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He Gao
Beijing Normal University, gaohe@bnu.edu.cn

Shun-Ke Ai
University of Nevada, Las Vegas

Zhou-Jian Cao
Beijing Normal University

Bing Zhang
University of Nevada, Las Vegas, bing.zhang@unlv.edu

Zhen-Yu Zhu
Xiamen University

See next page for additional authors

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Authors

He Gao, Shun-Ke Ai, Zhou-Jian Cao, Bing Zhang, Zhen-Yu Zhu, Ang Li, Nai-Bo Zhang, and Andreas Bauswein

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RESEARCH ARTICLE

Relation between gravitational mass and baryonic mass for non-rotating and rapidly rotating neutron stars

He Gao[1,†], Shun-Ke Ai[2,1], Zhou-Jian Cao[1], Bing Zhang[2], Zhen-Yu Zhu[3,4], Ang Li[3], Nai-Bo Zhang[5], Andreas Bauswein[6]

[1] *Department of Astronomy, Beijing Normal University, Beijing 100875, China*
[2] *Department of Physics and Astronomy, University of Nevada Las Vegas, NV 89154, USA*
[3] *Department of Astronomy, Xiamen University, Xiamen 361005, China*
[4] *Institute for Theoretical Physics, Max-von-Laue-Strasse 1, 60438 Frankfurt, Germany*
[5] *Shandong Provincial Key Laboratory of Optical Astronomy and Solar-Terrestrial Environment, School of Space Science and Physics, Institute of Space Sciences, Shandong University, Weihai 264209, China*
[6] *GSI Helmholtzzentrum für Schwerionenforschung, Planckstraße 1, 64291 Darmstadt, Germany*
Corresponding author. E-mail: [†] *gaohe@bnu.edu.cn*
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With a selected sample of neutron star (NS) equations of state (EOSs) that are consistent with the current observations and have a range of maximum masses, we investigate the relations between NS gravitational mass M_g and baryonic mass M_b, and the relations between the maximum NS mass supported through uniform rotation ($M_{\max}$) and that of nonrotating NSs ($M_{\rm TOV}$). We find that for an EOS-independent quadratic, universal transformation formula ($M_b = M_g + A \times M_g^2$), the best-fit A value is 0.080 for non-rotating NSs, 0.064 for maximally rotating NSs, and 0.073 when NSs with arbitrary rotation are considered. The residual error of the transformation is $\sim 0.1 M_\odot$ for non-spin or maximum-spin, but is as large as $\sim 0.2 M_\odot$ for all spins. For different EOSs, we find that the parameter A for non-rotating NSs is proportional to $R_{1.4}^{-1}$ (where $R_{1.4}$ is NS radius for $1.4 M_\odot$ in units of km). For a particular EOS, if one adopts the best-fit parameters for different spin periods, the residual error of the transformation is smaller, which is of the order of $0.01 M_\odot$ for the quadratic form and less than $0.01 M_\odot$ for the cubic form ($M_b = M_g + A_1 \times M_g^2 + A_2 \times M_g^3$). We also find a very tight and general correlation between the normalized mass gain due to spin $\Delta m \equiv (M_{\max} - M_{\rm TOV})/M_{\rm TOV}$ and the spin period normalized to the Keplerian period $\mathcal{P}$, i.e., $\log_{10} \Delta m = (-2.74 \pm 0.05) \log_{10} \mathcal{P} + \log_{10}(0.20 \pm 0.01)$, which is independent of EOS models. These empirical relations are helpful to study NS-NS mergers with a long-lived NS merger product using multi-messenger data. The application of our results to GW170817 is discussed.

Keywords gravitational waves

1 Introduction

The structure of neutron stars (NSs) depends on the poorly understood physical properties of matter under extreme conditions, especially the equation-of-state (EOS) of matter at the nuclear density [1, 2]. One way to diagnose the NS EOS is to constrain the NS maximum mass $M_{\rm TOV}$ of nonrotating stellar models. For instance, observing an NS with a sufficiently large mass could set an interesting lower limit on the NS maximum mass, thus ruling out a set of soft EOS models. Up to now, the largest well-measured NS mass is $\sim 2.01 M_\odot$ for PSR J0348+0432 [3].

Binary neutron star (BNS) mergers have been suggested to be good targets for constraining the NS maximum mass in two different approaches: i) For one particular BNS merger event, if the gravitational mass of the BNS merger remnant, its rotational properties, and its life time until black-hole formation can be inferred from gravitational wave (GW) and electromagnetic (EM) observations, one can directly infer the NS maximum mass [4–8]; ii) Supposing that the internal X-ray plateau followed by a very rapid decay as observed in some SGRBs signals the formation of a long-lived, supramassive NS after the merger [9–11], statistical analyses on historical SGRB X-ray afterglow data can be used to estimate the fraction for different BNS merger products, thus placing constraint on the NS maximum mass [11–15].

For both approaches, it is important to estimate the

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remnant mass for the merger product. In NS problems, two masses, i.e., the baryonic mass M_b and the gravitational mass M_g, are usually discussed. The former (M_b) is theoretically relevant, since it is directly connected to the mass of the iron core of the progenitor massive star. In the problems of NS-NS mergers, it is the baryonic mass that is conserved. The latter (M_g), on the other hand, is directly measured from observations, and is smaller than M_b due to the subtraction of the binding energy. Studying the general relationship between M_g and M_b becomes ever more important, as the accuracy of NS mass measurements becomes sufficiently precise to i) set interesting lower limits on the NS maximum mass [1, 16]; ii) to study the NS initial mass function [17–19]; iii) to study the neutrino emission from core collapse supernovae [20–23], and so on.

