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Measuring ‘Closeness’ in 3-Candidate Elections: Methodology and an Application to Strategic Voting

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Abstract

Past research suggests that voter behavior is influenced by perceptions of electoral competitiveness. For example, when an election is perceived to be close, voters will be more likely to turnout and/or cast strategic votes for their second-most preferred candidate. Operationalizing electoral competitiveness in three-candidate elections presents previously unrecognized methodological challenges. This paper first shows that many past strategies for measuring ‘closeness’ in three-candidate contests have violated at least one of three basic properties that any such measure should satisfy. We then propose a new measurement grounded in *probability ratios*, and prove formally that ratio-indices satisfy these axiomatic criteria. Empirical analyses using this new index provide novel and nuanced findings on the extent and causes of *strategic voting* in the 2010 British general election. The paper’s operational strategy should be generally applicable to research on voting in elections, legislatures, and organizations.

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1. Introduction

The notion that voter choice is driven by both voters' preferences *and* their expectations over the set of possible electoral outcomes figures prominently in the study of voter behavior. This is particularly true of research grounded in the *Calculus of Voting*, an expected utility model in which both voter turnout and voter choice are affected by the election's competitiveness, or *closeness*, and in particular the likelihood that voters are 'pivotal' in creating or breaking 1st place ties between candidates (Downs 1957; Riker and Ordeshook 1968; McKelvey and Ordeshook 1972; Black 1978; Hoffman 1982).[1] The argument is intuitive: when an election is close, voters will understand that their votes are more 'meaningful'. In turn they should be more likely to *turnout*,[2] and in some cases to cast *strategic votes* for their 2nd-most preferred candidate.[3]

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[1] Green and Shapiro (1994) famously criticize the Calculus of Voting for failing to explain *voter turnout*. For the interested reader, Part VI of the Supporting Information file argues that, even if this is true, expected-utility may nonetheless be a perfectly viable framework for modeling *voter choice*.

[2] Although by no means exhaustive, for additional work on turnout and competitiveness see Hinich and Ordeshook (1969), Rosenthal and Sen (1973), Ferejohn and Fiorina (1974), Cox (1988), Grofman et al. (1998), Endersby et al. (2002), and Adams et al. (2006).

[3] A large amount of past research on strategic voting is reviewed in the process of developing Sections 2 through 4 below. For a schematic review of more recent contributions see Cox (1997); Fey (1997); Alvarez and Nagler (2000); Blais (2000); Niou (2001); Fisher (2004); Fieldhouse et al. (2007); Merolla and Stephenson (2007); Myatt

Measuring closeness in 3-candidate plurality rule contests presents an under-appreciated methodological challenge. This paper's first substantive contribution is to unearth an implicit debate among scholars of voter choice as to the proper methodology. Sections 2a and 2b demonstrate that, among the many interesting and creative proposals (e.g. Rosenthal and Sen 1973; Black 1978; Cain 1978; Abramson et. al 1992; Ordeshook and Zeng 1997), all past *proxies* of closeness in 3-candidate contests violate at least one of three basic Properties that such measures should satisfy. Having identified the challenge, Section 2c demonstrates formally that a proxy measure grounded in *probability ratios* satisfies these axiomatic criteria.

Section 3 then presents the Calculus of Voting and its hypotheses regarding strategic choice, and proposes a step-by-step strategy for testing these hypotheses the proposed ratio measure. Using the 2010 British Election Study (BES),[4] Section 4 provides statistical tests of these hypotheses, whose results provide novel insight into the causes of strategic voting. Firstly, while past research has emphasized the pairwise comparison of one's most-preferred and second-most preferred candidates, our results suggest that strategic voting is driven almost entirely by viability comparisons involving one's *least-*

(2007); Herrmann and Pappi (2008); Kselman and Niou (2010); Meffert and Gschwend (2011); Kiewiet (2013); and Hillygus and Treul (2014), and Eggers and Vivyan (2017).

[4] For reasons discussed at greater length below, this data source and election present the ideal context for a complete test of the paper's methodology and core hypotheses. We conduct robustness checks on the 1988 Canadian General Election Study (CNES), which also meets the necessary criteria.

preferred candidate. Put otherwise, voters use their least-preferred candidate as an 'anchor point' when deciding who to vote for, rather than making explicit comparisons between their two preferred candidates. We also demonstrate that the impact of expectations on vote choice is contingent on a voter's preference profile. For example, among voters who only mildly prefer their second preference to their least-preferred candidate, increasing the closeness of the contest between these two candidates has little impact on the likelihood of strategic voting. Finally, we demonstrate that past estimates of strategic behavior may have been influenced by the presence of distinct forms of tactical choice such as *protest voting*; and propose a strategy for purging estimates of this potential bias. Taken together, these findings represent some of the most precise, but also nuanced, evidence to date in favor of expected-utility maximization as a model of voter choice.

The Concluding Section 5 discusses additional applications and extensions. While the current paper's application is to strategic voting in 3-candidate plurality rule elections, its methodology should be relevant to studies of strategic voting in more complex environments (e.g. N>3 candidates, coalition government, etc.). As well, it should be applicable to future studies of voter turnout, protest voting, and campaign contributions,[5] where electoral competitiveness is also a relevant consideration. Moving beyond the world of popular elections, our measure of closeness should be relevant for studies of instrumental and tactical voting in democratic legislatures (Enelow 1981; Calvert and Fenno 1994), as well as to the study of *shareholder elections* in private corporations, whose

[5] See Bouton et al. (2018) on the relationship between competitiveness and campaign contributions.

importance has increased over time (Yermack 2010). Finally, the Conclusion identifies avenues for future research into the theoretical and empirical comparison of *proxy* vs. *explicit* measures of closeness, a distinction to which we now turn.

1a. Proxy vs. Explicit Measures of Closeness

Our approach to measuring closeness follows most past research, in that it aims to create a reliable empirical *proxy* for the probability of casting a pivotal vote in a plurality election. Rather than proxy measures, a set of recent papers generate *explicit* values of the probability of being pivotal (Herrmann et al. 2016; Fisher and Myatt 2017; Eggers and Vivyan 2017). These papers model voter beliefs over electoral outcomes as a *probability density function* over the set of all possible 3-dimensional electoral outcomes.[6] One can then extract the closeness measures by integrating over the set of outcomes in which a voter is 'pivotal' for creating or breaking a 1st-place tie between two candidates. Although very small, these values are greater than '0'.

This approach presents the value of precision: it specifies a detailed belief structure for voters, and extracts closeness measures directly from that belief structure. It thus represents an improvement in our inductive ability to calculate closeness. That being said, just as with the proxy measures studied in this paper, there are a multitude of different functional forms these explicit measures can take, and the decision as to how to represent baseline voter expectations becomes important. For example, Herrmann et al. (2016) use a

[6] The density function's maximum occurs at the observed electoral outcome from the previous electoral cycle (or current cycle in the case of Eggers and Vivyan 2017), and its value decreases as one moves further and further from this observed outcome.

multinomial distribution to model voter beliefs, whereas and Fisher and Myatt (2017) and Eggers and Vivyan (2017) use a *dirichlet* distribution, each of which makes specific assumptions about voter expectations. Our aim here is to provide a logical framework with which to evaluate any functional form used to measure closeness, *whether proxy or explicit*. Just as with proxy measures, explicit measures grounded in probability density functions which meet the criteria developed in Section 2 would be judged superior. The Conclusion proposes future theoretical work which, although technically challenging, could serve to identify the general class of probability density functions which satisfy the theoretical criteria developed in Section 2.

All of this begs the question: what impact does functional form have on substantive empirical findings? Section 4 demonstrates this paper's proxy for closeness, which satisfies Section 2's theoretical criteria, performs both *differently* and *better* than closeness proxies from past studies, with one important exception and informative exception (Ordeshook and Zeng 1997). Future research should also compare the performance of our proxy measure with explicit measures of closeness. Our hunch, based on the evidence in this paper, is that all measurement strategies, whether proxy or explicit, that satisfy Properties 1-3 will generate similar substantive findings. We return to these issues in the Conclusion, which suggests that the proxy-based and explicit measurement approaches should be complementary and mutually informative; and that both will benefit from research on the processes by which voters translate expected vote shares into probabilities of winning.

