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Ming Qian
Virginia Tech Transportation Institute

Pushkin Kachroo
University of Nevada, Las Vegas, pushkin@unlv.edu

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MODELING AND CONTROL OF ELECTROMAGNETIC BRAKES FOR ENHANCED BRAKING CAPABILITIES FOR AUTOMATED HIGHWAY SYSTEMS

Ming Qian
Virginia Tech Center for Transportation Research
1700 Kraft Drive, Suite 2000
Blacksburg, VA 24061
Ph. #(540)231-7509, ming@ctr.vt.edu

Pushkin Kachroo
Bradley Department of Electrical and Computer Engineering
Virginia Tech, Blacksburg, VA 24061
Ph. #(540)231-8340, pushkin@vt.edu

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ABSTRACT
In automatic highway systems, a faster response and a robust braking system are crucial part of the overall automatic control of the vehicle. This paper describes electromagnetic brakes as a supplementary system for regular friction brakes. This system provides better response time for emergency situations, and in general keeps the friction brake working longer and safer.

A modified mathematical model for electromagnetic brakes is proposed to describe their static characteristics (angular speed versus brake torque). The performance of the modified mathematical model is better than the other three models available in the literature in a least-square sense. A robust sliding mode controller is designed that achieves wheel-slip control for vehicle motion. The objective of this brake control system is to keep the wheel slip at an ideal value so that the tire can still generate lateral and steering forces as well as shorter stopping distances. The system shows the nonlinearities and uncertainties. Hence, a nonlinear control strategy based on sliding mode, which is a standard approach to tackle the parametric and modeling uncertainties of a nonlinear system, is chosen for slip control. Simulation will be performed to confirm the effectiveness of the controller.

INTRODUCTION
The principle of braking in road vehicles involves the conversion of kinetic energy into heat. This high energy conversion demands a large rate of heat dissipation so that stable performance can be maintained. By using the electromagnetic brake as supplementary retardation equipment, the friction brakes can be used less frequently and therefore never reach high temperatures. The brake linings can have a longer life span, and the potential "brake fade" problem can be avoided. In this paper, a new mathematical model for electromagnetic brakes is proposed to describe their static characteristics (angular speed versus brake torque). The performance of the new mathematical model is better than the other three models available in the literature in a least-square sense. A robust sliding mode controller is designed that achieves wheel-slip control for vehicle motion. The objective of this brake control system is to keep the wheel slip at an ideal value so that the tire can still generate lateral and steering forces as well as shorter stopping distances. The system shows the nonlinearities and uncertainties. Hence, a nonlinear control strategy based on sliding mode, which is a standard approach to tackle the parametric and modeling uncertainties of a nonlinear system, is chosen for slip control. Simulation will be performed to confirm the effectiveness of the controller.

GENERAL PRINCIPLE OF ELECTROMAGNETIC BRAKES
The conventional friction brake can absorb and convert enormous energy values, but only on the condition that the temperature of the friction contact materials is controlled. Electromagnetic brakes work in a relatively cool condition and satisfy all the energy requirements of braking at high speeds. Electromagnetic brakes can be applied separately completely without the use of friction brakes. Due to their specific method of installation, electromagnetic brakes can avoid problems that friction brakes face as we mentioned before. Typically, electromagnetic brakes have been mounted in the transmission line of vehicles [1,5]. The propeller shaft is divided and fitted with a sliding universal joint and is connected to the coupling flange on the brake. The brake is fitted into the chassis of the vehicle by means of anti-vibration mounting.

The working principle of the electric retarder is based on the creation of eddy currents within a metal disc rotating between two electromagnets,
which activate a force opposing the rotation of the disc. If the electromagnet is not energized, the rotation of the disc is unaffected by the retarder and accelerates under the action of the weight to which its shaft is connected. When the electromagnet is energized, the rotation of the disc is retarded and the energy absorbed is converted into heating of the disc. A typical retarder consists of a stator and a rotor. The stator holds induction coils, energized separately in groups (e.g. four coils in a group). The stator assembly is supported through anti-vibration mountings on the chassis frame of the vehicle. The rotor is made up of two discs, which provide the braking force when subject to the electromagnetic influence when the coils are excited [5].