In order to calculate the remnant mass of BNS mergers, the conversion between the gravitational mass and baryonic mass for both non-rotating and rapidly rotating NSs is needed. Specifically speaking, it is generally assumed that NS-NS mergers conserve the baryonic mass, with a small fraction, $M_{\rm ej} \lesssim 10^{-1} M_\odot$, ejected during the merger [e.g., 24–28]. On the other hand, GW observations can only provide the total gravitational mass of the system with the infinite binary separation, $M_{g,\rm tot}$. In order to calculate the mass of the merger remnant, one needs to first convert $M_{g,\rm tot}$ to total baryonic mass $M_{b,\rm tot}$, then convert $M_{b,\rm tot} - M_{\rm ej}$ back to the remnant gravitational mass $M_{g,\rm rem}$. According to the observed galactic NS binary population, before the merger, the relatively low spin period is expected for both NSs in the binary [29, for details]. Therefore, the relation between M_g and M_b for non-rotating NSs should be enough in converting $M_{g,\rm tot}$ into $M_{b,\rm tot}$. However, since the newborn central remnant must be rapidly spinning, if the remnant is a uniformly rotating NS, the conversion from M_b to M_g for a rapidly rotating NS becomes essential[1)]. Finally, the maximum gravitational mass is significantly enhanced by rapid rotation. For some constraints on $M_{\rm TOV}$, the relation between the maximum NS mass supported through uniform rotation (marked as $M_{\rm max}$) and $M_{\rm TOV}$ is also required.

In this work, we consider rigidly rotating stellar equilibrium models (treated by numerical relativity methods) and aim to find relations between M_g and M_b, and between $M_{\rm max}$ and $M_{\rm TOV}$, for a selected sample of EOSs, which are consistent with current observations (i.e., $M_{\rm TOV}$ is larger than $\sim 2.01 M_\odot$) and have a range of maximum masses. We apply our results to GW170817 to estimate its merger remnant mass. Here we only consider rigidly rotating non-magnetized NSs at zero temperature. In the literature, the relationships between M_g and M_b or between binding energy $M_b - M_g$ and M_g have been investigated by many authors, most of which have focused on nonrotating non-magnetized NSs at zero temperature [e.g., 17, 20, 31, 32], with some studying rotating NSs [e.g., 33]. The comparisons between our results and previous results are discussed in detail. NS structural quantities in the differential rotation phase are not touched in this work, which has been discussed in previous works [e.g., 33–37].

1) It is worth noticing that besides knowing the relations between M_g and M_b, one still faces some problems such as extracting the ejecta mass information from EM observations or estimating the initial spin period of the rigidly rotating NS, especially when significant angular momentum loss (e.g., due to the strong viscous spin down [30]) is considered.

2 NS structure equations and NS EOS

We consider the equilibrium equations for a stationary, axially symmetric, rigid rotating NS, within a fully general relativistic framework. The spacetime metric can be written in the form

$$\begin{aligned} ds^2 = &-e^{2\nu}dt^2 + r^2\sin^2\theta B^2 e^{-2\nu}(d\phi-\omega dt)^2 \\ &+e^{2\alpha}(dr^2+r^2\,d\theta^2), \end{aligned} \tag{1}$$

where the potentials ν, B, ω, and α only depend on r and θ, and have the following asymptotic decay [38]:

$$\begin{aligned} \nu &= -\frac{M}{r} + \frac{B_0 M}{3r^3} + \nu_2 P_2(\cos\theta) + \mathcal{O}\left(\frac{1}{r^4}\right), \\ B &= 1 + \frac{B_0}{r^2} + \mathcal{O}\left(\frac{1}{r^4}\right), \\ \omega &= \frac{2I\Omega}{r^3} + \mathcal{O}\left(\frac{1}{r^4}\right), \end{aligned} \tag{2}$$

where M, I and Ω are the NS mass, moment of inertia, and angular frequency, respectively. B_0 and ν_2 are real constants.

We describe the interior of the NS as a perfect fluid, whose energy-momentum tensor reads

$$T^{\mu\nu} = (\rho+P)u^\mu u^\nu + P g^{\mu\nu}, \tag{3}$$

where ρ and P are the energy density and the pressure, and u^μ is the 4-velocity. Given a particular NS EOS, we use the public code `RNS` [39] to solve the field equations for the rotating NS.

Our selection of realistic (tabulated) EOSs (as listed in Table 1) are SLy [40], WFF1 [41], WFF2 [41], AP3 [42], AP4 [42], BSK21 [43], DD2 [44], MPA1 [45], MS1 [46], MS1b [46], with $M_{\rm TOV}$ ranging from $2.05 M_\odot$ to $2.78 M_\odot$.

3 Relation between M_g and M_b

Given a particular EOS, the baryonic mass (M_b) and gravitational mass (M_g) for a rigid rotating NS are determined by the values of central energy density (ρ_c) and spin period (P). In order to figure out the relationship between M_b and M_g, for each EOS, we calculate a series of M_b and M_g for different spin periods. We first present the results for the non-spin case and the maximally-spinning

Table 1 Characteristic parameters for various EOSs.

