Does Variable Wagering Affect the Ability of Reel Slot Players to Detect Differences in Pars by Way of Changes in Play Time?

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Abstract

The aim of this study was to better understand the role of a slot machine's house advantage (a.k.a. par) in the individual player's gambling experience. On this issue, the results challenged the inveterate wisdom of the industry. A battery of simulations comparing outcomes produced on slot machines with different pars failed to produce significant differences in play time, i.e., spins per losing player. These simulations were the first to accommodate variable wagering behavior, as identified by player tracking data donated by a Nevada casino operator. The results inform operators and game makers alike as to the ability of gamblers to detect differences in the house advantage, based solely on their results from play. This information is critical to the formulation of revenue optimization strategies, price positioning strategies, and marketing communications. Additionally, critical insight is provided on the slot machine experience, within a profit center vital to the success of many of the world's gaming properties. The absence of significant differences in play time for individual gamblers suggests potential for gains in aggregated slot revenue, without fear of "price" detection by individual gamblers. The findings add to a growing stream of research on the impacts of pay table metrics.

Keywords: slot machines; par; casino management; slot math; operations analysis

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Introduction

Slot revenues are critical to the success of many of the world's casinos. In the extreme, with its approximately 1,300 venues, the Australian club sector is almost completely reliant on casino revenue from electronic gaming devices, as live table games are prohibited. Although the casinos in Macao and Singapore are notable exceptions to a hyper-dependence on slots, even Nevada with its table-heavy Las Vegas Strip garnered 66% of its 2019 casino revenue from slots (Nevada Gaming Control Board, 2019). Further, in Nevada, the departmental operating profit margin in slots is often three to four times that of table games (Lucas & Kilby, 2012). Within the broader category of slots, it is reel games that produce the greatest revenues (i.e., as compared to video poker, video keno, electronic table games, etc.). Their broad appeal is likely due in part to ease of play and the lack of a skill requirement (Bishop, 2021).

This reliance on slot revenue throws all of the associated positioning and revenue optimization strategies into sharp relief. At the center of these strategy discussions is the debate related to the ability of players to detect the concealed house advantage on reel slots. If detected, this obfuscated "price" could result in the loss of play, in step with the Law of Demand. Others have claimed that higher pars will also damage the customer experience by noticeably reducing the play time (Hwang, 2019; Legato, 2019; Wyman, 2020). But a growing body of research suggests that players cannot detect the pars of reel slots, and that higher par games generate significantly greater revenues (Lucas & Spilde, 2019a, 2019b, 2020, 2021).

The outcomes of this study afford casino operators valuable insight on the ability of reel slot players to detect differences in pars. Such information is critical to the assessment of key positioning and revenue optimization strategies related to pars. It also informs on important issues surrounding the adoption and management of server-based gaming systems, where wholesale adjustments to pars can be quickly executed (Pollack, 2007). For game makers, the findings provide new information on the customer experience, which may lead to improvements in future game designs, or the validation of existing configurations. Academically, our results connect to the literature on cognitive bias and the illusory truth effect, as well as to the Law of Small Numbers and the Law of Demand.

Literature Review

Industry Positions

There are many heuristics in casino operations management; some are useful, but others are problematic. For example, one popular heuristic holds that a game with a 6% house edge will generally provide twice the number of spins (i.e., play time) as a game with a 12% house advantage, ceteris paribus (Legato, 2019). That is, the relationship between par and play time is believed to be linear, inverse and proportional, even in the short term. Others have expounded on the considerable limitations of this view (Dunn, 2004; Kilby & Fox, 1997; Lucas & Singh, 2021), yet it persists. Similarly, Hwang (2019) cites inevitable differences in the wagering volume (i.e., coin-in) as a reliable means for players to detect differences in pars. Hwang and others have claimed that increased pars present a critical threat to slot revenues in that players will eventually notice the "price" increases (Frank, 2017), especially those who are frequent gamblers (Meczka, 2017). At the center of these popular beliefs lies the assumption that individual players will be able to detect increased pars, via decreased play time (Hwang, 2019; Legato, 2019; Meczka, 2017; Wyman, 2020). Alternatively stated, the results from play will provide a reliable basis for detection of par.

Law of Demand

In short, the Law of Demand holds that increases in price will lead to decreases in the quantity demanded. It is not specific or helpful, i.e., beyond its basic premise. For example, it does not endeavor to explain the extent of the decrease in quantity demanded. It would be reasonable to assume that at least some would be willing to absorb a 10%

decline in quantity demanded, following a 25% increase in price. This would result in a 12.5% increase in sales, ceteris paribus.

