Estimating Kelly Fraction
Gambling & Risk Taking
Las Vegas

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Diffusion Model

Estimating $F$

Estimating $\sigma^2_F$

Warren Buffett

Further Work
Geometric Brownian Motion

F times Kelly betting is modeled by

$$dB = \left( F \frac{\mu}{\sigma^2} \right) \mu B dt + \left( F \frac{\mu}{\sigma^2} \right) \sigma B dW$$
Geometric Brownian Motion

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Letting $\theta = \frac{\mu}{\sigma}$ we have

$$dB = (F \theta^2) B dt + (F \theta) B dW$$
Itoh’s Lemma:

\[
\frac{dB}{B} = F \theta^2 dt + F \theta dW
\]

yields

\[
d \ln B = (F - \frac{1}{2} F^2) \theta^2 dt + F \theta dW
\]
Diffusion Model

**Estimating F**

Estimating $\sigma^2_{\hat{F}}$

Warren Buffett

**Further Work**
Estimating $F$

Set $\rho = \frac{F - 1}{2}$ and $\Sigma = F \theta$. Solving for $F$ yields $F = \frac{2\Sigma}{\rho^2 + \Sigma^2}$. 

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$$F = \frac{2\Sigma^2}{2\rho + \Sigma^2}$$
In order for our estimate to be useful in practice we need some idea of its statistical variability. Assume that for times $t_0, t_1, \cdots, t_N$ we know the bankrolls $B_0, B_1, \cdots, B_N$. Define $\Delta B_k = B_k - B_{k-1}$ and $\Delta t_k = t_k - t_{k-1}$. 
We use standard estimators for the parameters $\rho$ and $\Sigma^2 = \text{Var}(\ln(B))$:

\[
R = \frac{\ln(B_N) - \ln(B_0)}{t_N - t_0}
\]

\[
v = \frac{\sum_{k=1}^{N}[\ln(B_k) - \ln(B_{k-1}) - \Delta t_k R]^2}{t_N - t_0}
\]
Our estimator for the Kelly fraction $F$ is

$$\hat{F} = \frac{2\nu}{2R + \nu}$$

We estimate its standard deviation next.
Diffusion Model

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Estimating $\sigma_\hat{F}^2$

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Further Work
Estimating $\sigma_{\hat{F}}^2$

Taylor approximation of $\sigma_{\hat{F}}^2$

\[ \approx \left| \frac{\partial F}{\partial R} \right|^2 \sigma_R^2 + \left| \frac{\partial F}{\partial v} \right|^2 \sigma_v^2 \]
Estimating $\sigma^2_{\hat{F}}$

Taylor approximation of $\sigma^2_{\hat{F}}$

$$\approx \left| \frac{\partial F}{\partial R} \right|^2 \sigma^2_R + \left| \frac{\partial F}{\partial v} \right|^2 \sigma^2_v$$

$$= \left| \frac{-v}{(2R + v)^2} \right|^2 \left( \frac{v}{t_N - t_0} \right)$$
Estimating $\sigma^2_{\hat{F}}$

Taylor approximation of $\sigma^2_{\hat{F}}$

\[
\zeta \left| \frac{\partial F}{\partial R} \right|^2 \sigma^2_R + \left| \frac{\partial F}{\partial v} \right|^2 \sigma^2_v \\
= \left| \frac{-v}{(2R + v)^2} \right|^2 \left( \frac{v}{t_N - t_0} \right) \\
+ \left| \frac{R}{(2R + v)^2} \right|^2 \left( \frac{2v^2}{(t_N - t_0)^2} \sum_{k=1}^{N} (\Delta t_k)^2 \right)
\]
Estimating $\sigma^2_{\hat{F}}$

Taylor approximation of $\sigma^2_{\hat{F}}$ with $N$ equal time intervals
Estimating $\sigma^2_{\hat{F}}$

Taylor approximation of $\sigma^2_{\hat{F}}$ with $N$ equal time intervals

$$\frac{v^3 + 2R^2v^2}{N(R + \frac{1}{2}v)^4}$$
Diffusion Model

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Further Work
E. Thorp (2006) and W. Ziemba (2003) say that Warren Buffett seems to be a Full Kelly bettor. We find a Kelly Fraction of $F = 0.26 \pm 0.09$ ($\alpha = 0.05$) for closing prices 1980-present.
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Further Work
Methods in discrete time