Pair-a-Dice Lost: Experiments in Dice Control

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"We may throw the dice, but the Lord determines how they fall" (Bible.com).

This paper presents our findings from experiments designed to test whether we could use a custom-made dice throwing machine applying common dice control methods to produce dice rolls that differ from random. In earlier research we used a popular method of dice control to calculate how much "control" a shooter needs to overcome the casino's advantage (Smith and Scott, 2018). We found that a shooter only needs an 8.031% level of control (0% is random and 100% is perfect horizontal-axis control of both dice) to erase the casino's advantage of 1.41% for a standard pass line bet.[1] This finding supports a common claim among dice controllers that you do not need to throw the dice perfectly every time to win, simply throwing the dice with a little more control than a random throw is enough. The natural follow-up question is: Can a human being achieve the desired level of control under normal casino conditions? This question has not been answered elsewhere. Even documented evidence of extremely long craps hands is not as intuitively convincing as it may appear.

We decided to run experiments to see if a dice throwing machine that generally mimics the biomechanical properties of expert craps players (e.g., back spin, on-axis throw, repeatable throwing angle etc.) could achieve at least a break-even level of control. Using our machine (named "Lucky Lil") on a 6' foot craps table we filmed dice throws using a Phantom® VEO4K 990s high-speed digital camera that captured video in 4K resolution. After these initial observations we calibrated the machine ensuring the dice were spinning on a stable horizontal axis (rotating around numbers one and six axis) and recorded 7,557 craps throws. We use chi-squared tests to determine if we were able to produce non-random rolls and hypothesis tests to see if we achieved a statistically significant number of on-axis throw outcomes.

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[1] It is possible to reduce the casino's advantage below 1.41% by using Free Odds bets (e.g, 3-4-5X Odds) on a pass bet (at 100X Odds on a pass line bet the casino's advantage is 0.374% for bet made/bet resolved or 0.006% per roll)—see Wizard of Odds (2019).

The Mystique and Business of Dice Control

On May 23, 2009 at the Borgata Casino in Atlantic City, New Jersey, Patricia Demauro broke the longest craps roll world record.[2] She rolled 154 times for 4 hours and 18 minutes (Suddath, 2009). This New Jersey grandmother had only played craps once before her world record. The previous record was held by Stanley Fujitake who placed a $5 pass line bet and went on to roll 118 times over a period of 3 hours and 16 minutes at the California Hotel (The Cal) in Las Vegas, Nevada on May 28, 1989.[3] This record earned Fujitake the nickname "The Golden Arm." The casino reviewed the surveillance tape to verify Fujitake's roll. The Cal has a glass-encased bronze cast of Fujitake's hand holding a pair of dice from that night's historic roll. Nearby there are many small plaques commemorating shooters' membership into the Golden Arm Club, which requires at least one hour of successful craps rolls. There is also a platinum wall reserved for those who rolled for at least 90 minutes or have more than one one-hour roll. The Golden Arm himself is a multi-platinum member having rolled over one hour four times. Every year since 1989 The Cal hosts the Golden Arm Craps Tournament (during the end of April in recent years).

In Claire Suddath's Time (2009) article on Demauro the statement was made that the odds of her roll were "roughly 1 in 1.56 trillion." However, those are the odds of not throwing a seven during 154 rolls—i.e., $1/(5/6)^{154}$. But that is incorrect because in the come-out roll (first phase of the game) rolling a seven is a win not a loss and the game continues. The only way a craps hand (or shooter's hand) ends is throwing a seven during the point cycle (i.e., sevens out). Therefore, Demauro undoubtedly threw many sevens during her momentous craps hand, but never during the point cycle. Calculating the odds of a single craps roll is straightforward using simple probabilities (e.g., the odds of throwing two dice that total seven is 6/36 or 1/6). Determining the length of a craps hand, however, is non-deterministic, meaning it is not possible to calculate the odds of a world record-breaking craps hand using simple probability. Stewart Ethier and Fred Hoppe (2010) identified two methods for calculating the tail probabilities for the length of craps hands. The first method is by recursion and the second was developed by Peter Griffin based on a Markov chain.[4] Their calculations show the odds of having a craps hand last 154 rolls is one chance in 5.59 billion. The expected length of a shooter's hand is 8.52551 with a standard deviation of 6.785 rolls. This means a shooter can expect to roll 8 to 9 times during an average craps hand.