The Law of Demand also does not address the relationship between price and quantity demanded when price must be inferred, rather than expressly marked. This introduces the unique condition surrounding the reel slot player. If we assume that these players can detect changes in unmarked price, they may very well be subject to the Law of Demand. But what if they cannot? Hwang (2019) freely assumes the former when invoking the Law of Demand in his argument for "price" detection. As variance has been shown to have a profound effect on play time for reel slot players (Lucas & Singh, 2008; Lucas et al., 2007), it may become increasingly difficult to detect a difference in the pars of games when the variance in the outcome distribution is increased. One way for slot players to increase this variance is to deviate from the practice of a constant wager on each spin.

Cognitive Bias

The literature establishes the practice of gamblers varying their wagers by way of research on the presence of cognitive bias (Croson & Sundali, 2005; Sundali & Croson, 2006). These researchers observed wagering behavior consistent with the following forms of bias: gambler's fallacy, hot hand, hot outcome and stock-of-luck. All of these behaviors were observed from surveillance video recorded in a live casino environment, involving own-money wagering. This footage revealed a clear prevalence toward one or more forms of bias in wagering behavior.

The practice of varying the wager in an attempt to gain an advantage can be linked to each of the previously listed forms of bias (Sundali & Croson, 2006). For example, consider the gambler who observes the roulette ball settling in a black pocket on five consecutive spins. If this gambler succumbs to the gambler's fallacy she will assume that the probability of the ball landing in a red pocket is increased on the sixth/next spin. This incorrect assumption provides a compelling incentive to increase her wager on red.

In the field, other researchers have corroborated Sundali and Croson (2006) by observing the presence of wagering bias in over half of a 47-subject sample of blackjack players (Keren & Wagenaar, 1985). These players were found to vary their wagers based on the outcome of previous hands. Another lab study found subjects to vary their wagers based on the prior outcomes of a simulated blackjack game (Chau & Phillips, 1995). Specifically, the subjects wagered more after a string of winning wagers than after a spate of losing bets; thus, deviating from the pattern of a constant wager. Croson and Sundali (2005) observed a similar behavior in the field. They found that roulette players who won on a previous spin made more of the same type of bets on the ensuing spin, as compared to those who lost on the previous spin. To the contrary, Leopard (1978) found subjects in her lab study took more risk after losing than winning, over a series of gambling outcomes. She concluded that subjects who lost were taking more risk in an effort to recover their losses.

The previously reviewed research in the area of cognitive bias establishes the prevalence of wagering bias in both the lab and the field, and that it is directly associated with gamblers varying their bets within a single session of play. One aim of this paper is to see how this type of behavior (i.e., varying wagers) impacts the ability of gamblers to detect differences in pars, based solely on their results.

Law of Small Numbers

Tversky and Kahneman's (1971) Law of Small Numbers serves to connect the mechanisms of cognitive bias to the underlying distributions of the pay tables that are common to the high-variance, modern slot machine. First, Singh et al. (2013) describe the unusual nature of these pay table outcome distributions, noting the presence and impact of the high-variance structures. Such designs are ideal for producing small samples that are not representative of population parameters such as par (i.e., the mean). The Law of Small

Numbers is predicated on something its authors referred to as the representation hypothesis. In short, this describes our human tendency to rely on strongly-held yet incorrect conclusions about chance events.

Tversky and Kahneman (1971) demonstrated our willingness to assume that the results of inadequate/small samples are reflective of population parameters. Additionally, they established our tendency to conclude that all sequences of random outcomes are valid proxies for the true population parameters. This occurs even when sample sizes are demonstrably inadequate for drawing such conclusions. Further, they demonstrated that when a series of random outcomes does not resemble a known population parameter, it is expected to quickly self correct. This belief underlies the root source of behavioral phenomena such as the gambler's fallacy and hot outcome bias. Others have argued that it also lies beneath the hot hand and stock-of-luck biases (Gilovich, Vallone & Tversky, 1985).

In summary, we have a device prone to produce samples that are not reflective of the game's long-run design, combined with a human tendency to draw inaccurate and poorly founded conclusions about the representativeness of the results. In addition, the presence of cognitive bias would further complicate the detection of unmarked pars by introducing variable betting behavior. This behavior only increases the already formidable amount of variance in the outcome distribution, greatly increasing the difficulty of identifying a difference in pars.

Pay Table Studies

A review of the academic literature reveals three components of a slot machine's pay table with the potential to influence a gambler's play time. The first is hit frequency, followed by variance (a.k.a. volatility) and the house advantage (a.k.a. par). Hit frequency is defined as the percentage of spins that are expected to result in a payout of at least one credit. Based on the results of multiple simulations, Kilby and Fox (1997) found no evidence of a monotonic relationship between hit frequency and spins per losing player (i.e., SPLP). Notable contributions of this work included a focus on losing players, as winning players were assumed to be satisfied. That is, losing players are the ones most likely to draw on alternative notions of gaming value such as play time (a.k.a. time on device). Additionally, these simulations employed session-level play constraints such as a fixed beginning bankroll, a constant wager, and termination criteria that were reflective of actual play (e.g., quit after bankruptcy, or doubling the beginning bankroll).

