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Non-linear model for reinforced concrete under cyclic loading

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Most of the available shear models for reinforced concrete rely on empirical formulations. In this study, a rational shear stress function is used to define the shear stress–strain envelope for reinforced concrete. Cyclic rules are proposed to define the loading, unloading and reloading relationships for reinforced concrete under shear stress reversals. A normal stress function describing the cyclic relationship of concrete under axial stress is also introduced. The proposed functions are verified using experimental data of reinforced concrete panels tested under monotonic and cyclic loading. Subsequently, the normal and shear stress functions along with their cyclic rules are integrated in a non-linear finite element analysis code. The resulting model accounts for tension stiffening, crack opening and closing, compression hardening and softening, degradation of concrete strength and stiffness in the direction parallel to the crack, compression unloading and reloading, as well as non-linear steel behaviour (strain hardening and Bauschinger effect). The finite element model is then used to analyse two Portland Cement Association shear walls with different geometries tested under cyclic loading. The results show a good agreement between analytical and experimental data. The model showed an excellent capacity of predicting shear deformations of reinforced concrete elements under cyclic loading with minimal computational efforts.

Notation

		E_1, E_2	tangent moduli of elasticity in the principal directions
d_b	bar diameter (mm)	E_i	tangent modulus of steel in the i -direction
$d\varepsilon_1, d\varepsilon_2$	incremental normal strains in the principal directions	E_s	Young's modulus of steel
$d\gamma_{12}$	incremental shear strain in the principal directions	f_c	concrete stress
$d\sigma_1, d\sigma_2$	incremental normal stresses in the principal directions	f'_c	concrete strength
$d\tau_{12}$	incremental shear stress in the principal directions	f_{cr}	cracking stress of concrete
E_{c1}	initial tangent modulus	f_n	stress in steel bars embedded in concrete at the beginning of yielding
E_{c1}, E_{c2} and E_{c3}	tangent moduli of elasticity for the tri-linear stress–strain curve of concrete in compression	f_{unb}	unbalanced stress resulting from assuming the tangent stiffness of the descending branch equal to zero
		f_y	yield strength of bare steel bars
		f_{yh}	yield strength of transverse reinforcement in MPa
		G_{12}	shear modulus in the principal directions
		H'	width of concrete core measured to outside of stirrups
		K	parameter that accounts for the strength increase due to confinement
		S_h	centre-to-centre spacing of ties or hoop sets
		x, k and r	parameters defining the softening

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	branch of the concrete stress–strain curve in compression
Z	strain softening slope
α	parameter defining the tension softening branch of concrete stress–strain curve
γ_{12}	concrete shear strain
ε_c	concrete strain
ε'_c	concrete strain at peak stress f'_c
ε_{cr}	cracking strain for concrete
ε_o	concrete strain at maximum stress for confined concrete
ε_n	strain in steel bars embedded in concrete at the beginning of yielding
ε_s	steel strain
λ_t	parameter that controls the rate at which the response of concrete in tension decays
ν	Poisson's ratio
ρ	steel reinforcement ratio
ρ_i	reinforcement ratio in the i -direction
ρ_s	ratio of the volume of hoop reinforcement to the volume of concrete core measured to outside of transverse ties
τ_{12}	concrete shear stress

Introduction

With advancements in the state of knowledge on the behaviour of reinforced concrete (RC), more refined models are needed to predict the behaviour of RC structures under different types of loading.^{1,2} In particular, modelling the behaviour of RC under cyclic loading remains a challenge, and most of the research work in the literature on the non-linear finite element analysis of RC is confined to the case of monotonic loading. Research performed on the cyclic behaviour of RC is comparatively very limited. Computational and numerical problems associated with the complex rules describing the stress–strain relationships of concrete and steel under cyclic loading are among the major constraints to the development of more adequate design and analysis tools in this area. During the past 20 years, a limited number of refined models describing the behaviour of RC under cyclic loading have been developed. However, these models are generally based on empirical shear transfer functions.^{3–7}

