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Parametric study of stimulus-response behavior incorporating vehicle heterogeneity in car-following models

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PARAMETRIC STUDY OF STIMULUS-RESPONSE BEHAVIOR INCORPORATING VEHICLE HETEROGENEITY IN CAR-FOLLOWING MODELS

by

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ABSTRACT

Parametric Study of Stimulus-Response Behavior Incorporating Vehicle Heterogeneity in Car-Following Models

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The objective of this study was to develop a family of car-following models that address the shortcomings of car-following models developed by General Motors (GM) in the 1950s. The developed models consist of separate models for acceleration, deceleration, and steady-state responses for congested freeway traffic conditions. The study calibrated the models using individual vehicle trajectory data collected on a segment of Interstate 101 in Los Angeles, California. Furthermore, the study validated the models using individual vehicle trajectory data collected on a segment of Interstate 80 in Emeryville, California. The study used nonlinear regression with robust standard errors to estimate the model parameters and obtain the distribution of the model parameters across drivers and for different pairs of following vehicles. The stimulus response thresholds that delimit acceleration and deceleration responses were determined based on Signal Detection Theory.

The results indicate that average drivers' response time lag is significantly lower for deceleration response than for acceleration response. This is intuitive because deceleration response is generally related to safety, thus, drivers are expected to respond faster than for acceleration response. Acceleration is a response that is related to drivers' desire to attain maximum speeds which is the less urgent need than safety. Additionally,

drivers' response to negative stimulus is sometimes further aided by activation of brake lights for a leading vehicle that is braking. For similar safety reasons, the results show that average stimulus threshold is significantly lower for deceleration response than acceleration response and with higher magnitudes of parameters for deceleration response than acceleration response.

The results also indicate that drivers' behavior is significantly different for different vehicle being driven and/or followed. The results show that automobiles traveling behind large trucks have both lower magnitudes of acceleration and deceleration responses than when traveling behind other automobiles. These are unexpected results and could be due to inability of automobile drivers to see beyond large trucks in front of them.

Overall, the results confirm the need for separating models for acceleration and deceleration responses and for different pairs of following vehicles because they impact drivers' behavior differently. However, both the driver response time lags and stimulus thresholds are likely to depend on speed and vehicle separation. This research simplified the models and determined the driver response time lags and stimulus thresholds independent of these factors.

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DEDICATION

I dedicate this work to my late parents, sister, and brother. Their legacy has proven to me that there is no mountain higher as long as GOD is on my side.

CHAPTER 1

INTRODUCTION

1.1 Motivation

Traffic safety and operational problems have existed since early age of automobile use as a means of transportation. Since the early 1950s, traffic has continued to grow at a dramatic rate (Gazis, 2002). Consequently, transportation engineers and planners have been concerned with their resulting implications to traffic operations and safety of the traveling public. One of the major challenges that are facing transportation professionals is the accuracy of computer models they use for planning and operational analysis of highways. Analyzing operational impacts of the proposed short-term and long-term policies and strategies require traffic performance data. However, traffic performance data are typically either unavailable or too expensive to collect from the field. Therefore, with the increase in computing power, many transportation agencies rely on traffic microscopic simulation to analyze the performance of transportation systems.

Typically, traffic engineers use traffic simulation models to evaluate operational benefits and consequences of the proposed alternative policies and strategies for improving safety and traffic operational efficiency. However, the reliability of results and conclusions drawn from traffic simulation rely heavily on the accuracy of traffic simulation models used for evaluation and analysis. One of the key components in traffic simulation models are car-following models, which are designed to emulate drivers' car-following behavior in the same lane.

1.2 History of Car-Following Models

Studies on car-following behavior started in the early 1950s (Reuschel, 1950 and Pipes, 1953). Reuschel and Pipes both were independently inspired by the vehicle separation law of the California Vehicle Code, which states that "A good rule for following another vehicle at a safe distance is to allow yourself the length of a car (about fifteen feet) for every ten miles per hour you are traveling". They developed traffic dynamic models that emulate such driving behavior. The models expressed minimum separation of a following vehicle behind a leading vehicle as a linear function of speed. The developed models assumed that drivers responded instantaneously to the actions of a leading vehicle. Forbes (1963) addressed this limitation by incorporating a driver reaction time component into the model.

In the mid-1950s, researchers associated with the General Motors (GM) (Chandler et al., 1958) developed five series of models that modeled acceleration and deceleration response behavior of a following vehicle due to the driving actions of a leading vehicle. The concept of the models pursued by GM was similar to that of Reuschel, Pipes, and Forbes. However, the upgraded models assumed that a driver response to the actions of a leading vehicle as a function of driver sensitivity and stimulus. The GM models define as the relative speed between the two following vehicles. Negative relative speed, when the leading vehicle travels slower than the following vehicle, triggers a deceleration response. Conversely, a positive relative speed, when the leading vehicle travels faster than the following vehicle, triggers an acceleration response. Gazis et al. (1961) generalized stimulus-response models by further improving on the driver sensitivity term. The resulting was a nonlinear model that had the driver

sensitivity term proportional to speed of the following vehicle and inversely proportional to vehicle spacing.

Since then, numerous studies have attempted to modify parameters of the GM model with the aim of improving drivers' car-following behavior. Some of these studies include Eddie (1960), May and Keller (1967), Heyes and Ashworth (1972), Ceder and May (1976), Ceder (1976, 1978). Similarly, Aron (1988), Ozaki (1993), and Subramanian (1996) modified the structure of the GM model by separating acceleration and deceleration models. Ahmed (1999) further improved Subramanian's model by adding traffic density in the sensitivity term and assumed nonlinearity on the stimulus term. Similarly, Toledo (2003) re-estimated parameters of the nonlinear model proposed by Subramanian.

Car-following models are one of the important components of traffic simulation programs. The most commonly used programs for traffic simulation applications include VISSIM, PARAMICS, and CORSIM. They use car-following models that are similar to the stimulus-response models. However, VISSIM and PARAMICS use car-following models that delimit driving behavior into several regimes. Demarcation of the regimes is based on thresholds (i.e. limits) of human perception of differential speed and distance. For example, VISSIM uses a car-following model with thresholds that delimit driving process into four types of driving behavior (Wiedemann, 1974). The types include "un-influenced driving", "closing process", "following process", and "emergency braking". In each type, drivers behave differently when reacting to differential speed and distance. Therefore, each type has different procedures for calculating values of the acceleration or

deceleration responses. This brief history of car-following models indicates there is a need for further research in this area for improving drivers' car-following behavior.

In summary, the existing stimulus-response car-following models for emulating drivers' car-following behavior as reviewed above have three major shortcomings.

1. The models assume that drivers can detect even small stimulus, which is unrealistic. Drivers are expected to detect the stimulus if it exceeds a certain detectable threshold.
2. The models assume the same distribution of driver response time lags for all drivers and ignore differences between vehicle types. Drivers are expected to have different response time lags and sensitivity for similar magnitudes of stimulus.
3. The models estimate a single value for each of the other model parameters including speed, relative speed, and separation. Estimating a single value for each parameter of the model does not capture individual differences between different drivers and different vehicle type being driven and/or followed. For example, some drivers of automobiles may behave differently when driving behind a large truck partly because of real and/or perceived safety risks imposed by large trucks. On the contrary, large trucks have low acceleration and deceleration capabilities than automobiles. Therefore, large truck drivers generally try to compensate these limitations by leaving longer space headways than automobile drivers.

1.4 Objectives of the Study

The objective of this study is to develop a family of car-following models that, among other things, address the shortcomings of the GM models. This proposed family

of models consists of separate models for acceleration, deceleration, and steady-state responses for congested freeway traffic conditions. These models are designed to:

1. Determine driver response time lags for both acceleration and deceleration responses
2. Determine stimulus response thresholds for both acceleration and deceleration responses.
3. Incorporate vehicle heterogeneity in the models. For each acceleration or deceleration response, three car-following models are developed depending on the types of vehicles following each other. The models include "automobile following automobile", "automobile following large truck", and "large truck following automobile".
4. Capture heterogeneity in driving behavior across drivers by estimating distributions of drivers' response time lags, stimulus response thresholds, and other model parameters for speed, relative speed, and vehicle separation for acceleration and deceleration responses.

1.5 Hypotheses of the Study

This research aims to test four major hypotheses:

1. Driver response time lags are lower for deceleration response than for acceleration response. Deceleration is a response related to safety, therefore, one would expect a faster response time (i.e. small time lag). On the other hand, the acceleration response is related to driver's desire to attain maximum speeds which is not a critical need, therefore, does not require urgent response.

2. For the same reasons stated above, the stimulus response thresholds are lower for deceleration response than for acceleration response.
3. Drivers are likely to respond more aggressively when required to decelerate than when they want to accelerate. Therefore, a higher magnitude of the parameters is expected for the deceleration response than acceleration response.
4. Driving response behavior is different for different vehicle types being driven and/or followed. This is due to the fact that different vehicle types have different physical and performance characteristics that may impact drivers' response behavior.

1.6 Significance of the Study

The findings of this study are expected to significantly contribute to the understanding of drivers' car-following behavior on congested limited access highways. This knowledge will be useful in improving the accuracy of car-following models used in traffic simulation. To that end, this will assist traffic researchers and practitioners in modeling more accurately impacts of proposed alternative policies and strategies to improve safety and traffic performance of existing and future planned highways. Additionally, the estimated drivers' response time lags can be used in roadway design in calculating important design parameters such as stopping sight distance.

1.7 Dissertation Outline

This dissertation is divided into seven chapters. Chapter 1 describes the problems with the existing stimulus-response car-following models and presents the objectives and significance of the study. Chapter 2 presents the literature review on car-following

models. Chapter 3 describes in detail a family of car-following models proposed in this study. Chapter 4 discusses implementation methodology for estimating parameters of the models. Chapter 5 presents the results and discusses their implications in car-following behavior. Chapter 6 presents and describes the process for validating the models, results obtained and their implications. Chapter 7 presents conclusions and recommendations for future research in this area.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The aim of this chapter is to review the existing car-following models. It introduces the basic concepts of the models and focuses on efforts made in improving the models. In addition, it summarizes the significant shortcomings of the existing car-following models in emulating drivers' car-following behavior.

2.2 Traffic Simulation Models

There are three types of traffic models: macroscopic, mesoscopic, and microscopic. The macroscopic traffic models describe traffic flow behavior on a segment-by-segment basis in lieu of tracking individual vehicles. As a result, the models produce aggregate traffic stream parameters such as speed, flow, and density and their corresponding relationships. The models use the equation of conservation of vehicle flow to describe the relationships and how disturbances such as shockwave propagate in the traffic stream. Examples of macroscopic models include Greenshield (1935), Greenberg (1959), Underwood (1961), Edie (1961), and Bell Curve (Duke et al., 1990). The most prevalent benefit of such models is that they can describe the spatial and temporal extent of traffic congestion particularly that is caused by non-occurring incidents such as traffic crashes.

The microscopic traffic models describe the movement and interactions of an individual vehicle with a leading vehicle. The models track vehicles on at a certain time interval and produce observations of vehicle longitudinal and lateral positions, speed, and

acceleration/deceleration at each time interval. The models use fundamental rules of motion and rules of driving behavior such as lane changing and car-following behavior for moving vehicles in the system. The most important benefit of microscopic models is that they are used for evaluating traffic operational performance of the existing or future planned highways.

The mesoscopic traffic models describe individual vehicle interacting with other vehicles but aggregate parameters for all vehicles. In essence, the models combine both characteristics of microscopic and macroscopic models. For example, the model can be used to evaluate average travel time and speed of a certain highway segment using individual vehicles equipped with in-vehicle real-time travel information systems. The models are most beneficial in evaluating traveler information systems. Kinetic theory based models are typical examples of mesoscopic models (Prigogine, 1971).

2.3 Car-Following Models

This study uses the following definitions and notations in describing the car-following models. Consider two following vehicles traveling from left to right as shown schematically in Figure 2-1. Vehicle $n-1$ is a leading vehicle with length L_{n-1} and vehicle n is a subject vehicle. The subscript t denotes the time of observation of vehicle position, velocity, and acceleration/deceleration.

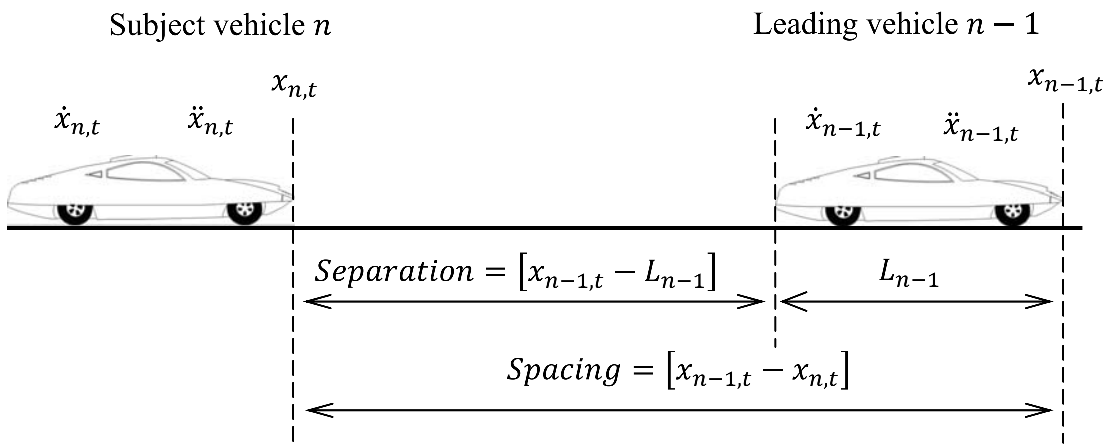


Figure 2-1. Definitions and notations used in car-following model.

The following are definitions of the variables resulting from Figure 2-1.

$x_{n-1,t}$ is the position of a leading vehicle $n-1$ at time t

$x_{n,t}$ is the position of a subject vehicle n at time t

$\dot{x}_{n-1,t}$ is the speed of the leading vehicle $n-1$ at time t

$\dot{x}_{n,t}$ is the speed of the subject vehicle n at time t

L_{n-1} is the length of the leading vehicle

$[x_{n-1,t} - x_{n,t}]$ is the spacing between the two vehicles at time t

$[x_{n-1,t} - x_{n,t} - L_{n-1}]$ is the separation between the two vehicles at time t

In the car-following mode the leading vehicle influences driving behavior of a subject vehicle. Therefore, the driver of the subject vehicle reacts to the perceived stimulus resulting from driving behavior of the leading vehicle. The stimulus could be a speed differences and/or separation between the two vehicles. Furthermore, the driver of the subject vehicle responds to the stimulus after a certain time lag. This study defines the

time lag as the driver response time lag. The driver response time lag is the interval of time between occurrence of stimulus and initiation of response.

Car-following models can be broadly divided into four main types:

- Safe distance car-following models,
- Stimulus-response car-following models,
- Psychophysical car-following models, and
- Fuzzy logic-based car-following models

2.3.1 Safe Distance Car-following Models

Reuschel (1950) and Pipes (1953) were the early pioneers who developed minimum safe distance models. They were both independently inspired by the law of vehicle separation stipulated in the California Vehicle Code, which states that "A good rule for following another vehicle at a safe distance is to allow yourself the length of a car (about fifteen feet) for every ten miles per hour you are traveling". They developed traffic models that emulate such driving behavior. The models expressed minimum safe distance maintained by a subject vehicle behind a leading vehicle as a linear function of speed. The models assumed that drivers of vehicles obeyed this rule at all times and derived model that emulate such driving behavior. The developed models assumed that drivers reacted instantaneously to the actions of a leading vehicle. Pipes model has the following form:

$$x_{n-1} = x_n + [b + T\dot{x}_n] + L_{n-1} \quad \text{2-1}$$

Where:

b is the prescribed legal distance when vehicles at standstill in feet and T in seconds is a time constant as prescribed by the California Driver Code i.e. ($T = \frac{15ft}{1.47 \times 10mph} = 1.023sec$)

This results in a minimum safe vehicle separation distance equal to:

$$d_{min} = [x_{n-1} - x_n] = c + T\dot{x}_n \qquad 2\text{-}2$$

Where $c = b + L_{n-1}$ is constant

From equation 2-2, the minimum theoretical time headway approaches T seconds when speed is at infinity. However, field measurements indicated slight variations in the minimum safe time headway derived according to Pipe's model at low and high speeds (May, 1990). The same study also showed that the minimum safe time headway does not decrease with speed at certain speed range. Furthermore, Pipe's model predicts that roadway capacity occur when speed is infinite, which is unrealistic.

Forbes (1958) developed a car-following model that incorporated a driver reaction time component. This was based on the fact that there is a time lag between occurrence of stimulus and initiation of response. The model assumed that a driver of a subject vehicle maintains minimum safe time headway at least equal to the driver reaction time. This time headway is the summation of the driver reaction time and the time taken to travel a distance equivalent to the length of a leading vehicle. Forbe's model is defined as follows:

$$h_{min} = \Delta t + \frac{L_{n-1}}{\dot{x}_n}$$

2-3

Where:

Δt is the driver reaction time

L_{n-1} is the length of a leading vehicle

$\dot{x}_{n-1}$ is the speed of a subject vehicle

Similarly, field results indicated considerable variations between the actual field measured and that obtained from Forbe's model and for different drivers (May, 1990). The field results showed that the minimum values of time headway ranged from 1 to 3 seconds. This model also has shortcomings similar to the ones discussed for Pipe's model.

Kometani and Sasaki (1958) investigated the dynamic equation developed by Pipes by introducing the driver response time lag. In this model, the spacing between two consecutive vehicles in queue is expected to depend on velocities of vehicles. For simplicity, the model assumed a linear function of the following the form:

$$\left[x_{n-1,t-\Delta t} - x_{n,t-\Delta t}\right] = \alpha \dot{x}_{n-1,t-\Delta t} + \beta \dot{x}_{n,t} + b \qquad \text{2-4}$$

Where:

Δt is the reaction time of driver

$\left[x_{n-1,t-\Delta t}\right]$ is the position of a leading vehicle $n-1$ at time $t-\Delta t$

$\left[x_{n,t-\Delta t}\right]$ is the position of a subject vehicle n at time $t-\Delta t$

$\dot{x}_{n-1,t-\Delta t}$ is the speed of the leading vehicle $n-1$ at time t

$\dot{x}_{n,t}$ is the speed of the subject vehicle n at time $t-\Delta t$

$\left[x_{n-1,t-\Delta t} - x_{n,t-\Delta t}\right]$ is the spacing between the two vehicles at time $t-\Delta t$

$\alpha, \beta,$ and b are constants

To simplify the model, the study assumed that vehicle separation is proportional to the speed of the subject vehicle. Differentiating both sides of the equation 2-4 with respect to t results in the following equation:

$$\left[\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}\right] = \beta \ddot{x}_{n,t} \qquad \text{2-5}$$

Kometani and Sasaki (1959) continued the efforts to determine safe separation based on Newtonian equations of motion. The model assumed that vehicle separation is proportional to both speed of the subject vehicle and the leading vehicle. The developed model has the following form:

$$\left[x_{n-1,t-\Delta t} - x_{n,t-\Delta t}\right] = \alpha\left[\dot{x}_{n-1,t-\Delta t}\right]^2 + \beta_1\left[\dot{x}_{n,t}\right]^2 + \beta_2\left[\dot{x}_{n,t}\right] + b \qquad \text{2-6}$$

Where β_1 and β_2 are constants and other notations are as defined in equation 2-4.

The parameters of the models were calibrated using data collected from pairs of test vehicles driving on a city street. The study collected 22 runs on a segment of 200 meters long with average speed of less than 45 kilometer per hour. The results indicated that driver reaction time value Δt of 0.5 seconds, β_1 value of -0.00028, and β_2 value of 0.585. Another experiment was conducted on the faster track and varied speeds between

40 and 60 kilometer per hour using two subjects. The results obtained showed that the value of Δt was 0.75 seconds, β_1 value of -0.00084, and β_2 value of 0.78. These parameters are larger than those obtained from a city street, suggesting significant variations in the two models. Further improvement of this model was made by Gipps (1981).

Gipps (1981) derived the model by setting the limits of performance of driver and vehicle and used the limits to calculate a safe speed with respect to a leading vehicle. In other words, the driver should not exceed his/her desired maximum speed and the vehicle should not exceed its maximum acceleration and deceleration capabilities. Furthermore, the study used additional safety margin to compensate for driver related errors equal to half of driver reaction time. The assumption made is that a driver of a subject vehicle maintains a speed which allows the driver to bring the vehicle to safe stop should a vehicle ahead come to a sudden stop. The model has the following form:

$$\left[x_{n-1,t} - x_{n,t} - L_{n-1}\right] \geq \frac{\left[\dot{x}_{n-1,t}\right]^2}{2b_{n-1}} + \left[\dot{x}_{n,t}\right]\frac{T}{2} + \left[\dot{x}_{n,t+T}\right]T - \frac{\left[\dot{x}_{n,t+T}\right]^2}{2b_n} \tag{2-7}$$

Where:

$\left[x_{n-1,t}\right]$ is the position of a leading vehicle $n-1$ at time t

$\left[x_{n,t-\Delta t}\right]$ is the position of a subject vehicle n at time t

$\dot{x}_{n-1,t-\Delta t}$ is the speed of the leading vehicle $n-1$ at time t

$\dot{x}_{n,t}$ is the speed of the subject vehicle n at time t

L_{n-1} is the length of the leading $n-1$ vehicle

$[x_{n-1,t} - x_{n,t} - L_{n-1}]$ is the separation between the two vehicles at time t

T is the reaction time of the driver

b_{n-1} is the most severe braking of the leading vehicle ($b_{n-1} < 0$)

b_n is the most severe braking of the subject vehicle ($b_n < 0$)

This model was validated by simulating a three lane lanes of divided highway. Each of the parameter was sampled from normal distributions. The results appeared to logically replicate the behavior and propagation of disturbance in traffic stream both for pairs of following vehicles and for platoon of vehicles.

2.3.2 Shortcomings of Safe Distance Car-Following Models

The structure of the models developed by Reuschel, Pipes, Forbes, Kometani and Sasaki, and Gipps were reasonable. However, the models have the following major shortcomings:

1. The models did not include other important variables such as relative speed which may influence how drivers maintain safe following distance. This may result in inaccurate modeling of drivers' acceleration and deceleration response behavior
2. The models assume the same driver response time lags and ignore differences between vehicle types.
3. The models assume similar acceleration and deceleration response aggressiveness, which is unrealistic. Drivers' behavior for acceleration and deceleration responses may be different because the need for acceleration and deceleration are different.

2.3.3 Stimulus-Response Car-Following Models

Researchers associated with the General Motors (GM) (Chandler et al., 1958) developed five series of models that described acceleration and deceleration response behavior of a subject vehicle due driving actions of a leading vehicle. The structure of the models pursued by GM was similar to that of Reuschel, Pipes, and Forbes. However, the upgraded models assumed that a driver response as a function of driver sensitivity and stimulus. The GM models define stimulus as the relative speed between the two following vehicles. Negative relative speed, when the leading vehicle travels slower than the following vehicle, triggers a deceleration response. On the contrary, a positive relative speed, when the leading travels faster than the following vehicle, triggers an acceleration response. The magnitudes of the acceleration/deceleration depend on sensitivity term which includes speed and vehicle spacing. The models have the following general form:

$$Response = f(sensitivity, stimuli) \qquad 2\text{-}8$$

Chandler et al. (1958) developed the first simple linear model that assumes acceleration/deceleration response of a subject vehicle is proportional to the relative speed between two following vehicles as shown below.

$$\ddot{x}_{n,t} = \alpha\left[\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}\right] \qquad 2\text{-}9$$

Where:

Δt is the driver response time lag

$\ddot{x}_{n,t}$ is the acceleration/deceleration of a subject vehicle n at time t

$\dot{x}_{n-1,t-\Delta t}$ is the speed of a leading vehicle $n-1$ at time $t-\Delta t$

$\dot{x}_{n,t-\Delta t}$ is the speed of the subject vehicle n at time $t-\Delta t$

$\left[\dot{x}_{n-1,t-\Delta t}-\dot{x}_{n,t-\Delta t}\right]$ is the relative speed between the two vehicles at time $t-\Delta t$

α is the driver sensitivity parameter

The stimulus term, that is, relative speed, at any time step can be either positive, zero, or negative resulting in drivers' response in form of acceleration, no response, or deceleration, respectively. The model assumes that the driver sensitivity is constant across driver population and/or vehicle types.

The study calibrated parameters of the model using instrumented cars on test track of the GM. The experiment involved two vehicles with a cable on a pulley connected with a wire wound around a reel mounted on the front of the leading vehicle. The experiment used eight drivers who drove the test cars while varying driving conditions of mean speed. The driver of the subject vehicle followed the leading vehicle while maintaining their desired safety distance. The correlation analysis between observed and estimated acceleration was used to estimate the parameters of the model. The values that produced the highest correlation were used as the estimate of the driver response time lag and sensitivity for a particular driver. Table 2-1 shows the estimated parameter values obtained from this study.

Table 2-1. Estimated Parameters (Chandler et al., 1958)

Measured Value	Response Time Lag (sec)	Sensitivity (sec-1)
Minimum	1.00	0.17
Average	1.55	0.37
Maximum	2.20	0.74

The results obtained from field experiment showed significant variation in the sensitivity values. The sensitivity term appeared to depend on the distance between the vehicles, therefore, suggested a modification of the sensitivity term.

Gazis et al. (1959) addressed this weakness of the model by incorporating spacing between two vehicles in the sensitivity term. The second model proposed that the sensitivity term should have two states depending on closeness between two following vehicles. This means that higher sensitivity value $\boldsymbol{\alpha}_1$ is applicable when the two vehicles are close together and lower sensitivity value $\boldsymbol{\alpha}_2$ when the two vehicles are far apart. This suggests that drivers are more sensitive at shorter following distance and less sensitive at larger following distance. The model is defined as shown below:

$$\ddot{x}_{n,t} = \alpha_1 \text{or } \alpha_2 \left[\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}\right] \qquad \text{2-10}$$

Where:

Δt is the driver response time lag

$\ddot{x}_{n,t}$ is the acceleration/deceleration of a subject vehicle n at time t

$\dot{x}_{n-1,t-\Delta t}$ is the speed of a leading vehicle $n-1$ at time $t-\Delta t$

$\dot{x}_{n,t-\Delta t}$ is the speed of the subject vehicle n at time $t-\Delta t$

$\left[\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}\right]$ is the relative speed between the two vehicles at time $t - \Delta t$

is the α_1 is the sensitivity parameter at smaller spacing

α_2 is the driver sensitivity parameter at bigger spacing

This model posed significant challenges in determining the values of $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$ and the problems associated with discontinuous state. These challenges necessitated further field experiments to determine the means of incorporating vehicle spacing into the sensitivity term. The results of the field experiments and numerical solutions showed that acceleration was inversely proportional to the spacing between the two vehicles. Therefore, the model was modified by incorporating the spacing into the second model resulting in third model which is defined as follows:

$$\ddot{x}_{n,t} = \frac{\alpha_0}{\left[x_{n-1,t-\Delta t} - x_{n,t-\Delta t}\right]} \left[\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}\right] \tag{2-11}$$

Where:

Δt is the driver response time lag

$\ddot{x}_{n,t}$ is the acceleration/deceleration of a subject vehicle n at time t

$\dot{x}_{n-1,t-\Delta t}$ is the speed of a leading vehicle $n - 1$ at time $t - \Delta t$

$\dot{x}_{n,t-\Delta t}$ is the speed of the subject vehicle n at time $t - \Delta t$

$\left[\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}\right]$ is the relative speed between the two vehicles at time $t - \Delta t$

$\left[x_{n-1,t-\Delta t} - x_{n,t-\Delta t}\right]$ is the spacing between the two vehicles at time $t - \Delta t$

α_0 is the driver sensitivity parameter

The sensitivity term in this model is a function of the constant $\boldsymbol{\alpha}_0$ and vehicle spacing. This model suggests that the magnitudes of the sensitivity term are higher when the vehicles are closer than when they are far apart. Similarly, field experiments were conducted to calibrate the parameter values using test drivers on the GM test track, Holland tunnel, and Lincoln tunnel. Table 2-2 shows the results obtained from correlation analysis for each test site.

Table 2-2. Estimated Parameters (Gazis et al., 1958)

Location	Number of Drivers	Response Time Lag (sec)	Sensitivity (sec-1)
GM test track	8	1.5	40.3
Holland tunnel	10	1.4	26.8
Lincoln tunnel	16	1.2	29.8

Gazis et al. (1959) further improved the sensitivity term by incorporating speed of the subject vehicle into the sensitivity term, thus, forming the fourth model. The concept of the model is based on the fact that as the speed increases, a driver of the subject vehicle becomes more sensitive to the relative speed than at lower speeds. The model is defined as shown below.

$$\ddot{x}_{n,t} = \alpha \frac{\dot{x}_{n,t-\Delta t}}{\left[x_{n-1,t-\Delta t} - x_{n,t-\Delta t}\right]} \left[\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}\right] \tag{2-12}$$

Where:

Δt is the driver response time lag

$\ddot{x}_{n,t}$ is the acceleration/deceleration of a subject vehicle n at time t

$\dot{x}_{n-1,t-\Delta t}$ is the speed of a leading vehicle $n-1$ at time $t-\Delta t$

$\dot{x}_{n,t-\Delta t}$ is the speed of the subject vehicle n at time $t-\Delta t$

$[\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}]$ is the relative speed between the two vehicles

$[x_{n-1,t-\Delta t} - x_{n,t-\Delta t}]$ is the spacing between the two vehicles

$\boldsymbol{\alpha}$ is the driver sensitivity parameter

Gazis et al. (1961) generalized stimulus-response models by further improving on the driver sensitivity term. The resulting was a nonlinear model that has the driver sensitivity term proportional to speed of the following vehicle and inversely proportional to vehicle spacing. The model is defined as follows:

$$\ddot{x}_{n,t} = \alpha \frac{[\dot{x}_{n,t-\Delta t}]^{\beta}}{[x_{n-1,t-\Delta t} - x_{n,t-\Delta t}]^{\gamma}} [\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}] \tag{2-13}$$

Where:

Δt is the driver response time lag

$\ddot{x}_{n,t}$ is the acceleration/deceleration of a subject vehicle n at time t

$\dot{x}_{n-1,t-\Delta t}$ is the speed of a leading vehicle $n-1$ at time $t-\Delta t$

$\dot{x}_{n,t-\Delta t}$ is the speed of the subject vehicle n at time $t-\Delta t$

$[\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}]$ is the relative speed between the two vehicles at time $t-\Delta t$

$[x_{n-1,t-\Delta t} - x_{n,t-\Delta t}]$ is the spacing between the two vehicles at time $t-\Delta t$

α is the driver sensitivity constant

β is the speed parameter

γ is the spacing parameter

In this model, the driver sensitivity term is proportional to speed raised to power β and inversely proportional to spacing raised to power γ. The parameter α represents the driver sensitivity constant. It is worthwhile mentioning that th first four models are the special cases of the generalized model. Furthermore, the macroscopic flow-speed relationship developed by Greenshields (1934) can be derived from the GM model by setting $\beta = 0$ and $\gamma = 2$.

