Vehicle Merging Control Design for an Automated Highway System

Pushkin Kachroo
University of Nevada, Las Vegas, pushkin@unlv.edu

Li Zhijun

VEHICLE MERGING CONTROL DESIGN FOR AN AUTOMATED HIGHWAY SYSTEM

Pushkin Kachroo
Bradley Department of Electrical and Computer Engineering
Virginia Polytechnic Institute and State University,
Blacksburg, VA 24061-0111

Zhijun Li
Center for Transportation Research,
Virginia Polytechnic Institute and State University,
Blacksburg, VA 24061-0536

Keywords Optimal Control, Sliding Mode

ABSTRACT
The merging process in an Automated Highway System (AHS) is divided into *a speed adjustment* stage and *a lane merging* stage. Three important parameters, namely, acceptability, availability and pursuability, are analyzed to characterize the AHS lane gap features for the ideal, smooth and safe merging of the ramp vehicles. Three control guidance laws, namely, linear, optimal and parabolic speed profiles, are developed to describe the desired behaviors of the merging vehicle based on the merging quality and safety. The desired states of the merging vehicle are generated through the outer loop by specified control guidance law. The tracking errors compared with desired states are eliminated by the proper design of controllers in the inner loop. Both longitudinal and lateral controllers are designed using sliding mode control theory that can handle the nonlinear and model uncertainties of the vehicle dynamics. The simulation results show encouraging results.

INTRODUCTION
There has been good deal of research in the area of AHS recently, such as Compendium of Research Summaries (1994). The merging control is an important AHS operation. Hence, its study and control design is very important. The subject of merging control under manual driving has been studied extensively by many traffic engineers in 1960s, Drew,(1968), Drew, et. al. (1965). Yang and Kurami (1993-a) proposed an automated merging system for potential application to ITS (Intelligent Transportation Systems). Yang & Kurami (1993-b) also proposed another merging control system that guides and controls the longitudinal motion of the merging vehicle to reach the target gap at the merging point. Kachroo (1993) applied the sliding mode control method to the vehicle longitudinal headway control. Hedrick, et. al. (1991) described a combined throttle/brake control algorithm designed to control vehicle space headway within a fully automated "platoon" vehicles. Peng and Tomizuka (1993) developed a lateral control system for vehicle lane keeping maneuver.

The problem statement for the control design is given as follows:
Given an acceptable gap **G** *formed by the highway lane contiguous with the ramp, with the speed* V_g *and upstream distance* D_g *to the merging point* **O** *in the highway lane at time* t_0, *a ramp merging vehicle* **M** *with current initial position* x_m *and speed* V_m *is to be driven safely, smoothly and efficiently under automatic control to the equivalent target position* **S** *with the gap speed* V_g *, in the estimated gap traveling time period* $T = t_s - t_0$. *Meanwhile the vehicle is continuously steered to stay in the center of the lane.*

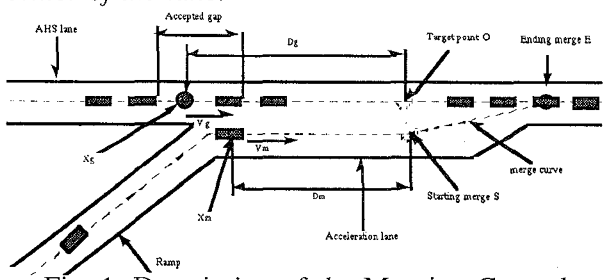

Fig. 1. Description of the Merging Control System

The various factors which play important role in merging process are "gap availability", and "gap persuability". These are defined next. The ideal acceptable gap is defined as:

$$G_{min} = 2 \cdot (L + V_g \cdot \tau) \qquad (1)$$

where, G_{min} is the minimum ideal accepted gap, τ is the total delay of the merging control system, V_g is the gap speed, and L the length of vehicle

Gap availability measures the average number of gaps available per mile for a vehicle in the ramp to be accepted in the merging process. It is a function of the actual traffic volume, given a derived AHS capacity. It has been shown by Li (1996) that if the maximum number of vehicles in a platoon is 10 and the traffic volume is less than 80% of the desired AHS capacity, the available accepted gaps per mile is greater than 27. In the possible case that no gap is available, the merging vehicle may stop on the ramp, or vehicle platoons on the highway may split to generate an acceptable gap, or one of the platoon vehicles may change the lane.