The rotating crack approach was used by Stevens *et al.*³ to model RC under monotonic and reversed cyclic loading. In this model, which was based on the modified compression field theory (MCFT),⁸ the direction of principal strains was taken as the axes of orthotropy at which the material properties are calculated. Accordingly, these axes change as the crack rotates. The rotating crack model usually yields better results for anisotropically reinforced concrete elements, in which

cracks change their direction as testing progresses.⁹ The crack rotation causes discontinuities in the stresses and strains in the crack direction from the end of one load step to the beginning of the next load step. This complicates the rules defining the stress–strain relationships under cyclic loading since more than one curve is needed to define a certain region of the response. Xu⁵ used a cyclic non-orthogonal multi-crack model in order to account for deficiencies of the fixed crack model while avoiding the complexity of the rotating crack model. His technique involved decomposing the total strain into concrete strain and crack strain, which allowed intact concrete and cracks to be modelled separately. Nevertheless, this approach involves substantial computational efforts for calculating the constitutive relations.

Despite the fact that the above-mentioned models^{3,5} were very successful at the element level, they were not employed in the analysis of complete RC structures due to numerical difficulties.⁶ Sittipunt and Wood⁶ were able to analyse complete RC structures under a large number of cyclic load reversals. They used a cyclic concrete model based on the fixed crack approach that does not account for compression degradation of concrete properties. Lee *et al.*¹⁰ used four-noded quadrilateral isoparametric and truss elements to model concrete and steel reinforcement, respectively. A plastic damage approach was used to define constitutive relations for concrete. The model showed good agreement with experimental data from cyclic load testing of a reinforced concrete column. More details on non-linear finite element analysis of RC structures subjected to cyclic loading can be found elsewhere.^{7,11}

In the current study, a refined yet simple non-linear finite element model employing twelve-noded elements is used for the analysis of RC structures subjected to cyclic loading. The fixed crack approach is adopted in the proposed model. The model is characterised by its capacity to strike a balance between simplicity and accuracy. Simple hysteretic rules defining the cyclic stress–strain curves of concrete and steel are used. The stiffness and strength degradation of cracked concrete is accounted for in the formulation of the model and a rational function is used to describe the shear behaviour of reinforced concrete.

Constitutive models

Several models exist that can simulate the non-linear behaviour of structural concrete. These models vary, for instance, in terms of the tension and compression unloading behaviour, compression strain softening, and the Bauschinger effect in steel. A description of the non-linear models adopted in this study is included below.

Material model for concrete

Concrete is modelled as an orthotropic material in

the principal strain directions and is treated as an incremental linear elastic material. At the end of each load increment, the material stiffness values are corrected to reflect the latest changes in the material properties. The incremental constitutive relationship referring to the principal axes is described as follows

$$\{d\sigma\} = [D]\{d\varepsilon\} \quad (1)$$

$$\begin{bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\tau_{12} \end{bmatrix} = \frac{1}{1-\nu^2} \begin{bmatrix} E_1 & \nu\sqrt{E_1E_2} & 0 \\ \nu\sqrt{E_1E_2} & E_2 & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\gamma_{12} \end{bmatrix} \quad (2)$$

where E_1 , E_2 are the tangent moduli of elasticity in the two principal directions; ν is Poisson's ratio; G_{12} is the shear modulus in the principal directions and is equal to $0.25 \times (E_1 + E_2 - 2\nu\sqrt{E_1E_2})$; $d\sigma_1$, $d\sigma_2$ are the incremental normal stresses in the principal directions; $d\tau_{12}$ is the incremental shear stress in the principal directions; $d\varepsilon_1$, $d\varepsilon_2$ are the incremental normal strains in the principal directions; and $d\gamma_{12}$ is the incremental shear strain in the principal directions.

For each load increment, the values of the material properties E_1 and E_2 are determined as a function of the state of stress and strain throughout the analysis procedure. In this model, it is assumed that only two cracks can form at a point. The two cracks are assumed to be orthogonal and the crack orientation is determined by the orientation of the first crack, with the second crack forming perpendicular to the first one. The orientation of the cracks is fixed during the entire computational process (fixed crack model). The effect of Poisson's ratio is neglected after cracking. Therefore, the material stiffness matrix after cracking can be expressed as follows

$$\begin{bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\tau_{12} \end{bmatrix} = \begin{bmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\gamma_{12} \end{bmatrix} \quad (3)$$

The normal stress function is used to calculate the concrete stresses σ_1 and σ_2 as well as the tangent moduli E_1 and E_2 . G_{12} and τ_{12} are calculated using the shear stress function.

