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Proactive Travel Time Predictions Under Interrupted Flow Condition

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Abstract

This research is focused on the development of a model for estimating arterial travel time by utilizing Automatic Vehicle Location (AVL) system-equipped bus as a probe vehicle. As an initial achievement, a prototype arterial travel time estimation model, applied to the bus arrival time estimation, was developed. The methodology adopted in this phase of the travel time estimation model was the on-line parameter adaptation algorithm. Three objectives were identified for this phase of the research. These were: 1) studying dynamics of bus behavior at a single bus stop, 2) extending the dynamics of bus behavior study to multiple bus stops, and 3) developing a prototype bus arrival time prediction model. The prototype travel time estimation was tested and evaluated through the simulation.

Introduction

Deterministic model of bus operation was first introduced by Newell and Potts [1]. Bell and Cowell [2] suggested the more descriptive dynamic model which covers bus journey times between the single bus stop and multiple bus stops, and expanded Newell and Potts' model by introducing recursive autoregressive model. However, one of the unrealistic assumptions that both of those former researchers had made was that the passenger arrival rate at bus stop and boarding rate were time independent values. In reality this assumption is not valid and, therefore, it is assumed in this research that passenger arrival rate and boarding rate are time dependent.

A prototype model development in this research consisted of three tasks. The first task was focused on the study of dynamics of bus behaviors at a single bus stop. Number of boarding passengers were simulated based on time varying passenger arrival rate and boarding time. The second task was based on the study of travel time estimation at multiple bus stops. The dynamics of bus behaviors at multiple stops were simulated based on ratio between passenger arrival rate and passenger boarding rate. The main variable focused on this simulation was the departure time headway. Bus bunching is discussed at the end of this task. Finally, arrival time prediction model based on parameter adaptation algorithm [3,4,5,6,7] was developed. In this model, least square parameter adaptation algorithm [7] with forgetting factors was adopted. Two simulations scenarios, one with constant and the other with varying parameters at each bus stop, were tested in order to identify the parameter update capability. The prediction model was analyzed according to parameter errors and estimation errors. Currently, discrete time version of sliding mode parameter estimation algorithm is being developed, for sliding mode technique can more effectively accommodate varying parameters. We will next study the impact of signalized intersections on travel time prediction, which will be a subject of future paper.

Single Bus at a Single Bus Stop

Model Formulation

The initial approach of development of travel time estimation model in this research relies on the extended autoregressive model of Bell and Cowell [2]. The characteristics of autoregressive model is that the most recent output affects the current status of model the most through the adaptive process. The enhancement made in this research was the adoption of time varying passenger

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arrival rate and boarding rate were considered in order to be more realistic and accurate model for interpreting the real-world behavior of transportation system.

The first modeling approach concentrated on the simplest architecture of the environment as illustrated in Fig. 1. Dynamics of bus operation in between the stops were investigated. Although the model is simple, it retains the essential dynamics of the bus operation. The formulation of model is as follows:

$$z_{i+1} = h_{i+1} - \alpha_{avg,i}x_i + \alpha_{avg,i+1}x_{i+1} \quad (1)$$

where, z_{i+1} = departure time headway between bus i and bus i+1 (minute/vehicle)

h_{i+1} = arrival time headway between bus i and bus i+1 (minute/vehicle)

$\alpha_{avg,i}$ = average boarding time at stop i (minute/passenger)

x_i = no. of passenger boarding on the bus at stop i (no. of passenger/vehicle)

From Fig. 1, $p_{avg,i+1}$ and $\alpha_{avg,i+1}$ can be calculated as

$$p_{avg,i+1} = \frac{\int_{t2}^{t4} p(t)dt}{z_{i+1}} \quad (2)$$

where, p(t) = passenger arrival rate at time t (passenger/minute)

