The House Edge and Play Time: Do Industry Heuristics Fairly Describe This Relationship?

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Abstract

Based on modified versions of licensed pay tables from reel slot machines, simulations of play failed to indicate a statistically significant difference in the spins per losing player (SPLP), despite a marked difference in the pars (i.e., 7.9% vs. 12.9%). To reflect a volume of play consistent with frequent gambling, the simulations included results from 1–4 visits per week, for the equivalent of one year. Additionally, this level of play was repeated for 100 "years," within multiple scenarios of buy-in amounts and stoppage-of-play criteria. Still, most outcomes indicated a negligible decline in SPLP, in spite of the 63%-increase in the par. These results were reproduced from a second pairing of games featuring a 100%-increase in par (i.e., 5.0% vs. 10.0%). The findings spotlighted considerable limitations of popular industry heuristics related to the relationship between par and play time. While additional studies are warranted, the outcomes suggested that operators may be overly mindful of the fallout from increased pars. These overbroad beliefs are likely to impede critical progress toward revenue optimization.

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Introduction

Slots are critical to the success of most casinos, but they often take on an exaggerated importance for Western-market operators catering to a frequently-visiting clientele. While there are some obvious and notable exceptions in Asia, the majority of casinos are heavily reliant on profits from slot machines. Therefore, much attention is given to the management of these complex devices. At the center of this attention is the role of par in the customer experience. Par represents the machine's programmed, long-run, house advantage. Unlike nearly all consumer products and services, this "price" is not marked on reel slots. And there is much debate related to the ability of gamblers to infer reel pars from play alone. Typically, the bulk of slot revenue comes from reel games, hence the increased level of concern for par/price detection.

Many operators and gaming insiders have sternly cautioned against the fallout from increasing pars (Frank, 2017; Gallaway, 2014; Hwang, 2019; Legato, 2019), but results from a series of recent field studies found increased pars to consistently produce greater game-level revenues (Lucas & Spilde, 2020b). Still, some have argued that any such gains come at the expense of the individual gambler's experience, contending that clear declines in play time would be inevitable (Hwang, 2019; Legato, 2019; Meczka, 2017; Wyman, 2020). Their primary concern is that frequent players will eventually notice that their bankroll does not last as long as it once did.

This study aims to better understand how increases in par affect the individual gambler's play time, ceteris paribus. Given the remarkable amount of variance in the outcome distribution of modern slot machines, along with the wallet- and time-related limitations of gamblers, this is far from settled science. The game-level results from the aforementioned field studies represent a key performance measure for operators, but they comprise many individual experiences. And it is possible for these individual contributions to vary. The current study shifts the focus from the collective clientele to the individual gambler, providing a deeper understanding of the impact of par on the gaming experience.

Our results will inform operators who want to further assess the ramifications of increased pars, and manufacturers who wish to design games that produce player experiences that are in step with the needs of casino operators. Academically, the findings will add to a growing literature on how the mathematical parameters of slot machine pay tables affect the player experience. Additionally, we examine the extent to which samples generated by individual players serve as valid proxies for known population parameters, through the lens of Tversky and Kahneman's (1971) Law of Small Numbers. Ultimately, these ends will allow for an objective evaluation of popular industry heuristics related to par and the player experience.

Literature Review

Industry Positions

Industry pundits, consultants, and operators have often conflated the long-term and short-term effects of par, usually failing to recognize the context of the individual gambler's experience (Frank, 2017; Gallaway, 2014; Hwang, 2019; Legato, 2015, 2019; Meczka, 2017; Tottenham, 2019; Wyman, 2020). This is understandable, as management is comfortable with and afforded many aggregated and/or long-term views of slot machine results. For example, one popular heuristic that is advanced as support for the short-term effects of par holds that a game with a 5% par can be expected to provide twice the average play time of a game with a 10% par, ceteris paribus. The following example reveals the assumptions that underlie this conclusion: (1) Game A has a 5% par; (2) Game B has a 10% par; (3) a player engages each of these two games with a fixed bankroll (e.g., \$100); and (4) the player places equal and constant wagers on both games until she loses her entire initial bankroll. In short, this argument hinges on the following simple equations: (1) Game A: $\$100/0.05 = \$2{,}000$ in total wagers; and (2) Game B: $\$100/0.10 = \$1{,}000$ in total wagers. Therefore, subscribers conclude that Game A can be expected to provide twice the play time as Game B, based on the difference in total wagers (i.e., coin-in).

Of course, not all players wager until they lose their entire buy in, and the games do not take a constant percentage of each wager – far from it. That is, this argument assumes a geometric distribution of outcomes and a mandatory bankruptcy condition. The former does not reflect how slot machines work and the latter assumes that no players win, or walk with any credits. The problem here is that games are not experienced in the long term. To the contrary, they are experienced in the extreme short term. Given the amount of variance in the outcome distribution of modern reels, even considerable differences in pars (i.e., population means) can be difficult to detect for an individual player (Singh et al., 2013). This is an issue of contextual congruency, as in viewing the short-term experience through the lens of long-run expectations.

Similarly, in his review of the related research, Hwang (2019) cited elevated wagering volume as a clear marker of differences in pars, but his game-level coin-in estimates were based on shortcut, long-term math that failed to separate the outcomes produced by winners from those generated by losers. It is important to note that the winning players have the capacity to generate disproportionate coin-in, from recycling jackpots (i.e., house money). Moreover, we contend that it is the losing players who would be most likely to invoke abstract measures of gaming value, such as play time. Therefore, it is critical to isolate the results of the losing players when attempting to understand measures of play time, also known as time on device (TOD). From the perspective of the individual player, game-level aggregation of results obscures this important distinction.

Figure 1 was constructed to summarize the views and concerns reflected in the previously cited trade literature. This figure is helpful in establishing the need for the current study, as our focus was on understanding the extent to which the individual gambler's play time is affected by changes in pars. This aim represents the crux of Figure 1, as all of the potential consequences and concerns related to increased pars stem from the assumption of noticeably decreased play time. It is important to limit the applicability of Figure 1 to markets that are characterized by a frequently-visiting clientele; as such concerns are muted in destination markets like the Las Vegas Strip.

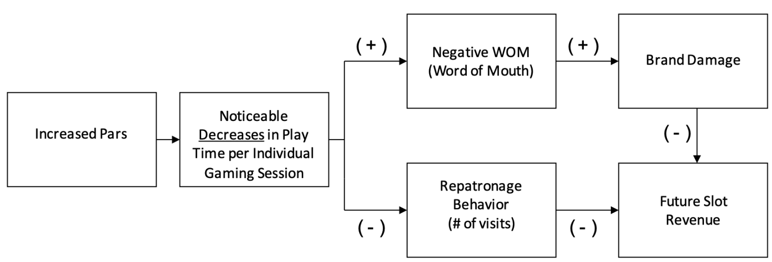

Figure 1
A framework for examining popular industry positions on par and the reel slot player experience in repeater markets.

In short, Figure 1 depicts the belief that increased pars will lead to noticeable differences in play time, which will in turn lead to increases in negative word of mouth among player populations. The increased negative WOM is thought to increase brand damage, resulting in declines in future slot win. Following the lower path of Figure 1, it is also believed that noticeable decreases in play time will lead to fewer visits from existing players, further reducing future slot win.

