

Analysis of Video Poker

Dr. Edward Gordon

*Risk Analysis Consultant
272 Tempus Circle
Arroyo Grande, CA 93420*

Abstract

Several analyses of video poker reported expected payouts in the vicinity of 99.6 percent. The analysis techniques they used were critically reviewed to validate those favorable results for Jacks or Better video poker. Improvements in playing strategy were discovered during the validation. The performance is now 99.7 percent expected payout. Wong (1988) called attention to an error in the technique earlier analysts used. However, it was found that this error amounted to only 0.1 percent in the overall expected payout.

Introduction

The popularity of video poker has been growing rapidly since it was introduced in the late 1970s. Today, many casinos have more video poker machines than all other types of slot machines combined. In part this is due to the miniaturization of computers. By 1970, it was feasible to use a microprocessor to generate a more attractive display than the traditional mechanical slot machine. Also contributing to the growth of video poker is its high level of consumer acceptance. Given the popularity of video poker, the purpose of this article is to critically review some of the mathematical analyses of video poker that have been conducted and to identify improved play strategies.

Video Poker Microprocessor Functions

These functions start with receiving an input from the person playing the machine via the DEAL push button provided. Alternatively, the input comes from receipt of a signal that the maximum number of coins expected were inserted. It causes the microprocessor to select five cards out of the poker deck simulated and to display the rank and suit of those five cards. The player decides which of the five cards to hold. The microprocessor displays the cards held as well as the cards it selected as replacements for the discards. If the resulting hand justifies a payout, the microprocessor initiates dispensing of the payout as coins or it adds to a credit balance for the player. The details of how these functions are performed are important to the machine manufacturer, machine owner, and to the gaming commissions. They are of little or no consequence to the video poker player so long as

Acknowledgment

A draft of this paper was presented at the 9th International Conference on Gambling & Risk-Taking on May 31, 1994, at the MGM Grand Hotel Casino & Theme Park in Las Vegas, Nevada. I gratefully acknowledge helpful comments from Dr. Wesley S. Roehl of UNLV, and from two anonymous reviewers on an earlier revised draft of this paper.

there is a reasonable assurance the computer program has not been modified since manufacture of the video poker machine.

Payouts for Jacks or Better

The most popular version of video poker is Jacks or Better. Other popular versions of video poker using a standard poker deck are Tens or Better and Deuces Wild. Joker Wild and Deuces Joker Wild have a joker added to the deck. For each of these versions, there is an optimal playing strategy dependent upon the payout schedule. The critical part of that strategy is a procedure for analyzing the five-card combination dealt to the player to determine which cards in that hand to hold. What Frome (1990) calls the "Full-Pay Schedule" for Jacks or Better can be expressed as multipliers for the amount bet as follow in Table 1.

Table 1. Full-Pay Schedule for Jacks or Better

Index	Hand	Payout multiplier
0	royal flush	800
1	straight flush	50
2	4-of-a-kind	25
3	full house	9
4	flush	6
5	straight	4
6	3-of-a-kind	3
7	two pair	2
8	high pair (jacks, queens, kings, or aces)	1

The 800 times the amount bet for a royal flush applies when the player has inserted the maximum number of coins the machine will accept. When a smaller number of coins is inserted, that payout is 250 times the amount bet.

Number of As Dealt Hands

For the Jacks or Better game, the first card dealt to the player can be any one of the 52 cards in a poker deck. Because the second card dealt must be different from the first card dealt, it can be any one of 51 cards. The third card dealt can be any one of 50 cards, the fourth any one of 49, and the fifth any one of 48. Thus, there are $52 \times 51 \times 50 \times 49 \times 48 = 311,875,200$ distinct permutations of five cards as dealt. For the payout schedule in Table 1, the order in which the cards are dealt has no effect on the outcome of the game. Any particular combination of five cards can have any one of the five cards first, any one of four second, any one of three third, any one of two fourth and only one fifth. Any combination of five cards can appear in $5 \times 4 \times 3 \times 2 \times 1 = 120$ permutations. The number of distinct combinations of five card as dealt hands is $311,875,200 / 120 = 2,598,960$. For those decks that

contain a joker, the first card can be any one of 53. The number of distinct five-card combinations changes to $53 \cdot 52 \cdot 51 \cdot 50 \cdot 49 / (5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) = 2,869,685$.