Is breaking the world record or gaining membership in the Golden Arm Club luck or skill? If enough hands of craps are played then a string of good rolls is inevitable, though the hand length becomes increasingly unusual as the number of rolls continues. The fact that someone beat long odds and rolled 154 rolls seems miraculous since 5.59 billion is a large number of craps hands and it seems unlikely that there have been so many attempts. However, using simple estimates, we find that the number of hands played at all craps tables across the world is large. Although a precise calculation of the number of attempts is impossible, we here attempt an order of magnitude calculation. There are over 3,000 casinos worldwide (World Casino Directory, 2019). If each casino averages five craps tables in play, there are 15,000 active tables worldwide. If each craps table averages 100 hands per day, there are 1,500,000 hands per day, or roughly 550,000,000 attempts per year. Over a twenty-year period, this results in roughly 11

[2] There is no official record keeper of craps hands, so the title of world record is ambiguous. As to accuracy, after Stanley Fujitake's historic craps hand the casino reviewed the surveillance tape to verify the rolls. It is expected casinos would verify any unusually long craps hands.

[3] There is a long-held rumor that the craps table that Stanley "The Golden Arm" Fujitake used (table 3) was later taken out back of the casino, chopped up and burned.

[4] Their analyses are not germane to this paper; however, it is encouraged that anyone interested in understanding the probabilities related to craps hands see Ethier and Hoppe's (2010) paper or Stewart Ethier's book "The Doctrine of Chances" (2010)—especially chapter 15.

billion craps hands. Although one might argue with the specifics given here, it appears that having a craps hand last 154 rolls, while still remarkable, is not miraculous (unless it happens to you, of course).

Dice controllers believe that dice can be thrown with enough precision to beat the odds (slightly). The argument by those who promote dice control is that a talented dice thrower who practices regularly is no different than a good putter in golf or three-point shooter in basketball—i.e., with enough practice and some skill, some manageable control is possible. An element of luck exists in all things; however, statistics gives us the ability to tease away the rare lucky/unlucky streaks from predictable results. There are many variations of dice control, but the intentions are all the same—produce dice rolls that result in desired numbers (or avoid losing numbers). There are many books, videos, computer programs, seminars and private instructors that teach dice control methods. Weekend dice control seminars can cost well over $1,500. Some people, therefore, have strong financial incentives to promote the idea that dice control works.

To our knowledge, no independent peer-reviewed study exists that attempts to prove or disprove the possibility of dice control in craps. There are at least two possible reasons for this: First, physicists, mathematicians, engineers et al. who are likely to test this theory understand the nature and probabilities associated with dice throws and have discarded the possibility out of hand. Second, people have run experiments, but for either personal or professional reasons have not published the results. For example, if someone found that dice control is possible, they might keep this information secret so that casinos do not adopt strategies to thwart their efforts or ban the practice. On the other hand, it is possible casinos or similar entities have tested the possibility of dice control and disproved it. Perhaps this explains why many dice control seminars are held in casinos, which may be analogous to having a safe-breaking seminar in a bank. No bank or casino would allow it, if the seminars were effective. But if the seminars generate interest in craps and convinces some people that they can control dice outcomes (even if they cannot) then casinos are smart to welcome dice control seminars.