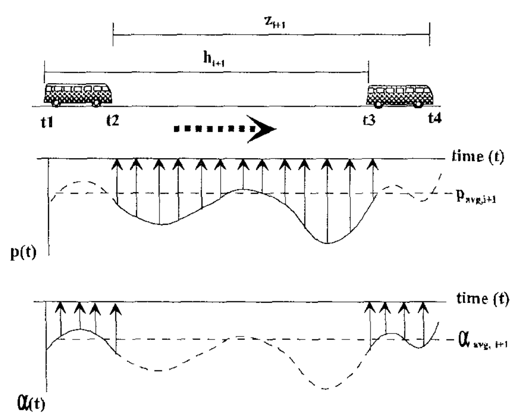

Fig. 1. Travel time estimation at single bus stop

$$\alpha_{avg,i+1} = \frac{\sum_{k=0}^{x_{i+1}} \alpha_k}{x_{i+1}} \quad (3)$$

where, α_k = passenger boarding time (minute/passenger).

The number of passengers boarding the $i+1^{th}$ bus will be the integration of passenger arrival rate between t_2 and t_4. As noticed here, passenger arrival rate (p) is considered to be a continuous function of time, but passenger boarding time (α) is discontinuous function of time. In reality, this is not the case, but we can perform analysis and simulation using this assumption, because these conditions functions emulate the actual behavior in an average sense.

$$x_{i+1} = \int_{t2}^{t4} p(t)dt \quad (4)$$

Thus, from (1) and (2), (4) can be rewritten as

$$x_{i+1} = p_{avg,i+1}z_{i+1} = p_{avg,i+1}(h_{i+1} - \alpha_{avg,i}x_i + \alpha_{avg,i+1}x_{i+1}) \quad (5)$$

$$(1 - \alpha_{avg,i+1}p_{avg,i+1})x_{i+1} = -\alpha_{avg,i}p_{avg,i+1}x_i + p_{avg,i+1}h_{i+1}. \quad (6)$$

Therefore,

$$x_{i+1} = \frac{-\alpha_{avg,i}p_{avg,i+1}}{(1 - \alpha_{avg,i+1}p_{avg,i+1})}x_i + \frac{p_{avg,i+1}}{(1 - \alpha_{avg,i+1}p_{avg,i+1})}h_{i+1}. \quad (7)$$

From (7), sensitivity analysis based on different α_{avg} (averaged passenger boarding time) and $p_{avg,i+1}$ (averaged passenger arrival rate) were performed to test the stability of the number of boarding passengers.

Model Simulation

The purpose of this simulation was to identify the behavior of buses at a bus stop. Specifically, sensitivity of total number of boarding passengers were investigated under the different sets of passenger arrival rate and boarding time. In order for the perturbation to be damped over successive buses, as it was suggested by Bell and Cowell [2], the passenger arrival rate should be less than half of the boarding rate. Simulation proved the above system stability condition. Another issue verified in the simulation was that at system equilibrium the number of boarding passengers equals the product of headway and passenger arrival rate (i.e., $x = ph$). The simulation results confirmed this condition.

In the first simulation, p = 0.1*(1+0.1*sin(1.5*i)) and α = 0.001*(1+0.001*sin(1.5*i)) were used. The result is shown in Fig. 2. In this case, passenger arrival rate is considerably smaller than the boarding rate. Fig. 2 shows that after a short transient, the variable reaches a steady state with minor oscillation.. When p = 0.51*(1+0.51*sin(1.5*i)) and α = 0.01*(1+0.01*sin(1.5*i)) were used. The result, as illustrated in Fig. 3, will obtained. Passenger arrival rate is smaller than boarding rate, but it is not significantly smaller than the previous case. Thus, even though its amplitude is large, it remains stable. However, when p = 0.51*(1+0.51*sin(1.5*i)) and α = 0.90*(1+0.90*sin(1.5*i)) were used, variable representing number of boarding passengers showed an oscillatory behavior as is illustrated in Fig. 4. In this case, passenger arrival rate is not smaller than half of the boarding rate, thus perturbation occurred. Its result is shown in Fig. 4.