Academic Studies

Figure 2 was constructed to frame (1) the delimitations of the current study; and (2) the subsequent discussion of the academic literature. As illustrated, our focus was restricted to the play time experienced by losing players, as it was assumed that the winning players would be satisfied by their outcome. Within this limited domain, we examined the studies that have addressed the play-time impacts of each variable appearing within the dotted-line box. Figure 2 is not offered as a valid model of the reel slot player's experience, but rather as a framework for the literature review.

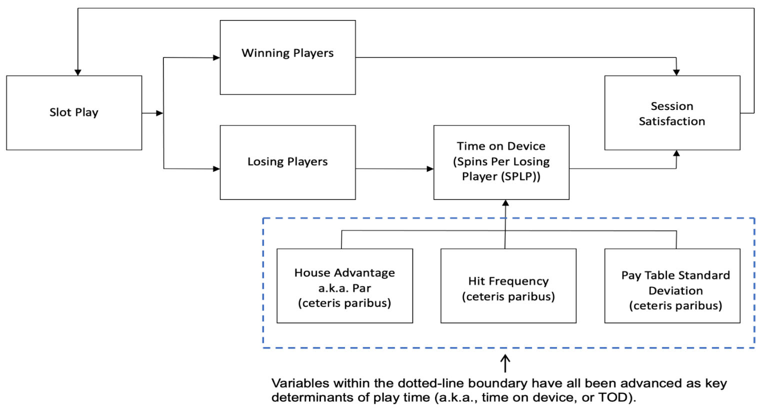

Figure 2
A framework for discussing satisfaction with the reel slot player's experience.

As shown, Figure 2 assumes that the effect of each variable within the dotted-line box is considered under the assumption that all other potential sources of impact on play time are held constant. Of course, variables other than play time could also affect the satisfaction level of losing players, yet it remains as the primary concern among industry insiders.

Hit Frequency & Play Time. Kilby and Fox (1997) sought to examine the then-popular notion that hit frequency was a primary driver of play time. Play was simulated on 10 reel slot machines, attempting to hold par constant. A total of 100,000 players wagered under fixed simulation constraints, on each of the 10 games. These virtual players engaged the games under the following three scenarios: (1) Start with 100 credits and wager until reaching 200 credits, or bankruptcy; (2) Start with 100 credits and wager until reaching 300 credits, or bankruptcy; and (3) Start with 200 credits and wager until reaching 400 credits, or bankruptcy. The results from losing players were separated from those produced by the winners, permitting the calculation of mean spins per losing player (i.e., SPLP) for each game. The results clearly failed to support a positive and monotonic relationship between hit frequency and play time. In hindsight, this was a logical result, as the hit frequency calculation ignores hit magnitude. For instance, its computation treats/considers a top-award jackpot the same as a single-credit payout. Laudable contributions from this work included establishing winning and losing players under real-world engagement criteria, and isolating the results of the losing players.

Standard Deviation & Play Time. Lucas, Singh and Gewali (2007) extended Kilby and Fox (1997) by examining the relationship between pay table variance and SPLP. Specifically, they observed a monotonic relationship between the standard deviation of the pay table and SPLP. As the standard deviation increased, SPLP was found to decrease. This result was produced by simulating play on six different reel games, with the pars held constant at 10%. The standard deviations ranged from 2.37 coins to 12.21 coins. The player engagement criteria were identical to those employed in Kilby and Fox (1997). The results from Lucas et al. held across all three of Kilby and Fox's engagement conditions.

Lucas and Singh (2008) simulated play on five, reel games with Game 1 offering a par of 11.20% and a standard deviation of 2.37 coins, and Game 5 featuring a par of 5.60% and a standard deviation of 7.21 coins. The par incrementally decreased by approximately 1.5 percentage points from Game 1 to Game 5, while the standard deviation incrementally increased by approximately 1.2 coins. Fifty thousand virtual players engaged each of the five games, with each one beginning with 200 credits and wagering until bankrupt, or reaching at least 400 credits. The greatest mean SPLP was generated by Game 1, i.e., the game with greatest par. In fact, the results were monotonic, with decreases in par resulting in decreases in SPLP. This result confounded the general applicability of the relationship between par and play time, as described in the trade literature (Frank, 2017; Hwang, 2019, Legato, 2019; Meczka, 2019; Tottenham, 2019; Wyman, 2020). Specifically, lower pars do not necessarily result in greater play time.

Par & Play Time. A simple example from Kilby and Fox (1997) demonstrated the severe limitations of the argument that lower pars on reel games must result in greater play time. They described a hypothetical reel slot that consisted of 254 blanks and one jackpot symbol, on each of four reels. The payout schedule was simple, consisting of a single line indicating a payout of 4,224,022,374 credits, for the only possible jackpot. This game featured 4,228,250,625 possible outcomes (i.e., 255^4). Given a one-credit wager on each spin, the game would produce a payback percentage of 99.9% (i.e., a 0.1% par). While this game features a remarkably low par, how much play time would you expect it to produce? How many players would call this game loose? It would very likely be the game with the least mean SPLP of any slot machine on the floor. Again, we cannot conclude that lower pars necessarily result in greater play time. Although this is an extreme example, it should raise some red flags for subscribers to the popular heuristic of low pars produce noticeably greater play time.

Staying with the previous example, consider the outcomes produced by 10,000 players who engage the 99.9% payback game with a $100 bankroll, and wager $1 per spin. In terms of the average number of spins, wouldn't you expect all of them to lose every spin? That is, the mean, median, and mode number of spins would all equal 100. After all, the only jackpot is expected to hit once in every 4.2 million spins. Even if one player were lucky enough to hit the jackpot, the median and mode number of spins would still be 100. But if you invoke the math advanced by the industry experts (Legato, 2019; Meczka, 2017; Hwang, 2019, Wyman, 2020), then you would expect the mean number of spins to be 100,000 (i.e., $100/0.001/$1). Of course, this would require (1) someone to hit the jackpot; and (2) continue play until losing all of the jackpot credits to the game. Even then, the mean number of spins would be heavily influenced by the lone winner. And it would seem reasonably safe to assume that this is the one player who would already be satisfied with her experience.

Still, to the best of our knowledge, there have been no published studies isolating the impact of par on play time, under gaming conditions reflective of actual play, on pay tables resembling actual games. Within this general domain, Harrigan and Dixon (2010) conducted simulations designed to compare variables such as the total number of spins per player. They simulated play on two, reel slots, with one par set at 2% and the other at 15%. While they did see significant increases in the mean number of spins on the 2% game, the effect diminished when they examined the median outcomes. They were careful to note this difference in their results. But their simulation required the virtual players to make constant wagers until going bankrupt. This condition forced the relatively few winners to play-off considerable credit balances, hence the difference in their results (i.e., between means and medians). That is, the origin of the significant differences in the means seemed to stem from requiring the outliers (i.e., those who hit jackpots) to wager their credits until reaching a zero balance.