Based on unusual patterns in the results of Kilby and Fox (1997), additional simulation studies were conducted by other researchers. Lucas, Singh and Gewali (2007) followed with simulations that held par constant across six different reel slot pay tables, while varying the amount of pay table variance. They found that spins per losing player (SPLP) consistently decreased with increases in the variance, i.e., a monotonic relationship emerged. Based on this finding, Lucas and Singh (2008) sought to test the veracity/limits of a popular industry heuristic, i.e., lower pars necessarily result in increased play time. Their simulations failed to support this notion. In fact, they demonstrated the opposite effect by manipulating the pay table variance. That is, the game with the greatest house edge produced the greatest session-level SPLP, due its low pay table variance. The game with the lowest par produced the least SPLP, because it featured the greatest amount of variance.

Harrigan and Dixon (2010) simulated play on reel slots with a 2% par and a 15% par. Initially, they identified a significant difference in the mean number of spins, in favor of the 2% game. In part, this result was an artifact of the simulation parameters. Specifically, all virtual gamblers played until bankrupt, i.e., no gambler was permitted to finish as a winner. Therefore, all top-award jackpots were required to be wagered until lost. This engagement protocol contributed to the significant difference in mean total spins by creating impactful outliers. The authors thoughtfully noted that the significant difference was absent when comparing the median number of spins.

Lucas and Singh (2011) sought to isolate the effect of par on an individual player's results, on a single visit, and over time. To do so, they created multiple pay tables with identical amounts of pay table variance, but different pars. In short, the results of their simulations failed to support the idea that players could to detect differences in the pars of otherwise identical games, even over decades of regular/frequent play. Moreover, their results indicated that it wasn't so much that players could not detect a difference, but rather there was no difference to detect. But their simulations featured a constant/equivalent wager and fixed number of spins on each game, as the aim of their study was not to measure the number of spins produced by games with different pars. Lucas and Singh (2021) produced similar results after removing the fixed number of spins constraint, but their simulations retained the constant wager assumption.

Dixon et al. (2013) conducted a lab study whereby gamblers played equal-appearing games with different pars (i.e., 2% vs. 15%). Subjects made constant wagers on both games, over a fixed number of spins on each one. These equal-play sessions were repeated over several weeks. Seven subjects completed the experiment, all of whom correctly identified the low-par game. It is worth noting that aside from Harrigan and Dixon (2010), the par gap of 13 percentage points represents the greatest true difference of any extant comparison. This holds for simulation, lab, and field studies.

A series of field studies were conducted to determine the ability of players to detect differences in the pars of otherwise identical games (Lucas & Spilde, 2019a, 2019b, 2020, 2021). One advantage of these quasi-experimental designs was the ability to gather results from players engaging in own-money wagering within a live gaming environment. In spite of differences as great 11 percentage points (i.e., a 4% game vs. a 15% game), the high-par games consistently outperformed their paired low-par counterparts. Nearly all of these experiments were conducted over periods of time ranging from six to twelve months, in casinos that were heavily reliant on a clientele of frequent gamblers. That is, regular players were afforded ample time to discover a difference in the pars. The results supported the ideas that increasing pars can significantly increase revenues, and that players are either not sensitive to differences in pars, or they cannot detect the differences. The latter two conclusions were based on the absence of play migration. Specifically, the observed differences in game-level revenue failed to dissipate over time, indicating that the frequently-visiting clientele was not responding to the egregious "price" shocks in a rational manner.

Hypothesis

All of the simulation and lab studies reviewed herein have featured a constant wager, due to the respective aims of the researchers; yet, we know from the cognitive bias literature that this wagering behavior is not likely. To the best of our knowledge, no one has allowed the wager to vary within a simulation of play on games with different pars. Doing so will present the challenge of detecting a difference in pars under conditions more reflective of actual gambling behavior. Additionally, the results will allow for valuable and insightful comparisons against the outcomes of extant fixed-wagering simulations. In step with these aims, the following hypothesis was advanced:

$$H_0 : \mu_{1ij} - \mu_{2ij} = 0.$$

Consistent with the approach of prior researchers (Kilby & Fox, 1997; Lucas et al., 2007; Lucas & Singh, 2008), only the outcomes of losing players will be tested. Within the null hypothesis, μ_{1ij} indicates the mean number of spins produced by a game with par 1, over i play sessions, under j wagering conditions. The μ_{2ij} term reflects the same, but the outcomes are produced from a game with par 2 (i.e., where par 1 $\neq$ par 2). Details related to the variable wagering parameters are forthcoming in the Methodology Section.

