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Improved parameterization to invert Rayleigh-wave data for shallow profiles containing stiff inclusions

Carlos Calderón-Macías¹ and Barbara Luke²

ABSTRACT

Inversion of shear-wave velocity profiles from phase-velocity measurements of Rayleigh-wave energy for sites containing stiff layers can be erroneous if such layers are not characterized in the starting or reference model. Incorporation of a priori knowledge then is key for converging upon a realistic or meaningful solution. Resolving soil profiles in desert regions where stiff layers cemented with calcium carbonate are intermixed with softer, uncemented media is an application for which locating shallow stiff inclusions has important implications. Identification of the stiff layers is critical for foundation design and cost estimating of excavations. A parameterization that seems adequate for this problem is to solve for anticipated high-stiffness layers embedded in a coarser (background) profile that captures the general shear-wave velocity trend of the study area. The optimization is accomplished by using simulated annealing. Uncertainty measures resulting from the inversion are helpful for describing the influence of the parameterization on final model estimates.

INTRODUCTION

Geophysical inversion is a well-established concept that has been used to find shear-wave velocities of homogeneous and isotropic horizontal layered models. The dispersive nature of surface waves has been used to estimate shear-wave velocities of the crust from earthquake-seismology data (Wiggins, 1988), to constrain shallower structure through ground-roll filtering in reflection seismology (Al-Eqabi and Herrmann, 1993), and to determine elastic properties of near-surface media from experimental group- and/or phase-velocity measurements (Stokoe et al., 1994; Xia et al., 1999). These

studies and others involving surface-wave inversion have supported the conclusion that Rayleigh waves are most sensitive to shear-wave velocity variations compared to variations in compressional-wave velocity and density.

Two active-source methods that have proved valuable for determining shallow shear-wave velocity profiles are the spectral analysis of surface wave (SASW) method (Stokoe et al., 1994) and the multi-channel analysis of surface waves (MASW) method (Park et al., 1999). In both, dispersion curves are interpreted from the measured data, and a shear-wave profile is updated through inversion by using a forward-modeling method to match the simulated dispersion curves with the measured curves. The SASW method uses receiver pairs and, through spectral phase correlation, generates an effective dispersion curve that comprises a superposition of surface-wave modes and other wave types (Gucunski and Woods, 1991), although historically, inversion of SASW data typically assumed that most of the energy making up the dispersion curve is associated with the fundamental-mode Rayleigh wave. The second method builds dispersion curves by transforming the receiver array to a domain that allows independent interpretation of the fundamental and higher-mode Rayleigh waves plus body waves. Xia et al. (2000) and Beaty et al. (2002) experimentally demonstrated with multichannel data that using the fundamental and higher modes provides better model resolution with increasing depth than inverting for the fundamental mode only.

Defining adequate model representations and setting up model constraints are key requirements for inverting Rayleigh-wave data. Xia et al. (1999) found that fixing the values of acoustic velocity and density of all layers at values that are within 25% of the actual values had little effect on the estimation of shear-wave velocity. Because the shear-wave velocity changes with depth and hence depends on layer thickness, the thicknesses are typically fixed in the inversion by using thin layers regularly spaced in depth. A different model representation consists of solving for a few coefficients of a smoothly varying function of depth instead of the velocities for each layer

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(Beaty et al., 2002). To obtain a credible model, the inversion should incorporate a priori knowledge in the form of (1) some physical insight, such as the customary trend for velocities to increase with depth or for velocities in hard rock to be higher than those in soil or (2) hard data such as constraining elastic properties with independent in situ measurements. Such anecdotal data can be used to define a reference or starting model that will be updated or refined through inversion. Two such practices are (1) defining the shear-wave velocity of the uppermost layer on the basis of the fact that phase-velocity dispersion curves typically have an asymptote at their short-wavelength limit that can be scaled by some constant to define the shallowest shear-wave velocity (Stokoe et al., 1994) and (2) recognizing that Rayleigh-wave phase velocity can be roughly mapped to a depth equal to one-third of the corresponding wavelength (Gazetas, 1992).

Estimation of near-surface shear-wave velocity by linearized inversion (LI) of Rayleigh-wave data has been widely studied by researchers such as Wiggins (1988), Nazarian (1984), Ganji et al. (1998), and Xia et al. (1999). However, straightforward LI is not adequate to converge on a correct solution for cases with complex layering. For example, in a two-step inversion process, Ganji et al. (1998) applied LI followed by a quasi-linear Newton's method to analyze data from relatively complicated velocity profiles that include sharp stiffness contrasts between adjacent layers. The rationale of the authors for applying a quasi-linear method is that this error-minimization method circumvents weak nonlinearity by using the curvature as well as the slope of the error function. But it is our experience that complicated error surfaces can require multiple computations with different reference models to find a solution that can be interpreted as realistic (Luke et al., 2003). Martínez et al. (2000) applied the nonlinear optimization method known as simulated annealing (SA) to estimate shear-wave velocities at the crustal scale and thus reduce the dependence of the solution on a sometimes arbitrary starting model. Beaty et al. (2002) applied an SA method for inverting multimodal Rayleigh-wave data to define the shallow structure for a study site; Beaty and Schmitt (2003) also performed SA inversion to test repeatability of phase-velocity measurements of a shallow lacustrine deposit and obtained consistent estimates of shear-wave velocities for the different periods.