Dixon et al. (2013) and Lucas and Singh (2011) also conducted studies on the differences in outcomes produced by games with different pars. Although Dixon et al. conducted a lab study with live gamblers and Lucas and Singh simulated play, both studies fixed the number of spins on each of the paired reel games. In the case of Dixon et al., their aim was to determine whether players could identify the lower par game, after equal play on both a 2% game and a 15% game. For Lucas and Singh, it was to determine whether the different pars on otherwise identical games would produce significantly different results, given equal play on both games. Dixon et al. reported that all 7 of the subjects who completed their study were able to identify the low-par game. Lucas and Singh found a paucity of significant differences in the outcomes of their paired games, but their pairings featured a maximum difference of 9 percentage points (vs. 13 for Dixon et al.). On balance, the results from Lucas and Singh suggested there was no significant difference in the outcomes, despite the difference in pars. By eliminating the fixed-number-of-spins constraint, the current study seeks to identify whether games with different pars will produce significantly different SPLP results.

In a related research stream, a series of field studies produced results that failed to support the ability of players to detect differences in the pars of otherwise identical games (Lucas & Spilde, 2019a, 2019b, 2020a, 2020b). This conclusion was based on a failure to observe play migration from the high-par games to the low-par games, despite the nearby location of the paired low-par games. Moreover, the majority of these results were observed over 180- to 365-day sample periods, in casinos that were heavily reliant on a frequently-visiting clientele. Additionally, the high-par games produced significantly greater revenues that were sustained over these sample periods. These results also indicated an inability of the frequently-visiting clientele to detect (1) a price shock; and (2) an obvious gaming value. Regarding the latter, one study pitted a 4% penny reel against a 15% version of the same game (Lucas & Spilde, 2020a). If players were able to detect pars from play alone, one would expect to see the 4% game demonstrate obvious performance gains, by nearly all measures. This did not occur, in spite of the game's unusually low par (i.e., for a penny reel), over the course of a 365-day sample.

Cognitive Bias

Many of the industry positions, as well as the results from academic studies, could certainly be influenced by way of cognitive bias. Tversky and Kahneman's (1971) facetiously dubbed Law of Small Numbers is particularly germane to this issue. The authors base this "law" on something they refer to as the representation hypothesis, which is theorized to govern strongly-anchored yet inaccurate intuitions about chance. In their paper, the authors demonstrate how even trained researchers overestimate the extent to which results from small samples represent population parameters. This is the very issue that underlies the explanations of slot play offered by many industry operators, consultants, and pundits (Frank, 2017; Hwang, 2019; Meczka, 2017; Legato, 2019; Tottenham, 2019; Wyman, 2020). That is, to what extent can we expect a gambler's sample of randomly generated outcomes to reflect the population from which it came?

Tversky and Kahneman (1971) note the prevalence of beliefs in the representativeness of random samples, regardless of sample size, or what sampling theory would predict. Further, they note and demonstrate how people tend to believe that every random sequence of events must be reflective of the population parameters, even in remarkably small samples/series. It follows that any sequence of outcomes that deviates from a known population parameter would be expected to quickly self correct, rather than slowly dissipate. Of course, the gambler's fallacy is the best known version of this bias; however, researchers have observed other similar forms in live gaming environments, including stock-of-luck, hot hand, and hot outcome (Croson & Sundali, 2005; Sundali & Croson, 2006). All of these biases stem from a belief in autocorrelation within a non-autocorrelated random sequence of outcomes (Sundali & Croson, 2006).

Regarding the issue of play time, many seem to believe that a single player's gambling activity will produce a sample of outcomes that would be generally sufficient to identify changes in a game's par, i.e., its true population parameter (Frank, 2017; Hwang, 2019; Meczka, 2017; Legato, 2019; Tottenham, 2019; Wyman, 2020). Granted, this sample could be produced over many visits. Still, with the considerable variance in the outcome distribution of the modern slot machine, this would likely require a larger-than-expected sample (Singh et al. 2013). In step with this concern, Tversky and Kahneman (1971) demonstrated the tendency of people to overestimate the degree to which small samples are similar to one another and to the population from which they were generated. With reel slots, even "large" samples may be too small.

Hypotheses

To the best of our knowledge, no published studies have empirically examined and/or demonstrated how differences in pars affect the number of spins experienced by losing players, assuming otherwise equal wagering parameters. These parameters were designed to reflect actual gaming behavior, with precise definitions found in the methodology section. With this end in mind, the following null hypothesis was advanced:

$$H_0 : \mu_{1ij} - \mu_{2ij} = 0.$$

Within H_0, μ_{1ij} represented the mean number of spins produced by a losing player, on a game with par level 1, over i sessions of play, under j wagering parameters. μ_{2ij} represented the same for par level 2. For example, the mean SPLP from play on a 7.9% par was compared against that from a 12.9% par, over 100 sessions of play on each game, where a gambler placed constant wagers from a starting bankroll of 100 credits, and terminated play after reaching either 200 credits or bankruptcy. While the pars varied, the wagering parameters remained constant in all tests of the null. Further details are forthcoming in the Methodology section.

Methodology

The Game

The initial simulations incorporated pay tables from two versions of the same garden-variety penny reel (See Appendix A, Table A1). For purposes of our research question we did modify the two pay tables, attempting to maintain the general structure of the original formats. We were not permitted to identify the title or manufacturer of the game, but we can report that both par versions were licensed for distribution in multiple markets, including Nevada. Additionally, the original pay tables were available in multiple titles (i.e., the math was skinned). More specifically, the art, theme, and graphics differed across the titles, but the game math and structure of play remained the same.

All licensed versions of this game included five, 50-70-stop reels, 40 pay lines, a free-spin feature, and a forced minimum wager of 60 credits per spin. Per the par sheet, the expected value was not affected by the amount wagered per line, or the number of lines played. No progressive jackpots were offered on either version of the game. The two simulated versions featured pars of 12.9% and 7.9%, respectively. This difference represented a 63% increase in the par, from the level of the low-par game. The high-par game was established at 12.9%, as the majority of penny pars range from 12% to 16% (Gallaway, 2014). Legato (2019) confirmed this position, noting the push to move beyond the 12% mark. The low-par game was reduced by five percentage points, to produce the desired par gap for this first look into the impact on SPLP.

The standard deviation was 11.96 credits for the 7.9% game, and 11.84 credits for the 12.9% game. The direction and magnitude of this difference was reflective of the designed game parameters from which operators must choose. That is, the standard deviations within the suite of licensed pars for a particular title are typically not held constant, often declining very slightly with increases in the par (Lucas & Spilde, 2019b). It was for this reason that this parameter was not held constant in the simulations, as the intent was to compare the results from play under real-world game conditions.

The Simulations

An individual gambler's play on each of two par versions was simulated, holding the following variables constant: Starting bankroll, wager per spin; number of gaming sessions on each version; termination criteria; and the number of experimental replications.

Simulations were conducted in R programming language (R Core Team, 2020). Figure 3 was provided to clarify the multiple grains of data aggregation within the simulations. As shown in Figures 2 and 3, and covered in the description of the null hypothesis, the analysis of outcomes was limited to those produced by losing players. This distinction was important, as the simulations did produce both winning and losing players.