Assuming that the current gap speed V_g is constant over the merging process, the estimated time duration T of the gap is calculated by (2). The limitation of using a constant gap velocity is countered by using a feedback mechanism to drive the errors caused by varying gap velocity to zero.

$$T = \frac{D_g}{V_g} = \frac{O - x_g}{V_g} \tag{2}$$

where :

T = The predicted time duration of the gap from current point to the target point **O**, x_g = The gap position at time t_0, D_g = The distance from gap position at time t_0 to the target point **O**, O = The projection point at AHS lane of **S** at acceleration lane.

Given an acceptable gap **G** at the time t_0, the position x_g with the speed V_g, and a merging vehicle **M** at the point x_m with the speed V_m, **G** is said to be (smoothly) pursuable by **M** at t_0, if there exists a trajectory $x_m(t)$ and corresponding speed profile $V_m(t)$ satisfying:

Limitation condition:

$$V_{min} \le V_m(t) = \dot{x}_m(t) \le V_{max} \tag{3}$$

$$b_{max} \le a_m(t) = \dot{V}_m(t) \le a_{max} \tag{4}$$

Two end-condition:

$$x_m(t_0) = x_m; \quad V_m(t_0) = V_m \tag{5}$$

$$x_m(t_s) = S, \quad V_m(t_s) = V_g \tag{6}$$

Distance condition:

$$D_m = S - x_m = \int_{t_0}^{t_s} V_m(t) \cdot dt \tag{7}$$

Time condition:

$$T = t_s - t_0 = \frac{D_g}{V_g} = \frac{O - x_g}{V_g} \tag{8}$$

The variable $x_m(t)$ is called *feasible pursuit trajectory* and its corresponding speed profile $V_m(t)$ is called *feasible speed profile* . The Merging vehicle **M,** when following this trajectory, is said to have smoothly intercepted target **S.**

We define a linear speed function L_{Dl} that is a line connecting the vehicle speed V_m and the gap speed V_g. The area under L_{Dl} is called *linear critical distance* D_{cl}. A vehicle speed line L_{Vl} is also defined. The area under it is the vehicle actual distance D_m, One end of it connects the gap speed V_g, and the another end is called the *linear critical speed* V_{cl}.

$$L_{Dl} = (V_g - V_m)\frac{t - t_0}{T} + V_m \quad \forall \quad t \in [t_0, t_s] \tag{9}$$

$$D_{cl} = \int_{t_0}^{t_s} L_{Dl} \cdot dt = \frac{(V_g + V_m) \cdot T}{2} = (\frac{V_m}{V_g} + 1)\frac{D_g}{2} \tag{10}$$

$$L_{Vl} = (V_g - V_{cl})\frac{t - t_0}{T} + V_{cl} \quad \forall \quad t \in [t_0, t_s] \tag{11}$$

$$V_{cl} = (\frac{2 \cdot D_m}{D_g} - 1) \cdot V_g \tag{12}$$

Variables D_{cl} and V_{cl} serve as the desired distance and speed of the vehicle to the merging point respectively. The speed adjustment is needed for the compensation of the errors between the actual and the desired distance and speed. The gap is not pursuable when the errors of the vehicle are too large. The errors combined with limitation of the vehicle reflect the degree of pursuability of a gap. Given the gap and the vehicle current states D_m, D_g, V_m, V_g and the limitation of the vehicle states V_{max}, V_{min}, a_{max}, b_{max}, we may develop a pursuability index to determine the pursuit ability as follows:

$$\psi = \frac{e}{e_{max}} = \frac{D_m - D_{cl}}{|D_{max} - D_{cl}|} = \frac{V_{cl} - V_m}{|V_{max} - V_m|} \tag{13}$$

where:

D_{max} = Maximum allowed distance of the merging vehicle, V_{max} = Maximum or minimum allowed speed of the merging vehicle, e = The actual error of the merging vehicle, e_{max} = The maximum allowed error of the merging vehicle If $|\psi|<1$, the gap is pursuable by the vehicle.