It was found that electromagnetic brakes can develop a negative power which represents nearly twice the maximum power output of a typical engine, and at least three times the braking power of an exhaust brake [1]. By using the electromagnetic brake as supplementary retardation equipment, the friction brakes can be used less frequently, and practically never reach high temperatures. The brake linings last considerably longer before requiring maintenance, and the potential "brake fade" problem can be avoided. In research conducted by a truck manufacturer, it was proven that the electromagnetic brake assumed 80 percent of the duty which would otherwise have been demanded of the regular service brake [1]. On the other hand, the electromagnetic brake prevents the dangers that can arise from the overuse of friction brakes beyond their capability to dissipate heat.

MATHEMATICAL MODEL OF ELECTROMAGNETIC BRAKES

The electromagnetic brake is a relatively primitive mechanism, yet it employs complex electromagnetic and thermal phenomena. As a result, the calculation theory is mainly empirical.

There are three models proposed in the literature on eddy current brakes [2-4]. Smythe’s approach [2] is to treat the rotating part as a disc of finite radius and obtain a closed-form solution by means of a reflection procedure specifically suited to the geometry of the problem. The first step is to calculate the magnetic induction, $B$, produced by the eddy currents induced in a rotating disk by a long right circular cylinder. After deriving the stream function, which is the current flowing through any cross section of the rotating disk from a point to its edge, the torque can be calculated by integrating the product of the radial component of the current by the magnetic induction and by the lever arm and integrating over the area of the pole piece. Since there is a demagnetizing effect such that permeable pole pieces of an electromagnet short-circuit the flux of the eddy current, the total flux in motion would be

$$\phi = \phi_0 - \frac{\beta^2 \gamma^2 \omega^2}{\phi} \frac{R}{R + \beta^2 \gamma^2 \omega^2}$$  \hspace{1cm} (1)

where $\phi_0$ is the flux penetrating the rotating disk at rest, and $\beta^2 \gamma^2 \omega^2 \phi / R$ represents the demagnetizing flux attained through dividing the demagnetizing magnetomotive force by the reluctance of the electromagnet. The final integration result of the brake torque is:

$$T = \omega \phi^2 \frac{\phi_0}{D} \frac{R^2 \phi_0^2 D}{(R + \beta^2 \gamma^2 \omega^2)^2}$$ \hspace{1cm} (2)

where

- $T = \text{brake torque}$
- $\omega = \text{angular velocity}$
- $\phi_0 = \text{flux penetrating the rotating disk at rest}$
- $D = \text{constant coefficient, depending on pole arrangement}$
- $R = \text{reluctance of the electromagnet}$
- $\beta = \text{constant coefficient}$
- $\gamma = 10^{-9}/\rho$, where $\rho$ is the volume resistivity of the disk

This model is good at low speed but decreases too fast in high speed compared with the experimental curve (see Figure 1). The asymptotic behavior shows a fall-off of the torque more rapid than $\omega^{-1}$ in the high speed region, which is in contradiction with experimental results. Smythe pointed out that this behavior could be due to other conditions, such as the degree of saturation of the iron in the magnet which will upset the assumed relations between magnetomotive force and flux ($\phi$) and may modify equations (1) and (2).

Schieber adapted a general method of solution to a rotating system which is different from Smythe’s approach [3]. The result is for low-speed only:

$$T = \frac{1}{2} \sigma \omega \pi R^2 m^2 B^2 Z \left[ 1 - \frac{(R/a)^2}{(1 - (m/a)^2)^2} \right]$$ \hspace{1cm} (3)

where

- $\sigma = \text{electrical conductivity}$
\[ \delta = \text{sheet thickness} \]
\[ \omega = \text{angular velocity} \]
\[ \pi = \text{coefficient} \]
\[ R = \text{radius of electromagnet} \]
\[ m = \text{distance of disc axis from pole-face center} \]
\[ a = \text{disk radius} \]
\[ B_z = z \text{ component of magnetic flux density} \]

