8-9-2018

# Circumbinary Discs around Merging Stellar-mass Black Holes

Rebecca G. Martin
*University of Nevada, Las Vegas*, rebecca.martin@unlv.edu

Chris Nixon
*University of Leicester*

Fu-Guo Xie
*University of Leicester; Chinese Academy of Sciences*

Andrew King
*University of Leicester; University of Amsterdam; Leiden University*



## Repository Citation
Martin, R. G., Nixon, C., Xie, F., King, A. (2018). Circumbinary Discs around Merging Stellar-mass Black Holes. *Monthly Notices of the Royal Astronomical Society, 480*(4), 1-6.
http://dx.doi.org/10.1093/mnras/sty2178

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

MNRAS **480,** 4732–4737 (2018) doi:10.1093/mnras/sty2178
Advance Access publication 2018 August 9

# Circumbinary discs around merging stellar-mass black holes

Rebecca G. Martin,[1★] Chris Nixon ,[2] Fu-Guo Xie[3,2] and Andrew King [2,4,5]

[1] *Department of Physics and Astronomy, University of Nevada, Las Vegas, 4505 South Maryland Parkway, Las Vegas, NV 89154, USA*
[2] *Department of Physics and Astronomy, University of Leicester, University Road, Leicester LE1 7RH, UK*
[3] *Key Laboratory for Research in Galaxies and Cosmology, Shanghai Astronomical Observatory, Chinese Academy of Sciences, 80 Nandan Road, Shanghai 200030, China*
[4] *Anton Pannekoek Institute, University of Amsterdam, Science Park 904, NL-1098 XH Amsterdam, the Netherlands*
[5] *Leiden Observatory, Leiden University, Niels Bohrweg 2, NL-2333 CA Leiden, the Netherlands*

Accepted 2018 August 8. Received 2018 August 6; in original form 2018 May 24

## ABSTRACT

A circumbinary disc around a pair of merging stellar-mass black holes may be shocked and heated during the recoil of the merged hole, causing a near-simultaneous electromagnetic counterpart to the gravitational wave event. The shocks occur around the recoil radius, where the disc orbital velocity is equal to the recoil velocity. The amount of mass present near this radius at the time of the merger is critical in determining how much radiation is released. We explore the evolution of a circumbinary disc in two limits. First, we consider an accretion disc that feels no torque from the binary. The disc does not survive until the merger unless there is a dead zone, a region of low turbulence. Even with the dead zone, the surface density in this case may be small. Secondly, we consider a disc that feels a strong binary torque that prevents accretion on to the binary. In this case there is significantly more mass in regions of interest at the time of the merger. A dead zone in this disc increases the mass close to the recoil radius. For typical binary-disc parameters we expect accretion to be significantly slowed by the resonant torque from the binary, and for a dead zone to be present. We conclude that provided significant mass orbits the binary after the formation of the black hole binary and that the radiation produced in recoil shocks can escape the flow efficiently, there is likely to be an observable electromagnetic signal from black hole binary mergers.

**Key words:** accretion, accretion discs – black hole physics – gravitational waves – hydrodynamics – binaries: general.

## 1 INTRODUCTION

The direct detection of gravitational waves (GW) from binary black hole mergers has naturally raised the question of whether there could be a corresponding electromagnetic (EM) signal. Recently de Mink & King (2017) suggested that the gravitational effect of a BH + BH merger on a circumbinary disc might lead to an EM signal delayed by the light-travel time from the merging binary to this disc (see also Rossi et al. 2010, where this effect is applied to mergers of supermassive black holes). Given the lack of simultaneity, identifying this latter signal would require spatial coincidence, and so positional information from several GW detectors. This should be available as new detectors become operational.

At least two detailed binary evolution scenarios, the classical common-envelope and chemically homogeneous pictures, show how a massive binary can remain bound as it evolves to a BH + BH binary (see the discussion in de Mink & King 2017, Section 2, and references therein). In both cases the two stars shed large fractions of their original masses. Unless all of this mass is expelled to infinity, or somehow completely re-accreted by the black holes, a residual circumbinary disc must remain, as the only stable configuration for the shed mass.

In this paper we consider the evolution of this circumbinary disc around a merging black hole binary. Specifically, we are interested in whether there is sufficient material in the disc at the time of the merger for an EM signal to be generated by the perturbation from the gravitational wave recoil and the sudden drop in mass of the binary.

The circularization radius of material falling back after a supernova explosion is very small (Perna et al. 2014; Perna, Lazzati & Giacomazzo 2016). This suggests that the angular momentum of the ejected material is small in the frame of the star. Thus the material is ejected with a velocity equal to that of the mass-losing star plus a velocity radial from the mass-losing star with magnitude $v_{\rm eject}$. For ejection velocities greater than the escape velocity from the binary, $v_{\rm eject} > v_{\rm esc}$, the material is lost. For $v_{\rm eject} \ll v_1$, where $v_1$ is the orbital velocity of the mass-losing star, the material does not

★ E-mail: rebecca.martin@unlv.edu

escape the Roche Lobe of the star. For $v_{\rm eject} \sim v_1$ the material has a spread of angular momentum around the angular momentum of the mass-losing star: the material at the back of the star has less, and the material at the front of the star has more. Thus we expect approximately half of the ejected material to have a velocity such that it ends up in a circumbinary configuration with a spread of angular momenta and thus circularization radii. Whether the material circularizes, or passes within the binary and cancels angular momentum with gas on similar but opposed orbits, to circularize at smaller radii, requires more detailed numerical simulations than we attempt here.