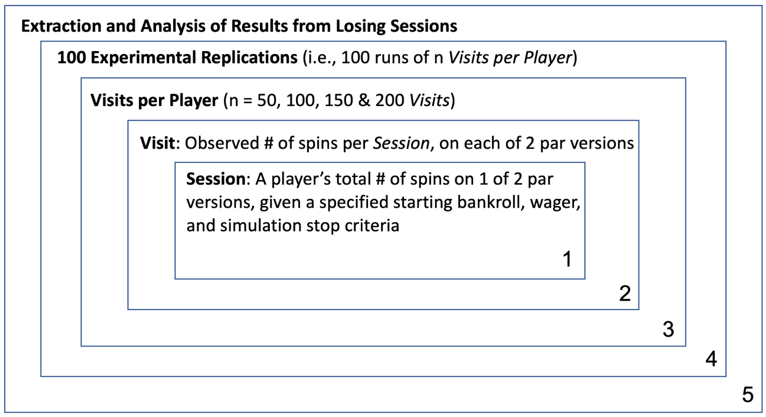

Figure 3
Grains of data aggregation.

At the center of Figure 3 is Frame 1, with its definition of a Session. This level referred to the number of spins produced by a player on a game with a distinct par, given specific engagement criteria (forthcoming). Once completed, this process was repeated for the second par, under the same engagement criteria. The Visit grain in Frame 2 included two observations, both of which represented the number of spins produced by a single Session on each version of par. Frame 3's Visits per Player accumulated output for a specified number of Visits. As shown in Frame 3, the simulations generated data from 50, 100, 150 and 200 Visits. That is, in the case of 50 Visits per Player, the virtual gambler would have produced 50, spin totals on each of the two, par versions. Frame 4 represented output from 100 replications of Frame 3. Continuing the last example, the virtual gambler would produce 99 more sets of 50 Visits, for a total of 10,000 Sessions (i.e., 50 Sessions x 2 par versions x 100 replications = 10,000 Sessions = 5,000 Visits). Alternatively stated, this would produce 100 sets of 50-Visit outcomes. Finally, Frame 5 isolates the results of the losing sessions, within the broader simulation constructs.

At 200 Visits per Player, the virtual gambler would produce 40,000 Sessions, or 20,000 Visits. If conducted by an actual gambler, this would require a considerable amount of time, money, and discipline. Such imposing resource commitments would be required for all versions of the simulations.

There were multiple versions of the player engagement criteria at the Session level. Specifically, the initial bankrolls included 50, 100 and 200 credits. The stop criteria for each level of initial bankroll were set to (1) bankruptcy; and (2) reaching a credit balance of at least twice the initial bankroll. A second version of the simulation included stop criteria of bankruptcy, or reaching a credit balance of at least three times the initial bankroll. All of these criteria were consistent with the parameters of related simulation studies (Kilby & Fox, 1997; Lucas et al., 2007; Lucas & Singh, 2008).

All versions of the simulations featured a constant wager of one credit. This resulted in an average bet equal to 2% of the initial bankroll when starting with 50 credits, but only 0.5% when beginning with 200 credits. The one-credit wager was determined after multiple conversations with casino operators in repeater markets, in order to reflect realistic bet-to-bankroll ratios for penny reels (i.e., where trip-level buy in served as a proxy for trip bankroll). Our fixed wager was also in line with the average bet estimates for penny reels, as reported by repeater-market operators in Legato (2019), i.e., c. US$0.80 per spin.

Data Analysis

All hypothesis testing was conducted by way of two-tailed, independent measures t-tests. As recommend by Welch (1947), unequal variances were assumed. The alpha was set at 0.05 for all hypothesis tests, but a Bonferroni Correction was necessary as the null was tested 100 times at each level of the engagement parameters. This resulted in an alpha of 0.0005 (i.e., 0.05/100). As the results of losing players were parsed from the larger set of outcomes, it did result in an unbalanced design. For example, at the simulation level of 100 visits per player, a gambler could produce 82 losing visits on one par, and 84 on the other. The two-samples t-test from Welch, however, is known to perform well under such conditions (McDonald, 2014).

From Figure 3, the Visits-per-Player levels were selected to represent the following annual patronage patterns: 50 = 1 visit per week; 100 = 2 visits per week; 150 = 3 visits per week; and 200 = 4 visits per week. It was assumed that play did not occur in 2 weeks of the year for reasons such as vacations, business travel, illness, or any other potential interruption of regular/normal visitation. These patronage levels were important, as results from frequent players have been cited as more likely to reveal differences in pars (Frank, 2017; Meczka, 2017; Wyman, 2020). The simulations produced 100 tests of the null, after a "year" of gambling, at each of these four levels of weekly visitation.

Results

There were 100 tests of each null, at each level of Visits per Player, for each set of player engagement criteria. Because of these layered conditions, there were far too many individual data sets to report descriptive statistics for each one. Instead, the results of the simulation outcomes were summarized, per 100 replications. Therefore, Table 1 describes 100 "years" of results produced by losing players, on each game, under each simulation scenario, and each level of visitation. Although generally reflective of the overall simulation results, it is important to note that an individual player would need to play both games according to the prescribed terms for 100 years, to access this aggregated level of output.

In the scenarios featuring a 50–credit bankroll, the mean SPLP on the 12.9% game generally decreased within the range of 0.5 - 7.0%, on a par increase of 63% (i.e., from 7.9% to 12.9%). For the remaining scenarios, the SPLP dropped in the neighborhood of 9-11%, on the same 63%-increase in par. With the average bet held constant at one credit in all scenarios, the bet-per-spin represented 2% of the initial bankroll in the 50-credit condition. Of course, this percentage declines as the initial bankroll increases. The aggregated results supported an inverse relationship between this percentage and the percentage change in SPLP. Figure 4 depicts an example of this relationship. Differences in the percentage of losing visits also seemed to be impacted by this ratio. That is, as the bet-to-bankroll percentage decreased, the difference in the percentage of losing trips generally increased. This gap expansion was most noticeable in the 200-credit bankroll condition.

In all scenarios the median SPLP was less than the mean, indicating something of a positive, yet uniform, skewness in the distributions of outcomes. The standard deviations were greater for the 7.9% game in all but one comparison, but did not appear importantly different.

The null hypothesis was not rejected in any of the 100 tests, within any simulation scenario. Alternatively stated, the null hypothesis was not rejected in any of the 2,400 tests. These results were produced in spite of a 63%-increase in par (i.e., from 7.9% to 12.9%). Given the absence of statistically significant differences in SPLP, the following probability density plots offer a visual representation of the results, demonstrating the challenge of identifying the difference in the pars from play alone (see Figures 5 & 6).