A complex soil profile that poses some challenges for inversion contains very high elastic impedance contrasts such as in desert regions where stiff layers cemented with calcium carbonate are intermixed with softer layers. To study this problem, we first used a least-squares approach and solved for a 1D profile with layers regularly spaced in depth and for a coarser (background) profile with layer thickness exponentially increasing with depth. The inverted data correspond to phase velocities from the fundamental mode of Rayleigh waves. We then implemented an SA approach (1) to address the nonlinearity of the problem and (2) as a vehicle for accommodating a priori information. To accommodate a priori information, we prefer SA over the least-squares solution because SA permits a more flexible representation of the model. We have found this capability to be particularly useful when inverting complicated phase-velocity data such as those from a profile containing layers with high elastic impedance contrasts. We tested the approach on SASW data from a site in Las Vegas, Nevada. With this method, phase velocities are extracted directly from frequency-domain averaging of phase information of the recorded traces; stacked time histories are not computed. Although our tests considered only the fundamental mode, the method can be readily modified to address higher-mode phase- and/or group-velocity data.

INVERSION METHODS

Linearized inversion (LI) method

The parameters that define an elastic layered medium of M layers are the density vector $\boldsymbol{\rho} = (\rho_1, \rho_2, \dots, \rho_M)$, where ρ_i is the density of layer i ; the acoustic velocity vector $\mathbf{v}_p = (v_{p1}, v_{p2}, \dots, v_{pM})$; the shear-wave velocity vector $\mathbf{v}_s = (v_{s1}, v_{s2}, \dots, v_{sM})$, and the thickness vector $\mathbf{h} = (h_1, h_2, \dots, h_{M-1})$. An equivalent parameterization for defining the same medium corresponds to $\boldsymbol{\rho}$, $\mathbf{v}_p$, $\mathbf{h}$, and Poisson's ratio, $\boldsymbol{\nu} = (\nu_1, \nu_2, \dots, \nu_M)$. Nazarian (1984) and Xia et al. (1999) show numerically that when inverting Rayleigh-wave velocity data, $\boldsymbol{\rho}$ and $\mathbf{v}_p$ are poorly resolved parameters, compared to $\mathbf{v}_s$ or $\boldsymbol{\nu}$. Nazarian also shows that $\boldsymbol{\nu}$ is a poorly resolved parameter when solving for $\mathbf{v}_s$ while fixing $\mathbf{h}$. Xia et al. (1999) found that fixing the values of $\mathbf{v}_p$ and $\boldsymbol{\rho}$ of all layers at values that are within 25% of the actual values had little effect on the estimation of $\mathbf{v}_s$. Because $\mathbf{v}_s$ changes with depth and hence depends on $\mathbf{h}$, thicknesses are typically fixed in the inversion by using many thin layers regularly spaced in depth. Similar to the inversion results published by Xia et al. (1999), it has been our experience that a good data fit is typically produced by applying to measured phase velocities a linearized inversion using the parameters just defined (Liu et al., 2002). But the estimated shear-wave velocity might not be so accurate, depending on the complexity of the soil profile being sampled.

Rayleigh-wave phase velocities sampled at wavelength j , c_{Rj} , can be written in terms of the elastic properties as

$$c_{Rj} = F(\lambda_j, \mathbf{v}_p, \mathbf{v}_s, \boldsymbol{\rho}, \mathbf{h}), \quad (1)$$

where λ_j is the wavelength with $j = 1, \dots, N$ and N corresponds to the number of observations. In practice, equation 1 is solved implicitly for c_{Rj} by using a numerical approach (e.g., Schwab and Knopoff, 1972). For computing phase velocities of the fundamental mode, we use the matrix formulation of Rössset and Foinquinos (Stokoe et al., 1994), which obtains stable solutions to equation 1 for profiles containing stiff inclusions.

The fundamental assumption of LI is that perturbations in the model are linearly related to perturbations in the data:

$$\Delta \mathbf{d} \approx \mathbf{G} \Delta \mathbf{m}, \quad (2)$$

where $\Delta \mathbf{d}$ is a column vector of N elements that represent perturbations in the data with respect to wavelength, $\Delta \mathbf{m} = (\Delta v_{s1}, \Delta v_{s2}, \dots, \Delta v_{sM})^T$ (where T means transpose) corresponds to a vector of M unknown shear-wave velocity perturbations from a reference model $\mathbf{m}_0 = (v_{s1}^0, v_{s2}^0, \dots, v_{sM}^0)^T$, and $\mathbf{G}$ is an N by M matrix of partial derivatives of data with respect to the model with elements given by

$$G_{j,i} = \left. \frac{\partial c_{Rj}}{\partial m_i} \right|_{\mathbf{m}=\mathbf{m}_0}. \quad (3)$$

Equation 2 is an approximation because it neglects second- and higher-order terms. Matrix $\mathbf{G}$ can be accurately computed from numerical differentiation, as described by Xia et al. (1999). Model perturbations can be estimated from

$$\Delta \mathbf{m}_{\text{est}} = \mathbf{G}^{\#} \Delta \mathbf{d}, \quad (4)$$

where $\mathbf{G}^{\#}$ corresponds to the linearized inverse operator, here computed from minimizing the L2 norm of the data-error vector

$$\text{DE} = \left[\frac{\Delta \mathbf{d} \cdot \Delta \mathbf{d}}{N} \right]^{1/2} = \left[\frac{1}{N} \sum_{j=1}^N (c_{Rj}^0 - c_{Rj}^t)^2 \right]^{1/2}, \quad (5)$$

where c_{Rj}^0 and c_{Rj}^t correspond to phase velocities computed for the reference model with equation 1 and those of the target (or observed) data. A least-squares solution to equation 4 is

$$\mathbf{G}^g = (\mathbf{G}^T \mathbf{G} + \varepsilon^2 \mathbf{I})^{-1} \mathbf{G}^T, \quad (6)$$

where ε^2 works as a damping factor by weighting the a priori information or by minimizing the solution error in equation 5 (Menke, 1989). The damping factor is estimated by trial and error, where the chosen factor results in the smallest data error.