Playable Hands

Because of the large number of five-card hands, it is cost-effective to make the card hold decisions utilizing the concept of playable hands. Each playable hand is a group of five or fewer cards from an as dealt hand which has a reasonable chance of yielding a payout. Analyses of the Jacks or Better game by Gerhardt and Korfman (1987, p. 17), Frome (1990, p. 36), D. Crevelt and L. Crevelt (1991, p. 53), and Paymar (1994, p. 48) reported overall expected payouts close to 99.6 percent. Those analysts have shown that the most common playable hand for Jacks or Better video poker is a pair with rank of ten or less, a low pair. A low pair should be selected as the playable hand in one fourth of all five-card hands. A low pair leads to a payout if the hand is improved to at least two pair or three-of-a-kind by the cards drawn, and its expected payout is 0.82 times the amount bet. Next in order of selection frequency is the high pair. It has an expected payout of 1.54 times the amount bet.

Payouts for Playable Hands

Calculation of Expected Payout

The procedure recommended by the author for analysis of video poker starts with selection of a set of playable hands by the analyst. Table 2 illustrates calculation of the expected payout for the playable hand described as KQJXi. The K designates a king, the Q a queen, the J a jack, and the X a 10 so that each card in the playable hand calls for one character. A lower case "i" following the combination of characters representing ranks of cards in a playable hand indicates all of the cards are in the same suit. Thus a suitable draw can yield a flush.

Table 2. Expected Payout Calculation for KQJXi

Card(s) drawn	Number	Result	Payout	EPC
Ai	1	royal flush	800	17.021271
9i	1	straight flush	50	1.063830
2-8i	7	flush	6	0.893617
Ao,9o	6	straight	4	0.510638
Ko,Qo,Jo	9	high pair	1	0.191489
others	23	no payout	0	0.000000
			Expected payout	19.680845

Note: EPC = Expected payout contribution

The number of draws that yield each payout is shown in the second column of Table 2. If the card is an ace in the same suit, that draw is designated as A_i . There is only one such card available, so the entry in the second column is 1. The resulting hand is the royal flush, $AKQJX_i$. Similarly, 9_i as the draw calls for 1 in the second column and the result is the straight flush, $KQJX9_i$. Mathematically, the i th expected payout contribution is the probability of occurrence of the i th payout for the j th playable hand times the value of the i th payout. The probability of occurrence for the i th payout for the j th playable hand is assigned the symbol $p(i,j)$ and the i th payout value assigned the symbol $v(I)$. Then, the contribution is $p(i,j) * v(I)$. The expected payout for the j th playable hand is the sum of these products as illustrated in Table 2 and written as follows:

$$EPNL(j) = \sum p(i,j) * v(I). \quad (1)$$

This summation includes all values of the index "I" from 0 to 8. The payout outcomes with zero occurrence probability are not shown in Table 2.

Selection of Playing Strategy

Other "4 from Royal Flush" Expected Payouts

The basis for Table 2 is the playable hand $KQJX_i$, which is a member of the group "4 from royal flush" in Table 3. Other members of the group are $AKQJ_i$, $AKQX_i$, $AKJX_i$, and $AQJX_i$. The expected payout calculation for the playable hand containing $AKQX_i$ is quite similar to that for $KQJX_i$ in Table 2. The most important difference is that $AKQX_i$ can not yield the straight flush, $KQJX9_i$, as $KQJX_i$ did. The corresponding entry in Table 2 (1.063830) is not present for $AKQX_i$. It is replaced by the contribution from one more ordinary flush, 0.127660. For $AKQX_i$, only a jack can complete the straight in contrast to either ace or 9 for $KQJX_i$. Thus, half of the 0.510638 for straights in Table 2 is lost. The $AKQX_i$ expected payout is $19.680845 - 1.063930 - 0.255319 + 0.127660 = 18.489256$. Playable hands $AKJX_i$ and $AQJX_i$ have the same expected payout as $AKQX_i$, even though the specific card drawn to complete a payout may be different. Playable hand $AKQJ_i$ differs in expected payout from $AKQX_i$ because $AKQJ_i$ has four high ranks, so it can produce three more high pairs for an increase of 0.063830—up to 18.553086 expected payout. Although these different payout values are treated separately in the analysis, they are close enough in expected payout to be treated as a single group for playing strategy purposes.

