$ is diagonal, and the relative degree of each output in (16) is one. This special structure will be utilized to design sliding mode control law for the diversion law. We can also analyze and design control when there are uncertainties in the system, which is bound to be happen in practice. When there are uncertainties in the system, the control law (21) can not be directly utilized, but it will have to be modified.

Let a single input nonlinear system be defined as

$$x^{(n)} = f(\mathbf{x},t) + b(\mathbf{x},t)u(t) \tag{23}$$

Here, $\mathbf{x}(t) = [x(t)\ \dot{x}(t) \dots x^{(n-1)}]^T$ is the state vector, u is the control input and x is the output state. The superscript n on x(t) signifies the order of differentiation. A time varying surface S(t) is defined by equating variable s(t) to zero, where

$$s(t) = \left(\frac{d}{dt} + \gamma\right)^{n-1} \tilde{x}(t) \tag{24}$$

Here, γ is a design constant and $\tilde{x}(t) = x(t) - x_d(t)$ is the error in the output state where $x_d(t)$ is the desired output state. The switching condition

$$\frac{1}{2}\frac{d}{dt}(s(t)^2) \le -\eta |s(t)|,\ \eta > 0 \tag{25}$$

makes the surface S(t) an invariant set. All trajectories outside S(t) point towards the surface, and trajectories on the surface remain there. It takes finite time to reach the surface S(t) from outside. Moreover the definition (24) implies that once the surface is reached, the convergence to zero error is exponential. Chattering is caused by non-ideal switching around the switching surface. Delay in digital implementation causes s(t) to pass to the other side of the surface, which in turn produces chattering.

Consider a second order system

$$\ddot{x}(t) = f(\mathbf{x},t) + u(t) \tag{26}$$

where $f(\mathbf{x},t)$ is generally nonlinear and/or time varying and is estimated as $\hat{f}(\mathbf{x},t)$, u(t) is the control input, and x(t) is the output, desired to follow trajectory $x_d(t)$. The estimation error on $f(\mathbf{x},t)$ is assumed to be bounded by some known function $F=F(\mathbf{x},t)$, so that

$$|\hat{f}(\mathbf{x},t) - f(\mathbf{x},t)| \le F(\mathbf{x},t). \tag{27}$$

We define a sliding variable according to (4)

$$s(t) = \left(\frac{d}{dt} + \gamma\right)\tilde{x}(t) = \dot{\tilde{x}}(t) + \gamma\tilde{x}(t) \tag{28}$$

The next two theorems give controls that guarantee the satisfaction of the switching condition (25).

Theorem 1 : For a single input second order nonlinear lumped parameter system, affine in control, given by (26), where $\mathbf{x} \in R^2$, $u \in R$, $x \in R$, and $f : R^2 \times R^+ \to R$, choosing control law as:

$$u(t) = \hat{u}(t) - k(\mathbf{x},t)\ \mathrm{sgn}(s(t)) \text{ with } k(\mathbf{x},t) = F(\mathbf{x},t) + \eta, \text{ and } \hat{u}(t) = -\hat{f} + \ddot{x}_d - \gamma\dot{\tilde{x}} \tag{29}$$

satisfies the invariant condition of Equation (25).

Results for a second order system with uncertain control gain are given by the following theorem. *Theorem 2* : For a single input second order nonlinear lumped parameter system, affine in control, given by

$$\ddot{x}(t) = f(\mathbf{x},t) + b(\mathbf{x},t)u(t) \text{ where} \tag{30}$$

$$0 \leq b_{min}(\mathbf{x},t) \leq b(\mathbf{x},t) \leq b_{max}(\mathbf{x},t)$$

where $\mathbf{x} \in R^2$, $u \in R$, $x \in R$, $b : R^2 \times R^+ \to R$, and $f : R^2 \times R^+ \to R$, control law

$$u(t) = \hat{b}(\mathbf{x},t)^{-1}[\hat{u}(t) - k(\mathbf{x},t)\ \mathrm{sgn}(s(t))] \tag{31}$$

where $k(\mathbf{x},t) \geq \alpha(\mathbf{x},t)(F(\mathbf{x},t) + \eta) + (\alpha(\mathbf{x},t) - 1)|\hat{u}(t)|$ (32)

$$\hat{b}(\mathbf{x},t) = \sqrt{b_{min}(\mathbf{x},t)b_{max}(\mathbf{x},t)} \tag{33}$$

and (34)

$$\alpha(\mathbf{x},t) = \sqrt{b_{max}(\mathbf{x},t)/b_{min}(\mathbf{x},t)}$$

ensures the invariant condition of Equation (5).