2. Measuring 'Closeness' in 3-Candidate Contests

In a 3-candidate plurality rule election, define p_j as the probability that a voter gives their j^{th} preference of winning the electoral contest ($j \in \{1,2,3\}$): p_1 is the probability their most-preferred candidate wins, p_2 is the probability their second-preference wins, etc. Measuring p_j for individual voters is an important first step in generating an index of the electoral competitiveness between two candidates. Some studies use state- or district-level vote-shares from previous electoral cycles as a proxy for these probabilities (e.g. Cain 1978; Alvarez and Nagler 2000): if party *j* received vote share 35% in a particular electoral district at time 't', then survey respondents from said district are assigned the value $p_j = 35\%$ in election 't+1'.[7] While presenting numerous advantages, these approaches eliminate the possibility for individual idiosyncrasies, and more generally for within district variance in expectations across voters.[8] To avoid these issues, some papers measure p_j with the probabilities of winning assessments that respondents provide in public opinion surveys (e.g. Abramson et al., 1992; Merolla and Stephensen 2007). This 'subjective' approach

[7] Recent innovations go a step further, using multi-level regression and post-stratification techniques to extract district-level forecasts from national surveys; and then using these vote forecasts as a measure of p_j (Hanretty et al. 2016, 2018; Eggers and Vivyan 2017).

[8] Cain (1978) and Alvarez and Nagler (2000) also assume that voters make a 1-to-1 translation of candidates' expected vote shares into their expected probabilities of winning. As addressed in Sections 4 and 5 below, the translation of expected vote shares into likelihoods of winning is characterized by non-linearities.

allows for within-district variance and greater precision, but is susceptible to respondents' biases and cognitive constraints.

In this paper, we remain agnostic as to the best strategy for measuring perceptions of a candidate's probability of winning p_j.[9] Our focus is rather on *how the raw probabilities of winning p_j and p_k are used to generate a closeness index p_{jk}*, capturing the perceived likelihood that a voter is pivotal in creating or breaking a 1st place tie between their j^{th} and k^{th} preferences. As such, p_{12} represents the probability of being pivotal between their first- and second-preferences, p_{13} is the probability of being pivotal between their first- and last-preferences, and p_{23} is the probability of being pivotal between their second- and last-preferences. One common and obvious proxy for the closeness in 2-candidate contests is $p_{jk} = 1 - |p_j - p_k|$, the absolute difference between the two competitors' expected probabilities of winning.[10] The indicator, which ranges from 0 to 1, is of course not a direct measure of p_{jk}, which will be small in large electorates. However, the measure works as a proxy because it captures the proper comparative statics: it increases as the race between the two candidates becomes closer, with the highest value '1' occurring when both candidates have a 50% chance of winning.

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[9] Indeed, the paper's core empirical analyses use data from the 2010 British Election Study (BES) precisely because this provides the best opportunity to replicate our analysis with *both* objective and subjective data.

[10] Cox (1988) analyzes different operational strategies for measuring closeness in 2-candidate contests.

In three candidate contests this difference measure no longer works. Take two situations, one where a voter's first- and second-preferences each have a 20% chance of winning ($p_1 = p_2 = 20\%$) while their least-preferred candidate has a 60% choice of winning ($p_3 = 60\%$); and one where a voter's first and second preferences each have a 50% chance of winning ($p_1 = p_2 = 50\%$) while their least-preferred candidate has a 0% choice of winning ($p_3 = 0\%$). In both cases the absolute difference measure yields $p_{12} = 1$, but the second voter is clearly more likely to be pivotal than the first, since the likelihood of a 1st place tie between her top two preferences is much higher. In words, a two-dimensional difference measure fails to incorporate information about the two candidates' *relative viability vis-à-vis the third candidate*, and thus is missing important information.

2a. An Axiomatic Approach

To generalize this argument, this paper takes an *axiomatic* approach to the challenge of measuring p_{jk} in 3-candidate contests: we begin by setting forth a series of properties which any measure of p_{jk} should satisfy, and then develop an operational strategy to satisfy these criteria. The first two properties relate to the *comparative statics* that any measure of closeness in 3-candidate contests should satisfy, and are labeled *Plurality Margin* and *Mutual Viability*.

- Property 1 (**Plurality Margin**): Assume *j* is the expected plurality winner. Then, if any of the three candidates' vote shares is held constant, the closeness of the race between *j* and *k* (p_{jk}) should *increase* as the margin separating these two candidates *decreases*.

To demonstrate the importance of Property 1, consider two voters who believe their most-preferred candidate will place first, their second-most-preferred candidate will place second, and their least-preferred candidate will place last in the election. The first voter has $p_1 = 60\%, p_2 = 30\%, p_3 = 10\%$ while the second voter has $p_1 = 50\%, p_2 = 40\%, p_3 = 10\%$. Both voters expect their least-preferred candidate to be the loser, and this candidate's probability of winning is held constant at 10%. However the probability of a 1st place tie between their first- and second-preferences p_{12} should be higher for the second voter, for whom the expected margin separating them is smaller.

- Property 2 (**Mutual Viability**): Holding their *expected probability separation constant*, the closeness of the race between *j* and *k* (p_{jk}) must *increase* with their shared likelihood of winning the election against the third candidate.

To demonstrate the importance of Property 2, return to the example above where one voter has expectations $p_1 = 20\%, p_2 = 20\%, p_3 = 60\%$ while a second has expectations $p_1 = 50\%, p_2 = 50\%, p_3 = 0\%$. In both cases the voters' first- and second-preference have equal probabilities of winning; however the probability of a 1st place tie between their first- and second-preferences p_{12} should clearly be higher for the second voter.

Creating a measure for p_{jk} which displays the proper comparative statics is a necessary but not sufficient condition for measuring closeness in 3-candiate contests. In addition, any such measure must satisfy the following *Absolute Size* criterion:

- Property 3 (**Absolute Size**): The probability of a 1st place tie between the two leading candidates must be higher than the probability of any 1st place tie which includes the trailing candidate.

For example, among voters who perceive their most-preferred candidate to be winning, their second-most preferred candidate to be in 2nd place, and their least-preferred candidate to be losing ($p_1 > p_2 > p_3$), Property 3 requires that $p_{12} > p_{13}, p_{23}$.

As discussed in Section 5 (and Supporting Information Part IV, pages 7-9), these three Properties could (and perhaps should…) be complemented with additional criteria. For the moment, they constitute a minimalistic and intuitive core. Holding any of the three candidates' expected vote shares constant, the probability of a 1st-place tie between the expected plurality winner and a second candidate should increase as the race between those two candidates tightens (Property 1). Holding the margin separating two candidates constant, the probability of a first-place tie should increase as their shared chances against the third candidate improve (Property 2). Finally, the likelihood of a 1st place tie between the top two candidates should be higher than the likelihood of a 1st place tie involving the trailing candidate (Property 3).

2b. Past Strategies for Measuring Closeness in 3-Candidate Contests

Before developing our measurement, we demonstrate that past *proxies* of p_{jk} in 3-candidate contests do not simultaneously satisfy Properties 1-3.[11] These studies have

[11] The Conclusion discusses the relationship between Properties 1-3 and past *explicit* measures of p_{jk}.

employed a wide variety of approaches, including an *absolute difference* approach (e.g. Cain 1978; Abramson et al. 1992; Blais and Nadeau 1996), a *conditional difference* approach (Rosenthal and Sen 1973), a *multiplicative* approach (Ordeshook and Zeng 1997), and *Euclidean geometry* (Black 1978; Herrmann and Pappi 2008). Abramson et al. (1992) use the absolute difference between the two candidates' probability of winning $|p_j - p_k|$ to capture closeness in multi-candidate American Primaries. While this is technically a measure of 'distance', it can be rescaled as in the two-party context to capture closeness: $p_{jk} = 1 - |p_j - p_k|$. In contrast, Cain (1978) uses the reciprocal of the margin $p_{jk} = \frac{1}{|p_j - p_k|}$ to rescale the margin as a measure of closeness.

