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S. Al-Nasur
Virginia Polytechnic Institute and State University

Pushkin Kachroo
University of Nevada, Las Vegas, pushkin@unlv.edu

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A Microscopic-to-Macroscopic Crowd Dynamic model

Sadeq Al-nasur* and Pushkin Kachroo**
Bradley Department of Electrical and Computer Engineering
Virginia Polytechnic Institute and State University
Blacksburg, VA 24060-0111, USA

Abstract—This paper presents a model for a two-dimensional pedestrian movement flow. The model is derived by extending a one-dimensional vehicle traffic flow model that uses two coupled partial differential equations (PDEs) to govern vehicular motion. This model modifies the vehicular traffic model so that bidirectional flow is possible, and also the pedestrian movement can be controlled to model different behaviors. The model satisfies the conservation principle and is classified as a hyperbolic PDE system. Analysis of the model is drawn based on the theoretical aspects as well as the numerical simulation results achieved using the finite volume method. The original vehicular macroscopic model was derived by extending a corresponding microscopic model. This paper follows the same strategy and shows how microscopic pedestrian behavior can be used to derive the macroscopic behavior.

I. INTRODUCTION

The development of pedestrians dynamic models is a growing research area. These models explore many issues, ranging from design and development to safety and management in public facilities such as buildings, theaters, sport centers, airports, and passenger ships. Simulating an evacuation strategy on the facility design before its implementation is a good way to test its effectiveness. It becomes even more important because the cost involves human death or injury, property damage, etc. In addition, a better understanding of crowd behavior is needed to improve safety procedures in various buildings and areas.

There are two ways to model pedestrian or crowd dynamic flow. One is a microscopic level modeling, which involves a detailed design that focuses on individual units and their characteristics such as individual interactions, direction, and speed. The movement in these models is based on changes in pedestrian surrounding environment. For example, if a table is in front of a pedestrian it is considered an obstacle, and the pedestrian would need to avoid it by moving around it. By doing so, pedestrian velocity is also affected by its surrounding environment. A macroscopic level modeling is concerned with group behavior and deals with a crowd as a whole. Therefore, detailed interactions are overlooked, and the model’s characteristics are shifted toward continuum density flow and average speed.

II. DEVELOPMENT

There are several microscopic simulation based analysis models as in [1]–[3] that are classified as physical force, cellular automata, and queuing network models. On the other hand, the macroscopic models in [4]–[6] are based on fluid flow theory. In this paper, we are interested in macroscopic way of modeling, which according to [7], the pedestrian traffic theory is similar to the vehicle traffic flow theory with the exception that vehicle traffic flow is a one-dimension problem while crowd dynamics represent a two-dimension problem. In [8] an attempt was made to extend a one-dimension microscopic traffic flow model to a two-dimension microscopic pedestrian model. These kinds of models have more design details, where you have to follow each person interactions, speed, and direction. Therefore, simulating pedestrians becomes more expensive as the number of pedestrians increases. In contrast, the macroscopic models are less detailed and therefore easier to handle in a simulation environment. In addition, they provide a good way to study crowd dynamics.

The aim of this study is to investigate crowd dynamic behavior in normal and emergency situations. For example, in an evacuation event we will have the ability to direct the flow and study the crowd reaction as they move toward an exit, or any other interaction criteria. In order to do so, we present a macroscopic system model that is derived from a microscopic car-following model. The model is an extension of a one dimensional macroscopic vehicle traffic flow model that was proposed by Zhang [9]. The model selection was based on its microscopic-to-macroscopic derivation property, which adds to the macroscopic model an important validation point. The coupled PDE equations provide conservation of continuity and movement (momentum). In addition, the model carries the desired anisotropic nature of traffic flow. This property is described as being the natural observed traffic flow behavior in which pedestrian movement is influenced by conditions from current and locations ahead only. This property is different than the isotropic property of fluids, where a fluid particle is influenced from all directions.