In this work, we distinguish between two types of profiles: (1) a normally dispersive profile in which velocity gradually increases with depth, with perhaps some relatively small local velocity inversions, and (2) an irregularly dispersive or complex profile that consists of a normally dispersive profile with a number of anomalously low- or high-velocity inclusions. The first velocity condition is appropriate for a soil profile developed through simple deposition processes and consolidated by self weight. Two examples of the second velocity condition correspond to an otherwise normally dispersive profile containing thin layer(s) of low-velocity loose, saturated sand, which have a high liquefaction potential, and a similar profile after extensive secondary deposition of calcium carbonate in discrete horizons, as is common in some desert areas.

To illustrate the inversion of a complex profile, we chose a test model that represents an unsaturated, normally dispersive deposit containing a heavily cemented layer. The profile has nine layers including the half-space, which falls at a depth of 37.8 m. Poisson's ratio is set to 0.3 for all layers, and density is set to 1500 kg/m³ and 2500 kg/m³ for the normally dispersive part of the profile and the stiff inclusion, respectively. Figure 1 shows the shear-wave velocity profile and the synthetically computed Rayleigh-wave phase velocities; 3% of uncorrelated noise from a normal distribution has been added to the computed phase velocities. Wavelengths for our synthetic simulations are logarithmically distributed to correspond to our data collection method, which provides a better definition of the profile at short wavelengths.

Selection of a reference shear-wave profile v_s^0

Two simplifying assumptions are incorporated for developing a reference profile: First, the depth associated with a given Rayleigh-wave velocity in the dispersion curve equals one-third of its wavelength (Gazetas, 1992); and second, the shear-wave velocity is equal to the Rayleigh-wave velocity. This second assumption is based on the fact that Rayleigh-wave velocities range from 89% to 95% of v_s for values of ν between 0.1 and 0.49, a range that encompasses virtually all earthen materials (Graff, 1975). We consider this difference to be of little significance with respect to the other gross approximations involved in creating a reference model. Representative phase velocities from the experimental data are thus assigned to each layer of the reference model. Depth of the half-space is conservatively fixed from one-sixth to one-third of the maximum measured wavelength, on the basis of simple trial-and-error sensitivity tests and the requirements of the investigation.

For the problem at hand, we select two profiles as reference models. The first consists of 20 layers with a constant thickness of 0.5 m for the first 7 m, 2 m for as far as 13 m, and 10 m for as far as 43 m. Layers for this profile are thinner than those of the target profile for

all depths. In the illustrations that follow, we refer to this profile as regularly spaced. We note that the stiff layer in the target profile is 2.6-m thick and begins at a depth of 2.3 m. The second reference profile, referred to as exponentially spaced, has nine layers that thicken exponentially, 0.5 m for the first layer and 24 m at the base of the half-space. Thus, in our experiment, layers of the reference profiles are thin for shallow depths, where higher resolution would logically be expected from a surface-based measurement and all layers are thicker than the actual ones. Test examples about generating a layer geometry in which thickness increases exponentially with depth can be found in Liu et al. (2002). Figure 1a depicts the two profiles along with the target profile, and Figure 1b shows the corresponding dispersion curves. The phase velocities obtained from the reference profiles match one another almost exactly, but deviate substantially from the target data. Note from Figure 1a that this straightforward scheme results in a profile in which the velocity at the approximate depth of the stiff inclusion shows a minor increase. Overall, the reference profiles provide a relatively good match with the velocity trend of the target. Velocity differences in absolute value between the target and reference models, with respect to the target profile lacking the stiff inclusion, range from 9% over the first 10 m to 13% at the depth of the half-space.

For evaluation purposes, we define model error (ME) as the mean velocity error between a test profile and the true or target profile. To compute ME, we first obtain an ordered set containing the depths of the profiles to be compared, in this case, vectors $\mathbf{z}^0$ and $\mathbf{z}^t$ for reference and target profiles, respectively. For instance, this set might contain the following elements: $\mathbf{z} = (z_0, z_1^0, z_1^t, z_2^0, z_2^t, \dots, z_h)^T$, where z_i^0 and z_j^t correspond to the depths of the reference and target profiles for layers i and j , respectively; depth z_0 corresponds to a reference depth such as the free surface; and z_h corresponds to the depth of the half-space. If both profiles had the same layer geometry between depths z_0 and z_h , then $\mathbf{z} = \mathbf{z}^0 = \mathbf{z}^t$. Equivalently, this set can be written as $\mathbf{z} = (z_0, z_1, z_2, \dots, z_K)^T$, where K corresponds to the number of elements within the set and $z_k > z_{k-1}$ for $k = (1, 2, \dots, K)$. Then ME can be computed from the following equation:

Figure 1. (a) Shear-wave velocity profile simulating near-surface layering with a stiff layer, labeled as the target, and reference profiles with exponential-layer and regular-layer geometry. (b) Modeled Rayleigh-wave phase-velocity curve sampled at 50 wavelengths for the target model plus 3% of uncorrelated noise, and corresponding phase-velocity curves for the reference profiles. ME and DE refer to model and data errors (in m/s) between target and reference.

$$ME = \frac{1}{z_h - z_0} \sum_{k=1}^K |v_s^0(z_k) - v_s^t(z_k)| \Delta z_k, \quad (7)$$

where $\Delta z_k = z_k - z_{k-1}$ and $v_s^0(z_k)$ and $v_s^t(z_k)$ correspond to shear-wave velocities of the reference and target profiles, respectively, evaluated at depth z_k . A small value of ME indicates a close match between the estimated and target profiles. ME can be useful in real situations if independent measurements of shear-wave velocity exist. Values of ME and DE obtained from comparing the reference and target profiles and data are displayed in Figure 1.