If two or more playable hands are present in a five-card hand as dealt, the player should choose the playable hand present with the largest expected payout.

Table 3. Optimum Playing Strategy for Jacks or Better Video poker

Discards	Playable hand groups in decreasing payout order
0	royal flush, straight flush, 4-of-a-kind, full house;
1	4 from royal flush - 4 in one suit, 10 or higher rank;
0	ordinary flush - 5 in one suit, at least 1 gap;
2	3-of-a-kind - 3 cards with same rank;
0	straight - 5 adjacent ranks;
1	4 from straight flush - 4 in one suit at most 1 gap;
1	two pair;
3	high pair - jacks, queens, kings, or aces;
2	3 from royal flush - 3 in one suit, 10 or higher rank;
1	4 from ordinary flush - 4 in one suit, more than 1 gap;
3	low pair - pair of rank 10 or lower;
1	4 from straight - 4 adjacent ranks, no ace, no gap;
2	3 from straight flush - 3 in one suit; no gap, or 1 gap and at least one jack or better; or 2 gaps and at least 2 jacks or better;
3	2 from royal flush - 2 in one suit and both jack or better;
1	4 from straight containing ace, king, queen, and jack;
2	3 from straight flush - 2 gaps & 1 jack or better or 1 gap;
1	4 from straight - 1 gap and 3 jacks or better;
2	3 high cards - king, queen, jack in different suits;
3	2 from royal flush - 10 plus jack or better;
4	1 or 2 high cards - keep only lower rank;
2	3 from straight flush - 2 gaps, no high cards;
5	5 unrelated low cards - discard all five cards.

Gaps in Potential Straights

The term "gap" used in Table 3 can be illustrated in terms of the "4 from straight flush" group. That group contains four consecutive ranks, such as QJX9i to 5432i with no gap, as well as corresponding playable hands with one gap, such as KQJ9i down to 532Ai. There are two cards in the suit that can be drawn to complete the straight flush if the hand has no gap. With one gap, there is only one way to complete the straight flush. Note that 432Ai can complete a straight flush only by drawing 5i.

Choose Playable Hand with Largest Expected Payout

If two or more playable hands are present in a five-card hand as dealt, the player should choose the playable hand present with the largest expected payout. The last entry in Table 3 is for those hands which are so unfavorable in terms of expected payout that the player is better off discarding all five cards. The technique used to determine an average expected payout for those hands is described in Gordon (1996).

Effect of Discards

Expected Payout Loss Due to Discards

The symbol *EPNL* is used in Equation (1) as a reminder that calculation is done on a no loss basis. The results in Table 4 cover the consequences when the discarded fifth card causes a loss of one of the favorable outcomes in Table 2. With 7 cards available to complete the flush, the probability that one of them is the fifth card is 7/48. (The denominator for the occurrence probability in Table 4 is 48 because there are 52 - 4 = 48 cards available from which the fifth card is selected.) Similarly, there are three other aces and three other nines for a total of six ways to produce a straight. There are three other cards in each of the king, queen, and jack ranks which can yield a high pair, for a total of nine ways. The average loss of 0.033 from a value close to 20 is not much over 0.1 percent, which is negligible for most practical purposes.

Table 4. Payout Losses for KQJXi

Phl	Payout	IPL	OP	CAL
flush	6	6/47=.127660	7/48	0.018617
straight	4	4/47=.085106	6/48	0.010638
high pair	1	1/47=.021277	9/48	0.003989
				Average loss = 0.033245

Note: Phl = Playable hand lost
 IPL = Individual payout loss
 OP = Occurrence probability
 CAL= Contribution to average loss

Wong's Approach

Wong (1988) used a different approach in his analyses of Progressive Jackpot games. In those games, the jackpot (payout for a royal flush) increases as additional games are played. Consequently, the expected payout for those playable hands which can produce a royal flush, such as "3 from royal flush," will be increasing as the jackpot increases. These hands will pass the expected payout for a high pair because high pair is above "3 from royal flush" in Table 3. Cross-overs will occur at jackpots greater than the royal flush payout in Table 1. A series of calculations varying the royal flush payout will quickly determine the cross-over payout in a five-card hand such as QJXi plus a queen or jack.