	$M_{\rm TOV}$ ($M_\odot$)	$R_{1.4}$ (km)	A^*	A_1^*	A_2^*	A^+	A_1^+	A_2^+	P_k (ms)	α ($10^{-10}{\rm s}^{-\beta}$)	β	a	b
SLy	2.05	11.69	0.083	0.041	0.023	0.066	0.042	0.011	0.55	2.311	−2.734	0.086	0.209
wff1	2.14	10.40	0.102	0.046	0.029	0.076	0.050	0.012	0.47	3.111	−2.729	0.106	0.279
wff2	2.20	11.1	0.092	0.043	0.026	0.070	0.046	0.011	0.50	2.170	−2.694	0.096	0.270
ap4	2.22	11.36	0.090	0.045	0.023	0.069	0.046	0.010	0.51	2.095	−2.721	0.094	0.251
Bsk21	2.28	12.55	0.079	0.039	0.020	0.061	0.038	0.010	0.60	1.958	−2.799	0.083	0.257
ap3	2.39	12.01	0.087	0.046	0.019	0.066	0.047	0.007	0.55	1.883	−2.780	0.091	0.288
dd2	2.42	13.12	0.077	0.046	0.014	0.059	0.044	0.006	0.65	2.269	−2.811	0.079	0.259
mpa1	2.48	12.41	0.082	0.046	0.017	0.063	0.046	0.007	0.59	2.584	−2.693	0.088	0.284
ms1	2.77	14.70	0.069	0.042	0.010	0.053	0.038	0.005	0.72	5.879	−2.716	0.072	0.263
ms1b	2.78	14.46	0.070	0.043	0.011	0.054	0.040	0.005	0.71	4.088	−2.770	0.074	0.283

*Best fit vales for non-spinning NS cases.

+Best fit values for Keplerian spinning NS cases.

(Keplerian) case. We then sample the spin period P as $\{1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 3.0, 4.0\}$ P_k, where P_k is the Keplerian period corresponding to $M_{\rm g,max}$. We consider both relations describing all the EOSs and the relations for individual EOSs.

3.1 Non-rotating NSs

For a non-rotating NS, transformation of the baryonic NS mass to gravitational mass is commonly approximated using the quadratic formula

$$M_b = M_g + A \times M_g^2, \tag{4}$$

where A is a constant, which is quoted as 0.075 by Timmes *et al.* [17]. Throughout the paper, M_b and M_g are in units of $M_\odot$. Here we treat A as a free parameter in order to find out its best-fit value for each adopted EOS (results are collected in Table 1). If each EOS adopting its own best fit A value, the residual error of the transformation between M_g and M_b is in order of a few $10^{-2}M_\odot$, see Fig. 1 for details. Putting (M_g, M_b) results for all adopted EOS together, the overall best fit A value is 0.080. In this case, the residual error of the transformation from M_g to M_b is as large as $0.12M_\odot$ and the residual error from M_b to M_g is as large as $0.09M_\odot$. By comparison, when A is adopted as 0.075, the residual error of the transformation from M_g to M_b is as large as $0.14M_\odot$ and the residual error from M_b to M_g is as large as $0.11M_\odot$.

In order to further reduce the residual error of the transformation, we test a new cubic formula

$$M_b = M_g + A_1 \times M_g^2 + A_2 \times M_g^3. \tag{5}$$

We determine the best fit values of A_1 and A_2 for each adopted EOS (collected in Table 1). If each EOS adopts its own best-fit values, the residual error of the transformation between M_g and M_b is in the order of a few $10^{-3}M_\odot$, which is almost one order of magnitude better than the quadratic cases. See Fig. 1 for details. Putting all EOSs together, the overall best fit A_1 and A_2 values are 0.0729 and 0.0032. In this case, the residual error of the transformation from M_g to M_b is as large as $0.12M_\odot$ and the residual error from M_b to M_g is as large as $0.09M_\odot$, which are in the same order as the quadratic case.

In the literature, some universal relations between the binding energy $M_b - M_g$ and the neutron star's compactness have been proposed [31, 32], which are also applicable for transformation between M_b and M_g. Here we find that for the quadratic transformation, one has $A \times R_{1.4} \approx 1$ for all adopted EOS, where A is the best fit value and $R_{1.4}$ is the NS radius (in unit of km) for $1.4M_\odot$. We thus propose a new universal relation

$$M_b = M_g + R_{1.4}^{-1} \times M_g^2, \tag{6}$$

with 1.8% relative error. We note that in our proposed relation, only $R_{1.4}$ is invoked in the relation, instead of using the compactness number proposed in previous works [e.g., 31, 32]. The accuracy is similar to the previous ones in terms of the residual error of the transformation, but our proposed relation is more practical since it does not involve the calculations of the radii corresponding to different masses. In case that the binding energy $M_b - M_g$ is measured independently, such a universal relation may be helpful to constrain $R_{1.4}$, which helps to distinguish NS EOSs [31].

3.2 Maximally rotating NSs

It is generally believed that the NS produced from NS-NS mergers may have an initial spin period close to the Keplerian period, which is supposed to be the minimum spin period of a rigidly rotating NS with a certain M_b. Also, M_g in this case is known as the maximum gravitational