Linearized inversion tests

Reference profiles of Figure 1a are used as starting models for LI, which is then carried out for as many as 12 iterations until the data error converges to a value that is considered to be at the noise level, and a damping factor of $\varepsilon^2 = 0.01$ is used for estimating the least-squares solution in equation 6. Densities and Poisson's ratios are assumed known and are held constant for all layers. Layer thicknesses, which are selected when building the reference profiles, are not modified during the inversion. The inverted profiles are shown in Figure 2a, and the corresponding data are given in Figure 2b. Note that inversions with both models — exponential-layer and regular-layer geometry — have predicted an increase in velocity in the vicinity of the actual stiff inclusion, but slightly deeper and of greater thickness and lower velocity than the actual stiff inclusion. Relative model-error reduction from the reference to inverted profiles is rather small, 1% and 6%, for the exponential-layer and regular-layer geometry models, respectively. This finding can be interpreted as failure of the inversions to correctly map local variations of S-wave velocity, even though the regularly spaced profile is composed of thinner layers than the actual stiff layer. In both cases the data fit is almost perfect, with $\sim 90\%$ of relative error reduction (slightly better for the profile with regular layering).

To improve our understanding of the inversion results, we compute the resolution matrix $\mathbf{R}$ (Tarantola, 1987) around reference model $\mathbf{m}_0$ as

$$\mathbf{R} = \mathbf{G}^s \mathbf{G}. \quad (8)$$

Note from equations 2 and 4 that $\Delta \mathbf{m}_{\text{est}} = \mathbf{R} \Delta \mathbf{m}$, where $\Delta \mathbf{m}$ corresponds to the perturbation needed for converging from $\mathbf{m}_0$ to the true model, and that $\mathbf{R}$ only depends on the forward-modeling and inverse operators. The computed model is perfectly resolved when the resolution matrix is the identity matrix. When off-diagonal terms are present, the computed model is a filtered version of the real model (Tarantola, 1987). To compute $\mathbf{R}$, we use singular-value decomposition on matrix $\mathbf{G}$ and then solve for equation 5.

Figure 3a displays the resolution matrix $\mathbf{R}$ obtained at the last iteration of the inversion for the exponential-layer geometry model case (the regular-layer geometry model displays a similar behavior). Cell sizes in the figure are proportional to thicknesses of the inverted layers. For a uniquely resolvable model, this matrix corresponds to the identity matrix under the assumption of Gaussian statistics and linearity of the solution near the reference model. Figure 3a indicates that intermediate depths from 4 to 10 m show strong off-diagonal elements. Figure 3b illustrates the resolution matrix obtained for the simpler case of a normally dispersive profile generated by removing the stiff inclusion of the target profile. The inverted profile from data generated for this simpler target profile uses the rules previously described for choosing a reference model, and the inversion is carried out for six iterations with the same damping factor used for the case of the stiff-layer inversion ($\varepsilon^2 = 0.01$). From Figure 3b, $\mathbf{R}$ is approximately an identity matrix. A close fit between inverted and target profiles and data, not shown for brevity, was observed in this case.

A qualitative interpretation of these results suggests that on average, the inverted shear-wave profiles are in relatively good agreement with the target profiles, but the local variations of the shear-wave velocity are poorly predicted. We note that the inversion method applied here does not modify the thicknesses of the layers. As a result, the measure of the variance of the estimated parameters would only indicate uncertainties of velocities, not layer depths. In some real problems, estimating thickness and depth of a particular element can be the aim of the inversion. In the desert setting where stiff, carbonate-cemented inclusions are encountered, the engineer needs to know depth of burial and thickness of the inclusions in order to properly lay out the site, prepare grading plans, and design foundations. Fixing layer geometry and solving with an unconstrained LI

Figure 2. (a) Inverted shear-wave velocity profiles using linearized inversion (LI). The starting models are those from Figure 1a. Note that MEs are slightly lower than those of the reference profiles in Figure 1a. (b) Corresponding Rayleigh-wave phase-velocity curves and DEs.

Figure 3. (a) Resolution matrix of the linearized inverted profile with exponential layering. Only the first 30 m are plotted. Cell sizes are proportional to layer thicknesses. (b) Resolution matrix from a normally dispersive target profile with the same layering as the original target profile but with no stiff inclusion.

method leads to nonunique solutions. We address this problem by exploring a different model parameterization and substituting LI with the global optimization method known as simulated annealing.

Improved parameterization and simulated annealing (SA) inversion

To successfully predict strong stiffness contrasts, we propose a model parameterization that is tailored for incorporating additional constraints on the basis of a priori knowledge. The model for inversion is subdivided into a background profile, into which one or more high-velocity layers are overprinted. More precisely, $\mathbf{m}_b = (v_{s1}^b, v_{s2}^b, \dots, v_{sM}^b)$ represents the background profile, where v_{si}^b is the shear-wave velocity of layer i and $\mathbf{m}_d = (v_{s1}^d, v_{s2}^d, \dots, h_1^d, h_2^d, \dots, h_L^d, z_1^d, z_2^d, \dots, z_L^d)$ consists of L distinctively (indicated by d) high- or low-stiffness layers, where v_{si}^d , h_i^d , and z_i^d are the shear-wave velocity, thickness, and depth of anomalous layer i , respectively. Figure 4 illustrates the process of combining $\mathbf{m}_b$ and $\mathbf{m}_d$ to build profile $\mathbf{m} = (\mathbf{m}_b, \mathbf{m}_d)^T$. We also note that $\mathbf{m}_d$ is composed of a small number of layers in comparison to the number of layers that define $\mathbf{m}_b$.

