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Investigating the Use of Kalman Filtering Approaches for Dynamic Origin-Destination Trip Table Estimation

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Abstract - This paper studies the applicability of kalman filtering approaches for network wide traveler Origin Destination estimation from link traffic volumes. The paper evaluates the modeling assumptions of the Kalman filters and examines the implications of such assumptions.

1. Dynamic Origin Destination Tables

Dynamic Origin-Destination (0-D) trip tables represent a time varying set of traffic demand patterns that occur for particular time periods between each Origin Destination pair. Dynamic O-D tables are an essential input for dynamic traffic assignment models and are useful for tracking time variable O-D patterns for on-line identification and control of traffic systems [6].

1.1 Approaches for Estimating Dynamic O-D Trip Tables

Till date two kinds of approaches for Dynamic O-D Estimation have been formulated: Parameter Optimization and Statistical approaches. Parameter optimization (Cremer and Keller (1987), [3], and Sherali et al [4]) formulations derive the O-D table by formulating objectives that minimize deviation between the actual traffic flow and that associated with the O-D table being derived. Statistical models may further be classified into statistical inference models (Maher [10], Nihan and Davis [7]) and kalman filtering models.

1.1.1 Kalman Filtering Approaches for O-D Estimation

The difference (transition) equation) for Kalman Filter is of the form

$$x_k = \phi_{k-1}x_{k-1} + \Gamma_{k-1}u_{k-1} + \Lambda_{k-1}w_{k-1} \qquad (1)$$

where ϕ_{k-1} is the matrix of autoregressive coefficients, u_k is a deterministic input, Γ_k the control gain, Λ_k the disturbance gain, and w_k is the disturbance input. The measurement equation is given by

$$x_k = H_k x_k + n_k \qquad (2)$$

where n_k is the error term. The filter computes the estimate of the state while minimizing the spread of the estimate error probability distribution function [5]. Assumptions used are:

The expected value of the initial state (x_0) and its covariance (P_0) are known:

$$E(x_0) = \hat{x}_0 \qquad (3)$$

$$E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] = P_0 \qquad (4)$$

The disturbance input is a white zero mean Gaussian sequence:

$$E(w_k) = 0 \qquad (5)$$

$$E(w_k w_k^T) = Q_k \qquad (6)$$

$$E(w_k w_k^T) = 0 \quad j \neq k \qquad (7)$$

The measurement error n_k is a white zero mean Gaussian random sequence that is uncorrelated with the disturbance input, i.e.

$$E(n_k) = 0 \qquad (8)$$

$$E(n_k n_k^T) = R_k \qquad (9)$$

$$E(n_k n_j^T) = 0 \quad j \neq k \qquad (10)$$

$$E(w_k n_j^T) = 0 \quad \forall j,k \qquad (11)$$

For the system described by (1) and (2), under the conditions expressed in equations (3) to (11) the covariance and state estimation update equations may be written as:

$$P_k = \left\{ \left[\phi_{k-1}P_{k-1}\phi_{k-1}^T + \Lambda_{k-1}Q_{k-1}\Lambda_{k-1}^T \right]^{-1} + H_k^T R_k^{-1} H_k \right\}^{-1} \qquad (12)$$

$$K_k = P_k H_k^T R_k^{-1} \qquad (13)$$

$$\hat{x}_k = \left[\phi_{k-1}\hat{x}_{k-1} + \Gamma_{k-1}u_{k-1} \right] + \\ K_k \left\{ z_k - H_k \left[\phi_{k-1}\hat{x}_{k-1} + \Gamma_{k-1}u_{k-1} \right] \right\} \qquad (14)$$

State Space O-D Models - Group I

Several KF models for Dynamic O-D estimation have been developed. Cremer and Keller (1987), [3], and Nihan and Davis (1987), [11] have developed KF approaches in which they assume that the dynamics of the split parameter b_{jk} (defined as the portion of traffic volume ($q_i(k)$) entering the ith entrance during the kth time interval that leaves through exit j) vary as :

$$b_{ij}(k+1) = b_{ij}(k) + w_{ij}(k) \qquad (15)$$

The measurement equation relates the traffic flow to the split parameters and may be written as:

$$y_j(k) = q^t(k)b_j(k) + v(k) \qquad (16)$$

They assume that $w_{ij}(k)$ is a white zero mean Gaussian sequence. They, however, do not explicitly incorporate the flow conservation restrictions on the split parameters in the form of constraints. Chang and Wu (1984) [8] use the same dynamic model for split parameters and assume travel time to vary as a function of link flows creating nonlinearities in the measurement equations thus prompting the use of Extended Kalman Filter (EKF).