When the GM researchers were developing these models, at the same time the researchers associated with the Port of New York were also developing and evaluating macroscopic flow model of speed as a function of traffic density. Greenberg (1959) used fluid dynamic theory to derive macroscopic model relating speed and traffic stream density. They developed a macroscopic model known as Greenberg model and is defined as follows:

$$u = u_0 ln \left({}^{k_j}/_{k} \right) \tag{2-14}$$

Where:

u is the space mean speed in miles per hour

u_0 is the optimum speed in miles per hour

k_j is the jam density in vehicles per lane-mile

k is the traffic density in vehicles per lane-mile

The results of this model motivated Gazis et al. (1959) to develop a relationship between microscopic car-following model and macroscopic traffic model. The translation

of the microscopic car-following model into macroscopic relationship was performed by integrating both sides of equation 2.11 assuming steady-state as follows:

$$\int \ddot{x}_t dt = \int \alpha \frac{\left[\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}\right]}{\left[x_{n-1,t-\Delta t} - x_{n,t-\Delta t}\right]} dt \tag{2-15}$$

$$u = c + \alpha ln \left[x_{n-1,t-\Delta t} - x_{n,t-\Delta t}\right] \tag{2-16}$$

Replacing $\left[x_{n-1,t-\Delta t} - x_{n,t-\Delta t}\right] = \frac{1}{k}$ yields:

$$u = c + \alpha ln \left(\frac{1}{k}\right) \tag{2-17}$$

At $k = k_{j,}\ \ u = 0$

$$u = \alpha ln \left(k_j\right) \tag{2-18}$$

$$u = \alpha ln \left({}^{k_j}/_{k}\right) \tag{2-19}$$

Equation 2-19 is identical to the macroscopic model derived by Greenberg (1959). This bridge between the GM third microscopic car-following model and Greenberg macroscopic model was a very important discovery.

Edie (1961) argued that the model proposed by Chandler et al. (1959) was unrealistic in modeling traffic at low density. The rationale was that at extremely low traffic density, there is no interaction between vehicles. Further, speed and density

relationship derived yields infinite speed as density approaches to zero. According to Edie, lack of upper limit on traffic stream speed exhibit loss of realism because as density approaches zero the speed approaches infinity. Edie stated that the sensitivity of a driver varies with his absolute speed; the faster the driver is traveling, the greater the driver's sensitivity. Therefore, Edie further modified car-following model as follows:

$$\ddot{x}_{n,t} = \alpha \frac{\dot{x}_{n,t-\Delta t}}{\left[x_{n-1,t-\Delta t} - x_{n,t-\Delta t}\right]^2}\left[\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}\right] \tag{2-20}$$

Where:

Δt is the driver response time lag

$\ddot{x}_{n,t}$ is the acceleration/deceleration of a subject vehicle n at time t

$\dot{x}_{n-1,t-\Delta t}$ is the speed of a leading vehicle $n-1$ at time $t-\Delta t$

$\dot{x}_{n,t-\Delta t}$ is the speed of the subject vehicle n at time $t-\Delta t$

$\left[\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}\right]$ is the relative speed between the two vehicles at time $t-\Delta t$

$\left[x_{n-1,t-\Delta t} - x_{n,t-\Delta t}\right]$ is the spacing between the two vehicles at time $t-\Delta t$

$\boldsymbol{\alpha}$ is the driver sensitivity parameter

This model results in macroscopic traffic model which has demonstrated to be realistic at low densities. The resulting macroscopic model is shown below.

$$u = u_f e^{-ck} \tag{2-21}$$

May and Keller (1967) further extended the general model proposed by Gazis et al. (1961) shown in equation 2-13. This study used regression analysis to estimate parameters of the model considering both integer and non-integer values using macroscopic dataset. The results showed that that non-integer values of β and γ produced higher correlation coefficient than integer values. Table 2-3 shows the results obtained from the study.

Table 2-3. Estimated Parameters (May and Keller, 1967)

Parameter	Estimates with integer β, γ	Estimates with non-integer β, γ
Sensitivity, α	1.35×10^{-4}	1.33×10^{-4}
Speed, β	1.0	0.8
Spacing, γ	3.0	2.8
Free speed, u_f (mph)	48.7	50.1
Jam density, k_j (vpm)	∞	220
Optimum speed, u_0 (mph)	29.5	29.6
Optimum density, k_0 (vpm)	60.8	61.1
Maximum flow, q_{max} (vph)	1795	1810
Macroscopic model	$u = u_f e^{-0.5\left[\frac{k}{k_0}\right]^2}$	$u = u_f \left[1 - \left({}^{k}/_{k_j}\right)^{1.8}\right]^5$

Heyes and Ashworth (1972) questioned the assumptions made in the stimulus-response car-following models for using only relative speed as stimulus. One of the reasons stated is that in practice the stimulus is difficult to accurately measure. Thus, Heyes and Ashworth suggested using the rate of change of visual angle. This was based on the study on human perception of motion conducted by Michaels (1963). This study found that the dominant perception factor was the rate of change of visual angle. Thus,

Michaels suggested that the sensitivity as an inverse function of time headway and the model form can be written more generally as:

$$\ddot{x}_{n,t} = \alpha \left[\frac{\dot{x}_{n,t-\Delta t}}{x_{n-1,t-\Delta t} - x_{n,t-\Delta t}}\right]^P \left[\frac{\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}}{\left(x_{n-1,t-\Delta t} - x_{n,t-\Delta t}\right)^2}\right] \quad \text{2-22}$$

Where:

Δt is the driver response time lag

$\ddot{x}_{n,t}$ is the acceleration/deceleration of a subject vehicle n at time t

$\dot{x}_{n-1,t-\Delta t}$ is the speed of a leading vehicle $n-1$ at time $t - \Delta t$

$\dot{x}_{n,t-\Delta t}$ is the speed of the subject vehicle n at time $t - \Delta t$

$\left[\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}\right]$ is the relative speed between the two vehicles at time $t - \Delta t$

$\left[x_{n-1,t-\Delta t} - x_{n,t-\Delta t}\right]$ is the spacing between the two vehicles at time $t - \Delta t$

α is the driver sensitivity parameter

P is the constant

The parameters of the model were calibrated from speed-density observations recorded using data-logger built for the study. The range of values of P between 0.70 and 0.90 were used for constructing fundamental diagrams of speed-density data corresponding to stable car-following for different locations. The best fit parameters were determined using regression analysis. The results and visual observation of theoretical equations clearly indicated that the value of P of 0.80 consistently reproduced the observed data regardless of location.

Ceder and May (1976) extended the GM model (equation 2-13) by evaluating single and two-regime traffic flow models. The models for both single and two-regimes were investigated using a sample of 32 sets of speed-concentration measurements. The paper evaluated the predictions using 13 new sets of data. For the single-regime model, the study found that optimum value of β was 0.6 and γ was 2.4. For the two-regime model, the study found that for congested, the value of β approached 0 while γ value was between 0 and 1. On the other hand, for the free-flow two-regime, the value of β was 0 and γ was 3.

Furthermore, Ceder (1976, 1978) proposed improvement on the sensitivity component of the GM general proposed by Gazis et al. (1961). The proposed model replaced the traditional sensitivity form of:

$$\alpha \frac{[\dot{x}_{n,t}]^{\beta}}{[x_{n-1,t-\Delta t} - x_{n,t-\Delta t}]^{\gamma}} \text{ by } \alpha \frac{A^{-\frac{s_j}{s_t}}}{{s_t}^2} \qquad \text{2-23}$$

Where s and s_j are spacing and jam spacing and A is non-dimensional weighing factor. The proposed model was analyzed using a sample of 45 data sets and validated using a sample of 13 data sets. The study concluded that for the two-regime model, the proposed model was superior to the generalized car-following model, particularly in simplicity and clarity.

Ozaki (1993) modified the structure of the GM model (equation 2-13) by separating acceleration and deceleration response models. Furthermore, the study defined four components of the driver reaction time as shown in Figure 2-2. The components were defined as follows:

1. Time taken to attain zero acceleration given the relative speed is zero,
2. Time taken to start accelerating given the speed of the leading vehicle is greater than speed of the subject vehicle,
3. Time taken to attain zero deceleration given the relative speed is zero, and
4. Time taken to decelerate given the speed of the leading vehicle is greater than speed of the subject vehicle.

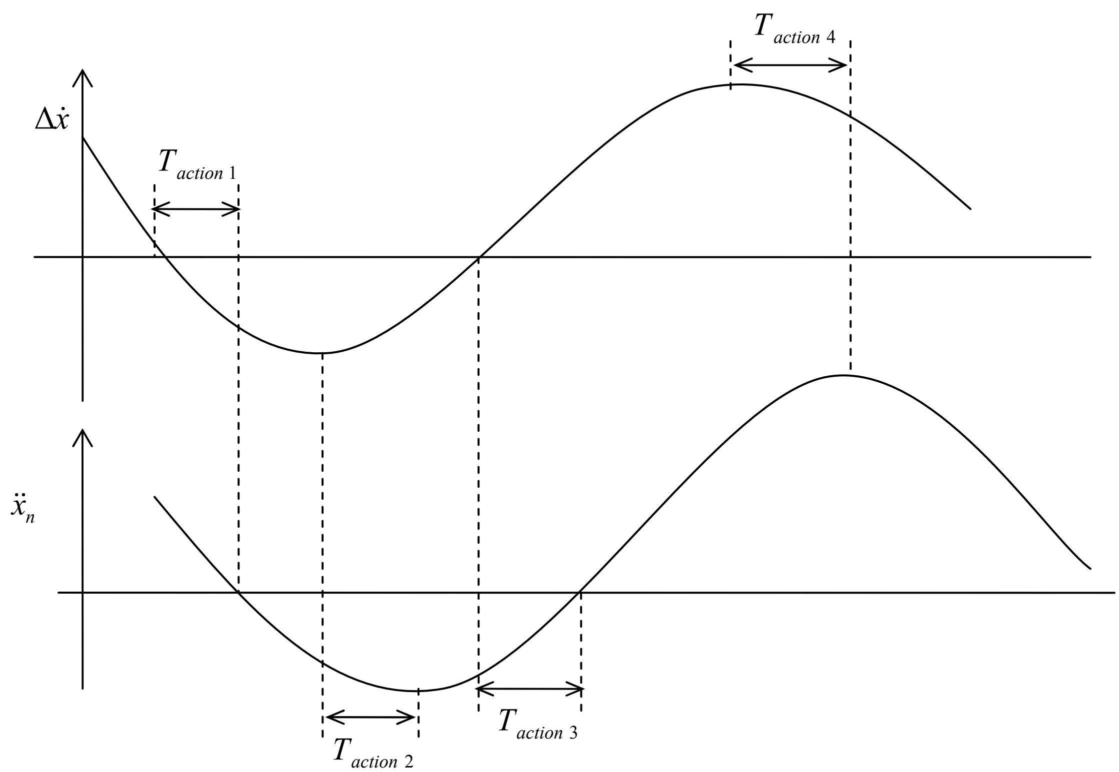


Figure 2-2. Four actions to measure reaction time (Ozaki, 1993).

The study used video camera to collect microscopic data on a freeway in Japan and conducted correlation analysis between the observed reaction time Δt and driving conditions at time $t - \Delta t$. Driving conditions evaluated for every driver included relative

speed, rate of acceleration/deceleration of the leading vehicle, spacing, speed of the subject vehicle, and the reciprocal of time headway. Table 2-4 shows the estimated parameters of the models for both acceleration and deceleration. The study found that spacing and acceleration of the leading vehicle were significant in explaining the reaction time. Furthermore, the research used simple linear regression to estimate reaction time reaction time using spacing and acceleration of the leading vehicle as independent variables. The major weakness of this model is that, in practice, a driver of a following vehicle may not be able to detect small change in acceleration of a leading vehicle.

Table 2-4. Estimated Parameters (Ozaki, 1993)

Parameter	Acceleration model	Deceleration model
$\boldsymbol{\alpha}$	1.1	1.1
β	-0.2	0.9
γ	0.2	1.0
Δt	$1.5 + 0.01[x_{n-1,t} - x_{n,t}] - 0.6\ddot{x}_{n-1,t}$	$1.3 + 0.02[x_{n-1,t} - x_{n,t}] + 0.7\ddot{x}_{n-1,t}$

Subramanian (1996) extended the GM model (equation 2-13) in order to capture drivers' acceleration behavior in both car-following and free-flow regimes. The study developed two separate models for replicating both free flow and car-following regimes. In the car-following regime drivers follow their leader whereas in the free-flow regime drivers try to attain their desired speed. Furthermore, the study developed separated acceleration and deceleration response models. In addition, the model incorporated the variations in driver reaction time across drivers in order to capture individual driver characteristics and aggressiveness. The study also assumed that $\varepsilon_{n,t}$ and Δt followed normal and truncated lognormal distributions, respectively.

The parameters of the models were calibrated using data collected in 1983 along a section of Interstate 10 Westbound in Los Angeles. The study used panel data employing the maximum likelihood technique to estimate jointly the parameters of the model, distributions of reaction time, desired speed, and spacing threshold. Table 2-5 shows results obtained from this study. The study concluded that the spacing thresholds have significant impact on estimation of parameters of the model. However, the results obtained in this study show that the estimated mean reaction time for deceleration response was higher than that for the acceleration response, which is counterintuitive.

Table 2-5. Estimated Parameters (Subramanian, 1996)

Parameter	Acceleration Model		Deceleration Model	
	Estimate	*t-statistic*	*Estimate*	*t-statistic*
Sensitivity, $\boldsymbol{\alpha}$	9.210	1.237	15.24	4.282
Speed, β	-1.667	5.201	1.086	3.901
Spacing, γ	-0.884	3.818	1.659	9.077
Mean, Δt (sec.)	1.97	1.97	2.29	2.29
Std. dev., Δt sec.	1.38	1.38	1.42	1.42

Ahmed (1999) addressed the limitations of the GM model (equation 2-13) by incorporating traffic density of traffic into the sensitivity term and allowed for non-linearity in the stimulus term. Furthermore, the model assumed different reaction times for the sensitivity and stimulus terms. The model is defined as follows:

$$\ddot{x}_{n,t} = \alpha \frac{\left[\dot{x}_{n,t-\xi\Delta t}\right]^{\beta}}{\left[x_{n-1,t-\xi\Delta t} - x_{n,t-\xi\Delta t}\right]^{\gamma}} \left[\kappa_{n,t-\xi\Delta t}\right]^{\lambda} \left[\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}\right]^{\rho} + \varepsilon_{n,t}$$

2-24

Where:

Δt is the driver reaction time

$\ddot{x}_{n,t}$ is the acceleration/deceleration of a subject vehicle n at time t

$\dot{x}_{n-1,t-\Delta t}$ is the speed of a leading vehicle n at time $t - \Delta t$

$\dot{x}_{n,t-\Delta t}$ is the speed of the subject vehicle $n - 1$ at time $t - \Delta t$

$\left[\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}\right]$ is the relative speed between the two vehicles

$\left[x_{n-1,t-\Delta t} - x_{n,t-\Delta t}\right]$ is the spacing between the two vehicles

$\left[\kappa_{n,t-\xi\Delta t}\right]$ is the density of traffic at time $t - \xi\Delta t$

α is the driver sensitivity constant

β is the speed parameter

γ is the spacing parameter

λ is the traffic density parameter

ρ is the relative speed parameter

$\xi \in [0,1]$ is the parameter of sensitivity time lag

$\varepsilon_{\ n,t}$ is the error term associated with the n^{th} vehicle at time t

The study assumed that the parameter ξ would capture the influence of traffic conditions in driver perception in decision-making process. In this case, ξ=1 implies that the time lag for the sensitivity and stimulus are equal. This means that drivers do not update their perception of traffic conditions. On the other hand, ξ<1 implies that time lag for the sensitivity term is smaller than that of stimulus meaning that drivers update their perception due to traffic conditions. The study also hypnotized that there is more uncertainty involved in predicting position and speed of the leading vehicle at high traffic density than at lower density. The expectation was that drivers are more conservative at

higher traffic densities than at low densities. At high density, the subject vehicle is likely to accelerate at a lower rate, while decelerate at higher rate, hence ρ could be positive or negative respectively.

The study further assumed that spacing threshold follows truncated normal distribution with truncation on both sides. In addition, it was assumed that reaction time followed a truncated lognormal distribution as proposed by Subramanian (1996). However, the study estimated the parameters non-parametrically due to complexity in the formed likelihood function. The study used data collected in 1995 and 1997 from a section of Interstate 93 in the southbound direction in Boston using video. Table 2-6 summarizes results obtained from this study.

Table 2-6. Estimated Parameters (Ahmed, 1999)

Parameter	Acceleration Model		Deceleration Model	
	Estimate	*t-statistic*	*Estimate*	*t-statistic*
Sensitivity, $\boldsymbol{\alpha}$	0.0225	1.08	-0.0418	-1.20
Speed, β	0.722	4.67	-	-
Spacing, γ	0.242	6.31	0.151	5.32
Traffic density, λ	0.682	4.20	0.804	4.21
Relative speed, ρ	0.600	7.20	0.682	10.71

The results indicated that sensitivity constants for both the acceleration and deceleration models for the car-following state were statistically insignificant at 5 percent level. For the deceleration model, speed parameter was insignificant (*t-statistic = 0.64*) and has counterintuitive sign, therefore, speed was removed from the model specification. For the acceleration, the results indicate that acceleration increases with speed, density,

and relative speed and decreases with spacing. The sign of speed and traffic density are counterintuitive because one would expect that the higher the speeds or traffic density, the lower the magnitude of acceleration response.

Furthermore, Ahmed pointed out that the average acceleration value estimated in this study was smaller than those predicted by Subramanian (1996) model. He mentioned lack of variability in the data with acceleration observations or the influence of the geometric characteristics of the Boston data collection site as the possible reasons. In addition, he indicated that the difference in data collection years and sites might have contributed to the differences observed in the estimates.

Toledo (2003) pointed out that the existing driving behavior models had several major limitations. First, the existing models separate different behavior and therefore do not capture inter-dependencies. For example, the models ignore the effect of lane changing in acceleration and deceleration response behavior. This study addressed the limitations by proposing an integrated driving behavior that is based on the concepts of short-term goals and short-term plans. The study was an extension of the model proposed by Ahmed (1999) shown in equation 2-24. The parameters of the model were calibrated using data collected in 1983 by Federal Highway Administration (FHWA) along a four-lane section of Interstate 395 in the southbound in Arlington, Virginia.

Table 2-7 shows the results for both acceleration and deceleration response models. For the acceleration response model, results indicated the acceleration response of the subject vehicle increases with its speed, density, and relative speed and decreases with vehicle spacing. The signs for parameter of speed, density, and spacing are counterintuitive. Toledo expected that acceleration is lower for higher speed and density

and higher for bigger spacing. The study also indicated the parameter for sensitivity constant and spacing were statistically insignificant at 5 percent level.

For the deceleration response model, the results showed that deceleration of the subject vehicle increased with, density, and relative speed and decreased with spacing. The results also showed that speed of the following was insignificant, thus, speed was removed from the deceleration model. This is inconsistent with intuitive expectation. This result contradicts previous results obtained by Subramanian (1996) and Ozaki (1993) that showed that speed as a significant factor in deceleration response model.

Table 2-7. Estimated Parameters (Toledo, 2003)

Parameter	Acceleration Model		Deceleration Model	
	Estimate	*t-statistic*	*Estimate*	*t-statistic*
Sensitivity, $\boldsymbol{\alpha}$	0.0355	1.21	-0.86	-3.92
Speed, β	0.291	5.64	-	-
Spacing, γ	0.166	1.68	0.565	9.51
Traffic density, λ	0.550	2.50	0.143	2.04
Relative speed, ρ	0.520	7.97	0.834	12.68

2.3.4 Shortcomings of Stimulus-Response Car-Following Models

The stimulus-response models as reviewed in this chapter have several major limitations in replicating drivers' car-following behavior. Although numerous studies have attempted to address the limitations of the GM model (Chandler et al., 1961) there still remain three major shortcomings:

1. The models assume that the reaction time is the same for all drivers and ignore differences between vehicle types.

2. The models assume drivers can detect even small magnitudes of stimulus, which is unrealistic. Drivers are expected to respond to the driving actions of the leading vehicle only if the perceived stimulus exceeds a certain threshold.
3. The models estimate a single value for each of the other model parameters including speed, relative speed, and vehicle separation. Estimating a single value for each parameter of the models does not capture individual differences between different drivers and different vehicle type being driven and/or followed. For example, drivers of automobiles may behave differently when driving behind large trucks as opposed to when driving behind other automobiles.

2.3.5 Psychophysical Car-Following Models

The concept of the psychophysical car-following models is similar that of the stimulus-response models. That is, a leading vehicle influences longitudinal movement of a following vehicle in the same lane. A driver perception of relative movement of the leading vehicle, changes in separation and speed difference both influences the characteristics of the following vehicle. According to Wiedemann (1974), drivers perceive changes in separation only if the physical impulse exceeds a certain minimum value called threshold. The impulse is the seen size of a leading vehicle. Drivers perceive these changes depending on how fast the image of the leading vehicle changes. These changes are also function of differential speed and distance (Wiedemann, 1974.

Todosiev (1963) was motivated by previous studies undertaken on steady-state car-following and started to study vehicle following processes. This study collected typical vehicle trajectories and derived distance, speed, relative speed, and relative acceleration variables. The study found that the relative acceleration was almost constant

in certain portions and driver changed them at certain action points called thresholds (i.e. limits). Todosiev conducted simulator experiments to determine the thresholds and human perception thresholds causing them. The results obtained were compared from similar results obtained from psychophysical investigations in human perception of moving objects. The study used action point density as a function of relative speed and relative acceleration for different speed levels to determine speed thresholds. The speed thresholds defined limits for drivers to detect relative speed with a certain fixed probability at a given separation. The study found that the threshold for positive relative speed was greater than the threshold for negative relative speed for the same separation.

Michaels (1965) studies three car-following modes: closing process with constant relative speed, steady-state following, and responses to acceleration of a lead vehicle. According to Michaels, in the closing process, a horizontal visual angle subtended by the lead vehicle keeps changing continuously. The rate of change of angular speed correlates to the movement of the leading vehicle, hence relative speed between the two vehicles. The rate of change of angular speed is defined as:

$$\frac{d\theta}{dt} = k\frac{\dot{x}_{n-1} - \dot{x}_n}{[x_{n-1} - x_n]^2} \tag{2-25}$$

Where:

$\dot{x}_{n-1}$ is the speed of a leading vehicle

$\dot{x}_n$ is the speed of a subject vehicle

x_{n-1} is the position of the leading vehicle

x_n is the position of the subject vehicle

The study found that human threshold of angular speed ranged from 3×10^{-4} to 10×10^{-4}, with mean around 6×10^{-4}. During car-following state, Michaels proposed three distinct phases: when angular speed is below threshold, when angular speed is above threshold, and when angular speed is equal to threshold. In each phase, driver detects angular speed and adjusts driving condition accordingly. At angular speed above the threshold, the driver simply employs detection of changes in separation to determine whether a leading vehicle is traveling slower. On the other hand, when angular speed is above the threshold, the driver responds by reducing speed sufficiently to keep angular speed at or near the driver absolute threshold of detection. Otherwise, the driver tries to keep relative speed to zero and maintain minimum safe separation. For steady-state following, the angular speed between two consecutive vehicles was assumed below the threshold of detection. In this case, the results showed that the distance change required for detection was less for closing than for opening situation.

Hoefs et al. (1972) conducted field measurements on motorways in order to calibrate models developed by Todosiev (1963) and Michaels (1965). The result of the measurements showed the action points in the closing behavior, that is, when following driver starts to adapt his driving behavior to the slower leading vehicle by decelerating. Moreover, the measured separation and speed difference gave researchers in-depth insights on driving behavior under different conditions: deceleration behavior in closing processes and duration of oscillation processes in close following situations.

Wiedemann (1974) combined measurements and investigations conducted by Todosiev (1963), Michaels (1965), and Hoefs (1972) in defining the basic functions of the psychophysical model. The psychophysical model emulates driver perceptions and

reactions using a defined set of thresholds and desired distances. These thresholds delimit different modes that define different situations of interactions between a subject vehicle and a leading vehicle. The modes include free driving, approaching, following, and emergency braking. Different driving procedures are associated with different modes representing driving behavior under given situations and drivers behave differently.

Figure 2-3 shows different thresholds developed by Wiedemann that characterize separation for different driving modes between two following vehicles. In this figure, horizontal axis represents the difference in speed while the vertical axis shows separation between the two following vehicles. The positive difference in speed characterizes a closing process, that is, speed of leading vehicle is lower than a subject vehicle and negative difference in speed characterizes opening process.

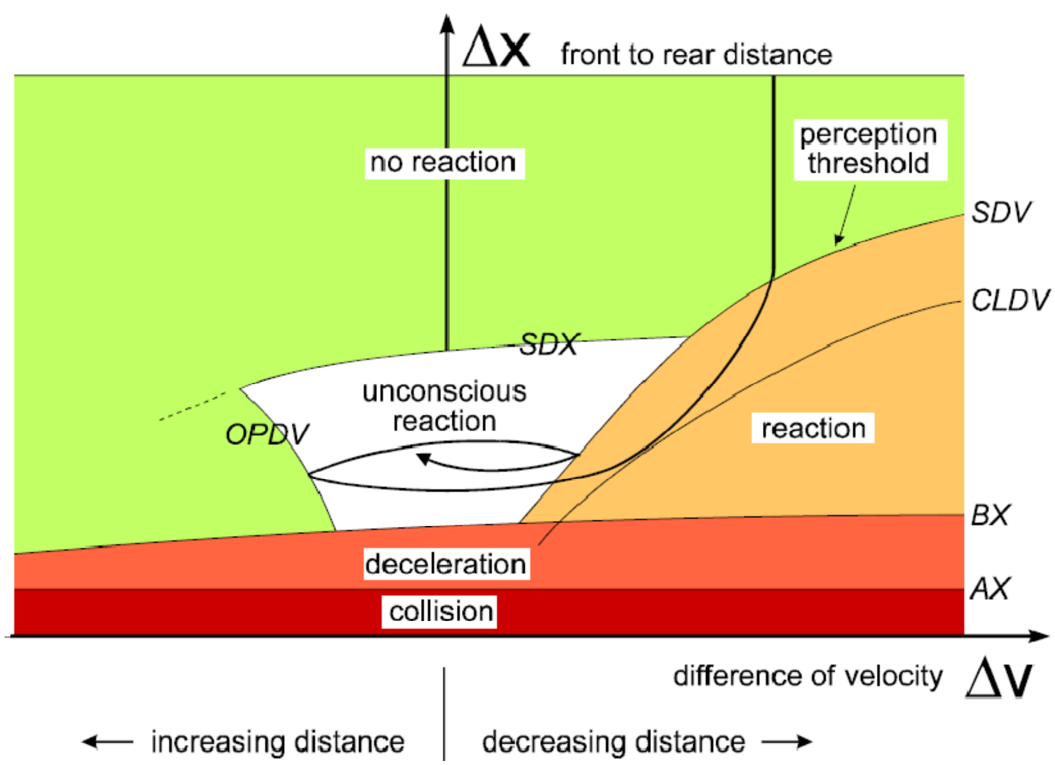


Figure 2-3. Car-following logic (Wiedemann, 1974).

The following is the definitions of the thresholds shown in Figure 2-3 above:

AX: desire distance for standing vehicles, that is, front-to-front distance and is given by:

$$AX = L + AXadd + RND1(I).AXmult$$

Where *AXadd* and *AXmult* define the range of desired minimum separation.

ABX: desired minimum following distance at low speed differences and is given by:

$$ABX = AX + BX$$

$$BX = (BXadd + BXmult.RND1(I))\sqrt{V}$$

Where *BXadd* and *BXmult* define the range of variation.

SDV: perception threshold of speed difference at long distances and defined as:

$$SDV = \frac{DX - AX}{CX}^{2}$$

$$CX = CXconst(CXadd + CXmult(RND1(I) + RND2(I)))$$

Where *CXconst, CXadd* and *CXmult* define the range of the thresholds.

SDX: perception of growing distance in following process and is calculated as follows:

$$SDX = AX + EX.BX$$

$$EX = EXadd + EXmult(\mathrm{NRND} - \mathrm{RND2(I)}$$

Where *NRND* and *RND2 (I)* are random parameters.

CLDV: perception threshold for recognizing small speed differences at short, decreasing distances and is defined as:

$$CLDV = SDV.EX^{2}$$

OPDV: perception threshold for recognizing small speed differences at short but increasing distances and is defined as:

$$OPDV = CLDV(-OPDVadd - OPDVmult.NRND)$$

Where *OPDVadd* and *OPDVmult* define the range of the parameter while *NRND* parameter represents variations for the same driver.

As previously stated, the thresholds described above delimit driving behavior into four modes: un-influenced driving, closing process, following process and emergency braking. In each mode, different procedures are used for estimating the parameter of driving behavior such as values of acceleration in longitudinal direction. In the following process a driver in this mode was assumed be following a leading vehicle at quite the same speed. The driver does not consciously react to movements of leading vehicle but tries to keep acceleration low.

2.3.6 Shortcomings of Psychophysical Car-Following Models

The psychophysical models reviewed above are designed to emulate drivers' car-following behavior based on limits of human perception of moving objects. The models, however, have four major shortcomings:

1. Wiedemann (1974) mentioned that there was no exact knowledge of human distributions for different parameters was available when the models were developed. Therefore, random parameters including *RND1 (I), NRND*, and *RND4 (I)* were assumed to be normally distributed with mean value of 0.5 and standard deviation value of 0.15 (i.e. N (0.5, 0.15).
2. In the following process, drivers are assumed do not consciously react to the movement of a leading vehicle but tries to keep acceleration low, which is opposite of what is expected. Drivers are expected to be more conscious in the following process because of safety related reasons.

3. The models do not incorporate driver response time lags of a following vehicle. In other words, the models assume drivers react instantaneously after attaining their desired thresholds.
4. The thresholds of the models are the same for different drivers and ignore differences between vehicle types. Thus, the models do not capture differences in driving behaviors of different drivers depending on the type of vehicle being driven and/or followed.

2.3.7 Fuzzy Logic-Based Car-Following Models

Fuzzy logic-based and neuro-fuzzy logic-based models have recently been proposed for modeling driving behavior in car-following models. The basic concept of fuzzy logic-based model is that it transforms input factors into linguistic forms using certain membership functions. In other words, the models use normal language-based driving rules. On the other hand, neural network approach is similar to fuzzy logic-based but it incorporate past driving behavior through the process of learning and training. For example, Kikuchi and Chakroborty (1992) proposed car-following based on fuzzy inference system. Drivers' responses were based on transformed sets of driving rules into linguistic terms. For example, a typical rule for a driver response can take the following form:

IF	Separation is ADEQUATE
	Relative speed is NEAR ZERO,
	Acceleration response of a leading vehicle is MILD
THEN	Subject vehicle should accelerate MILDLY

Inokuchi et al. (1999) presented similar approach by incorporating neural networks and fuzzy logic. This neuro-fuzzy model employed similar linguistic definitions of IF and THEN fuzzy logic rules. The study also incorporated a neural network to enhance the control algorithm. The neural network model is representation of the process that learns by experience and examples. In the proposed neuro-fuzzy car-following model, the characteristic of the following driver was observed from a data and the learning process was represented by changing the weight of the synapse or connection between neurons in the model. The neural network was incorporated with the fuzzy rules and produced an estimation of the actions of the driver. However, driver response time lag was considered as a constant in this model. Several other research efforts also considered fuzzy application of car-following theory including fuzzification of the MISSION model (Rekersbrink, 1995) and information of the MITRAM model (Yikai et al., 1993).

2.3.8 Shortcomings of Fuzzy Logic-Based Car-Following Models

The fuzzy logic-based car-following models have the following limitations:

1. The models assume constant driver response time lags and ignore differences between vehicle types.
2. It is difficult to calibrate the models and to select the function type and their limiting values.
3. In practice, it is difficult to determine the thresholds based on linguistic driving rules such as ADEQUATE vehicle separation. For example, different drivers may define ADEQUATE vehicle separation differently for the same magnitudes of stimuli.

4. The models indicate important parameters in replicating drivers' car-following behavior but do not indicate which parameters are statistically significant.

CHAPTER 3

PROPOSED MODELS AND ESTIMATION METHODOLOGY

3.1 Introduction

This chapter describes in detail the stimulus-response models proposed for this research. The models are an extension of the GM models (Gazis et al. (1961). Unlike the GM models, the models proposed for this study delimit car-following behavior into three separate modes: acceleration, deceleration, and steady-state responses. Generally, during peak periods, traffic is congested and there are therefore limited opportunities for drivers to attain their desired free flow speeds. It is however as noted by other previous researchers (Ozaki, (1993), Subramanian (1996)) the characteristics of the responses of the drivers depend on whether they are reacting to a stimulus that requires an acceleration response or one that requires a deceleration response.