Methodology

Proxies of actual games were created to simulate play on slot machines with different pars. The revised pay tables were modified versions of actual licensed pay tables. The modifications allowed for comparison of more precise pars gaps, while attempting to maintain the fundamental structure of the actual games. The pay table data are shown in Table 1.

Table 1
Pay Table Data

	5% Par ($\sigma = 18.69$)		10% Par ($\sigma = 17.82$)	
Event	p(Event)	Pays	p(Event)	Pays
E1	0.00001017	4000	0.00001017	4000
E2	0.00043741	500	0.00043741	500
E3	0.00045436	400	0.00045436	300
E4	0.00127095	12	0.00127095	10
E5	0.00115095	10	0.00115095	8
E6	0.00230809	6	0.00230809	6
E7	0.00263227	5	0.00263227	5
E8	0.00153534	3	0.00153534	3
FS10	0.00476218	20	0.00476218	20
FS12	0.00293704	16	0.00293704	16
B1	0.00817326	15	0.00817326	15
B2	0.09257128	2	0.09257128	2
E13	0.88134432	0	0.88134432	0

Note: Pay tables as shown in Lucas and Singh (2021). "E" represents Event, "FS" represents Free Spin, and "B"represents Bonus.

The design of the simulations mirrored that employed by Lucas and Singh (2021), differing with respect to the constant wager constraint. The current simulations included variable wagering behavior, based on the records of reel slot play from a live gaming environment. This insight was made possible via proprietary information provided by a Nevada casino operator with a heavy reliance on frequent/repeat gamblers. For purposes of the simulations, it was critical to understand (1) the ratio of the average bet to the starting bankroll; and (2) the general extent to which the bets varied across spins. The process of expressing these two items is unpacked in the following two paragraphs.

As the actual visit-level bankroll of players is generally fluid and cannot be precisely known, the observed actual loss served as the most accurate proxy. For example, players may enter the casino with an intended bankroll of $50, but outcomes such as quicker-than-expected losses or big wins can alter their original intentions. Therefore, the average actual loss serves as the best available representation of a visit-level bankroll. Further, the focus of this study is on losing players, adding to the utility of this particular bankroll proxy. The casino's data indicated that the average bet on a reel slot was 2% of the average loss per day, per player. This ratio was identical to one of the scenarios simulated in Lucas and Singh (2021), i.e., the simulations featuring a 50-credit bankroll and a constant wager of one credit. As a result, the outcomes of the current simulations will provide revealing points of comparison, while also reflecting realistic bankroll and wagering parameters.

The operator's wagering data also indicated an elevated standard deviation in the bet per spin, indicating positive skewness in the distribution. This is an expected artifact of the reel slot design, as forced minimum wagers create a floor in the distribution, while the maximum wager per spin is far less restricted. Regarding the latter, there is a generous and intentional capacity for greater wagers created by the combination of (1) the considerable number of allowable betting lines; and (2) the permissible number of credits wagered per line. To reflect the observed skewness in the wagering behavior, we employed the following the simulation parameters: (1) a 20% chance of a wager at 50% of the average bet; (2) a

60% chance of an average bet; (3) a 10% chance of a wager at 150% of the average bet; and (4) a 10% chance of a wager at 200% of the average bet. This distribution of wagers was clearly influenced by the low-denomination, multi-line games that dominated the slot floor of the donor casino. Still, such a distribution would be generally applicable to most slot floors in markets with a heavy reliance on a repeat clientele.

The previously described constraints reflect a somewhat conservative representation of the observed variance in the wagering behavior of reel slot players. The precise behavior could certainly vary by market and market segment, but the larger point of the simulations is to demonstrate the general impact of variable wagering on the results produced by individual gamblers. This will further inform operators as to the capacity of losing players to detect differences in the pars of games, based solely on the number of spins. Alternatively stated, it will provide insight on the relationship between par and play time, under wagering behavior generally reflective of actual gamblers.

For purposes of comparison against the results produced via a constant wager, our simulations followed the design from Lucas and Singh (2021). Rather than review the details of their design here, we offer the following example of one scenario. Virtual players begin with 50 credits and play until they either triple their credit balance or lose it all. This protocol is applied to each of two games with different pars, producing n spins on each game. This is considered to be one visit, within the quasi-experimental design. Each virtual gambler continues play according to these parameters for a total of 150 visits, i.e., the equivalent of 3 visits per week for 50 weeks. At the end of the 150 visits, a *t* test is performed on the number of visit-level spins produced by each game, but only the losing visits on each game are eligible for inclusion. For example, there may be 122 losing sessions on Game A and 120 losing sessions on Game B. This entire process is repeated 100 times, for a total of 100 *t* test results.