A limited amount of research has been conducted to estimate network wide dynamic O-D trip tables from link counts. The models that fall in this category generally have the following features: (1.) they all consider general urban networks, (2.) require prior O-D Estimation, (3.) directly estimate O-D flows rather than the proportions, and (4.) rely on a traffic assignment model to provide a flow distribution matrix. Okutani, [2], has developed a state-space model that employs an auto-regression function for representation of consecutive O-D flows. Ashok and Ben-Akiva, [1], present a modified search space model for network wide dynamic O-D estimation. This model assumes the existence of an accurate nominal dynamic O-D pattern (seed or prior table). The state space model is then used to estimate the deviations of the O-D flows from the nominal flow values. Okutani, [2], assumes a dynamic model for O-D that is of the form

$$x_{h+1} = \sum_{p=h-q}^{h} f_h^p(x_p) + v_h \qquad (17)$$

where x_h is the vector of O-D flows departing during the time period h, f_h^p is an $n_{O-D}*n_{O-D}$ matrix of effects of (x_p) on x_{h+1}. The measurement equation for the model is obtained by relating the network link volumes with the O-D flows through the following relationship.

$$y_h = \sum_{p=h-p}^{h} a_h^p(x_p) + v_h \qquad (18)$$

where y_h is the (n_l*1) vector of link flows, a_h^p (assignment matrix) is an (n_l*n_{O-D}) matrix of contributions of x_p to y_h. The transition equation employed by Ashok and Ben-Akiva accounts for structural information among O-D trip patterns by use of the idea of deviations from historic values of O-D flows. The transition equation is of the form

$$x_h - x_h^H = \sum_{p=h-q}^{h} f_h^p(x_p - x_p^H) + w_h \qquad (19)$$

and the measurement equation is given as

$$y_h - y_h^H = \sum_{p=h-p}^{h} a_h^p(x_p - x_p^H) +$$

$$\qquad (20)$$

$$\sum_{p=h-p}^{h} a_h^p x_p^H - y_p^H + v_h$$

Both models, suitably augment the above equations before they are used in the filter. If in the above equations, one makes the assumption that the structure of the autoregressive equation remains constant then f_h^p would only depend on the difference $(h-p)$ and not on individual values of h and p. Since the assignment matrices usually vary with time, the state space models of Okutani (1987), [2], and Ashok and Ben-Akiva (1993), [1], may be classified as Linear Time Varying (LTV) estimation problems. For convenience in presentation, these models are here after referred to as LTV(O-D) and LTV(differences), respectively.

2 Objective

The objective of this paper is to investigate the use of state space models for O-D estimation. Using real data, we explore the characteristics of the error terms in the underlying dynamic process of O-D departures. The aim of our data analysis is to check for inconsistencies between observed O-D flow patterns and Kalman filter modeling assumptions. This is crucial in determining the appropriate design of the filter. In the light of our data analysis, we present potential strategies that account for some limitations of existing filters, and the resulting improvements are demonstrated.

3 Dynamics of O-D Flow and Its Implications in Filter Design

Models developed for freeway O-D estimation have used the form given in (15), while models developed for Generalized Networks have used either (17) or (19). The following analysis illustrates the relationship between the dynamics assumed in (17) and (15).

Equation (15) can alternatively be written as

$$\frac{x_{ij}^{k+1}}{q_i^{k+1}} = \frac{x_{ij}^k}{q_i^k} + w_{ij}^{k-1} \qquad (21)$$

Multiplying both sides of (21) by q_i^{k+1} we get

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$$x_{ij}^{k+1} = x_{ij}^k \frac{q_i^{k+1}}{q_i^k} + q_i^{k+1} w_{ij}^{k-1} \qquad (22)$$

Thus one may view such assumed dynamical behavior to be a special case of (17) where

$$\dot{q} = 1, \quad f_{h,ij}^p = \frac{q_i^{k+1}}{q_i^k} \quad \forall ij \in O - D, \quad \text{and}$$

where q_i^{k+1} plays the role of the disturbance gain.

In this section we study data obtained from the Massachusetts Turnpike Authority. While the same network has been tested in previous literature [1], the data sets were for different days.

3.1 Massachusetts Turn Pike

Three days worth of O-D data for the peak period [7:00-11:00 am] was obtained for the Massachusetts Turnpike, a toll freeway stretching from the New York State Border to Weston [Route 128]. The network has 16 entry exit ramps, hence the network has 240 possible entry-exit or O-D pairs. For the time period from 7:00 am to 11:00 am, for three consecutive days of the week, information on the origins of travelers at each exit was available at fifteen minute time intervals. To validate the dynamic relationships given in (15), (17), (19) it is necessary to perform regression analysis on time series of departure flow data. To obtain departure flow data we employed a back calculation scheme as in [1] assuming some speed distribution on the freeway links.