Regardless of the functional form adopted, approaches grounded in the absolute difference fail to satisfy Properties 2 and 3. Regarding Property 2 we've already mentioned the comparison of two voters one with $p_1 = 20\%, p_2 = 20\%, p_3 = 60\%$ and a second with $p_1 = 50\%, p_2 = 50\%, p_3 = 0\%$. The difference measure $p_{jk} = 1 - |p_j - p_k|$ would assign these voters identical scores of $p_{12} = 1$, but a 1st-place tie between one's first- and second-preferences should clearly be lower for the first voter, since in this case their third-preference is the clear favorite to win. As well, consider a voter with expectations $p_1 = 60\%, p_2 = 30\%, p_3 = 10\%$. By the absolute difference measure $p_{jk} = 1 - |p_j - p_k|$ the value of $p_{12} = .7 < p_{23} = .8$, which violates Property 3: the probability of a 1st place tie must be higher for the two leading candidates than for the two trailing candidates.

In their piece on turnout and abstention in French elections, Rosenthal and Sen (1973) present a *conditional difference* measure which is consistent with Properties 1 and 3, but which does not rectify the above problem regarding Property 2. Their measure conditions the difference in probability of winning between candidates j and k on their shared closeness to the expected plurality winner, denoted by $\hat{p}$. In particular, their function for the probability of a 1st place tie between *j* and *k:*

$$p_{jk} = \left(1 - |p_j - p_k|\right).\left(1 - \hat{p} + \max\,[p_j, p_k]\right). \qquad \textbf{(1)}$$

Note that when either j or k is the candidate with the highest probability of winning, equation (1) reduces to $p_{jk} = 1 - |p_j - p_k|$, the same difference measures as above, which thus suffers from similar problems regarding Property 2. For example, compare two voters one with $p_1 = 52.5\%, p_2 = 47.5\%, p_3 = 0\%$ and a second with $p_1 = 40\%, p_2 = 35\%, p_3 = 25\%$. The conditional measure would assign these voters identical scores of $p_{12} = .95$, but by Property 2 a 1st-place tie between one's first- and second-preferences should be higher for the first voter, since in this case the race is an entirely two-way race between those two candidates.

Black (1978) measures the closeness of a race between candidates *j* and k as the Euclidean distance between a voter's *actual* position and the position which would make these two candidates equal, holding the third candidate's vote share constant.[12] More particularly, in a 3-candidate race with candidates *j*, *k*, and *l*, define a voter's expectation vector as $\{p_j, p_k, p_l\}$, and consider the Euclidean distance between this vector and a second vector $\{\hat{p}_j, \hat{p}_k, p_l\}$, where the latter represents the vector in which j and k would be tied for the 1st place, holding p_l constant. For example, for the starting vector $\{.5, .3, .2\}$, the associated vector would be $\{.4, .4, .2\}$. The Euclidian distance between these two vectors is:

[12] The equations therein contain a minor error, which was corrected by Herrmann and Pappi (2008).

$$p_{jk} = [(p_j - \hat{p}_j)^2 + (p_k - \hat{p}_k)^2 + (p_l - p_l)^2]^{\frac{1}{2}}. \quad \textbf{(2)}$$

This represents a creative effort at incorporating the third candidate's position into the measure, and thus is promising vis-à-vis Properties 2 and 3. However, let $p_j > p_k$. Then, since $\hat{p}_j = \hat{p}_k = \frac{p_j + p_k}{2}$, equation (2) can be simplified to $p_{jk} = \frac{p_j - p_k}{\sqrt{2}}$. Put otherwise, in the end this formula can be reduced to a difference-based measure, which in turn does not satisfy Properties 2 and 3 (demonstration omitted for redundancy).

Adopting a very different approach, Ordeshook and Zeng (1997) propose a *multiplicative* measure $p_{jk} = p_j p_k$ which solves the above problems associated with Properties 2 and 3, but in exchange introduces new problems with regards to Property 1. To see this, compare a voter with expectations $p_1 = 60\%, p_2 = 30\%, p_3 = 10\%$ to a voter with expectations $p_1 = 40\%, p_2 = 30\%, p_3 = 30\%$. By the multiplicative measure p_{12} will be higher for the first voter than for the second voter: $(.6 \times .3) > (.4 \times .3)$. In fact, by Property 1 it is the reverse that should obtain: the first voter perceives the front-runner to be much further ahead (30% ahead vs. 10% ahead), while the second voter perceives a much closer 1st-place race between her first- and second-preferences.

2c. An Axiomatically Sound Measure of Closeness

We now present a distinct proxy measure which satisfies all three properties, and which has the benefits of simplicity and transparency. It is grounded in *probability ratios* of the form $p_{jk} = \frac{p_j}{p_k}$ where $p_j < p_k$ such that $p_{jk} \leq 1$. It is straight-forward to show that probability ratios satisfy Properties 1 and 2 (theoretical proofs in Supporting Information Part I). Regarding Property 1, consider one voter with expectations $p_1 = 60\%, p_2 =$

$30\%, p_3 = 10\%$ and a second voter with $p_1 = 40\%, p_2 = 30\%, p_3 = 30\%$. In contrast to the multiplicative measure, p_{12} will clearly be higher for the second voter than for the first voter: $\frac{.3}{.4} > \frac{.3}{.6}$. These ratio measures also satisfy Property 2. Again compare two voters, the first of whom believes $p_1 = 52.5\%, p_2 = 47.5\%, p_3 = 0\%$ and the second of whom believes $p_1 = 40\%, p_2 = 35\%, p_3 = 25\%$. Although the margin separating the voter's first- and second-preference is 5% for both voters, p_{12} will be higher for the second voter, since $\frac{.475}{.525} > \frac{.35}{.4}$.

Given this discussion, it is tempting to use these simple ratios to measure the closeness of a race between any two candidates. However, it is also straightforward to see that simple ratios may, under certain circumstances, violate Property 3. For example, consider a voter with $p_1 = 24\%, p_2 = 26\%, p_3 = 50\%$. If we use simple ratios to measure p_{jk} then $p_{12} = \frac{.24}{.26} > p_{23} = \frac{.26}{.5}$, violating Property 3. To avoid this violation, we need to develop a slightly modified approach to measuring the closeness of the race between the election's *two trailing candidates*.

Suppose that among candidates *j*, *k*, and *l*, candidate *l* is the expected plurality winner. Consider the following measurement strategy:

$$\left\{ p_{jl} = \frac{p_j}{p_l}, p_{kl} = \frac{p_k}{p_l}, p_{jk} = \frac{p_j p_k}{p_l^{\,2}} \right\}.^{13} \quad \textbf{(3)}$$

[13] For example, the specific values for a respondent with expectations $p_3 > p_2 > p_1$ would be $\left\{ p_{23} = \frac{p_2}{p_3}, p_{13} = \frac{p_1}{p_3}, p_{12} = \frac{p_1 p_2}{p_3^{\,2}} \right\}$.

By conditioning the closeness score for the two trailing candidates on the size of the plurality winner's lead, we ensure that (3) satisfies Property 3. The following Proposition, proven in Part I of the Supporting Information file, confirms the viability of the ratio measurement strategy codified in (3):

*** Proposition 1:** Suppose that among candidates *j*, *k*, and *l*, candidate *l* is the expected plurality winner. Then $\left\{ p_{jl} = \frac{p_j}{p_l}, p_{kl} = \frac{p_k}{p_l}, p_{jk} = \frac{p_j p_k}{p_l^2} \right\}$ satisfies Properties 1-3.