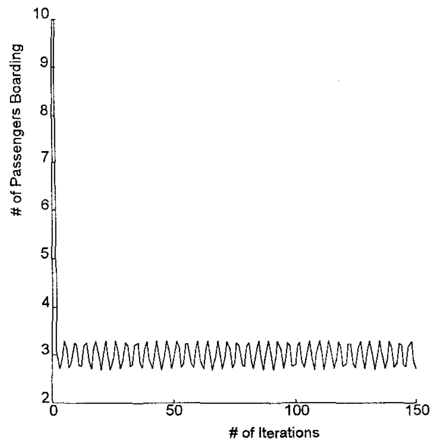

Fig. 2. Number of passenger boarding (stable condition)

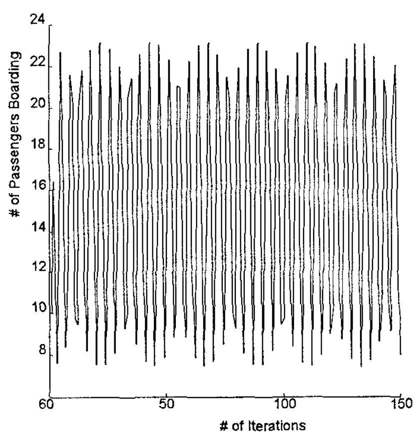

Fig. 3. Number of passenger boarding (stable condition)

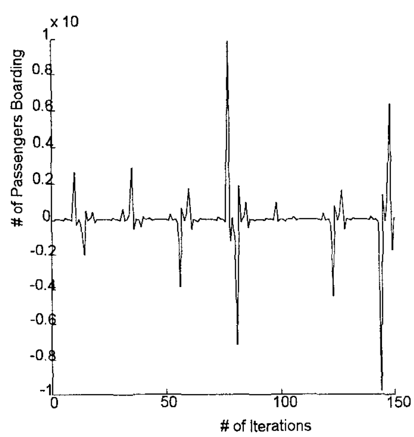

Fig. 4. Number of passenger boarding (unstable condition)

TRAVEL TIME PREDICTION ON MULTIPLE BUSES AT MULTIPLE BUS STOPS

Formulation

The formulation of simulation model, based on Newell and Potts' model [2], assumes the time independence of passenger arrival rate and bus boarding time. Thus, number of passenger boarding on bus m at stop n equals the product of arrival time and arrival rate, and the product of loading time and loading rate. This statement leads to the following result.

$$k_{mn} = \frac{\text{loading time}}{\text{passenger arrival time}} = \frac{\text{passenger arrival rate}}{\text{loading rate}} \quad (8)$$

In here, m denotes bus number, and n stands for stop number.

According to (8), the following can be derived.

$$t_{mn} - t_{mn-1} - T_{mn} = k_{mn}(t_{mn} - t_{m-1n}) \quad (9)$$

$$t_{mn} = \frac{t_{mn-1}}{1-k_{mn}} - \frac{k_{mn}}{1-k_{mn}} t_{m-1n} + \frac{T_{mn}}{1-k_{mn}} \quad (10)$$

t_{mn} is the time when the bus m leaves the stop n, τ denotes the headway of bus, and T_{mn} is the travel time of bus m between stop n and previous stop n-1.

Simulation

Simulation of dynamics of bus behavior at multiple stops was based on the dynamics (10). t_{m-1n} was assumed to be known from the AVL probe vehicle data, and we assumed. values of loading time and time taken from i^{th} to $i+1^{th}$. First, steady state simulation (constant loading time and k) was performed for four bus stops with twenty buses. Fig. 5 illustrates dynamics of bus behaviors on steady state (constant α and k). Then, simulation for nonsteady state was performed with the same assumptions except that α and k were taken as time vary 9by adopting random numbers. Fig. 6 illustrates dynamics of bus behavior for nonsteady condition.