The paper is organized in the following manner. In section II, we present the extension to the Zhang model from vehicle dynamic flow into crowd dynamics flow model, and we also explore the velocity-density relation used in this model. Section III shows the derivation procedure from microscopic model to macroscopic model. In section IV, we write our model in conservation form and find its eigenvalues and
eigenvectors. Section V gives an analysis of the model characteristics followed by simulation results based on finite volume method in Section VI. Finally, we provide discussion for future work.

II. MODEL DESCRIPTION

This model is based on the high order model by Zhang [9], which is classified as a traffic flow hyperbolic system of two coupled partial differential equations. Zhang model is derived from a microscopic car-following model. The model established a link between micro-to-macro models which we also establish for the crowd model we present here by starting from two dimensional microscopic version of the car-following model. The nature of this model is anisotropic, i.e. pedestrians respond to front conditions only. This feature will also be preserved when we extend the model to a two-dimensional space assuming that pedestrian motion is influenced conditions at the front.

The first equation is a two-dimensional conservation of continuity which conserve mass (pedestrians) that can be derived from Fig. 1, where the conserved density will change according to the change in flow at the boundary endpoints only. The conservation equation is given by

\[ \rho_t + (\rho v)_x + (\rho u)_y = 0, \]  

where \( \rho(x, y, t) \) is the density, \( v(x, t) \) and \( u(y, t) \) are the two components of the velocity in the \( x \)-axis and \( y \)-axis directions respectively. The flux flow rate in both directions is represented by \( \rho v \) and \( \rho u \). The above equation is defined on \( x \in \mathbb{R}, y \in \mathbb{R} \) and time \( t \in \mathbb{R}^+ \).

The second set of equations is similar to the momentum equations in two-dimension. They represent the pedestrian motion dynamics and they are derived for \( x \) and \( y \) directions from the microscopic model. They are given by

\[ v_t + vu_x + uv_y + \rho V'(\rho)(v_x + u_y) = \frac{V(\rho) - v}{\tau} \]  

\[ u_t + vu_x + uv_y + \rho U'(\rho)(v_x + u_y) = \frac{U(\rho) - u}{\tau} \]

where the velocity \( V(\rho) \) and \( U(\rho) \) are the desired velocities functions meant to mimic pedestrian behavior given by the velocity-density relation (4), and \( V'(\rho) \) and \( U'(\rho) \) is the traffic sound speed at which small traffic disturbances are propagated relative to the moving traffic stream. The relaxation term \( (V(\rho) - v)/\tau \) is added to the model to keep speed concentration in equilibrium, where \( \tau \) is this process relaxation time. Initial conditions are \( \rho(x, y, 0) \geq 0 \), \( v(x, 0) = \pm v_f \), and \( u(y, 0) = \pm v_f \). Changing the initial values of \( V(\rho) \) and \( U(\rho) \) will change pedestrian speed and direction.

A. Velocity-Density Relation

In the above model, \( V(\rho) \) and \( U(\rho) \) are the velocity-density functions that relate the desired pedestrian velocity to its density profile. There are many functions that can describe a traffic flow. In our model we used Greenshild model [12], which assumes that the velocity is a linearly decreasing function of density as shown in Fig. 2, and it is given by

\[ V(\rho) = v_f(1 - \frac{\rho}{\rho_m}) \]  

where \( v_f \) is the free flow speed. The magnitude and sign of \( v_f \) determines the speed and direction (controlling parameter) for the pedestrian flow. The maximum density (jam density) is given by \( \rho_m \), and \( V'(\rho) \leq 0 \) is

\[ V'(\rho) = -\frac{v_f}{\rho_m} \]

where this is part of the system characteristics (eigenvalues) that will be discussed in Section V.