The suggested parameterization can be easily incorporated by a global optimization method such as SA. SA relies on a cooling schedule, in an analogy to the physical annealing process of metals, to drive a search through model-parameter space, and a Metropolis rule for selecting models. The fact that the method does not require computation of a derivative matrix linking model perturbations with data perturbations is an advantage over the LI method, given the proposed parameterization. Rather, SA uses random perturbations in the model and checks whether some suitable measure of error increases or decreases — in our case, the data error in equation 5. Updates to the model solution are carried out on the basis of probability (Sen and Stoffa, 1995). Model parameters are drawn from a model search space defined by lower- and upper-parameter bounds. Shear-wave velocity ranges input as constraints to our implementation of the SA inversion method are based on the same assumptions used for LI; velocity-search limits are set by using the reference model described earlier. The minimum and maximum bounds are obtained, for instance, by halving and doubling the velocities of the reference model, respectively. The geometry of the coarsely layered (background) profile is built according to the observed data, as described in a previous section.

A composite profile (Figure 4c) is obtained at each iteration of SA by perturbing model vectors $\mathbf{m}_b$ (Figure 4a) and $\mathbf{m}_d$ (Figure 4b) within predefined search ranges. The proposed parameterization has an important practical consequence: Model constraints based on a priori information help reduce the search space from the most permissive parameterization in which any layer might have a very high or low shear-wave velocity. Note that this parameterization can yield models that have different numbers of layers at different stages of the inversion, depending on the thickness and location of the inclusion layer(s) with respect to the layers defining $\mathbf{m}_b$. This possibility alone can cause improved resolution with respect to a solution that is restricted to a fixed layer geometry. The method has several interesting applications. Materials that could be investigated include the already-mentioned cemented layers in desert soils and potentially liquefiable layers, plus unconsolidated layers in landfills or beneath landslides, unknown pavement layering, buried engineered features on previously developed sites, and frozen gas-bearing hydrates in unconsolidated seafloor sediments.

Simulated annealing tests

We apply this parameterization scheme through the use of, for brevity, only the reference model with exponential layering (Figure 1a) to define search guides for the normally dispersive part of the profile. A stiff inclusion is assumed to exist within the depth range from 1.0 to 7.0 m, taking as reference the depth to the middle of the inclusion (in the true profile, the stiff inclusion is at 3.6 m), within a thickness range from 0 to 4 m (the true thickness is 2.6 m), and within the shear-wave velocity range from 1250 to 2000 m/s (the true velocity is 1534 m/s). The velocity range corresponds to knowledge of stiff carbonate inclusions formed in desert soil (Stone and Luke, 2001). We note that a lower limit for thickness can be fixed, e.g., below 0.3 m, to reject a solution model with a very thin inclusion that might be of no consequence in terms of data sensitivity or engineering significance and therefore removed in the interpretation of the final model. Poisson's ratio and density for all layers are assumed known. For this experiment, SA is run until the computed L2-norm error stops changing for some preselected number of iterations. The inverted shear-wave velocity profile and the search-velocity range are shown in Figure 5a, and the corresponding computed phase-ve-

Figure 4. Model parameterization for the SA method. (a) Parameterization of background profile with thicknesses of the layers assumed fixed. (b) Layer of anomalous stiffness. The unfilled and filled arrows indicate search limits for thickness and depth, respectively. (c) Composite profile.

Figure 5. SA inversion results. (a) Inverted V_s (dashed line), target (solid line) profiles, and search limits for the stiff inclusion (shaded areas). (b) Target and modeled inverted data.

locity curve is shown in Figure 5b. The inverted velocity profile presents a very close match with the target profile (a 28% of ME reduction from reference to inverted), and equally important, the high-velocity layer has a depth and thickness that approximately matches that of the actual stiff inclusion. The match with the target data, although not as close as the LI result (Figure 2b), is also very good.

We provide two numerically computed measures of resolution that make use of the stochastic nature of SA: approximated posterior model covariance matrix and marginal probability distributions.

In the first measure, the proposed scheme assumes that the posterior probability function is simple and well behaved and that by repeating several different runs, we are able to sample the most significant part of the distribution. The posterior model covariance matrix, $\mathbf{C}'_M$, is computed with the following equation (Sen and Stoffa, 1996):

$$\mathbf{C}'_M \equiv \frac{1}{K} \sum_{i=1}^K (\mathbf{m}_i - \langle \mathbf{m} \rangle)(\mathbf{m}_i - \langle \mathbf{m} \rangle)^T, \quad (9)$$

where the $\mathbf{m}_i$ values correspond to models sampled by SA and $\langle \mathbf{m} \rangle$ to the mean model of K sampled models; T indicates matrix transpose. Equation 9 is an approximation to the integral

$$\mathbf{C}'_M = \frac{1}{M} \int (\mathbf{m} - \langle \mathbf{m} \rangle)(\mathbf{m} - \langle \mathbf{m} \rangle)^T \sigma_M(\mathbf{m}|\mathbf{d}) d\mathbf{m}, \quad (10)$$

where $\sigma_M(\mathbf{m}|\mathbf{d})$ is the conditional probability density function of the model in terms of the observed data and M is the norm of σ_M (Tarantola, 1987). In equation 10, σ_M is approximated by drawing models according to the Gibb's distribution that SA uses to attain equilibrium or convergence to a minimum. For evaluating $\mathbf{C}'_M$, we rely on σ_M

being simple and on conducting sufficient model evaluations to adequately describe the function and obtain stable mean and covariance estimates.