The acceleration mode occurs when the stimulus is positive and exceeds the driver's threshold. On the other hand, the deceleration mode occurs when the stimulus is negative and exceeds the driver's threshold. The steady-state mode occurs when the stimulus is within the thresholds. Therefore, the response of a driver of subject vehicle n at time t is defined as follows:

$$\text{Response}_{n,t} = \begin{cases} \text{acceleration} & \text{if} \quad \left[\dot{x}_{n-1,t-\Delta t_1} - \dot{x}_{n,t-\Delta t_1}\right] \geq z_1 \\ \text{deceleration} & \text{if} \quad \left[\dot{x}_{n-1,t-\Delta t_2} - \dot{x}_{n,t-\Delta t_2}\right] \leq z_2 \\ \text{steady-state} & \text{otherwise} \end{cases} \qquad \text{3-1}$$

Where:

Δt_1 is the driver response time lag for the acceleration response

Δt_2 is the driver response time lag for the deceleration response

z_1 is the relative speed threshold for the acceleration response

z_2 is the relative speed threshold for the deceleration response

$\dot{x}_{n,t-\Delta t}$ is the speed of a subject vehicle n at time $t - \Delta t$

$\dot{x}_{n-1,t-\Delta t}$ is the speed of a leading vehicle $n - 1$ at time $t - \Delta t$

$\left[\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}\right]$ is the relative speed between the two vehicles at time $t - \Delta t$

$z_1 > 0$

$z_2 < 0$

The drivers' response time lags for both the acceleration and deceleration responses are assumed to be different. The thresholds are also assumed to be different for different drivers and also for the same driver the magnitude of z_1 and z_2 may be different. Todosiev (1963) found that the positive response threshold is greater than the negative response threshold for a given vehicle separation. Similarly, Michaels (1965) also found that the distance for detecting a slower leading vehicle is smaller compared to the one for detecting a faster leading vehicle. These findings indicate that drivers are more sensitive under deceleration response than acceleration response. However, the thresholds are likely to be a function of speed and separation. That is, at slower speeds and smaller separations the threshold may be smaller than the thresholds at higher speeds and bigger separations. This research simplified the models by determining one value of threshold that is independent of these factors but the values of thresholds are different for acceleration response and deceleration response.

3.2 Acceleration and Deceleration Response Models

As previously stated in this research, the proposed family of car-following models is based on the GM model shown in Equation 2-13. The proposed models have four significant differences with the GM model:

1. Delimit the car-following behavior into three separate modes: acceleration, deceleration, and steady-state responses. It is hypothesized that in each mode, drivers behave differently and have different response aggressiveness needs.
2. Determine the driver response time lags and stimulus thresholds for acceleration and deceleration responses.
3. Incorporate vehicle heterogeneity in the models by estimating sub-models depending on type of vehicle being driven and/or followed. The sub-models include automobile following automobile, automobile following large truck, and large truck following automobile.
4. Calibrate the acceleration and deceleration response models for each driver separately, obtain the distributions of these parameters, and aggregate the results.

The proposed general form for the acceleration and deceleration response models is shown in the following equation.

$$\ddot{x}_{n,t} = \beta_0\left[\dot{x}_{n,t-\Delta t}\right]^{\beta_1}\left[x_{n-1,t-\Delta t} - x_{n,t-\Delta t} - L_{n-1}\right]^{\beta_2}\left[\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}\right]^{\beta_3} \qquad 3\text{-}2$$

Where:

Δt is the driver response time lag

$\ddot{x}_{n,t}$ is the acceleration/deceleration of a subject vehicle n at time t

$\dot{x}_{n,t-\Delta t}$ is the speed of a subject vehicle n at time $t - \Delta t$

$\dot{x}_{n-1,t-\Delta t}$ is the speed of a leading vehicle $n - 1$ at time $t - \Delta t$

$x_{n-1,t-\Delta t}$ is the position of the leading vehicle $n - 1$ at time $t - \Delta t$

$x_{n,t-\Delta t}$ is the position of the subject vehicle n at time $t - \Delta t$

L_{n-1} is the length of the leading vehicle

$\left[x_{n-1,t-\Delta t} - x_{n,t-\Delta t} - L_{n-1}\right]$ is the vehicle separation at time $t - \Delta t$

$\left[\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}\right]$ is the relative speed between the two vehicles at time $t - \Delta t$

β_0 is the driver sensitivity constant

β_1 is the speed parameter

β_2 is the relative speed parameter

β_3 is the vehicle separation parameter

As shown in the Equation 3-2, the acceleration and deceleration response modes have similar model form. In both models the acceleration or deceleration response is a function of driver sensitivity and a stimulus. The stimulus that a driver responds to is represented by the relative speed i.e. $\left[\dot{x}_{n-1,t-\Delta t} - \dot{x}_{n,t-\Delta t}\right]$ while the driver sensitivity is a function of vehicle speed and vehicle separation and is represented by the term $\beta_0\left[\dot{x}_{n,t-\Delta t}\right]\left[x_{n-1,t-\Delta t} - x_{n,t-\Delta t} - L_{n-1}\right]$.

It is assumed that the parameters β_j are different for different drivers. The variations are attributed to aggressiveness and capabilities of individual drivers in estimating differential speeds and separations with the leading vehicle. Further, as discussed earlier in this study, the parameters are expected to be different for the acceleration and deceleration modes and for different vehicle types (automobiles versus large trucks).

3.2.1 Expectation of the Signs of the Parameters

For the acceleration response, the larger the positive relative speed, the larger the magnitude of the expected acceleration for the following vehicle. Hence, the sign of the relative speed parameter β_3 is expected to be positive. It is also hypothesized that drivers are less aggressive when accelerating from a higher speed than from a lower speed and also that vehicle acceleration capabilities are lower at higher speeds. Therefore, the magnitude of the acceleration response is expected to be lower at higher speeds and higher at lower speeds. This suggests that the expected sign for speed parameter β_1 is negative. Equally, the magnitude of the acceleration is expected to be lower for bigger vehicle separation than for smaller separation between the two following vehicles. Hence, the sign of the vehicle separation parameter β_2 is expected to be positive.

Similarly, for the deceleration response, the larger the negative relative speed, the larger the magnitude of the expected deceleration for the following vehicle. Hence, the sign of the relative speed parameter β_3 is expected to be positive. It is also hypothesized that, for safety reasons, drivers will respond with higher deceleration rates at higher speeds than at lower speeds. This implies that the expected sign for speed parameter β_1 is positive. For similar reasons, when the vehicle separation is smaller, the magnitude of deceleration response is expected to be higher. Therefore, the expected sign of vehicle separation parameter β_2 is negative. It should be noted that the signs for parameters β_1 and β_2 are different from those of the acceleration response.

3.3 Steady-State Response Model

As previously stated in this study, the steady-state mode occurs when the stimulus is within the thresholds. In this case, speed differences are lower and undetectable to the

drivers. In situations, drivers are more concerned about maintaining safe vehicle separation with the leading vehicle. Thus, a driver will either accelerate or decelerate depending on whether there is a need to increase or decrease the separation between the two vehicles. Therefore, the stimulus is no longer the relative speed but it is both the vehicle separation and speed. The concept of this model is based on assumption that at higher speeds or smaller vehicle separations, drivers are more likely to decelerate in lieu of accelerating and vice versa. To emulate this driving behavior, the following model is proposed:

$$\ddot{x}_{n,t} = \beta_0 - \left[\dot{x}_{n,t-\Delta t}\right]^{\beta_1} + \left[x_{n-1,t-\Delta t} - x_{n,t-\Delta t} - L_{n-1}\right]^{\beta_2} \qquad 3\text{-}3$$

Where:

Δt is the driver response time lag

$\ddot{x}_{n,t}$ is the acceleration/deceleration of a subject vehicle n at time t

$\dot{x}_{n,t}$ is the speed of the subject vehicle n at time $(t - \Delta t)$

$x_{n-1,t-\Delta t}$ is the position of a leading vehicle $n-1$ at time $(t - \Delta t)$

$x_{n,t}$ is the position of the subject vehicle n at time $t - \Delta t$

$\left[x_{n-1,t-\Delta t} - x_{n,t-\Delta t} - L_{n-1}\right]$ is the vehicle separation at time $(t - \Delta t)$

L_{n-1} is the length of the leading vehicle

β_0 is the constant

β_1 is the speed parameter

β_2 is the vehicle separation parameter

3.3.1 Expectation of the Signs of the Parameters

Similar to the acceleration and deceleration response models, the parameters β_j are assumed to be different for different drivers. Furthermore, this study hypothesize that the driver response has a negative relationship with the speed and a positive relationship with vehicle separation. This means that the higher the speed, the higher the likelihood of a deceleration response. On the other hand, the bigger the vehicle separation, the higher the likelihood of an acceleration response. Therefore, the signs of both parameters β_1 and β_2 are expected to be positive.

At steady-state, the subject vehicle is moving at a constant speed which results in zero acceleration/deceleration response i.e.

$$0 = \beta_0 - \left[\dot{x}_{n,t}\right]^{\beta_1} + \left[x_{n-1,t} - x_{n,t} - L_{n-1}\right]^{\beta_2} \tag{3-4}$$

Rearranging the equation result in:

$$\left[\dot{x}_{n,t}\right]^{\beta_1} = \beta_0 + \left[x_{n-1,t} - x_{n,t} - L_{n-1}\right]^{\beta_2} \tag{3-5}$$

Further simplification yields the following relationship:

$$\dot{x}_{n,t} = \left(\beta_0 + \left[x_{n-1,t} - x_{n,t} - L_{n-1}\right]^{\beta_2}\right)^{\frac{1}{\beta_1}} \tag{3-6}$$

But $\left[x_{n-1,t-\Delta t} - x_{n,t-\Delta t}\right]$ is the spacing between the two vehicles and is the reciprocal of traffic density k. Thus, substituting the spacing by density and converting speed into miles per hour and density into vehicles per mile establishes the relationship between microscopic and macroscopic traffic flow model of the following form:

$$u = \frac{1}{1.47}\left(\beta_0 + \left[{}^{5280}/_{k} - L\right]^{\beta_2}\right)^{\frac{1}{\beta_1}} \tag{3-7}$$

Where:

u is the space mean speed in miles per hour

k is the traffic density in vehicles per mile-lane

L is the average length of the vehicles in feet

The traffic flow rate, q in vehicle per hour is given as:

$$q = ku = \frac{1}{1.47}k\left(\beta_0 + \left[{}^{5280}/_{k} - L\right]^{\beta_2}\right)^{\frac{1}{\beta_1}} \tag{3-8}$$

The traffic jam density, k_j which occurs when vehicles are at standstill e.g. at zero speed is obtained by equating equation 3-7 to zero as follows:

$$0 = \frac{1}{1.47}\left(\beta_0 + \left[{}^{5280}/_{k_j} - L\right]^{\beta_2}\right)^{\frac{1}{\beta_1}} \tag{3-9}$$

Rearranging 3-9 gives the expression of traffic jam density shown below:

$$k_j = \frac{5280}{\left[(-\beta_0)^{1/\beta_1} + L\right]} \qquad \text{for} \qquad \beta_0 \leq 0 \tag{3-10}$$

In the equation 3-10, the term $(-\beta_0)^{1/\beta_1}$ represents the average vehicle separation when vehicles are stopped, that is, jam separation.

3.4 Estimation of Disaggregate Parameters of the Models

This study estimates the parameters of the models in two stages. The first stage estimates the disaggregate parameters for each individual subject vehicles. The second stage estimates the aggregate parameters for all vehicles selected in this research and is discussed in detail in Section 3.5. The equations of the models proposed in this research are nonlinear in parameters. The disaggregate parameters of the models for each individual vehicles are estimated using nonlinear least squares regression technique. The models proposed can be rewritten in general form as:

$$f(x_{n,t}) = f(\beta, \boldsymbol{X}_{n,t-\Delta t}) + u_{t-\Delta t}, \qquad (t = 1,2, \ldots, p) \tag{3-11}$$

Where:

$f(x_{n,t})$ is the response variable at time t

β is the k-vector of unknown parameters

$\boldsymbol{X}_{n,t-\Delta t}$ is the vector of explanatory variables at time $t - \Delta t$

$u_{t-\Delta t}$ is the error term at time $t-\Delta t$

p is the number of observations

The error term accounts for the unobserved factors and for estimation purpose it is assumed to be normally identically distributed random variable with mean zero and constant variance i.e. $u_{t-\Delta t} \sim NID(0,\sigma^2)$, $E(u_{t-\Delta t})=0$, and $Var(u_t)=\sigma^2$).

In a nonlinear model the unknown parameters of the models are estimated by maximizing log likelihood function. The log likelihood function for the nonlinear regression equation is defined as:

$$\ell(\beta,\sigma^2)=\frac{1}{(2\pi\sigma^2)^{n/2}}e^{\left\{-\frac{\sum_{i=1}^{N}\left[f(x_{n,t})-f(\beta,\boldsymbol{X}_{n,t-\Delta t}\right]^2}{2\sigma^2}\right\}}$$

3-12

The log likelihood is maximized when the sum of squared residuals, $s(\beta)$ is minimized.

$$s(\beta)=\sum_{t=1}^{N}\left[f(x_{n,t})-f(\beta,\boldsymbol{X}_{n,t-\Delta t})\right]^2$$

3-13

Differentiating the objective function, $s(\beta)$ with respect to β and equating it to zero yields:

$$\frac{\partial s(\beta)}{\partial\beta}=-2\sum_{t=1}^{N}\left[f(x_{n,t})-f(\beta,\boldsymbol{X}_{n,t-\Delta t})\right]\frac{\partial s(\beta,\boldsymbol{X}_{n,t-\Delta t})}{\partial\beta}=0$$

3-14

Setting the partial derivatives to zero produces equations for estimating the parameters of the regression equation. The equations formed do not have closed form solution, thus, they require solution by numerical optimization method. This study uses the Stata program to estimate parameters of the models. The Stata implements a modified Gauss-Newton method in estimating parameters of the models (Baum, 2008). The modified Gauss-Newton method finds the minimum of the sum of squares:

$$s(\beta) = \sum_{i=1}^{n} {r_i}^2 \tag{3-15}$$

The modified Gauss-Newton method starts with an initial guess β^0 for the minimum squares and proceeds by the iterations and generates a better estimate of the unknown parameters. The iterations continue until the values of the parameters do not change significantly within a specified precision as follows:

$$\beta^{k+1} = \beta^k + \alpha\Delta \tag{3-16}$$

Where k superscript refer to iteration number, α is the fraction of increment vector that limits divergence to occur and takes the values in interval $0 < \alpha < 1$. The increment Δ is the solution of normal equations:

$$\left(\boldsymbol{J_r}^{\boldsymbol{T}}\boldsymbol{J_r}\right)\Delta = -\boldsymbol{J_r}^{\boldsymbol{T}}\boldsymbol{r} \tag{3-17}$$

Where T superscript denotes the matrix transpose, $\boldsymbol{r}$ is the vector of function of r_i , and $\boldsymbol{J_r}$ is the $m \times n$ Jacobian matrix of $\boldsymbol{r}$ with respect to β both evaluated at β^k. The Jacobian $\boldsymbol{J_r}$ is the vector of residuals r_i with respect to unknown parameters β_j and is defined as:

$$\boldsymbol{J_r} = \frac{\partial r_i}{\partial \beta_j}, \qquad (i = 1, .. n, \;\; j = 0, \dots, p)$$

3-18

Where the i subscripts refer to a particular data point, the j subscripts refer to a particular fitting parameter.

3.5 Estimation of Variance of the Parameters

The parameter variances may be estimated from linearized version of the model based on asymptotic covariance matrix as shown in the equations that follows:

Let $X_{tj} = \frac{\partial f(\hat{\beta}, \boldsymbol{X}_{n,t-\Delta t})}{\partial \hat{\beta}_j}$ and $\boldsymbol{X} = X_{tj}$

Then, the estimate of asymptotic covariance matrix of regression parameters is given as:

$$Var(\hat{\beta}) = s^2(\boldsymbol{X}'\boldsymbol{X})^{-1} \qquad \text{3-19}$$

Where:

s^2 is the estimate of the variance of the error term

$(\boldsymbol{X}'\boldsymbol{X})^{-1}$ is the covariance matrix.

The resulting standard error of the parameter estimate is given as follows:

$$se(\hat{\beta}) = s\sqrt{(\boldsymbol{X}'\boldsymbol{X})^{-1}} \qquad 3\text{-}20$$

3.6 Hypothesis Test

Hypothesis test is used for making statistical inferences and comparing parameters of the models. In the context of this research, the comparisons are made using null hypothesis H_0, that is, there is no statistical difference in the means of the parameters of the interest. Rejecting H_0 means accepting the alternative hypothesis H_A, that is, there is evidence of statistical difference in the means of the parameters of the interest. The hypothesis test is defined as follows:

$$H_0: \beta_i - \beta_j = 0$$

$$H_A: \beta_i - \beta_j \neq 0 \qquad 3\text{-}21$$

The hypothesis test is conducted using *t-statistic* test. The *t-statistic* for nonlinear regression is commonly referred as *pseudo-t* because it does not have Student's t distribution with $n-k$ degrees of freedom in finite samples when $f(\beta, \boldsymbol{X}_{n,t-\Delta t})$ is nonlinear in the parameters. Furthermore, $f(\beta, \boldsymbol{X}_{n,t-\Delta t})$ depends on lagged values of $\ddot{x}_{n,t}$ and error $u_{t-\Delta t}$ are not normally distributed. In this case, *pseudo-t* is calculated as follows:

$$t_{\hat{\beta}_j} = \frac{\hat{\beta}_i - \beta}{se(\hat{\beta}_j - \beta)}$$

3-22

The approximate confidence interval (C.I.) of parameter estimate is computed as:

$$C.I. = \hat{\beta}_j \pm t\alpha_{/2} se(\hat{\beta}_j)$$ 3-23

Where $t\alpha_{/2}$ is the critical value of t which is 1.96 for two tail test at 5 percent significance level. Therefore, the 95 percent confidence interval of the parameter is simplified as follows:

$$C.I. = \hat{\beta}_j \pm 1.96 se\left(\hat{\beta}_j\right)$$ 3-24

The *p-value* which is the probability associated with the *t-statistic* value indicates the significance level of accepting or rejecting the null hypothesis and is calculated as:

$$p\text{-}value = Probability\ (t > t_{\alpha/2})$$ 3-25

At 5 percent significance level, the null hypothesis H_0 is accepted when the calculated *p-value* is greater than 5 percent. On the other hand, the null hypothesis is rejected when the calculated *p-value* is less than 5 percent and accepts the alternative hypothesis H_A.

3.7 Measures of Goodness-of-Fit of the Model

Goodness-of-fit measures how well the estimated model able to explains the variation of the observed values. In this research, the scalar measure of the model fit is the based on adjusted R^2 and is calculated as shown in the following equations below:

$$R^2 = 1 - \frac{SS_{residual}}{SS_{total}} = 1 - \frac{\sum\left[f(x_{n,t}) - f(\boldsymbol{X}_{n,t})\right]^2}{\sum\left[f(x_{n,t}) - f(\overline{x}_{n,t})\right]^2} \tag{3-26}$$

$$Adjusted\ R^2 = 1 - (1 - R^2)\frac{n-1}{n-k} \tag{3-27}$$

Where:

n is the number of observations

k is the number of parameters in the model.

3.8 Aggregation of Parameters of the Model

Parameter and variances estimates in the above equations are for individual drivers. To obtain the aggregate parameter and variance estimates for all drivers require pooling the estimates of the individual drivers. The literature review indicated that there are numerous methods used for aggregating parameter estimates and their corresponding variances. This study assumes that estimates have similar distribution and uses simple mean and variance to aggregate the estimates as follows

The mean parameter estimate is calculated using the following equation:

$$\hat{\beta}_{mean} = \frac{\sum_{i=1}^{n} \hat{\beta}_i}{n}$$

3-28

Where n is the number of vehicles used in estimation.

The standard deviation std of the mean of the parameter is calculated as follows:

$$Std\ (\hat{\beta}_{mean}) = \sqrt{\frac{\sum_{i=1}^{n} (\hat{\beta}_i - \hat{\beta}_{mean})^2}{n-1}}$$

3-29

3.9 Validation of the Models

Validation of the model is important because it shows the transferability ability of the model to a different spatial locations or time periods. The most widely used statistical measures found in the literature that have demonstrated to produce reasonable results in practical applications include Root Mean Square Error (RMSE) and Theil Inequality Coefficients (*U*). These measures provide a gauge of how well the estimated values replicate the corresponding field observed values. In other words, the measures indicate the degree of goodness-of-fit of the estimated values in emulating the field observed values.

RMSE measures the deviations between the estimated and field observed values and is defined as follows:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i - y_i}{y_i}\right)^2} \qquad 3\text{-}30$$

Where:

n is the number of observations

x_i is the estimated responses from the model

y_i is the field observations of acceleration/deceleration responses

U measures the scaled root mean squared difference between the estimated and field observed values. This measure was introduced by Henry Theil in 1967 for economic forecasting. The most prevalent benefits of the *U* measure are: First, it allows the comparison of different pairs of datasets at different scales, with respect to a broad concept of inequality. Second, the inequality coefficient *U* can be decomposed to provide additional information of its main statistic factors such as difference in mean, difference in variability, and lack of correlation. Theil inequality coefficient (*U*) is defined as follows:

$$U = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - x_i)^2}}{\sqrt{\frac{1}{n}\sum_{i=1}^{n} {y_i}^2} + \sqrt{\frac{1}{n}\sum_{i=1}^{n} {x_i}^2}} \qquad 3\text{-}31$$

Where:

n is the number of observations

x_i is the estimated responses from the model

y_i is the field observed values

The numerator represents the mean squared difference and the denominator provides the scaling factor. As a result of scaling factor, the value of U statistic lies between zero and unity. The value U is zero when the two datasets are identical, meaning there is a perfect fit between the two datasets. Similarly, the higher the value of U indicates disagreement between the two datasets. In addition, the coefficient can be separated into three components that contribute to the overall inequality between the two datasets. The components are important because they provide additional information on the difference in means between the two datasets (U_m), difference in variance (U_s), and (3) lack of correlation (U_c). These three components are defined as follows:

$$U_m = \frac{n(\bar{y} - \bar{x})^2}{\sum_{i=1}^{n}(y_i - x_i)^2} \tag{3-32}$$

$$U_s = \frac{n\left(\sigma_y - \sigma_x\right)^2}{\sum_{i=1}^{n}(y_i - x_i)^2} \tag{3-33}$$

$$U_c = \frac{2(1 - r)n\sigma_y\sigma_x}{\sum_{i=1}^{n}(y_i - x_i)^2} \tag{3-34}$$

$$r = \frac{1}{n - 1}\sum_{i=1}^{n}\left[\frac{(x_i - \bar{x})(y_i - \bar{y})}{\sigma_x\sigma_y}\right] \tag{3-35}$$

Where $\bar{y}$, $\bar{x}$, $\sigma_{y,}$ and σ_x are means and standard deviations of the field observed values and estimated values, respectively, whereas r is their correlation coefficient. The three components sum to one, i.e.

$$U_m + U_s + U_c = 1 \tag{3-36}$$

The U_m measure indicates a systematic error, that is, it measures the degree to which the average estimated values and field observed values deviate from each other. A large value would indicate the presence of a systematic bias, therefore, the need for revising the models. The U_s measure indicates the ability of the model to capture the degree of variability between the estimated values and field observed values. Similarly, a large value of U_s indicates that the estimated values have considerable variations compared to the field observations suggesting model revision. The U_c measure indicates the asymmetric error, that is, it measures the remaining error after accounting for the deviations from the field observed values. A small value of U_c suggests that that there is large correlation between the two datasets, thus, the need further revisions of the model.

CHAPTER 4

IMPLEMENTATION

4.1 Introduction

The focus of this chapter is to present a detailed implementation procedure used for estimating parameters of the models. To meet the research objectives, as previously stated, the study applied nonlinear least square regression with robust standard errors to estimate driver response time lags and other parameters of the models including speed, relative speed, and separation. In order to incorporate vehicle heterogeneity in the acceleration, deceleration, and steady state response models, three separate models are developed for the following pairs of following vehicles:

- "Automobile following automobile",
- "Automobile following large truck", and
- "Large truck following automobile".

Due to data limitation, no model for "large truck following large truck" is calibrated. For example, there were only two pairs of large trucks that satisfied selection criteria for inclusion in this research. Using the two pairs would not produce results that are representative of the observed driving behavior of large trucks traveling behind other large trucks. This research used vehicle trajectory data that was collected as part of the FHWA's Next Generation Simulation (NGSIM) project (NGSIM, 2008) to calibrate parameters of the models.

4.2 Data Description

The family of car-following models developed is calibrated using vehicle trajectory dataset collected on a segment of Interstate 101 (Hollywood Freeway) in Los Angeles, California. The dataset contains 45 minutes of vehicle trajectory data collected on Wednesday June 15, 2005 from 7:50 am to 8:35 am. The time period from 7:50 am to 8:05 am represented build up to congestion while 8:05 to 8:35 am represented primarily congested traffic conditions (Cambridge Systematics, Inc., 2005). The benefits of using the dataset include very detailed field vehicle trajectory data collected to date by FHWA for traffic microsimulation research and development (FHWA, 2005). A full detailed description of methodology and technology used to collect and process the data are available at the NGSIM Web site at http://ngsim.fhwa.dot.gov. A brief summary of study area and other characteristics of the data are explained in the subsections that follow.

4.3 Study Site Characteristics

The vehicle trajectory data was collected on a 2100 feet long section in the southbound direction of the freeway. The section has five through lanes (lanes 1 to 5) and one auxiliary lane (lane 6). The leftmost inner lane, that is, lane 1 is the High-Occupancy Vehicle (HOV) lane. The auxiliary lane is approximately 698 feet long and connects the on-ramp at Ventura and off-ramp at Cahuenga Boulevard. The data was collected using eight synchronized digital video cameras installed on adjacent 36-storey building (10 Universal City Plaza). Figure 4-1 shows a schematic diagram of the study site and camera coverage area.

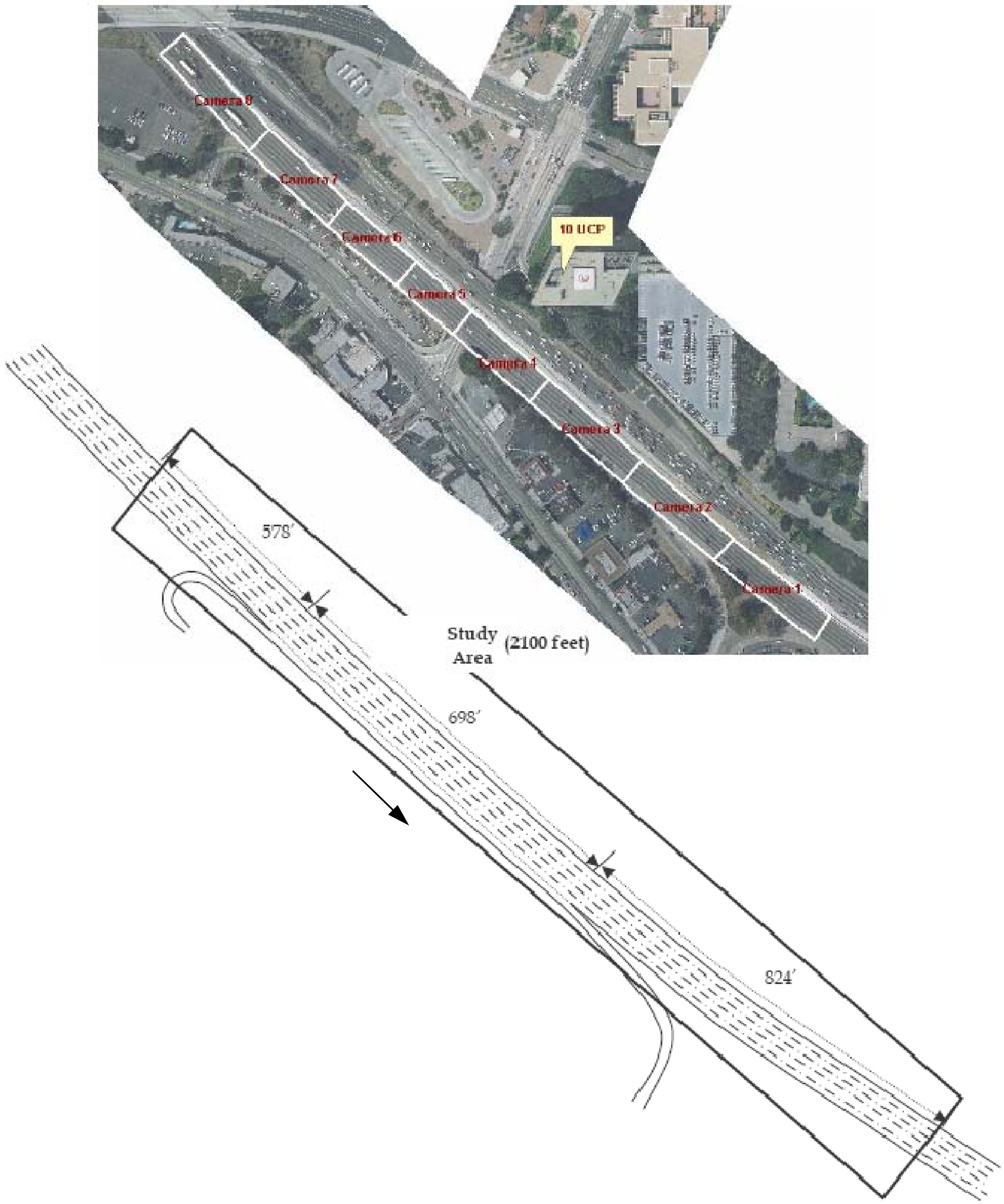


Figure 4-1. Study site and camera coverage (Cambridge Systematics, Inc., 2005).

4.4 Data Characteristics

To process the collected video data, a special computer program was used to translate the recorded video images into vehicle trajectory data. This program automatically detected and tracked individual vehicles from the recorded images from the beginning to the end of the study site. However, in situations where the translated vehicle trajectory was inaccurate, manual corrections were made. Figure 4-2 shows a flow chart summarizing the process used for detecting and tracking vehicles.

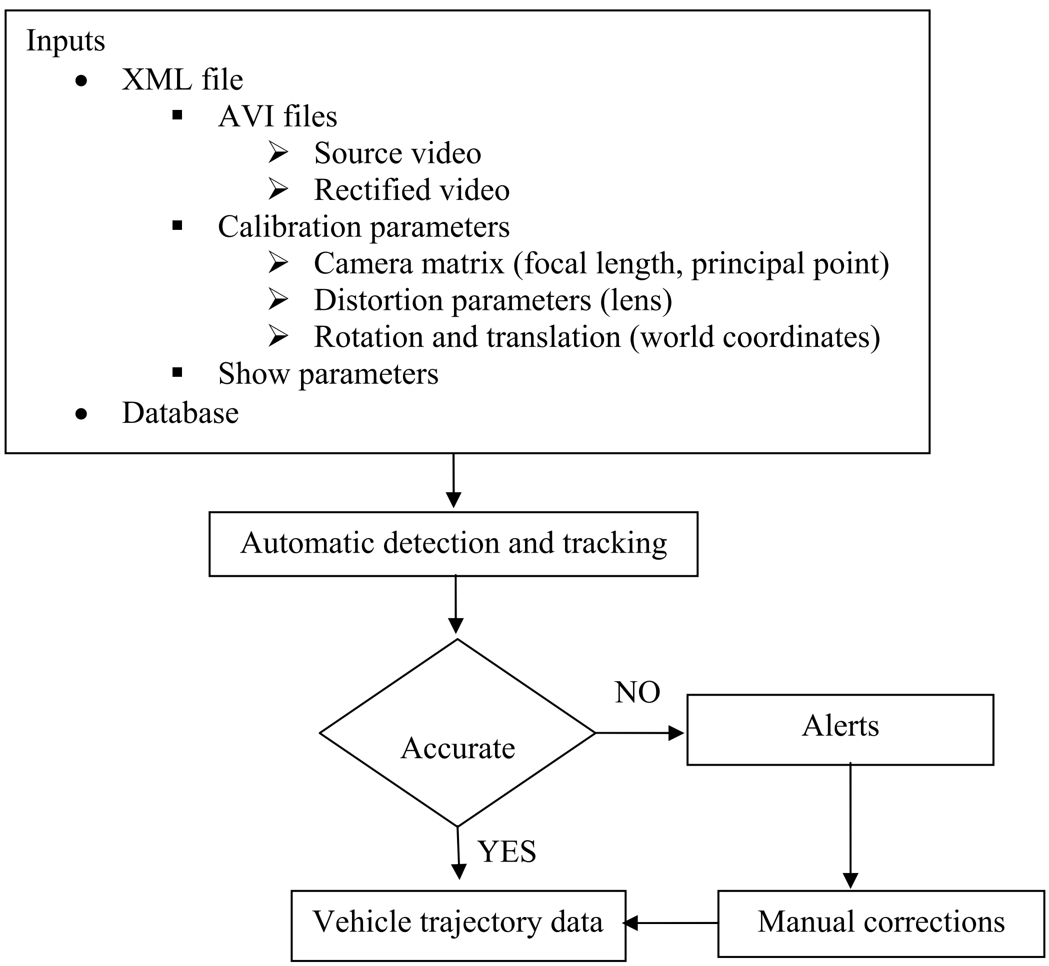


Figure 4-2. Vehicle detection and tracking process (Cambridge Systematics, Inc., 2005).