CONTROL LAWS OF THE MERGING VEHICLE

We can design control laws in various ways. We show three different control laws in the following sections for comparative study. The *linear speed control law* in terms of the vehicle position, speed and acceleration is chosen as follows:

$$D_{desl} = (\frac{V_m}{V_g} + 1)\frac{D_g}{2} \tag{14}$$

$$V_{desl} = (\frac{2 \cdot D_m}{D_g} - 1) \cdot V_g \tag{15}$$

$$\dot{V}_{des1} = \frac{V_g - V_m}{T} - \frac{V_m - V_{des1}}{T} + \frac{V_{des1}}{V_g}\dot{V}_g \quad (16)$$

It is noteworthy that the speed error and the distance error are not independent. The relation between them is given by:

$$V_m - V_{des1} = \frac{2}{T}(x_m - x_{des1}) = -\frac{2}{T}(D_m - D_{des1}) \quad (17)$$

We can also use calculus of variation to design the control. See Fox (1997), and Stengel (1994) for introductory material to calculus of variation techniques. We consider the vehicle merging control system as a class of *simple isoperimetric problem*:.

$$\min \sigma^2 = \frac{1}{T}\int_{t_0}^{t_s}[\dot{V}_m(t) - a_{ave}]^2 dt = \frac{1}{T}\int_{t_0}^{t_s}\dot{V}_m(t)^2 dt - a_{ave}^2 \quad (18)$$

$$a_{ave} = \frac{1}{T}\int_{t_0}^{t_s}\dot{V}_m(t)\cdot dt = \frac{V_g - V_m}{T} \quad (19)$$

$$\text{s.t. } D_m = \int_{t_0}^{t_s} V_m(t)\,dt \quad (20)$$

$$V_m(t_0) = V_m, \quad V_m(t_s) = V_g$$

where:

σ = The acceleration noise

a_{ave} = The average acceleration

The acceleration noise is defined as standard deviation of the accelerations. It reflects the smoothness of the vehicle travel and is a good measure of vehicle merging quality. Thus, it serves as the objective of the merging problem. By solving the problem, the optimal speed profile is parabolic curve. The acceleration of this optimal speed profile at *t* serves as the desired acceleration. It should be noted that the desired speed and position are just V_m and D_m. In summary, we get the *optimal control guidance law* as follows:

$$D_{des2} = D_m \quad (21)$$

$$V_{des2} = V_m \quad (22)$$

$$\dot{V}_{des2} = \frac{(V_g - V_m)}{T} - \frac{3(V_m - V_{c1})}{T} \quad (23)$$

We define a parabolic speed curve L_{D3} over $[t_0, t_s]$, which is connected to V_m and V_g with $a_m(t_s)=0$ at **S**. The area under the L_{D3} which composes of a distance that merging vehicle traveled is called *parabolic critical distance* D_{c3}. We also define a parabolic speed curve L_{V3} such that the area under it comprises of the actual distance of the vehicle D_m. One end of it connects V_g with $a_m(t_s)=0$ and the other end is called *parabolic critical speed* V_{c3}:

$$L_{D3}: V_m(t) = -(V_g - V_m)[(\frac{t-t_0}{T})^2 - 2(\frac{t-t_0}{T})] + V_m \quad \forall \quad t \in [t_0, t_s] \quad (24)$$

$$D_{c3} = \int_{t_0}^{t_s} L_{D3}\cdot dt = \frac{(V_m + 2\cdot V_g)\cdot T}{3} = (\frac{V_m}{V_g} + 2)\frac{D_g}{3} \quad (25)$$

$$L_{V3}: V_m(t) = -(V_g - V_{c3})[(\frac{t-t_0}{T})^2 - 2(\frac{t-t_0}{T})] + V_{c3} \quad \forall \quad t \in [t_0, t_s] \quad (26)$$

$$V_{c3} = \frac{3\cdot D_m}{T} - 2\cdot V_g = (\frac{3\cdot D_m}{D_g} - 2)\cdot V_g \quad (27)$$

Smooth merging is anticipated if $V_m = V_{c3}$ or $D_{c3} = D_m$ over the time interval T. When $V_m \neq V_{c3}$ or $D_{c3} \neq D_m$, it is desirable to drive the vehicle from V_m to V_{c3}. Hence, D_{c1} and V_{c1} serve as the desired distance and speed for the vehicle to follow. Moreover, the derivative of V_{c3} is taken as the desired acceleration. In summary we may get the Parabolic speed control law in terms of the vehicle position, speed and acceleration:

$$D_{des3} = (\frac{V_m}{V_g} + 2)\frac{D_g}{3} \quad (28)$$

$$V_{des3} = (\frac{3\cdot D_m}{D_g} - 2)\cdot V_g \quad (29)$$

$$\dot{V}_{des3} = \frac{2(V_g - V_m)}{T} - \frac{V_m - V_{c3}}{T} + \frac{V_{c3}}{V_g}\dot{V}_g \quad (30)$$

The Parabolic control law is better than the other two control laws in describing the vehicle desired behavior because the smooth merging concept is extended by tracking the gap acceleration.