Normal stress function. The normal stress function describes the stress-strain relationship for concrete in the direction of cracks and perpendicular to cracks (one and two axes, respectively) using uniaxial stress-strain relationships. The calculation of σ_1 and σ_2 from ε_1 and ε_2 accounts for the effect of biaxial stress through the degradation of the concrete properties in the direction parallel to the crack.⁷ It has been established that the concrete monotonic curve defines the envelope for the cyclic normal stress-strain curve.¹² Hence, defining the monotonic envelopes is usually the first step in developing hysteretic models.

Concrete tension envelope. The adopted concrete tension envelope consists of two parts. The first part is before cracking in which concrete is assumed to be linearly elastic and is represented using the following relation

$$f_c = E_{c1}\varepsilon_c \quad (4)$$

where E_{c1} is the initial tangent modulus, f_c is the concrete stress, and ε_c is the concrete strain.

For the second part, which is after cracking, the relation developed by Stevens *et al.*,³ taking into account tension stiffening, is adopted

$$\frac{f_c}{f_{cr}} = (1 - \alpha)e^{-\lambda_t(\varepsilon_c - \varepsilon_{cr}) + \alpha} \quad (5)$$

where f_{cr} and ε_{cr} are the cracking stress and strain, respectively, ε_c is the concrete strain, $\alpha = 75(\rho/d_b)$ (mm), ρ_s is the steel ratio and d_b is the bar diameter (mm). The parameter λ_t controls the rate at which the response decays and is equal to

$$\lambda_t = \frac{270}{\sqrt{\alpha}} \quad \lambda_t \leq 1000 \quad (6)$$

The concrete tension envelope is shown in Fig. 1. The cyclic tension rules used in this study are described in greater detail elsewhere.⁷

Concrete compression envelope. The ascending branch of the stress-strain curve of concrete under uniaxial compression is modelled in this study using a simplified form of a curve given by Saenz.¹³ This simplified form, shown in Fig. 2, consists of a tri-linear curve defined by the following equations

$$f_c = E_{c1}\varepsilon_c < 0.75f'_c \quad 0 < \varepsilon_c < 0.375\varepsilon'_c \quad (7)$$

$$f_c = 0.75f'_c + E_{c2}(\varepsilon_c - 0.375\varepsilon'_c) \leq f'_c \quad 0.375\varepsilon'_c < \varepsilon_c < 0.80\varepsilon'_c \quad (8)$$

$$f_c = f'_c \quad 0.80\varepsilon'_c < \varepsilon_c < \varepsilon'_c \quad (9)$$

Fig. 1. Stress-strain envelope for concrete in tension

where E_{c1} is the initial tangent stiffness and is equal to $2f'_c/\epsilon'_c$, $E_{c2} = 0.3E_{c1}$, $E_{c3} = 0$ and ϵ'_c is the strain at peak stress f'_c given by $f'_c{}^{1/4}/1153$ (MPa).

This tri-linear simplification reduces the computational effort and ensures that the tangent stiffness is equal to E_{c1} at low strain levels.

The strain softening branch of the stress–strain curve of concrete in compression (shown in Fig. 2) is defined by Collins and Mitchell¹⁴ for unconfined concrete and is given by the following equation

$$f_c = \left(\frac{rx}{r-1+x^{kr}} \right) f'_c \quad (10)$$

where $x = \epsilon_c/\epsilon'_c$

$$r = 0.8 + \frac{f'_c}{17} \text{ (MPa)} \quad (11)$$

and

$$k = 0.67 + \frac{f'_c}{62} \text{ (MPa)} \quad (12)$$

The tangent stiffness modulus of the descending branch is assumed to be zero, thus the unbalanced stress (f_{unb}) is redistributed in the next load increment as shown in Fig. 2.