6. SAMPLE PROBLEM (TWO ALTERNATE ROUTES WITH ONE SECTION)

In order to illustrate the ideas discussed above, we have designed a feedback control law for two alternate routes problem with single section each. The space discretized flow equations used for the two alternate routes are:

$$\dot{\rho}_1 = -\frac{1}{\delta_1}\left[v_{f1}\rho_1(1-\frac{\rho_1}{\rho_{m1}}) - \beta u\right] + \omega_1, \tag{35}$$

$$\dot{\rho}_2 = -\frac{1}{\delta_2}\left[v_{f2}\rho_2(1-\frac{\rho_2}{\rho_{m2}}) + \beta u - u\right] + \omega_2 \tag{36}$$

,

where ω_1 and ω_2 are the terms representing uncertainties of the system equations. We have considered a simple first order travel time function, which is obtained by dividing the length of a section by average velocity of vehicles on it. According to that, the travel time can be calculated as

$$\chi_1(k) = d_1/[v_{f1}(1-\frac{\rho_1}{\rho_{m1}})], \tag{37}$$

$$\chi_2(k) = d_2/[v_{f2}(1-\frac{\rho_2}{\rho_{m2}})], \tag{38}$$

where, d_1 and d_2 are section lengths, v_{f1} and v_{f2} are the free flow speeds of each section, and ρ_{m1} and ρ_{m2} are the maximum (jam) densities of each section. Since we need to equate the travel times in the two freeways, we take the new transformed state variable y as the difference in travel times. Differentiating the equation representing y in terms of the state variables introduces the input split factor into the dynamic equation.

The variable y is equal to the difference in the travel time on the two sections.

$$y = \frac{k_1}{(k_2 - \rho_1)} - \frac{k_3}{(k_4 - \rho_2)} \tag{39}$$

where

$$k_1 = \frac{d_1 \cdot \rho_{m1}}{v_{f1}}, k_2 = \rho_{m1}, k_3 = \frac{d_2 \cdot \rho_{m2}}{v_{f2}}, k_4 = \rho_{m2}.$$

This equation can be differentiated with respect to time to give the travel time difference dynamics.

$$\dot{y} = \frac{k_1\dot{\rho}_1}{(k_2-\rho_1)^2} - \frac{k_3\dot{\rho}_2}{(k_4-\rho_2)^2} \tag{40}$$

By substituting (35) and (36) in (40) we obtain

$$y = -\frac{k_1\left(v_{f1}\rho_1(1-\frac{\rho_1}{\rho_{m1}}) - \beta u + \delta_1\omega_1\right)}{\delta_1(k_2-\rho_1)^2} + \frac{k_3\left(v_{f2}\rho_2(1-\frac{\rho_2}{\rho_{m2}}) + \beta u - u + \delta_2\omega_2\right)}{\delta_2(k_4-\rho_2)^2} \tag{41}$$

This equation can be rewritten in the following form.

$$\dot{y} = F + G\beta + f \tag{42}$$

where

$$F = \left[-\frac{k_1 v_{f1}\rho_1}{\delta_1(k_2-\rho_1)^2}(1-\frac{\rho_1}{\rho_{m1}}) + \frac{k_3}{\delta_2(k_4-\rho_2)^2}\left((1-\frac{\rho_2}{\rho_{m2}})v_{f2}\rho_2 - u\right)\right] \tag{43}$$

$$G = \left(\frac{k_1}{\delta_1(k_2-\rho_1)^2} + \frac{k_3}{\delta_2(k_4-\rho_2)^2}\right)u \tag{44}$$

$$f = \frac{k_1\omega_1}{(k_2-\rho_1)^2} - \frac{k_3\omega_2}{(k_4-\rho_2)^2} \tag{45}$$

Hence a feedback linearization control law can be designed to cancel the non-linearities and provide the desired error dynamics. The sliding mode control law used is

$$\beta = G^{-1}\left(-F - \hat{f} - k\,\mathrm{sgn}(y)\right), \quad k \geq \phi \tag{46}$$

where ϕ is a known function which gives the error estimate bound on f. If there were explicit uncertainties on G, then we would use control law similar to (31).