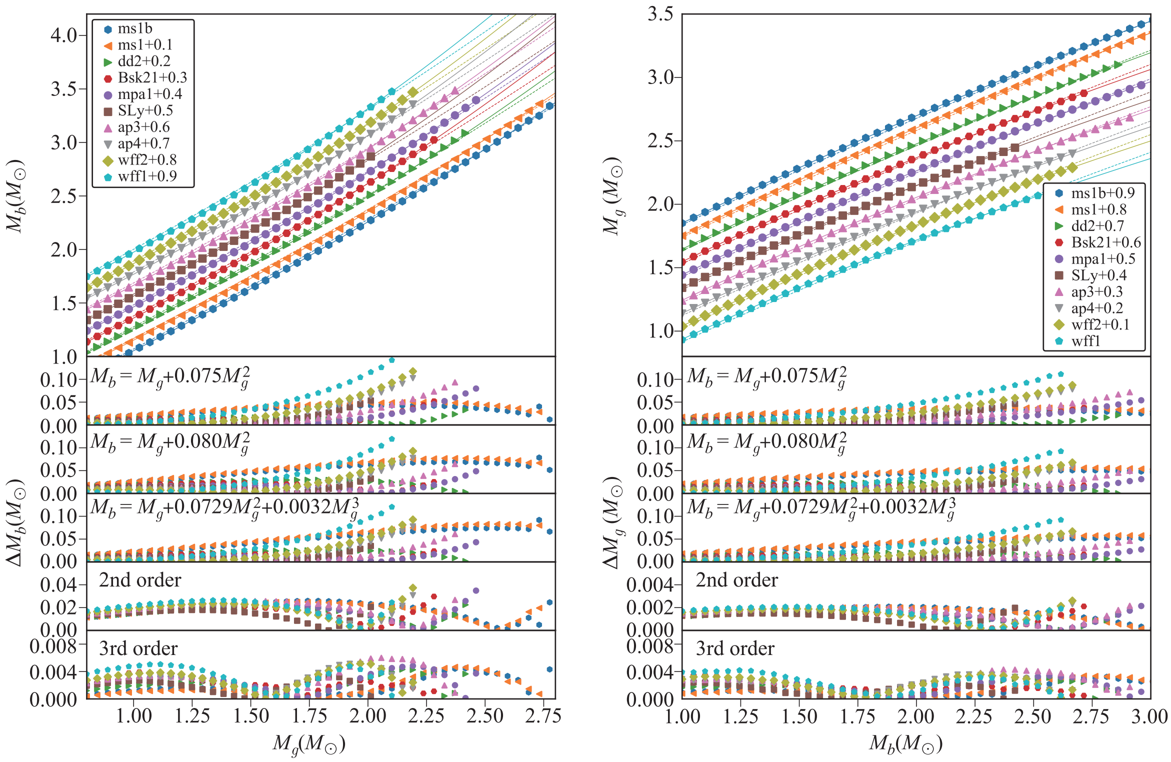

Fig. 1 The relation between the baryonic mass (M_b) and gravitational mass (M_g) for a non-rotating NS and the residual error for different transformation formulae. Different colors denote different EOSs. The solid and dashed lines represent the best fitting results with the quadratic and cubic formulae, respectively, with the best-fit parameters adopted for each EOS.

mass for that M_b. We still apply the quadratic and cubic formulae to fit the relation of M_b vs. M_g. For the quadratic formula, it is worth noting that if $A = 0.075$ is adopted, the residual error of the transformation from M_g to M_b could be up to $0.22M_\odot$, and that of the transformation from M_b to M_g could be up to $0.15M_\odot$, which is imprecise to estimate the remnant mass of NS-NS mergers. Here we find that in the maximal-rotation case, the overall best fitting value is $A = 0.064$ with the residual errors $0.11M_\odot$ and $0.08M_\odot$, respectively, for transformation from M_g to M_b and vice versa. The newly proposed coefficient could significantly reduce the residual error of the transformation. For the cubic formula, the overall best fitting value are $A_1 = 0.0561$ and $A_2 = 0.0033$. But it cannot reduce the residual error effectively compared with the quadratic formula.

When we adopt the best-fit parameters for each EoS, the residual error of transformation between M_g and M_b is in the order of a few $10^{-2}M_\odot$ for the quadratic formulae and $10^{-3}M_\odot$ for the cubic formulae. See details in Fig. 2.

Similar to the non-spin case, we find $A \times R_{1.4} \approx 0.78$ for the quadratic formula. We thus have a universal formula in the maximally rotation case as

$$M_b = M_g + 0.78R_{1.4}^{-1} \times M_g^2 \tag{7}$$

with a 1.3% relative error.

3.3 General NSs with arbitrary rotation

For individual EOSs, we also fit their M_b and M_g relations with both the quadratic and cubic formulae for the situations with different rotation periods. We find following conclusions which could be applied to all adopted EOSs: i) The relation between M_b and M_g is different for different spin periods. ii) If one adopts the best-fit parameters for each spin period, the cubic formula is almost one order of magnitude better than the quadratic formula in terms of the residual error of the transformation. For instance, the residual error of the transformation from M_g to M_b is up to $0.06M_\odot$ for the quadratic case and is up to $0.01M_\odot$ for the cubic case. iii) Putting together the (M_g, M_b) results for different spin periods, with the overall best-fit values, the cubic formula is no better than the quadratic formula. For instance, the residual error of the transformation from M_g to M_b is up to $0.068M_\odot$ for the quadratic case and is up to $0.062M_\odot$ for the cubic case. For a comparison, when A is fixed to 0.075, the residual error of the transformation from M_g to M_b is as large as $0.11M_\odot$. The best-fit values for the quadratic and cubic formulae are collected in Table 2.

Considering (M_g, M_b) results for different EOSs and different spin periods together, the overall best-fit value is $A = 0.073$ for the quadratic formula and $A_1 = 0.078$ and $A_2 = -0.0018$ for the cubic formula. In this case, the