For confined concrete, the strain softening branch used in this study is developed by Scott *et al.*¹⁵ based on a previous model by Kent and Park.¹⁶ This modified Kent and Park model is simple yet accurate. The model defines the strain softening branch using the following expression

$$f_c = Kf'_c[1 - Z(\epsilon_c - \epsilon_0)] \geq 0.2Kf'_c \quad (13)$$

where

$$\epsilon_0 = \epsilon'_c K \quad (14)$$

$$K = 1 + \frac{\rho_s f_{yh}}{f'_c} \quad (15)$$

Fig. 2. Stress–strain envelope for concrete in compression

$$Z = \frac{0.5}{\frac{3 + 0.29f'_c}{145f'_c - 1000} + 0.75\rho_s \sqrt{\frac{h'}{S_h}} - \epsilon_c k} \quad (16)$$

In the equations above ϵ_0 is the concrete strain at maximum stress for confined concrete, K is a factor that accounts for the strength increase due to confinement, Z is the strain softening slope, f_{yh} is the yield strength of transverse reinforcement in MPa, ρ_s is the ratio of the volume of hoop reinforcement to the volume of concrete core measured to outside of transverse ties, h' is the width of concrete core measured to outside of stirrups, and S_h is the centre-to-centre spacing of ties or hoop sets. The cyclic compression rules used in this study are described in greater detail elsewhere⁷ along with an outline of interaction between tension and compression models.

Shear stress function. Several researchers have studied the non-linear shear stress–strain relationship of cracked concrete. For instance, various empirical formulae for the shear modulus of cracked concrete based on the fixed crack approach have been proposed.^{8,17,18} In the smeared crack model, two approaches have been used to represent the shear stiffness of cracked concrete. In the first, which is referred to as the reduced shear stiffness approach, shear stiffness is reduced by a retention factor μ usually valued between 0 to 1. This technique was employed in recent research.^{19–26} In the second technique, which is referred to as the varying shear stiffness approach, the shear stiffness of cracked concrete is assumed to be a function of the strain normal to the crack. Several researchers^{17,27–29} proposed different functions based on this approach to represent the shear stiffness of cracked concrete.

Monotonic shear stress function. In this model the rational shear modulus formulated by Zhu *et al.*³⁰ is used to describe the monotonic shear stress–strain curve of cracked concrete. This rational shear modulus has the advantage of being determined solely from the stress–strain curves of concrete in tension and compression, independently from data of experimental testing of RC in shear. The secant shear modulus is given as follows

$$G_{12} = \frac{\sigma_1 - \sigma_2}{2(\epsilon_1 - \epsilon_2)} \quad (17)$$

Using the secant shear modulus, the shear stress of concrete is expressed as follows

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2(\epsilon_1 - \epsilon_2)} \gamma_{12} \quad (18)$$

Cyclic shear transfer model. Several studies investigated the shear transfer mechanisms for both

reinforced and un-reinforced concrete subjected to cyclic loading.^{31–34} However, only a limited number of analytical models are available for such behaviour.^{4–6,35} To account for the continually varying stiffness and energy absorption characteristics of concrete under cyclic loading, a suitable hysteretic model is needed. The model proposed in this study for cyclic shear transfer is shown in Fig. 3.

The monotonic shear transfer model described in the previous section provides the envelope curve for the cyclic shear model. Fig. 3 shows a typical reversed shear cycle. The loading starts in the positive shear stress direction and follows the monotonic envelope described in the previous section. At point A, the load direction is reversed. Unloading from point A follows a straight line shooting for point B whose coordinates are $(0.85\gamma_A, 0)$. As loading proceeds in the negative direction, a significant reduction in the tangent stiffness occurs and this accounts for the pinching effect experienced by RC structures under cyclic loading. Unloading continues without any change in stress until point C is reached, at which loading in the negative direction starts following the negative envelope curve. As the loading is re-reversed at point D, the unloading stiffness to point E is calculated in the same way as for the positive direction. After point E, positive loading

continues with a reduced stiffness, calculated based on the coordinates of point A, as shown in Fig. 3, until the positive unloading branch is reached at point F. Reload- ing then follows that of the previous unloading branch (between A and B) until the positive envelope curve is reached.

Material model for steel reinforcement

For each reinforcement component, a constitutive matrix $[D]_i$ is set in the reinforcement direction as follows

$$D_i = \begin{bmatrix} \rho_i E_i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (19)$$

where ρ_i is the reinforcement ratio and E_i is the tangent modulus. The evaluation of the stress and tangent modulus for steel components in each direction is carried out using the non-linear cyclic model for reinforcing steel described below.