The angular momentum of the material forming the circumbinary disc may not necessarily be initially aligned to the binary angular momentum. For a circular black hole binary, differential precession driven by the binary torque combined with viscous dissipation causes the disc to move towards alignment or counteralignment with the binary depending only on the initial inclination (e.g. King et al. 2005; Nixon, King & Pringle 2011b; Foucart & Lai 2013, 2014). As the disc angular momentum is usually significantly smaller than the binary angular momentum (Nixon 2012), the alignment or counteralignment condition is straightforward: if the disc begins closer to alignment, it aligns, and if it begins closer to counteralignment it counteraligns. How this alignment proceeds depends upon properties of the disc. Papaloizou & Pringle (1983) showed that the propagation of warps occurs in two distinct ways, the dominant mechanism determined by the relative magnitudes of the disc viscosity and pressure. If the Shakura & Sunyaev (1973) viscosity parameter is less than the disc aspect ratio, $\alpha < H/R$, then the warp propagates through waves, otherwise the warp evolves diffusively (see Nixon & King 2016, for a review of warped disc physics). Since the disc aspect ratio of circumbinary discs around black holes is likely to be small, the inner parts of the disc are likely to be aligned (or counteraligned) while the outer parts of the disc may be strongly warped.

The strength of the tidal torque from the binary on the inner parts of the disc depends upon the inclination of the angular momentum of the material, being strongest when it is aligned to the binary angular momentum (e.g. Lubow, Martin & Nixon 2015; Miranda & Lai 2015). For a retrograde circumbinary disc around a circular binary the tidal torque from Lindblad resonances is zero (Nixon et al. 2011a). Nixon & Lubow (2015) showed that resonances do occur in retrograde circumbinary discs around eccentric binaries, but that they are weak enough that they only modulate the accretion flow on to the binary rather than providing a torque of sufficient strength to arrest accretion.

The accretion rate from the inner edge of the circumbinary disc on to the components of a binary depends upon the strength of the tidal torque and properties of the disc. This has received significant attention as it impacts a variety of astrophysical systems (e.g. Artymowicz & Lubow 1994, 1996; Roedig et al. 2012; Shi et al. 2012; D'Orazio, Haiman & MacFadyen 2013; Farris et al. 2014). The gas is subject to two competing torques. The binary transfers angular momentum to the gas, causing the inner disc to move outwards, and viscosity redistributes that angular momentum to larger radii in the disc, allowing the inner disc to shrink. Thus discs subject to a weak torque (or strong viscosity) can move in and accrete on to the binary components. In contrast a strong torque (or weak viscosity) leads to the disc being held out. For the small disc aspect ratio expected in black hole binaries, the accretion rate is significantly suppressed (Ragusa, Lodato & Price 2016). If the tidal torque is sufficiently strong, accretion on to the binary may be completely halted. Disc solutions in this case correspond to *decretion* discs (Pringle 1991) rather than traditional *accretion* discs in which material flows freely through the inner boundary (Pringle 1981).

In Section 2 we first consider how much material is required in a circumbinary disc for an observable EM signal to be generated. In the rest of this work, we then estimate how much material is present in an evolving circumbinary disc. Because of the uncertainties in the strength of the tidal torque and the resulting accretion flow on to the binary, we consider two extreme limits. In Section 3 we consider the disc evolution in the case that the binary provides no torque on the disc and material freely flows inwards and is accreted on to the binary components. The disc is a traditional accretion disc. In Section 4 we examine the case where the binary torque is strong enough to prevent all flow on to the binary. We draw our conclusions in Section 5.

## 2 ELECTROMAGNETIC SIGNAL FROM A CIRCUMBINARY DISC

Gravitational wave emission during a binary black hole coalescence produces a sudden drop in the total central mass. The weaker potential causes circumbinary disc orbits to expand (e.g. Rosotti, Lodato & Price 2012). This excites density waves which dissipate their energy, giving an EM signal. However, as the black holes merge the merged hole suffers a momentum kick due to the asymmetry of the gravitational wave emission. As the hole is kicked, the specific angular momentum of the gas in the circumbinary disc is altered, leading the gas orbits to become eccentric and causing shocks, dissipation and circularization on the local orbital time-scale. This can give a rather stronger EM signal (Rossi et al. 2010).