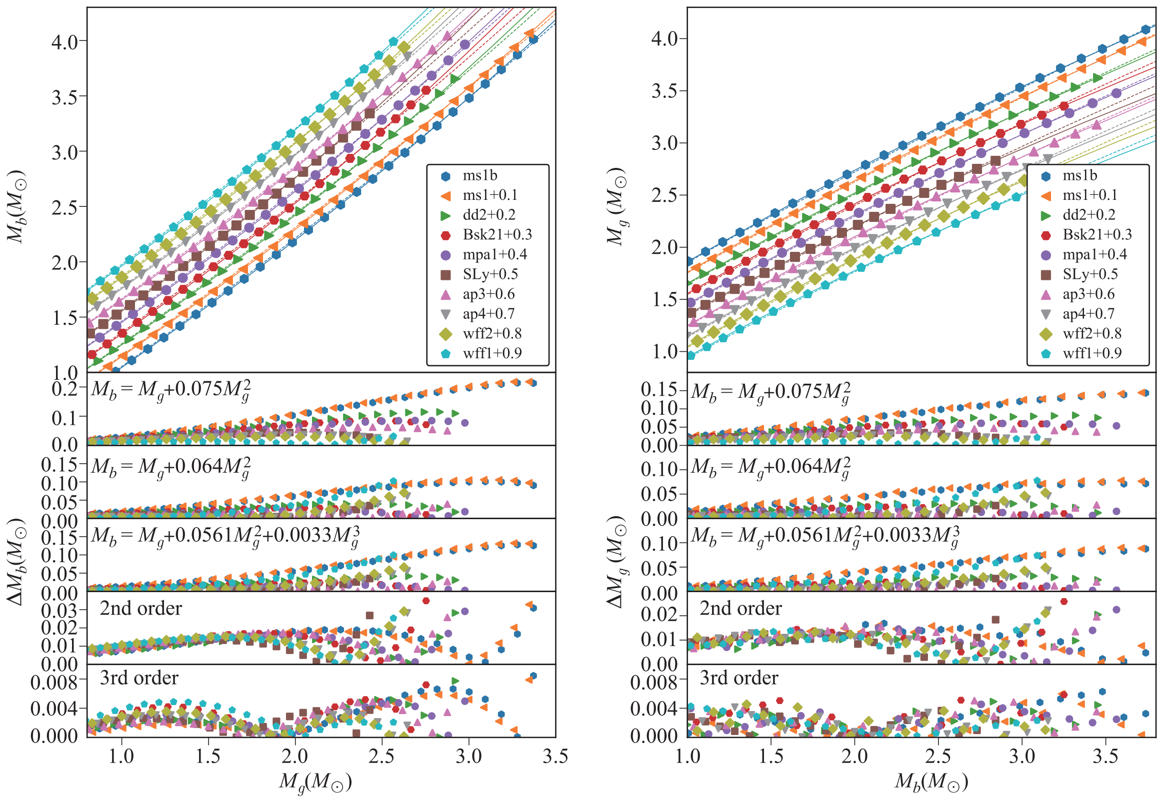

Fig. 2 The relation between the baryonic mass (M_b) and gravitational mass (M_g) for a maximally rotating NS and the residual error for different transformation formulae.

Table 2 Best fitting results for different EoSs with different spin periods.

	SLy			wff1			wff2			ap4			Bsk21		
	A	A_1	A_2	A	A_1	A_2	A	A_1	A_2	A	A_1	A_2	A	A_1	A_2
$P = 1.3P_k$	0.072	−0.318	0.177	0.085	−0.172	0.112	0.079	−0.327	0.171	0.077	−0.214	0.123	0.068	−0.179	0.101
$P = 1.4P_k$	0.073	−0.112	0.087	0.084	−0.058	0.066	0.079	−0.111	0.083	0.077	−0.075	0.067	0.068	−0.058	0.054
$P = 1.5P_k$	0.072	−0.046	0.058	0.085	−0.020	0.050	0.080	−0.049	0.058	0.078	−0.029	0.049	0.067	−0.017	0.038
$P = 1.6P_k$	0.073	−0.016	0.045	0.086	0.002	0.042	0.079	−0.017	0.045	0.077	−0.004	0.039	0.068	0.001	0.032
$P = 1.7P_k$	0.073	0.000	0.038	0.086	0.015	0.037	0.079	0.000	0.039	0.078	0.009	0.034	0.068	0.012	0.028
$P = 1.8P_k$	0.073	0.011	0.034	0.086	0.024	0.033	0.079	0.012	0.034	0.078	0.018	0.031	0.069	0.019	0.025
$P = 1.9P_k$	0.073	0.018	0.031	0.088	0.028	0.032	0.080	0.019	0.031	0.079	0.023	0.029	0.069	0.024	0.023
$P = 2.0P_k$	0.074	0.022	0.029	0.088	0.033	0.030	0.081	0.024	0.030	0.079	0.028	0.026	0.069	0.028	0.022
$P = 3.0P_k$	0.076	0.038	0.023	0.091	0.047	0.026	0.083	0.041	0.024	0.082	0.043	0.022	0.072	0.037	0.019
$P = 4.0P_k$	0.077	0.041	0.022	0.093	0.049	0.026	0.085	0.044	0.024	0.083	0.046	0.022	0.073	0.039	0.019
	ap3			dd2			mpa1			ms1			ms1b		
	A	A_1	A_2	A	A_1	A_2	A	A_1	A_2	A	A_1	A_2	A	A_1	A_2
$P = 1.3P_k$	0.072	−0.292	0.140	0.064	−0.100	0.064	0.069	−0.112	0.070	0.058	−0.095	0.052	0.059	−0.104	0.055
$P = 1.4P_k$	0.073	−0.095	0.068	0.064	−0.028	0.038	0.070	−0.039	0.043	0.058	−0.030	0.031	0.059	−0.031	0.032
$P = 1.5P_k$	0.073	−0.033	0.045	0.065	−0.002	0.029	0.070	−0.005	0.032	0.058	−0.006	0.024	0.059	−0.004	0.023
$P = 1.6P_k$	0.073	−0.006	0.035	0.065	0.013	0.023	0.070	0.010	0.027	0.058	0.007	0.020	0.059	0.009	0.019
$P = 1.7P_k$	0.074	0.008	0.030	0.066	0.022	0.020	0.071	0.019	0.023	0.059	0.015	0.017	0.060	0.017	0.017
$P = 1.8P_k$	0.074	0.017	0.027	0.066	0.027	0.019	0.071	0.026	0.021	0.059	0.020	0.016	0.060	0.023	0.015
$P = 1.9P_k$	0.074	0.024	0.024	0.066	0.031	0.017	0.072	0.031	0.020	0.060	0.024	0.015	0.061	0.027	0.014
$P = 2.0P_k$	0.075	0.029	0.023	0.067	0.034	0.017	0.073	0.033	0.019	0.060	0.027	0.015	0.061	0.030	0.013
$P = 3.0P_k$	0.078	0.043	0.018	0.070	0.043	0.014	0.076	0.044	0.016	0.062	0.037	0.012	0.064	0.039	0.011
$P = 4.0P_k$	0.080	0.046	0.018	0.071	0.045	0.014	0.077	0.046	0.016	0.064	0.039	0.011	0.065	0.041	0.011