The generated vehicle trajectory data contained a detailed longitudinal and lateral positions of each vehicle in the study site every at one-tenth of a second interval. The available vehicle trajectory data from NGSIM has 18 vehicle trajectory variables that include:

1. Vehicle identification number,
2. Frame identification number,
3. Total frames—total number of frames which the vehicle appears in the dataset,
4. Global time (Epoch time)—elapsed time since January 1, 1970 (milliseconds),
5. Local X—lateral (X) coordinate of the front center of the vehicle with respect to the left-most edge of the section in the direction of travel (feet),
6. Local Y—longitudinal (Y) coordinate of the front center of the vehicle with respect to the entry edge of the section in the direction of travel (feet),
7. Global X—X coordinate of the front center of the vehicle based on CA State Plane III in NAD83 (feet),
8. Global Y—Y coordinate of the front center of the vehicle based on CA State Plane III in NAD83 (feet),
9. Vehicle Length (feet),
10. Vehicle Width (feet),
11. Vehicle Class (text),
12. Vehicle velocity—instantaneous velocity of vehicle (ft/sec),
13. Vehicle acceleration—instantaneous acceleration of vehicle (ft/sec2),
14. Lane Identification (number),
15. Preceding Vehicle (number),

16. Subject vehicle (number),
17. Spacing (spacing headway) (feet), and
18. Headway (time headway) (seconds)

However, the vehicle trajectory variables used in this study for each subject vehicle include the following:

1. Time of observation in seconds,
2. Longitudinal position (local Y) in feet,
3. Class,
4. Length in feet,
5. Speed in feet per second,
6. Acceleration/deceleration in feet per second square,
7. Lane number (lane 2, lane 3, and lane 4),
8. Spacing in feet, and
9. Leading vehicle class, speed, acceleration/deceleration, length, and spacing,

Additional post processing of the data was made to generate other variables including relative speed and separation between pairs of following vehicles.

4.5 Traffic Characteristics

Table 4-2 shows traffic composition during the study period as compiled by Cambridge Systematics, Inc. (2005). The table shows that the traffic mix consisted of approximately 0.7 percent motorcycles, 97.0 percent automobiles, and 2.3 percent trucks and buses.

Table 4-1. Vehicle Types

Time Period	Motorcycles	Automobiles	Trucks & Buses	Total
7:50 a.m. - 8:05 a.m.	30	2,086	53	2,169
8:05 a.m. - 8:20 a.m.	10	1,963	44	2,017
8:20 a.m. - 8:35 a.m.	5	1,870	40	1,915
Total	45	5,919	137	6,101
Percentage	0.7%	97.0%	2.3%	100%

Table 4-2 shows a summary of traffic flow rates and speeds at 15-minute intervals. The results show significantly low flow rates and speeds indicating congested freeway traffic conditions.

Table 4-2. Traffic Flow Rate and Speed in 15-minutes

Time Period	Vehicle flow rate (vph)	Space Mean Speed mph
7:50 a.m. - 8:05 a.m.	8,612	25.66
8:05 a.m. - 8:20 a.m.	8,016	21.59
8:20 a.m. - 8:35 a.m.	7,604	18.34

Table 4-3 shows a summary of the traffic flow rates and speeds per lane during the study period. On the average, it appears there were no appreciable differences in speeds on the through lanes. However, auxiliary lane had considerably higher difference in speeds compared to the through lanes.

Table 4-3. Traffic Flow Rate and Speed per Lane

	Vehicle flow rate (vph)			Space mean speed (mph)		
Lane	*7:50 a.m. to 8:05 a.m.*	*8: 05 a.m. to 8:20 a.m.*	*8: 20 a.m. to 8:35 a.m.*	*7:50 a.m. to 8:05 a.m.*	*8: 05 a.m. to 8:20 a.m.*	*8: 20 a.m. to 8:35 a.m.*
Lane 1	1,528	1,474	1,394	21.45	21.84	16.39
Lane 2	1,676	1,574	1,460	25.45	20.88	16.90
Lane 3	1,660	1,474	1,390	26.68	20.90	17.07
Lane 4	1,620	1,518	1,374	26.27	21.19	17.02
Lane 5	1,664	1,512	1,490	27.70	23.22	19.55
Lane 6	464	464	496	37.45	34.51	31.62
Total	8,612	8,016	7,604	26.21	22.35	18.34

Figure 4-3 shows the number of lane change made by all vehicle types during the study period. The results show that number of lane changes on the through lanes decreased from the rightmost lane to leftmost lane. In other words, lane 5 had the highest number of lane changes and lane 1 had the fewest number of lane changes. This could be related to vehicles merging from on-ramp and exiting onto off-ramp.

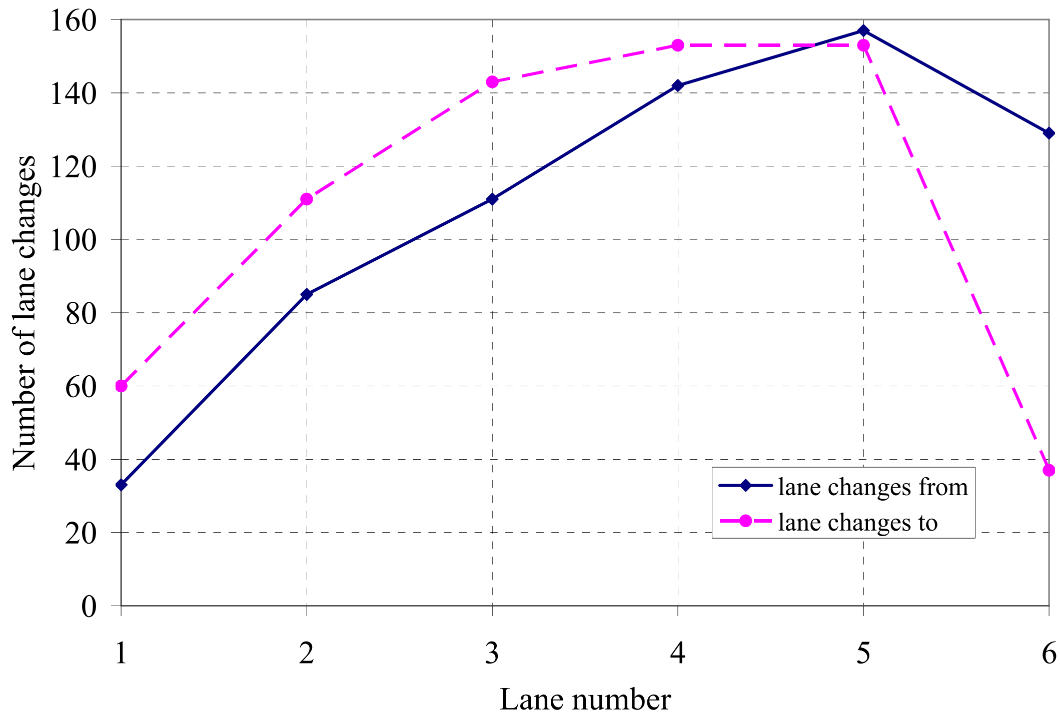


Figure 4-3. Vehicle lane changes by lane.

Figure 4-4 shows a portion of vehicle trajectory on lane 4. It indicates interactions between pairs of following vehicles such as propagation of disturbances in the traffic stream and lane changes. For example, a broken line indicates that the vehicle changed lane. From the graph, it is observed that generally large trucks on the average had bigger spacing than automobiles. This may be due to the fact that trucks require longer stopping and lane changing distances than automobiles. Further observations indicate that automobiles had similar spacing regardless of whether they were travelling behind large trucks or behind other automobiles.

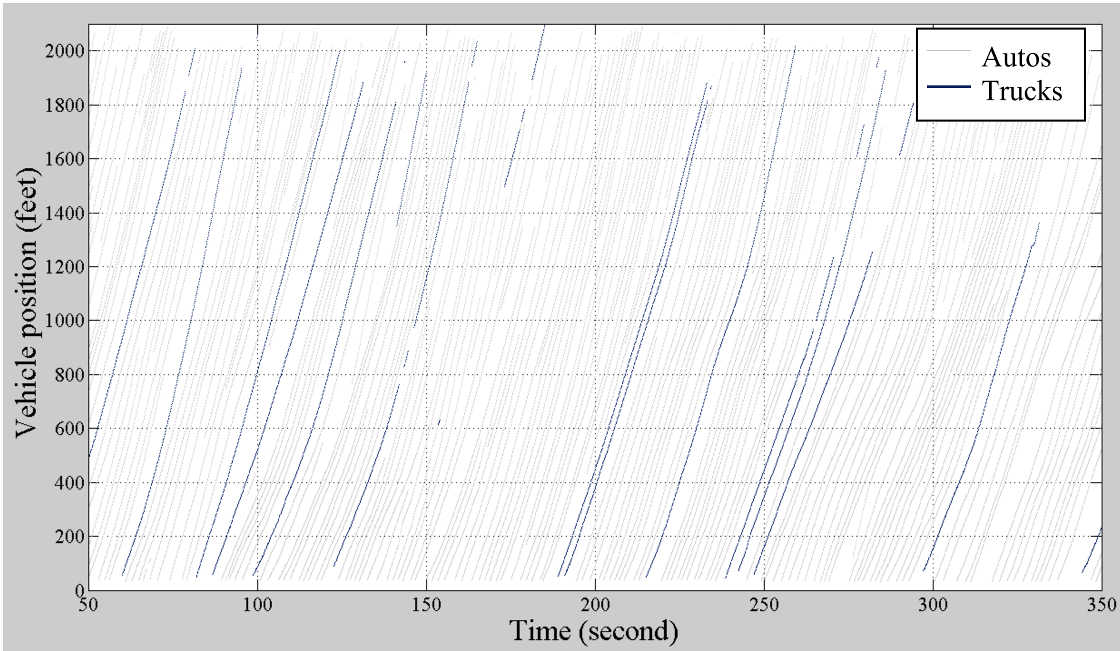


Figure 4-4. A portion of vehicle trajectories on lane 4.

4.6 Preparation of Calibration Data

Vehicle pairs selected for this study had to satisfy the following criteria:

1. Only pairs of vehicles that were following each other over the entire section without changing lanes and being interrupted by another vehicle were selected. The rationale of selecting only vehicles that did not change lanes is based on the assumption that drivers when changing lanes may exhibit different characteristics from those of car-following behavior.
2. Only vehicles in the three middle lanes were considered in order to avoid the impact of weaving movements on the auxiliary lane and the rightmost lanes as well as the leftmost lane, which is an HOV lane.

This process resulted in 749 pairs of “automobile following automobile”, 25 pairs of “automobile following large truck”, 32 pairs of “large truck following automobile”, and two pairs of “large truck following large truck”. However, for “automobile

following automobiles" model, only 75 out of 749 pairs were selected at random for use in this study. The reason for using only 75 pairs of vehicles was to reduce the workload required to complete the study. Furthermore, it should be noted there was no enough pairs of vehicles to calibrate the model for "large truck following large truck".

For each subject vehicle and the leading vehicle selected based on the criteria above, the following variables were extracted at each time interval:

1. Time of observation,
2. Acceleration/deceleration,
3. Speed of vehicles,
4. Relative speed between the two vehicles, and
5. Separation between the two vehicles.

To minimize the random fluctuations of the instantaneous trajectory data, this data was further filtered by taking the moving averages for each of the variables over 0.5 seconds. The problem of using unfiltered data was also observed by Treiber et al. (2008).

4.7 Descriptive Statistics of the Variables

This section presents the descriptive statistics of the response, stimulus, and sensitivity variables. Figure 4-5 shows distributions of acceleration/deceleration responses for different pairs of following vehicles and fitted normal density. The figure indicates symmetrical distributions with high peaks near zero for all pairs of following vehicles. Although, these high peaks may indicate drivers were traveling at constant speed, they may also be due to drivers' incidental responses. To account for this, this research assumed that the responses within ±0.05 were due to drivers' incidental

responses. In addition, it appears that there was no appreciable difference in the acceleration and deceleration responses between automobiles and large trucks.

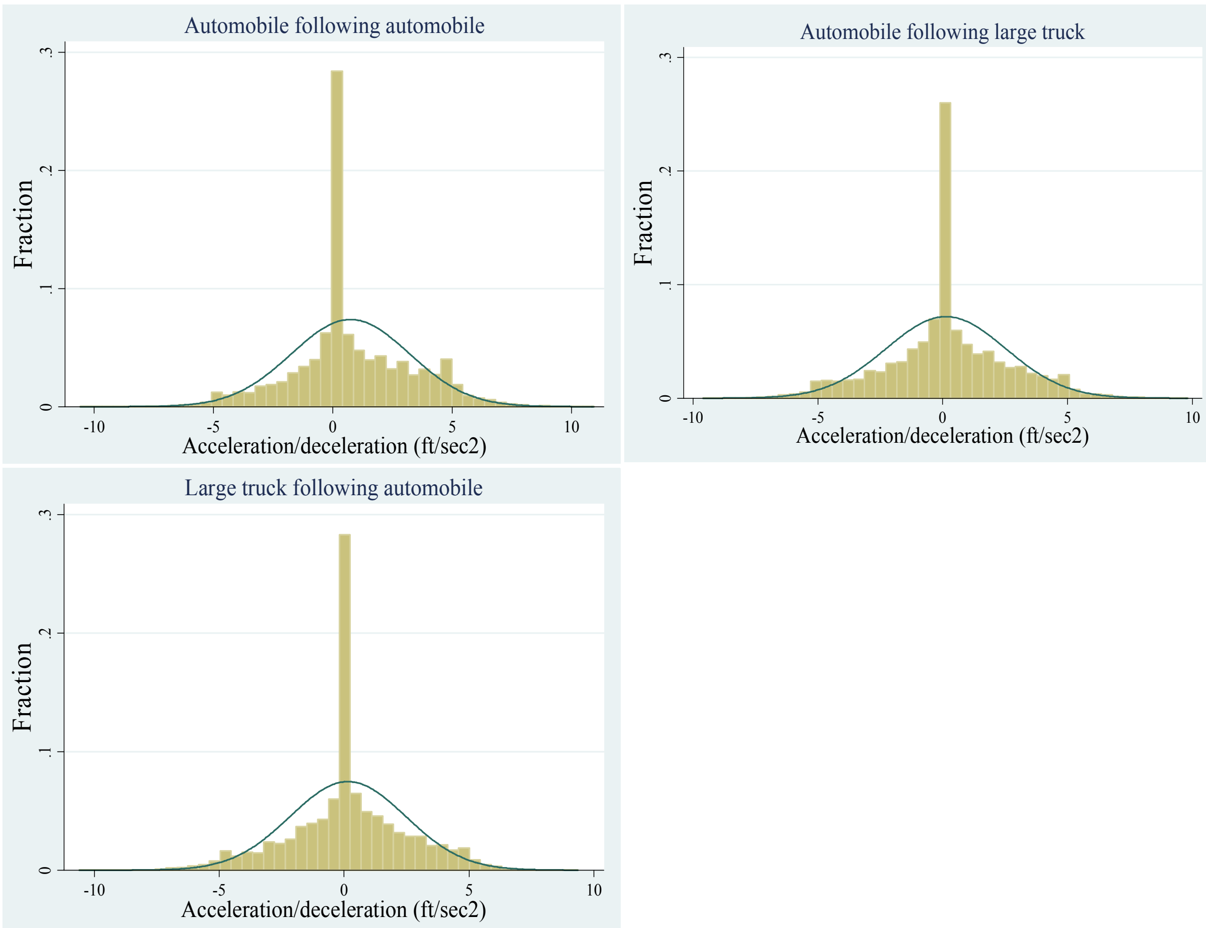


Figure 4-5. Acceleration/deceleration distributions.

Figure 4-6 shows distributions of relative speeds for different pairs of following vehicles and fitted normal density curve. The figure shows automobiles traveling behind large trucks or behind other automobiles had similar distribution of relative speeds. However, large trucks following automobiles have more spread distribution than

automobiles. This may suggest that large trucks maintained relatively higher relative speeds compared to automobiles.

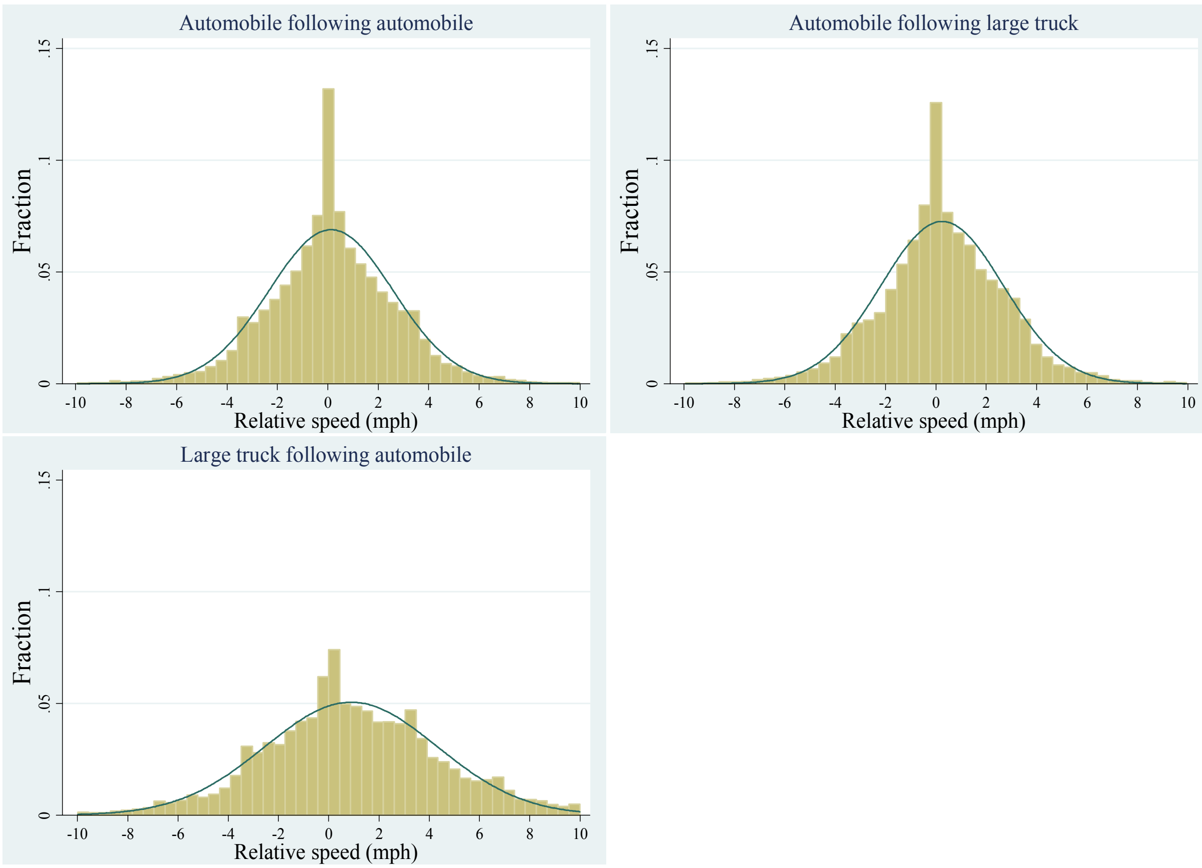


Figure 4-6. Relative speed distributions.

Figure 4-7 shows distributions of speed of a subject vehicle and fitted lognormal density curve for different pairs of following vehicles. The figure indicates that the speed distribution for “automobile following automobile” is skewed to the left while other pairs of following vehicles are approximately normally distributed. Furthermore, on the average, there are no considerable differences on the average speeds between different pairs of following vehicles. This is not surprising because congestion forces vehicles to

travel at relatively lower speeds, thus, it is unlikely for certain vehicle types to outpace the others.

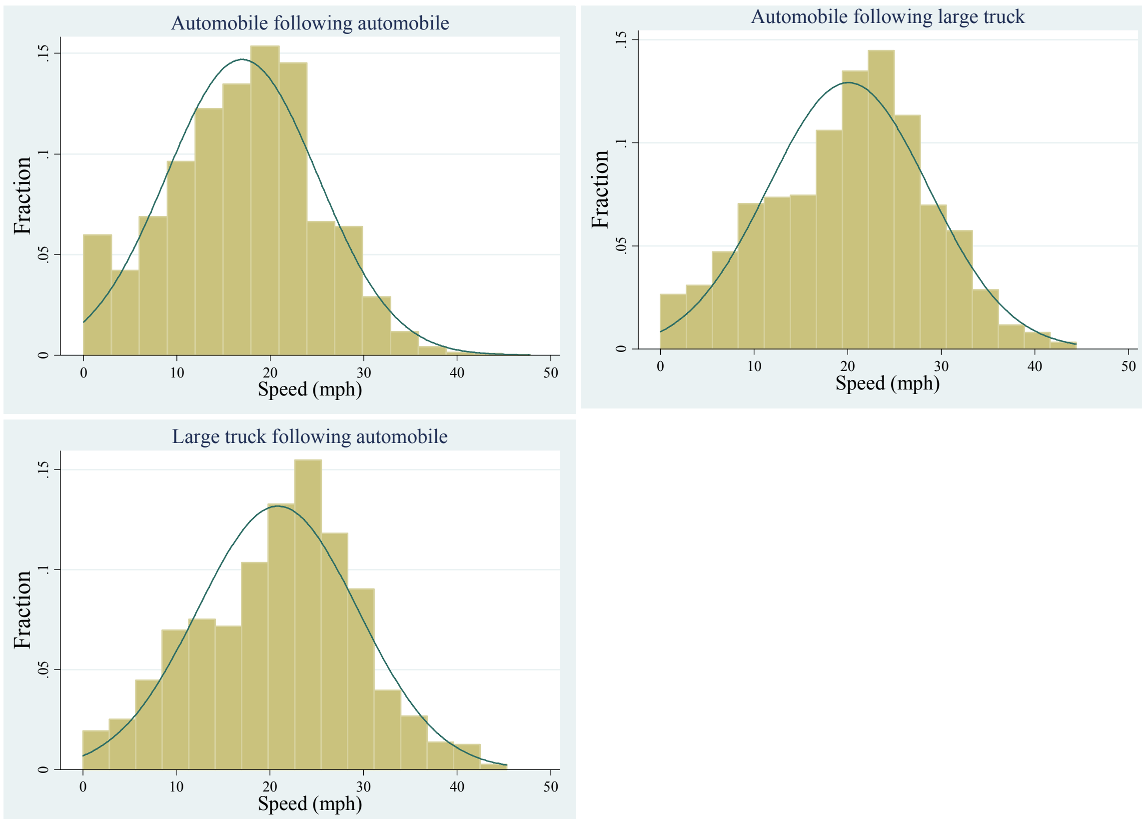


Figure 4-7. Speed distributions.

Figure 4-8 shows distributions of vehicle separation maintained for different pairs of following vehicles. A lognormal fitted curve is superimposed on each distribution. From the figure, it appears that automobiles traveling behind other automobiles maintained smaller separation compared to automobiles behind large trucks. For example, automobiles behind other automobiles had average separation of 47 feet while automobiles traveling behind large trucks had average separation of approximately 50 feet. Further observation indicates that large trucks traveling behind automobiles had

separation of about 67 feet, which appears to be significantly bigger than automobiles. This observation was expected because large trucks generally need bigger safe stopping distance compared to automobiles.

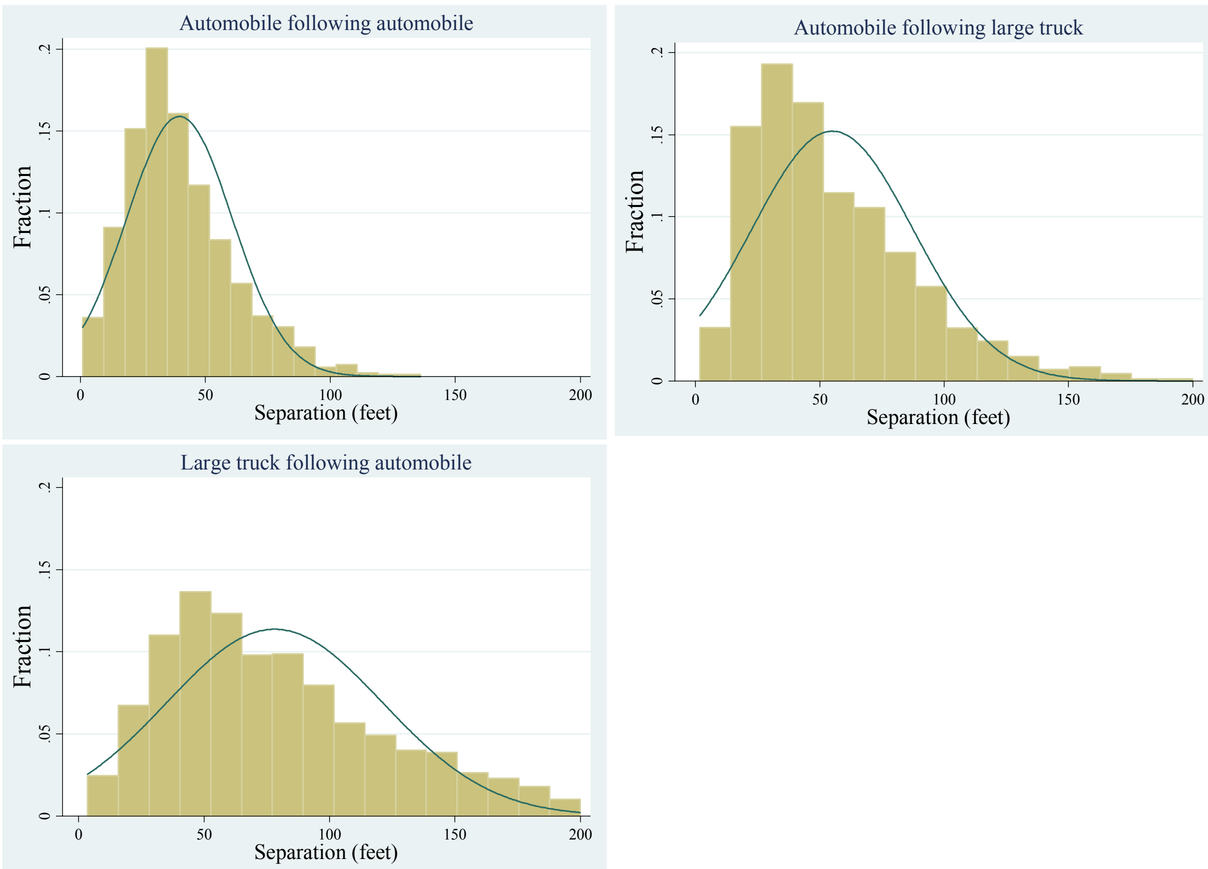


Figure 4- 8. Vehicle separation distributions.

Table 4-4 summarizes descriptive statistics of the variables for different pairs of following vehicles. The results indicate that the mean value of the acceleration response is higher than deceleration response for different pairs of following vehicles. Similarly, the positive relative speed has higher mean value than the negative relative speed. Furthermore, the observation indicates almost similar average speeds for different pairs

of following vehicles. As expected, the results indicate that large trucks traveling behind automobiles have bigger separation than automobiles.

Table 4-4. Descriptive Statistics of Variables

Automobile following automobile				
Variable	*Mean*	*Std*	*Minimum*	*Maximum*
Acceleration (ft/sec2)	2.03	1.80	0.05	11.04
Deceleration (ft/sec2)	1.91	1.81	0.05	11.00
Positive relative speed (mph)	1.95	1.74	0.00	14.45
Negative relative speed (mph)	1.85	1.66	0.00	17.91
Speed (mph)	16.40	8.14	0.00	46.17
Vehicle separation (ft)	46.66	24.41	0.78	237.05
Automobile following large truck				
Acceleration (ft/sec2)	2.04	1.76	0.05	9.61
Deceleration (ft/sec2)	1.37	1.44	0.05	8.98
Positive relative speed (mph)	1.96	1.67	0.00	2.10
Negative relative speed (mph)	1.78	1.65	0.00	16.29
Speed (mph)	20.08	8.57	0.00	44.41
Vehicle separation (ft)	49.77	27.82	1.83	150.40
Large truck following automobile				
Acceleration (ft/sec2)	2.01	1.76	0.05	9.07
Deceleration (ft/sec2)	1.57	1.67	0.05	9.36
Positive relative speed (mph)	3.79	3.87	0.00	32.24
Negative relative speed (mph)	2.48	2.24	0.00	20.83
Speed (mph)	20.85	8.57	0.00	45.30
Vehicle separation (ft)	66.44	35.67	7.88	191.64

Table 4.5 shows the same data in Table 4-4 but arranged by different pairs of following vehicles. When comparing acceleration responses between pairs of the following vehicles, it appears that there is no difference in the means of the acceleration responses for different pairs of following vehicles. However, the deceleration responses

appear to be different. Similarly, automobiles traveling behind other automobiles have relatively higher difference in speed than other pairs of following vehicles. Furthermore, observation indicates that automobiles traveling behind other automobiles have higher speed than when traveling behind large trucks. Also the results indicate that, on the average, large trucks have bigger vehicle separation compared to automobiles. More interestingly, automobiles traveling behind large trucks have almost maintained the same vehicle separation regardless of vehicle being followed.

Table 4-5. Descriptive Statistics of the Variables by Pairs of Following Vehicles

Variable	Statistic	Automobile following automobile	Automobile following large truck	Large truck following automobile
Acceleration (ft/sec^2)	Mean	2.03	2.04	2.01
	Std	1.81	1.76	1.76
	Minimum	0.000	0.05	0.05
	Maximum	11.04	9.61	9.07
Deceleration (ft/sec^2)	Mean	1.92	1.37	1.57
	Std	1.81	1.44	1.67
	Minimum	0.00	0.05	0.05
	Maximum	11.01	8.98	9.36
Positive relative speed (mph)	Mean	1.95	1.96	3.79
	Std	1.74	1.67	3.87
	Minimum	0.00	0.05	0.05
	Maximum	14.45	2.10	32.24
Negative relative speed (mph)	Mean	2.74	1.78	2.48
	Std	2.472	1.65	2.24
	Minimum	0.05	0.05	0.05
	Maximum	26.33	16.29	20.83
Speed (mph)	Mean	28.81	20.08	20.85
	Std	13.05	8.57	8.57
	Minimum	0.05	0.05	0.05
	Maximum	74.51	44.41	45.30
Vehicle separation (ft)	Mean	46.66	49.77	66.44
	Std	24.41	27.82	35.67
	Minimum	0.78	1.83	7.88
	Maximum	237.05	150.40	191.64

4.8 Estimation of Disaggregate Parameters

This section discusses in detail methodology used for estimating the driver response time lags for both acceleration and deceleration responses for each individual driver. Figure 4-9 shows the field observed speed profiles of two following vehicles. The figure shows a portion of the speed profiles when the two vehicles were decelerating, accelerating, and traveling at somewhat constant speed. The figure indicates a similar trend of the speed profiles between the following vehicle and the leading vehicle. This implies that a driver of the following vehicle was being impacted by the driving actions of the leading vehicle. Also the figure shows the following vehicle had higher peaks than the leading vehicle, suggesting that the driver of the following vehicle was unable to estimate accurately the speed of the leading vehicle.