Proposition 1 applies to voters who express *strict expectation rankings*. For voters who believe that one candidate is in the lead and that the other two candidates have an equal probability of winning, the following measures satisfy Properties 1-3:

$$\left\{ p_{jl} = p_{kl} = \frac{p_j = p_k}{p_l}, p_{jk} = \frac{p_j p_k}{p_l^2} \right\}$$

For voters who believe that two front-runner candidates have equal chances of winning, the following measures satisfy Properties 1-3:

$$\left\{ p_{jl} = p_{jk} = \frac{p_j}{p_k = p_l}, p_{kl} = 1 \right\}.$$

3. Closeness and Strategic Voting

Section 2 developed a new strategy for measuring p_{jk}, the probability of creating or breaking a tie between one's candidates j^{th} and k^{th} preferences in 3-candiate contests. We now demonstrate that these closeness parameters are crucial for the study of *strategic voting*. At its foundation, the logic of strategic voting is grounded in expected utility maximization: I choose a candidate who I prefer less, but who has a better chance of winning, to avoid 'wasting' my vote. The *Calculus of Voting* (COV) is an expected utility model of voting behavior applicable to winner-take-all elections, and was first proposed by Downs (1957) and formalized by Riker and Ordeshook (1968) and McKelvey and Ordeshook (1972). While, the COV was originally applied to the question of turnout, it can also be used to derive predictions as to the conditions under which voters will cast strategic votes for their second-preference in 3-candidate contests.

According to the COV a voter will cast a strategic vote *if and only if* the *expected utility* of choosing their second-preference is higher than that of choosing their first-preference. Define U_j as a voter's utility for having their j^{th} preference win the election, such that by construction $U_1 \geq U_2 \geq U_3$. Furthermore, define E_j as a voter's expected utility for choosing candidate j. From past formal research (Black 1978; Hoffman 1982; Fisher 2004; Kselman and Niou 2010) we reproduce the following condition for strategically choosing their second-preference:

$$E_2 > E_1 \ \textit{if and only if}$$

$$p_{23} \cdot (U_2 - U_3) > 2p_{12} \cdot (U_1 - U_2) + p_{13} \cdot (U_1 - U_3) \,. \tag{4}$$[14]

When the left-hand side is greater than the right-hand side, the voter's optimal choice should be to choose their second-most-preferred candidate rather than their most-preferred candidate. To simplify the result for empirical analysis, we now implement a standard expected utility normalization (Von Neumann and Morgenstern 1953). Define $\mathbf{U_1}$, $\mathbf{U_2}$, and

[14] Inequality (5) employs the assumption that *adjacent probabilities in comparable outcome spaces are equal* (see also McKelvey & Ordeshook 1972, p. 51). This assumption is purely expository, and in no way affects result's implications or generality.

$\mathbf{U}_3$ as the standardized value of U_1, U_2, and U_3 respectively. In turn, let $\mathbf{U_1} = 1$, $\mathbf{U_3} = 0$, and $\mathbf{U}_2 = \frac{U_2 - U_3}{U_1 - U_3} \equiv \sigma$. We can then rearrange (4) above as follows:

$$E_2 > E_1 \text{ if and only if}$$

$$\sigma p_{23} - 2(1-\sigma)p_{12} - p_{13} > 0 \; . \qquad \textbf{(5)}$$

Henceforth, we will refer to a voter's first-preference as '**1**', her second-preference as '**2**', and her third-preference as '**3**'. A voter will cast a strategic vote for **2** if the left-hand side of (5) is greater than 0. The likelihood of strategic voting should thus decrease in p_{13}, i.e. choosing **2** becomes less likely when **1** and **3** are in a close race for first place. As well, the likelihood of strategic voting should increase in p_{23} and decrease in p_{12}, i.e. choosing **2** becomes more (less) likely when **2** and **3** (**1** and **2**) are in a close race for first place. However, and importantly, *these effects should be dependent on the size of* σ. For large σ, at which voters are fairly indifferent between **1** and **2**, increases in p_{23} should have a strong positive effect on the likelihood of choosing **2**, since their biggest concern is with keeping **3** out of office. In contrast, for lower values of σ where voters are more indifferent between **2** and **3**, the positive effect of p_{23} should be less pronounced. As for p_{12}, at larger values of σ it should have little negative effect on the likelihood of choosing **2**, since voters are fairly indifferent between **1** and **2**. The negative effect should become more pronounced at smaller values of σ, when **1** is highly preferred to **2**.

3a. Measurement and Operationalization

While some research has studied strategic voting using aggregate-level data (Cain 1978; Galbraith and Rae 1989; Johnston and Pattie 1991; Cox 1997; Bawn 1999),[15] the most promising tests of an individual-level theory like the COV are grounded in individual-level analyses.[16] In particular, to test the above inequality we need survey data from a 3-candidate plurality-rule election which allows us to operationalize: a.) a respondent's utility for a candidate (U_j); b.) the 'closeness' of a contest between two candidates (p_{jk}); and c.) the respondent's vote choice.[17] For reasons which will become clear below, the most promising data set for testing our model is the 2010 *British Election Study* (BES), although

[15] Aggregate level evidence is suggestive but suffers from problems of ecological inference (Freedman et al. 1991; Achen and Shivley 1995; King 1997).

[16] A distinct set of papers uses respondents' *self-reported motivations* (Heath et al. 1991; Niemi et al. 1992; Franklin et. al 1994; Fisher 2004; Blais et al. 2005; Artabe and Gardeazabal 2014), identifying as strategic anyone who reports having been motivated by considerations of 'viability' and/or the desire to avoid 'wasted-votes'. Alvarez and Nagler (2000) suggest that the higher overall rates of strategic behavior often uncovered by such studies may result from response bias in post-election surveys.

[17] We use *expected* vote choices from pre-election surveys rather than *reported* vote choices in post-election surveys. The results are qualitatively identical with data on reported vote choice in post-election surveys (available upon request).

the 1988 *Canadian National Election Study* (CNES) also meets the core requirements, and serves to validate the robustness of statistical results.[18]

We use *Feeling Thermometers*, which ask survey respondents to report their general 'attraction' to a party or candidate on a 1-100 scale, to measure a voter's utility for a particular party (Cain 1978; Black 1978; Abramson et all 1992; Herrmann and Pappi 2008).[19] We then create a dummy variable V_2 which assumes the value '0' if the respondent chooses her first preference and '1' if she chooses her second preference.[20] The core inputs into measures of the closeness parameters p_{12}, p_{13} and p_{23} are the individual probabilities p_1, p_2, and p_3, candidates **1**, **2**, and **3**'s respective probabilities of winning the contest. Unlike other studies in the BES series, the 2010 survey contains a *subjective measure of*

[18] Part II of the Supporting Information file reproduces all of the specific survey items and data sources used in this study.

[19] Alvarez and Nagler (2000) and Alvarez et al. (2006) estimate a Multinomial Probit (MP) model whose right-hand side includes demographic, programmatic, and regional inputs to raw voter preferences. Blais and Nadeau (1996), Blais et al. (2001) and Blais et al. (2005) employ a composite feeling thermometer which simultaneously incorporates preferences for candidates, party leaders, and parties.

[20] On other hand if she chooses her third preference she is eliminated from the analysis. In the British data, only 15 out 6,830 respondents included in our analysis choose their least-preferred candidate. Discarding these respondents is a necessary operational step: the '0' category would be polluted were it to contain not only the choice for **1** but also the tiny subset of respondents which chooses **3**.

voter expectations: respondents were asked to assign parties a likelihood of winning on a 1-10 scale.[21] We will use the probability-ratio strategy presented in equation (3) to convert raw p_j scores into closeness parameters p_{jk}. After presenting analyses in which p_{jk} is constructed from survey respondents' subjectively reported probabilities of winning p_j, we confirm the robustness of our results when p_{jk} is constructed using district-level vote shares from the previous electoral cycle, an *objective* proxy for p_j (section 4a).