In order to compare resolution from LI and SA tests, we compute the correlation matrix whose elements c^{ij} measure the interdependence between model parameters i and j :

$$c^{ij} = \frac{C'^{ij}_M}{\sqrt{C'^{ii}_M C'^{jj}_M}}. \quad (11)$$

Factors from equation 11 can be qualitatively compared with the resolution matrix obtained by using LI.

The second measure, the marginal probability distribution of model parameter m_i , is approximated by binning the models drawn by the SA method within the minimum and maximum limits allowed in the inversion for each model parameter i :

$$\sigma_M(m_i|\mathbf{d}) \cong \sum_{\mathbf{m}_1} \sum_{\mathbf{m}_2} \cdots \sum_{\mathbf{m}_{i-1}} \sum_{\mathbf{m}_{i+1}} \cdots \sum_{\mathbf{m}_M} \sigma_M(\mathbf{m}|\mathbf{d}). \quad (12)$$

Both of the described measures — posterior covariance and marginal probabilities — are expected to be biased estimates because the SA method tends to sample models more densely in those areas where the error converges to a minimum. Sen and Stoffa (1996) show that this deviation was relatively small when compared with an exhaustive grid search for a resistivity-sounding problem. In our tests, estimates of the two measures are obtained by exhaustively running multiple independent SA inversions, thus reducing dependency on the finite search.

Figure 6 displays binned frequency distributions scaled by number of model evaluations of (a) velocity (v_s), thickness (h), and

depth (z) of the stiff inclusion (model vector $\mathbf{m}_d$) and (b) shear-wave velocities at three different depths for the background profile (vector $\mathbf{m}_b$). The distributions in the figure comprise summed results from 12 independent runs. From Figure 6a, velocity of the stiff layer is the poorest resolved parameter, as the distribution presents two modes with the most probable solution at ~ 1800 m/s. Standard deviations of velocity, thickness, and depth (σ_{v_s} , σ_h , and σ_z in Figure 6a), computed from the square root of the diagonal elements of the covariance matrix (equation 9), are displayed in the figures. Note that the standard deviation for the velocity of the stiff inclusion is larger than the standard deviation for the velocity of the encasing layers in the background profile shown in Figure 6b. This observation can be related to the fact that thickness and depth are also being inverted for the stiff inclusion. Also note that standard deviations of the inverted velocities generally increase with depth, which can be interpreted as decrease of resolution with depth. Figure 6c displays the approximated marginal posterior probability distribution of the inverted velocities (shading; see gray scale on right), mean model (white line), and target profile (dashed line).

Figure 6. (a) Frequency distributions normalized by number of sampled models for the stiff-inclusion search parameters of velocity, thickness, and depth and computed standard deviations σ_{v_s} , σ_h , and σ_z , respectively. (b) Frequency distributions of shear-wave velocities of the background profile at three different depths and corresponding standard deviations. (c) Approximated marginal posterior probability distribution of the inverted velocities (shading; see gray scale on right), mean model (white line), and target profile (dashed line).

velocity trend of the target profile and approximately matches the high-velocity layer in thickness and depth.

Finally, maps of correlation factors, in absolute value, for the inverted velocities, for the composite profile (Figure 7a), and for the inverted parameters of the stiff inclusion, model vector $\mathbf{m}_d$ (Figure 7b), are shown. These factors were also obtained from summing statistics of the 12 independent runs. Figure 7a demonstrates that resolution decreases for depths corresponding to the depth of the stiff inclusion (~ 5 m). A strong correlation exists between the stiff layer and the layer below (fourth and fifth layers, respectively), consistent with the resolution matrix for the LI case (Figure 3a). A moderate correlation is also present between the inverted stiff inclusion and the layer that starts at ~ 10 m, indicating that inverting for the stiff inclusion somewhat decreases resolution at greater depths. Interpretation of Figure 7b tells us that constraints on thickness, rather than depth, might have a greater impact for reducing uncertainty for estimating velocity of the stiff layer, as these two parameters have a higher correlation coefficient. Hence the importance of constraining thickness and/or velocity of the stiff inclusion in order to avoid an outcome containing an unrealistically thick layer with a velocity that is substantially lower than the target-layer velocity.

LAS VEGAS SPRINGS PRESERVE TEST DATA CASE

The inversion process is illustrated for a test site at the Las Vegas Springs Preserve, an interpretive site surrounding the original artesian springs of Las Vegas, Nevada. The site serves as a valuable resource for research on surface-based geophysical methods. The preserve exhibits features such as subsidence, fissuring, and cemented layers common in dry desert soils (Sundquist and Luke, 2001). The site remains an active well field, and the investigation described here was undertaken to support expansion of a reservoir for surface storage of pumped groundwater. Site investigations included shear-wave velocity profiling with the SASW method (Stokoe et al., 1994) and with the crosshole method (D4428, ASTM, 1984). Drill logs generated for the crosshole test are also meaningful for comparison.