Aside from the variable betting component, the simulation design was identical to the one from Lucas and Singh (2021). There was also one difference in the engagement parameters, i.e., the scenarios featuring the 200-credit buy in with a one credit average bet were not replicated. The wagering data supplied by the Nevada casino operator indicated that this bet-to-buy in ratio (1:200, or 0.005) was not generally reflective of actual gaming behavior.

As for the start/stop criteria, the simulations terminated a player's gambling session after reaching bankruptcy or doubling the initial bankroll. A second set of simulations stopped play after reaching bankruptcy or tripling the initial bankroll. These criteria were based on input from multiple slot mangers operating in repeater markets. Additionally, these start/stop parameters were consistent with those implied by previous researchers (see Kilby & Fox, 1997; Lucas & Singh, 2008, 2021; Lucas et al., 2007)

Like Lucas and Singh (2021), the null was tested via two-tailed, independent samples *t* tests with the unequal variance assumption in place. All two-tailed hypothesis tests were conducted at 0.05 alpha, but a Bonferroni Correction was necessary due to 100 repeated tests of the null. This adjustment reduced alpha to 0.0005 (i.e., 0.05 $\div$ 100). Because only losing visits were included in the *t* tests, it resulted in an unbalanced design. Use of the unequal variance assumption was based on the recommendation in Welch (1947), with McDonald (2014) noting the efficacy of Welch's test in an unbalanced design.

Results

Table 2 provides summary-level statistics for 100 replications of each simulated scenario of play. For example, consider the simulation scenario labeled "50/0/100" at the 150-visit level. In this case, a player would engage each of the two games according to the prescribed engagement parameters, over 150 visits to the casino. This play scenario would be repeated 100 times, resulting in 1,500 sessions on each of two games. Table 2 reports the descriptive statistics for each game, based on the outcomes from these 1,500 sessions. For instance, the mean number of SPLP on the 5.0% game was 134, over 1,500 sessions, while the mean SPLP for the 10.0% game was 124. The far-right column of Table 2 reports the percentage of those 1,500 sessions that resulted in bankruptcy. For the 5.0% game, 84.5% of the 1,500 sessions resulted in the player losing the entire 50-credit buy in, with that percentage increasing to 87.2% for the 10.0% game.

Table 2
Descriptive statistics: Results of 100 replications of each simulated scenario.

			Spins per losing player (SPLP)[1]					
Simulation Scenario[2]	# of Visits	Par	Mean	Median	St. Dev.	Min.	Max.	% of Losing Visits[3]
50/0/100:	50	5.0%	133	110	77	50	856	84.2
		10.0%	124	103	67	50	655	86.7
50/0/100:	100	5.0%	134	110	78	50	684	84.7
		10.0%	124	104	67	50	782	87.2
50/0/100:	150	5.0%	134	110	78	50	780	84.5
		10.0%	124	104	67	50	685	87.2
50/0/100:	200	5.0%	134	111	78	50	900	84.5
		10.0%	125	104	68	50	672	87.2
50/0/150:	50	5.0%	142	111	93	50	872	87.7
		10.0%	128	106	76	50	1,016	89.1
50/0/150:	100	5.0%	145	114	96	50	971	87.4
		10.0%	130	105	77	50	1,164	88.7
50/0/150:	150	5.0%	145	114	98	50	1,464	87.7
		10.0%	129	106	76	50	820	89.1
50/0/150:	200	5.0%	144	113	98	50	1,291	87.8
		10.0%	129	106	76	50	937	89.4
100/0/200:	50	5.0%	291	256	137	105	1,217	78.5
		10.0%	260	232	112	106	998	80.1
100/0/200:	100	5.0%	290	252	138	106	1,350	77.1
		10.0%	262	232	115	100	1,265	80.3
100/0/200:	150	5.0%	290	255	139	105	1,708	77.1
		10.0%	261	233	112	106	1,196	79.4
100/0/200:	200	5.0%	290	252	141	105	1,799	77.2
		10.0%	259	231	111	105	1,212	79.7
100/0/300:	50	5.0%	303	258	164	107	1,590	78.2
		10.0%	270	235	137	105	1,707	81.7
100/0/300:	100	5.0%	302	256	165	105	1,575	78.5
		10.0%	267	234	129	105	1,450	80.6
100/0/300:	150	5.0%	301	256	163	107	1,940	79.3
		10.0%	267	232	132	110	1,715	81.3
100/0/300:	200	5.0%	304	257	171	107	2,124	78.8
		10.0%	268	233	131	105	1,576	81.3

Notes: [1] All five SPLP statistics are expressed in terms of outcomes produced at the session grain. [2] The first number represents the starting bankroll (i.e., number of credits), the second number represents bankruptcy stop condition, and the third number represents credit value (i.e., winning) stop condition. [3] Percentage of losing visits per *n* number of visits, where *n* equals 50, 100, 150 & 200 (all repeated 100 times). The table structure was adapted from Lucas and Singh (2021) to facilitate a direct comparison of results.