In addition, craps players are susceptible to the illusion of control, which is a concept developed by Ellen Langer (1975; 1977). She found that people had greater confidence in an outcome when they were in control of initial conditions. Coincidentally, one of her experiments involved throwing dice and whether being the thrower of the dice influenced their confidence or not (which, of course, it did). Thus, craps is the perfect game to induce illusion of control—it is the only casino game someone has control over the gambling objects (dice), so the feeling of control is higher than in a game such as roulette where bettors are mere bystanders to the croupier. The gambler's fallacy is also applicable in craps. Future rolls are not influenced by previous rolls. The dice combinations are fixed and thus unchanging. The idea that certain numbers appear more/less often is a result of small sample sizes and variance more than any "behavior" of the dice. This notion is closely associated with the hot hand fallacy in basketball, which states that if a player successfully makes multiple shots early in a game then that player is considered hot and thus more likely to make future shots (Gilovich, Tversky & Vallone, 1985). When a craps shooter has a hand with multiple rolls (and many pass line bet wins) then the shooter is considered "hot," and some people will bet based on this belief. The hot hand fallacy shows that random sequences (or variance) can induce an illusion of control—craps shooters revert to expected means just like basketball players. Streaks happen and under the right circumstances can appear correlated (i.e., a streak followed by another streak), which can further propagate the illusion of control.

Dice Control in Craps

There are several methods of dice control. The one we studied in our previous research (Smith & Scott, 2018) involves two primary factors.[5] First is dice setting, which requires arranging the dice so the front-facing numbers (four per die—sixteen outcomes in total) are more likely to produce desired rolls by minimizing the chances that the side numbers end up on top. Casino dice are cubes with the same dot (or pip) orientation—opposing sides always sum to seven (i.e., six is always opposite the one and five is always opposite the two). Dice controllers (in general) believe this pip orientation creates an opportunity. For example, if we want to reduce the chance of rolling a seven during the point cycle then we could use a variety of dice sets where the front-facing numbers are less likely to add up to seven when thrown on their horizontal axis, thus having a greater likelihood of settling on a front-facing number (minimizing side numbers) producing desired results. Second, is a backward spinning on-axis throw that hits the back (pyramidal lined) wall with minimal force. The explanations for backspin include (a) keeping dice rotating on-axis (think: gyroscopic wheel) and (b) slowing the dice when they hit the table, thus reducing their energy before hitting the back wall.

Why Dice Control Might Be Possible

While statistics and physics makes it clear that dice control is a myth; there are modern examples of games that were considered unbeatable succumbing to superior analysis. Roulette, once considered unbeatable, is now debunked. In the 1970s a group of graduate student physicists nicknamed the Eudaemons developed models that when combined with computers could predict where a roulette ball would land in a certain quadrant of the roulette wheel (see Bass, 1985; Poundstone, 2005; Small and Tse, 2012). You need the wheel speed, ball speed and a few other parameters to reduce randomness. Improving predictability even a small amount is profitable because roulette's payoff is large. A trio in 2004 were detained by police for using a laser-enabled cell phone at a casino in London to *win* £1.3 million (Kucharski, 2016). Persi Diaconis, Susan Holmes and Richard Montgomery (2007) used a high-speed camera to study coin tosses and found that using a machine they could generate exact results every time; thus, "coin tossing is 'physics' not 'random'" (p. 211). Marcin Kapitaniak et al. (2012) show that dice rolls are deterministic (thus not chaotic) if one knows the initial conditions. This is the first analysis we are aware of that finds dice rolls are not random. The study does, however, throw the dice via a machine that slides the dice to a small drop-off onto a smooth glass surface. These conditions are quite dissimilar from a casino craps table, but it is the first peer-reviewed article that finds this result using a high-speed camera and basic mechanics.

Our earlier research shows that if 0% control is random and 100% control is perfect horizontal axis control a shooter only needs to control the dice at a level of 8.031% to break even when using a standard pass line bet. Thus, not much control is needed, so it sounds possible. Many people accept that sports require skill and with practice and good coaching people can become better at physical activities such as golf, tennis, darts, billiards etc. So, dice control may be a matter of enough purposeful practice.