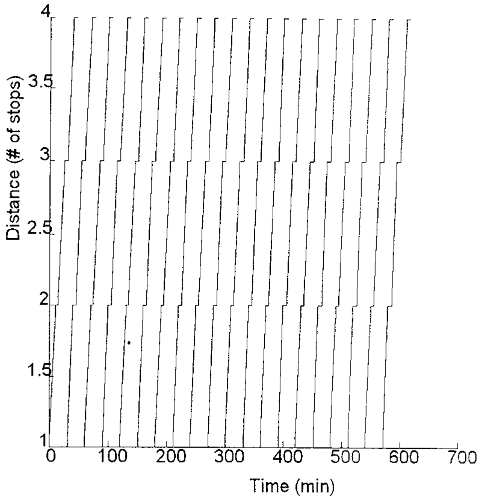

Fig. 5 Dynamics of bus behavior at steady state

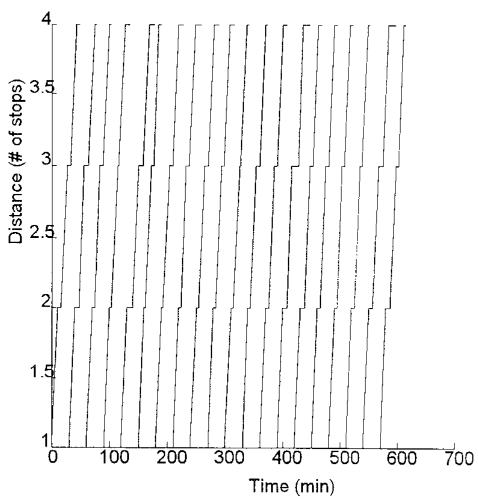

Fig. 6 Dynamics of bus behavior at nonsteady state

In the nonsteady state condition, when the loading time of bus gets longer due to delay, then the loading time of following bus at the same stop gets shorter. On the other hand, since the previous bus left early because of less passengers, the following bus at the same stop stays longer, and so on. This disturbance due to bus bunching, can be seen in Fig. 6. The varying loading time of nonsteady state is descriptively shown in Table 1.

The next step in the analysis is the prediction of bus arrival time at stops. Least square parameter estimation algorithm [7] was adopted for identification of the system for this analysis.

Table 1 Nonsteady state loading time

no. of buses	*loading time at the second stop*	*loading time at the third stop*	*loading time at the forth stop*
1	4.0948	3.2352	6.3943
2	3.1012	3.2077	5.8573
3	3.0400	5.7365	3.4025
4	8.3906	8.2457	13.8082
5	5.5854	2.544	2.3059
6	7.7199	11.6253	13.1391
7	2.7395	4.3987	0.9239
8	9.1491	13.6022	9.3529
9	8.7883	5.1909	3.9725
10	2.7438	5.4893	7.7680
11	6.2809	9.5145	7.2424
12	4.3502	3.8504	2.3321
13	9.9517	12.4607	5.2881
14	5.0404	3.2215	2.0689
15	6.1930	5.0543	4.1420
16	3.0860	5.4645	4.9753
17	10.6106	8.3153	8.4602
18	2.3969	4.5456	3.2545
19	3.9459	3.1066	8.3737
20	7.6035	10.0346	10.4576

A Prototype Arrival Time Model

Adopting from Bell and Cowell's model [2], travel time between two locations including a single stop can be expressed as

$$t_i = c + \alpha x_i \qquad (11)$$

where, c = departure time headway that excludes passenger loading time at a bus stop

αx_i = passenger loading time at a bus stop.

Substituting x_i from (7) to (11), the general form of recursive arrival time estimation model becomes

$$t_i = \frac{-\alpha p}{1-\alpha p} t_{i-1} + \frac{c + \alpha p h}{1-\alpha p}. \qquad (12)$$

This equation describes the recursive approach for travel time estimation. The current travel time between the stops relies on the most recent travel time.