Canadian elections provide a second example of plurality-rule elections with more than two parties. Indeed, since 1993 the Canadian party-system has been even more fragmented than the British system, as the effective number of parties in Canada has vacillated between four and five. This began with the rise of *Reform Party* and the *Bloc Quebecois* in 1993, and was continued with the rise of the *Green Party* since 2004. As discussed in the Conclusion, both the COV and our axiomatic approach to measuring closeness can be extended to party systems with more than three parties. However, these extensions are not trivial, and will require careful proofs which are beyond the scope of the current paper. While the 1993-2015 elections in Canada will thus be useful for future work, for the present paper we focus on the 1988 Canadian National Election Study (CNES), which was a genuinely 3-party contest, and in which respondents were asked to assign parties a probability of winning in their local riding. The sample size is much smaller, but analysis of the 1988 CNES data represents a second useful robustness check.

4. Statistical analysis of Strategic Voting

[21] Following Abramson et. all (1992), we normalize the p_j scores to generate the indices $\mathbf{p_1} = \frac{p_1}{p_1+p_2+p_3}$, $\mathbf{p_2} = \frac{p_2}{p_1+p_2+p_3}$, and $\mathbf{p_3} = \frac{p_3}{p_1+p_2+p_3}$.

Direct tests of strategic voting should be grounded in the reduced form inequality (5) above. Past empirical studies overlook the fact that the hypotheses regarding p_{12} and p_{23} which emerge from inequality (5) are *interactive* with the parameter σ. Take a voter who is fairly indifferent between candidates **2** and **3**, i.e. for whom σ is small. By the result in (5), increases in p_{23} should have little impact on this voter's likelihood of casting a strategic vote. On the other hand, if a voter greatly prefers candidate **2** to candidate **3** (i.e. σ is large), then increases in p_{23} should have a strong impact on this voter's likelihood of casting a strategic vote. We thus implement the following *logistic* regression, whose dependent variable is the binary variable 'Vote for **2**':[22]

$$Prob\ (V_2 = 1) = \begin{cases} \beta_0 + \beta_1 \cdot p_{13} + \beta_2 \cdot p_{23} + \beta_3 \cdot p_{12} + \beta_4 \cdot \sigma \\ \quad + \ \beta_5 (p_{23} \cdot \sigma) + \beta_6 (p_{12} \cdot \sigma) + \varepsilon \end{cases} \qquad \textbf{(6)}$$

Per inequality (5), we would expect p_{13} (coefficient β_1) to have a negative effect on the likelihood of strategic voting. As well, we would expect p_{23} and σ (coefficients β_2 and β_4) to have a positive effect on the likelihood of strategic voting, while p_{12} (coefficient β_3) to have a negative effect. Given that the regression is *logistic*, we have no *a priori* expectation

[22] The difference between (5) and (6) comes from the inclusion of individual regressors p_{12}, p_{23}, and σ on the right-hand side, alongside to the multiplicative terms $p_{23} \cdot \sigma$ and $p_{12} \cdot \sigma$. As with any theoretical model whose hypotheses imply interaction effects, the regression model must include these individual regressors in order to generate point estimates for marginal effects at different levels of the conditioning variable.

as to the sign of the interactive coefficients β_5 and β_6. Our hypotheses pertain rather to the interactive effects of expectations and preferences described above.

Part III of the Supporting Information presents summary statistics and correlations for all variables. In the tables below we label the p_{jk} indicators *Closeness J-K*, and the σ indicator as *Utility Differential 2-3*. Table 1 presents baseline empirical estimates of voter choice as a function only of their utility for the respective candidates.

Table 1: Baseline Model (Preferences Only)

	Model 1 UK Full	Model 2 UK Reduced	Model 3 Canada Full
Utility Differential 2-3	6.266*** (0.338)	5.598*** (0.415)	2.920*** (0.487)
Constant	-6.494*** (0.239)	-4.741*** (0.282)	-3.866*** (0.291)
Observations	6,815	1,580	1,418
Predicted%	93.5%	79.1%	91.3%
Log Likelihood	-1406.46	-685.54	-399.23

Note: *p<0.1; **p<0.05; ***p<0.01

As already noted, the paper's primary evidence will come from analysis of the 2010 BES data, whose results are presented in column 1. Although the data set is much smaller, column 3 conducts identical analyses on the 1988 CNES (ignore column 2 for now). Not surprisingly, *Utility Differential 2-3* (σ) has a positive and statistically significant effect: voters are more likely to cast a strategic vote for **2** when they greatly prefer **2** to **3**. The following table introduces the closeness parameters from the COV as specified in (6).

Table 2: Fully-Specified Analysis

	Model 1 UK Full	Model 2 UK Reduce	Model 3 Canada Full
Closeness 1-3	-3.341***	-2.452***	-2.875***
	(0.282)	(0.642)	(0.467)
Closeness 2-3	4.763***	2.708***	2.795***
	(0.672)	(0.875)	(0.910)
Closeness 1-2	-2.180**	-2.915**	-0.448
	(.0926)	(1.248)	(0.919)
Utility Differential 2-3	6.409***	5.860***	2.630***
	(0.565)	(0.850)	(0.911)
Closeness 2-3 X Differential	-2.357**	-2.138*	0.239
	(0.988)	(1.284)	(1.553)
Closeness 1-2 X Differential	2.459*	2.843*	0.381
	(1.264)	(1.718)	(1.514)
Constant	-6.547***	-4.865***	-3.532***
	(0.398)	(0.584)	(0.531)
Observations	6,815	1,580	1,418
Predicted%	93.7%	81.3%	91.5%
Log Likelihood	-1178.113	-638.61	-363.28

Note: *p<0.1; **p<0.05; ***p<0.01

Column 1 contains the core analysis from the 2010 BES. Not surprisingly, the percentage of correct predictions rises only marginally from Table 1 to Table 2, given the fact that strategic voting is a rare event, i.e. that the overwhelming choice of all voters is to choose their first preference **1**. That said, the log likelihood of the model improves significantly upon including the closeness parameters, and likelihood ratio tests demonstrate the superior 'fit' of the regression analysis in Table 2 as compared to Table 1.[23]

Beyond this increasing goodness of fit, the expectation parameters are largely consistent with theoretical expectations.[24] Firstly, the 'Closeness 1-3' measure (row 1) exerts a significant negative effect on the likelihood of casting a strategic vote: when **1** and **3** are in a close race for first place, respondents are much more likely to choose **1** than **2**. This is true for both the British (column 1) and Canadian (column 3) data; Figure 1 presents the substantive size of the respective effects.

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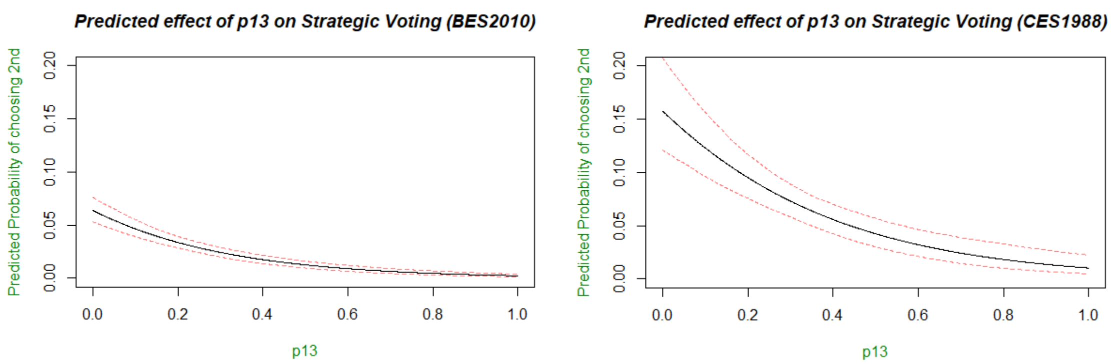
[23] The likelihood ratio statistics comparing the models in Table 2 to those in Table 1 are equal to 456.6985, 93.8361, and 89.16258 for Columns 1, 2, and 3 respectively. While the model fit improves the most for column 1 models, which have by far the most observations, the improvement is statistically significant at $p<.001$ in all three models.