Why Dice Control Might Not Be Possible

From the standpoint of quantum-mechanical physics all outcomes are inherently probabilistic and inherently not totally predictable. However, when one considers large objects such as dice, the probability of deviation from predictions made using Newtonian (deterministic) mechanics is negligibly small. In theory, replication of the initial conditions would result in almost certain replications of the outcomes. However, exact replication of the initial conditions is never possible by machine or human. So, the key

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[5] A possible third factor is the grip; however, this varies between dice controllers. While many different grips exist, the most common is an overhand three-finger grip where the dice are together, and the thumb is on the middle seam of the dice with the middle and ring fingers on the front of each die.

question is: Under casino rules can a human, or in our case a machine, replicate initial conditions 'close enough' to attain a replication of outcomes (at least probabilistically)?

Intuitively a well-designed machine can replicate initial conditions much better than any human. So intuitively, if humans can control dice, a machine can be built that will control dice. That is why a properly designed and built machine can possibly shed light on whether or not human dice control is possible. For example, there are machines that shoot basketballs as well or better than humans (see Salo, 2019).

If we are assuming deterministic equations of motion, an important question is how closely do the initial conditions need to be replicated for the outcomes to be replicated? Many deterministic equations (such as used for weather prediction) over a long enough time period diverge unrecognizably based on miniscule changes in the initial conditions (the so-called "butterfly effect"). Are the equations of motion for dice mechanics within casino rules in this category?

A bouncing object with sharp edges and corners intuitively is extremely sensitive to extremely small changes in initial conditions on each bounce. These changes further compound based on movement differences over time between the bounces. An analogy might be to have a pitching machine pitch to a hitting machine that hits balls to another hitting machine. By the time the ball got to the second hitting machine any small changes in the previous trajectory would become highly magnified. Having a third hitting machine would make it even worse. Bouncing casino dice may be equally chaotic.

Previous Dice Control Tests

The only verified real world test of dice control was undertaken by Stanford Wong in 2004. Stanford Wong (gambling pseudonym of John Ferguson) is a popular gambling author and creator/operator of the website BJ21.com. His book "Wong on Dice" (2005) details his interest in craps that started in 2004 and is what generated our interest in this topic. After practicing dice control techniques Wong learned dice control at a seminar and from private instruction then he started playing at casinos. He recorded the results of his rolls and showed he was able to throw non-random dice to a statistically significant degree. His patrons (called green chip members—green chips at casinos are typically valued $25) were not convinced. So, Wong posted the following challenge on his website:

> "I propose doing it in real games, on full-sized tables in various Las Vegas Casinos. I'll have some way of signaling to the monitor ahead of time when I want the next roll to count. Those rolls will be by either me or other crapshooters at the table. We won't have to argue over whether the dice bounce off the back wall; if the casino accepts the roll, we will accept it for our challenge. I propose monitoring 500 such rolls, a number that probably can be reached in two days. 500 random rolls are expected to include 83.3 sevens." (Wong, pp. 103-104).

The over/under was set at 79.5 sevens over 500 rolls with caps set by bettors—Wong allowed a cap of up to $100,000 on the overs (i.e., people betting against him). So, if the total number of sevens was 78 (1.5 under the expected value) and someone bet $1,000 on the under then that person would win $1,500 (1.5 times his/her bet). Volunteer monitors were recruited, and bets were taken. Wong enlisted rolling help from Little Joe Green—one of his green chip members. Rolls were recorded by multiple monitors and all rolls were legal casino craps throws (none were called "no rolls" by the dealers nor disputed by the monitors). Wong and Green only tried to throw non-sevens to make the challenge move more quickly and ease interpretation. The results are in Table 1.

Table 1

Results from Stanford Wong's dice challenge in 2004 (exact date withheld by Wong).

Shooter	Number of Rolls	Number of Sevens	Percent Sevens
Stanford Wong	278	45	16.2%
Little Joe Green	222	29	13.1%
Total	500	74	14.8%

Source: Wong (2004).