Now, let's consider the case of travel time prediction between two locations that include two buses. According to the Bell and Cowell's suggestion [2], we have to add two consecutive travel times in two stops to forecast bus arrival time for the following bus. However, even though current travel time is based on that of the previous one, it can not be simply summed up for forecasting the next bus. It is explained in the following statements. If we add two travel times on a route, the total travel time can be expressed as

$$t_{i,1} + t_{i,2} = \frac{-\alpha p}{1-\alpha p} t_{i-1,1} + \frac{c + \alpha p h_{i,1}}{1-\alpha p} + \frac{-\alpha p}{1-\alpha p} t_{i-1,2} + \frac{c + \alpha p h_{i,2}}{1-\alpha p} \qquad (13)$$

where, $t_{i,1}$ = time taken the i^{th} bus to pass the first stop

$t_{i,2}$ = time taken the i[th] bus from the first stop to the second stop

$h_{i,1}$ = arrival time headway between i-1 and i[th] bus at the first stop

$h_{i,2}$ = arrival time headway between i-1 and i[th] bus at the second stop

Equation (13), however, can not be expressed as one time variant equation. For example,

$$\alpha_1 x_1 + \alpha_2 x_2 = \alpha_3(x_1 + x_2)$$
$$\alpha_3 = \frac{\alpha_1 x_1 + \alpha_2 x_2}{x_1 + x_2}$$
$$\alpha_3 = f(x_1, x_2) \tag{14}$$

From (14), α_3 is the function of two variables (x_1 and x_2). Therefore, travel time on multiple stops should be estimated separately in the stepwise manner, instead of summing them up for one route to calibrate predicted travel time. For this, segment by segment calibration method is introduced. The description of new approach in this research is as follows.

From Fig. 7, estimated arrival time headway in the first stop will be expressed as

$$\hat{h}_{i,1} = \tau + c - \xi_{i-1,1} \tag{15}$$

where, $\hat{h}_{i,1}$ = estimated arrival time headway for bus i at stop no.1

τ = initial arrival time headway

$\xi_{i-1,1}$ = arrival time of previous bus (i-1) at stop no. 1

Then travel times on the s^{th} stop will be estimated as

$$\hat{t}_{i,1} = \frac{-\alpha p}{1-\alpha p} t_{i-1,1} + \frac{c + \alpha p \hat{h}_{i,1}}{1-\alpha p} \tag{16}$$

$$\hat{h}_{i,2} = t_{i,1} + c - \xi_{i-1,2} \tag{17}$$

$$\hat{t}_{i,2} = \frac{-\alpha p}{1-\alpha p} t_{i-1,2} + \frac{c + \alpha p \hat{h}_{i,2}}{1-\alpha p} \tag{18}$$

...

$$\hat{h}_{i,s} = t_{i,s-1} + c - \xi_{i-1,s} \tag{19}$$

$$\hat{t}_{i,s} = \frac{-\alpha p}{1-\alpha p} t_{i-1,s} + \frac{c + \alpha p \hat{h}_{i,s}}{1-\alpha p} \tag{20}$$

where, i = bus number

s = stop number

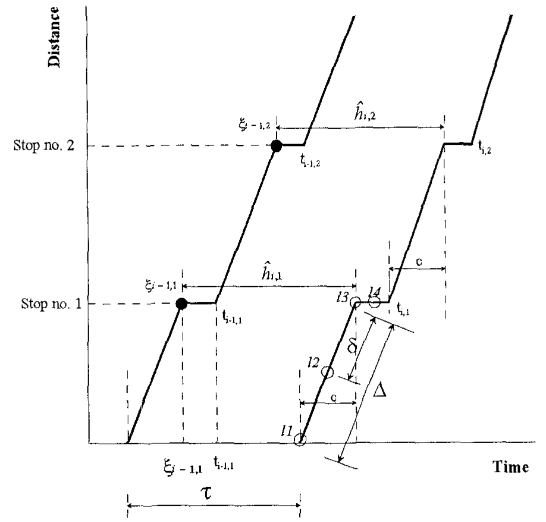

Fig. 7. A prototype arrival time estimation model in time-space domain

Parameter Estimation

The underlying assumption in parameter adaptation algorithm [3,4,5,6,7] is that system structure is known, but parameter values are unknown. In general, two approaches, off-line and on-line computational methods, are utilized for the system identification in order to estimate the parameters. Since parameters are usually time-dependent for the bus dynamics, application of on-line estimation is appropriate.