III. DERIVATION OF MACROSCOPIC MODEL FROM MICROSCOPIC MODEL IN 2-D

The second equation of the model can be derived from the microscopic car-following model that is used to represent vehicle traffic flow in one-dimension. The idea is that a pedestrian in one-dimension can move and accelerate forward based on two parameters; the headway distance between him and the pedestrian in front, and their speed difference. Hence, it is called following, where the pedestrian from behind follow the one in front, and this is the anisotropic property. For two-dimensions we use the same idea and start our derivation from a homogeneous microscopic model (since the relaxation term can be added later) given by

\[ \tau(s_n(t)) \dot{x}_n(t) = x_{n-1}(t) - x_n(t), \]
where
\[ s_n(t) = x_{n-1}(t) - x_n(t), \] (7)
and \( s_n(t) \) is a function of the local spacing between pedestrians. \( x_n(t) \) is the two dimension position of the \( n \)th pedestrian, \( \dot{x}_n(t) \) is the velocity, and \( \tau(s_n(t)) \) is the average pedestrian response time to the headway distance. Using the above notations, we rewrite (6) for the \( x \) component where the local spacing \( s \) is in the \( x \) direction only, and define the velocity as \( v(x,t) = \dot{x}(t) \) to obtain the following
\[ \tau(s(x(t),t)) \frac{dv(x,t)}{dt} = \frac{d(s(x(t),t))}{dt}, \] (8)
and by using convective derivative \( \partial_t + v \partial_x + u \partial_y \) on the \( x - axis \) direction velocity component (for \( y - axis \) use \( u \) component instead of \( v \)), we get
\[ \tau(s) (v_t + v v_x + u v_y) = (s_t + s v_x + u s_y). \] (9)

From the conservation law (2), let \( \rho = 1/s \), and by using the following derivative form
\[ D_x \left( \frac{a}{b} \right) = \frac{b D_a - a D_b}{b^2}, \] (10)
we get
\[ s_t + s v_x + u s_y = s v_x + u v_y, \] (11)
and by direct substituting in the right hand side of equation (9) we obtain our desired equation in the following form
\[ (v_t + v v_x + u v_y) = \frac{s}{\tau(s)} (v_x + u_y), \] (12)
where
\[ \frac{s}{\tau(s)} = -C(\rho) = -\rho V' \rho \geq 0, \] (13)
is the sound wave speed. This completes the derivation of the two-dimensional macroscopic model from its microscopic counterpart for the \( x - axis \). The relaxation term can be added to get equation (2). We follow similar steps for the \( y - axis \). The conservative form of this model for the \( x - axis \) is derived next.

IV. CONSERVATION FORM AND EIGENVALUES

To find the eigenvalues of the system and check if the system is hyperbolic we write the model in conservation form which is also the same form being used in the simulation section. Equation (1) is rewritten as,
\[ \rho_t + \rho v_x + \rho_x v + \rho u_y + \rho y u = 0, \] (14)
and by substituting in equation (2) and then multiplying by \( \rho \) we get
\[ \rho(v - V(\rho))_t + \rho v (v - V(\rho))_x + \rho u (v - V(\rho))_y = \rho \frac{V(\rho) - v}{\tau}. \] (15)

For simplicity the density \( \rho \) is dropped from the velocity functions \( V(\rho) \) and \( U(\rho) \). Using product rule for
\[ (\rho (v - V))_t = \rho_t (v - V) + \rho (v - V)_t, \] (16)
\( (\rho (v - V))_x, \) and \( (\rho (v - V))_y \) we substitute in (15). Finally we use the conservation law (1) to write the model in conservation form (we follow similar steps for the \( y \) component using \( U(\rho) \) instead of \( V(\rho) \)). By doing so we get our model in conservation form as
\[ \rho_t + (\rho v)_x + (\rho u)_y = 0 \] (17)
\[ \rho(v - V)_t + (\rho v (v - V))_x \]
\[ + (\rho u (v - U))_y = \frac{V - v}{\tau} \] (18)
\[ (\rho (u - U))_t + (\rho v (u - U))_x \]
\[ + (\rho u (u - U))_y = \frac{U - u}{\tau}. \] (19)