[24] One potential problem with the British data emerges in Scotland, where in some constituencies the *Scottish National Party* (SNP) represents a genuine 4th option. We have re-run all analyses dropping respondents from the Scottish constituencies (Supporting Information Part V, Table A11). The results are identical.

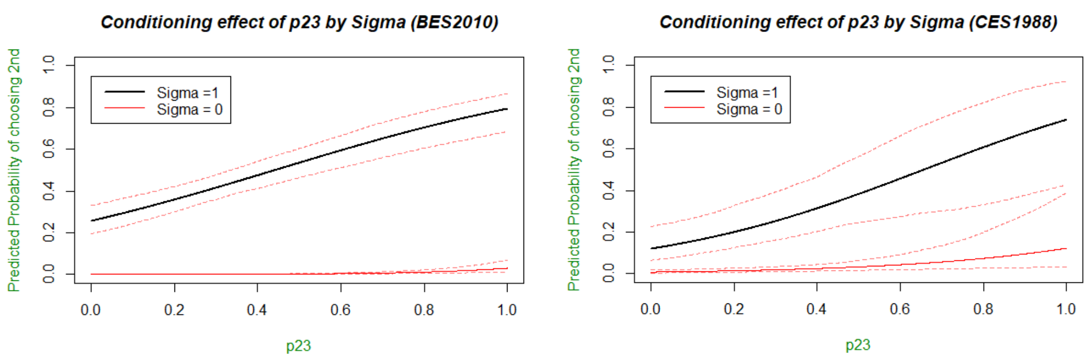
Figure 1: Closeness of the Race between 1 and 3



In the British data, holding all other variables at their means, moving from a value of '0' to a value of '1' on p_{13} leads to a drop of roughly 6.4% in the likelihood of choosing **2**. Furthermore, respondents who have a value of '1' on this variable essentially *never* vote strategically. As can be seen in the second plot from Figure 1, this effect is more or less replicated in the Canadian sample. The difference is that in the Canadian sample the effect is stronger: moving from a value of '0' to a value of '1' on p_{13} leads to a drop of roughly 15.7% in the likelihood of choosing **2**. The findings point to a systematic negative effect of p_{13} on the likelihood of strategic voting in 3-candidate plurality rule elections, which holds across distinct electoral contexts, and which is exactly what one would expect based on an expected-utility model of voter choice.

Rows 2 and 4 from Table 2 report coefficient estimates β_2 and β_4, associated with the individual regressors p_{23} and σ. In both columns 1 and 3 the coefficient values β_2 and β_4 are positive and significant, which is once again consistent with theoretical expectations. However, as already noted, these parameters' impact should be interactive, as captured by the coefficient β_5 (row 5). To derive a more complete picture of the effect, Figure 2 plots the *marginal effect* of p_{23} on the likelihood of choosing **2** at the highest ('1') and lowest ('0') possible values of σ.

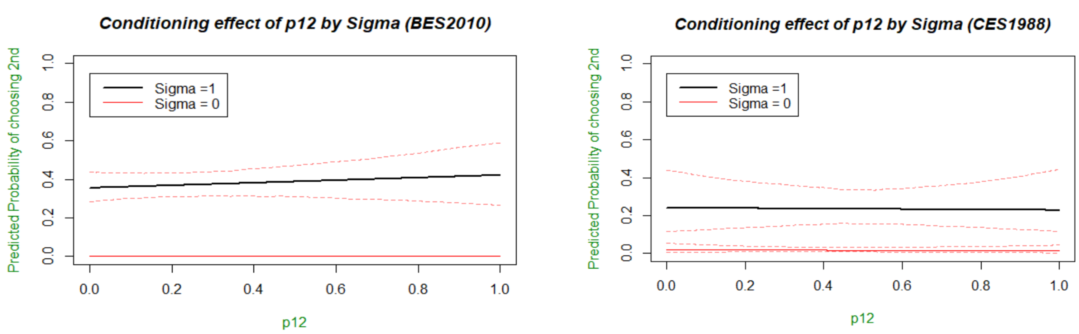
Figure 2: Closeness of the Race between 2 and 3



The results are consistent with predictions. When $\sigma = 0$, such that voters are essentially indifferent between candidates **2** and **3**, increasing p_{23} has almost no effect on the likelihood of strategic voting, which is close to '0' across the board. On the other hand when $\sigma = 1$, and voters are indifferent between candidates **1** and **2**, increases in p_{23} have a strong and consistent effect. For example, moving from $p_{23} = 0$ to $p_{23} = 1$ leads to an increase of 53% in the likelihood of choosing **2** in the British data. In the Canadian data, the confidence intervals are not as tight, which explains the lack of statistical significance of β_5 in column 3. However, the qualitative findings are identical: when $\sigma = 0$ increasing p_{23} has no effect on the likelihood of choosing **2**; but when $\sigma = 1$ moving from $p_{23} = 0$ to $p_{23} = 1$ leads to an increase of 62% in the likelihood of choosing **2**.

Coefficients β_3 and β_6 are associated with the variables p_{12} and the interaction $p_{12} \cdot \sigma$ respectively (rows 3 and 6 in Table 2). Consistent with inequality (5)'s predictions, Black (1978) and Ordeshook and Zeng (1997) find that as p_{12} increases the likelihood of strategic voting decreases. In contrast, Abramson et al. (1992) find that increases in p_{12} *increase* the likelihood of strategic voting.[25] Rows 3 and 6 from Table 2 help to shed light on this issue. Beginning again with column 1, we see that the coefficients β_3 and β_6 are statistically significant; but that the effects are much weaker than those associated with the coefficients β_1, β_2, β_4, and β_5. The effect in column 3 is even weaker: in the 1988 CNES analyses neither β_3 and β_6 are statistically significant. The marginal effect plots confirm that changes in p_{12} have essentially '0' effect on the likelihood of choosing **2**.

Figure 3: Closeness of the Race between 1 and 2



[25] Abramson et al. (1992) use a measure of 'distance' rather than 'closeness', and their dependent variable is 'Vote for **1**'. The estimated coefficient thus has a positive sign: as the margin between **1** and **2** increases, the likelihood of choosing **1** also increases.

This non-finding suggests the presence of a *heuristic* short-cut by which voters simplify the calculus captured in (5): namely, by focusing only on viability comparisons which include their least-preferred candidate **3**. A large literature exists on the cognitive short-cuts voters use to deal with informational constraints and complex choice environments (Popkin 1991; Lupia and McCubins1998; Kuklinsky and Quirk 2001; Lau and Redlawsk 2001).[26] One general message emerging from this literature is that heuristics allow voters to behave *as if* rationally, without actually having to engage in expected-utility calculations. The message here is slightly different: discounting p_{12} is done not to *avoid entirely*, but rather to *simplify*, the task of making expected-utility calculations.[27]

4a. Robustness Checks

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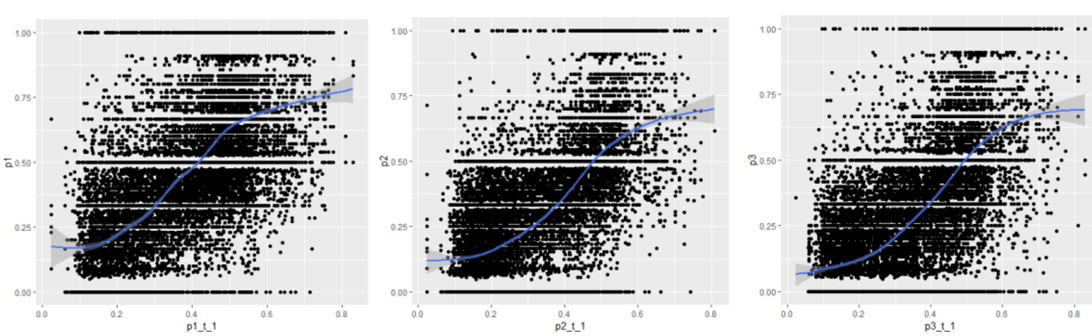
[26] Related to the current paper, Lago (2008) argues that supporters of far-left parties in Spain will cast strategic votes when this most-preferred party has no current district seats (and is thus seen as 'non-viable'). Van der Straeten et al. (2010) argue that voters can avoid complex expected-utility calculations in multi-party environments by identifying a restricted subset of viable candidates; and then choosing their favorite candidate from among this subset.