The unders won their bets with room to spare. The odds of rolling so few sevens is 14.8% (simple odds—binomial test); however, this is not quite a statistically significant result. The standard deviation of a random 500 craps rolls is $\sqrt{np(1-p)}$ or $\sqrt{500 * \frac{1}{6}(\frac{5}{6})}$, which equals $8.\overline{33}$. This finding shows that Wong's dice challenge outcomes were a little more than one standard deviation less than the expected value (z-score of -1.12) of a random roll. Using the binomial distribution, we know that the probability of rolling 74 or fewer sevens over 500 rolls is 14.41%—or using a normal distribution the probability is 13.1%. Typically, empiricists want a p-value of ≤ 0.05 (or $\leq 5\%$). Thus, while Wong's challenge did produce fewer rolls than expected (i.e., 1.12 standard deviations below the mean), the difference was not statistically significant. The results, however, do show promise and they did produce fewer sevens than expected—costing several knowledgeable gamblers some money.

Stanford Wong's challenge begs the question of whether dice control is actually possible with a large number of consistent throws. We could program a computer to run simulations, which is interesting, but not real enough to be convincing. The only way to see if dice control is possible is to test it in real-world conditions using a machine that can produce consistent throws that adhere to the tenets of most dice controllers.

Dice Throwing Machine: Lucky Lil'

In order to test whether dice control is possible, we needed a machine that, as much as is reasonably possible, could generate throws that mimic the techniques of experienced dice throwers. We considered many different designs. The machine needed to fill certain criteria: First, it needed to produce backspin since this is an important component of dice control—thus producing an on-axis spin. Second, it needed to be adjustable—particularly the angle of the throw (flat (0-degrees) to 45-degrees), speed of the throw and rate of backspin. Third, it needed to be sturdy, so it could last through thousands of throws consistently. Fourth, it needed to be simple to operate.

We needed a machinist with an understanding of craps. On a recommendation from a friend, we posted an ad on The Home Machinist! listing what we were looking to do. Fortunately, Tom, a retired engineer living near Atlantic City (go figure) answered our ad. We came to him with vague ideas of how the machine should function and what materials we wanted. He then went to work and produced Lucky Lil', a spring loaded, aluminum dice throwing machine that exceeded our expectations.[6] Figures 1 and 2 below offer a side view and top view of Lucky Lil', respectively.

[6] Named by Tom in honor of his granddaughter.

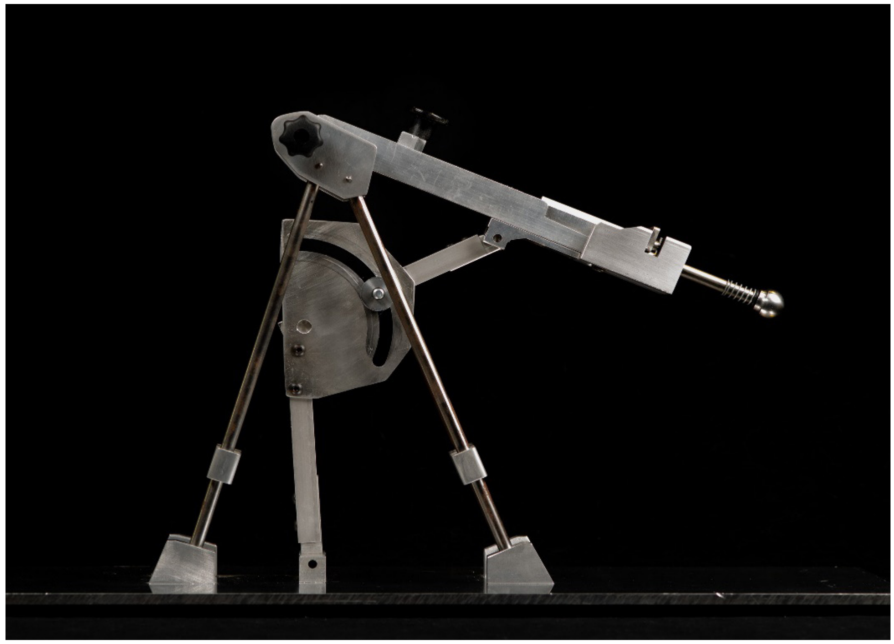

Figure 1: Side view of Lucky Lil'
Source: Photograph taken at Monmouth University by Prof. Mark Ludak (July 1, 2019).