Next we write the system in two dimensional vector form
\[ Q_t + F(Q)_x + G(Q)_y = S, \] (20)
where \( Q \) is the conservative variables (states), and \( F \) and \( G \) are the fluxes in the \( x \) and \( y \) directions respectively, and \( S \) can be considered as the source term. These are given by
\[ Q = \begin{bmatrix} \rho \\ \rho v \\ \rho (v - V) \end{bmatrix}, \]
\[ F(Q) = \begin{bmatrix} \rho v \\ \rho v (v - V) \end{bmatrix}, \]
\[ G(Q) = \begin{bmatrix} \rho u \\ \rho u (v - V) \end{bmatrix}, \]
\[ S = \begin{bmatrix} 0 \\ s_1 \\ s_2 \end{bmatrix}. \]

Next we rewrite the system in the general quasi-linear form given by
\[ Q_t + A(Q)Q_x + B(Q)Q_y = 0, \] (21)
where the source term is zero and the flux Jacobian matrices are found from \( A(Q) = \partial F(Q)/\partial Q \), and \( B(Q) = \partial G(Q)/\partial Q \). For this system the matrices and their corresponding eigenvalues and eigenvectors are found by first setting the conservative values (states) as
\[ Q = \begin{bmatrix} \rho \\ w \\ z \end{bmatrix}, \]
where \( \rho(v - V) = w \Rightarrow v = w + V \)
\[ \rho(u - U) = z \Rightarrow u = \frac{w}{\rho} + U \]
We rewrite the fluxes \( F(Q) \) and \( G(Q) \) as
\[ F(Q) = \begin{bmatrix} w + \rho V \\ \frac{w^2}{\rho} + wV \\ \frac{w^2}{\rho^2} + zV \end{bmatrix}, \]
\[ G(Q) = \begin{bmatrix} z + \rho U(\rho) \\ \frac{w^2}{\rho^2} + wU \end{bmatrix}. \]
The corresponding Jacobian matrices are found to be
\[ A = \begin{bmatrix} V + \rho V' & 1 & 0 \\ \frac{w^2}{\rho^2} + wV' & 2\frac{w}{\rho} + V & 0 \\ \frac{w^2}{\rho^2} + zV' & \frac{w}{\rho} + V \end{bmatrix}, \] (22)
\[ B = \begin{bmatrix} U + \rho U' & 0 & 1 \\ \frac{w^2}{\rho^2} + wU' & \frac{w}{\rho} + U \end{bmatrix}. \] (23)
Solving \( A(Q) \) for the eigenvalues and eigenvectors we get
\[
\lambda_1^A = v + \rho V'(\rho), \quad \lambda_2,3^A = v, \quad (24)
\]
\[
e^1_A = \begin{bmatrix} 1 \\ v - V \\ u - U \end{bmatrix}, \quad e^2_A = \begin{bmatrix} 1 \\ v - V - \rho V' \\ 0 \end{bmatrix}, \quad e^3_A = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]
and for \( B(Q) \) matrix to obtain
\[
\lambda_1^B = u + \rho U'(\rho), \quad \lambda_2,3^B = u \quad (25)
\]
\[
e^1_B = \begin{bmatrix} 1 \\ v - V \\ u - U \end{bmatrix}, \quad e^2_B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad e^3_B = \begin{bmatrix} 1 \\ 0 \\ u - U - \rho U' \end{bmatrix}
\]

From the above, we see that in each Jacobian, the eigenvalues are real and that one is a repeated pair. We also note that the first eigenvalue in both \( A \) and \( B \) is always less or equal to the second one which is due to \( \rho V' \) effect. From this fact we acknowledge the anisotropic property of the system. This shows that information can not travel faster than the actual wave, i.e. pedestrian movement is influenced from current and front stimuli only. Secondly, despite repeated eigenvalues, each matrix has linearly independent eigenvectors.