We consider a black hole binary system of total mass $M$, composed of two black holes of mass $M_{\rm bh} = 30\,{\rm M}_{\odot}$. Heating by shocks occurs on the dynamical time-scale

$$t_{\rm dyn} = \frac{GM}{v_{\rm rec}^3} = 2.2 \left(\frac{M}{60\,{\rm M}_{\odot}}\right) \left(\frac{v_{\rm rec}}{1000\,{\rm km\,s^{-1}}}\right)^{-3} {\rm h}, \tag{1}$$

where $v_{\rm rec}$ is the recoil velocity imparted to the centre of mass. The gas in the disc orbits the central mass at Keplerian angular frequency $\Omega = \sqrt{GM/R^3}$, where $R$ is the radius from the binary centre of mass. The recoil radius is where the Keplerian velocity of the disc is equal to the recoil velocity

$$R_{\rm rec} = \frac{GM}{v_{\rm rec}^2} = 11.5 \left(\frac{M}{60\,{\rm M}_{\odot}}\right) \left(\frac{v_{\rm rec}}{1000\,{\rm km\,s^{-1}}}\right)^{-2} {\rm R}_{\odot} \tag{2}$$

(de Mink & King 2017). The magnitude of the recoil velocity depends sensitively on the pre-merger black hole spin vectors (e.g. Blecha et al. 2016). If the spins are within a few degrees of alignment then the kick may be small, $\lesssim 600\,{\rm km\,s^{-1}}$. However, if the spins are not aligned larger kicks up to several thousand ${\rm km\,s^{-1}}$ can occur (Campanelli et al. 2007; Lousto & Zlochower 2011). Spin alignment is unlikely in X-ray binaries (e.g. Martin, Pringle & Tout 2007, 2009; Martin, Reis & Pringle 2008b; Martin, Tout & Pringle 2008a; King & Nixon 2016) or in SMBH binaries (Lodato & Gerosa 2013), but could perhaps be facilitated in close binaries where the stellar spins have been synchronized with the binary orbit. In this work, we assume that kick velocities of order 500–1000 ${\rm km\,s^{-1}}$ are possible, and that some ejected material settles into a circumbinary disc. We are interested in how much material may still be present at the recoil radius at the time of the merger.

The luminosity generated by the kick is calculated with rate of dissipation of kinetic energy as

$$L = \frac{f M_{\rm rec} v_{\rm rec}^2}{t_{\rm dyn}}, \tag{3}$$

where $M_{\rm rec}$ is an estimate of the mass of the disc within the region that feels the kick strongly and $f$ is a scaling factor. The scaling factor is calibrated to the numerical simulations by Rossi et al. (2010) who found $f \approx 0.1$ for a kick angle $\theta = 15^\circ$. The peak luminosity is somewhat insensitive to the kick angle. For typical parameters the luminosity is

$$L = f\frac{M_{\rm rec}v_{\rm rec}^2}{t_{\rm dyn}} = 1.5\times10^{43}\left(\frac{f}{0.1}\right)\left(\frac{M_{\rm rec}}{0.001\,M}\right)\left(\frac{v_{\rm rec}}{1000\,{\rm km\,s^{-1}}}\right)^5 {\rm erg\,s^{-1}}. \quad (4)$$

There is a radius outside of which all particles in the disc will become unbound after the kick, this is given by

$$R_{\rm ub} = \left(\cos\theta + \sqrt{\cos^2\theta + 1}\right)^2 R_{\rm rec} \quad (5)$$

(Rossi et al. 2010), where $\theta$ is angle between the kick velocity direction and the disc plane. This is maximal if the kick is in the plane of the disc. We approximate this radius with

$$R_{\rm ub} = kR_{\rm rec}. \quad (6)$$

We find the average parameter of $k = 3.64$ by assuming an isotropic distribution for $\theta$ and integrating equation (5) over $0 < \theta < 90^\circ$. This corresponds to $\theta = 46.4^\circ$. We use this in our estimates for the mass $M_{\rm rec}$ in the rest of this work.

A further constraint on the circumbinary disc is that it must survive until the black hole merger takes place. This depends sensitively on the initial separation of the binary at the time when the black hole binary forms. The time-scale for the decay of the orbit of an equal mass binary as a result of gravitational waves is

$$t_{\rm GW} = 1.1\times10^8\left(\frac{a_{\rm b}}{10\,{\rm R}_\odot}\right)^4\left(\frac{M}{60\,{\rm M}_\odot}\right)^{-3}{\rm yr} \quad (7)$$

(Peters 1964), where $a_{\rm b}$ is the separation of the binary. The inspiral may be accelerated by the torque from a prograde circumbinary disc (e.g. Armitage & Natarajan 2005; Lodato et al. 2009) and even more significantly from a retrograde disc (Nixon et al. 2011a,b).

The separation of the binary does not affect the structure of the disc at large radii from the binary. In this work we assume that the disc remains in quasi-steady state as the disc and the binary separation evolve.

## 3 ACCRETION DISC MODEL

We now consider the evolution of a circumbinary disc freely accreting on to the binary components. The total mass of the disc is taken as $M_{\rm d} = 0.001\,M$. This otherwise arbitrary choice is designed to illustrate that a detectable EM signal is possible even from a residual disc of far lower mass than that lost ($\sim M$) in the earlier evolution of the binary (see de Mink & King 2017, section 2). Since $M_{\rm d} \propto M_{\rm rec}$, equation (4) shows the effect of different choices. We take a simple model in which we assume that the disc aspect ratio $H/R$ is constant over the radial extent of the disc.