Figure 2: Top view of Lucky Lil'
Source: Photograph taken at Monmouth University by Prof. Mark Ludak (July 1, 2019).

Lucky Lil' weighs 16 pounds (7.3 kg). Its dimensions (height, width and length) are 13.5" x 11.3" x 26.3". The height of the machine matches the release height of the dice using an overhand grip with backhand swinging throw (as is common among dice controllers). The machine has three speed settings that are notches that lock the spring pull-back rod: speed notch 1 throws the dice about 5-6 miles per hour (8-9.7 kph); speed notch 2 is 10-11 mph (16-17.7 kph); and speed notch 3 is 19-20 mph (30.6-32.2 kph). The machine's angle can be adjusted from 0-degrees (flat) to 45-degrees—the effective launch angles are higher because of the rammer bar that pushes the dice up at launch. The last feature is the backspin, which is generated by using a rounded aluminum bar that adjusts in height from position A (0.05"); B (0.125"); C (0.25"); D (0.373") and E

(0.465”). Position A produced maximum spin, but little forward motion, so we never used it. Position B produces ~520 RPM of backspin at launch, which did not match our practice throws by hand. Position E produces a small amount of forward spin, which we do not want. Position D produces no spin (flat or knuckleball). All of our throws (except later ones) are set at position C that generated a desirable rate of backspin (angular velocity vector) at around 280 rpm at launch.

We used AAA grade casino dice (bought from a supplier that sells the same dice to casinos) and used a digital micrometer to measure the dimensions of each die (19 mm) and a digital scale to weigh each die (less than one milligram difference at ~8.925 grams each) so that we were using consistent dice.

Once we had Lucky Lil’ we used a Phantom VEO4K 990s high-speed camera to calibrate throws on a 6 foot craps table.[7] We found the speed needed to match a normal throw (around 10-11 miles per hour), spin rate (position C) and other minor tweaks and adjustments ensuring smooth delivery of the dice that best mimicked what we observed from dice controllers—particularly an on-axis (around the ones and sixes) backward spinning throw.

Experiments and Findings

Our strategy (like Wong above) was to simplify our outcomes so they were easy to test. Thus, we used a dice set with the numbers one and six on the sides. If Lucky Lil’ can successfully throw the dice on their horizontal axes (i.e., around the ones and sixes) consistently then we should observe statistically significantly fewer ones and sixes. We used Chi-square tests to measure our observed values against expected values.[8] The null hypothesis of a Chi-square test is that the observed values are no different from the expected values to a statistically significant degree. The alternative hypothesis is that the observed values do differ from expectations. If our machine can control the dice in a way that minimizes ones and sixes, then we should get low p-values. We also developed hypothesis tests that measure if the machine was able to produce break-even fewer numbers of ones and sixes—thus verifying the dice are landing on-axis more often than expected.

We launched our dice together (contiguous or side-by-side) as most dice controllers recommend. We did use two different colored dice so we could analyze each die individually (or as a pair). Our first set of throws (n=1,400) were short rolls (i.e., they did not hit the back wall) on a six-foot craps table with Lucky Lil’ set at an angle of 20-degrees (producing a launch angle of 35-degrees) with position C spin of ~280 rpm and speed notch 1 (5-6 mph). We used a dice set that put the ones and sixes on the side of the dice—with the expectation that if on-horizontal axis throws were successful we would get fewer ones and sixes than the other numbers. This also allowed us to look at the combined total of the dice as well as capture each die’s results individually. The first 1,001 rolls (combining both dice gives us 2,002 outcomes to test) was the only series in our experiment to produce a p-value low enough to be statistically significant (0.06) showing results that diverged from random. The results are below in Table 2:

[7] Andy Kubit (Ametek/Vision Research that makes Phantom cameras) went above and beyond to help us take many excellent videos of Lucky Lil’ tossing dice.