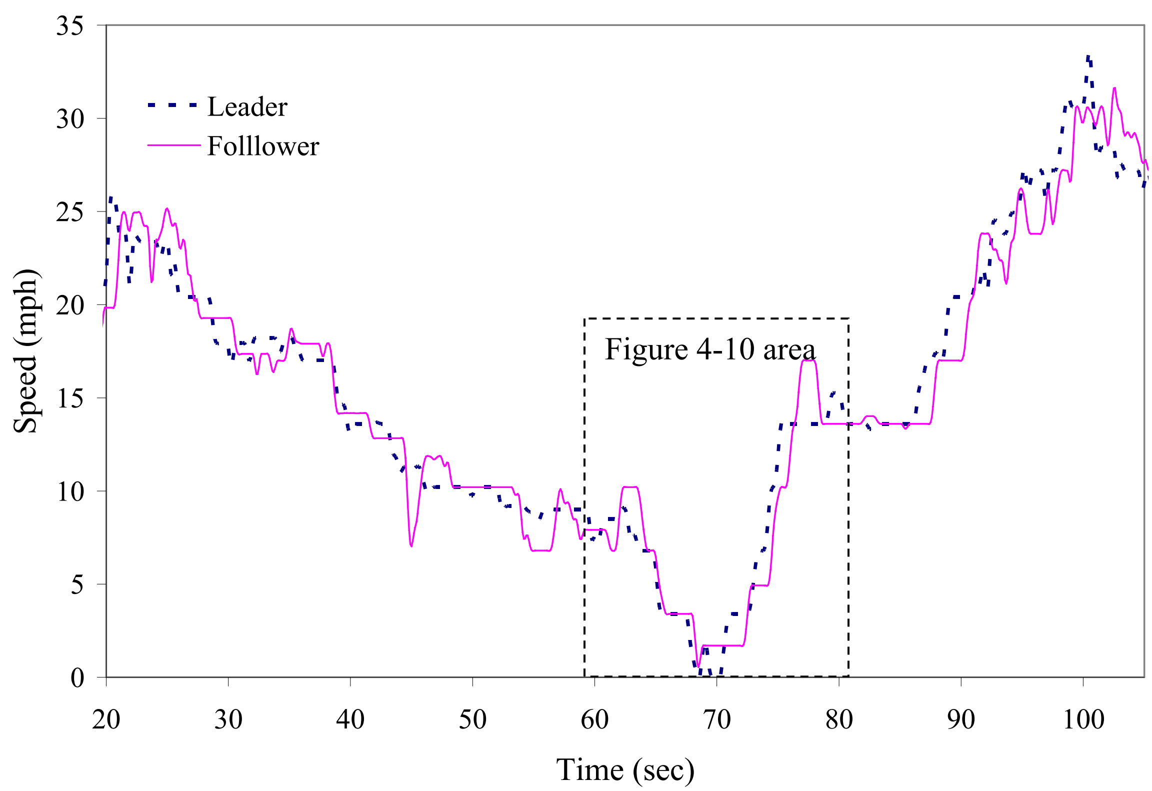


Figure 4-9. Field observed speed profiles of two following vehicles.

Figure 4-10 is the section of the graph highlighted is shown in Figure 4-9 above. The figure shows the time lag that the driver of the following vehicle responded to the driving actions of the leading vehicle. This time lag represents the driver response time lag. As shown on parts of the figure, the driver response time lag varied for the driver. However, this study assumes that the driver response time lag is the same for each individual driver but may be different for different drivers depending on the type of vehicle being driven and/or followed.

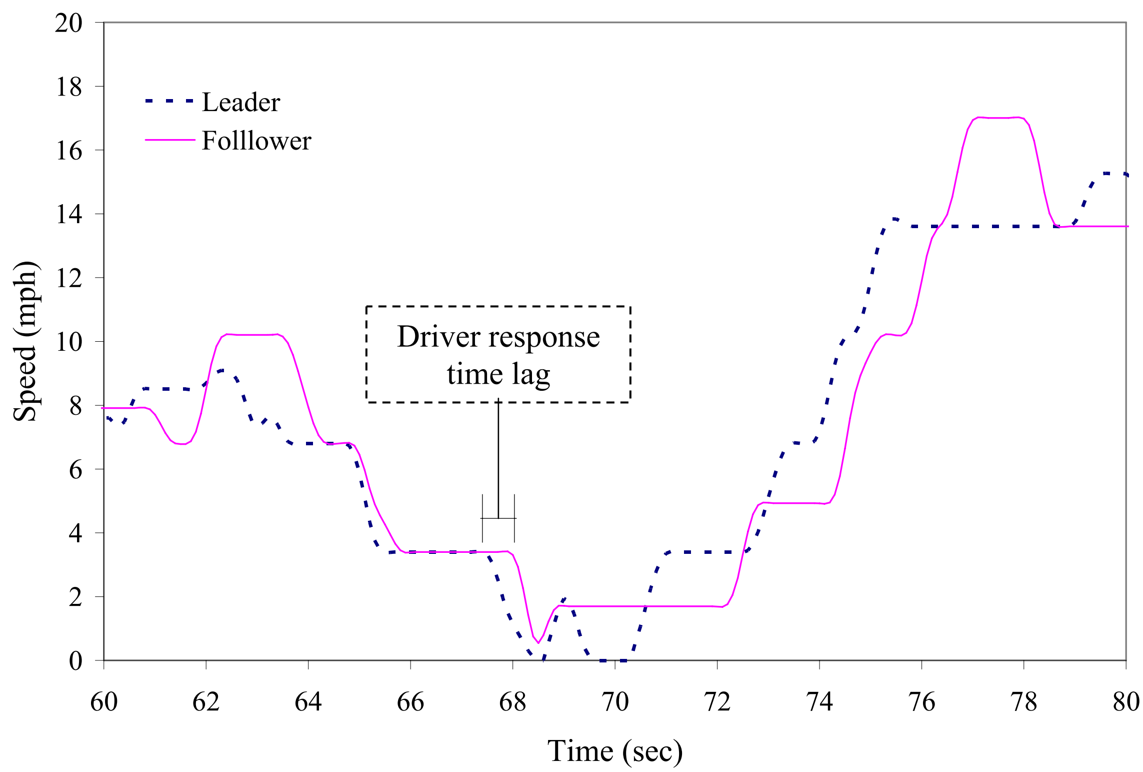


Figure 4-10. Example of a driver response time lag.

In practices, the driver response time lag cannot be estimated from the graph because it is laborious and time consuming. Therefore, this research estimates the driver response time lag using a different approach. Figure 4-11 is the continuation of the

concept for estimating the driver response time lag. The figure shows how the driver time lag can be estimated based on stimulus, which is the relative speed between the two vehicles. In the figure, three cases of the driver time lags are plotted: smaller, optimal, and bigger. The first plot represents the case where a driver response time lag is 0.0 seconds. In this case, the driver is reacting instantaneously to the driving actions of the leading vehicle. The second plot represents the driver response time lag of 0.70 seconds. The third plot represents the case where a driver response time lag is 2.0 seconds. As can be seen from the figure, in both the smaller and bigger driver response time lags, the observed accelerations and deceleration responses are more scattered and less correlated with corresponding R^2 of 0.1712 and 0.0437, respectively. On the other hand, 0.70 seconds of the driver response time lag shows the responses are less scattered and more correlated with corresponding R^2 of 0.5329. This suggests that there is an optimal time lag that maximizes the correlation between acceleration/deceleration responses and the stimulus.

This study assumes that the driver response time lag is the time that produces the maximum correlation as measured by adjusted R^2. The driver response time lag depends on individual driver's stimulus response threshold for acceleration and deceleration responses. The section that follows describes in detail the methodology used for determining the stimulus response thresholds for acceleration and deceleration responses for each individual driver.

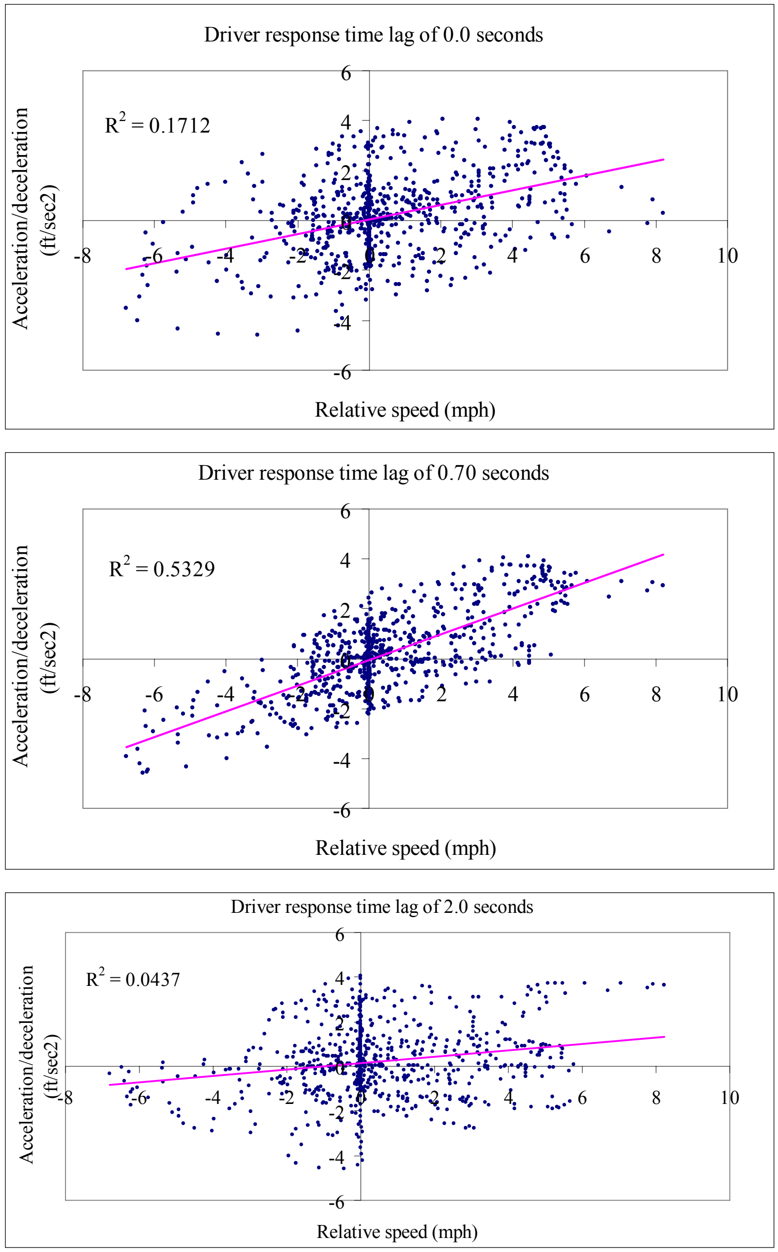


Figure 4-11. Graphical method of estimating the driver response time lags.

4.9 Determination of Driver Stimulus Response Thresholds

One of the important model parameters is the driver stimulus response threshold. This is the minimum difference in speed detectable by a following driver that will trigger a response. This threshold is likely to be different depending on whether the response is

deceleration or acceleration. As has been previously discussed, it is expected that drivers are more sensitive under deceleration response than acceleration response. Therefore, a lower magnitude of the threshold is expected for deceleration response than for acceleration response. In this study, these thresholds were determined using signal detection theory (SDT). The SDT theory has been used widely in situations with two or more discrete states of the world that cannot be easily discriminated (Wickens and Hollands, 2003). For the car-following situations, a driver is normally faced with three possible scenarios of the stimulus, namely, positive relative speed, zero relative speed, and negative relative speed. The driver is expected to respond by accelerating when faced with positive stimulus, drive at constant speed when the stimulus is zero, and decelerate when faced with a negative stimulus. This section discusses the methodology used to determine the threshold values for the stimulus that would trigger these expected responses.

The combination of state of the stimulus and three possible driver responses is shown in Table 4-6. Intuitively, when the stimulus is positive, zero, or negative, the driver is expected to respond with acceleration, keep constant speed, or deceleration, respectively. However, since the stimulus may be too small to be detected, or for other reasons, unexpected responses will occur, and so field data will generally show observations in all the six cells of Table 4-6.

Table 4-6. Outcomes of the State of Stimulus and Responses

		State of the Stimulus		
		Negative	*Zero*	*Positive*
Response	*Acceleration*	unexpected	unexpected	expected
	Constant speed	unexpected	expected	unexpected
	Deceleration	expected	unexpected	unexpected

Table 4-7 shows the observed responses of a selected driver in the dataset used in this study. For example, the table shows that the driver was faced with a total of 87 situations when the relative speed was -1.4 miles per hour. In 47 of those situations, the driver decelerated as expected. However, in 40 of those situations the driver remained in steady-state or accelerated the responses that were unexpected.

Table 4-7. Observed Responses of Selected Driver from the Dataset

		Stimulus (miles per hour)									
		-2.7	*-2.0*	*-1.4*	*-0.7*	*0.0*	*0.7*	*1.4*	*2.0*	*2.7*	*2.7*
Response	*Acceleration*	0	6	18	27	65	60	41	35	44	35
	Constant speed	0	9	22	39	63	38	21	2	1	0
	Deceleration	26	32	47	47	66	38	12	4	0	0
	Total responses	26	47	87	113	192	136	74	41	45	35

Figure 4-12 is a plot of the data shown in Table 4-7 expressed as proportions. Based on Signal Detection Theory, the threshold values X_{acc}. and X_{dec}. are the points on the curves where the driver made equal numbers of expected and unexpected responses. These thresholds delimit the acceleration, steady-state, and deceleration responses for the

driver. The thresholds depends on the driver response time lags, therefore, each driver response time lag may have different threshold values.

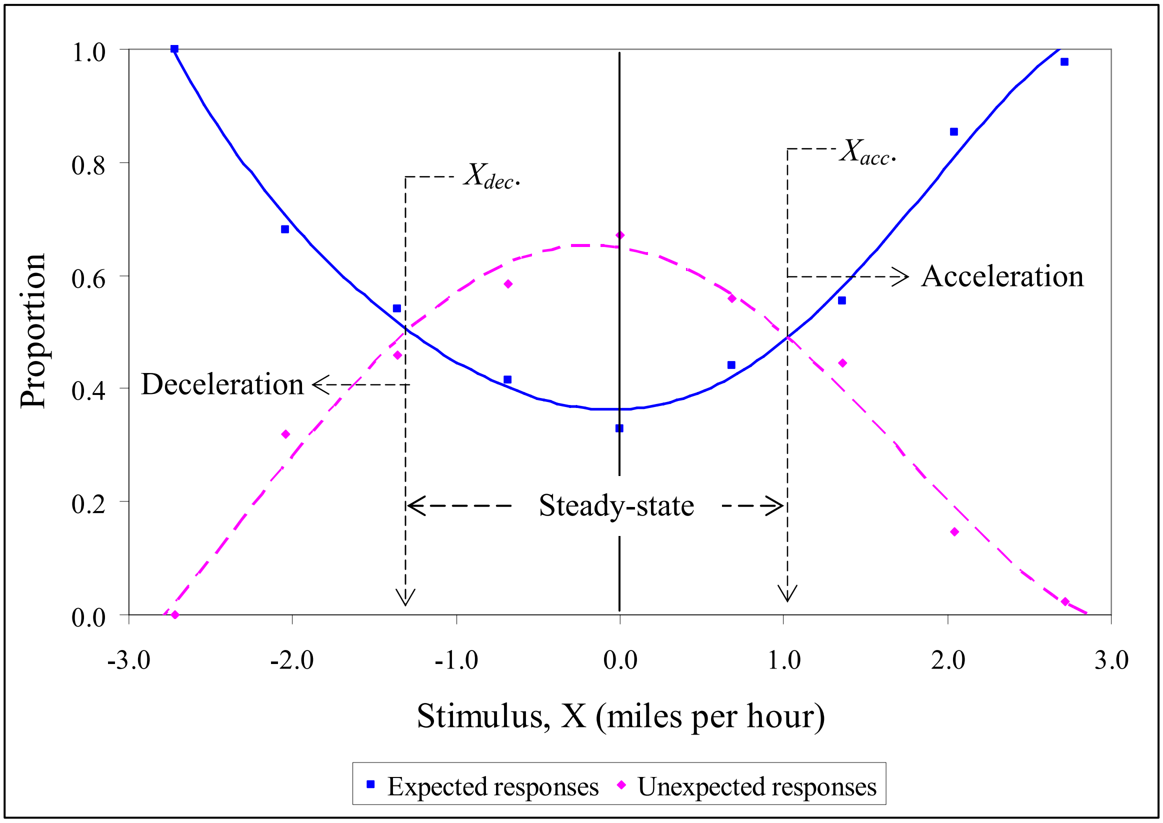


Figure 4-12. Distributions of expected and unexpected responses of an actual driver.

The methodology presented for estimating the driver response time lags and the stimulus response threshold is based on relative speed as the stimulus. Similarly, analysis of the driver response time lags and stimulus response thresholds can be determined using vehicle separation as the stimulus. Due to different possibilities of using different stimulus in estimating the driver response time lag and stimulus response thresholds, analysis performed in this study for determining the optimal driver response time lags is based on the combined effect of speed difference, speed, and vehicle separation. This was

done by running the models in Equation 3-2 for different driver response time lags, each with different stimulus response thresholds using Stata statistical program. In other words, the driver response time lag was estimated jointly with other parameters of the models. The time lag that produced the best statistically model fit as measured by adjusted R^2 at 5 percent significance level represents the driver response time lag. The stimulus response thresholds that result from the best model is used as the stimulus response thresholds for that particular driver.

Figure 4-13 shows a plot indicating variation of goodness-of-fit of the model for different driver response time lags as measured by adjusted R^2 for different stimulus response threshold values. From the figure, the estimated driver response time lag for the acceleration response is 0.80 seconds resulting from the stimulus threshold value of 1.0 mph. For the deceleration response, the driver response time lag is 0.6 seconds resulting from the stimulus threshold value of 0.54 mph.

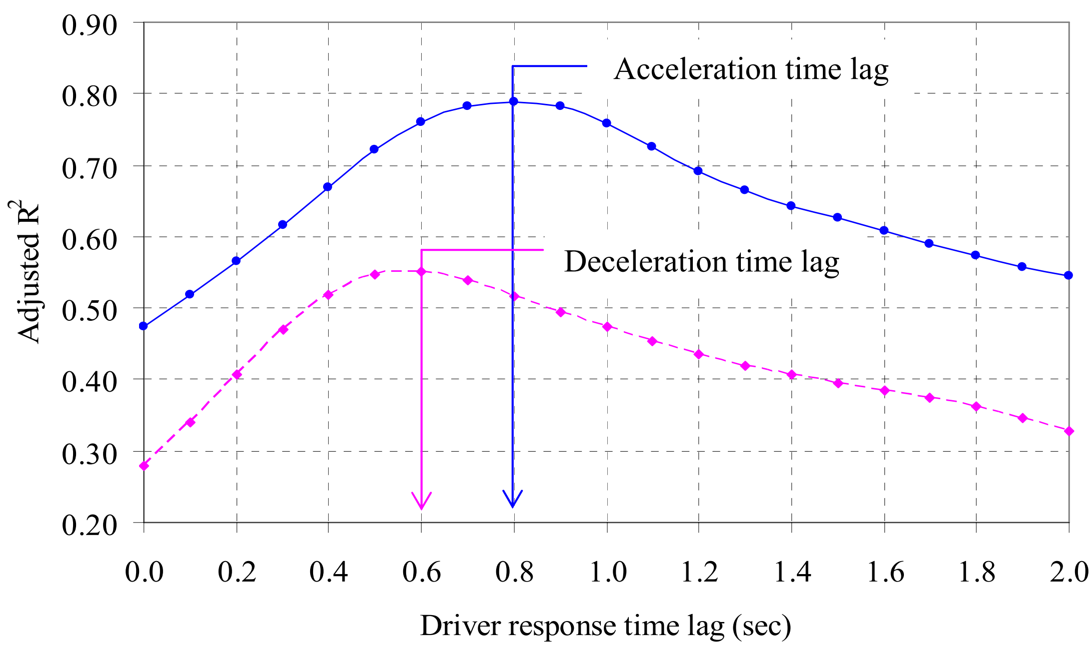


Figure 4-13. Optimal driver response time lags for a single driver.

Table 4-8 shows the results for a single driver for both the acceleration and deceleration responses for “automobile following automobile” model based on equation 3-2.

Table 4-8. Results for Acceleration/Deceleration Response Models for Single Driver

Parameter	Response	Coeff.	Std error	t-stat.	p-value
Driver sensitivity constant, β_0	*Acceleration*	1.000	-	-	-
	Deceleration	0.523	0.282	1.85	0.065
Speed, β_1	*Acceleration*	-0.469	0.105	-4.45	0.000
	Deceleration	0.771	0.201	3.84	0.000
Vehicle separation, β_2	*Acceleration*	0.434	0.122	3.54	0.000
	Deceleration	-1.208	0.201	-6.01	0.000
Relative speed, β_3	*Acceleration*	0.902	0.079	11.35	0.000
	Deceleration	1.853	0.151	12.22	0.000
Number of observations	*Acceleration*	447			
	Deceleration	364			
Adjusted R^2	*Acceleration*	0.631			
	Deceleration	0.518			

For this particular driver, the results indicate that in both the acceleration and deceleration responses, coefficients have the expected signs. The acceleration response shows that the speed parameter β_1 has negative sign, indicating that acceleration response is lower at higher speeds and higher at lower speed. The results also shows that vehicle separation parameter β_2 has positive sign implying that acceleration response is lower at smaller separation and higher at bigger separation. Similarly, the positive sign of relative speed parameter β_3 indicate that acceleration magnitude is higher at higher speed difference than at lower speed. It should be noted that the driver sensitivity constant for the acceleration model was insignificant at 5 percent level.

The deceleration model shows that the speed parameter β_1 has positive sign, indicating that deceleration response is higher at lower speeds and vice versa. The results also shows vehicle separation parameter β_2 has negative sign indicating that the deceleration magnitude is higher when separation at smaller and lower when separation is bigger. Similarly, the positive sign of relative speed parameter β_3 indicate that the deceleration response is higher at higher speed difference.

The results also show that all parameters have higher magnitudes for the deceleration response than acceleration response. As expected, this may be related to the drivers' desire for both safety and mobility. Drivers when decelerating, they are trying to keep their desired safe distance partly due to safety related reasons. On the contrary, drivers when accelerating, they are trying to attain their desired maximum speed, which is less critical and urgent than safety.

Table 4-9 shows the results for the steady-state response for a single driver from the data set based on equation 3-3. The speed parameter is β_1 while vehicle separation parameter is β_2. The sign of the parameters are both positive and according to intuitive expectation. This implies that at higher travel speeds, the response is likely to be deceleration while at lower speeds the response is likely to be acceleration. On the other hand, at bigger vehicle separation, the response is likely to be acceleration and vice versa.

Table 4-9. Result for Steady-State Response Model for Single Driver

Parameter	Coefficient	Std error	t-stat.	p-value	95% C.I.
β_0	-1.296	0.216	-6.00	0.000	[-1.721, -0.871]
β_1	0.338	0.060	5.67	0.000	[0.220, 0.454]
β_2	0.495	0.035	14.30	0.000	[0.426, 0.563]
Adjusted R^2		0.172			
Number of observations		447			

4.10 Modeling Process

As explained above, it is clear that parameters of the models are interrelated and cannot be estimated independently. This study estimated the parameters jointly in an iterative process. Figure 4-14 summarizes the entire process of estimating the model parameters.

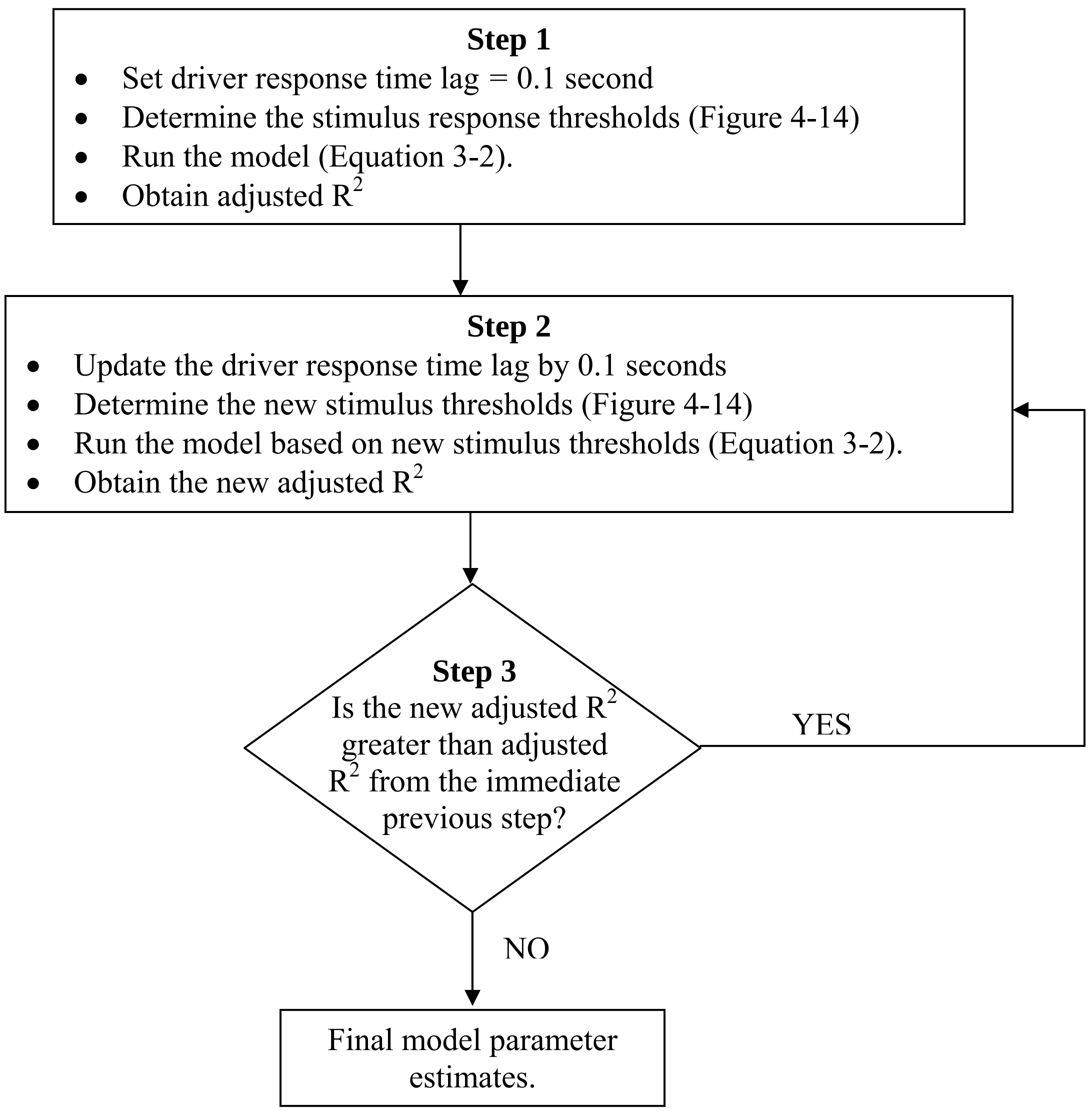


Figure 4-14. Modeling process.

CHAPTER 5

MODEL RESULTS AND DISCUSSIONS

5.1 Introduction

This chapter presents and discusses results of models developed for acceleration, deceleration and steady-state responses. It is worthwhile mentioning that measurements taken over time such as vehicle trajectory generally are serially correlated. This violates homoskedasticity assumption on error term. Even with stringent model specifications, error term in trajectory data similar to that used in this study will exhibit heteroskedasticity. The presence of heteroskedasticity will inflate test statistics used for making inferences and hypothesis testing of parameters of the models. The Stata program handles this problem using robust command. The robust estimation produces *t-statistic* of parameters based on asymptotic covariance matrix, therefore, accounting for heteroskedasticity in the error term. Furthermore, the results for each of the models are separated depending on type of vehicle being driven and/or followed. This include: "automobile following automobile", "automobile following large truck", and "large truck following automobile".

5.2 Results for Acceleration and Deceleration Response Models

Table 5-1 summarizes the results of parameter estimates for the acceleration and deceleration response models with their corresponding standard deviations in parenthesis. The results are for models for: "automobile following automobile", "automobile following large truck", and "large truck following automobile". The sign of the parameters in the table is based on equation 3-2.

Table 5-1 Results for Acceleration and Deceleration Response Models

Response	Parameter	Automobile following automobile	Automobile following large truck	Large truck following automobile
Acceleration	*Stimulus threshold,* z_1 *(mph)*	1.29 (0.64)	1.24 (0.57)	1.33 (0.77)
	Driver response time lag, Δt_1 *(sec)*	0.80 (0.25)	0.82 (0.25)	0.78 (0.20)
	Driver sensitivity constant, β_0	1.839 (3.24)	0.906 (0.24)	1.492 (1.58)
	Speed, β_1	-0.961 (1.06)	-1.012 (1.07)	-1.447 (2.11)
	Vehicle separation, β_2	0.737 (0.50)	0.746 (0.94)	0.672 (1.45)
	Relative speed, β_3	0.667 (0.51)	0.778 (0.61)	0.844 (0.85)
Deceleration	*Stimulus threshold,* z_2 *(mph)*	-0.96 (0.56)	-1.03 (0.65)	-1.06 (0.54)
	Driver response time lag, Δt_2 *(sec)*	0.71 (0.18)	0.68 (0.14)	0.67 (0.15)
	Driver sensitivity constant, β_0	-3.247 (4.81)	-1.161 (0.77)	-1.224 (1.00)
	Speed, β_1	1.298 (1.38)	1.766 (1.68)	2.329 (3.99)
	Vehicle separation, β_2	-1.544 (1.22)	-1.975 (1.60)	-2.352 (2.90)
	Relative speed, β_3	1.243 (0.62)	1.226 (0.91)	1.490 (1.46)

Table 5-2 shows the results used for comparing difference means of parameter estimates between the acceleration response and deceleration response models. The table indicates for each parameter, the mean, standard deviation, mean difference, pooled standard deviation, and *p-value.* The section that follows discusses in detail the interpretations and implications of these results in emulating the observed field car-following behavior.