Based on the works of Schieber and Smythe, Wouterse tried to find the global solution for the high-speed region as well as the low-speed region [4]. Wouterse proposed the following expression for low speed:

\[ Fe = \frac{\pi D^2 B_z^2 c v}{4 \rho} \]

\[ c = \frac{1}{2} \left[ 1 - \frac{1}{4 \left( 1 + \frac{R}{D} \right)^2 \left( \frac{A - R}{R} \right)^2} \right] \]

where \( Fe \) is the braking force and \( v \) is the speed. The other variables are parameters that can be evaluated based on different types of eddy current brakes. The formula completely agrees with Smythe's result in the low-speed region. Wouterse's study on the air gap magnetic field at different speeds produced three remarkable phenomena:
- At very low speeds, the field differs only slightly from the field at zero speed.
- At the speed at which the maximum dragging force is exerted, the mean induction under the pole is already significantly less than \( B_0 \).
- At higher speeds, the magnetic induction tends to further decrease.

Based on this observation, Wouterse proposed the following solution at the high speed region:

\[ Fe(v) = Fe \left( \frac{2}{\sqrt{v_k + \frac{v}{v_k}}} \right) \]

\[ Fe = \frac{1}{\mu_0} \sqrt{\frac{c}{\xi} \frac{\pi D^2 B_0^2}{4} \sqrt{\frac{x}{D}}} \]

\[ v_k = \frac{2}{\mu_0} \sqrt{\frac{1}{c \xi} \frac{\rho}{d} \sqrt{\frac{x}{D}}} \]

where
\[ \rho = \text{specific resistance of disc material} \]
\[ d = \text{disc thickness} \]
\[ D = \text{diameter of soft iron pole, for noncircular pole shape D denotes the diameter of the circle with the same area as pole face} \]
\[ \xi = \text{ratio of zone width, in asymptotic current distribution around poles, to air gap} \]
\[ c = \text{ratio of total contour resistance to resistance of contour part under pole} \]
\[ v = \text{tangential speed, measured at center of pole} \]
\[ v_k = \text{critical speed} \]
\[ B_0 = \text{air gap induction at zero speed} \]
\[ x = \text{air gap between pole faces including disc thickness or coordinate perpendicular to air gap} \]
\[ R = \text{distance from center of disc to center of pole} \]

Wouterse also made use of another known phenomenon of the high-speed region in his proposal: the drag force becomes proportional to \( v^{-1} \). The model turns out to be much closer to the experimental result in the high-speed region.

While Wouterse's model gives a global solution which is good at high speed as well as at low speed, it must use two different expressions for the low-speed and high-speed regions. From a simulation or control perspective, there are difficulties involved in determining the critical speed or transitional region at which to split the low- and high-speed region. As Wouterse pointed out, the proportionality factor \( \xi \) in equation (5) is not exactly known; It is estimated to have a value of about unity. The 10-20% estimate error of \( \xi \) would cause about a 10% error in equation (5). A uniform model is needed to represent the function at both regions in one expression and reduce the estimation error further.

Our approach is to modify Smythe's model according to Wouterse's observation. As Smythe pointed out himself [2], his model gives too rapid a falling off at high speeds because the degree of saturation of the iron in the magnet upset many of his assumptions. To overcome this problem, we treat reluctance (R in formula (2)) as a function of speed instead of a constant for representing the aggregate result of all those side effects that upset Smythe's assumptions to deduce his formula. This aggregate effect can be called "reluctance effect." The expression of reluctance should also reflect Wouterse's observations on the high-speed region:
(a) The drag force becomes proportional to \( v^{-1} \), and (b) The original magnetic induction under the pole tends to be canceled by the current induced around it in the disc. We found that to represent reluctance as

\[ R = \frac{C_1 + C_2 \omega + C_3 \omega^3}{1 + C_4 \omega^2} \]

is a good approximation that satisfies the above requirement.
Substituting this reluctance function in Smythe’s formula, we are proposing the following uniform model which conforms to the experimental values for the electromagnetic brake operation at low as well as high speed:

\[ T = \frac{k_1\omega}{k_2\omega^2 + k_3\omega^3} \left(1 + \frac{k_4\omega + k_5\omega^2}{1 + k_4\omega + k_5\omega^2}\right)^2 \]

This model agrees with all the proposed models except for the high velocity range of Smythe’s formula, which is inaccurate anyway. This model represents the correct behavior at high speeds. Parameters \( k_1-k_5 \) can be evaluated for specific types of electromagnetic brakes.