Staying with the previous example, it is important to note that the statistics reported in Table 2 would require a player to visit the casino three times per week for 50 weeks a year, for 100 years. Additionally, s/he would need to play both games on each visit. Given that each of the listed simulation scenarios was repeated 100 times, it was not practical to provide descriptive statistics at the level of the *t* test (i.e., the unit of analysis). No individual player would be likely to ever see a summary of this many outcomes. Also, it's important to point out that just because the mean SPLP is generally greater for the 5.0% game in Table 2, it is possible if not quite likely that the 10.0% game produced the greater

SPLP in several of the 100 "years" of play. This conclusion is based on the magnitude of the standard deviations relative to the difference in the mean SPLPs. Notwithstanding this caveat, the mean, median, and standard deviation of the SPLP was greater for the 5.0% game in every comparison.

Table 3 contains the results of the formal hypothesis tests. There was an average of 0.9 rejections of the null across the 16 simulation scenarios. This was down from the 4.2 rejections produced by Lucas and Singh (2021), i.e., with play simulated on the same two pay tables, under the same simulation scenarios. The reduction in the global rejection rate (i.e., from 4.2% to 0.9%) was likely due to increases in the outcome distribution's variance, resulting from the variable betting protocol. The far right column of Table 3 was included to facilitate the comparison of the variable wagering results against those produced by a constant wager on the same two games, under the same player engagement protocol. This comparison demonstrates a clear reduction in the number of rejected nulls in each of the four sections (i.e., 50/0/100; 50/0/150; 100/0/200; and 100/0/300).

Table 3
Summary of Null Hypothesis Test Results for Losing Players.
SPLP: 5.0% Par vs. 10.0% Par

Buy In (in credits) on each of 2 Games, on each Visit	Sim. Stop Criteria (in Credits) for Play on each Game	# of Visits to the Casino by the Player (for Play on both Games)	# of Rejected Nulls (out of 100 tests)	# of Rejected Nulls (out of 100 tests in Lucas & Singh)
50	0 or 100	50	1	0
50	0 or 100	100	0	0
50	0 or 100	150	1	3
50	0 or 100	200	0	1
50	0 or 150	50	0	1
50	0 or 150	100	1	1
50	0 or 150	150	1	2
50	0 or 150	200	1	3
100	0 or 200	50	0	1
100	0 or 200	100	2	6
100	0 or 200	150	0	9
100	0 or 200	200	6	17
100	0 or 300	50	1	0
100	0 or 300	100	0	5
100	0 or 300	150	0	7
100	0 or 300	200	1	11

Note: Table structure adapted from Lucas and Singh (2021) to facilitate a direct comparison of results.

Discussion

The Table 3 results can be interpreted by way of the following example. In the third line, only one of 100 players rejected the null hypothesis. That result comes from each of 100 gamblers playing both games on 150 visits. The outcomes (i.e., SPLP) from each of those games were used to test the null hypothesis. That is, after 150 sessions on each of two games, each of the 100 players employed the results from their losing sessions to test the null. Again, there was only one rejection. Therefore, the entries in the column labeled "# of Rejected Nulls (out of 100 tests)" can be used to compute empirical *p* values (e.g.,

$(100-1)/100=0.99$).

Staying with Table 3, only one scenario produced more than 2 rejections, with six players rejecting the null in the scenario defined as 100/0/200, 200 visits. Even at six rejections, the resulting empirical p value was 0.94. Of course, with a sample size of 200 visits, rejection becomes more likely. Still, across all of the simulated scenarios, it would be difficult to conclude that the players would be able to detect the considerable difference in pars solely from the outcomes of their play.

Alternatively, the Table 3 results could be interpreted as 100 annual tests of the null hypothesis, after n visits by a single player. That is, the result expressed in the third line of Table 3 also represents a single rejection over 100 consecutive annual tests, with each one comprised of 150 visits per year. Of course, it is not likely that any actual gambler would live long enough to produce a body of such results. Still, the outcomes provide insight regarding the informational wherewithal of gamblers to detect differences in pars over time. This is a concern advanced in Meczka (2017).

Lucas and Singh (2008, 2011, 2021) demonstrated the limitations associated with the idea that lower pars necessarily result in significant differences in play time. The results of the current study extended our understanding of the relationship between par and play time, by removing the constant wager assumption featured in those three studies. The variable-betting protocol added to the variance in the outcome distribution, which generally reduced the number of rejected nulls, i.e., in comparison to the rates observed by Lucas and Singh (2021).