Table 5-2. Statistical Comparison of Parameters of the Models

Automobile following automobile							
	Acceleration model		Deceleration model		Comparison statistics		
Parameter	*Mean*	*Std dev.*	*Mean*	*Std dev.*	*Mean diff.*	*Pooled Std dev.*	*p-value*
Driver response time lag (sec)	0.80	0.26	0.70	0.18	0.10	0.22	0.025
Stimulus threshold (mph)	1.29	0.64	0.96	0.56	0.33	0.60	0.001
Driver sensitivity, β_0	1.839	3.247	-3.247	4.808	5.086	4.113	0.000
Speed, β_1	-0.961	1.062	1.298	1.379	-2.259	1.234	0.000
Vehicle separation, β_2	0.737	0.501	-1.544	1.216	2.281	1.018	0.000
Relative speed, β_3	0.667	0.507	1.243	0.617	-0.576	0.523	0.000
Automobile following large truck							
Driver response time lag (sec)	0.82	0.25	0.68	0.14	0.14	0.28	0.016
Stimulus threshold (mph)	1.24	0.57	1.03	0.65	0.21	0.62	0.210*
Driver sensitivity, β_0	0.906	0.242	-1.161	0.769	2.067	0.571	0.120*
Speed, β_1	-1.012	1.066	1.766	1.681	-2.778	1.368	0.000
Vehicle separation, β_2	0.746	0.943	-1.975	1.599	2.729	1.289	0.000
Relative speed, β_3	0.778	0.613	1.226	0.914	-0.262	0.784	0.084
Large truck following automobile							
Driver response time lag (sec)	0.78	0.20	0.67	0.15	0.11	0.170	0.04
Stimulus threshold (mph)	1.33	0.77	1.06	0.54	0.27	0.655	0.058
Driver sensitivity, β_0	1.492	1.583	-1.224	1.000	2.716	8.297	0.026
Speed, β_1	-1.447	2.113	2.329	3.991	-3.776	3.205	0.000
Vehicle separation, β_2	0.672	1.453	-2.352	2.895	3.024	2.294	0.000
Relative speed, β_3	0.844	0.851	1.490	1.458	-0.646	1.202	0.054

Note: * indicate the difference in means is statistically insignificant

5.3 Discussion of the Parameters

The following is a discussion of each of the parameter values and their distributions for acceleration and deceleration responses. The parameters include the driver response time lags, stimulus thresholds, driver sensitivity constant, parameters for speed, relative speed, and vehicle separation. As previously stated in this study, the

acceleration and deceleration responses have similar model form. Therefore, the results of each parameter of the models are superimposed in one plot. This is helpful when comparing drivers' behavior for the acceleration and deceleration responses.

5.3.1 Driver Response Time Lags

Figure 5-1 shows the estimated distributions of drivers' response time lags for both the acceleration and deceleration responses and for different pairs of following vehicles. On the average, the results for "automobile following automobile" indicate that the driver response time lag for acceleration response is 0.80 seconds with standard deviation of 0.25 seconds. For the deceleration response, the average driver response time lag is 0.70 with standard deviation of 0.18 seconds.

Hypothesis test conducted to evaluate if there is statistical differences in the means for the acceleration and deceleration responses is shown in Table 5-2. The null hypothesis in this context is that there is no difference in means between the acceleration response and deceleration response. Using a 5 percent significance criterion, the mean difference in the drivers' response time lags for the acceleration and deceleration was found to be statistically different from zero. The results are in agreement with intuitive expectation, that is, drivers have lower response time lag when decelerating than accelerating partly due to safety reasons. This could be related to drivers' aggressive need to maintain safe vehicle separation when decelerating as opposed to the less critical need to attain desired maximum speed related to acceleration response. In addition, drivers' response to negative stimulus is further aided by the activation of brake lights for the leading vehicle that is braking.

These results contradict results obtained by Subramanian (1996) that drivers' response time lag was higher for the deceleration response than acceleration response. Furthermore, the mean values are closer to 0.70 seconds for expected situations reported in the studies by Colbourn (1978) and Ma and Andreasson (2006). In congested freeway traffic conditions, drivers are more cautious and they are likely to expect driving actions of the leading vehicle.

Similar analysis was conducted for "automobile following large truck" and "large truck following automobile" models. The results as shown in Figure 5-1 also indicate similar distributions for the driver response time lags. This suggests that during congested freeway traffic conditions, drivers have similar response time lags regardless of type of vehicle being driven and/or followed. The results are intuitive because under congested traffic conditions, drivers are more concerned with longer travel times as a result of lower travel speeds. Furthermore, drivers usually maintain smaller separation when driving in congested freeway traffic conditions than uncongested traffic conditions. Consequently, drivers are expected to be more vigilant and respond with smaller response time lags when decelerating than when accelerating partly due to safety related reasons.

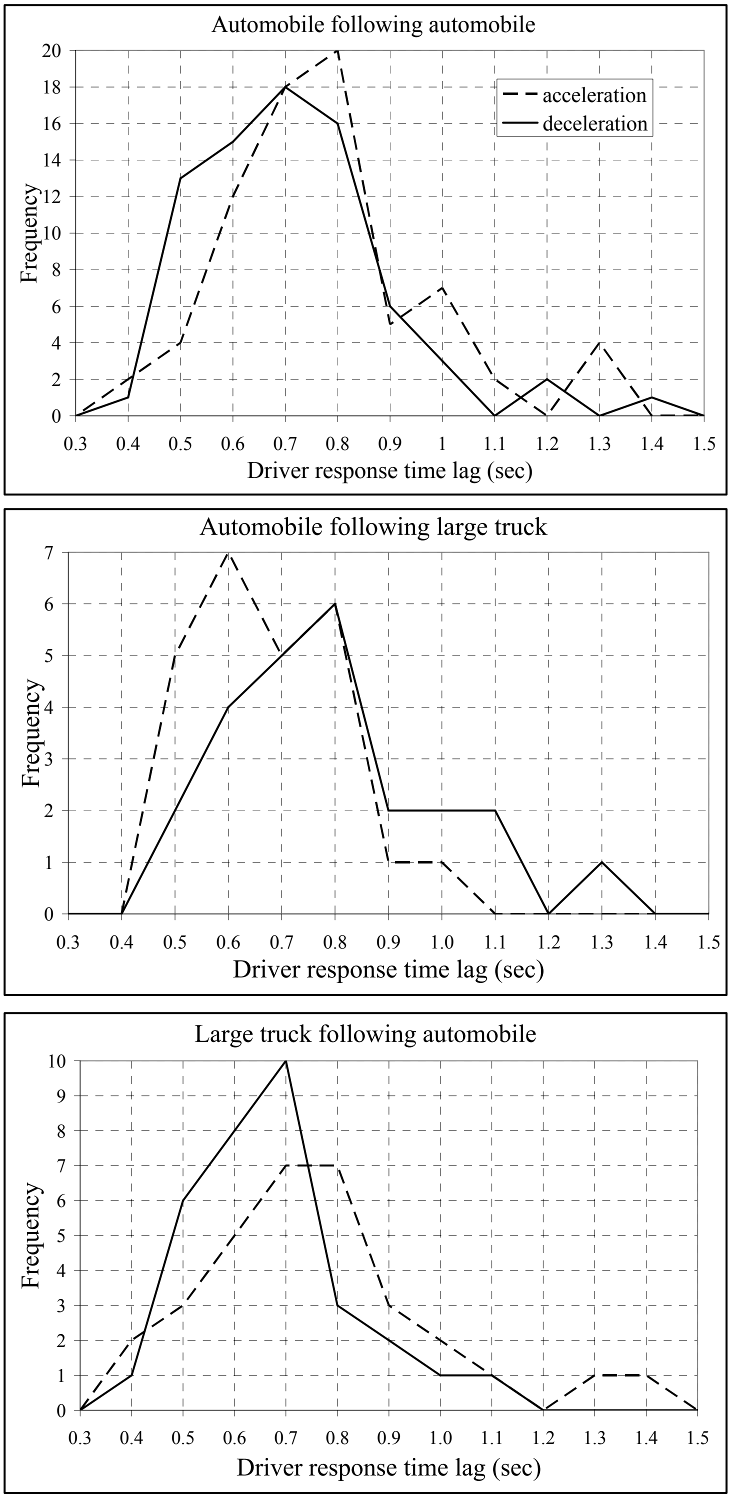


Figure 5-1. Distributions of the driver response time lags.

5.3.2 Driver Stimulus Response Thresholds

Figure 5.2 shows the distributions of stimulus response thresholds for both the acceleration and deceleration responses and for different pairs of following vehicles. For the "automobile following automobile" model, the average threshold for the acceleration response is 1.29 miles per hour with standard deviation of 0.64 miles per hour. For the deceleration response, the average threshold is -0.96 miles per hour with standard deviation of 0.56 miles per hour. The mean difference is statistically different at 5 percent significant criterion. As expected, these results are intuitive, in that, drivers are expected to respond to smaller stimulus when decelerating than when accelerating. The reason for this is similar to the one for the driver response time lags discussed above.

These results are in line with those obtained by Todosiev (1963) and Michaels (1965). However, these threshold values are significantly lower than the ones reported in study by Evans and Rothery (1974) which found that under optimal driving conditions in a field the lowest perceptible closing relative speed was 3.0 miles per hour with a probability of 0.99 of correct detection at 197 feet over a 4.0 seconds observation period. However, individual differences in ability to detect motion are large and dependent on speed of the subject vehicle and separation.

Comparison of the results for the acceleration and deceleration responses for different pairs of following vehicles, indicate insignificant difference in the means. This means that in congested freeway traffic conditions drivers detect almost the same magnitudes of stimulus that triggers a response regardless of vehicle being driven and/or followed. This could be partly due to the activation and deactivation of brake light from the leading vehicle that is braking.

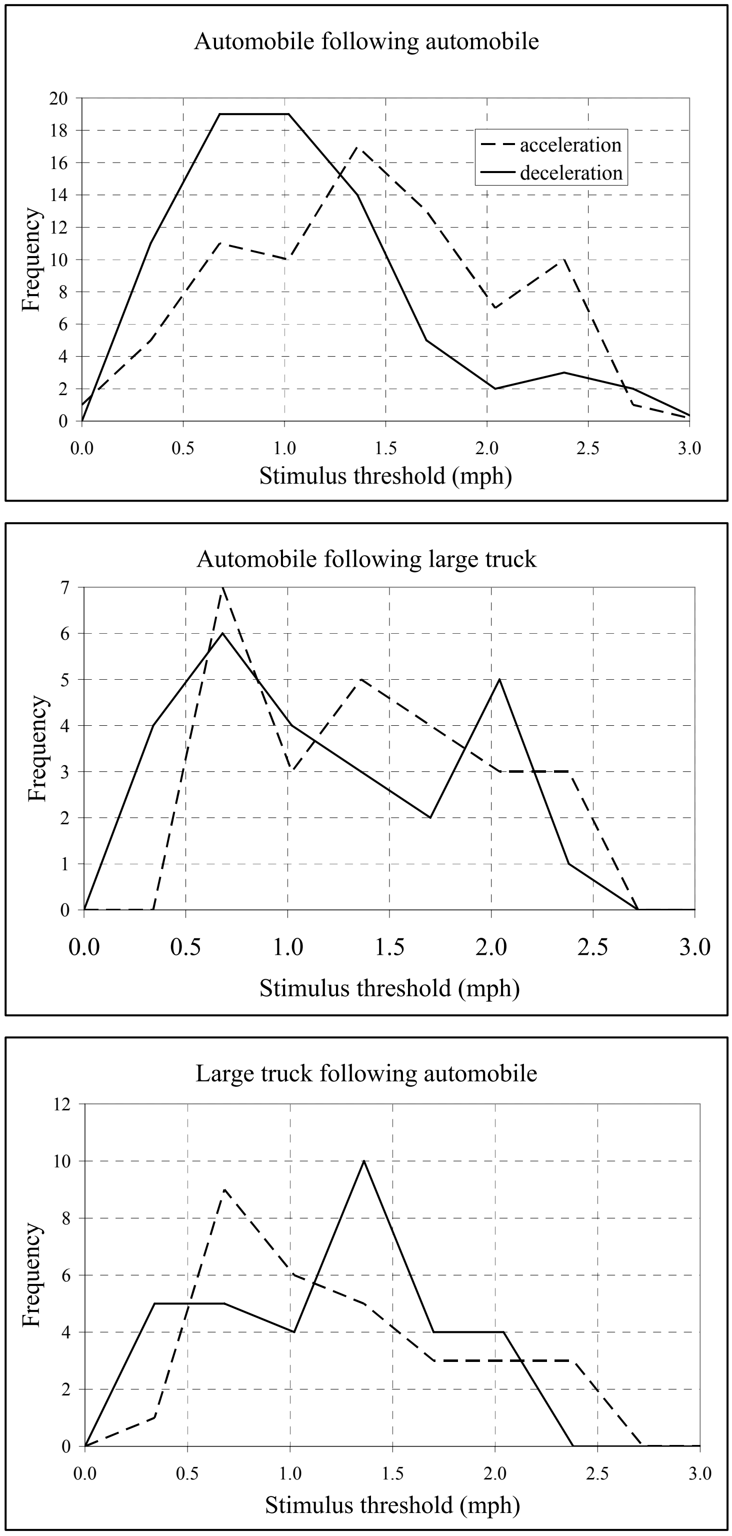


Figure 5-2. Distributions of the stimulus response thresholds.

5.3.3 Driver Sensitivity Constant, β_0

Figure 5-3 show the magnitude of distributions of the driver sensitivity constant, β_0 (refer equation 3-2) for the acceleration and deceleration response models. The figure shows very similarly skewed distributions of the driver sensitivity constant for both the acceleration response and deceleration responses and for different pairs of following vehicles. For "automobile following automobile" the average driver sensitivity constant for the acceleration response is 1.839 with standard deviation of 3.25. For the deceleration response, the mean value is 3.06 with standard deviation of 4.81. The results confirm the expectation that drivers are likely to be more sensitive for deceleration response than for acceleration response due to safety concerns and activation of brake lights of leading vehicle that is braking. However, the distributions are skewed to the left creating a dilemma whether the mean, mode, or median should be used as the measure of the driver sensitivity constant. This study used the mean values as the estimate of the driver sensitivity constant. The mean difference is statistically significant at 5 percent level.

When comparing the driver sensitivity for different pairs of following vehicles, the results for acceleration response indicate insignificant difference in the mean values of the driver sensitivity constant. For the deceleration response, the results show that automobiles traveling behind other automobiles have significantly higher mean values than other pairs of following vehicles. However, the driver sensitivity is interrelated with other sensitivity parameters such as speed and separation, thus, the results may be inconclusive in explaining the drivers' sensitivity to stimulus.

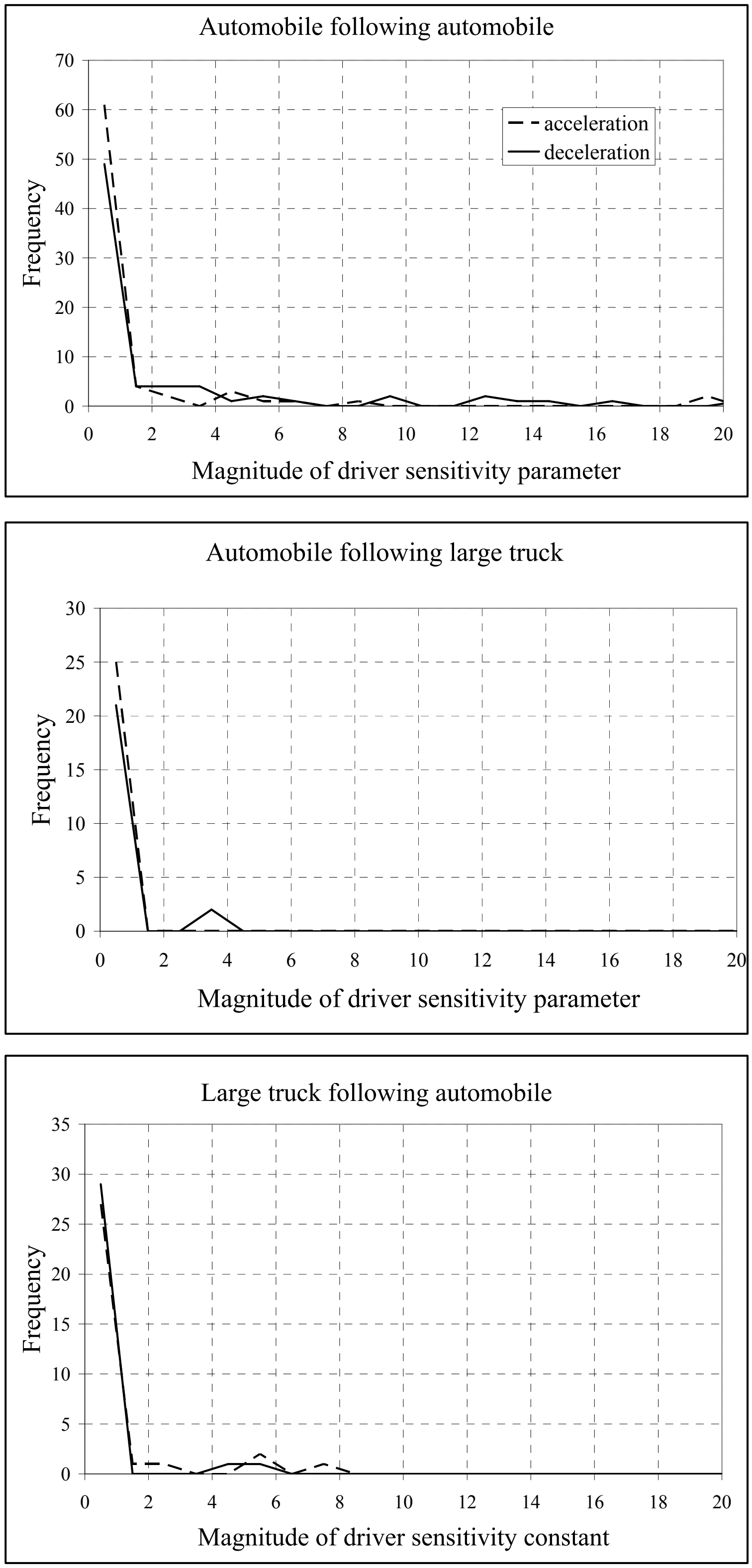


Figure 5- 3. Distributions of the driver sensitivity constant, β_0.

5.3.4 Speed Parameter, β_1

Figure 5-4 shows plots of distributions of speed parameter, β_1 for both the acceleration and deceleration response models and for different pairs of following vehicles. For "automobile following automobile" model, the average parameter value for the acceleration response is -0.961 with standard deviation of 1.06 and that for deceleration response is 1.298 with standard deviation of 1.38. The mean difference is statistically significant at 5 percent level. As expected, signs of the parameters for both the acceleration response and deceleration response are intuitive. For the acceleration response, the negative sign implies that at higher speeds drivers have lower magnitudes of acceleration response than at lower speeds. The reasons for this include reduced vehicle acceleration capability and less drivers' desire to drive faster at higher speeds. For the deceleration response, the positive sign indicates that drivers have higher magnitudes of deceleration response at higher speeds than at lower speeds. The higher magnitude for the deceleration response compared to acceleration response suggests that drivers are more sensitive to speed when decelerating than accelerating. These results contradict those obtained by Ahmed (1999) and Toledo (2002) who found that speed was statistically insignificant in the deceleration response.

Comparison of the results for different pairs of following vehicles, indicate insignificant difference in the mean values for the acceleration response. For the deceleration response, the results indicate large truck traveling behind automobiles have significantly higher mean value of speed parameter than automobiles. This is intuitive result because large trucks are heavier and require longer stopping and lane changing

distances than automobiles. Therefore, large trucks are likely to be more sensitive to speed when decelerating compared to automobiles.

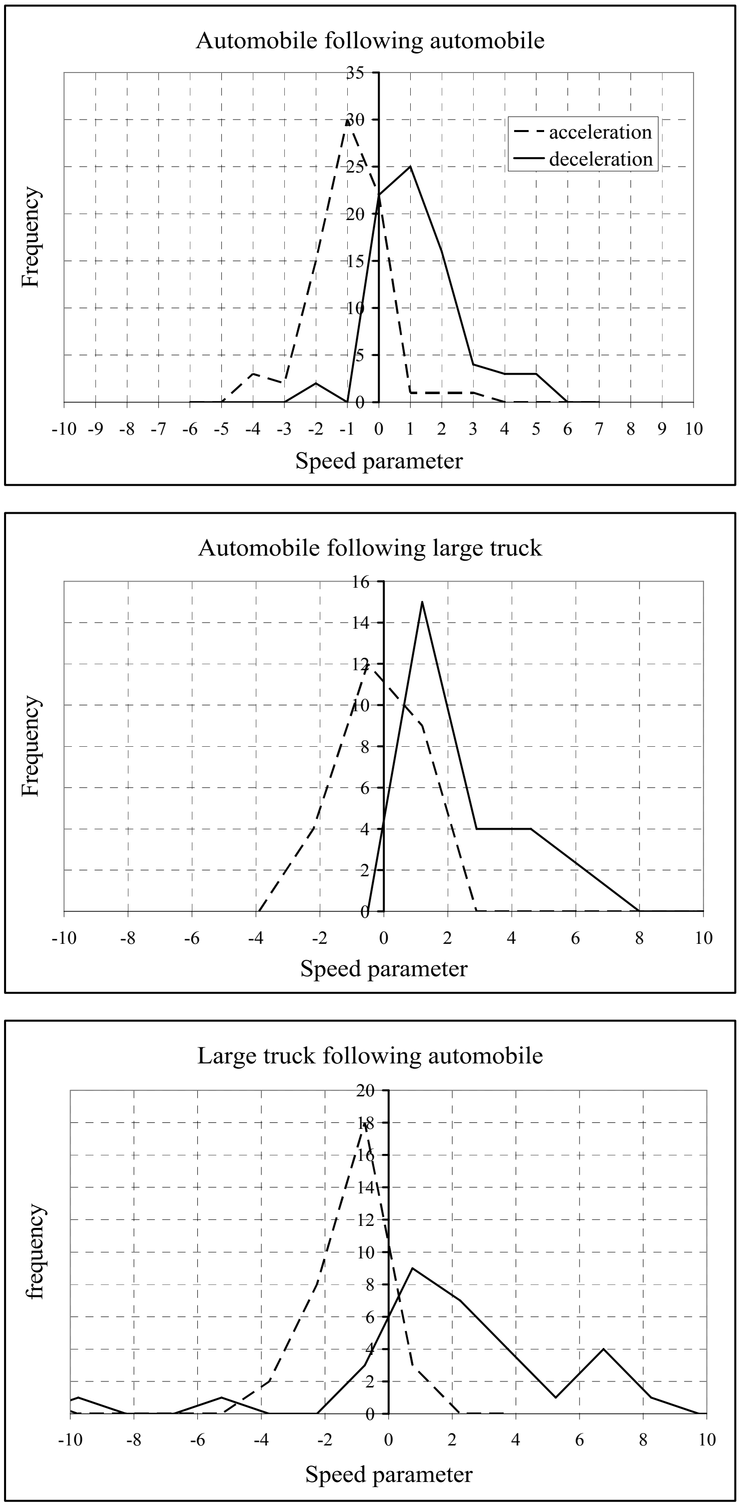


Figure 5- 4. Distributions of the speed parameter, β_1.

5.3.5 Vehicle Separation Parameter, β_2

Figure 5-5 is a plot of distributions of calibrated vehicle separation parameter, β_2 for both the acceleration and deceleration responses and for different pairs of following vehicles. For "automobile following automobile", the average value for acceleration response is 0.737 with standard deviation of 0.50 and, for deceleration response it is -1.544 with standard deviation of 1.22. As expected, the signs obtained for this parameter is intuitive, with the positive sign for the acceleration indicating that drivers have higher magnitudes of acceleration response when separation is bigger and lower when separation is smaller. On the contrary, the negative sign for the deceleration response indicates that drivers apply higher magnitudes of deceleration response when the separation is smaller and lower when separation is bigger. The mean difference is statistically different from zero at 5 percent significance level.

Comparison of the results for different pairs of following vehicles indicated no difference in the mean values of the vehicle separation parameter for acceleration response. For the deceleration response, results showed that large trucks traveling behind automobiles have significantly higher mean of parameter value than automobiles. The results are intuitive because generally drivers of large trucks are more safety cautious than automobiles due to their awareness of large trucks limitations.

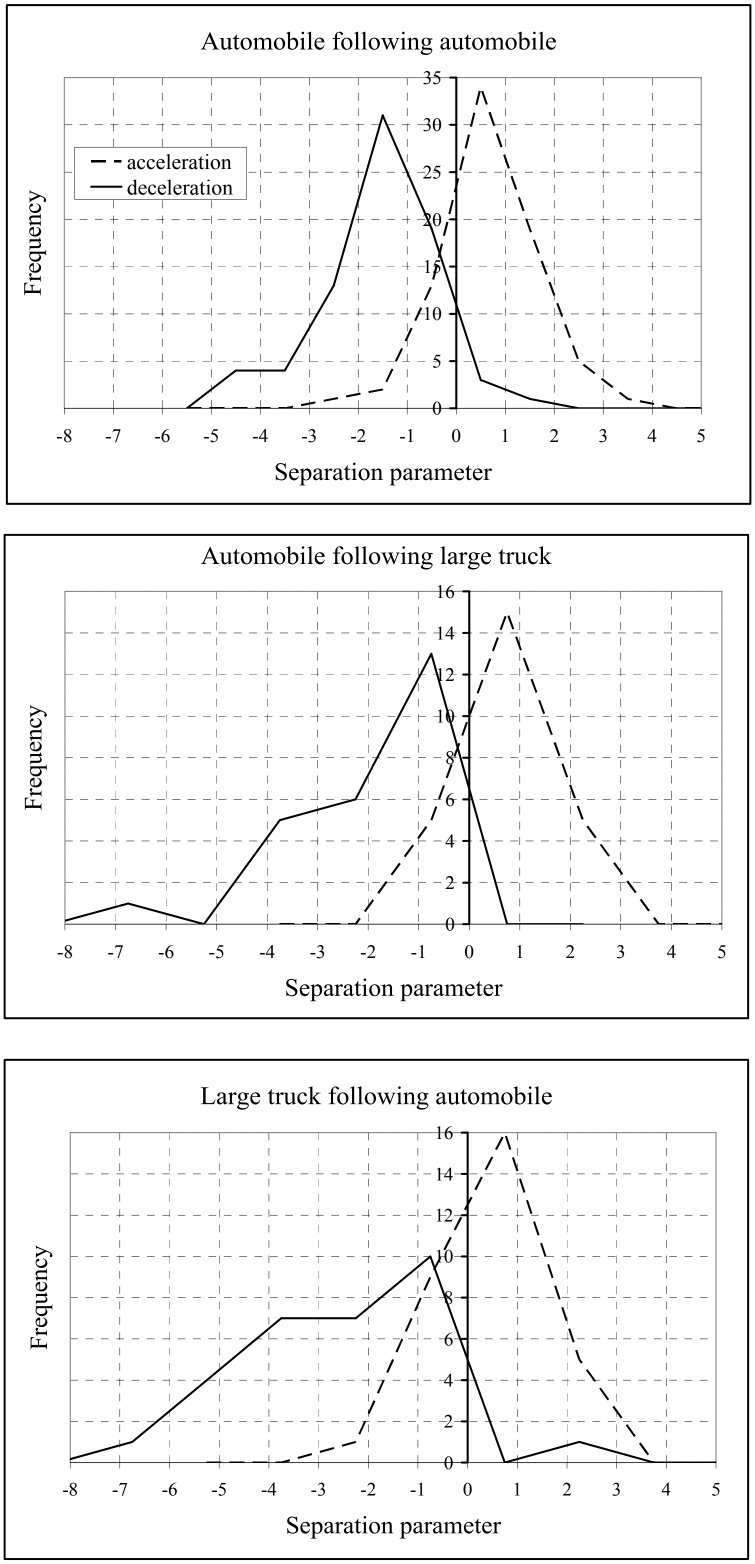


Figure 5- 5. Distributions of the vehicle separation parameter, β_2.

5.3.6 Relative Speed Parameter, β_3

Figure 5-6 shows distributions of the calibrated relative speed parameter, β_3 both for the acceleration and deceleration response models and for different pairs of following vehicles. As expected, the parameter is positive for both the acceleration and deceleration responses. This means that the bigger the magnitude of the stimulus, that is, relative speed, the bigger the magnitude of the response, regardless whether it is acceleration or deceleration response. However, the average magnitude of parameter value for the deceleration response is bigger than acceleration response. For example, the results for "automobile following automobile" indicate the mean value for the acceleration is bigger at 1.243 compared to 0.667 for acceleration response. With corresponding standard deviations of 0.62 and 0.51, the difference in magnitude is statistically significant at 5 percent level. This difference in the magnitudes of the parameter confirms that driver drivers are more likely to respond with higher magnitude when decelerating than when accelerating. Furthermore, both average values are statistically different from one as proposed in the GM models with corresponding *t-statistic* values of 5.74 and 3.41 for the acceleration response for deceleration response, respectively.

Comparison of the results for different pairs of following vehicles, for both acceleration and deceleration responses indicates insignificant difference in the mean values. The results could be due to smaller separation that drivers generally maintain when driving in congested freeway traffic conditions.

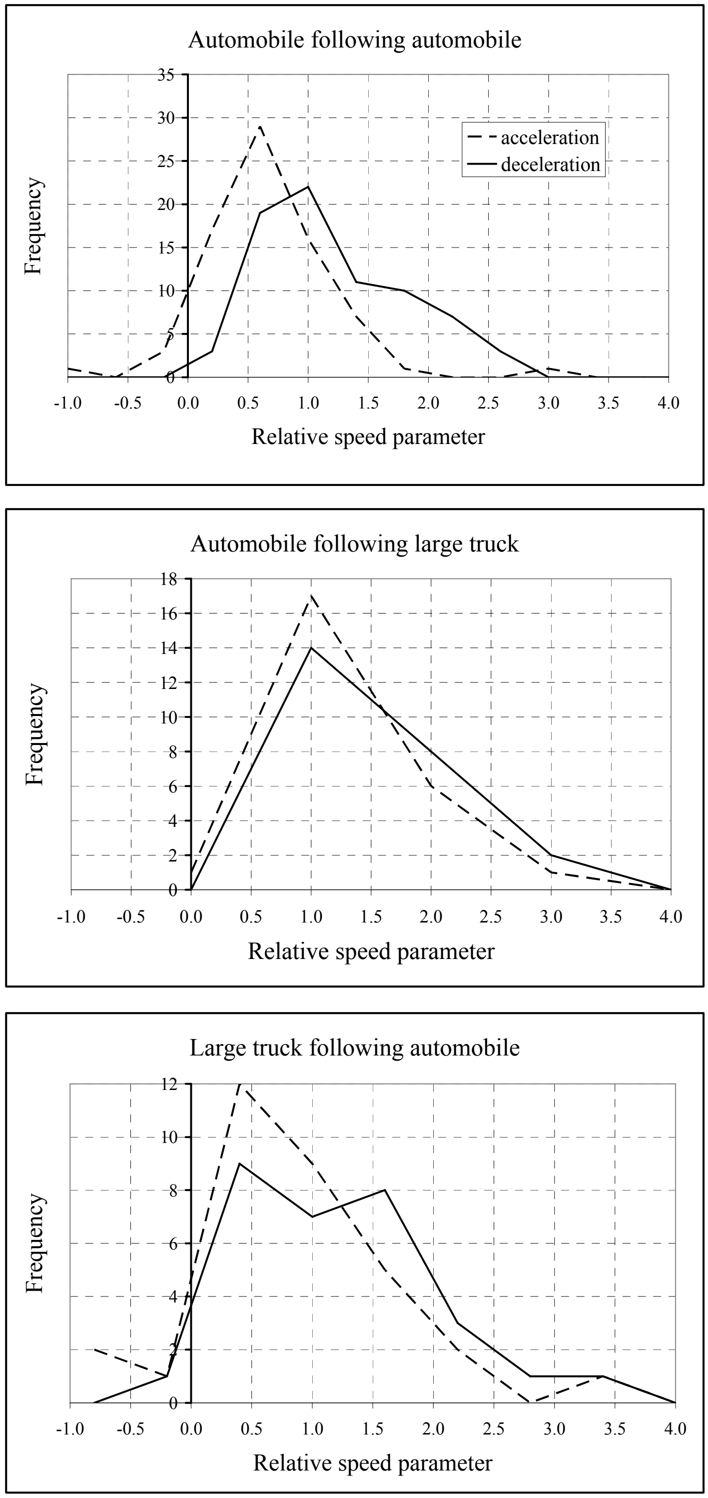


Figure 5- 6. Distributions of relative speed parameter, β_3.

5.3.7 Comparison of the Parameters with Other Studies

Table 5-3 compares the results of parameter estimates obtained from this study with other previous similar studies. For comparison purpose, the values in the table represent the average values for all vehicle types. This creates the basis for comparison because other studies estimated aggregate values for all vehicle types. The signs of the parameters are based on equation 3-2 and the parameters in italics are those that have counterintuitive signs and/or magnitudes. For example, Subramanian's study indicates that drivers' response time lag is smaller for acceleration than deceleration, which contradicts intuitive expectation. As explained previously in this research, deceleration is a response that is related to safety and therefore, drivers are more likely to respond faster, which results in smaller time lags as opposed to acceleration response. Acceleration is a less critical response as it is related to drivers' need for attaining their desired maximum speeds.