Angular momentum transport in accretion discs is thought to be driven by turbulence. In many cases, turbulence can be driven by the magnetorotational instability (MRI, Balbus & Hawley 1991). A critical level of ionization in the disc is required for this mechanism to operate: if the disc is sufficiently hot, it may be sufficiently thermally ionized. In parts of the disc with a lower temperature, it may be that the surface layers of the disc are MRI-active because they are ionized by external sources such as cosmic rays (e.g. Glassgold, Najita & Igea 2004). A 'dead zone' forms at the disc midplane where the MRI does not operate (Gammie 1996; Gammie & Menou 1998; Perna et al. 2016). In this section we first consider a disc model in which the MRI operates throughout, and then we consider a disc model with a dead zone.

### 3.1 Fully turbulent disc

If the disc is fully MRI active, the lifetime of the disc is approximated by the viscous time-scale at the circularization radius of the gas. The viscous time-scale is

$$t_\nu = \frac{R^2}{\nu}, \quad (8)$$

where the viscosity is parametrized with

$$\nu = \alpha c_{\rm s} H, \quad (9)$$

with $\alpha$ is the dimensionless Shakura & Sunyaev (1973) viscosity parameter. The scale height of the disc is $H = c_{\rm s}/\Omega$, where $c_{\rm s}$ is the sound speed, and so

$$\nu = \alpha\left(\frac{H}{R}\right)^2 R^2\Omega \quad (10)$$

and for typical values we find the viscous time-scale to be

$$t_\nu = 6.5\times10^5\left(\frac{\alpha}{0.1}\right)^{-1}\left(\frac{H/R}{0.01}\right)^{-2}\left(\frac{M}{60\,{\rm M}_\odot}\right)^{-\frac{1}{2}}\left(\frac{R}{10^4\,{\rm R}_\odot}\right)^{\frac{3}{2}}{\rm yr}. \quad (11)$$

The lifetime of a fully MRI active disc with no binary torque is relatively short, and in this case there may be no circumbinary material left at the time of the merger unless the binary black holes are formed with a separation $a_{\rm b} \lesssim 1\,{\rm R}_\odot$.

### 3.2 Disc with a dead zone

A dead zone forms at the disc mid-plane if two conditions are satisfied. First, the temperature must be lower than the critical required for the MRI to operate, $T_{\rm crit} \approx 800$ K (Umebayashi & Nakano 1988). Above this temperature, the disc is thermally ionized and MRI active. Secondly, the surface density must exceed the critical that can be sufficiently ionized by external sources, $\Sigma_{\rm crit}$. For a central black hole binary, the only source of external ionization is cosmic rays. These may ionize a maximum surface density of $100\,{\rm g\,cm^{-2}}$ (e.g. Gammie 1996; Matsumura, Pudritz & Thommes 2009; Zhu, Hartmann & Gammie 2009). Effects such as recombination may lead to a much smaller active layer (e.g. Wardle 1999; Fleming, Stone & Hawley 2000; Balbus & Terquem 2001; Martin et al. 2012a). In this work we take a fiducial value of $\Sigma_{\rm crit} = 10\,{\rm g\,cm^{-2}}$.

The inner edge of the dead zone is fixed by the radius at which the temperature of the fully turbulent steady state disc model is equal to the critical temperature required for the MRI. The sound speed in the disc is given by

$$c_{\rm s} = \sqrt{\frac{kT}{\mu m_{\rm H}}}, \quad (12)$$

where $k$ is the Boltzmann constant, $T$ is the temperature, $\mu = 2.3$ is the mean molecular weight, and $m_{\rm H}$ is the mass of a hydrogen atom. Thus the disc temperature is

$$T = 3.3\times10^4\left(\frac{H/R}{0.01}\right)^2\left(\frac{M}{60\,{\rm M}_\odot}\right)\left(\frac{R}{10\,{\rm R}_\odot}\right)^{-1}{\rm K}. \quad (13)$$

Setting $T = T_{\rm crit}$ we find the radius of the inner edge of the dead zone as

$$R_{\rm dz} = 417 \left(\frac{H/R}{0.01}\right)^2 \left(\frac{M}{60\,{\rm M}_\odot}\right) \left(\frac{T_{\rm crit}}{800\,{\rm K}}\right)^{-1} {\rm R}_\odot. \quad (14)$$

Note that both the temperature and the inner radius of the dead zone depend sensitively upon the disc aspect ratio.

The surface density of the disc in the dead zone is higher than that of a steady accretion disc since material builds up there over time (e.g. Martin & Lubow 2011, 2013; Martin et al. 2012b). The radius of interest here, $R_{\rm ub}$, is likely too hot to be in a dead zone, unless the disc aspect ratio is smaller than 0.01. The dead zone acts as a bottleneck in the accretion flow. We assume that the viscosity in the dead zone is zero, although in practice there may be a small amount of turbulence there (e.g. Martin & Lubow 2014). The surface density at radii inside the dead zone radius is smaller than that of a disc without a dead zone, and the reduced accretion on to the binary means that the disc survives longer. The bottleneck occurs at the innermost dead zone radius, $R_{\rm dz}$. The accretion rate through the active surface layer is approximately given by

$$\dot{M}_{\rm dz} = 3\pi\nu\Sigma_{\rm crit}. \quad (15)$$

so

$$\dot{M}_{\rm dz} = 7.2 \times 10^{-9} \left(\frac{\alpha}{0.1}\right) \left(\frac{H/R}{0.01}\right)^3 \left(\frac{M}{60\,{\rm M}_\odot}\right) \times \left(\frac{\Sigma_{\rm crit}}{10\,{\rm g\,cm^{-2}}}\right) \left(\frac{T_{\rm crit}}{800\,{\rm K}}\right)^{-1/2} {\rm M}_\odot\,{\rm yr}^{-1}. \quad (16)$$