VEHICLE DYNAMIC MODELING

Basic assumptions for the vehicle and wheel dynamic modeling are: 1) The vehicle is assumed to have only three degrees of freedom namely in longitudinal, lateral, and yaw directions; 2) the effect of the road super-elevation is neglected. 3) The vehicle has front wheel steering and four wheel driving. All wheels have the same dynamic and geometric properties and the two front wheels have the same steering angles. 4) The vehicle's steering angle and the wheel lateral slip angle are small. Using the assumptions just stated, the following vehicle dynamic model can be obtained by applying Newton's law and analysis of the various forces involved.

$$\dot{V}_x = -\frac{k_x \cdot V_x^2}{M} + \Omega\cdot V_y + (\mu_x - \eta - \phi)\cdot g \quad (31)$$

$$\dot{V}_y = -\frac{2\cdot\dot{\mu}_y\cdot g}{V_x}V_y - \Omega\cdot V_x + \frac{2(\dot{\mu}_y + \mu_x - \eta)\cdot l_r\cdot g}{L}\delta \quad (32)$$

$$\dot{\Omega} = \frac{2\cdot l_f\cdot l_r\cdot M\cdot g}{I_x}(-\frac{\dot{\mu}_y}{V_x}\Omega + \frac{\dot{\mu}_y + \mu_x - \eta}{L}\delta) \quad (33)$$

$$\dot{\omega} = \frac{1}{G^2 I_e + 4I_w}[-(\mu_x - \eta)R_w\cdot M\cdot g + (G\cdot T_e - T_{bw})] \quad (34)$$

$$\dot{T}_e = K_e\cdot(P_{max}\cdot TC(\Phi) - G\cdot\omega\cdot T_e) \quad (35)$$

$$\tau_b\cdot\dot{T}_{bw} + T_{bw} = K_p\cdot R_w\cdot\mu_{x\,max}\cdot M\cdot g\cdot PC(\Psi) \quad (36)$$

where:

M = vehicle mass, L = vehicle length, I_z = vehicle inertia about **C**, I_w = wheel inertia about the wheel axle, l_f = distance from front wheel axle to **C**, l_r = distance from rear wheel axle to **C**, T_{bw} = total brake torque, T_e = total engine torque, G = gear ratio, g = acceleration due to gravity, δ = steering angle, R_w = wheel radius, I_e = engine inertia, ω = wheel angular velocity, k_x = longitudinal air resistant coefficient, η = rolling resistance coefficient, ϕ = slope ratio of the road, μ_x = longitudinal adhesion coefficient, TC(Φ) = throttle characteristic, PC(ψ) = pedal characteristic, Φ = throttle angle, ψ = pedal angle, τ_b = actuator delay, $K_b = \mu_s (\lambda = 1) / = \mu_{xmax}$, $\dot{\mu}_y$ = derivative of lateral adhesion coefficient at α=0.

The vehicle dynamic model has four longitudinal state variables V_x, ω, T_e, T_{bw}, and two lateral state variables V_y and Ω.. The longitudinal dynamics are nonlinear and lateral dynamics are linear under the small angle assumption. The system has three control inputs TC, PC, and δ.

DESIGN OF THE VEHICLE CONTROLLERS

There are two feedback loops involved in the longitudinal control for the merging process. In the outer feedback loop, the desired behavior of merging vehicle is obtained by specifying *control guidance law* based on the objectives of the control system (see the section on control laws of the merging vehicle previously discussed). In the inner feedback loop, the final control inputs to the vehicle plant are generated through proper controller design. The objective of the controller design is to eliminate the state errors of the vehicle and to track the desired states of the vehicle. The control design methodology used for the longitudinal and lateral control for the inner loop is illustrated below.