Furthermore, for acceleration response, Ozaki (1993), Ahmed (1999), and Toledo (2003) indicate that vehicle separation parameter, β_2 is negative. This implies that the magnitude of acceleration response is lower when the separation is bigger and lower when separation is smaller, which are counterintuitive results. Bigger separation is more likely to entice drivers to accelerate at higher magnitude in order to attain their desired maximum speeds. Similarly, studies by Ahmed and Toledo indicate that the speed parameter, β_1, is positive, meaning that acceleration is higher when the speed is higher, which are also unexpected results. Logically, drivers are unexpected to accelerate faster at higher speeds for two reasons. First, the need to accelerate is less at higher speeds than

at lower speeds. Secondly, the vehicle capability to accelerate is less at higher speeds than at lower speeds.

Furthermore, both studies by Ahmed and Toledo left out speed in the deceleration response because it was statistically insignificant. Having a deceleration response that is not a function of speed is counterintuitive. In addition, both studies did not determine drivers' response thresholds that delimit acceleration and deceleration responses. On the basis of these results, it is clear that this study obtained intuitive results that addressed significant shortcomings of the previous similar studies.

Table 5-3. Comparison of Parameter Estimates with other Studies

		Study				
Parameter	*Model*	*Ozaki (1993)*	*Subramanian (1996)*	*Ahmed (1999)*	*Toledo (2003)*	*Siuhi (2009)*
Driver response time lag (sec)	*Acceleration*	-	***1.97***	-	-	0.80
	Deceleration	-	***2.29***	-	-	0.70
Stimulus threshold (mph)	*Acceleration*	-	-	-	-	1.30
	Deceleration	-	-	-	-	1.00
Driver sensitivity constant, β_0	*Acceleration*	1.1	9.21	0.0225	0.0355	1.578
	Deceleration	1.1	15.24	0.0418	0.860	2.274
Speed, β_1	*Acceleration*	***-0.2***	-1.667	***0.722***	***0.291***	-1.088
	Deceleration	0.9	1.086	-	-	1.637
Vehicle separation, β_2	*Acceleration*	-0.2	0.884	***-0.242***	***-0.166***	0.723
	Deceleration	-0.9	-1.659	-0.151	-0.565	-1.822
Relative speed, β_3	*Acceleration*	1	1	0.600	0.520	0.731
	Deceleration	1	1	0.682	0.143	1.300

5.3.8 Comparison of Performance of the Models

This section compares the results for different pairs of following vehicles in emulating the field observed driver's car-following behavior. Figure 5-7 is the plot of field observed values and estimated acceleration/deceleration responses for automobiles traveling behind other automobiles. These response values are for average speed of 20.4 miles per hour and vehicle separation of 40 feet. The figure indicates that the estimated values for both the acceleration and deceleration responses emulate reasonablly the field observed values. As expected, the results indicate that both acceleration and deceleration response magnitude increases as the relative speed increases.

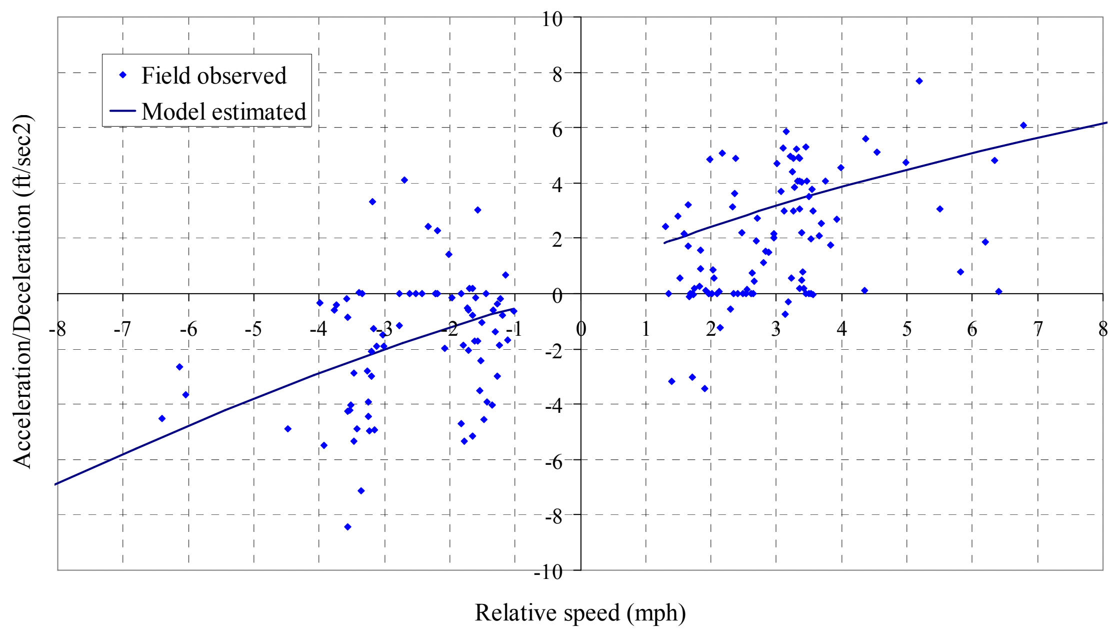


Figure 5- 7. Observed and estimated responses for "automobile following automobile".

Figure 5-8 shows the field observed and estimated acceleration and deceleration responses for automobiles traveling behind large trucks. Similarly, the results show that

the acceleration and deceleration magnitude increases as relative speed increases. This figure also indicates that on the average the magnitudes of the deceleration response are higher than acceleration response at the same value of stimulus.

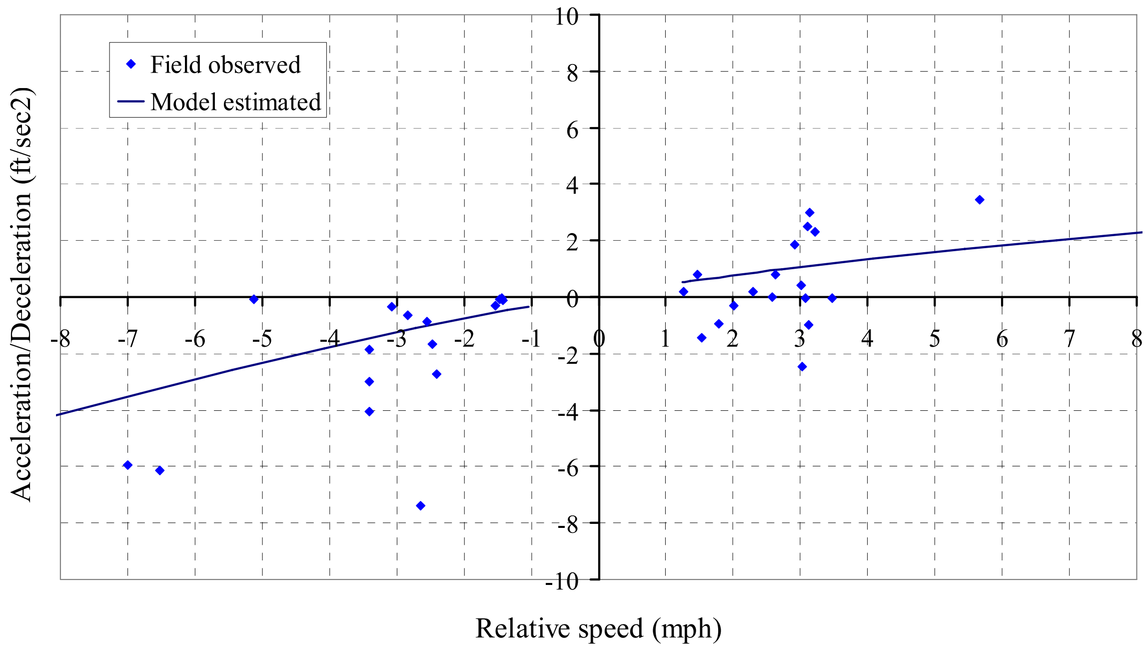


Figure 5- 8. Observed and estimated responses for "automobile following large truck".

Figure 5-9 is a display of the field observed and estimated values of the acceleration and deceleration responses for large trucks traveling behind automobiles. The results also indicate higher magnitudes of the deceleration response compared to acceleration response for the same value of stimulus.

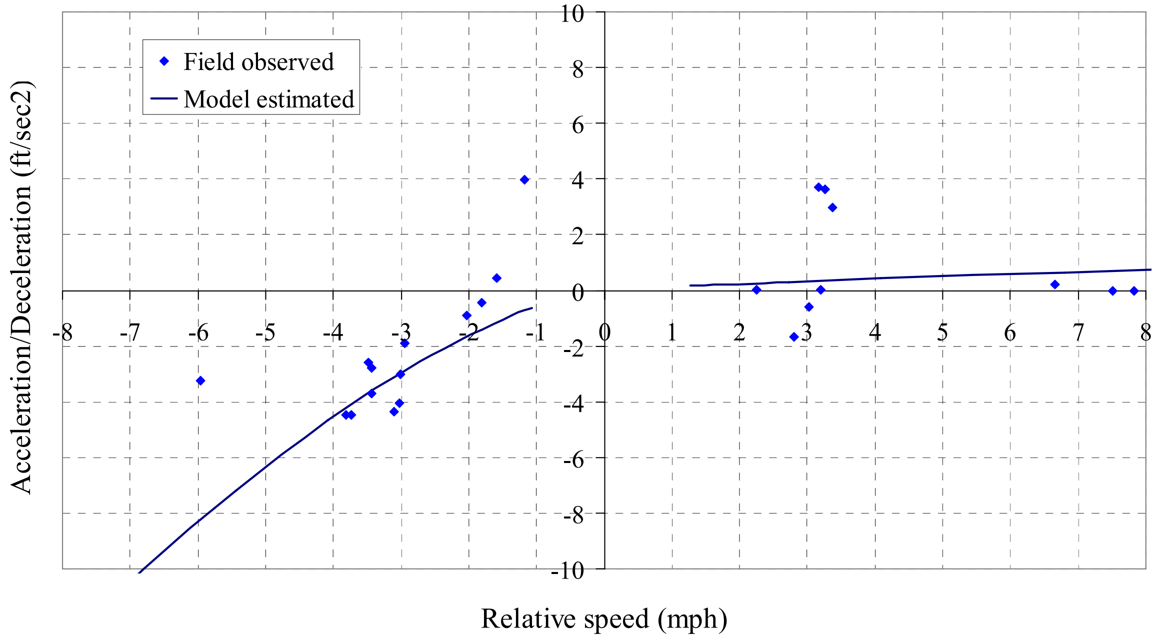


Figure 5-9. Observed and estimated responses for "large truck following automobile".

Figure 5-10 shows superimposed plots for different pairs of following vehicles. For the acceleration response, the results show that automobiles traveling behind other automobiles have higher acceleration response, followed by automobile traveling behind large trucks, and then large trucks traveling behind automobiles. These are intuitive results because automobiles generally have higher acceleration capabilities, shorter stopping and lane changing distances compared to large trucks. However, the results indicate lower acceleration response for automobiles traveling behind large trucks. This could due the fact that large trucks blocks visibility of drivers of automobiles to see in front of the large trucks.

For the deceleration response, the results indicate that large trucks have higher deceleration response compared to automobiles. The results could be associated to the facts that large trucks are heavier and need longer stopping distances compared to

automobiles. Therefore, large truck drivers are more cautious than automobiles. More importantly, the results indicate that automobiles traveling behind large trucks have lower deceleration responses than when traveling behind other automobiles. This finding could be due to two reasons: First, drivers of the automobiles may feel to be safer when traveling behind large trucks because drivers of large trucks are more cautious than drivers of automobiles. Second, large trucks have longer dimensions that block visibility of automobile drivers' traveling behind, thus, the drivers of automobiles do not respond to vehicles in front of the large trucks.

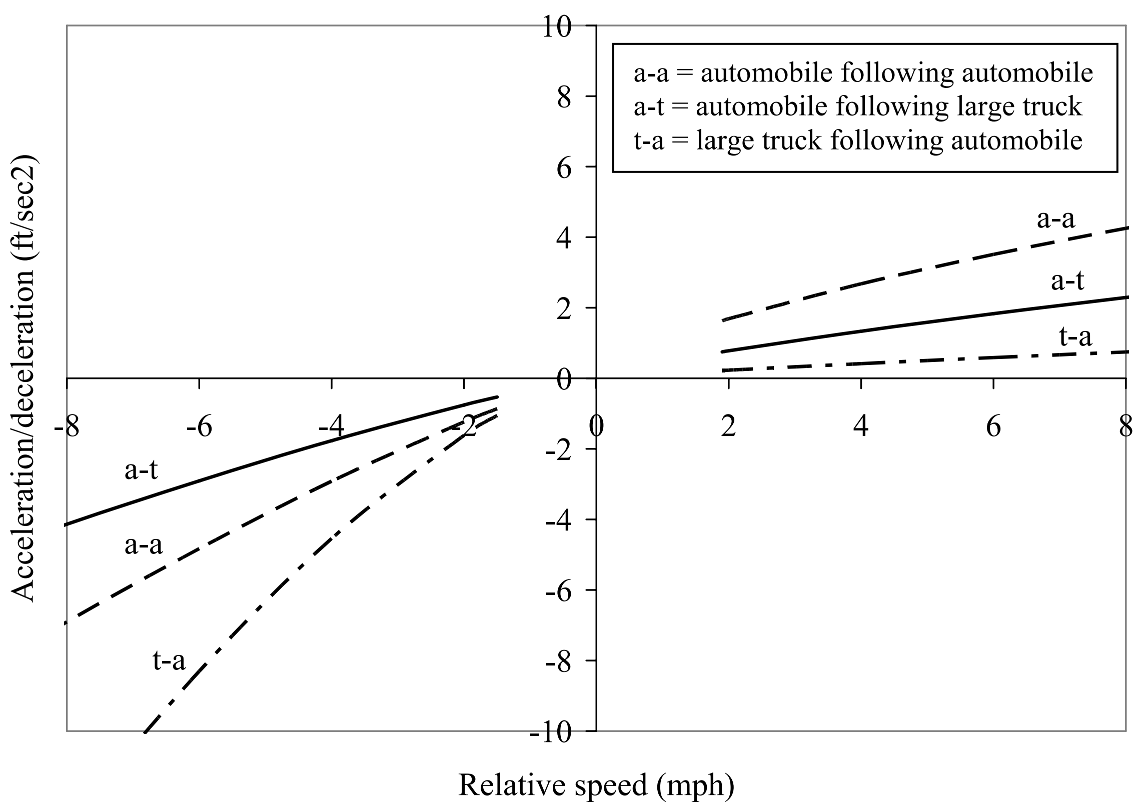


Figure 5-10. Impact of relative speed on acceleration/deceleration responses.

Figure 5-11 shows the field observed and estimate response values for average relative speed of 3.4 miles per hour and vehicle separation of 40 feet. The results also indicate that automobiles traveling behind other automobiles have higher acceleration response, followed by automobile behind large truck, and then large truck behind automobile. Likewise, the same reasons mentioned above apply in this case. For the deceleration response, as expected, the deceleration response increases as speed increases.

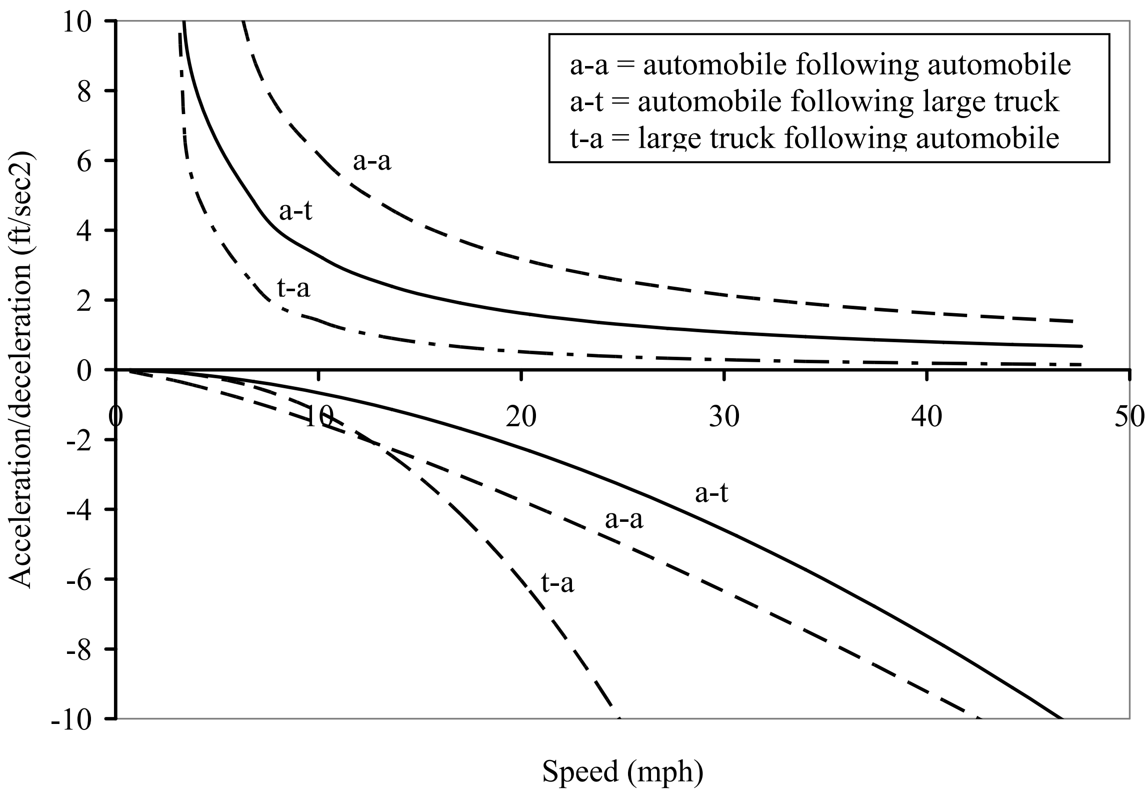


Figure 5-11. Impact of vehicle speed on acceleration/deceleration responses.

Figure 5-12 also shows the driver responses for the average relative speed of 3.4 miles per hour and speed of 20.4 miles per hour. For the acceleration response, the

results show that acceleration response increases as separation increases. Furthermore, the results indicate that automobile have higher acceleration response compared to large trucks. Similarly, this could be related to large trucks' limited acceleration capability and higher weight to horse power ratio compared to automobiles. For the deceleration response, the results indicate that large trucks have higher deceleration than automobiles for the same stimulus. This could be due to longer stopping distance that large trucks need than automobiles.

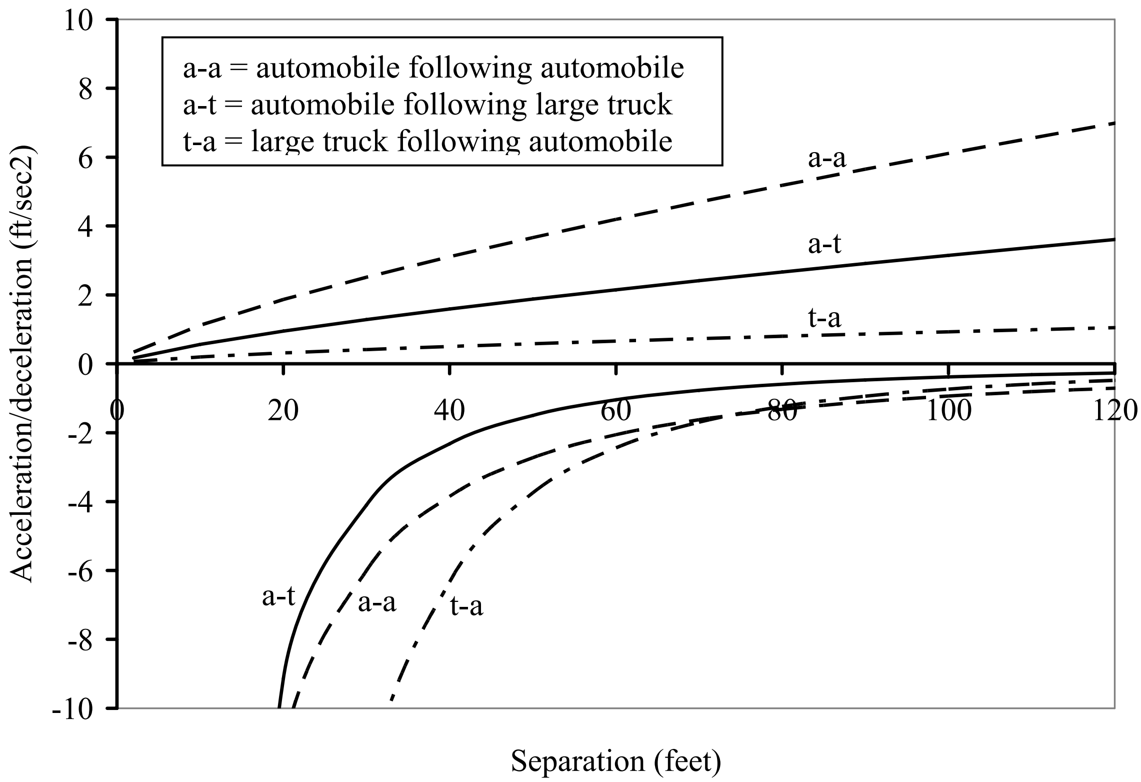


Figure 5-12. Impact of vehicle separation on acceleration/deceleration responses.

5.3.9 Statistical Measures of Model Performance

The graphs presented above showed the performance of the models in replicating the field observed drivers' car-following behavior. The discussion was based on visual observations of how well the models emulated the observed response values. This section further evaluates the performance of the models using statistical measures of performance typically used in such statistical analysis. Table 5-4 shows results of statistical measures of performance of the models. The measures include Root Mean Square Error (RMSE) and Theil inequality coefficient (U) and its main components U_m, U_s , and U_c.

Table 5-4. Statistical Measures of Performance of the Models

		Statistical Measure				
Model	*Response*	*RMSE*	*U*	U_m	U_s	U_c
Automobile following automobile	*Acceleration*	8.401	0.400	0.274	0.044	0.681
	Deceleration	8.626	0.395	0.260	0.094	0.645
Automobile following large truck	*Acceleration*	6.048	0.378	0.057	0.055	0.887
	Deceleration	4.318	0.403	0.021	0.045	0.934
Large truck following automobile	*Acceleration*	3.942	0.440	0.096	0.008	0.893
	Deceleration	7.928	0.434	0.155	0.193	0.651
Recommended threshold (Hourdakis et a., 2002)		< 15%	< 0.3	≤ 0.1	≤ 0.1	≥ 0.9

The results for the RMSE indicate that all models have prediction error of less than 10 percent. These values lie within the recommended thresholds by Hourdakis et al. (2002). This implies that there is statistical agreement between the field observed and estimated responses. The results for the Theil Inequality Coefficient (*U*) values are closer to 0.40, which is slightly higher than recommended threshold value of less than 0.3. The

results also indicate that the components of U, that is, “bias proportion”, U_m and “variance proportions”, U_s are small than the “covariance proportions”, U_c for all models. This suggests that the prediction errors are concentrated on U_c, which is desired for model prediction. Although, the some of the measures obtained violated these thresholds, the values may be acceptable. This is based on the fact that Hourdakis et al. (2002) did not state the rationale and the basis thereof of selecting the range of the thresholds. In addition, it appears from the literature that the thresholds for deciding whether the model is acceptable or unacceptable are study specific. In other words, the analyst should decide the thresholds based on the accuracy needed for the model.

5.4 Results for Steady-State Response Model

This section presents and discusses the results obtained for the steady-state model. It should be noted here that the result are for all vehicle types regardless of vehicle being driven and/or followed. This aggregation of parameters was deemed appropriate because the purpose of the model development is to translate microscopic behavior into macroscopic traffic flow characteristics. The result for steady-state model is summarized in the equation below:

$$\ddot{x}_{n,t} = -1.743 - \left[\dot{x}_{n,t}\right]^{0.50} + \left[x_{n-1,t} - x_{n,t} - L_{n-1}\right]^{0.50} \tag{5-1}$$

Where:

$\ddot{x}_{n,t}$ is the acceleration/deceleration of a subject vehicle n at time t

$\dot{x}_{n,t}$ is the speed of the subject vehicle n at time t

$x_{n-1,t}$ is the position of the leading vehicle $n-1$ at time t

$x_{n,t}$ is the position of the subject vehicle n at time t

$[x_{n-1,t} - x_{n,t} - L_{n-1}]$ is the vehicle separation at time t

L_{n-1} is the length of the leading vehicle

5.4.1 Discussion of the Parameters

The parameters shown in the equation 5-1 emulate the observed driving behavior near steady-state mode. This model is transformed into macroscopic traffic flow characteristics. Based transformed equation the speed—density relationship (equation 3-7) is given as follows:

$$u = \left(-1.743 + \left[{}^{1}\!/_{k} - 15\right]^{0.50}\right)^{2.0} \qquad 5\text{-}2$$

Where u the speed of a subject vehicle in feet per second is, k is the density of traffic stream in vehicles per feet, and 15 is the average length of vehicles in feet used for estimating the parameters. Converting the speed to miles per hour and density to vehicles per mile yields the following relationship:

$$u = \frac{1}{1.47}\left[-1.743 + \sqrt{\left({}^{5280}\!/_{k} - 15\right)}\right]^{2} \qquad 5\text{-}3$$

The resulting traffic flow in vehicles per hour is given as:

$$q = ku = u = \frac{1}{1.47} k \left(-1.743 + \left[{}^{5280}/_{k} - 15 \right]^{0.50} \right)^{2.0}$$

5-4

The traffic jam density (k_j) which occurs when vehicles are at standstill—e.g. when speed is zero is given by:

$$k_j = \frac{5280}{[1.743^{2.0} + 15]} \approx 292 \; vpm$$

5-5

The resulting macroscopic traffic flow model was compared to the field observed traffic flow parameters extracted from the same vehicle trajectory data. The parameters extracted included space mean speed and traffic density per lane. The speed and density were measured by taking snapshots of the entire length of the study site at every 30 seconds time intervals for duration of 45 minutes. The density was measured as the number of vehicles occupying the length at each time intervals, expressed in vehicle per mile. Figure 5-13 shows an example of the field observed traffic densities at the same location at 30 seconds time interval

.

Figure 5-13. Variation of traffic density at the same at different time.

Figure 5-13 shows fundamental diagrams of the field observed traffic flow parameters and those estimated from the model. The results indicate a closer agreement between the observed parameter values and the model estimates. The statistical measures of performance produce the values that are within the recommended thresholds. For the speed comparison, the results show *RMSE* value of 0.135, U value of 0.073, U_m value of 0.042, U_s value of 0.004, and U_c value of 0.956. For the traffic flow rate comparison, the *RMSE* value of 0.135, U value of 0.071, U_m value of 0.074, U_s value of 0.060, and U_c value of 0.869. These results indicate that the model emulates the observed field parameters quite well. However, the model captures the speed—density relationship at high traffic densities.

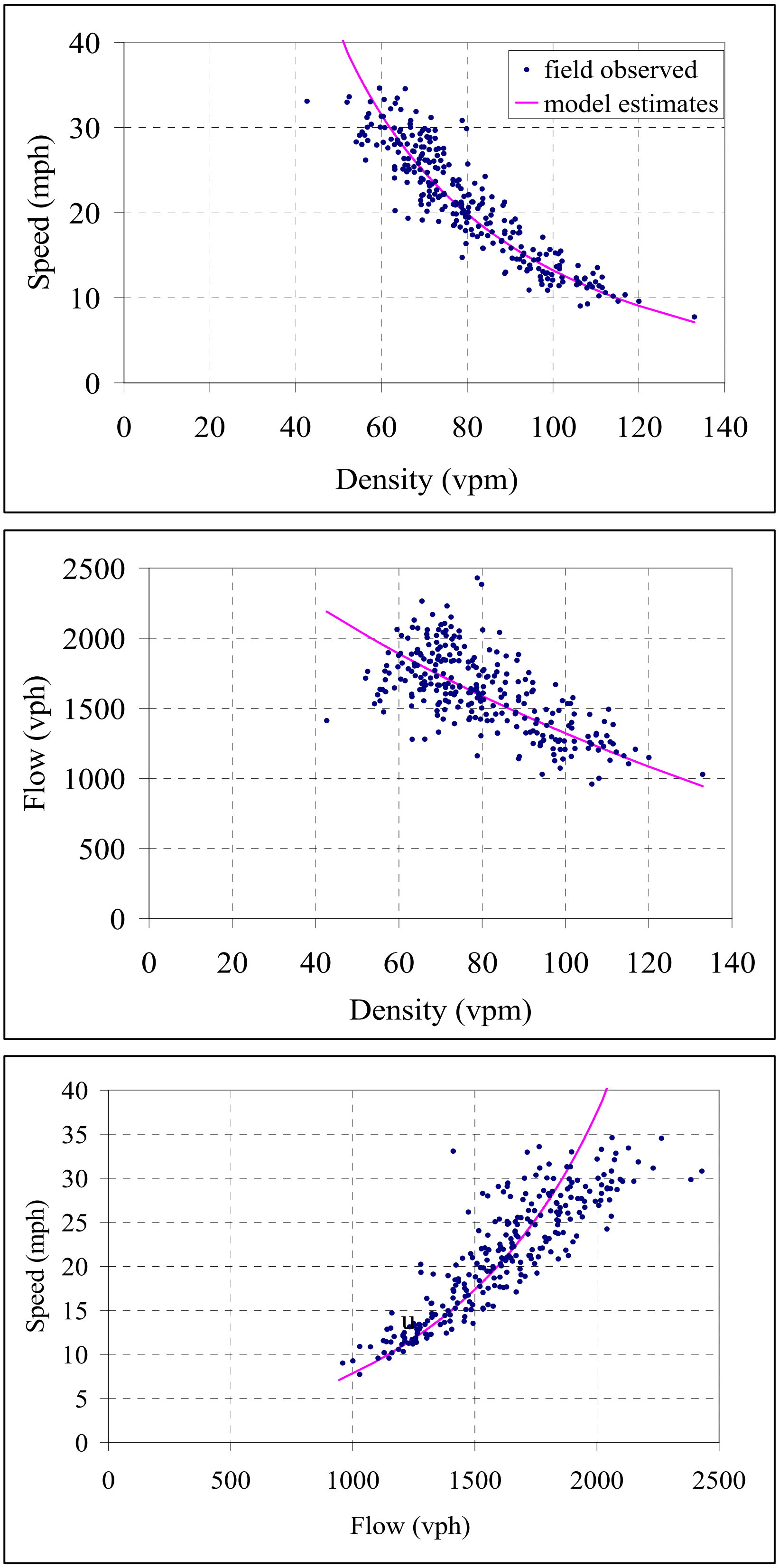


Figure 5- 14. Fundamental traffic flow diagrams.

5.4.2 Comparison of the Model with Other Macroscopic Traffic Models

Further analysis was conducted to compare the results of the model to other existing macroscopic traffic models. The models compared with this model include the Greenshields model, Greenberg model, Underwood model, and Bell Curve model. A brief description of the models is presented in the following paragraph.

Greenshields (1934) proposed a linear speed-density relationship based on field observations. The model form is as follows:

$$u = u_f \left(1 - {}^{k}/_{k_j}\right) \tag{5-6}$$

Where u is the speed, k is the traffic density, u_f is the free flow speed, k_j is the jam density. The advantage of this model form is that it is simple and straight forward but field observations indicated that the speed-density relationship is not perfectly linear.

The Greenberg model assumes a logarithmic relationship for the speed-density relationship of the following form:

$$u = u_c ln \left(\frac{k_j}{k}\right) \tag{5-7}$$

Where u is the speed, k is the traffic density, u_c is the critical speed, k_j is the jam density. The critical speed is the speed when the flow is maximal. The major disadvantage of this model is poor at estimating speeds at low densities.