We estimate the lifetime of the disc as

$$t_{\rm lifetime} = \frac{M_{\rm d}}{\dot{M}_{\rm dz}}, \quad (17)$$

where $M_{\rm d}$ is the total mass of the disc. For our typical parameters this is

$$t_{\rm lifetime} = 8.4 \times 10^6 \left(\frac{M_{\rm d}}{0.001\,M}\right) \left(\frac{\alpha}{0.1}\right)^{-1} \left(\frac{H/R}{0.01}\right)^{-3} \times \left(\frac{\Sigma_{\rm crit}}{10\,{\rm g\,cm^{-2}}}\right)^{-1} \left(\frac{T_{\rm crit}}{800\,{\rm K}}\right)^{1/2} {\rm yr}. \quad (18)$$

Again, this depends very sensitively on the disc aspect ratio. The lifetime of this disc may be sufficiently long for material to remain around until the merger.

Inside the inner dead zone radius, $R_{\rm dz}$, the disc behaves as a steady accretion disc fed with accretion rate $\dot{M}_{\rm dz}$. Away from the inner boundary, the steady state surface density of the disc is approximately $\Sigma = \dot{M}_{\rm dz}/(3\pi\nu)$. So if the recoil radius is smaller than the radius of the inner edge of the dead zone, the surface density at the recoil radius is

$$\Sigma(R_{\rm rec}) = 64.6 \left(\frac{M}{60\,{\rm M}_\odot}\right)^{1/2} \left(\frac{\Sigma_{\rm crit}}{10\,{\rm g\,cm^{-2}}}\right) \left(\frac{T_{\rm crit}}{800\,{\rm K}}\right)^{-1/2} \times \left(\frac{H/R}{0.01}\right) \left(\frac{R_{\rm rec}}{10\,{\rm R}_\odot}\right)^{-1/2} {\rm g\,cm^{-2}}. \quad (19)$$

This surface density is constant in time as long as the dead zone has not been accreted. The mass of the disc up to a radius $R_{\rm ub}$ is

$$M_{\rm d,rec} = 5.6 \times 10^{-7} \left(\frac{\Sigma_{\rm crit}}{10\,{\rm g\,cm^{-2}}}\right) \left(\frac{T_{\rm crit}}{800\,{\rm K}}\right)^{-1/2} \left(\frac{M}{60\,{\rm M}_\odot}\right)^2 \times \left(\frac{H/R}{0.01}\right) \left(\frac{v_{\rm rec}}{1000\,{\rm km\,s^{-1}}}\right)^{-3} \left(\frac{k}{3.64}\right)^{3/2} {\rm M}_\odot. \quad (20)$$

This is quite small, but this is a lower limit since if the inner edge of the dead zone is smaller than $R_{\rm ub}$ the surface density must be higher than this.

In summary, a disc around a black hole that provides no tidal torque on the disc is unlikely to survive until the time of the black hole merger, unless it contains a dead zone. The dead zone significantly extends the lifetime of the disc since material accretes on to the binary at a much lower rate. However, the surface density in the inner parts of the disc that feel the kick is low, and may not produce a detectable signal at the expected source distances. Comparing (20) with (4) suggests a peak luminosity of $\sim 2 \times 10^{38}\,{\rm erg\,s^{-1}}$.

## 4 DECRETION DISC MODEL

We now consider the case of a binary torque strong enough to prevent all accretion on to the binary. Angular momentum is added to the inner parts of the disc. The disc spreads outwards while the surface density decreases in time. We assume that the disc is fully MRI active in this case.

### 4.1 Quasi-steady state disc

The disc evolves as a quasi-steady state decretion disc. The steady state decretion disc solution satisfies

$$\nu\Sigma = \frac{(-\dot{M})}{3\pi}\left[\left(\frac{R_{\rm out}}{R}\right)^{1/2} - 1\right], \quad (21)$$

where $R_{\rm out}$ is the outer disc edge and $-\dot{M}$ is the outward accretion rate through the disc. Since we are interested in the inner parts of the disc, we use the approximation $\nu\Sigma \propto R^{-1/2}$ that is valid for $R \ll R_{\rm out}$. We find the constant of proportionality from the total mass of the disc that remains constant in time. We note that this approximation leads us to underestimate the mass in the inner parts of the disc so our calculated masses can be taken as a lower limit. We find the surface density

$$\Sigma = 3.94 \times 10^4 \left(\frac{M}{60\,{\rm M}_\odot}\right) \left(\frac{M_{\rm d}}{0.001\,M}\right) \left(\frac{R}{10\,{\rm R}_\odot}\right)^{-1} \left(\frac{R_{\rm out}}{10^4\,{\rm R}_\odot}\right)^{-1} {\rm g\,cm^{-2}}. \quad (22)$$

