Game Volatility
At Baccarat

Andrew MacDonald

Abstract

The authors discuss the volatility of table games using Baccarat as an example. The expected deviation from the win, in percentage terms, will decrease as the number of decisions is increased. In absolute terms, the deviation is likely to increase as decisions increase. Through the formulas presented in the article the author shows there is no natural evening-up process. That is if a casino lost one million dollars on Baccarat play last month, the casino manager cannot expect to make the theoretical win plus an additional one million next month. The article will help managers understand and plan for fluctuations in table game play.

Keywords: actual win, Baccarat, casino management, game volatility, table games, theoretical win, variance.

A question often asked by Analysts and Senior Management within the casino industry is: "... when will we hit our theoretical win percentage?"

This question is generally reserved for when results are below expectation and pressure is being exerted for the results to "turn" in the casinos favour.

Some labour under the impression results *must* "turn" so that the average will be achieved over time. "A run of ill fortune means that it is "our turn" and we are more likely to win." Such thoughts should obviously be reserved for those on the other side of the gaming tables. There is no "evening up". The game has no memory, with each result being effectively independent. What will occur as the number of decisions is increased is that the "expected" deviation from the mean in percentage terms will decrease. In absolute terms, however, the deviation will increase as decisions increase. This can best be shown in Table 2:

Number of Decisions	One Standard Deviation	Percentage Deviation	Theoretical Win
n	$\sqrt{n}$	$\dfrac{\sqrt{n}}{n}$	$TW = (A \times n)^{1}$
100	10	10%	1.25
1000	31.6	3.16%	12.5
10,000	100	1.0%	125
1,000,000	1000	0.1%	12,500

Where n equals the number of decisions and the square root of n represents one standard deviation from the expected result and where the expected result equals 1.25% x n for the game of Baccarat.

An interesting further point is that if we were to conduct an experiment where an unbiased coin were to be thrown 100 times with the results of each throw recorded and the experiment temporarily stopped after the first 10 throws with 10 heads having been thrown,

Andrew MacDonald
Executive General Manager
Crown Limited
Melbourne, Australia

it would now of course be more likely for heads to still be ahead at the conclusion of the 100 trials. Remember, the coin has no memory so for the 90 throws we would expect a 45/45 split with a standard deviation from the mean being the square root of the number of decisions multiplied by the probability of winning multiplied by the probability of losing.

$$\text{Probability (Heads)} = 0.5 \times 90 = 45$$

$$\text{One Standard Deviation} = \sqrt{npq} = \sqrt{90 \times 0.5 \times 0.5} = 4.74$$

Where p = the probability of a win and q = the probability of a loss.

95% of results will fall within +/- 1.96 standard deviations of the mean.

Therefore, for the next 90 throws we would expect the following:

$$45 \text{ hands} +/- 1.96 \times 4.74$$

ie. 95% of results will fall between <u>35.70</u> and <u>54.30</u>

Add this to the first 10 heads and it becomes obvious that at the conclusion of the 100 coin tosses it is highly likely heads will still outweigh tails if the first 10 results were all heads. No evening up, in fact the ability only to project that if we start out behind we are more likely to end up behind and if we start out ahead we are more likely to end up ahead.

What is useful to understand for casinos conducting high end play is how many decisions would be required for a certain departure from theoretical to occur for a given confidence interval.

The purpose of this paper is therefore to establish and verify a formula to determine the number of hands played which would result in a specified tolerance level of departure from the mean win percentage at the game of Baccarat for a given confidence interval.

Determination based upon the above of a level of turnover which should generate results replicating the theoretical win percentage at the game.

The number of decisions (n) at which the win percentage will deviate by a given fraction from the mean may be calculated using the following formula:

$$n = \left[\frac{1}{\left(\dfrac{\text{Tolerance level}}{\text{Confidence interval}} \right)^2} \right] \times \text{Variance of the game}$$

where n = resolved decisions (hands)

tolerance level (T) = dispersion from the mean in %.

confidence interval (C) = the z score relative to the desired confidence interval of a normal distribution curve (z = 2.33 for a 98% confidence interval)

Variance (V) = the variance of the game in question (ie the average squared result)^2

Therefore:

$$n = \left[\cfrac{1}{\left(\cfrac{T}{C}\right)^2}\right] \quad x \ V$$

$$= VC^2/T^2$$

A desirable level of tolerance for which to calculate may be one where breakeven is achieved for the specified confidence limit.