Table 1

Descriptive statistics: Results of 100 replications of each simulated scenario

			Spins per losing player (SPLP)[1]					
Simulation Scenario[2]	# of Visits	Par	Mean	Median	St. Dev.	Min.	Max.	% of Losing Visits[3]
50/0/100:	50	7.9%	167	136	107	50	976	74.1
		12.9%	161	132	101	50	1,182	80.7
50/0/100:	100	7.9%	165	135	104	50	975	74.7
		12.9%	161	130	100	50	927	79.7
50/0/100:	150	7.9%	162	135	100	50	963	74.2
		12.9%	161	132	102	50	1,337	79.6
50/0/100:	200	7.9%	165	135	104	50	1,059	75.0
		12.9%	161	131	100	50	951	79.0
50/0/150:	50	7.9%	190	142	149	50	1,368	81.0
		12.9%	178	136	133	50	1,171	84.9
50/0/150:	100	7.9%	194	143	157	50	1,648	81.6
		12.9%	182	140	137	50	1,346	83.8
50/0/150:	150	7.9%	191	142	148	50	1,618	80.8
		12.9%	180	137	134	50	1,990	83.8
50/0/150:	200	7.9%	194	145	151	50	1,712	81.3
		12.9%	180	137	136	50	1,463	84.4
100/0/200:	50	7.9%	406	337	241	106	2,333	67.0
		12.9%	367	310	206	103	1,832	72.8
100/0/200:	100	7.9%	401	332	243	103	2,365	67.5
		12.9%	368	312	207	109	2,299	72.5
100/0/200:	150	7.9%	401	332	237	106	2,189	67.6
		12.9%	372	313	211	106	2,387	71.9
100/0/200:	200	7.9%	401	331	241	109	2,644	67.9
		12.9%	371	315	208	106	2,632	72.7
100/0/300:	50	7.9%	476	355	359	112	3,283	74.5
		12.9%	428	329	320	112	3,099	78.1
100/0/300:	100	7.9%	484	362	368	100	4,215	73.8
		12.9%	435	328	325	115	3,356	78.4
100/0/300:	150	7.9%	481	355	373	106	4,248	74.7
		12.9%	433	330	321	106	4,215	78.0
100/0/300:	200	7.9%	479	355	373	109	4,479	74.2
		12.9%	434	329	319	106	3,863	78.5
200/0/400:	50	7.9%	1,016	831	622	260	6,894	57.3
		12.9%	911	748	544	278	5,728	63.9
200/0/400:	100	7.9%	1,011	822	613	266	5,912	56.7
		12.9%	909	755	521	251	5,122	63.7
200/0/400:	150	7.9%	996	810	601	249	6,287	57.2
		12.9%	904	748	520	258	5,988	64.2
200/0/400:	200	7.9%	1,004	823	608	270	6,574	56.7
		12.9%	908	750	514	257	4,969	64.8
200/0/600:	50	7.9%	1,351	949	1,104	233	8,919	65.5
		12.9%	1,203	838	997	254	9,893	73.5
200/0/600:	100	7.9%	1,346	932	1,129	253	10,432	66.2
		12.9%	1,202	833	993	247	10,056	73.5
200/0/600:	150	7.9%	1,338	932	1,116	263	10,097	65.9
		12.9%	1,185	832	966	249	10,253	73.6
200/0/600:	200	7.9%	1,331	930	1101	269	12,336	66.0
		12.9%	1,197	838	970	263	10,746	74.0

Notes: [1] All five SLP statistics are expressed in terms of outcomes produced at the session grain. [2] First number represents the starting bankroll (i.e., number of credits), second number represents bankruptcy stop condition, and third number represents credit value (i.e., winning) stop condition. [3] Percentage of losing visits per *n* number of visits, where *n* equals 50, 100, 150 & 200.

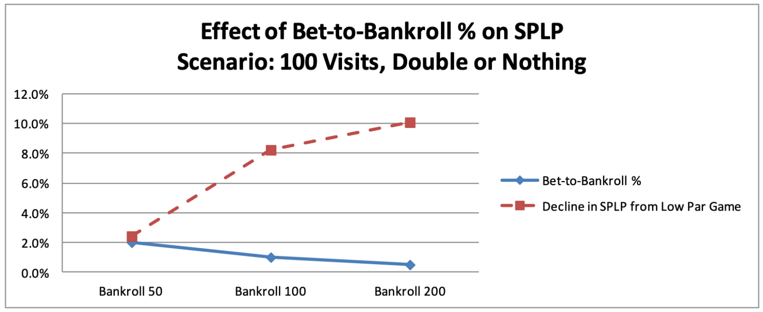

Figure 4
Example of the inverse relationship between Bet-to-Bankroll percentage and SPLP.

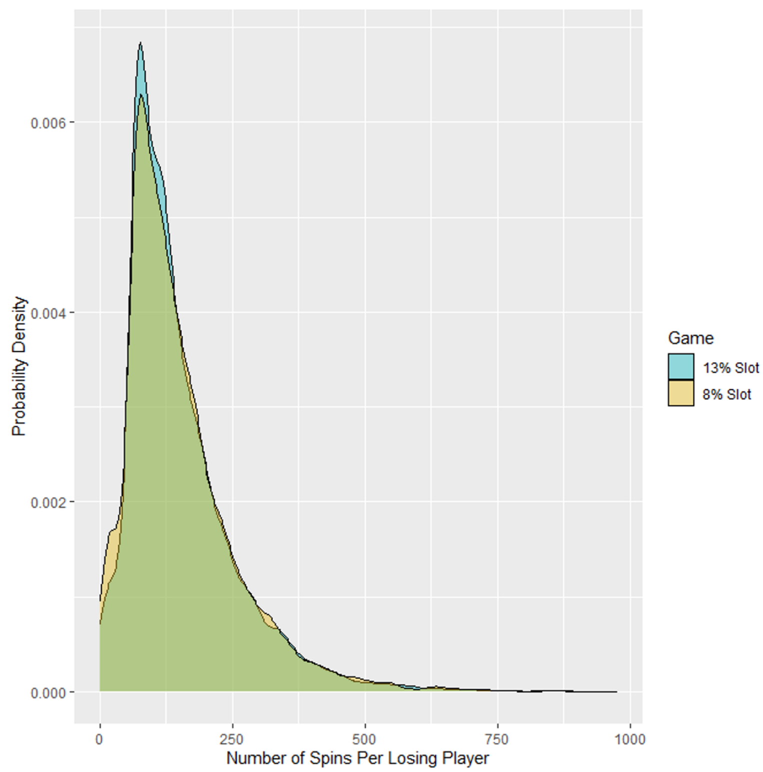

Figure 5
Probability density plot of observed SPLP from 100 replications of 100 sessions, on each game (i.e., 12.9% and 7.9%). Session-level enagagement parameters: Starting bankroll = 50 credits; stop criteria $\geq$ 100 credits, or 0 credits.

With the considerable increase in the pars (i.e., 63%) and the complete lack of rejected nulls, a second pairing of games was simulated under identical engagement parameters to further assess the extent to which the results could be reproduced. These alternative pay tables were also adjusted versions of licensed games, with one par set at 5.0% and the other at 10.0%. This second comparison resulted in a percentage increase in the pars of 100%, considerably greater than the 63%-increase in the original pairing. The pay table variance was greater for both games in the second pairing. Additional pay table details can