3.2 Analysis of O-D data

We used one of the representative O-D pairs (from Framingham to Boston, a distance of about 22 miles) for our studies. Our analysis of the transition equation (15) which is assumed by most Kalman filtering models developed for freeways showed that the noise had a zero mean but was not Gaussian.

The study of equations (17) and (19) called for regression analysis. Our analysis assumed that the autoregressive structure remained constant over the whole day (permitting to have enough observations to estimate these parameters) and that the flow between an O-D pair for a period was related to previous O-D flows for that pair alone. One day's data was used for the LTV (0-D) system and two days data were used to obtain a time series of deviations for the LTV (deviations) system.

The regression correlation coefficients were lower for the LTV (0-D) model. This indicates that the autoregressive equation of Okutani seems

to better describe the stochastic O-D departure process. However, a crucial assumption in the design of state space models rests on the nature of the error terms w_k, which are the residuals of the regression. The error terms w(k) are assumed to be a white gaussian noise with zero mean. We performed the Anderson-Darling test for the hypothesis that the error terms conform to a normal distribution. The results showed that the hypothesis was accepted at the 95% confidence level for both LTV(O-D) and LTV(Deviation) models. To test the whiteness of the noise the error covariance matrices of the regression were studied. The test specifically focused on checking to see if equation (7) was satisfied. This necessary condition may alternatively be stated as

$$E\left[w_h w_{h+1}'\right] = 0 \quad \forall l \neq 0 \qquad (23)$$

The study of the error covariance matrices showed that the $E\left[w_h w_{h+1}'\right] = 0 \quad \forall l \neq 0$ is quite comparable to $E\left[w_h w_h'\right]$. However, this conclusion has to be treated with some caution because the number of data points is relatively small. Also note that the high values of the regression correlation coefficients for high values of $\dot{q}$ may well be due to the reduction in the degrees of freedom for the regression. (Since we had only 16 data points increasing the number of independent variables is bound to increase the regression correlation coefficient.) In order to overcome these limitations, we carried out the regress ion by merging all three days data[1] Here, again, the regression correlation coefficients showed that O-D flow data for a given interval can be related to flows in the past interval only with a reasonable amount of confidence through Okutanis autoregressive structure. It also showed that relating deviations for the present period to those of the past does not really carry much statistical significance as reflected in the low values of the Regression Coefficients. More importantly, the data lead us to suspect the whiteness assumption of the noise for both LTI(O-D) and LTI(Differences) models. For

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[1] While the data for all three days starts at 07:00 in the morning and ends at 11:00 the merged data treats the time series as continuous. Since the peak period ends at 11:00 and starts at 07:00 the merging of the second days begining peak data at the end of the first days peak period makes the time series of flows reasonable.

LTI(O-D) model we clearly note that while

$$E[w_h w_{h+1}] \quad \forall h \neq k < E[w_h w_k] \quad \forall h = k,$$

there is no reason to believe that $E[w_h w_k] = 0 \quad \forall h \neq k.$ For the LTI(Differences) model an interesting observation can be made. While

$$E[w_h w_k] \quad \forall (h \neq k \text{ and } h < q') <<$$

$$E[w_h w_k] \quad \forall h = k \text{ for } h > q'$$

again, there is no reason to claim that

$$E[w_h w_k] = 0 \quad \forall h \neq k \ .$$

3.2.1 Findings from Data Analysis
From the above analysis of departure time data the following conclusions can be stated:
1. The noise terms for equation (15) had a zero mean but was not Gaussian
2. The noise terms for equations (19) and (17) were found to have gaussian but colored noise.

The next section examines the implications of colored and Non-Gaussian Noise, and this is followed by enhancements to account for some of these deficiencies.

3.2.2 Implications of Colored and Non-Gaussian Noise:
When noise is colored the Kalman Filters designed in the literature cannot be used for estimating O-D flows. In this section we present techniques to account for colored noise.

Case I: Noise is not Gaussian: In such a case, as found to be true for the autoregressive equation (15) the assumed dynamic equations needs to enhanced by inclusion of additional terms in the transition equation that will better capture the dynamics of O-D flow.