Least square parameter adaptation algorithm is one of the most popular parameter estimation algorithms. The function of adaptation mechanism is as follows:

$$y(k) = -ay(k-1) + bu(k) = \theta^T \phi(k-1) \tag{21}$$

where, $\theta^T = [-a \;\; b]$ and $\phi(k-1) = \begin{bmatrix} y(k-1) \\ u(k) \end{bmatrix}$

Since time taken from m-1 to m^{th} stop was formulated as (20), it can be rewritten as

$$t_m = -at_{m-1} + bh_m + c \tag{22}$$

where, $\frac{-\alpha p}{1-\alpha p} = -a$, $\frac{\alpha p}{1-\alpha p} = b$, and $\frac{c}{1-\alpha p} = d$.

Thus,

$$\theta^T = [-a \;\; b \;\; d] \text{ and } \phi(k-1) = \begin{bmatrix} t_{m-1} \\ h_m \\ 1 \end{bmatrix}.$$

The estimated output would be identified by an estimate of parameter vector $\hat{\theta}$. It can be written as

$$\hat{y}(k) = \hat{\theta}^T(k)\phi(k-1) \tag{23}$$

The estimation output error would be

$$e(k) = y(k) - \hat{y}(k) \tag{24}$$

Least square estimate (LSE) minimizes the summation of the squared prediction errors, i.e,

$$E = \sum_{k=1}^{n}\left[y(k) - \hat{\theta}^T(n)\phi(k-1)\right]^2 \quad (25)$$

The LSE can be found by setting the partial derivative of E with respect to $\hat{\theta}$ to zero.

$$\frac{\partial E}{\partial \hat{\theta}} = -2\sum_{k=1}^{n}\left[y(k) - \hat{\theta}^T(n)\phi(k-1)\right]\phi(k-1) \quad (26)$$

Therefore,

$$\hat{\theta}(k) = F(k)\sum_{k=1}^{n}\phi(k-1)y(k) \quad (27)$$

$$\text{where, } F(k) = \left[\sum_{k=1}^{n}\phi(k-1)\phi^T(k-1)\right]^{-1} \quad (28)$$

$\hat{\theta}(k+1)$ in the recursive form will be

$$\hat{\theta}(k+1) = \hat{\theta}(k) + \textit{correction term} \quad (29)$$

Note that

$$F^{-1}(k+1) = F^{-1}(k) + \phi(k)\phi^T(k) \quad (30)$$

and

$$\sum_{k=1}^{n}\phi(k-1)y(k) = \hat{\theta}(k)F^{-1}(k) \quad (31)$$

Therefore,

$$\hat{\theta}(k+1) = F(k+1)[\{F^{-1}(k+1) - \phi(k)\phi^T(k)\}\hat{\theta}(k) + \phi(k)y(k+1)]$$

$$= F(k+1)F^{-1}(k+1)\hat{\theta}(k) + F(k+1)\phi(k)[y(k+1) - \hat{\theta}^T(k)\phi(k)]$$

$$= \hat{\theta}(k) + F(k+1)\phi(k)[y(k+1) - \hat{\theta}^T(k)\phi(k)] \quad (32)$$

Since $\hat{\theta}^T(k)\phi(k)$ represents a priori predicted output based on the parameter estimate vector at time k, it can be replaced with

$\hat{y}^0(k+1) = \hat{\theta}^T(k)\phi(k)$ and $\hat{y}(k+1) = \hat{\theta}^T(k+1)\phi(k)$. (33)

Prediction errors can be expressed as

$e^0(k+1) = y(k+1) - \hat{y}^0(k+1)$ and

$$e(k+1) = y(k+1) - \hat{y}(k+1). \quad (34)$$

Therefore,

$$\hat{\theta}(k+1) = \hat{\theta}(k) + F(k+1)\phi(k)e^0(k+1) \quad (35)$$

Equation (31) is the recursive form of the parameter adaptation algorithm.