The sliding mode control is a robust feedback control approach which can be used to tackle the parameter and modeling uncertainties of a class of nonlinear systems. The vehicle longitudinal dynamic model includes four state variables. We use the multiple sliding surface method in the controller design. The output generated by one sliding surface is transmitted to the next sliding surface sequentially as the desired state so that final desired control input is obtained. Four sliding variables and corresponding surface are defined consequently:

$$s_1(t) = V_t - V_d \tag{37}$$

$$s_2(t) = \omega - \omega_d \tag{38}$$

$$s_3(t) = T_e - T_{ed} \tag{39}$$

$$s_4(t) = T_b - T_{bd} \tag{40}$$

Using the sliding mode control design method, Kachroo and Tomizuka (1994) and (1996), we obtain the feedback inputs TC_d and BC_d.

The lateral dynamics are represented by two differential equations with two state variables V_y, and Ω.. The front wheel steering angle δ is the only control input to both dynamic equations. The lateral dynamics can be expressed in the following forms:

$$\dot{V}_t = f_1(\mathbf{x}) + b_1\delta \tag{41}$$

$$\dot{\Omega}_e = f_2(\mathbf{x}) + b_2\delta \tag{42}$$

where, $f_1(\mathbf{x})$, $f_2(\mathbf{x})$, b_1, and b_2 are the corresponding transformed terms.

Combining two dynamic equations by adding one dynamic equation to the other with the positive weight ξ, we define a new state variable $Z = y_\rho + \xi\,\varphi$ which is the linear combination of y_ρ and φ. Let $f(\mathbf{x}) = f_1(\mathbf{x}) + \xi\, f_2(\mathbf{x})$, and $b = b_1 + \xi\, b_2$, we obtain:

$$\ddot{Z} = (\dot{V}_\rho + \xi \cdot \dot{\Omega}_e) = f(\mathbf{x}) + b\delta \tag{43}$$

Defining sliding variable $s(t) = \dot{Z} + \lambda \cdot Z$, we obtain the lateral control law for δ which holds the sliding condition:

$$\delta = \hat{b}^{-1} \cdot [-\hat{u}(\mathbf{x}) - k(\mathbf{x}) \cdot \mathrm{sign}(s)] \tag{44}$$

$$\hat{u}(\mathbf{x}) = -\hat{f}(\mathbf{x}) - \lambda \cdot \dot{Z} \tag{45}$$

$$k(\mathbf{x}) \geq \beta(\mathbf{x})(F(\mathbf{x}) + \eta) + (\beta(\mathbf{x}) - 1) \,|\, \hat{u}(\mathbf{x}) \,| \tag{46}$$

The sliding variable s(t) will be attracted to and remain on the surface (defined by s(t)=0) once it is on it. Finally, Z, $\dot{Z}$, and $\ddot{Z}$ will be exponentially convergent to their desired zero values. The original problem requires both y_ρ and φ to reach zero. It is possible for the case when Z=0, $y_\rho \neq 0$, and $\varphi \neq 0$. But we can show that both y_ρ and φ will be attracted to zero along the straight line specified by Z=0. Because y_ρ and φ are always opposite in sign, the vehicle will be attracted to the center of the lane in any situation if Z=0, $y_\rho \neq 0$, and $\varphi \neq 0$. If and only if vehicle touches and is tangent to the center line of the lane, both y_ρ=0 and φ =0 are obtained. Since s(t)=0 is guaranteed by the control law, Z=0 is guaranteed by linear deferential feature of the surface s(t)=0, y_ρ=0 and φ =0 is guaranteed by Z=0, we may conclude that s(t)=0 Z=0 and $y_\rho = \varphi$ =0 are all invariant sets.

SIMULATION

MATLAB was used for performing simulation studies. Simulations were performed and the system performance was compared for various initial conditions, different schemes of the control guidance laws, and different ramp curves. In this paper we present only a subset of the simulation results. Simulation results for all the cases with detailed explanation are given in [9].

Fig. 2 shows actual vehicle velocity, gap velocity, and the desired velocity obtained from the outer loop. The figure shows that for various initial conditions the vehicle is able to track the gap velocity using the linear control law. The same is shown in Fig. 3 and Fig. 4 when optimal control and parabolic control laws are used. Note that, in Fig. 4 that the final actual velocity is tangent to the gap velocity, which implies that this law also tracks gap acceleration.