The models have been evaluated based on a data chart given by Omega Technologies. The CC 250 type of electromagnetic brake is chosen to do the evaluation. Since the model is nonlinear and five unknown parameters need to be solved, the least squares method is used to obtain the value of \( k_1-k_5 \).

By applying the least squares method on the new proposed model, Smythe’s model, and J.H. Wouterse’s model (at high speed), we obtain the results shown in Figure 1. It can be seen that the new model has better performance in approximating the original curve in the least-squares sense. For simulation and control purposes of this paper, the new model can be used.

The dynamic equation for the angular motion of the wheel is

\[ \dot{\omega}_w = \frac{T_c - T_b - R_w F_t - R_w F_w}{I_w} \]

where the variables are defined in [7]. The tire tractive (braking) force is given by

\[ F_t = \mu(\lambda)N_v \]

The adhesion coefficient \( \mu(\lambda) \) is a function of wheel slip. Wheel slip is defined as

\[ \lambda = (\omega_w - \omega_v)/\omega_v, \omega_v \neq 0 \]

where \( \omega_v = V/R_w \) is the vehicle angular velocity which is defined as equal to the linear vehicle velocity, \( V \), divided by the radius of the wheel. The variable \( \omega \) is defined as

\[ \omega = \max(\omega_w, \omega_v) = \begin{cases} \omega_w & \text{for } \omega_w \geq \omega_v \\ \omega_v & \text{for } \omega_w < \omega_v \end{cases} \]

In our simulation, the function

\[ \mu(\lambda) = \frac{2\mu_p \lambda}{\lambda^2 + \lambda_p^2} \]

is used for a nominal curve, where \( \mu_p \) and \( \lambda_p \) are the peak values. The peak value for the adhesion coefficient may have values between 0.1 (icy road) and 0.9 (dry asphalt and concrete).

The dynamic equation for the vehicle motion is

\[ \dot{V} = [N_w F_t - F_v]/M_v. \]

Refer to [7] for variable definitions. Using equations (7) and (11), and defining the state variables as

\[ x_1 = V/R_w \]
\[ x_2 = \omega_w \]

and denoting \( x = \max(x_1, x_2) \), we obtain

\[ \dot{x}_1 = -f_1(x_1) + b_1N\mu(\lambda) \]
\[ \dot{x}_2 = -f_2(x_2) + b_2N\mu(\lambda) + b_3T \]

Wheel slip is chosen as the controlled variable for braking control algorithms because of its strong influence on the braking force between the tire and the road. By controlling the wheel slip, we control the braking force to obtain the desired output from the system. In order to control the wheel slip, we can have system dynamic equations in terms of wheel slip. During deceleration, condition \( x_2 \leq x_1 \) is satisfied, and therefore wheel slip is defined as:

\[ \lambda = (x_2 - x_1)/x_1 \]
\( \dot{x} = [(1 + \lambda) f_1(x_1) - f_2(x_2)] \) 

\[-\frac{b_2 N + (1 + \lambda) b_1 N \mu + b_3 T}{x_1} \]

gives the wheel slip dynamic equation for deceleration. This equation is nonlinear and involves uncertainties in its parameters. The nonlinear characteristics of the equation are due to the following:
- the relationship of wheel slip with velocity where velocity is nonlinear
- the \( \mu - \lambda \) relationship is nonlinear
- there are multiplicative terms in the equation
- functions \( f_1(x_1) \) and \( f_2(x_2) \) are nonlinear.