With the variable betting assumption more accurately portraying the behavior of actual gamblers, it may help explain the field study results observed by Lucas and Spilde (2019a, 2019b, 2020, 2021). Specifically, variable betting behavior would increase the variance in the outcome distribution and, in turn, increase the difficulty in detecting a difference in the pars of otherwise identical games. In part, this could have allowed the high-par games to consistently outperform their paired, low-par counterparts. Additionally, variable betting likely contributed to the lack of observed play migration, in spite of the substantial increases in the pars of their paired games.

Our results contrasted those from Harrigan and Dixon (2010), in terms of producing significant differences in the mean number of spins. Potential reasons for the difference included their use of a constant wager, the 13 percentage-point gap in their pars, and the requirement for all gamblers to wager until bankrupt (i.e., no winners were permitted). Dixon et al. (2013) also produced outcomes different from ours, with respect to detecting differences in pars. Like Harrigan and Dixon, they examined games with a 13-point par gap and imposed a constant-wager requirement. Additionally, their lab experiment included a fixed number of spins on each game, rather than imposing variable stop parameters.

Regarding the Law of Demand, the results of the simulations suggested that players would likely not be able to detect an increase in the obfuscated “price” (par) based solely on their outcomes. The ability to do so is the cornerstone of Hwang’s (2019) contention that noticeable differences in the wagering volume (i.e., coin-in) would occur on games with different pars. Further, he warned that the resulting decline in play time would signal a price increase, resulting in decreased casino patronage/revenue. It is important to note here that he was using coin-in as a proxy for play time, which is equivalent to using spins, ceteris paribus. Hwang’s concern for consequences stemming from the Law of Demand were not supported by our study, due to a lack of significant differences in the outcomes produced on games with different “prices.”

Managerial Implications

While industry concern for the ability of players to detect differences in the concealed pars of reel slots is well established, it may be based in part on subscription to the Law of Small Numbers. As seen from the results of this study and several others, outcomes produced by individual players are not sufficient to discriminate between even considerable differences in pars. This is because these small samples are not representative of the population parameter known as par (i.e., the mean).

Based on our results, we hope that managers will consider that the relationship between par and play time is not necessarily linear, inverse and proportional. Specifically, a 4% game will likely not provide an individual player twice the spins as an 8% game, ceteris paribus. The small samples produced by individual players simply do not allow for this long-term, highly aggregated relationship to manifest. Factors such as variable betting behavior, the amount of pay table variance, and the failure of players to record and test outcomes would all contribute to the difficulty of detecting differences in pars.

It may be helpful to consider the reality of imposing the simulation requirements on an actual player, when attempting to understand the challenge of detecting differences in pars. For example, the player would need to play both games the same number of times over the course of a full year. This player would have to strictly adhere to the prescribed buy-in, betting, and termination criteria. All session-level outcomes would need to be recorded for each game. The appropriate statistical procedure would need to be employed to test for a difference in means. Any deviation or failure to comply with these terms would render the test inaccurate. When all of this is considered, it seems unlikely that players could detect such differences by way of casual observation, yet this view endures.

If operators were to consider an explanation contrary to the inveterate wisdom of the industry, revenue gains could be possible if not likely. There is a cost to a misplaced fear of par detection, in the form of less-than-optimal revenues. A battery of research results suggests that the customer experience (e.g., play time) is not affected by par in the manner described in the prevailing view. A first step in moving toward optimizing slot revenue is overcoming the fear of “price” detection, where par serves as a proxy for price. This transition would open the door to a new way of thinking, potentially leading toward noticeable gains in revenue performance, and even decreasing negative disconfirmation with the gaming experience. For example, the numerous Nevada billboards proclaiming the availability of loose slots suggest that a player’s bankroll will last longer at the advertised casino. But what happens when it does not? And by the way, it likely will not. Such an experience would negatively disconfirm the player’s expectation and undermine the trust construct — a condition central to customer loyalty.

Par detection is also an important issue for the adoption and management of server-based gaming and mobile gaming systems (Pollack, 2007). This technology allows operators to readily increase or decrease the pars of reel slots. Should the fear of “price” sensitivity prevent them from doing so, they may not be able to optimize revenue during peak business periods (i.e., weekends, holidays, special events, etc.). Due the relative ease of changing pars in the server-based model, an accurate understanding of the associated impacts will be a critical element of any optimization strategy.

Illusory Truth Effect

There remains a considerable space between the industry positions on this topic and the results of this study as well as those from several others, featuring a variety of methodologies and designs. Given this collection of robust results from the academic literature, it is reasonable to consider the presence of the illusory truth effect. This may in part explain the staying power of these industry arguments.