SASW testing at the Las Vegas Springs Preserve was conducted at geophone separations ranging from 0.5 to 80 m. Seismic sources used included sledgehammers and the motion of a small tracked bulldozer. Mark Products geophones with resonant frequencies of 4.5 and 1 Hz and a Stanford Research Systems dynamic signal analyzer were used. The SASW method uses phase-difference data between geophone pairs. A signal analyzer generates phase and coherence data from time histories, in the field, by calculating cross-power spectra for the signal pairs. Multiple data sets are averaged in the frequency domain: time-domain stacks are not computed. Random-energy vibrations can be used as input. Tests at short geophone separation with low-energy sources resolve the high-frequency component of the dispersion relationship, and vice versa. The most subjective step in data processing is unwrapping the phase data and masking the parts that are not instructive. The wavelength corresponding to a given frequency is determined by the ratio of geophone separation to phase difference. The dispersion curve is obtained from this wavelength-frequency relationship. Examples of wrapped and masked/unwrapped phase data at 4- and 64-m geophone spacings for the example case are shown in Figure 8. The 4-m data were collected by using a sledgehammer source and 4.5-Hz geophones and the 64-m data were collected by using the bulldozer source and 1-Hz geophones. Dispersion data collected from the different geophone separations

are superimposed to form a master dispersion curve, which is then down-sampled and smoothed (Figure 9a).

Crosshole measurements were made by using three boreholes, nominally 3 m apart, located at the center of and inline with the SASW array. The borehole logs showed sporadically cemented silty sands and gravels in the upper 3 m, over a deep clay deposit, also containing cemented inclusions. The data set in its final condensed, smoothed form (Figure 9a) is the basis for inversion. Recall that the SASW method yields an effective dispersion curve, which superimposes contributions from all modes.

In using a fundamental-mode-only model to match the data, we are making the simplifying assumption that the wavefield is dominated by fundamental-mode surface-wave energy. Higher-mode contributions and body-wave scattering become noise. We anticipate that, with this data set, use of a forward model that captures the entire effective wavetrain should improve resolution or further reduce ambiguity. The partitioning of surface-wave energy among modes in the presence of irregularly stratified layers has been illustrated through numerical simulations by Gucunski and Woods (1991) and experimentally by Jin and Luke (2006). Incorporating

Figure 7. Maps of correlation factors estimated with SA inversion for (a) inverted velocities with cell sizes proportional to layer thickness and (b) stiff-inclusion model parameters.

Figure 8. Phase measurements and unwrapped phase for two pairs of stations, at (a) 64-m geophone spacing and (b) 4-m geophone spacing, used for developing the interpreted dispersion curve shown in Figure 9a.

more realistic forward modeling in surface-wave studies is a subject area receiving attention in current research (Xia et al., 2000; Beaty et al., 2002; Rydén, 2004).

Because the crosshole method is an independent means to determine the shear-wave velocity profile in situ, this interpreted profile becomes a surrogate for our target profile of study (Figure 9b). However, we would not expect the surface-wave inversion to generate a perfect match to this target, primarily because of differences in volume of media sampled by the two tests. We can also compare results against borehole lithology.

The SA inversion parameterization comprised a background profile with nine layers thickening exponentially with depth and having two stiff inclusions. The exponential law used to define layer geome-

try depends on a single parameter that is adjusted by using a simple minimization process that tries to match synthetically computed phase velocities with the observed ones, through the use of the empirical laws described in a previous section to assign a velocity to each layer (Liu et al., 2002). Densities are again set at 1500 kg/m^3 for all but the proposed stiff layers, which have densities set at 2500 kg/m^3 . Poisson's ratios for all layers are kept at 0.3. Search ranges for the shear-wave velocity of the background profile and velocity and depth of the two stiff inclusions are indicated by the shaded areas in Figure 9b. Search ranges for the two stiff layers are 0.2–1 m and 0.4–3 m for thickness, 0.2–3 m and 6–10 m for depth, and 1000–2000 m/s for velocity. The search ranges are designed on the basis of borehole data. We chose not to invert for a third stiff layer at a greater depth relative to the expected resolution of the SASW data. The SA inversion was run for 4000 iterations. In this test, the final profile was used as a starting model for LI, to provide a closer data fit, but the major model perturbations were performed by the SA method.

Figure 9b displays both the crosshole-interpreted data and the inverted profile. In general, the data sets are in agreement. The position of the upper layer from SA matched the position of the shallowest cemented layer from borehole logs nearby, but appears shallower than the velocity peak registered with the crosshole data at $\sim 2.5 \text{ m}$. At $\sim 4\text{-m}$ depth, the inverted profile shows a mild increase in velocity just below the velocity peak of the crosshole data. The deeper inverted stiff inclusion matched the approximate depth and thickness of both the borehole and crosshole data. For the rest of the profile, the inverted profile appears to match the crosshole measurements for intermediate depths (from 4 to 9 m), not including the velocity peak at $\sim 16 \text{ m}$. At greater depths, there appear to be greater discrepancies, which is normal because the surface-based measurement technique lacks resolution for inverting thinner layers at those depths. Figure 9a indicates a good match between the observed and fitted phase velocities, except for the shortest wavelengths.

Figure 10 shows frequency distributions for the shallow and deeper stiff layers from model vector $\mathbf{m}_d$ obtained by running the SA method 12 times. The inverted velocity of the shallow inclusion has a slightly larger standard deviation compared to the deeper layer. But the fact that thickness and depth are constrained better for the shallow stiff inclusion implies a loss of resolution with increasing depth for locating stiff layers. Figure 11a shows occurrence distributions for velocities at three different depths from vector $\mathbf{m}_b$, and Figure 11b displays the approximated marginal probability density function of the composite model $\mathbf{m}$. Overall, the width of the velocity distributions increases with depth; also the two stiff inclusions show wider distributions than their encasing layers. Figure 11b demonstrates that the depth of the deeper inverted stiff inclusion is at $\sim 8.5 \text{ m}$, but the best model in Figure 9 shows a depth of $\sim 7 \text{ m}$, perhaps a local maximum in the distribution for depth observed in Figure 10b. Again this result can be interpreted as a lack of confidence for locating the deeper stiff layer in depth.