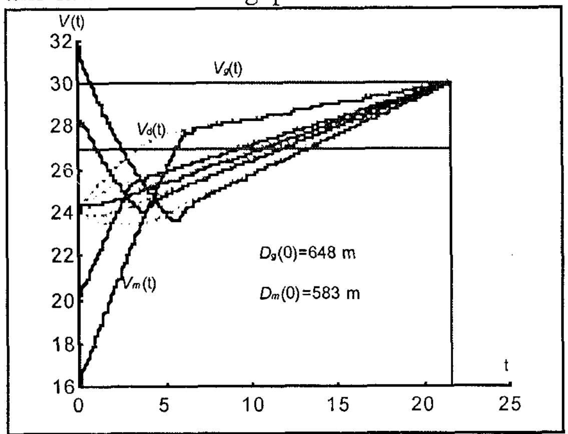

Fig. 2. The Speed Profile with Various Speed Conditions & Linear Control Law

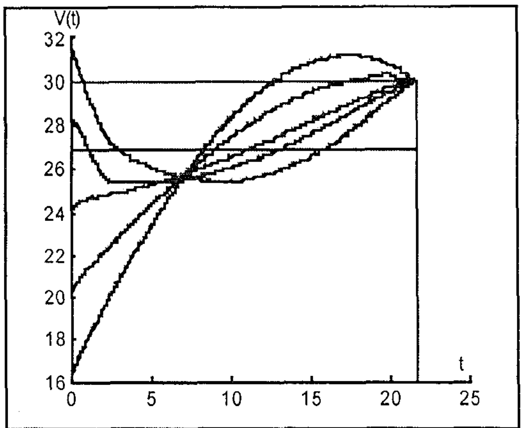

Fig. 3. The Speed Profile with Various Speed Conditions & Optimal Control Law

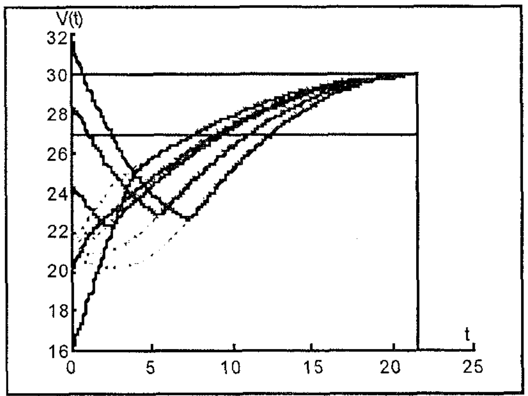

Fig. 4. The Speed Profile with Various Speed Conditions & Parabolic Control Law

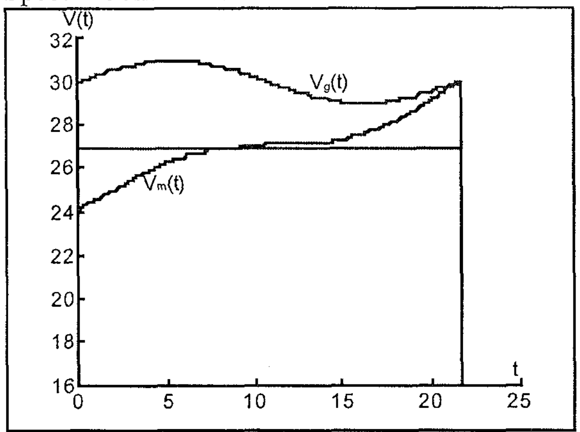

Fig..5. The Speed Profile with Gap Speed Change & Linear Control Law

Fig. 5 shows velocity tracking for one initial condition when the gap velocity is time varying and the linear control law is used. Fig. 6 shows the same for the case of parabolic control law. Finally Fig. 7 shows how the distance between the gap and the vehicle changes with time when the linear control law is used.