While this paper does not offer a direct test of the illusory truth effect, it can be helpful in explaining the resistance to the results of this work and those of prior researchers. As it applies to this study, the illusory truth effect describes a behavioral phenomenon whereby validity judgments of statements increase when a statement is repeated multiple times, even when that statement is false. Hasher, Goldstein and Toppino (1977) first demonstrated this effect by exposing subjects to plausible yet false statements. This may explain why a considerable number of studies have shown that reel slot players either cannot detect par, do not have the results-based wherewithal to detect par, or are unusually insensitive to increases in par (e.g., Lucas & Singh, 2011, 2021; Lucas & Spilde, 2019a, 2019b, 2020, 2021).

This effect translates to the industry via well-intended heuristics created to help operators understand and/or simplify the considerable complexities of the modern reel slot machine. These explanatory shortcuts are useful in certain contexts, but are often overapplied to incompatible questions, problems, and conditions. For example, consider the aforementioned belief that a game with a 5% par will provide a player twice the coin-in (play time) of a game with a 10% par. This popular heuristic breaks down, when factors such as visit-level bankroll, available leisure time, and the wagering behavior of the individual slot player are considered (Lucas & Singh, 2021). Further, other studies have demonstrated that the effect of the pay table variance is far more impactful on the gambler's play time, especially at the visit level (e.g., Lucas & Singh, 2008). Yet this par-play time heuristic has been frequently repeated for decades, which has resulted in many adopting the notion that par is a valid proxy for a gambler's expected play time (Frank, 2017; Hwang, 2019; Legato, 2019). It follows that operators and industry pundits would contend that players have the ability to identify changes in the pars of games (Frank, 2017; Hwang, 2019; Legato, 2019; Meczka, 2017).

It may be the frequency of these messages that has led to the ardent defense of their validity, despite the growing number of academic studies that suggest otherwise. This would be in line with the illusory truth effect. In the words of Hasher et al. (1977), "If people are told something often enough, they'll believe it." Moreover, Arkes, Hackett and Boehm (1989) demonstrated that the strength of the illusory truth effect is increased when people believe that they are familiar with or knowledgeable about the subject. This familiarity condition may have further increased resistance to the body of academic results. Additionally, researchers have discovered that increases in the fluency of a statement can increase perceived validity (McGlone & Tofighbakhsh, 2000; Reber & Schwartz, 1999), where fluency represents the ease with which a message is decoded by the brain. Therefore, it stands to reason that the simplicity of the previously mentioned par-play time heuristic could aid in its adoption as a true and accurate explanation.

Despite the preponderance of findings that stem from a variety of designs, methodologies, and explanations that falsify the application of par heuristics to the individual player's gaming experience, they seem to endure. These challenges of logic combined with the triangulation of empirical and simulation results makes for a compelling yet contradictory set of conclusions. Still, many resist them.

Limitations and Future Research

Our results are a function of the assumptions governing the simulations. While we believe these assumptions to be conservative, they are necessarily specific. Future researchers may attempt to acquire proprietary data that more precisely describe the variation in the wagering practices of gamblers. Such data could even be game specific.

It is worth noting here that the cognitive bias literature and the proprietary wagering data supplied by the Nevada casino operator both established a considerable presence of variable wagering in live gaming environments. Therefore, future attempts to understand the outcomes from reel slot play under such conditions should address this reality. Of course, differences in coded wagering behavior could produce different results.

Differences in pay tables may also produce differences in outcomes. We chose pay tables previously simulated in Lucas and Singh (2021) to demonstrate how varying the bet could affect the outcomes produced by players, as compared to results produced by a constant wager. The intent was to isolate the effect of variable wagering. Still, simulating play on different pay tables would increase our understanding of the relationship between par and play time. For example, differences in the par gaps of paired games may provide additional insight. The same could be said for differences in the bonus and free-spin features in pay tables.

The effects observed in this study could be muted by high-denomination games that considerably restrict the range of allowable wagers. For example, a maximum allowable wager of five credits would constrain the ability of the gambler to vary her wager. Therefore, the amount of variance added to the outcome distribution would be decreased, potentially increasing the possibility of a significant difference in SPLP on such games. While our focus was on the experience of losing slot players, we acknowledge that it is possible for certain winning players to also be dissatisfied. Although a much smaller population and arguably less of a pressing concern, future researchers may wish to study the judgments of the outcomes produced by winning players. Going forward, there are a nearly endless number of experimental configurations to examine, but the results of this work demonstrate that the relationship between par and an individual gambler's play time may be importantly different from the popular understanding. Still, more work is needed to determine the limits of our conclusions.

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