[8] $$\chi^2 = \sum \frac{(observed - expected)^2}{expected}$$

Table 2

First experiment results (n=2,002)

Dice	1	2	3	4	5	6
Observed	325	367	353	316	295	346
Expected	333.667	333.667	333.667	333.667	333.667	333.667
Difference (Observed-Expected)	-8.667	33.333	19.333	-17.667	-38.667	12.333

The p-value on the Chi-square test was significant at a 10% level (0.06), but we see in Table 2 that fewer ones were thrown than expected (8.667 fewer), but more sixes (12.333 more). It appears the variation among the other numbers was driving the Chi-square results (half higher than expected (2s and 3s) and half were lower than expected (4s and 5s). Since we wanted to reduce the number of ones and sixes (keeping the dice on axis), we developed a hypothesis test where:

Ho: Observed number of 1s and 6s ≤ Break-even number of 1s and 6s
Ha: Observed number of 1s and 6s > Break-even number of 1s and 6s

The total number of 1s and 6s in Table 2 is 671, which is higher than the expected number (667.33), and the break-even number of ones and sixes is 613.74 [(1/3-0.08031/3)*2002]. The difference between 671 and 613.74 (and divided by the standard deviation)[9] generated a Z-score of 2.776, which equates to a p-value of 0.002755 (or 0.2755%). Thus, we reject the null hypothesis, finding we did not achieve a low enough number of ones and sixes to break-even with this set of rolls. Thus, our machine was unable to throw the dice on axis to a statistically significant degree. Worse, we produced more ones and sixes (in total) than expected from random, which is the opposite of our goal.

Our second set of throws (n=1,300) were short rolls with an effective launch angle of 55-degrees using notch 1 and position C still for backspin. Combining results of both dice gave us a sample of 2,600. Results are in Table 3 below.

Table 3

Experiment 2 results (n=2,600)

Dice	1	2	3	4	5	6
Observed	450	413	469	444	423	401
Expected	433.33	433.33	433.33	433.33	433.33	433.33
Difference (Observed-Expected)	16.67	-20.33	35.67	10.67	-10.33	-32.33

We can see in Table 3 there were more ones thrown than expected, but far fewer sixes. The Chi-square test generated a p-value of 0.185, which is not significant, but when combined with the results in Table 2 suggest that if dice control has any chance of working, it is with short rolls.

We replicate the break-even hypothesis test used after Table 2 above and find that the number of ones and sixes observed in Table 3 is 851. While 851 is less than the expected number of ones and sixes (866.67), it is not near the critical value of 797.1 to achieve a break-even level of control. Our total of 851 ones and sixes compared to the break-even level of 797.1 generated a Z-score of 2.294, which equates to a p-value of

[9] Standard deviations are calculated using: $\sqrt{np(1-p)}$

0.0109 (or 1.09%). Again, we did not achieve a break-even level of control with these rolls—though the p-value was higher than in the first set of rolls in Table 2.

Our third set of throws (n=3,400) were legal craps throws from four feet with at least one die hitting the back wall. The machine was set at an effective launch angle of 40-degrees, speed notch 2 (10-11 mph) and position C for backspin. We combined the two dice for a total of 6,800 outcomes—presented in Table 4.