[27] To deal with possible multi-collinearity and feature space complications, we reran all analyses such that they included only one of the two interaction terms (results available upon request). The findings were identical: the interaction between σ and p_{23} is statistically and substantively significant, while the parameter p_{12} and its interaction with σ have essentially no effect on the likelihood of choosing **2.**

Overall, Figures 1-4 uncover striking parallels between the predictors of strategic voting in two very different electoral contexts. In most ways, the choice to cast a strategic vote for **2** is strongly consistent with the COV's expectations: voters choose **2** over **1** when **2** has a better chance of defeating **3**, i.e. when p_{13} is low and p_{23} is high; and the latter effect is enhanced when **2** is greatly-preferred to **3**. The one result which is not in line with the COV, pertaining to the non-effect of p_{12}, is also consistent across the British and Canadian analyses, and suggests the presence of a common short-cut that voters use to simplify the calculus derived in (5). Taken together, these results represent some of the most convincing, but also nuanced, evidence to date in favor of expected-utility maximization as a model of voter choice. We now conduct a series of robustness checks.

First, we rerun our core model using the previous election's outcome rather than self-reported expectations to measure the raw p_j scores. The following Figures plot the bivariate scatter plots of the self-reported expectations (labeled p_j) on outcomes from the previous election (labeled $p_j\_t\_1$), along with the associated best fit lines.

Figure 4: Self-Reported Expectations and Previous Election Outcomes (BES 2010)



The bivariate correlations between p_j and p_j*_t_1* are *nearly identical* for $j \in \{1,2,3\}$ (.641, .647, and .648 respectively), as is the S-shape of the best-fit line. In general, candidates whose party received 0-15% vote share in the previous election are given low probabilities of winning. Estimated probabilities of winning then begin to increase more or less linearly with previous vote shares up to the 65% mark, where the 'S' once again flattens. We return to these findings, and associated avenues for future research, in the Conclusion. Table 3 replicates regression model (6) on the BES 2010 data set, using the p_j*_t_1* measures to create the Closeness indicators p_{jk}. Using past election returns leads to less missing data and thus a larger sample.

Table 3: Fully-Specified Analysis – t-1 Expectations

	Model 1 UK Full	Model 2 UK Reduced
Closeness 1-3	-1.811***	-1.457***
	(0.092)	(0.201)
Closeness 2-3	1.217***	0.782*
	(0.276)	(0.474)
Closeness 1-2	-0.601**	-0.698
	(0.289)	(0.542)
Utility Differential 2-3	1.348***	1.531**
	(0.286)	(0.603)
Closeness 2-3 X Differential	1.313***	0.707
	(0.482)	(0.818)
Closeness 1-2 X Differential 2-3	0.315	0.914
	(0.507)	(0.938)
Constant	-0.952***	-4.865***
	(0.169)	(0.584)
Observations	10,298	2,971
Log Likelihood	-6249.97	-1904.45

Note: *p<0.1; **p<0.05; ***p<0.01

The estimates in column 1 (ignore for the moment column 2) strongly reinforce the findings above. The p_{13} measure exerts a statistically significant negative effect on the likelihood of casting a strategic vote; the substantive effect is in fact a bit larger than that in Table 2. As well, when σ is large (small) increases in p_{23} have a strong (weak) impact on the likelihood of choosing **2**. Finally, the variable p_{12} and its interaction with σ once again have very little impact on the likelihood of strategic voting. These results add confidence that the above findings were not driven by our particular choice of raw data.

As a second robustness check, recall that we have not yet discussed the results from column 2 of Tables 2 and 3. These results were generated by estimating (6) on the reduced subset of *potentially strategic voters.* Consider a voter who believes her most-preferred candidate **1** will place 1st in the election, and her second and third preferences **2** and **3** will place 2nd and 3rd in the election respectively. By definition, *this voter should never cast a strategic vote*, since **1** is a perfectly viable candidate and **3** is expected to place last. More generally, Kselman and Niou (2010) prove that strategic voting is possible *only if* $p_2 > p_1$, i.e. only if **2** has a better chance of winning than **1**.[28] Roughly 26% (115/444) of respondents in the British data choose **2** despite the fact that they do not meet this necessary condition.[29] These voters present an interesting empirical and theoretical puzzle: why would they choose **2** when **1** is a more viable candidate? One distinct possibility is *protest*

[28] This result formalizes an argument originally made by Blais and Nadeau (1996).

[29] Franklin et al. (1994) and Herrmann et al. (2016) also document the presence of voters who cast "…strategic votes in the 'wrong' direction – away from otherwise viable candidates." (Herrmann et al. 2016; p. 583)

voting: using one's vote to cast a signal of disaffection with one's most-preferred party (Bowler and Lanoue 1992; Kang 2004; Kselman and Niou 2011), a topic we return to in more detail below.

For the moment, note simply that empirical analyses conducted on the entire sample of respondents may thus generate coefficients of 'strategic' behavior which are impacted by other types of tactical decisions. Column 2 from Table 2 demonstrates that our findings are largely robust to running the analysis only on the pool of potential strategic voters. Although the coefficients' substantive size varies slightly, the qualitative implications captured in Figures 1-2 are reproduced almost entirely.[30] The one slight difference between the results in columns 1 and 2 of Table 2 concerns the effect of p_{12}. Recall that, according to (5), strategic voting should become *less likely* when **1** and **2** are in a close race (especially when **1** is greatly preferred to **2**). To the extent that p_{12} has any effect at all in column 1, it is thus in the *wrong direction* (see Figure 3 above): when σ is sufficiently large voters become slightly *more likely* to choose **2** when the race between **1** and **2** becomes closer (see also Abramson et al. 1992).

Things change slightly when we 'purge' the data of potential protest voters. Most basically, in column 2 of Table 2, the effect of p_{12} continues to be much weaker than those associated with p_{13} and p_{23}, and thus the core finding above on heuristics and the discounting of p_{12} remains unchanged. However, to the extent that p_{12} has any effect at all

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[30] Column 2 from Table 3 conducts the restricted analysis on data using past election returns to measure raw expectations. The findings are identical: the coefficients on p_{13} and p_{23} are significant and in the right direction; while the results on p_{12} are weak and unstable.

in column 2, it now moves in the expected direction: voters become less likely to choose **2** as p_{12} increases. The Conclusion suggests that the mixed-findings on p_{12}, both in the current paper and the broader literature, likely emerge from the presence of protest voting behavior, and points to future empirical research on electoral signaling which could help to complete our understanding of instrumental voter choice.

As a final robustness check, Part IV of the Supporting Information file (Tables A5-A10) compares the performance of our probability ratios with the *multiplicative* and *difference* indices from past research.[31] Note that the multiplicative measure, which to our knowledge has been used in only one study (Ordeshook and Zeng 1997), is highly correlated with our probability ratios.[32] Indeed, when we replicate our analysis with the multiplicative measure we uncover essentially identical qualitative results on all coefficients (Table A5). As discussed in the Supporting Information, Ordeshook and Zeng's measure only violates one subset of cases associated with Property 1, and is the most axiomatically sound of the measures used in past research. The parallels between the results thus provide further evidence that this paper's findings are not an artefact of any single functional form for p_{jk}, but rather of its axiomatic properties.

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[31] As demonstrated above, the *compound difference* and *Euclidean* indices are essentially more nuanced versions of the pure difference measure.

[32] The bivariate correlation for the p_{12} measures is r=.837, for the p_{13} measures is r=.850, and for the p_{23} measures is r=.869. The bivariate correlations for the corresponding Canadian data are r=.798, r=.830, and r=.884 respectively.