Wong's VPEXACT program was used to develop playing strategies for fixed payout games. Paymar (1994, pp. 46—48) describes how he applied this approach. From this description, it is apparent that hundreds of computer runs, some of which took as long as two hours, were required to generate an optimal playing strategy for a specified payout schedule. In contrast, the computer program used in this

study determines an optimal playing strategy and its overall expected payout in less than one minute on a 486-40 IBM compatible personal computer (PC).

Overall Expected Payout

Calculation of Overall Expected Payout

Determination of an overall expected payout starts with the calculations using Equation 1. Those no loss calculations are performed for each of the playable hands specified by the analyst. The individual playable hand no loss expected payouts, $EPNL(j)$, are combined into an overall expected no loss payout using

$$OEPNL = (\sum EPNL(j) * freq(j)) / 2598960 \quad (2)$$

where $freq(j)$ is the frequency of selection for the j th playable hand. Frome (1992) suggested determining the desired frequencies, $freq(j)$, by processing each as-dealt five-card hand to determine which playable hand has the highest expected payout from that five-card hand and tallying the selection frequencies. The next step in the calculations is an analysis of those playable hands which have a potential for creating a payout loss due to a discard. As illustrated in Table 4, two of those hands contain 4 cards from a royal flush plus one additional card, so that there is also a straight or flush present. The first result desired for these hands is their individual payout loss, IPL , which is shown in Table 4.

Can Discard Affect Choice?

Using the index "T" for a playable hand with the potential discard, the quantity $EPNL(j) - IPL(I)$ is compared with $EPNL(I)$. When the latter expression is less, as it is in every case in Table 4, the discard should occur. When the latter expression is at least as great, the discard should not occur. One example of the "greater" situation is when the "j" refers to a lone ace and the "I" refers to AX_i , which will be covered shortly. If the five-card hand considered contains a pair, the quantity $EPNL(j) - IPL(p)$ is compared with $EPNL(p)$ for the pair to be broken. If $EPNL(p)$ is less, the pair should be broken. Then the loss $IPL(p)$ occurs because one of the possible high-pair outcomes is eliminated. If $EPNL(p)$ is at least as large, the pair is retained and there is no payout loss.

Ace Plus Ten in Same Suit, AX_i

For the payouts in Table 1, ace alone has a no-loss expected payout of 0.471987, which is slightly higher than the expected payout of 0.460561 for the two card combination AX_i . Discard of the ten makes a royal flush outcome impossible and reduces the probability that the straight AKQJX or a flush will occur. After the ten is discarded, a straight can result from drawing any one of four kings, any of four queens, any of four jacks, and any of the three remaining tens. Thus, the straight occurs in $4*4*4*3 = 192$ draw combinations. There are 11 more of that

suit available if no other card in AXi suit is discarded. To complete the flush, there are $11 \cdot 10 \cdot 9 \cdot 8 = 7,920$ draw permutations. Each combination appears in $4 \cdot 3 \cdot 2 \cdot 1 = 24$ permutations, so the flush has $7,920/24 = 330$ draw combinations. If the ten had not been discarded, there would be $12 \cdot 11 \cdot 10 \cdot 9/24 = 495$ combinations, so 165 draw combinations for producing a flush were lost due to this discard. Similarly, there are $47 \cdot 46 \cdot 45 \cdot 44/24 = 178,365$ draw combinations for the ace alone. Thus, the expected payout loss due to discard of the ten is $(1 \cdot 800 + 165 \cdot 6 + 192 \cdot 4)/178365 = 0.014341$. This loss (*IPL*) is greater than the difference between the expected payout for ace alone versus AXi, which is 0.011426. Thus, the player is better off retaining the AXi. This is the only case for the payouts in Table 1 where loss due to a discard is sufficient enough to cause a playable hand with a lower *EPNL* to be retained.