Case II: Noise is Gaussian but time Correlated: When the noise is described by

$$E(w_k) = 0 \tag{24}$$

$$E(w_k w_k^T) = W_k \tag{25}$$

$$E(w_k w_{k-1}^T) = V_k \neq 0 \tag{26}$$

the time correlated disturbance can itself be treated as a first order Gauss-Markov sequence[2] that can be added to the original model [5, Colored Noise]. That is, the colored noise is obtained as the output of a linear system that is excited by a white noise. This may be shown as

$$w_k = A_{k-1} w_{k-1} + \eta_{k-1} \tag{27}$$

where η_{k-1} is a white noise sequence[3] . The time correlated disturbance can then be added to the original model, thus defining an augmented state vector $[x_k \quad w_k]^T$. We carried out such an enhancement to both the LTV (0-D) and LTV (difference)) models described here in. The tests and results are described below:

4 Enhancements for Colored Noise
4.1 Enhanced LTV (0-D) Model

$$x_{h+1} = \sum_{p=h-p'}^{h} f_h^p(x_p) + w_h \tag{28}$$

where $w_h = A_{k-1} w_{k-1} + \eta_{k-1}$ (29)

4.2 Enhanced LTV (Diff) Model
This system is described by

$$x_h - x_h^H = \sum_{p=h-q'}^{h} f_h^p(x_p - x_p^H) + w_h \tag{30}$$

where $w_h = A_{k-1} w_{k-1} + \eta_{k-1}$ (31)

and the measurement equation is given in (20).

4.3 Tests on Enhanced Models
To illustrate our enhancements to account for the colored noise it was enough to consider just one O-D pair. The O-D was connected by just one link which had a travel time equal to 30 minutes (twice the time length of the 15 minute interval for which departures from Framinhgam were known.) Based on the regressions carried out we chose $q' = 2$ for this illustrative test.

Figure-1 shows the true O-D flow data, the values predicted by the LTV(O-D) Model and that predicted by the enhanced LTV(O-D) model. It is seen that the enhancements to account for colored noise considerably improves the LTV(O-D) Model. This is also reflected in the RMS values for the error in prediction which are 16.74 and 5.12 for the LTV(O-D) and enhanced LTV(O-D) model respectively. Similar conclusions can be drawn for the LTV(Differences) and enhanced LTV(Differences) model from Figure-2. The RMS values for the error in prediction for this case are 9.77 and 4.02 for the LTV(Differences) and Enhanced LTV(Differences) model, respectively. Thus it is seen that accounting for colored noise has considerably improved the estimates of the Filter.

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[2] If the noise is a higher order time corelation then we can model it through higher order Gauss-Markov sequences. ([5])

[3] If η_{k-1} is not found to be white then one may account for its coloredness using the same procedure described here.

5. Conclusions

This paper has presented an investigation into the use of Kalman Filtering approaches for dynamic O-D estimation through a study of the dynamic relationships that are assumed in current literature. The analysis of the nature of error terms of currently assumed dynamics, using real data, revealed that the noise is not a white Gaussian sequence. Enhanced Kalman estimators that can filter the colored noise in the system were designed and tested to show the improvement.

6. Acknowledgments

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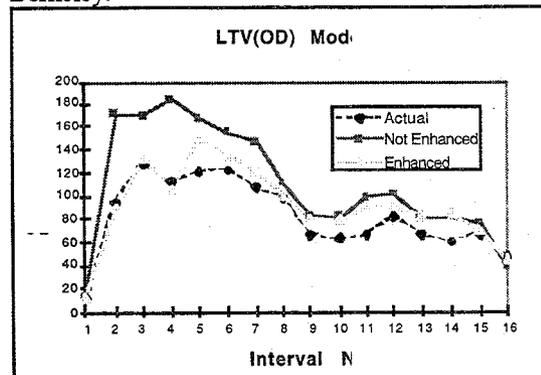
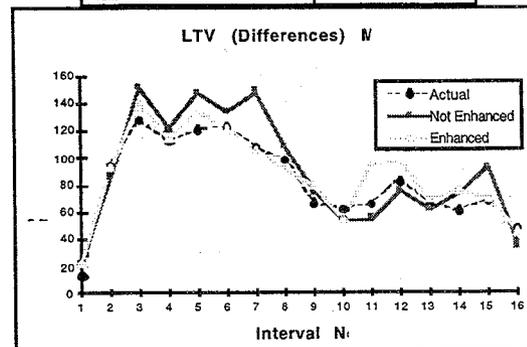Figure 1.   Tests on LTV(OD) Model.

Not Enhanced:	16.74
Enhanced:	5.12

Figure 2.   Tests on LTV(Differences Model).

Not Enhanced:	9.77
Enhanced:	4.02