We note that the initial outer radius of the disc is determined by the angular momentum of the material that forms the disc. We show in Section 4.2 that for late times, the initial outer radius does not significantly affect the properties of the disc. The mass within radius $R_{\rm ub}$ is

$$M_{\rm d,rec} = 2.5 \times 10^{-4} \left(\frac{M}{60\,{\rm M}_\odot}\right)^2 \left(\frac{M_{\rm d}}{0.001\,M}\right) \left(\frac{k}{3.64}\right) \times \left(\frac{R_{\rm out}}{10^4\,{\rm R}_\odot}\right)^{-1} \left(\frac{v_{\rm rec}}{1000\,{\rm km\,s^{-1}}}\right)^{-2} {\rm M}_\odot. \quad (23)$$

For our typical parameters, this is more than two orders of magnitude larger than the accretion disc mass given in equation (20). Comparing equation (23) with (4) suggests a peak luminosity of $\sim 10^{41}\,{\rm erg\,s^{-1}}$. However, since the decretion disc spreads outwards in time, we must consider how this mass changes in time.

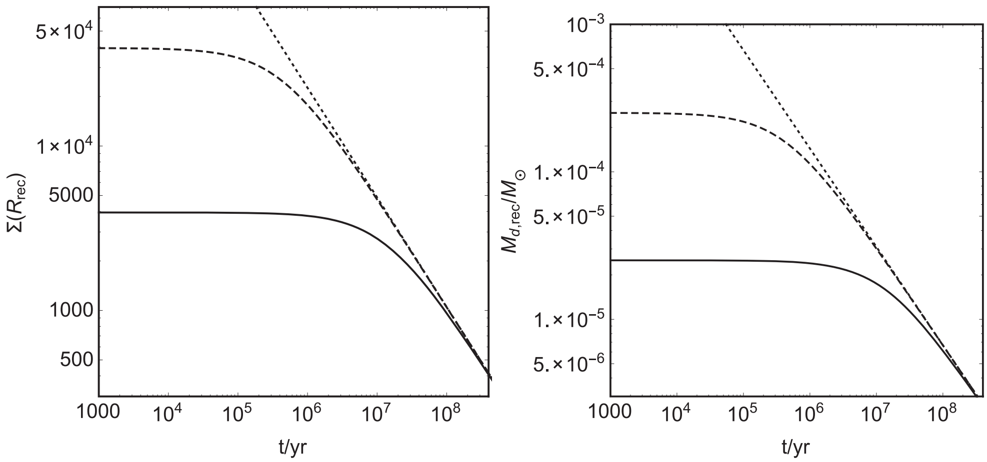


**Figure 1.** The decretion disc. Left: surface density at the recoil radius $R_{\rm rec} = 10\,{\rm R}_\odot$ as a function of time. Right: mass of the disc in $R < R_{\rm ub}$. The initial outer edge of the disc is at $R_0 = 10^5\,{\rm R}_\odot$ (solid line) and $R_0 = 10^4\,{\rm R}_\odot$ (dashed line). The dotted lines show the analytic fits given in equations (28) and (29).

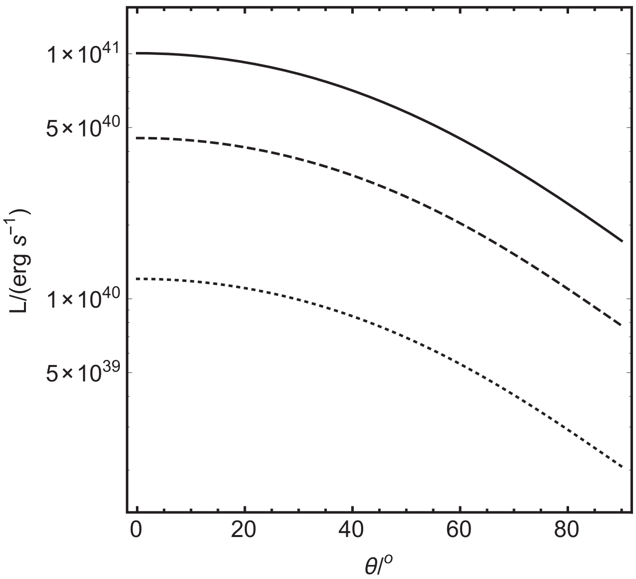


**Figure 2.** The luminosity of the decretion disc. The initial outer edge of the disc is at $R_0 = 10^4\,{\rm R}_\odot$. The times shown are $t = 0$ (solid line), $t = 10^6$ yr (dashed line), and $t = 10^7$ yr (dotted line).