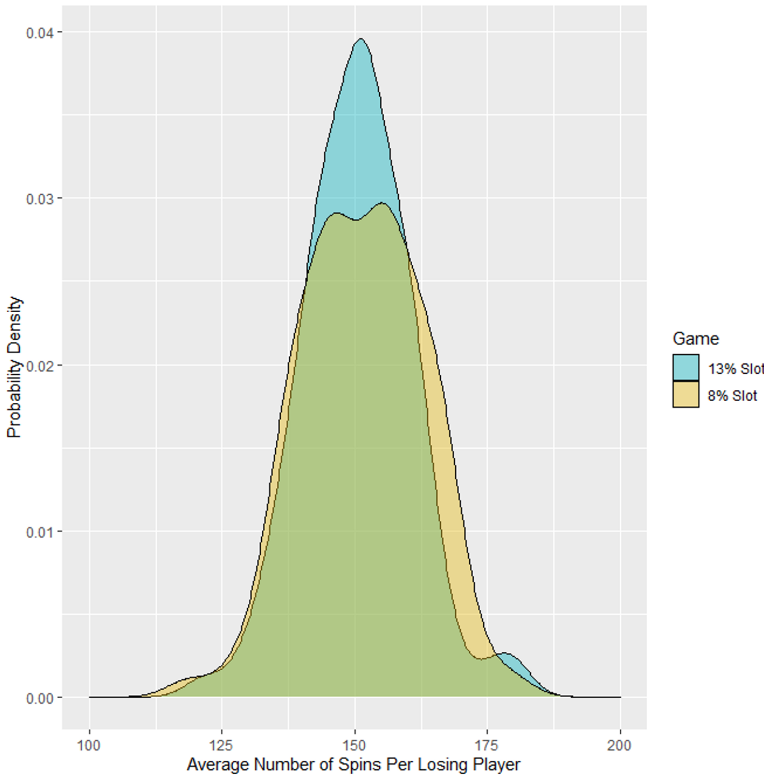

Figure 6
Probability density plot of the mean SPLP for 100 iterations of 100 sessions on both games (i.e., the 12.9% game and the 7.9% game). Starting bankroll = 50 credits; stop criteria ≥ 100 credits, or 0 credits.

be found in Appendix A (see Table A2).

The tables containing the descriptive statistics for Pairing 2 and the hypothesis test results can be found in Appendix B. As shown in Table B1 of Appendix B, the outcomes of the Pairing 2 simulations produced few rejections. In summary, the null hypothesis was rejected an average of 3.2 times per 100 tests, across the 24 simulated scenarios. In comparison to the original pairing, the Pairing 2 games featured lower pars, a greater par gap, and elevated pay table variances. These attributes could have been contributing factors to the differences in the results.

In Pairing 2, the gap in the percentage of losing visits was greatly reduced for all levels of the engagement parameters, posting a maximum of 3 percentage points through the first 16 of the 24 comparisons (i.e., through 100/0/300; 200 Visits). Additionally, despite their lower pars, the SPLP on each of the Pairing 2 games noticeably declined from the comparable Pairing 1 levels. The increased variance in the Pairing 2 pay tables may have been responsible for the declines in both the losing-visit gaps and the SPLP. See Tables B1 and B2 in Appendix B for comparisons against the results from Pairing 1.

Discussion

Given the overall rejection rates, the results appear to support another instance of Tversky and Kahneman's (1971) Law of Small Numbers. With reasonable stop criteria in place, the samples produced by losing players generally did not allow them to produce a statistically different number of spins. This was true for Pairing 1, with its 63%-increase in par, and again for Pairing 2, which featured an increase of 100%. Even with these considerable increases in the pars, the individual player samples were insufficient to detect a difference in the SPLP. These results held through the 150- and 200-visit scenarios, equating to 3 and 4 visits per week, respectively, for an entire year. Within the domain of losing visits, our findings failed to support the popular view that a frequent gambler's play time will noticeably decline with increases in par (Frank, 2017; Gallaway, 2014; Hwang, 2019; Legato, 2015, 2019; Meczka, 2017; Wyman, 2020). As demonstrated, the idea that samples from frequent gamblers are generally sufficient to identify differences in pars seems akin to a belief in the Law of Small Numbers.

The results from Lucas and Singh (2011) were bolstered, in that they profoundly failed to find significant differences in the actual outcomes of play on games with different pars, ceteris paribus. It follows that one would not expect to find a significant difference in the number spins on paired games with similar par differences, i.e., as compared to those analyzed in Lucas and Singh. The paucity of significant differences in the SPLP were in line with the game-level results from the field study research stream (Lucas & Spilde, 2019a, 2019b, 2020a, 2020b). Specifically, no difference in SPLP would deter rational play migration to the lower-par games. It would also allow the higher-par games to subtly accumulate greater revenues, as consistently observed in the field studies.

The stop criteria in the simulations spotlighted the limitations of the popular heuristic regarding the relationship between par and play time, for individual gamblers. Specifically, many industry experts have contended that a 10%-increase in pars will (or must) result in a conforming decrease in spins, ceteris paribus (Legato, 2019; Meczka, 2017; Wyman, 2020).[1] But individual players cannot amass enough trials for this math to bear out, and it unrealistically assumes that all players will lose their entire buy in, on each visit. Similarly, Hwang (2019) argued that lower-par games would produce noticeably different experiences, in terms of play time. But his premise was also based on the manifestation of a long-run average within the individual gambler's experience. He also neglected to make the distinction between the experiences of winning and losing players.

Undoubtedly, some players will produce an outcome that reflects the game's long-run edge, but this would be the exception to the general rule. Most reel games will produce two groups of reasonably similar outcomes – many players who lose their entire buy in, and a few who win big. Taken together, over a sufficient number of trials, their outcomes will produce the game's designed advantage, i.e., its par. To the contrary, it's very likely that this long-term average experience will not be representative of the single-player experience. This would be true for spins, theoretical win, and the dollar value of wagers placed (i.e., coin-in). Similar to the argument advanced in Syed (2019), sometimes the mean describes no one. It is not always an effective measure of central tendency. This appears to be one of those cases.

The Real Reel Experience

It is difficult to imagine that actual players would be able to mentally store more than a year's worth of visit-level outcomes, hence the design of the current study. Plus, reel slot players are not likely to limit play to two games (with the same title), maintain a constant

[1] We offer the following empirical equation to describe their explanation of how a change in par is expected to affect coin-in (i.e., CI) and ultimately, the number of expected spins: $|\%\Delta\text{CI}| = \%\Delta\text{PAR}/(1+\%\Delta\text{PAR})$, where $PAR_{Game\ A} \neq PAR_{Game\ B}$. Necessary assumptions include (1) Games A & B receive identical buy ins (i.e., starting bankrolls); (2) all buy-in credits are wagered until lost; (3) expected CI for each game is computed by dividing the buy-in amount by the game's par; and (4) a constant and equivalent wager on Games A & B. Given these constraints, CI becomes a proxy for the expected number of spins.

wager and buy in, strictly adhere to prescribed stop criteria, and record the number of spins produced on each game. Any failure to comply with these considerable engagement criteria would only make par detection more difficult, in both the comparative and individual game assessment conditions. This is no small point to make, as gamblers are likely to stray from all of these critical parameters.

Given that most reel players are engaging in an entertainment-based activity, it seems unlikely that they would adhere to such disciplined behavior, or have the awareness and knowledge to test the appropriate hypothesis, by way of the appropriate statistical method. It is fair to say, that would be asking a lot of the typical reel slot player.

The more likely scenario would entail the application of heuristics, which are crude measures that ignore important differences in experimental constraints. Additionally, the nature of these heuristics would surely be expected to vary across individual players. All of this would likely occur while oscillating between titles, varying wagers and buy ins, and interrupting play to attend to the occasional phone call or text message. Of course, there is long list of other possible distractions and confounds. And we have not yet considered the presence of cognitive bias among gamblers, as observed in live gaming environments (Croson & Sundali, 2005; Sundali & Croson, 2006). Such beliefs could certainly cause players to stray from the experimental design. Above all, they want to win, and will likely do whatever they believe gives them the best chance to do so.

In summary, the outcomes produced by our simulations were unaffected by the likely undisciplined conditions of actual play. When such conditions are considered, it is difficult to conclude that actual players would be more likely to detect a difference in the number of spins produced by the games, and ultimately, their respective pars. To the contrary, these likely deviations from the experimental script would only impair their chances of noticing a difference. This is an important frame, as slot play does not occur in a lab.