7. SOLUTION FOR THE GENERALIZED DTR PROBLEM FOR MULTIPLE ROUTES WITH MULTIPLE SECTIONS

In this section, we give a generalize solution for the n alternate route DTR problem described in section 3.1. The space discretized flow equations used for the n alternate routes and n sections are given by (12) and (13). Number of sections for each alternate route I is denoted by n_i. We are considering full state observation, which is used for estimating (sensing) the travel times on the various alternate routes. The dynamics can be written as

$$\dot{\rho}_{ij} = \frac{1}{\delta_{ij}}\left[v_{fij-1}\rho_{ij-1}\left(1-\frac{\rho_{ij-1}}{\rho_{mij-1}}\right) - v_{fij}\rho_{ij}\left(1-\frac{\rho_{ij}}{\rho_{mij}}\right)\right] + \omega_{ij} \quad (47)$$

$$\text{when}\,(i,j) = ((1,2),\ldots,(1,n_1),(2,2),\ldots,(2,n_2),\ldots,(n,2),\ldots,(n,n_n))$$

$$\dot{\rho}_{ij} = \frac{1}{\delta_{ij}}\left[\beta_i u - v_{fij}\rho_{ij}\left(1-\frac{\rho_{ij}}{\rho_{mij}}\right)\right] + \omega_{ij} \;\; \text{when}\,(i,j) = ((1,1),(2,1)\ldots(n,1)) \quad (48)$$

We have considered a simple first order travel time function, which is obtained by dividing the length of a section by average velocity of vehicles on it. According to that, we approximate travel time for a route as

$$\chi_i(t) = \sum_{j=1}^{n_i} \frac{d_{ij}}{v_{fij}\left(1-\frac{\rho_{ij}}{\rho_{mij}}\right)} \quad (49)$$

The system can be written in the standard nonlinear input affine form

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x},t) + \mathbf{g}(\mathbf{x},t)\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{h}(\mathbf{x},t) \quad (50)$$

where

$$\mathbf{x} = [\rho_{11}\cdots\rho_{1n_1}\cdots\rho_{n1}\cdots\rho_{nn_n}]' \quad ,\mathbf{u}(t) = [\beta_1\ldots\beta_{n-1}] \quad (51)$$

The output vector is denoted by $\mathbf{y}$, and is given by:

$$\mathbf{y} = [y_1\; y_2 \cdots y_i \cdots y_{n-1}] \quad (52)$$

where,

$$y_i = \chi_{i+1}(t) - \chi_i(t) \quad (53)$$

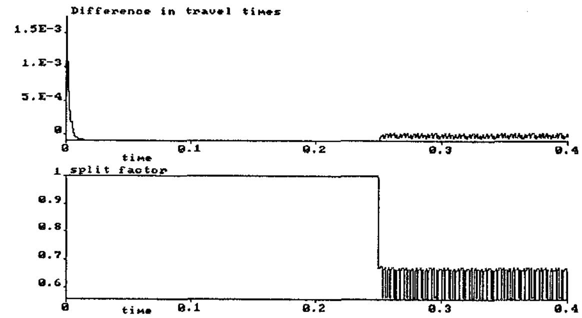
This equation can be differentiated with respect to time to give the travel time difference dynamics.

$$\dot{y}_i = \sum_{j=1}^{n_{i+1}} \frac{k_{i+1j1}\dot{\rho}_{i+1j}}{k_{i+1j2}\left(1-\frac{\rho_{i+1j}}{\rho_{mi+1j}}\right)^2} - \sum_{j=1}^{n_i} \frac{k_{ij1}\dot{\rho}_{ij}}{k_{ij2}\left(1-\frac{\rho_{ij}}{\rho_{mij}}\right)^2} \quad (54)$$

where k_{ijp} denotes a constant p=1 or 2 that belongs to section j of route i, similar to the constant k described for (39). This system is input-output square and can be solved since each output equation is decoupled and can be solved by the sliding mode control law.

8. SIMULATION

The test network consists of two alternate routes. The demand function is a constant flow of 325 vehicles per 15 minutes and it is then increased to 1000 vehicles per fifteen minutes. We have assumed full compliance of users for this sample case. The results shown in the figure below illustrate that the travel times become equal after the successful implementation of the controller developed in this paper. One drawback which is evident from the figure is the high frequency chattering encountered using the sliding mode control. To eliminate this problem, we can use the boundary layer method to replace a continuous controller inside the boundary layer [23], which essentially produces a filter for the high frequency content of the error signal.



CONCLUSIONS

In this paper, we have addressed the real-time traffic control problem for point diversion. A feedback model is developed for control purposes, and sliding mode technique is used to design this feedback controller. First, the simplest case with two alternate routes consisted of a single section each, is studied and a feedback controller using sliding mode technique is developed. Second, the case with two alternate routes with two discrete sections is analyzed and a feedback controller using sliding mode technique is also developed. Finally, the general case with multiple alternate routes divided into multiple sections is analyzed and a general solution is proposed. To illustrate the feasibility of above models, a simulation run is performed for a network topology of two alternate routes. The feedback controller developed for this test network performed well.

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