Since this is a two-dimensional problem, in order to verify that the system is hyperbolic, we need to check that our eigenvalues found earlier are for any combination of the roots of \( \hat{A} = \hat{n}.\hat{A} = n^x A(Q) + n^y B(Q) \), where \( \hat{n} = (n^x, n^y) \) is a unit vector. The eigenvalues are found to be
\[
\lambda_1 = \hat{v} + \rho \hat{V}', \quad \lambda_{2,3} = \hat{v} \quad (26)
\]
where \( \hat{v} = n^x v(x, t) + n^y u(y, t) \), and \( \hat{V}' = n^x V' + n^y U' \).

This will return the propagation in the \( x \) and \( y \) directions for \( \hat{n} = (1, 0) \) and \( \hat{n} = (0, 1) \). Since the eigenvalues above are real, we conclude that our model is hyperbolic, and the fact that we also have repeated eigenvalues our model is not strictly hyperbolic. The two matrices coefficients do not commute, i.e. \( AB \neq BA \) and the system matrices \( A \) and \( B \) do not have the same eigenvectors [14], therefore they can be diagonalized separately
\[
A = R^x \Lambda(R^x)^{-1}, \quad B = R^y \Lambda(R^y)^{-1}, \quad (27)
\]
where the two matrices have different eigenvectors. This means that the PDE equations are more intricately coupled. For each eigenvalue the corresponding eigenvectors are given by
\[
e^1_A = \begin{bmatrix} 1 \\ v - V \\ u - U \end{bmatrix}, \quad e^2_A = \begin{bmatrix} \hat{v} - \hat{V}' \\ n^x \\ 0 \end{bmatrix}, \quad e^3_A = \begin{bmatrix} 0 \\ 1 \\ n^y \end{bmatrix}
\]
\[
e_3^A = \begin{bmatrix} 0 \\ -n^y \\ 1 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 1 \\ \hat{v} - \hat{V}' \\ n^y \end{bmatrix}
\]

VI. MODEL ANALYSIS

A model that is aimed at studying crowd dynamics has been presented in this paper. The model is derived from one-dimension traffic flow system, which has a microscopic-to-macroscopic link. If we take a look at the model we notice that it is a nonlinear hyperbolic system of PDE’s with distinct characteristics. The terms \( V(\rho(x, y, t)) \) and \( U(\rho(x, y, t)) \) change the momentum equation to mimic crowd dynamic flow. They are used as anticipation terms to find the macroscopic response of pedestrians to traffic density. At the same time these terms help to keep velocity in equilibrium. If a pedestrian velocity is greater than the preferred value, these terms will tend to reduce the velocity and vice versa.

The use of the convective derivative for the velocity terms, which leads to the sound wave speed \(-C(\rho) = -\rho V'(\rho)\), enables the prediction of the expected response of crowd behavior. The system eigenvalues indicate the type of crowd behavior. The model has a total of six real eigenvalues, two of which are distinct, and two are repeated in each direction. Although we have repeated eigenvalues, the corresponding eigenvectors are linearly independent. This enabled us to perform simulations using numerical finite volume methods.

The system has an anisotropic nature that appears from its eigenvalues
\[
\lambda_1 \leq \lambda_{2,3}, \quad (28)
\]
This means that all information moves at a speed equal to or less than the velocity of the corresponding state. This feature is very attractive for traffic flow in general where pedestrian movement response is mostly due to conditions ahead as in normal conditions. Moreover, we can relate a panic crowd situation to this property from the fact that in most panic situations with high density crowd, pedestrians tend to slowly form groups and move together without looking back. They just follow, assuming that the leader of this group knows or sees the way out.