From the matrix inversion lemma, gain term would be expressed as

$$F(k+1) = F(k) - \frac{F(k)\phi(k)\phi^T(k)F(k)}{1+\phi^T(k)F(k)\phi(k)} \quad (36)$$

Multiplication of $\phi(k)$ in (33) will yield

$$F(k+1)\phi(k) = \frac{F(k)\phi(k)}{1+\phi^T(k)F(k)\phi(k)} \quad (37)$$

Therefore, parameter updating law will be

$$\hat{\theta}(k+1) = \hat{\theta}(k) + \frac{F(k)\phi(k)}{1+\phi^T(k)F(k)\phi(k)}e^0(k+1) \quad (38)$$

Relationship between e^0 and e can be identified by manipulating (32) and (33).

$$e(k+1) = y(k+1) - \hat{\theta}^T(k+1)\phi(k) \quad (39)$$

$$= y(k+1) - [\hat{\theta}(k) + \frac{F(k)\phi(k)}{1+\phi^T(k)F(k)\phi(k)}e^0(k+1)]^T\phi(k)$$

$$= y(k+1) - \hat{\theta}^T(k)\phi(k) - \{[F(k)\phi(k)e^0(k+1)]^T[1+\phi^T(k)F(k)\phi(k)]^{-1T}\phi(k)\}$$

$$= e^0(k+1) - \frac{e^{0T}(k+1)\phi^T(k)F^T(k)\phi(k)}{1+\phi^T(k)F(k)\phi(k)}$$

$$= \frac{e^0(k+1)}{1+\phi^T(k)F(k)\phi(k)}$$

Forgetting Factor

The vector θ was assumed to be time invariant in the derivation of least square formula. However, in the time dependent parameter identification problem the time decreasing adaptation gain factor, F(k), is not suitable because of its weak adaptation functionality. Due to this reason, forgetting factor, λ, is introduced. Therefore, new performance index will be

$$E = \sum_{k=1}^{n}\lambda^{n-k}[y(k) - \hat{\theta}^T(n)\phi(k-1)]^2 \quad (40)$$

where, $0 < \lambda < 1$

From adopting Landau and Silveira's general formula, F(k) can be represented as

$$F^{-1}(k+1) = \lambda_1(k)F^{-1}(k) + \lambda_2(k)\phi(k)\phi^T(k) \quad (41)$$

where, $F(0) > 0$,

$0 < \lambda_1(k) \le 1$ and $0 < \lambda_2(k) \le 2$.

According to the matrix inversion lemma, *F(k)* will be updated as

$$F(k+1) = \frac{1}{\lambda 1(k)}[F(k) - \frac{F(k)\phi(k)\phi^T(k)F(k)}{\lambda 1(k)/\lambda 2(k) + \phi^T(k)F(k)\phi(k)}] \quad (42)$$

Results

Bus arrival time estimations and estimation errors for each stops were the main outputs to be analyzed in this simulation. Actual parameters *a, b,* and *d* were assumed to be following continuous functions.

*a = -0.2+0.05*sin(0.13*(i-1))*

*b = 0.2+0.05*sin(0.13*(i-1))*

*d = 12+0.75*sin(0.13*(i-1))*

where, *i* = number of simulating buses

First Simulation

The validity of the estimation scheme for bus arrival times described above was tested using MATLAB. In the simulation plot, x axis represents the clock time (sequential time), and y axis represents absolute time. 100 buses with four bus stops were simulated so that estimation of arrival time can be predicted for nineteen buses (arrival time of the 1st bus was assumed to be known) at three stops (2nd, 3rd, and 4th stops). Figure 8 illustrates estimated arrival time of the second bus. In the figure, the first plot illustrates the arrival time of bus at the second stop, the second plot shows arrival time at the third stop, and the third plot shows arrival time at the forth stop.

The bus was assumed to be located at the zero coordinates at the initial time. Hence, estimated arrival time was predicted as 40 minutes (headway 30 minutes + travel time 10 minutes) in absolute time in the top plot in Fig. 8. In the second plot, the jump in the value of arrival time indicates the update of the estimated values by the algorithm. In the bottom plot, two updates were made because of the information from the previous bus at previous bus stops (i.e., information was obtained two times). Thus, the accuracy of prediction improves with time.