Stress–strain relationship for reinforcing steel

The monotonic stress–strain curve for reinforcing steel consists of three regions; the linear region, the yield plateau, and the strain-hardening region. A bi-linear curve is considered as an acceptable approxi-

Fig. 3. Proposed stress–strain curve for concrete in shear and adopted cyclic function

mation for the monotonic curve. The adopted cyclic model for steel accounts for the Bauschinger effect.³⁶ The cyclic function for reinforcing steel used in this study is illustrated in Fig. 4 and explained in more detail elsewhere.⁷ The average tensile stress–strain relationship of steel embedded in concrete is calculated using a simplification proposed by Zhu *et al.*³⁰

$$f_s = E_s \varepsilon_s \quad \varepsilon_s \leq \varepsilon_n \quad (20)$$

$$f_s = f_y \left[(0.91 - 2B) + (0.02 + 0.25B) \frac{\varepsilon_s}{\varepsilon_y} \right] E_s \varepsilon_s \quad \varepsilon_s > \varepsilon_n \quad (21)$$

where

$$\varepsilon_n = \varepsilon_y (0.93 - 2B) \quad (22)$$

B is a parameter defined as

$$B = \frac{1}{\rho} \left(\frac{f_{cr}}{f_y} \right)^{1.5} \quad \rho \geq 0.5\% \quad (23)$$

and f and ε are the stress and strain of steel bars, respectively. The suffixes s, n and y are used for bars embedded in concrete at any point on the curve, for

Fig. 4. Stress–strain curve for steel and adopted cyclic function

bars embedded in concrete at yielding, and for yield values of bare steel bars, respectively.

Validation of the proposed model

The non-linear material models for concrete and steel reinforcement described above have been integrated into PC-ANSR,³⁷ which is a general-purpose structural analysis computer program. Experimental data from tests on reinforced concrete panels were used to verify the monotonic aspects of the model, while data from two walls tested at the Portland Cement Association (PCA) were used to verify the model's cyclic behaviour capabilities. The two PCA walls used for the verification had rectangular (R1) and Barbell (B2) cross-section geometries.

Verification of monotonic behaviour of the model

Concrete panels (PV10, PV12, PV19, and PV23), originally tested at the University of Toronto,³⁸ were subjected to different types of monotonic loading. Table 1 shows reinforcement ratios as well as material and loading properties of the examined panels. The panels represent a variety of isotropically and anisotropically reinforced units and were analysed as basic membrane elements. Figs 5 and 6 show a comparison between the experimental shear stress–strain behaviour of panels PV12 and PV23, the theoretical analyses using the proposed model, and results of the MCFT.⁸ For panels PV10 and PV19, additional verification was made using the longitudinal and transverse strain plots. Figs 7 and 8 show the shear stress plotted versus longitudinal strain (ε_x), transverse strain (ε_y), and shear strain (γ_{xy}) for panels PV10 and PV19, respectively. The results of the model are compared to those of the MCFT⁸ and the distributed stress field model (DSFM)³⁹ in the case of panel PV10. For panel PV19, the comparison was made to results of the model proposed by Belletti *et al.*⁴⁰ The model proposed in this study showed a good agreement with experimental results and compared well with predictions of other models in the literature, as shown by results displayed in Table 1. Although it offered a similar accuracy to that of the MCFT and a somewhat lower accuracy compared to that of the DSFM method, the proposed model is simpler and requires much less computational effort.

Table 1. Material and loading properties of the examined panel specimens³⁸

Panel	f'_c : MPa	ρ_x : %	ρ_y : %	Loading: $\sigma_x:\sigma_y:\nu$	ν_{u-exp} : MPa	$\nu_{u-theor}/\nu_{u-exp}$		
						MCFT	DSFM	Proposed model
PV10	14.5	1.79	1.00	0 : 0 : 1	3.97	0.947	0.957	1.023
PV12	16.0	0.74	0.74	0:0:1	3.13	1.016	0.958	1.091
PV19	19.0	1.79	0.71	0:0:1	3.95	1.043	1.023	1.066
PV23	20.5	1.79	1.79	-0.39:-0.39:1	8.87	0.812	0.902	0.858