V. SIMULATION AND RESULTS

The model derived above is a two-dimensional system of nonlinear hyperbolic PDE’s. These equations admit discontinuous solutions like shocks, where fast moving pedestrians reach a slower crowd and they are forced to slow down. Another type is called the fan solution, where pedestrians in front are moving faster and the crowd from behind accelerate to catch up with them. These two kind of solution can be observed in vehicle traffic as well as pedestrian traffic flow. Analytical solution to nonlinear hyperbolic PDE’s is very difficult to find especially for two-dimensional space. Therefore, we solve our model numerically by finite volume methods. To obtain numerical solution for this model we need to use conservative schemes that can be found in finite volume methods as in [13], [14]. First-Order Centered (FORCE) method is classified as a first order total variation diminishing scheme that provides us with a good idea of the solution.
A. Initial and Boundary Conditions

1) Initial Condition: In our test problems we use Gaussian distribution for the density in two-dimensional space given by
\[ \rho(x, y, 0) = \rho_0 \exp \left( -\left( x - a \right)^2 - \left( y - b \right)^2 \right), \]  
where \(a\) and \(b\) are the initial density concentration locations. The free flow average speed (preferred speed) is taken to be \(v_f = 1.36 \, \text{ms}^{-1}\). Initial velocity profile is calculated from Greenshild's velocity-density model (4), and the velocity direction is specified by \(\pm v_f\).

2) Boundary Conditions: The domain chosen for the simulations is a square with a 20 x 20 \(m^2\) area where pedestrians can move freely inside, and at the boundary no-slip conditions are enforced except at the exit point \((x_e, y_e)\) (i.e., large room, one exit, without any obstacles). By no-slip we mean closed walls where no pedestrian (density) can pass through, but they can move tangent to the walls. We use ghost cells as in [14] (i.e., imaginary extra cells) to simulate such boundary condition.

The exit is treated as free flow, where if a pedestrian reaches the exit cell, his velocity is equal to the free flow velocity \(v_f\) and density is zero. For the crowds to move toward the exit, we simply force them to follow the desired direction. For the \(x\) direction (\(y\) direction use \(\sin \theta\)) it is done by updating the velocity-density function by
\[ V = V(\rho) \cos \theta = V(\rho) \frac{x_d - x_i}{\sqrt{(x_d - x_i)^2 + (y_d - y_i)^2}}, \]  
(30)
This drives the system velocities to their desired direction as given by the equilibrium relation system of PDE's.

B. Results

The first simulation test problem is to check the model response to direction change command, where \(y - axis\) velocity is positive and \(x - axis\) is negative (i.e. \(-v_f\) for the initial condition in the \(x\) direction). The result of the density profile is shown in Fig. 3, and its corresponding contours in Fig. 4, where as time progresses, crowd (density) is moving as expected towards the left upper corner, while at the same time density distribution is changing. From this we can conclude that after some time the crowd starts to diffuse as the pedestrians move together. This is a reasonable response, since from the anisotropic property we don’t expect pedestrians to stick with their initial density profile. Especially if there is a space where they can increase their speed, and at the same time stay in a group. The second test results are given in Fig. 5, and Fig. 6, where \(v_f\) is lowered which slows the pedestrian speed as shown. For the third test, we apply positive velocities in the \(x\) and \(y\) directions, and direct the crowd towards an exit by (30). The response we achieved from the system is shown in Fig. 7, and Fig. 8. The simulation predicts jam density at the exit which is verified by the observed behavior in emergency evacuation situation near exits.
VII. CONCLUSIONS AND FUTURE WORK

The evacuation problem is a two-dimensional multi-variable problem and this work is an attempt to understand the crowd dynamic flow in normal and panic situations. A macroscopic model is developed based on observed collective behavior in normal and emergency situations where detailed design of interactions is overshadowed by group behavior. We have presented a two dimensional microscopic-to-macroscopic model of crowd dynamic flow. The macroscopic model is a nonlinear hyperbolic system of PDE’s. The model carries the anisotropic behavior to the macroscopic setting from its microscopic origin. This feature is important in studying crowd behavior as shown by the results. For Future work, comparing the model developed with empirical data will be useful to validate the model and to find the sensitivities of its parameter.

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REFERENCES