Practical Significance

Aside from the statistical significance, there is the issue of economic/practical significance. It is equally important to review the results from this perspective. Regarding the 200-visit level of the 50/0/100 scenario in Pairing 1, Table 1 shows us that the 12.9% game provided 4 fewer spins than the 7.9% game (i.e., a 200-visit mean SPLP of 161 vs. 165, respectively). As for Pairing 2, the difference was 3 spins per visit, in the same scenario. Keep in mind, these mean SPLP results were computed from 100 replications of 200 annual visits, with play on both games. Staying with Pairing 1, it is difficult to believe that an actual player would ever notice a difference of 4 spins per visit, especially when considering the aforementioned manner in which games are actually played. At 500 spins per hour, this equated to 29 seconds of play. For additional perspective, this level of visitation comprises 4 sessions a week on each game, for 50 weeks. Such patronage would likely exceed Wyman's (2020) guidelines for frequent visitation in regional markets, i.e., "…upwards of 30 to 40 visits per year." Further, in conversations with repeater-market operators, and based on our own experience, this would be a valuable player. Remember, this result would be the same for a player with a buy in of 100 credits, and a constant 2-credit wager. While the SPLP difference was negligible, there was a 5.9%-increase in the percentage of losing trips (i.e., from 75.0% to 79.0%). But this difference also carries a standard error, so it may not be statistically significant. The same 200-visit scenario for Pairing 2 produced no difference in the percentage of losing trips.

In Pairing 1, the difference in the mean SPLP for the 200-visit level in the 50/0/150 scenario expanded, with the 12.9% game producing 14 fewer spins per visit. This equated to a decrease in play time of 101 seconds, assuming 500 spins per hour. But the mean difference in the proportion of losing visits declined to 3.8% (i.e., as compared to the 200-visit-level in the 50/0/100 scenario). Again, there would be an associated standard error for this difference, diminishing the capacity of the player to detect it. In comparison, the same scenario for Pairing 2 generated a mean difference of 16 spins per visit, with losing trips increasing by 1.1%.

We offer these analyses of the 50-credit buy in scenarios, as we believe a bet-to-buy in ratio of 2% to be most reflective of the wagering behavior of actual reel players. There is some support for this assumption in Lucas and Kilby's (2008) description of the credit-granting process for a table game player with little or no history of tracked play. Under such conditions the average observed wager on the player's current trip is divided by either 1.5% or 2%, to arrive at the amount of the player's initial credit line.

Managerial Implications

As others have astutely noted, the play time on reels is considerably impacted by factors other than par (Legato, 2015; Wyman, 2020). More specifically, the amount of play time experienced by a gambler operating from a fixed bankroll could be materially impacted by the following factors: (1) increases in pay table variance; (2) increases in the required minimum wager and/or the cost to cover all pay lines; and (3) increases in the game processing and reel-spin speed. Still, on balance, the industry focus seems to remain on increased pars, as the trade literature is replete with cautionary tales of its impact on the individual gambler's play time (Legato, 2019; Meczka, 2017; Hwang, 2019; Wyman, 2020).

In Pairing 1, our aggregated results do support the admonition from industry experts that there will be an increase in the proportion of losing visits, as par increases (See Gallaway, 2014; Wyman, 2020). But the question remains as to whether players would notice. The differences that we observed were not substantial, and again, the ability of players to notice them would be impeded by the previously-described battery of confounds. Moreover, the bulk of results from Pairing 2 do not support the claims of noticeable increases in the proportion of losing visits (See Table B2). Those outcomes suggest there are conditions that substantially limit this concern. In any case, we are not suggesting that operators make drastic wholesale increases to reel slot pars. To the contrary, any changes should be carefully carried out in a stepped fashion. There are surely limits to the extent to which reel pars can be increased, without significantly affecting SPLP, the proportion of losing trips, or overall revenues.

It is our hope that this paper elucidates the shortcomings of managing "price" and/or the "gaming value" of reel slots according the heuristic that lower pars will provide significantly more play time. As demonstrated under realistic conditions there are severe limitations to this rule of thumb, even within the context of frequent slot play. While there may be other conditions in which it does apply, research is needed to discover these scenarios. So far, all we have are warnings from industry experts regarding the dangers and consequences of increased pars. Again, there may be some, but studies are needed to specifically identify them.

Given the pressure to increase revenues, operators must strike a careful balance between game performance and protecting the customer experience. Clearly, these two ends are closely related, but not mutually exclusive. Based on the results of this study, the findings from Lucas and Singh (2011), and those from a host of field studies (Lucas & Spilde, 2019a, 2019b, 2020a, 2020b), there appears to be some room for improvement with minimal exposure to risk. At some point, the popular assumption that players can detect differences in pars from play alone becomes a managerial liability. Specifically, it will prevent operators from optimizing slot revenues.

The combination of the statistical and economic significance of our results should at least diminish fears of experimenting with increases in pars. Operators may be able to make considerable gains in revenues without destroying the customer experience. The path to such gains is through an improved understanding of how these complex devices produce their outcomes. This path involves a critical evaluation of existing beliefs. Testing the underlying assumptions expands our collective understanding of the individual player's experience.

Limitations & Future Research

The results of this simulation were limited to two modified versions of two, reel slot titles. Still, these pay tables serve as reasonable proxies for the general math structure of many other penny reel games. Across the category, many of the base game, free-spin, and bonus structures are similar, as math constraints and market forces dictate a somewhat homogenous design. Additionally, successful math is often copied with only minor alterations, resulting in a considerable level of similarity.

Of course, it is possible that certain structural differences in pay tables could produce different results. As correctly noted by others, there are multiple ways to manipulate pars (Lucas & Singh, 2011; Lucas & Spilde, 2020a; Wyman, 2020). Additionally, and perhaps more importantly, some of these alterations could materially affect the pay table variance. Therefore, continued simulations such as the ones conducted here would provide valuable insight related to the effects of specific pay table alterations on the player experience. For example, how would changes to the frequency of the following events affect play time: Low-end payouts, entry into free spin and bonus features, and high-end payouts? Also, expansions in the par gap beyond 5.0 percentage points may produce different results. This represents an obvious path for future research, given that the recent stream of field study results have found increased par gaps to produce increased gains in game-level revenues (Lucas & Spilde, 2019a, 2019b, 2020a, 2020b).

Although replicating the simulations with alternate game math would certainly provide additional insight, it can be difficult to obtain and sufficiently decipher the par sheets of licensed games. As is often the case with gaming research, gaining permission and/or access to the necessary data can be challenging. But this is understandable, as the game math is proprietary. Given these constraints, simulations on reasonable proxies of actual games could be sufficiently informative regarding the effects of specific pay table components and structures.

The results of any simulation are a function of the engagement parameters. The outcomes produced here challenge the idea that players would be able to detect differences in pars under the simulated conditions. While the findings of the current study render the general par-play time heuristic equivocal, we cannot generalize our results beyond the assumed pay tables and the assumed player engagement conditions. For example, altering the ratio of the bet-per-spin to the starting bankroll could affect the results. Revisions to the assumptions governing wagering behavior and the simulation's start/stop criteria could also impact the results. While we believe our assumptions were generally reflective of real-world gambling behavior, much opportunity remains for further exploration.