residual error of the transformation from M_g to M_b is up to $0.20M_\odot$ for the quadratic case and is up to $0.19M_\odot$ for the cubic case. It is interesting to note that the overall best fit A value is very close to 0.075 as proposed in Ref. [17].

For a given EOS, the quadratic transformation parameter A is a rough function of the spin period (see Fig. 3),

$$A \approx a\mathrm{e}^{-\frac{b}{\mathcal{P}}}, \tag{8}$$

where $\mathcal{P} \equiv P/P_k$ is the characterized NS spin period normalized to the Keplerian period. The best-fit values for a and b are collected in Table 1. Interestingly, we find that $a \times R_{1.4} \approx 1$ and $b \approx 1/4$ for all the adopted EOSs. We thus have a new universal relation for a rotating NS,

$$M_b = M_g + R_{1.4}^{-1}\mathrm{e}^{-\frac{1}{4\mathcal{P}}} \times M_g^2, \tag{9}$$

with a 3.3% relative error. This equation is consistent with Eq. (7) and Eq. (6) when $\mathcal{P} = 1$ and $\mathcal{P} \to \infty$ are adopted. Two EOS related parameters $R_{1.4}$ and P_k are invoked in this relation, which may be constrained if M_b and the binding energy $M_b - M_g$ for a NS could be independently measured.

4 M_{max} and M_{TOV} relation

It has been proposed to parameterize M_{max} as a function of the spin period of the central star [12],

$$M_{\mathrm{max}} = M_{\mathrm{TOV}}(1 + \alpha P^{\beta}), \tag{10}$$

where P is the spin period of the NS in units of second. Using the RNS code, we calculate the numerical values for α and β for our adopted EOSs. The results are presented in Table 1.

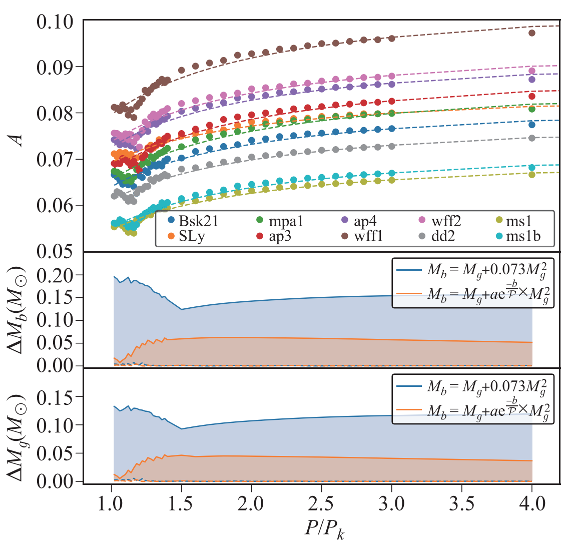

Fig. 3 Relation between the quadratic transformation coefficient A and the spin period of the NS. The dashed lines represent the best fitting results with formula $A \approx a\mathrm{e}^{-\frac{b}{\mathcal{P}}}$, with the fitting uncertainty showing in the lower panels.

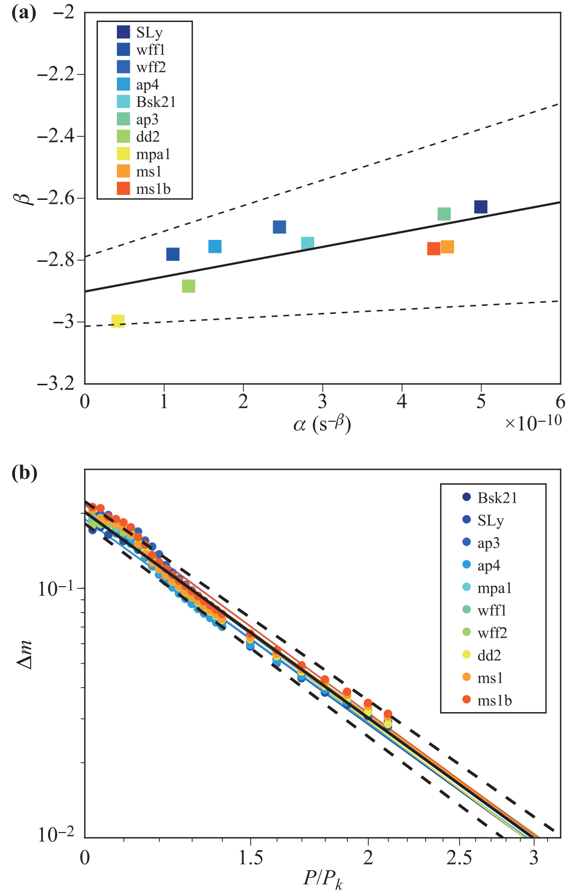

Fig. 4 **(a)** The correlation between α and β for our selected EOSs. The solid and dashed lines present the regression line and its 2-σ region. The correlation coefficient between α and β is 0.75. **(b)** The correlation between the normalized mass gain due to spin $\Delta m \equiv (M_{\mathrm{max}} - M_{\mathrm{TOV}})/M_{\mathrm{TOV}}$ and the normalized spin period $\mathcal{P} \equiv P/P_k$. The thin colored solid lines represent the results when specific α and β values are adopted for each EOS. The thick black solid and dashed lines represent the universal relation proposed by Eq. (12) and its 2-σ region.