### SLIDING MODE CONTROLLER DESIGN

The electromagnetic brake dynamics can be described as a first-order system:

\[ \dot{T} = (K_b P_b - T) / \tau \]  \hspace{1cm} (17)

where \( T \) denotes the braking torque generated, \( K_b \) denotes the brake gain or "brake effectiveness" (e.g. \( K_b = 0.1 \) psi/m/s/s denotes that for a pressure of 100 psi, one obtains a deceleration of 0.1 m/s/s). Electromagnetic brakes are different from regular brakes in that brake gain \( K_b \) is related to the speed due to its static characteristic. \( P_b \) is the brake pedal pressure and \( \tau \) is the rise time characteristic of the brake dynamics. For regular friction brakes, \( \tau \) usually has the value of 150-200ms [6] where time delays are caused mainly by the times needed to fill the calipers, booster spring pre-loads, and the reaction washer hysteresis. Due to different working mechanism, the time delays for electromagnetic brakes come from solenoid response time and the response time of exciting current. Generally, it can be assumed that electromagnetic brake have similar dynamic timing characteristics of regular friction brakes. By taking the derivative of (17) and substituting (18) as \( \dot{T} \), the dynamic equation can be written as:

\[ \dot{\lambda} = f + b \cdot u \]

where

\[ f = -[(1 + \lambda) f_1(x_1) - f_2(x_2)] - \frac{b_2 N + (1 + \lambda) b_1 N \mu + b_3 T}{x_1} \]

\[ \dot{\lambda} = b_3 N \mu + \frac{b_3 T}{\tau x_1} \]

\[ b = \frac{b_3 K_b}{\tau x_1} \]  \hspace{1cm} (19)

For a second-order system in the form of

\[ \ddot{x}(t) = f(x, t) + b(x, t) u(t) \]  \hspace{1cm} (20)

where \( b(x, t) \) is bounded as

\[ 0 \leq b_{\min}(x, t) \leq b(x, t) \leq b_{\max}(x, t), \]

it can be proven that the control law

\[ u(t) = \dot{\lambda}(x, t)^{-1} [\dot{\lambda}(t) - k(x, t) \text{sgn}(s(t))] \]  \hspace{1cm} (21)

with

\[ k(x, t) \geq \beta(x, t)(F(x, t) + \eta + (\beta(x, t) - 1) \dot{\lambda}(t)) \]  \hspace{1cm} (22)

satisfies the sliding condition [7] where

\[ \dot{\lambda}(x, t) = \sqrt{b_{\min}(x, t) b_{\max}(x, t)} \]

is the best estimate of control gain,

\[ \dot{\lambda}(t) = \dot{\lambda}(x, t) + \dot{\lambda}(t) - \gamma(\dot{x}(t)) \]  \hspace{1cm} (23)

is the best estimate of the equivalent control. For chattering \( \text{sgn}(s) \) in (21) is replaced by \( s / \phi \) inside the boundary where \( s \) is the sliding variable (error from the desired wheel slip), and boundary thickness, \( \phi \), is taken to be 0.005.

### SIMULATION RESULT

Figures 9-12 show the results of simulations for which the initial and desired values of the wheel slip are 0.02 and -0.12. Different road conditions have been simulated by using different \( \mu_p \) and \( \lambda_p \) in function

\[ \mu(\lambda) = \frac{2 \mu_p \lambda_p}{\lambda_p^2 + \lambda^2} \]

on dry concrete road (\( \mu_p = 0.8, \lambda_p = 0.2 \)) and slippery road (\( \mu_p = 0.2, \lambda_p = 0.15 \)) [7] to show extreme road conditions. Figure 9 shows the simulation performed on a slippery road, and figure 10 shows the simulation performed on dry concrete. We chose nominal road condition for the control design purposes such that the \( \mu_p \) and \( \lambda_p \) values would be the average of the values for extreme conditions. Figure 11 shows the simulation performed on a nominal road (\( \mu_p = 0.5, \lambda_p = 0.175 \)). All figures show good wheel slip tracking in spite of modeling errors in the parameters (we assumed 10% estimation error on \( b \) and \( f \) in the simulation). Figure 12 shows the simulation result on the vehicle decelerating along the dry concrete road at the first second, along the slippery road at the next second, and along the nominal road at the third second.
Every time the road condition changes, the controller output is quickly changed to compensate. Based on the simulation result, the controller performance is satisfactory.

CONCLUSION
The sliding mode controller gives satisfactory results for the application of electromagnetic brake control. By using electromagnetic brakes, the braking system has an increased capability to produce the required torque.

REFERENCES
[5] Technical literature of electromagnetic brakes, Omega Technologies