Second Simulation

Estimation errors (difference between actual and estimated arrival time) were plotted for each stop against each bus. Figure 9, 10, and 11 show estimation errors for all the stops plotted against the number of buses. Estimation errors converged to zero as bus goes on. The results obtained seem encouraging.

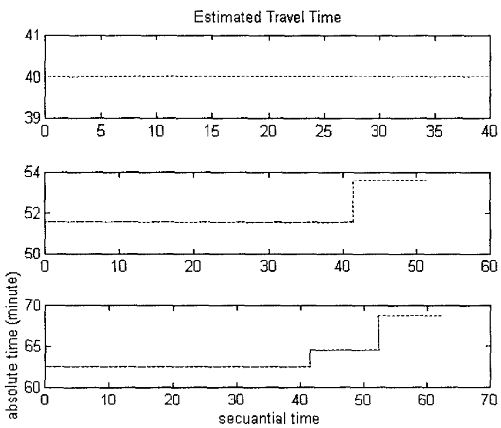

Fig. 8. Estimated travel time for the second bus

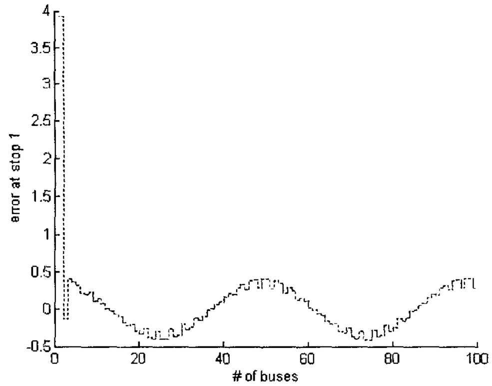

Fig. 9. Estimation error at the second stop

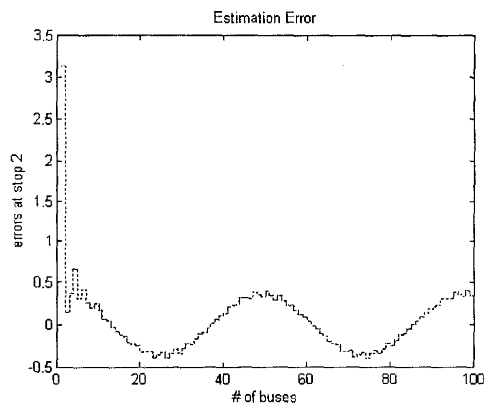

Fig. 10. Estimation error at the third Stop

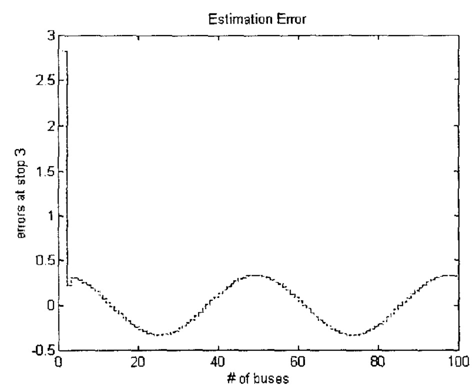

Fig. 11. Estimation error at the forth stop

CONCLUSION

We devised a new scheme based on least square estimation with forgetting factor for bus arrival time estimation. The theoretical foundation was laid, and confirmed by computer simulations. The results were encouraging, and the results can be further improve by appropriately modifying the techniques shown in this paper.

Accurately predicted short-term travel time is a good Measure of Effectiveness (MOE) for proactive mode in establishing control strategies for ATMS, route guidance for ATIS, and high quality of user services for APTS. The outcome of this project will provide both accurate and reliable forecasting of bus arrival time information to the regular and potential bus transit users. Travel time model development for normal traffic on the arterial road is the final goal of this research. By considering the conversion factors of lane usage of buses and dynamic characteristics of bus behaviors, and by excluding passenger loading times, a platform can be established to interpret normal traffic travel time from bus travel time information.

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