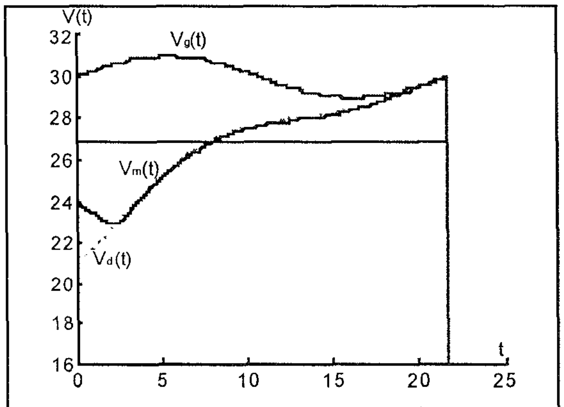

Fig..6. The Speed Profile with Gap Speed Change & Parabolic Control Law

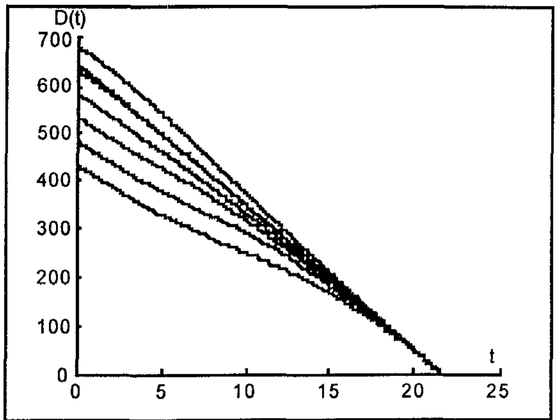

Fig..7. The Distance Profile with Various Distance Conditions & Linear Control Law

In general, the simulation results show that the merging performance of the developed system is excellent for the various initial conditions. The merging vehicle has perfect performance in tracking the gap speed. However, only the parabolic control law has the perfect tracking in terms of acceleration. The control guidance laws are robust against the disturbance of the gap speed. The pursuability index is convergent to zero in the range of [-1, 1] for various initial conditions. The merging vehicle can tightly track the road curves of linear, parabolic, circle and sinusoid forms. These results verify that both the longitudinal controller design and the lateral controller design are robust.

6. CONCLUSION

The merging control system in an AHS scenario is developed first assuming some parameters to be constant and then feedback loops are designed to account for the variation of the actual system from those assumptions. The design is used to accomplish smooth, safe and optimal behavior of the system. Simulation results provide encouraging results.

REFERENCES

Compendium of Research Summaries, *Automated Highway System Precursor Systems Analyses*, Staff members, Information Dynamics, Inc., Virginia, 1994

Drew, D. R., *Traffic Flow Theory and Control,* McGraw-Hill Book Company, New York, 1968

Drew, D. R., McCasland, W. R., and Wattleworth J. A., *Inbound Gulf Freeway Ramp Control Study 1,* Texas Transportation Inst. Res. Rept., Texas A&M Univ., College Station, 1965

Fox, C. *An Introduction to The Calculus of Variations* , Dover Publications, Inc., New York, 1987.

Hedrick, J. K., McMahon, D., Narendran, V. K., and Swaroop, D., *Longitudinal Vehicle Controller Design for IVHS Systems*, Proceedings of the American Control Conference, MA, 1991

Kachroo, P., *Nonlinear Control Strategies and Vehicle Traction Control,* Ph. D. dissertation, UC at Berkeley, CA, 1993

Kachroo, P. and Tomizuka, M., "Vehicle Traction Control and its Applications" Technical Report UIPRR-94-08, University of California at Berkeley, Institute of Transportation, 1994.

Kachroo, P. and Tomizuka, M., "Chattering Reduction and Error Convergence in the Sliding Mode Control of a Class of Nonlinear Systems", IEEE Transactions on Automatic Control, vol. 41, no. 7, July 1996.

Li, Z. J., *Vehicle Merging Control System for an Automated Highway System*, Ph. D dissertation, Dept. of Civil Engineering, Virginia Tech, VA, 1996

Peng, H. & Tomizuka, M., *Preview Control for Vehicle Lateral Guidance in Highway Automation,* ASME Journal of Dynamic Systems, Measurement and Control, Vol. 115, No. 4, 1993

Stengel, R. F., *Optimal Control and Estimation* , Dover Publications, Inc., New York, 1994.

Yang, C., et al., *A longitudinal Control Concept for Merging of Automated Vehicles*, Proc. Intelligent Vehicle 93 Symposium, Tokyo, Japan, July 1993

Yang, C. and Kurami K., *Longitudinal Guidance and Control for the Entry of Vehicle onto Automated Highways*, Proc, 32nd IEEE Conf. Decision & Control, San Antonio, Dec., 1993