Fig. 5. Model verification with experimental results of panel PV12

Fig. 6. Model verification with experimental results of panel PV23

Verification of the cyclic behaviour of the model

Limited experimental work was performed on the cyclic behaviour of RC under shear loading. Some RC specimens were tested under shear stress by Mattock,³⁴ Jimenez *et al.*,³⁵ Laible *et al.*³² and Stevens *et al.*³ Work performed by Stevens *et al.*³ investigated the behaviour of RC panels under cyclic loading, whereas other researchers studied the shear transfer behaviour of concrete specimens across a single crack. Panel SE8³ was used to verify the performance of the proposed model under cyclic loading. Fig. 9 shows the experimental and analytical shear stress–strain relationship for panel SE8. It can be observed that the model was able to capture the general trend of the experimental behaviour satisfactorily. The envelope of the analytical hysteresis is in good agreement with experimental results in terms of trend and strength.

Cyclic behaviour of PCA shear walls

Table 2 shows material properties and reinforcement ratios⁴¹ for the concrete walls used in the verification. The finite element mesh used to discretise the walls in the current analysis is shown in Fig. 10. Twelve-noded quadrilateral plane stress elements are employed in modelling the concrete walls. A total of thirty-two elements are used to represent each wall. The use of a twelve-noded element with its cubic displacement field allowed the selection of such a coarse mesh. The maximum aspect ratio for all elements is kept less than three to avoid numerical errors. Nodes at the base of the wall are restrained against horizontal and vertical translations. The top slab is modelled as rigid in order to distribute the load to the entire wall's cross-section. Steel reinforcement is modelled using a smeared representation over the element. The experimental cyclic

Fig. 7. Model verification with experimental results of panel PV10 plotted as shear stress (τ_{xy}) versus (a) shear strain (γ_{xy}), (b) longitudinal strain (ϵ_x), and (c) transverse strain (ϵ_y).

displacement-controlled load history for each specimen was applied at the upper left corner of the wall. The displacement-controlled history consisted of gradually increasing the top displacement of the specimen during each load cycle, and then increasing the displacement level during subsequent cycles. The importance of the load-displacement plot stems from the fact that the seismic design procedure for shear walls mainly depends on strength, lateral stiffness and energy dissipation, which can all be obtained from the load displacement plot.

Figures 11 and 12 show the experimental and analytical load-deflection relationships for the two walls investigated. The total deflection at a given point on the wall is the sum of two deformation modes: flexural and shear. The flexural mode is more desirable and is responsible for energy dissipation, whereas the shear mode is less desirable since it causes pinching that leads to lower energy dissipation. The experimental

load-deflection curves show typical pinched hysteretic loops indicating a shear governed behaviour. This type of behaviour represents a rigorous test for the cyclic constitutive model proposed in the present study since the flexural response of RC can be predicted using simple RC models while a shear-dominated response imposes more complicated analysis. It can be observed in the figures that the model is capable of capturing the trend of the hysteretic loops. The stiffness deterioration of the walls caused by the application of the reversed cyclic loading is also well represented analytically. The analytical peak values of the lateral loads are almost of the same magnitude as those recorded experimentally. Similar success was achieved by other models such as in Sittipunt and Wood⁶ but such models used finer meshes and more intensive calculations compared with those of the proposed model.

The capacity of the model to accurately represent the behaviour of zones with high shear deformation was

Fig. 8. Model verification with experimental results of panel PV19 plotted as shear stress (τ_{xy}) versus (a) shear strain (γ_{xy}), (b) longitudinal strain (ϵ_x) and (c) transverse strain (ϵ_y).