The same cannot be said of the difference index, the most commonly applied index in past studies of strategic voting. The first thing to note about this index is that all three p_{jk} measures cannot be included in a single regression, since $p_{23} = p_{13} - p_{12}$. To deal with this, Tables A6-A8 run separate models including each possible pair of closeness measures, before including the 'Utility Differential' and interaction terms in Tables A9-A10 (see also Abramson et al. 1992). The exercise leads to a number of conclusions, all discussed at greater length in Supporting Information Part IV. Firstly, the models using difference measures register significantly worse log-likelihood ratios in all cases. Indeed, adding the difference proxies leads to only a marginal improvement in model fit over a regression using only 'Utility Differential'. Secondly, the models tend to generate coefficients which move in the *wrong direction* in full sample analyses, but which switch to the right direction when run on the restricted sample of potential strategic voters. Finally, in the restricted sample analyses the effect of p_{12} tends to outweigh the effects of p_{23} and p_{13}. Indeed, it may be that the emphasis put in the competitiveness between candidates **1** and **2** in past analyses emerges from the use of difference-based measures. While in no way an outright condemnation of past studies, this exercise suggests that much past evidence of strategic voting may need to be revisited and reinterpreted.

5. Concluding Discussion

This paper's contributions are both methodological and substantive. At the theoretical level, we develop an axiomatic approach to measuring the closeness of a race between two candidates in a three-candidate election. Methodologically, we propose a new proxy measure for closeness, and demonstrate that this new measurement satisfies core axiomatic criteria. Substantively, we use this index to demonstrate some of the most consistent but also nuanced evidence to date in favor of expected-utility maximization as a model of strategic voting. The choice to cast a strategic vote is driven by viability comparisons involving one's *least-preferred candidate*, and not by comparing one's top two preferences. Furthermore, the impact of expectations on vote choice is contingent on a voter's preference profile, and in particular the size of their relative preference for **1** over **2** and **2** over **3**. Finally, we demonstrate that past estimates of strategic behavior may have been influenced by the presence of *protest voting*, and propose a strategy for purging estimates of this potential source of bias.

In this Conclusion, we comment on the broader applicability of our methodological contribution, and point to new avenues for research which emerge from the paper's substantive results. Note first that the paper's strategy for measuring closeness can be extended to winner-take-all elections with $N>3$ candidates. Consider for argument's sake a 4-candidate contest, in which voters may not only choose their second-most-preferred candidate, but also their *third-most-preferred* candidate. This would occur when this 3rd preference is in a close race for first place with the voter's least-preferred candidate or party, and when this third preference is greatly preferred to this least-preferred candidate. No paper has yet derived the full conditions for strategic voting in 4-candidate elections, i.e. the equivalent to (5) above, which will be necessary to guide empirical analyses. In this core prediction the parameters p_{12}, p_{13}, and p_{23} will be joined by p_{14}, p_{24}, and p_{34}, and the importance of Properties 1 and 2 above will persist: the parameters should increase when the 1st-place contest between two candidates tightens, and increase when the candidates' relative standing *vis-á-vis* the remaining two candidates improves. As noted above, the set of Canadian elections from 1993-2015 will provide a fertile ground for testing the model, and thus for determining whether expected-utility continues to be a viable model of voter choice in more complex environments.

Furthermore, an emerging literature suggests that voters may also engage in expected-utility calculations in Proportional Representation systems, where coalition government is the norm (Baron and Diermeier 2001; Indriadson 2011; Kedar 2011; Herrmann 2014). As emphasized by Herrmann (2014), voters in such systems may choose their second-most preferred party so as to secure a more favorable coalition government. Herrmann's expected-utility comparisons generate predictions grounded in the probability of being pivotal for distinct coalition outcomes, not unlike the pivotal probabilities which emerge from the traditional COV. The current paper's axiomatic framework, and the ratio measures derived in Section 2b, will be important for measuring these pivotal coalition probabilities when there are more than two possible coalition outcomes.

The paper's methodology will also be important for future empirical work on protest voting and electoral signaling. Recall from above that a non-negligible number of voters choose their second-preference **2** despite the fact their first-preference **1** is perfectly viable. Table 2 suggests that, as long as σ is large enough, this subset of voters become more likely to choose **2** as p_{12} increases. While this goes against the traditional COV's predictions, it is consistent with the extension to the COV developed in Kselman and Niou (2011): increasing p_{12} will increase the expected benefit of protest voting (especially when voters are fairly indifferent between **1** and **2**), as it will make the loss of a vote especially costly for one's first-preference, and thus be more likely to induce this first preference to improve their platform and/or performance. In future work we look forward to studying this subset of voters in more detail, with the aim of painting a more complete picture of instrumental voter choice in plurality-rule elections.

Note that issues of strategic voting and protest voting are not confined to voting in general elections. Although evidence from the US Congress suggests that sophisticated voting is rare (Krehbiel and Rivers 1990; Poole and Rosenthal 1997; Groseclose and Milyo 2010), it seems to be present in some notable votes (e.g. Enelow 1981; Calvert and Fenno 1994). As well increasing evidence suggests the presence of instrumental and tactical voting behavior in shareholder elections (Yermack 2010; Harris 2011). Consider the following quotation from Yermack's excellent review of the literature:

"Shareholder voting provides an effective means for shareholders to communicate with the board of directors, and boards usually take action in response to clear *protest voting*." (p. 121, italics added)

To the extent that voting behavior in legislatures and shareholder elections is tactical, and grounded on expectations over outcomes, this paper's methodology will be important in future empirical studies.

We now return to the introductory discussion of proxy vs. explicit measures of closeness. Comparative analysis of proxy and explicit measures constitutes an important avenue for future research. On the technical front, there is room for a broad investigation of the axiomatic properties of the distinct probability distributions authors could use to derive explicit measures (multinomial, dirichlet, etc.). Firstly, which distributions, or class of distributions, satisfy Properties 1-3 above? More ambitiously, do the distinct possible distribution functions embody distinct axiomatic assumptions about how voters calculate closeness? If the answers is yes, such work might help move beyond the minimalist criteria established in this paper.

Beyond this theoretical research, empirical work should also investigate the comparative performance of explicit as opposed to proxy measures in regression analyses. Based on the results in Section 4 and Supporting Information Part IV, our suspicion is that explicit measures which satisfy Properties 1-3 will also generate similar substantive findings. Of course, science does not advance on the basis of suspicion, and the question remains: when faced with multiple ways of measuring p_{jk}, all of which satisfy Properties 1-3, how much does the choice of a specific functional form affect regression results? Answering this question in future work will provide valuable new insight into the strategies and heuristics voters use in making vote choices, and should thus further enrich the axiomatic approach developed here.

As a final point we return to the issue of measuring p_j, the simple probability that candidate *j* wins the election. Figure 4 from Section 4a seems to uncover a fairly systematic process by which voters translate district-level vote share expectations into probabilities of winning. Parties with previous vote shares in the range of roughly 0-15% receive low probabilities of winning; and voters differentiate very little between parties who received 1% and those who received 15%. Above the threshold of 15%, a party's probability of winning increases more or less linearly with their previous vote share up until it reaches 65%, at which point the 'S' once again flattens. While Figure 4 itself is only suggestive of these patterns, the authors have generated some preliminary experimental results which point in the same direction.

Although further research is necessary, Figure 4 hints at a fairly simple, closed-form algorithm by which the average voter translates vote shares in the probabilities of winning p_j. If confirmed, future research can use this algorithm to extract more accurate measures of subjective p_j from objective vote shares. As well, for those studies which develop explicit measures of p_{jk}, future work could assess whether the values p_j which emerge from different density functions look similarly 'S'-shaped, and prioritize those density functions that satisfy this potentially systematic facet of voter choice. As a general conclusion, this paper's overarching framework points in a number of novel directions regarding future empirical research on instrumental voter choice.

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