We find that α and β are not independent of each other (see Fig. 4 for details). If we define

$$\Delta m \equiv (M_{\mathrm{max}} - M_{\mathrm{TOV}})/M_{\mathrm{TOV}} = \alpha P^{\beta}, \tag{11}$$

as the normalized mass gain due to spin, as shown in Fig. 4, a very tight and general correlation between Δm and $\mathcal{P}$ could be found as

$$\log_{10}\Delta m = (-2.74 \pm 0.05)\log_{10}\mathcal{P} + \log_{10}(0.20 \pm 0.01), \tag{12}$$

which is essentially independent of the EOS models. It is worth noticing that when $\mathcal{P} = 1$, i.e., when NS spin period equals to the Keplerian period, one has $\Delta m \simeq 0.2$, or $M_{\mathrm{max}} \simeq 1.2M_{\mathrm{TOV}}$, which is well consistent with the previous numerical simulation results [47–49].

5 Conclusion and discussion

In this work, we solved the field equations for the rotating NS with a selected sample of EOSs. For each EOS, we calculated a series of M_b and M_g for different spin periods. Our results could be summarized as follows:

- For non-rotating NSs, if one intends to apply an EOS-independent universal quadratic or cubic transformation formula to all the EOSs, one has the best-fit formulae $M_b = M_g + 0.080M_g^2$ and $M_b = M_g + 0.0729M_g^2 + 0.0032M_g^3$. The residual error of the transformation from M_g to M_b is as large as $0.12M_\odot$, and the residual error from M_b to M_g is as large as $0.09M_\odot$. There is no advantage for the higher order formula. However, for individual EOSs, if one adopts its own best-fit values for the coefficients, the cubic formula is much better than the quadratic formula, with the residual error of the transformation less than $0.01M_\odot$.
- For maximally rotating NSs, if one intends to apply an EOS-independent universal quadratic or cubic transformation formula to all the EOSs, one has the best-fit formulae $M_b = M_g + 0.064M_g^2$ and $M_b = M_g + 0.0595M_g^2 + 0.0017M_g^3$. The residual error of the transformation from M_g to M_b is as large as $0.10M_\odot$, and the residual error from M_b to M_g is as large as $0.07M_\odot$. There is no advantage for the higher order formula. Also, for individual EOSs, if one adopts its own best-fit values for the coefficients, the cubic formula is much better than the quadratic formula, with the residual error of the transformation less than $0.01M_\odot$.
- For general NSs with arbitrary rotation, the relation between M_b and M_g is different for different spin periods. If one intends to apply an EOS-independent universal quadratic or cubic transformation formula to all the EOSs for all spin periods, one has $M_b = M_g + 0.073M_g^2$ and $M_b = M_g + 0.078M_g^2 + 0.002M_g^3$. The residual error of the transformation is up to $0.22M_\odot$. There is still no advantage for the higher order formula. Given an EOS, if one intends to apply an EOS-independent universal quadratic or cubic transformation formula to all spin periods, the residual error of the transformation is up to $0.068M_\odot$. Again no advantage for the higher order formula. However, if one adopts the best-fit parameters for each spin period, the cubic transformation residual error is less than $0.01M_\odot$, almost one order of magnitude better than the quadratic formula.
- For quadratic transformation, we find three EOS insensitive relations: $M_b = M_g + R_{1.4}^{-1} \times M_g^2$ for non-rotating NSs, $M_b = M_g + 0.78R_{1.4}^{-1} \times M_g^2$ for maximally rotating NSs, and more generally, $M_b = M_g + R_{1.4}^{-1}\mathrm{e}^{-\frac{1}{4\mathcal{P}}} \times M_g^2$ for rotating NSs with spin period $P = \mathcal{P} \times P_k$. In principle, the first two relations for non-rotating NSs (which may be also applied to slowly spinning NSs) and maximally rotating NSs could be used to give constraints on the NS radius $R_{1.4}$, and hence, the NS equation of state, once M_b and M_g for a NS could be independently measured [31]. It becomes less straightforward for the relation including spin, since two EOS related parameters $R_{1.4}$ and P_k are invoked and the NS spin period may be difficult to determine.
- With our calculations, we also find a very tight and general correlation between the normalized mass gain due to spin $\Delta m \equiv (M_{\max} - M_{\mathrm{TOV}})/M_{\mathrm{TOV}}$ and normalized spin period $\mathcal{P}$ as $\log_{10}\Delta m = (-2.74 \pm 0.05)\log_{10}\mathcal{P} + \log_{10}(0.20 \pm 0.01)$, which is independent of EOS models. Note that this universal relation is valid only for rigidly rotating NSs.