## 4.2 Spreading of the disc

The time-scale for outward spreading of the disc is the viscous time-scale at its outer edge. We model this by assuming that

$$\frac{{\rm d}R_{\rm out}}{{\rm d}t} = \frac{R_{\rm out}}{t_\nu(R_{\rm out})}. \tag{24}$$

We write the viscous time-scale as

$$t_\nu = t_0 \left(\frac{R_{\rm out}}{R_0}\right)^{3/2}, \tag{25}$$

where $R_0$ is the initial outer disc radius and $t_0$ is the initial viscous time-scale at that radius, given by equation (8). Solving equation (24) with equation (11) we find that the outer radius of the disc evolves as

$$R_{\rm out} = R_0 \left(\frac{3}{2}\frac{t}{t_0} + 1\right)^{2/3}. \tag{26}$$

For an initial disc outer radius of $R_0 = 10^4\,{\rm R}_\odot$, the viscous time-scale there is $t_0 = 6.5 \times 10^5$ yr and for $R_0 = 10^5\,{\rm R}_\odot$, the viscous time-scale there is $t_0 = 2 \times 10^7$ yr. The left-hand panel of Fig. 1 shows the surface density of the disc at the recoil radius as a function of time for initial outer disc radii $R_{\rm out} = 10^4\,{\rm R}_\odot$ and $R_{\rm out} = 10^5\,{\rm R}_\odot$ for our fiducial parameters $M = 60\,{\rm M}_\odot$, $M_d = 0.001\,M$ and $R_{\rm rec} = 10\,{\rm R}_\odot$. The surface density in the decretion disc is significantly higher than that of the accretion disc with a dead zone, given in equation (19). The right-hand panel of Fig. 1 shows the mass of the disc in $R < R_{\rm ub}$.

For times greater than a few $t_0$, the outer disc edge is approximately

$$R_{\rm out} = R_0 \left(\frac{3}{2}\frac{t}{t_0}\right)^{2/3}. \tag{27}$$

We can find the power-law decay of the surface density and the mass of the disc for late times with this expression. The surface density of the disc at the recoil radius is

$$\Sigma(R_{\rm rec}) = 4.9 \times 10^3 \left(\frac{M}{60\,{\rm M}_\odot}\right)^{2/3} \left(\frac{M_{\rm d}}{0.001\,M}\right) \left(\frac{\alpha}{0.1}\right)^{-2/3} \left(\frac{H/R}{0.01}\right)^{-4/3} \times \left(\frac{R_{\rm rec}}{10\,{\rm R}_\odot}\right)^{-1} \left(\frac{t}{10^7\,{\rm yr}}\right)^{-2/3} {\rm g\,cm^{-2}}. \tag{28}$$

Further, the mass of the disc within $R_{\rm ub}$ is approximately

$$M_{\rm d,rec} = 2.5 \times 10^{-5} \left(\frac{M}{60\,{\rm M}_\odot}\right)^{5/3} \left(\frac{M_{\rm d}}{0.001\,M}\right) \left(\frac{\alpha}{0.1}\right)^{-2/3} \left(\frac{H/R}{0.01}\right)^{-4/3} \times \left(\frac{k}{3.64}\right) \left(\frac{v_{\rm rec}}{1000\,{\rm km\,s^{-1}}}\right)^{-2} \left(\frac{t}{10^7\,{\rm yr}}\right)^{-2/3} {\rm M}_\odot. \tag{29}$$

The dotted lines in Fig. 1 confirm that these fits to the surface density and the mass are good for times greater than about a viscous time-scale at the outer edge of the disc.

The mass in this decretion disc, for our fiducial parameters, is larger than that of the accretion disc given in equation (20). Further, if there is a dead zone in the decretion disc, the surface density may be higher than that predicted here for the fully turbulent decretion disc. The structure would depend sensitively on the initial conditions. We will model this scenario with time-dependent simulations in a future paper in order to understand the disc evolution.

Fig. 2 shows the luminosity of the decretion disc as a function of the angle between the velocity kick and the disc plane, $\theta$, as calculated by equation (4). We include the $\theta$ dependence from equation (5). The luminosity is maximal for a kick that is in the plane and decreases by almost an order of magnitude for a kick that is perpendicular to the disc.

## 5 CONCLUSIONS

We have explored the evolution of a circumbinary disc around a merging black hole system in the two extreme limits of an accretion disc and a decretion disc. The accretion disc feels no torque from the binary. The viscous time-scale of such a disc is short and it is unlikely that significant circumbinary material remains at the time of the merger unless there is a dead zone in the disc. The dead zone restricts the accretion flow through the disc and the disc lifetime is significantly extended. However, the surface density of the disc around the recoil radius is low, unless the dead zone extends into this region.

In the opposite limit, where a strong binary torque prevents accretion, the disc behaves as a decretion disc. The surface density at the recoil radius is much larger than in the accretion disc, even without a dead zone.

The physical conditions in a disc around a BH–BH binary probably conform to the case of a decretion disc, held out by tidal torques from the binary, and containing dead zones. As we have seen, there is significant mass close to the recoil radius in this case. We conclude that dynamical readjustment of the disc after the BH merger is likely to release significant energy in EM form. Since all of the disc matter, including its outer skin, is shocked simultaneously, it appears unlikely that this energy is significantly trapped within the shocked disc. The prompt appearance of an EM counterpart, delayed by the light-travel time to the recoil radius (a few hours) therefore seems promising (cf. de Mink & King 2017).

## ACKNOWLEDGEMENTS

RGM acknowledges the hospitality of Leicester University during a visit where parts of this work were completed. RGM acknowledges support from NASA through grant NNX17AB96G. CN is supported by the Science and Technology Facilities Council (grant number ST/M005917/1). FGX is supported in part by National Program on Key Research and Development Project of China (grant number 2016YFA0400804), the Youth Innovation Promotion Association of CAS (grant number 2016243), and the Natural Science Foundation of Shanghai (grant number 17ZR1435800). The Theoretical Astrophysics Group at the University of Leicester is supported by an STFC Consolidated Grant.