Fig. 9. Model verification with experimental results of panel SE8.³ (a) Model prediction; (b) experimental results

Table 2. Material properties and reinforcement ratios of PCA wall specimens⁴¹

	Wall B2	Wall R1
Cross-section shape	Barbell	Rectangular
Concrete compressive strength f'_c : MPa (psi)	53.6 (7780)	46.5 (6490)
Yield stress of boundary elements reinforcement: MPa (ksi)	410.3 (59.5)	450.2 (74.2)
Yield stress of vertical web reinforcement: MPa (ksi)	532.3 (77.2)	535.1 (77.6)
Yield stress of horizontal web reinforcement: MPa (ksi)	532.3 (77.2)	535.1 (77.6)
Boundary element reinforcement ratio	3.67	1.47
Vertical web reinforcement ratio	0.29	0.25
Horizontal web reinforcement ratio	0.63	0.31

Fig. 10. Nominal dimensions of the PCA wall specimens and finite element discretisation. (a) Nominal dimensions of test specimen; (b) reinforcement details; (c) finite element mesh. All dimensions are in cm

Fig. 11. (a) Analytical and (b) experimental load-displacement plots for wall R1⁴¹

examined through calculation of the average shear distortion for the lower part of the wall. The average shear distortion angle, γ_{avg} , of the lower 1828.8 mm (6 ft) of the wall is calculated based on horizontal and vertical displacements of the four corner nodes located on the web of the wall, as shown in Fig. 13, using the following equations given by Sittipunt and Wood.⁶

$$d'_1 = \sqrt{(h + V2)^2 + (l + U2)^2} \quad (24)$$

$$d'_2 = \sqrt{(h + V1)^2 + (l + U1)^2} \quad (25)$$

$$\gamma_{avg} = \frac{(d'_1 - d_1)d_1 - (d'_2 - d_2)d_2}{2hl} \quad (26)$$

where γ_{avg} is the average vertical and horizontal shear distortion angle, l , h are the horizontal and vertical dimensions of the studied part of the wall, $U1$, $V1$ and $U2$, $V2$ are the horizontal and vertical displacements at the corner joints of the studied part of the wall, respec-

Fig. 12. (a) Analytical and (b) experimental load-displacement plot for wall B2⁴¹.

tively, and d_1 , d_2 and d'_1 , d'_2 are the diagonal dimensions of the studied part of the wall before and after loading, respectively.

This shear distortion comparison provides a direct verification of the proposed shear stress function and associated cyclic rules since the shear distortion is directly related to the shear transfer mechanisms of RC. Figs 14 and 15 show the analytical and experimental shear distortion angle versus load plots for walls R1 and B2, respectively. The analytically calculated shear distortion is in good agreement with that measured experimentally for both walls. The success of the proposed model manifests through the close correlation between the predicted and the actual response of the shear walls. The analytically predicted hysteresis follows the same trend observed experimentally. The stiffness deterioration of the walls under reversed cyclic loading is well represented analytically. The analytical peak values of the lateral loads are almost of the same magnitude as those experimentally recorded.

Fig. 13. Estimation of shear distortion in the walls (adopted from Sittipunt and Wood⁶)

Fig. 14. (a) Analytical and (b) experimental load-shear distortion plot for wall R1⁴¹

Conclusions

A non-linear finite element model for RC has been developed and verified. The model offers a reliable analytical tool and is capable of capturing the major characteristic features of the behaviour of RC that is

Fig. 15. (a) Analytical and (b) experimental load-shear distortion plot for wall B2⁴¹

subjected to reversed cyclic loading. The model adopts the fixed crack approach and uses simple hysteretic rules to define the cyclic stress-strain curves of concrete and steel reinforcement. The effect of cracked

concrete stiffness and strength degradation is included in the model. The verification of the model on the basic membrane element level shows a good agreement between experimental and theoretical results, confirming the effectiveness of the proposed model for predicting the behaviour of several anisotropically reinforced concrete panels.

When implemented in a non-linear finite element procedure, the model showed good agreement between analytical and experimental results of RC walls having different cross-sections and loading histories. The model was able to successfully predict the stiffness degradation, the peak load–deflection values, and the shear distortions of the walls under the effect of reversed cyclic loading.

The use of high-power elements, such as the twelve-noded quadrilateral elements along with smeared reinforcement, allowed the use a much smaller number of elements to represent the behaviour of RC members. Added to the use of simple RC constitutive models, this can help to increase the practical applications of the model for the analysis of complete RC structures with reasonable computational effort, even for those structures considered as complicated. The proposed RC model is currently being used in predicting the behaviour of lightly reinforced concrete structures under the effect of earthquake loading.

Conversion factors

1 kip = 4.448 kN
1 in = 25.4 mm
1 psi = 0.00689476 MPa

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Discussion contributions on this paper should reach the editor by 1 November 2005