Correlation coefficients in absolute value are plotted in Figure 12 for the composite shear-wave

Figure 9. (a) Measured Rayleigh-wave phase velocities (dots) and modeled data (x) from the SA inverted profile. (b) Crosshole velocity measurements (open circles), inverted profile (x), and velocity-search limits (shaded areas) for background profile and stiff inclusions.

Figure 10. Frequency distributions normalized by number of sampled models for the search parameters for (a) shallow and (b) deeper stiff inclusions. Standard deviations are obtained from the posterior covariance matrix estimated from summing 12 independent SA runs.

velocity profile and for the model parameters defining the inclusions with high stiffness. The top inclusion shows a strong velocity dependence on the layers above (Figure 12a). The deeper stiff layer appears as better resolved, but shows some mild correlation with the layer immediately below. Furthermore, some crosstalk exists for the estimated velocities among the two stiff inclusions, according to the map of Figure 12b. From this map also, the submatrix involving only model parameters for the deeper stiff layer has smaller off-diagonal elements than those of the shallow stiff layer. An interpretation of this result is that better velocity constraints for the shallowest layers might help reduce nonuniqueness of the overall solution. Velocity and thickness show a higher correlation than velocity and depth for the shallow stiff layer, whereas for the deeper one, thickness and depth appear to have a similar effect on velocity. Also, from Figure 12a, note that below 10 m, inverted layers show relatively high cor-

relation coefficients among adjacent layers; this fact points toward the difficulty of inverting stiff inclusions that are relatively deep in the profile, considering that standard deviations of inverted parameters for both the background profile and the stiff inclusions gradually increase with depth. We also observe that, similar to the previous synthetic example, thickness and depth show relatively small correlation factors for both inclusions. Thus, fixing shear-wave velocities of the cemented layers — possibly through a priori independent measurements or expectations based on similar studies — could provide better estimates, thereby improving the ability to resolve locations of the anomalous inclusions.

CONCLUSIONS

Soil profiles that include anomalously stiff or soft inclusions are difficult to invert from Rayleigh-wave phase-velocity data because of the nonuniqueness of the inverse problem. We use a two-stage inversion method that results in a more realistic solution model. The first stage builds reference profiles based on a set of simple empirical rules applied to the measured phase-velocity data. The second uses the simulated annealing optimization method for perturbing the reference profile and incorporating layers with distinctive, narrowly defined velocity, depth, and thickness. This parameterization has the advantage of reducing the search space with respect to a parameterization in which layers of anomalous stiffness might exist at any depth. Constraints for resolving a profile are based on (1) suspected layer boundaries as might be found in pavement systems, landfills, or landslide debris or (2) results of independent geophysical tests or direct observation.

In our synthetic study and real data set, uncertainties in shear-wave velocity are larger when thickness and depth are allowed to vary than when thickness and depth are kept fixed for inversion. Thus, to improve resolution of anomalously stiff inclusions, the user should constrain the shear-wave velocity and/or thickness and depth of the anomalous layers as tightly as possible.

This effort is only possible if information is available from other in situ measurements or from experience. The approach may not be an option in very heterogeneous soils or other sites with few a priori expectations.

The depth range over which we are currently able to resolve the inclusions roughly corresponds to the depth range of primary interest to design engineers: cemented strata in the upper few meters are most significant for load-transfer capability or as obstacles to excavation. As foundations and excavations go deeper, the stiffness and bearing capacity of the uncemented soil increases, lessening the contrast in mechanical response between the uncemented soil and the cemented inclusions. A possible outcome of the proposed inversion scheme is a profile with no sharp stiffness contrast. If relatively shallow stiff layers are indeed present at the study site, such a profile is an unlikely result, providing that (1) the dispersion data set is sensitive to sharp contrasts that are relatively shallow and (2) the assigned search ranges encompass the correct solution.

Figure 11. (a) Occurrence distributions for the shear-wave background profile at three different depths along with corresponding standard deviations. (b) Marginal probability density function (shading; see gray scale on right), mean profile (white line), and interpreted crosshole velocities (open circles).

Figure 12. Correlation factor maps of (a) inverted shear-wave velocities and (b) inverted parameters (velocity, thickness, and depth) for the shallow (subscript 1) and deeper (subscript 2) stiff inclusions.

For the experimental test discussed in this paper, borehole data revealed two stiff layers at depths of less than 10 m, and that information was used as a priori information for the inversion. As depth increases, the surface-based method loses resolution. We note that the forward modeling adopted in this work is a plane-wave approximation of fundamental-mode surface-wave propagation through layered media. A more detailed forward model might reduce ambiguity of solutions. Particularly in the subject case, where large, abrupt stiffness contrasts exist, the potential for energy partitioning to higher modes and body-wave conversions is strong. Further work should test how resolution improves, particularly at depth, when the forward-modeling solution is enhanced to accept additional data such as phase velocities from higher modes. The inversion method described here is equally appropriate for more detailed forward modeling.

Resolution and uncertainty measures obtained numerically from repeated sampling of the model space are powerful tools for qualitative interpretation of computationally tractable problems. Estimates of resolution and marginal probability density function for the synthetic and real data help corroborate the suitability of the parameterization used.

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