For instance, holding the bet constant does aid in the isolation of the par effect on SPLP, but may not be reflective of actual behavior. It is far more likely that gamblers do not hold their wagers constant. Therefore, multiple levels of betting could be introduced into future simulations to determine the impact on SPLP. This would certainly increase the variance in any player's outcome distribution, which would most likely make rejection of our null hypothesis less frequent, ceteris paribus. Still, the specific impacts of variable wagering behavior on SPLP remain unknown.

Finally, it would be interesting to see what would happen if the variance in the pay table were manipulated, while holding par constant. Based on the results of Lucas and Singh (2008, 2011), significant differences in the SPLP would seem more likely. Of course, the results would depend on the magnitude of the difference in the variances.

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Appendix A

Table A1
Pay table data: Pairing 1

	7.9% Par ($\sigma = 11.96$)		12.9% Par ($\sigma = 11.84$)	
Event	p(Event)	Pays	p(Event)	Pays
E1	0.00001017	2000	0.00001017	2000
E2	0.00043741	400	0.00043741	400
E3	0.00045636	200	0.00045636	200
E4	0.00066446	100	0.00066446	100
E5	0.00073898	75	0.00073898	45
E6	0.00900695	8	0.00900695	6
E7	0.00127095	7	0.00127095	4
E8	0.00115095	5	0.00115095	4
E9	0.00230809	4	0.00230809	3
E10	0.00263227	3	0.00263227	2
E11	0.00153534	2	0.00153534	2
FS10	0.00476218	12	0.00476218	12
FS12	0.00293704	14	0.00293704	14
FS25	0.00817326	15	0.00817326	15
E12	0.09257128	2	0.09257128	2
E13	0.87134432	0	0.87134432	0

Table A2
Pay table data: Pairing 2

	5.0% Par ($\sigma = 18.69$)		10.0% Par ($\sigma = 17.82$)	
Event	p(Event)	Pays	p(Event)	Pays
E1	0.00001017	4000	0.00001017	4000
E2	0.00043741	500	0.00043741	500
E3	0.00045436	400	0.00045436	300
E4	0.00127095	12	0.00127095	10
E5	0.00115095	10	0.00115095	8
E6	0.00230809	6	0.00230809	6
E7	0.00263227	5	0.00263227	5
E8	0.00153534	3	0.00153534	3
FS10	0.00476218	20	0.00476218	20
FS12	0.00293704	16	0.00293704	16
B1	0.00817326	15	0.00817326	15
B2	0.09257128	2	0.09257128	2
E13	0.88134432	0	0.88134432	0

Appendix B

Table B1
Pairing 2.
Summary of null hypothesis test results for losing players.
SPLP: 5.0% Par vs. 10.0% Par

Buy In (in credits) on each of 2 Games, on each Visit	Sim. Stop Criteria (in Credits) for Play on each Game	# of Visits to the Casino by the Player (for Play on both Games)	# of Times Experiment was Repeated	# of Times Null was Rejected (out of 100 tests)
50	0 or 100	50	100	0
50	0 or 100	100	100	0
50	0 or 100	150	100	3
50	0 or 100	200	100	1
50	0 or 150	50	100	1
50	0 or 150	100	100	1
50	0 or 150	150	100	2
50	0 or 150	200	100	3
100	0 or 200	50	100	1
100	0 or 200	100	100	6
100	0 or 200	150	100	9
100	0 or 200	200	100	17
100	0 or 300	50	100	0
100	0 or 300	100	100	5
100	0 or 300	150	100	7
100	0 or 300	200	100	11
200	0 or 400	50	100	0
200	0 or 400	100	100	0
200	0 or 400	150	100	3
200	0 or 400	200	100	4
200	0 or 600	50	100	0
200	0 or 600	100	100	0
200	0 or 600	150	100	0
200	0 or 600	200	100	3

Table B2

Pairing 2.

Descriptive statistics: Results of 100 replications of each simulated scenario.

			Spins per losing player (SPLP)[1]					
Simulation Scenario[2]	# of Visits	Par	Mean	Median	St. Dev.	Min.	Max.	% of Losing Visits[3]
50/0/100:	50	5.0%	129	110	69	50	756	87
		10.0%	128	110	67	50	668	87
50/0/100:	100	5.0%	127	109	66	50	673	87
		10.0%	128	109	66	50	660	88
50/0/100:	150	5.0%	128	110	67	50	803	87
		10.0%	127	109	66	50	672	88
50/0/100:	200	5.0%	129	110	67	50	721	88
		10.0%	126	108	66	50	1,103	88
50/0/150:	50	5.0%	146	120	90	50	1,009	88
		10.0%	130	109	70	50	632	89
50/0/150:	100	5.0%	146	119	92	50	981	88
		10.0%	129	108	70	50	841	89
50/0/150:	150	5.0%	146	118	92	50	1,108	87
		10.0%	129	109	70	50	820	88
50/0/150:	200	5.0%	145	118	91	50	988	87
		10.0%	129	108	72	50	1,014	88
100/0/200:	50	5.0%	291	260	129	109	1,350	77
		10.0%	258	235	100	109	958	78
100/0/200:	100	5.0%	291	260	130	106	1,270	76
		10.0%	259	237	101	106	1,131	79
100/0/200:	150	5.0%	293	261	132	109	1,503	76
		10.0%	259	237	101	107	1,165	78
100/0/200:	200	5.0%	290	260	128	103	1,348	76
		10.0%	259	236	101	103	986	78
100/0/300:	50	5.0%	294	260	134	112	1,406	76
		10.0%	258	235	100	109	1,094	79
100/0/300:	100	5.0%	289	259	128	106	1,183	76
		10.0%	260	237	101	103	1,096	79
100/0/300:	150	5.0%	293	261	131	106	1,314	76
		10.0%	259	235	102	106	1,172	78
100/0/300:	200	5.0%	291	259	131	103	1,396	76
		10.0%	259	237	100	106	1,085	78
200/0/400:	50	5.0%	603	550	242	245	2,672	60
		10.0%	560	502	236	239	2,539	65
200/0/400:	100	5.0%	613	555	265	236	3,741	60
		10.0%	557	503	232	251	2,706	64
200/0/400:	150	5.0%	615	557	261	248	3,774	61
		10.0%	558	551	236	237	2,856	65
200/0/400:	200	5.0%	609	551	249	248	2,689	60
		10.0%	558	499	240	242	3,388	65
200/0/600:	50	5.0%	776	593	548	239	5,265	68
		10.0%	688	529	476	251	4,482	74
200/0/600:	100	5.0%	790	590	573	239	6,702	68
		10.0%	689	525	479	239	5,514	73
200/0/600:	150	5.0%	787	588	577	250	6,910	69
		10.0%	693	529	489	239	7,451	73
200/0/600:	200	5.0%	778	584	565	240	6,304	68
		10.0%	688	522	478	236	5,177	73

Notes: [1] All five SLP statistics are expressed in terms of outcomes produced at the session grain. [2] First number represents the starting bankroll (i.e., number of credits), second number represents bankruptcy stop condition, and third number represents credit value (i.e., winning) stop condition. [3] Percentage of losing visits per *n* number of visits, where *n* equals 50, 100, 150 & 200.