Breakeven may be calculated as follows:

Costs divided by turnover	=	E
Gaming Tax	=	G
Breakeven	=	E / (1 – G)

To consider this issue further it may also be useful to consider game bet distribution patterns and actual results derived over time.

The number of hands (resolved decisions) required based upon the above may be calculated as follows:

$$n = \left[\cfrac{1}{\left(\cfrac{A - (E/(1\text{-}G))}{2.33}\right)^2}\right] \quad x \ 0.975$$

The number of decisions played for the theoretical win % to be achieved within 0.04% on the same basis is astronomical.

$$n = 0.975 \ x \ 2.33^2 \ / \ 0.0004^2$$

n = 33,082,359 for the theoretical win % to fall within <u>1.225</u>% and <u>1.305</u>% at a 98% confidence limit.

If flat bets of $200,000 were considered, the volume of turnover required to generate such results would be:

For n = 33,082,359	turnover =	$6,616,471,800,000
		(ie. $6.62 tn)

Clearly not all bets occur at the table maximum and thus it may be appropriate to consider both the average bet likely to be achieved and the variance of the associated bet distribution.

If the following bet distribution is considered indicative of future playing patterns then it is possible to forecast the level of turnover which needs to be generated for results to fall within a certain tolerance level of the mean, with a particular degree of certainty.

Bet Size	% of Wagers	Weighted Average
$200,000	0.5%	$ 1000
$150,000	0.5%	$ 750
$100,000	1.0%	$ 1000
$ 75,000	4.0%	$ 3000
$ 50,000	4.0%	$ 2000
$ 25,000	9.0%	$ 2250
$ 10,000	11.0%	$ 1100
$ 5,000	70.0%	$ 3500
TOTAL	100.00%	$14600

The number of decisions required may be calculated using the additional variable of the variance of the bet distribution which for the above is 3.86. (The second moment over the squared mean of wagers = Vb).

$$n = \left[\left(\frac{1}{\frac{T}{C}} \right)^2 \right] \times V \times Vb$$

$$= VVbC^2 / T^2$$

If the degree of tolerance which is desired is equal to 0.04% then:

$$n = 0.975 \times 3.86 \times 2.33^2 / 0.0004^2$$

With an average bet of $14,600 as described previously, a turnover of $1.86 tn is required.

To calculate the period required to generate this volume of turnover it is a simple case of dividing this number by the future forecast annual turnovers volumes.

Based on forecast annual turnover volumes, this degree of certainty would be achieved after $1.86tn divided by the annual turnover volumes.

Conclusion

It is clear, based upon the above calculations that the variation in win percentage from theoretical should decrease as turnover volumes increase. The quantum of turnover required for the actual win percentage achieved to fall within a certain level of tolerance can be calculated based on the following formula:

$$n = \left[\left(\cfrac{1}{\cfrac{T}{C}} \right)^2 \right] \times V \times Vb$$

$$= VVbC^2 / T^2$$

where

n	=	number of decisions	
T	=	absolute deviation from the mean	
C	=	z score associated with a desired confidence interval	
V	=	Variance of a single game (average squared result)	
Vb	=	Second moment over the squared mean of wagers	

turnover   =   n x average bet

This same formula may be modified depending on which variables may be known. For example, it is useful if full play details are known to calculate the probability of the result. To do this the formula is solved to determine the value C.

$$C = \cfrac{T}{\cfrac{\sqrt{V \times Vb}}{n}}$$

$$= T\sqrt{n} / \sqrt{VV_b}$$

Once C is calculated a z score has been established representing the standard deviation from the mean of a normal distribution curve. z score statistical tables may then be reviewed to determine the probability of the actual result occurring.

This information once calculated can be very useful in establishing that the games are being conducted legitimately. A highly improbable result may be worthy of follow up and investigation.

As has been shown, the actual win percentage will tend to converge on the theoretical percentage over time. The level of turnover required for the actual win percentage to fall within a small level of tolerance from the mean can, however, be astronomical, so Analysts and Senior Management need to have very strong stomachs if involved in Baccarat high end play.

[1] TW = Theoretical Win
  A = House Advantage
  n = Decisions

[2] The Theory of Blackjack – author: Peter A. Griffin (page 167)