1 vs 4—One High Card Playable Hands

The individual expected payouts are 0.489883 for jack alone, 0.483918 for queen alone, and 0.477953 for king alone. All of these are higher than the ace alone. All of the previously published analyses of the Jacks or Better game recommend discard of the ten from AXi. They all combined the jack, queen, king, and ace high card playable hands into a composite one high card playable hand.

Any average of these four individual expected payouts is higher than for the ace alone. With a higher expected payout, the discard loss for a ten is not sufficient to justify retaining the ten. If the sum $IPL(I) + EPNL(I)$ for AXi is used instead of $EPNL(I)$ in the sort to put the playable hands in decreasing expected payout order, the sort will put AXi above A alone and adjust the sort for the effect of this discard. This ranking is used in the frequency analysis as well as the basis for the playing strategy in Table 3.

A slightly higher value is to be expected because the use of jack alone, queen alone, king alone, and ace alone instead of one high card as a playable hand does create a significant improvement in the playing strategy.

Correction for Discard Losses

Once the frequency values $freq(I)$ are determined for every playable hand, then Equation (2) can be used to determine *OEPNL*. For the payout values in Table 1, *OEPNL* is 99.87. Next, the expected loss due to discards,

$$CORR = \sum freq(I) \cdot IPL(I) \quad (3)$$

for those potential losses which do occur. As noted above, the loss does not occur for the playable hand AXi because the ten should not be discarded. A loss occurs for the three cases covered in Table 4 and the *IPL(I)* values are in the table. The corresponding frequency values are calculated as follows for the playable hands in Table 4.

When the fifth card comes from the same suit as the “4 from royal flush” in Table 4, this fifth card can have any rank from 2 to 8. There are four possible suits times the seven possible ranks for 28 five-card hands with this loss. For the other four playable hands in “4 from royal flush,” the fifth card can have any rank from 2 to 9 so there are $4 \times 4 \times 8 = 128$ more such hands for a total of 156, the desired value of $fri(I)$. When the fifth card in Table 4 creates a straight, it can be any ace or 9 from another suit, generating six possibilities. That six is multiplied by the four possible suits for a product equal to 24. For the other four “4 from a royal flush,” only one rank can complete the straight in $4 \times 4 \times 3 = 48$ ways. The number of occurrences of this loss is $24 + 48 = 72$ ways.

When the fifth card creates a high pair, there are three high cards from other suits available for each of the three high ranks in Table 4. When multiplied by the four possible flush suits, the result is 36. The same result applies for AKQXi, AKJXi, and AQJXi, whereas AKQJi yields 48 for a total of 192. The group “4 from straight flush” also has a greater expected payout than a high pair so any high pairs present in that group are broken. There are 36 high pairs lost for KQJ9i. For those hands with two high cards, the loss is 24 each for KQX9i, KJX9i, QJX9i, QJX8i, and QJ98i adding 156 cases. The straight flushes with one high card lose 12 high pairs each for QX98i, JX98i, JX97i, JX87i, J987i, A234i, A235i, A245i, and A345i adding 108 cases. The total is now 456 high pairs lost.

If the expected payout for the hand with no discard exceeds the expected payout for the hand after the discard by more than its individual payout loss, the discard and the loss will occur. Similar calculations are performed for each of the other playable hands containing a discard that generates a loss, $IPL(I)$.

Using the Correction

The products $fri(I) * IPL(I)$ where the loss occurs are summed and the total divided by 2,598,960 to yield the desired value for $CORR$, which was 0.12 for the payouts in Table 1. The desired overall expected payout,

$$OEP = OEPNL - CORR. \quad (4)$$

The result, 99.75, is slightly higher than the overall expected payouts reported by earlier studies. A slightly higher value is to be expected because the use of jack alone, queen alone, king alone, and ace alone instead of one high card as a playable hand does create a significant improvement in the playing strategy. That validates the favorable overall expected values reported by other studies. The system published by Cohen (1980) was not included in the above analysis because his book did not contain any playable hand frequencies.