## REFERENCES

Armitage P. J., Natarajan P., 2005, ApJ, 634, 921
Artymowicz P., Lubow S. H., 1994, ApJ, 421, 651
Artymowicz P., Lubow S. H., 1996, ApJ, 467, L77
Balbus S. A., Hawley J. F., 1991, ApJ, 376, 214
Balbus S. A., Terquem C., 2001, ApJ, 552, 235
Blecha L. et al., 2016, MNRAS, 456, 961
Campanelli M., Lousto C. O., Zlochower Y., Merritt D., 2007, Phys. Rev. Lett., 98, 231102
D'Orazio D. J., Haiman Z., MacFadyen A., 2013, MNRAS, 436, 2997
de Mink S. E., King A., 2017, ApJ, 839, L7
Farris B. D., Duffell P., MacFadyen A. I., Haiman Z., 2014, ApJ, 783, 134
Fleming T. P., Stone J. M., Hawley J. F., 2000, ApJ, 530, 464
Foucart F., Lai D., 2013, ApJ, 764, 106
Foucart F., Lai D., 2014, MNRAS, 445, 1731
Gammie C. F., 1996, ApJ, 457, 355
Gammie C. F., Menou K., 1998, ApJ, 492, L75
Glassgold A. E., Najita J., Igea J., 2004, ApJ, 615, 972
King A., Nixon C., 2016, MNRAS, 462, 464
King A. R., Lubow S. H., Ogilvie G. I., Pringle J. E., 2005, MNRAS, 363, 49
Lodato G., Gerosa D., 2013, MNRAS, 429, L30
Lodato G., Nayakshin S., King A. R., Pringle J. E., 2009, MNRAS, 398, 1392
Lousto C. O., Zlochower Y., 2011, Phys. Rev. Lett., 107, 231102
Lubow S. H., Martin R. G., Nixon C., 2015, ApJ, 800, 96
Martin R. G., Lubow S. H., 2011, ApJ, 740, L6
Martin R. G., Lubow S. H., 2013, MNRAS, 432, 1616
Martin R. G., Lubow S. H., 2014, MNRAS, 437, 682
Martin R. G., Pringle J. E., Tout C. A., 2007, MNRAS, 381, 1617
Martin R. G., Tout C. A., Pringle J. E., 2008a, MNRAS, 387, 188
Martin R. G., Reis R. C., Pringle J. E., 2008b, MNRAS, 391, L15
Martin R. G., Pringle J. E., Tout C. A., 2009, MNRAS, 400, 383
Martin R. G., Lubow S. H., Livio M., Pringle J. E., 2012a, MNRAS, 420, 3139
Martin R. G., Lubow S. H., Livio M., Pringle J. E., 2012b, MNRAS, 423, 2718
Matsumura S., Pudritz R. E., Thommes E. W., 2009, ApJ, 691, 1764
Miranda R., Lai D., 2015, MNRAS, 452, 2396
Nixon C. J., 2012, MNRAS, 423, 2597
Nixon C., King A., 2016, in Haardt F., Gorini V., Moschella U., Treves A., Colpi M., eds, Lecture Notes in Physics, Vol. 905, Astrophysical Black Holes. Springer-Verlag, Berlin, p. 45
Nixon C., Lubow S. H., 2015, MNRAS, 448, 3472
Nixon C. J., Cossins P. J., King A. R., Pringle J. E., 2011a, MNRAS, 412, 1591
Nixon C. J., King A. R., Pringle J. E., 2011b, MNRAS, 417, L66
Papaloizou J. C. B., Pringle J. E., 1983, MNRAS, 202, 1181
Perna R., Duffell P., Cantiello M., MacFadyen A. I., 2014, ApJ, 781, 119
Perna R., Lazzati D., Giacomazzo B., 2016, ApJ, 821, L18
Peters P. C., 1964, Phys. Rev., 136, 1224
Pringle J. E., 1981, ARA&A, 19, 137
Pringle J. E., 1991, MNRAS, 248, 754
Ragusa E., Lodato G., Price D. J., 2016, MNRAS, 460, 1243
Roedig C., Sesana A., Dotti M., Cuadra J., Amaro-Seoane P., Haardt F., 2012, A&A, 545, A127
Rosotti G. P., Lodato G., Price D. J., 2012, MNRAS, 425, 1958
Rossi E. M., Lodato G., Armitage P. J., Pringle J. E., King A. R., 2010, MNRAS, 401, 2021
Shakura N. I., Sunyaev R. A., 1973, A&A, 24, 337
Shi J.-M., Krolik J. H., Lubow S. H., Hawley J. F., 2012, ApJ, 749, 118
Umebayashi T., Nakano T., 1988, Progr. Theor. Phys. Suppl., 96, 151
Wardle M., 1999, MNRAS, 307, 849
Zhu Z., Hartmann L., Gammie C., 2009, ApJ, 694, 1045

This paper has been typeset from a TEX/LATEX file prepared by the author.