Table 4
Third experiment results (n=6,800)

Dice	1	2	3	4	5	6
Observed	1150	1125	1117	1141	1173	1094
Expected	1133.33	1133.33	1133.33	1133.33	1133.33	1133.33
Difference (Observed-Expected)	16.67	-8.33	-16.33	7.67	39.67	-39.33

The Chi-square test on these rolls generated a p-value of 0.65. We tried combining the dice results to see if perhaps the two together retained some level of correlation; however, this generated a Chi-square p-value of 0.99! We also tested each die individually thinking that perhaps the machine was throwing one die better than the other; however, this did not result in any significant outcomes.

Using our hypothesis test again to test if we achieved a break-even level of ones and sixes (on-axis throws), we see the total number of ones and sixes was 2,244, which is less than the expected number of ones and sixes (i.e., 2,266.67). However, it is not statistically significantly lower than the break-even number of 2084.63. The Z-score was 4.192, which is a p-value of <0.001. Once again, we did not achieve a break-even number of ones and sixes, so the machine failed to keep the dice on axis more than would happen randomly.

Our fourth and final set of throws (n=1,557) were all extreme attempts to produce non-random rolls. We tried setting the machine flush with a hard table with craps felt, but that did not work. Then we made rolls that were flush with a smooth Formica table, but this too did not work. Then we made a series of throws with the machine set at the most extreme spin rate (and various launch angles) to produce as much gyroscopic force as possible to produce on-horizontal axis throws—but none of these extreme experiments produced dice rolls that were different from random nor did they achieve a statistically significant on-axis level of control needed to break-even.

Conclusion

This paper outlines the results from our experiments using a machine (Lucky Lil') we had designed and custom built to throw dice that spin on their horizontal axis producing more front-facing numbers (2s, 3s, 4s, and 5s) and reducing the number of ones and sixes (side numbers). Using a Phantom VEO4K 990s high-speed digital camera we calibrated Lucky Lil' to produce throws that best matched our observations of real craps throws—keeping the dice spinning on their one/six axis. We then recorded 7,557 craps rolls using various settings that matched human dice throws. Most of our rolls used position C for the backspin that created ~280 rpm at launch. Our first tests were short rolls at a launch angle of 35-degrees at speed notch 1 (5-6 mph) that did not hit the back wall. Our first 2,002 rolls using this setup were the only series to produce outcomes that differed from random using Chi-square tests (p-value was 0.06). However, the differences in the results were not the result of fewer ones and sixes (as hoped). We then changed the launch angle of Lucky Lil' to an effective level of 55-degrees. These 1,300 rolls (2,600 total dice outcomes) did not produce results significantly different from random; however, the p-value was closer to showing non-randomness at 0.185. But again, we did not produce a statistically significant fewer ones and sixes Next, we made legal craps throws that hit the pyramidal backwall from four feet away using speed notch 2 (10-11 mph) at an effective launch angle of 40-degrees still using position C for backspin. These rolls were not significantly different from random and did not produce fewer ones and sixes. Lastly, we tried many extreme throwing methods to try and produce rolls that resulted in fewer ones and sixes, but none of our efforts worked.

This paper outlines our attempts to answer the question: Can a dice throwing machine attain the desired level of control under normal casino conditions? The relation to the human question is the fact that a machine can often be built to attain a level of consistency in mechanical outcomes far beyond abilities of the best humans. Our machine failed to attain the desired consistency we hoped to produce. However, even in this failure, we believe that the effort did shed considerable light on the possibility of either a human or a machine attaining such a level of control. Although a more sophisticated machine can be built, or the parameters further tweaked, we are much more skeptical than before we started that such control by either a machine or a human is possible. Considerable effort was expended to replicate throwing conditions by Lucky Lil' that might make control possible. Not only is there a lack of statistical support for control, but our high-speed camera videos show the chaotic behavior of the dice being thrown off-axis after a very few bounces. Our throws were consistently on-axis, such that if the dice landed on a super soft surface (think mud) the dice would land on the front-facing numbers (2s, 3s, 4s and 5s); but once the dice hit the craps table they quickly scrambled. While we do not expect our experimental findings to be the final word on dice control, we do hope it stimulates further discussions and experiments.

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