The Underwood model assumes exponential relationship between speed-density of the following form:

$$u = u_f exp\left(-k/_{k_c}\right) \tag{5-8}$$

Where k_c is the critical density and it is the density at maximum traffic flow. This model produces reasonable speed at low densities but is unreliable at higher densities where speed asymptotically approaches zero.

The bell curve model developed by Duke et al. (1967) assumes a bell-shaped curve for the speed-density relationship of the following form:

$$u = u_f exp\left[-0.5\left(k/_{k_c}\right)^2\right] \tag{5-9}$$

The disadvantage of this model is similar to that of the Underwood model, that is, the speed asymptotically approaches zero as speed increases.

Figure 5-15 is a plot of fundamental traffic flow diagrams resulting from different models from the same parameters obtained in the calibration. The parameters of other models were calibrated using the same data using Stata program. The figure shows that all models capture the speed-density relationship reasonably. However, it appears from the figure that the model developed in this research produces a better agreement with field observed speed-flow and flow-density relationships than other models. The model developed can be used for analysis of macroscopic traffic flow characteristics including freeway level of service, ramp metering control, etc.

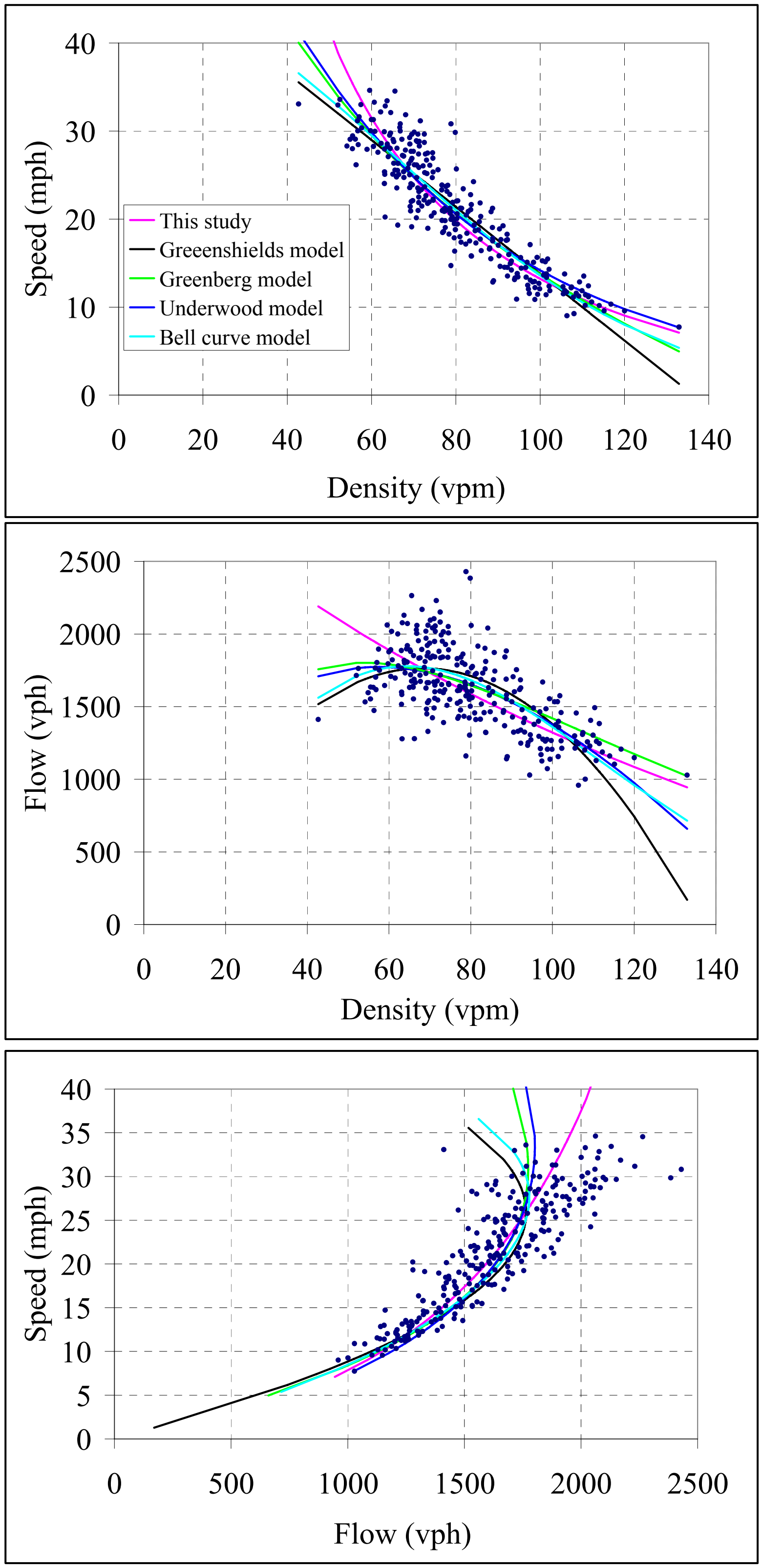


Figure 5-15. Comparison of fundamental traffic flow diagrams for different models.

CHAPTER 6

VALIDATION OF THE MODELS

6.1 Introduction

This chapter presents and discusses the validation process and the results obtained for the family of car-following model developed. The aim of validating the models is to determine whether the parameters calibrated can be transferred to other limited access highways with relatively comparable characteristics. This study used the field data collected from a different site. This study used vehicle trajectory data collected on a segment of Interstate 80 in Emeryville, San Francisco, California. The subsections that follows describe in detailed the data used, characteristics of the study site, comparison of the site with calibration site, that is, Interstate 101 in Los Angeles, and results of statistical measures of model performance.

6.2 Data Description

The family of car-following models developed was validated using vehicle trajectory data collected on a segment of Interstate 80 in Emeryville, San Francisco, California. The dataset was also collected as part of the FHWA's Next Generation Simulation (NGSIM) project. The dataset contains 45 minutes of vehicle trajectory that was collected in the afternoon peak hour on Wednesday April 13, 2005 from 4:00 pm to 4:15 pm and from 5:00 pm to 5:30 pm. The time period from 4:00 pm to 4:15 pm represented a transitional traffic from uncongested to congested traffic conditions whereas the period from 5:00 pm to 5:30 pm represented congested freeway traffic conditions. The models were calibrated using the 45 minutes data. A full detailed

description of methodology and technology used to collect and process the data are available at the NGSIM Website (http://ngsim.fhwa.dot.gov.)

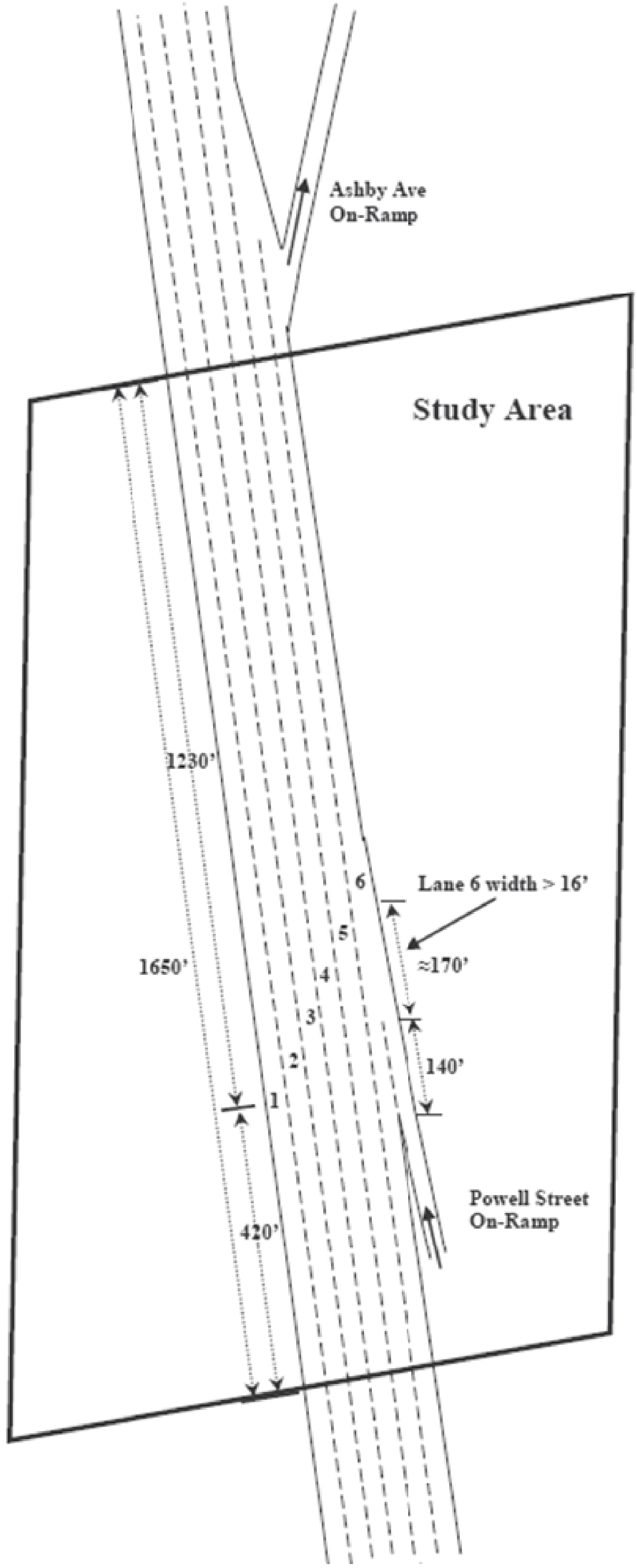
6.3 Study Site Characteristics

The vehicle trajectory data was collected on a 1,650 feet long section in the northbound direction on the freeway. The section has five through lanes (lanes 1 to 5) and one auxiliary lane (lane 6). The leftmost inner lane—i.e. number lane 1 is the High-Occupancy Vehicle (HOV) lane. The auxiliary lane is approximately 1,230 feet long. This data was collected using seven synchronized digital video cameras installed on an adjacent 30-storey building (Pacific Park Plaza). Figure 6.1 shows a schematic diagram of the study site and camera coverage area.

Similar procedures and criteria used for selection of vehicles used for calibration of the model parameters were also used for preparation of the validation data. From the dataset selected and based on the selection criteria, similar variables were extracted for each time step for each subject vehicle including:

1. Time of observation,
2. Acceleration/deceleration response,
3. Speed of a subject vehicle,
4. Relative speed between the two vehicles, and
5. Separation

Based on the vehicle selection criteria, the resulting sample consisted of 675 “automobile following automobile”, 37 “automobile following large truck”, and 23 “large truck following automobile”.



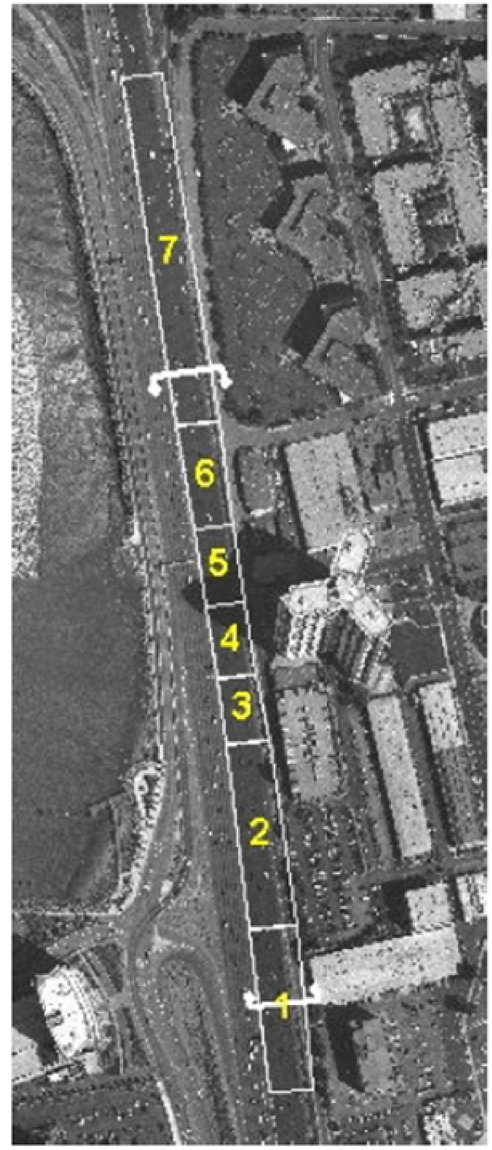


Figure 6-1. Study site and camera coverage (Cambridge Systematics, Inc., 2005).

6.4 Traffic Characteristics

Table 6-1 shows the summary statistics of traffic mix at the validation site during the study period. The statistics show that 1.0 percent of all vehicles were motorcycles, 95.2 percent automobiles, and 3.8 percent trucks and buses.

Table 6-1. Vehicle Types

Time Period	Motorcycles	Automobiles	Trucks & Buses	Total
4:00 p.m. - 4:15 p.m.	14	1942	96	2052
5:00 p.m. - 5:15 p.m.	24	1742	70	1836
5:15 p.m. - 5:30 p.m.	17	1724	49	1790
Total	55	5408	215	5678
Percentage	1.0%	95.2%	3.8%	100%

Table 6-2 summarizes the average traffic flow rates and space mean speeds at 15-minute time intervals. The table shows low vehicle speeds indicating congested freeway traffic conditions.

Table 6-2. Traffic Flow Rate and Speed

Time Period	Flow (vph)	Space mean speed mph
4:00 p.m. - 4:15 p.m.	8,144	17.86
5:00 p.m. - 5:15 p.m.	7,288	14.04
5:15 p.m. - 5:30 p.m.	7,048	11.93

Table 6-3 is a summary of average traffic flow rates and spacing mean speeds by lane. The table indicates that through lanes had less flow rates and speeds compared to

auxiliary lane. This may suggests that congestion on the middle lanes force a significant number of vehicles to shift to the auxiliary lane.

Table 6-3. Traffic Flow Rate and Speed per Lane

	Vehicle flow rate (vph)			Space mean speed (mph)		
Lane	*4:00 p.m. to 4:15 p.m.*	*5:00 p.m. to 5:15 p.m.*	*5:15 p.m. to 5:30 p.m.*	*4:00 p.m. to 4:15 p.m.*	*5:00 p.m. to 5:15 p.m.*	*5:15 p.m. to 5:30 p.m.*
Lane 1	1,420	1,592	1,544	30.03	23.51	22.62
Lane 2	1,042	1,196	1,042	20.28	11.28	10.32
Lane 3	900	952	900	20.55	10.06	8.45
Lane 4	1,036	1,032	1,036	14.50	10.53	9.12
Lane 5	1,094	1,080	1,094	15.18	11.24	7.23
Lane 6	1,432	1,436	1,432	14.41	10.81	9.39
Total	8,144	7,288	7,048	19.17	13.58	11.93

6.4 Comparison of the Interstate 80 Site with Interstate 101 Site

Table 6-4 compares the characteristics of the study sites namely, Interstate 101 in Los Angeles used for calibrating the parameters of the models and Interstate 80 in Emeryville used for validating the models. As can be seen from the table, the two study sites have two major differences in geometric characteristics: First, Interstate 101 study site has longer segment length compared to that of Interstate 80 site. Second, Interstate 80 study site has longer length of weaving segment than that of Interstate 101 site. The observation of traffic characteristics indicate Interstate 80 site had lower average speed and flow rate than Interstate 101 site. Furthermore, Interstate 80 site had 4 percent of large truck compared to 2 percent for Interstate 101 site. On the overall, the two study sites have comparable geometric and traffic characteristics.

Table 6-4. Comparison of the Study Sites Characteristics

Variable	Interstate 101	Interstate 80
Length of segment (feet)	2,100	1,650
Length of weaving segment (feet)	698	1,230
Number of through lanes	5	5
Number of auxiliary lanes	1	1
Time-of-day	7:50-8:35 am	4:00-4:15 pm, 5:00-5:30 pm
Duration of study (minutes)	45	45
Freeway segment type	weaving	weaving
Average flow rate (vph)	8,016	7,493
Average speed (mph)	21.59	14.77
Peak 15-minutes flow rate (vph)	7,428	7,048
Peak 15-minutes speed (mph)	17.94	12.40
Large trucks	2%	4%

6.5 Statistical Measures of the Model Validity

The statistical measures used for evaluating the performance of the models include Root Mean Square Error (RMSE) and Theil Inequality Coefficient (U). Table 6-5 shows the validation results for different statistical measures considered in this study. The table also contains the range of the recommended thresholds by Hourdakis et al. (2002) for calibrating and validating microscopic traffic simulation models.

Using the *RMSE* measure, the results indicate all models have values less than 10 percent, which is less than the recommended threshold of 15 percent. Furthermore, the Theil inequality coefficient, *U* produces the values that higher than the recommended values of less than 0.3 for all models. However, both models indicated major proportion of *U* is concentrated on the covariance proportion, which is a desirable characteristic. As previously discussed in this study, no justification was stated for selecting the thresholds. Although some models violated these thresholds still the results may be acceptable for the intended purpose of the developed models. On the overall, the validation results

indicated that the models can be transferred to different sites with relatively comparable geometric and traffic characteristics and emulate reasonably drivers' car-following behavior.

Table 6-5. Statistical Measures of Performance of the Models

Model	*Response*	Statistical Measure of Performance Measure				
		RMSE	*U*	U_m	U_s	U_c
Automobile following automobile	*Acceleration*	7.778	0.430	0.230	0.069	0.700
	Deceleration	9.401	0.460	0.217	0.121	0.661
Automobile following large truck	*Acceleration*	6.048	0.378	0.057	0.055	0.887
	Deceleration	4.318	0.403	0.021	0.045	0.934
Large truck following automobile	*Acceleration*	3.942	0.440	0.096	0.008	0.895
	Deceleration	7.928	0.434	0.155	0.193	0.651
Recommended threshold (Hourdakis et al. (2002))		< 15%	< 0.3	≤ 0.1	≤ 0.1	≥ 0.9

The steady-state response model was also validated using the same data. Figure 6-2 shows the fundamental traffic diagrams of the field observed and model estimates. The figure shows that the model emulates the observed macroscopic traffic flow characteristics reasonably. However, the results indicate that the model seems to underestimate speed-flow relationship.

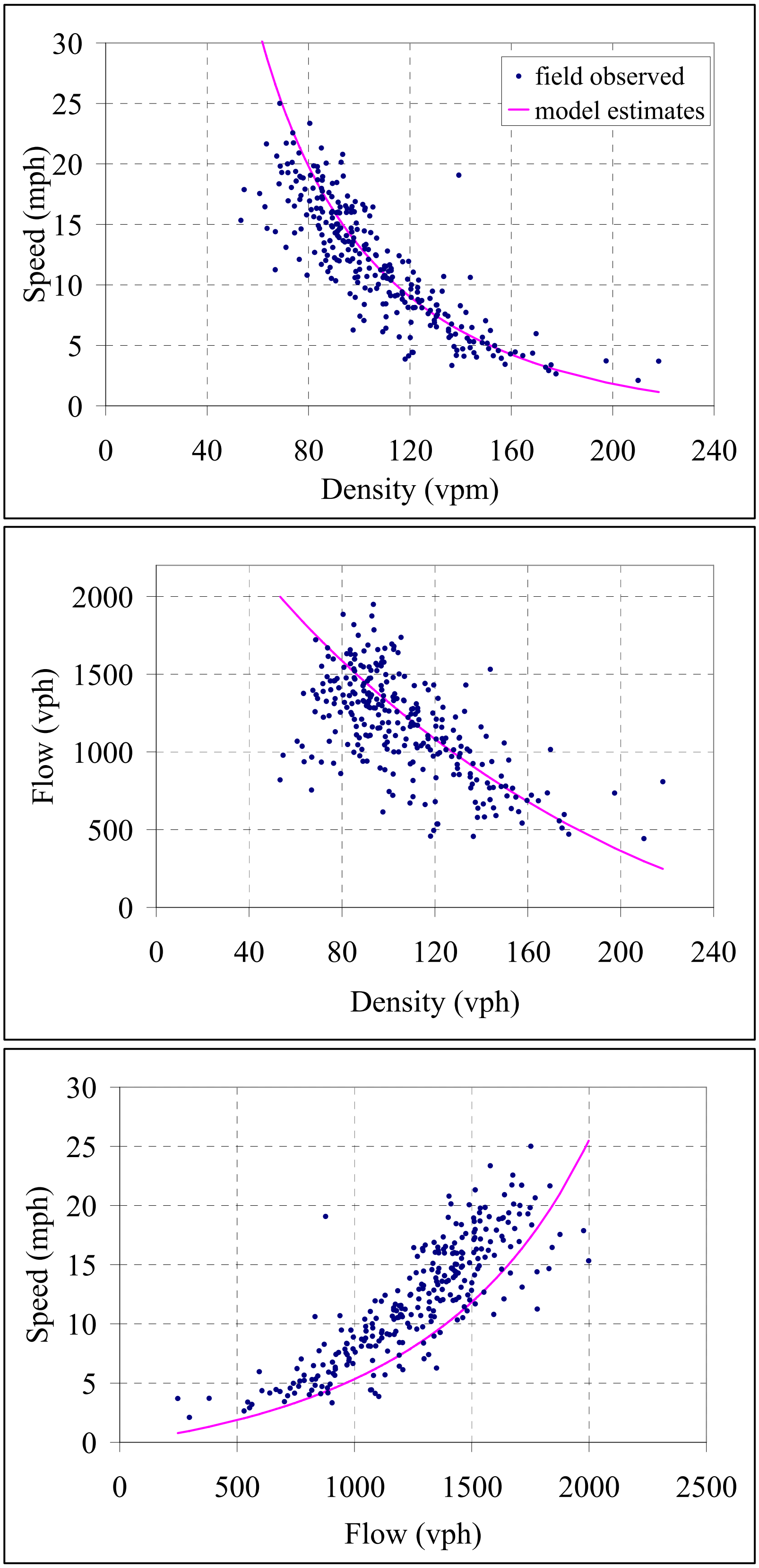


Figure 6- 2. Fundamental traffic flow diagrams.

6.6 Comparison of Model Transferability with other Models

Figure 6.2 shows the fundamental traffic diagrams of the field observed and estimated values by different macroscopic models. The parameters of each model used are the same as ones obtained in the calibration of the models using data from Interstate 101 site. From the figure, the results indicate that the model developed in this study and the Underwood model capture well the field observed values at higher densities compared to other models. It is tempting to conclude that the model developed in this study and Underwood model can be transferred to a different site and emulate well macroscopic traffic flow characteristics.

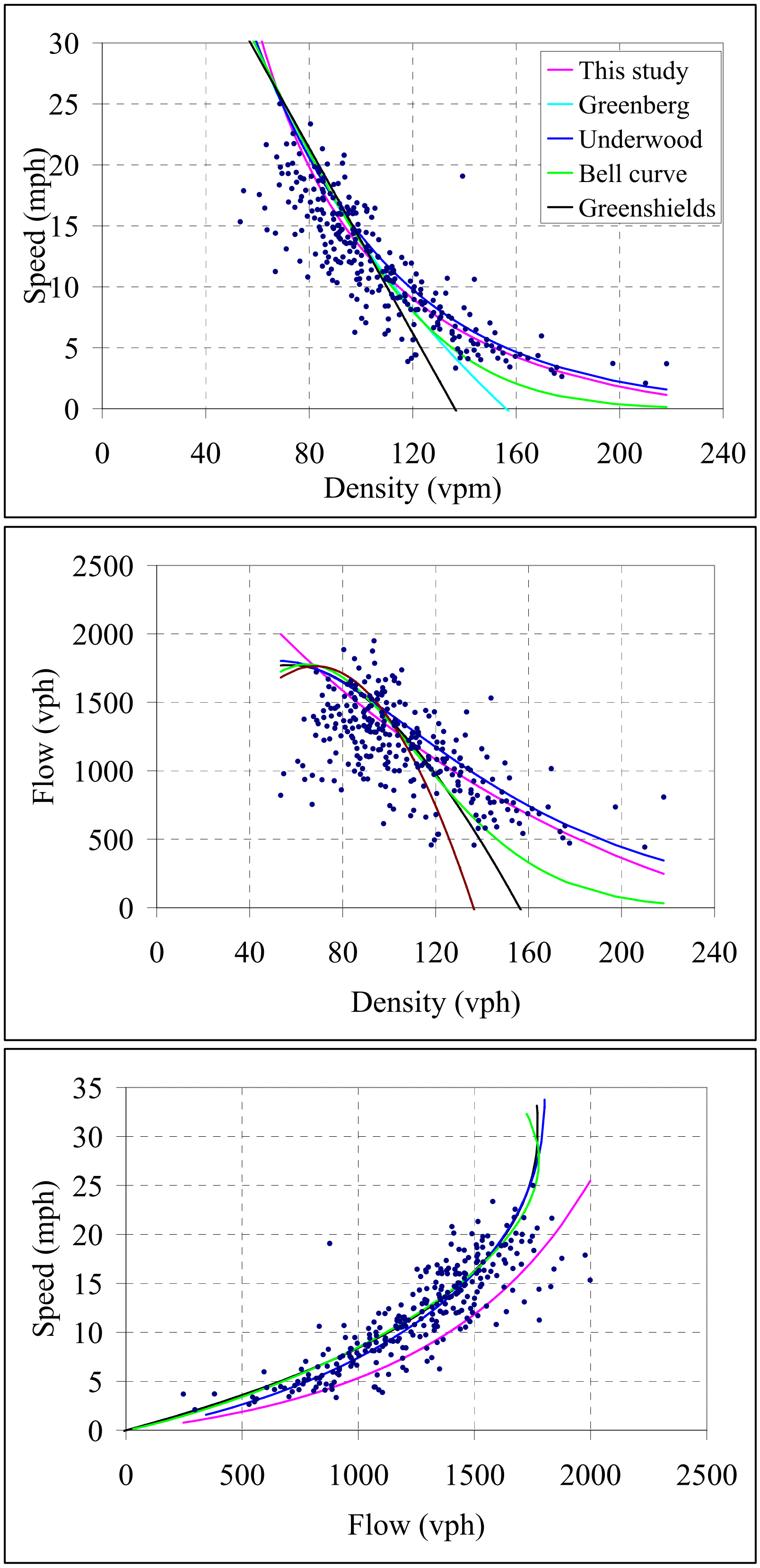


Figure 6-3. Comparison of fundamental traffic flow diagrams for different models.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 Introduction

This study developed a family of car-following models that address shortcomings of the existing stimulus-response car-following models. The developed models consist of separate models for acceleration, deceleration, and steady-state responses. The objectives of the study were to address the following four shortcomings of the existing stimulus-response models:

1. To determine driver response time lags for both acceleration and deceleration responses.
2. To determine stimulus response thresholds for both the acceleration and deceleration responses.
3. To incorporate vehicle heterogeneity in the models. For each acceleration or deceleration response, three car-following models were developed depending on the types of vehicles following each other. The models include "automobile following automobile", "automobile following large truck", and "large truck following automobile".
4. To capture heterogeneity in driving behavior across drivers by estimating distributions of the driver response time lags, stimulus response thresholds, and other model parameters for speed, relative speed, and vehicle separation for both acceleration and deceleration responses.

This study calibrated the parameters of the models using 45 minutes of individual vehicle trajectory data collected on a segment of Interstate 101 in Los Angeles,

California. The study used nonlinear regression with robust standard errors implemented in Stata statistical program to estimate parameters of the models and obtain the distribution of each of parameters across drivers and for different pairs of following vehicle types. The Stata program implements a modified Gauss-Newton method for estimating the parameters of the model that minimizes sum of squared residuals. The parameters used in this study include the driver response time lags, driver sensitivity constant, parameters for speed, relative speed, and vehicle separation.

The stimulus response thresholds that delimit acceleration and deceleration responses were determined based on Signal Detection Theory. The driver response time lags and other model parameters for relative speed, speed and separation were calibrated based on the combined effect of speed, relative speed, and vehicle separation.

The study also validated the models using 45 minutes of vehicle trajectory data collected on a segment of the Interstate 80 in Emeryville, California. The statistical measures used for assessing the validity of the models included Root Mean Square Error (RMSE) and Theil Inequality Coefficient (U).

7.2 Conclusions

On the overall, the results demonstrate the need to use separate models for the acceleration and deceleration responses, since the stimulus, relative speed, speed, and vehicle separation impact these responses differently. Furthermore, the results confirm the need to use separate models depending on type of vehicle being driven and/or followed such as "automobile following automobile", "automobile following large truck", and "large truck following automobile". The results show that drivers'

acceleration and deceleration responses are significantly different for different pairs of following vehicles. These conclusions are made based on the following observations:

1. As expected, driver response time lag is lower for deceleration response than for acceleration response. The average values are 0.70 seconds and 0.80 seconds for deceleration and acceleration responses, respectively. The difference is statistically significant at 5 percent significant criterion. These results are intuitive, since drivers' deceleration response is generally related to safety and, therefore, one would expect drivers to be more sensitive in responding to negative stimulus as opposed to positive stimulus. Additionally, drivers' response to negative stimuli is sometimes further aided by the activation of brake lights for a leading vehicle that is braking. The acceleration response is less critical as it is related to drivers' need to attain their desired maximum speed.
2. For similar safety reasons, the stimulus response threshold value is lower for deceleration response than for acceleration response. The thresholds are about 1.0 miles per hour and 1.3 miles per hour for deceleration and acceleration responses, respectively.
3. The models confirm the intuitive expectation that for the same magnitudes of speed and separation, drivers are more aggressive under deceleration response than acceleration response. This is indicated by the higher magnitudes of the model parameters for the deceleration response than for acceleration response. This observation is consistent regardless of type of vehicle being driven and/or followed.
4. The results show that there are significant differences in driving behavior between different types of pairs of following vehicles. They show that under similar positive

stimulus conditions, drivers of automobiles respond with higher acceleration rates compared to drivers of large trucks. On the other hand, under similar negative stimulus response conditions, drivers of large trucks respond with higher deceleration rates than drivers of automobiles. It appears that drivers of large truck drivers are more safety conscious and respond more aggressively under deceleration response.

5. The results also show that drivers of automobiles traveling behind large trucks have both lower acceleration and deceleration response magnitudes than when traveling behind other automobiles. In other words, automobiles drivers respond more aggressively when behind automobiles than when traveling behind large trucks. This could be related to the fact that large trucks block the visibility of drivers of automobiles traveling behind them due to their large dimensions compared to automobiles. This limits the ability of automobile drivers to see beyond large trucks when traveling them.
6. Comparisons of the results for different pairs of following vehicles indicate insignificant differences in the means of the driver response time lags and stimulus response thresholds for both acceleration and deceleration responses.
7. The results for steady-state response model show the same magnitudes of parameters for speed and vehicle separation. These results are intuitive because at steady-state drivers are traveling near constant speeds, therefore, less aggressive response to stimuli such as speed and vehicle separation.
8. By validating the models using similar data from a different site, the results show that the models were able to emulate the field observed driver behavior and macroscopic

traffic flow characteristics reasonably. Based on these results, the models demonstrate the potential for transferability between different sites or locations.

7.3 Limitations of the Study and Recommendations for Future Research

The family of the models developed in this study addresses some of the shortcomings of the existing stimulus-response car-following models for the observed driver car-following behavior in congested freeway traffic conditions. The following is a summary of the limitations of this study and recommendations for future related research.

1. Both driver response time lags and stimulus response thresholds are likely to be a function of speed of the vehicle and vehicle separation. This research simplified the models by estimating the driver response time lags and stimulus response thresholds independent of these factors.
2. The data used in this study were collected on a segment with adjacent weaving section. Drivers' behavior in vicinity of weaving section may be different from their behavior in basic freeway segments that are reasonably far from diverging and merging areas.
3. The family of car-following models developed is primarily calibrated for freeway congested traffic conditions. The models may not be appropriate for use under uncongested freeway traffic conditions. Therefore, there is a need to calibrate the models for such uncongested conditions.
4. Due to data limitations, this study did not calibrate model for "large truck following large truck". Drivers' behavior for such situations may be significantly different from other pairs of following vehicles calibrated in this study.

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