Other Considerations

Comparison of Frequency Results

The frequency results for playable hands reported by Frome (1990) and Paymar (1994) were critically compared with the new results. The largest differ-

ences in playable hand frequencies are due to splitting one high card into four distinct playable hands in this study. The group containing two high cards from differing suits is reduced to 251,196 from 386,148 for Frome and 391,116 for Paymar. The group containing one high card is increased to 500,436 from 402,528 for Frome and 406,704 for Paymar. The group containing "2 from royal flush" with one of them high is increased to 67,116 from 30,072 for both Frome and Paymar. Paymar had a value of 5,664 for straights containing AKQJ versus 3,072, with the increase coming from "2 from royal flush." Paymar's straight flush with one high and two gaps had 8,892 versus 18,036. The expected payouts were in agreement, so Paymar's errors had no effect on playing strategy.

Weber and Scruggs Analysis

Weber and Scruggs (1990) used a technique somewhat similar to Wong's. However, they randomly generated the five-card hands to be analyzed in contrast to Wong's analysis of each hand in sequence. Instead of 2,598,960 hands to be processed, Weber and Scruggs used 500,000,000. Unfortunately, the random generation of hands introduced significant errors in the results, which the larger number of hands processed can not eliminate. They used the 250 payout for a royal flush rather than the 800, which Frome and Paymar used, so the overall expected payout is reduced to 98.56 percent, not the 99.24 percent which they reported.

Summary and Conclusions

Validation of Favorable Results

The new results generated by this analysis of video poker have validated the payouts in the neighborhood of 99.6 percent previously reported. A refinement in this analysis is due to use of four distinct high card playable hands, jack, queen, king, and ace, rather than one composite high card playable hand. That introduced a small but significant gain, raising the expected payout to 99.75 percent.

Several errors were found in playable hand frequency results reported by Paymar. They were not large enough to affect the overall expected payout significantly. The individual playable hand expected payouts were generally in good agreement with results published by Cohen (1980). Cohen did not publish any hand frequency results so his value of 95 percent for overall payout apparently was an estimate based on judgment rather than analysis.

Errors due to Discard Losses

The errors due to ignoring expected payout losses due to discards from playable hands which were of concern to Wong are generally negligible. Completely ignoring these errors, as Gerhardt and Korfman (1987) did, introduced an error of 0.1 percent in their overall expected payout. It did not affect the playing strategy. It is obvious an error this small has no practical significance.

On the other hand, proper consideration of the consequences of these losses due to discards substantially increased the effort required to develop the computer

program used in this analyses. The computer program developed by Wong (1991) requires more than a thousand times as much computer time as the program used for this study, (days versus less than one minute) to develop a playing strategy and determine its overall expected payout.

No Improvement in Accuracy

The technique used by Weber and Scruggs (1990) had the additional disadvantage that the random variations present introduced significant uncertainties in all of their results. In addition, their use of 500,000,000 hands in the analysis rather than the 2,980,960 Wong used calls for a substantial further increase in computer time. The net effect is that their goal of improved analytical accuracy was not attained.

References

- Cohen, R. Carl. (1980). *Beating the poker slot machines*. Philadelphia, PA: R. Carl Cohen.
- Crevelt, D. & Crevelt, L. (1991). *Video poker mania*. Grand Rapids: MI Gollehon Books.
- Frome, L. (1990). *Expert video poker for Las Vegas*. Las Vegas, NV: Compu-Flyers.
- Frome, L. (1992). Private conversation.
- Gerhardt, D. & Korfman, T. (1987). *Video poker - Playing to win*, Las Vegas, NV: Gaming Books Incorporated.
- Gordon, E. (1996). *Video poker analyses*, Arroyo Grande, CA: Dr. Edward Gordon.
- Paymar, D. (1994). *Video poker - Precision Play*, (8th Edition), Las Vegas, NV: Enhanceware.
- Weber, G. & Scruggs, W. T. (1990). Mathematical and computer analysis of video poker. Paper presented at *8th International Conference on Gambling and Risk-Taking*, London, England.
- Wong, S. (1988). *Progressive video poker*, La Jolla, CA. Pi Yee Press.
- Wong, S. (1991). *Video poker software*, La Jolla, CA. Pi Yee Press.