In Table 3, we summarize all the transformation formulae between M_b and M_g proposed in previous works and in this work. For a fair comparison, we calculate the the maximum and average residual error of each transformation formula with the same $M_b - M_g$ sample (third column of Table 3). In general, the novel findings of this work compared with previous ones include: i) We study in detail how M_b and M_g are correlated for rapidly rotating NSs and derived the residual errors of the proposed transformation formulae; ii) The accuracy of our newly proposed universal relation between M_b and M_g is similar to the previous ones, but only $R_{1.4}$ is invoked in our relation instead of using the compactness number proposed in previous works [e.g., 31, 32]. As a result, our proposed relation is more practical since it does not involve the calculations of the NS radii corresponding to different NS masses; iii) For non-rotating NSs, if one adopts its own best-fit coefficients for individual EOSs (provided in Table 2), our proposed cubic transformation formula is much better than the quadratic formula, with the residual error of the transformation less than $0.01M_\odot$.

It has been proposed that binary neutron star merger events could give tight constraints on the NS maximum mass, as long as we could calculate the mass of the merger remnant and justify whether the remnant is a BH or a long-lasting NS. For instance, it has been claimed that the multi-messenger observations of GW170817 could provide an upper bound on M_{TOV} [6–8], e.g., $M_{\mathrm{TOV}} \lesssim 2.16\ M_\odot$ in Ref. [6]. However, this result sensitively depends on the assumption that the merger remnant of GW170817 is a not-too-long-lived hypermassive NS. This is still subject to debate, see supporting arguments by Margalit *et al.* [6–8] and counter opinion by Yu *et al.* [50–53]. This is due to the lack of GW detection in the post-merger phase and the insufficient capability to distinguish the merger product using electromagnetic (EM) observations only. If the low-significance flare-like feature at 155 days in the X-ray

Table 3 M_b–M_g relations proposed in the literature.

	Non-rotating NS		Rotating NS		N EoSs
	M_b–M_g relation	Residual error[A]	M_b–M_g relation	Residual error[A]	
Lattimer & Yahil (1989) [20]	$M_b = M_g + 0.084 \times M_g^2$	4.1% (1.7%)	–	–	7
Timmes *et al.* (1996) [17]	$M_b = M_g + 0.075 \times M_g^2$	5.8% (1.5%)	–	–	0[B]
Lattimer & Prakash (2001) [31]	$(M_b - M_g)/M_g = 0.6\beta/(1-0.5\beta)$, where $\beta = GM_g/Rc^2$	2.8% (0.92%)	–	–	14
Coughlin *et al.* (2017) [32]	$M_b/M_g = 1 + 0.89 \times (M_g/R)^{1.2}$	2.6% (0.56%)	–	–	19
This work	–	–	Maximally rotating NSs	–	–
	$M_b = M_g + R_{1.4}^{-1} \times M_g^2$	1.8% (1.1%)	$M_b = M_g + 0.078R_{1.4}^{-1} \times M_g^2$	1.3% (0.74%)	10
	$M_b = M_g + 0.080 \times M_g^2$	5.0% (1.5%)	$M_b = M_g + 0.064 \times M_g^2$	3.3% (1.2%)	10
	$M_b = M_g + A_1 \times M_g^2 + A_2 \times M_g^3$	0.45% (0.16%)	$M_b = M_g + 0.056 \times M_g^2 + 0.003 \times M_g^3$	3.5% (1.0%)	10
	–	–	General NSs with arbitrary rotation	–	–
	–	–	$M_b = M_g + R_{1.4}^{-1} e^{-\frac{1}{4P}} \times M_g^2$	3.3% (1.0%)	10
	–	–	$M_b = M_g + 0.073 \times M_g^2$	6.0% (1.6%)	10
	–	–	$M_b = M_g + 0.078 \times M_g^2 + 0.002 \times M_g^3$	4.0% (0.95%)	10

[A]The maximum residual error of the transformation between M_b and M_g, with the average residual error showing in the brackets.

[B]Timmes *et al.* [17] claims that the M_b–M_g relations used in their paper is based on private communications with J. M. Lattimer.

afterglow of GW170817 as claimed in Ref. [53] is true, or if the argument by Li *et al.* [51] that an additional energy injection from the merger product is required to interpret the blue component of AT2017gfo (GW170817 optical counterpart) is valid, the central remnant of GW170817 has to be a long-lived NS. The constraints on $M_{\rm TOV}$ would be reversed to a lower bound. Note that before GW170817 was detected, it was proposed that the statistical observational properties of Swift SGRBs favors NS EOS with $M_{\rm TOV}$ greater than 2.3 $M_\odot$ [14, 15], if the cosmological NS mass distribution follows that observed in the BNS systems in our Galaxy.

In the case that the merger remnant of GW170817 was a long-lasting, rigidly rotating NS, the remnant gravitational mass could be estimated as follows: the total gravitational mass of the binary system is determined as $m_{g,1} + m_{g,2} = 2.74M_\odot$ based on the inspiral phase GW signal [29]. The mass ratio of the binary is bound to 0.7–1 under the low dimensionless NS spin prior. We consider both the size and direction of the uncertainty. Without any prior for the NS EOS, the total baryonic mass could be estimated as[2)] $3.04M_\odot$ by using the EOS-independent universal quadratic formula for non-rotating NSs (i.e., $M_b = M_g + 0.08 \times M_g^2$). The uncertainty for the transformation is $[-0.05M_\odot, +0.10M_\odot]$. With the EM counterpart observations, the ejecta mass for GW170817 is estimated as $\sim 0.06M_\odot$, with uncertainty in the order of $10^{-2}M_\odot$ [54, and reference therein]. In this case, the baryonic mass for the merger remnant could be estimated as $2.98M_\odot$, with conservative uncertainty $\sim [-0.06M_\odot, +0.11M_\odot]$. If the remnant is a rapidly rotating NS, although it is difficult to determine its inital spin period, we can transform its baryonic mass to the gravitational mass as $2.52M_\odot$, by using the EOS-independent universal quadratic formula for arbitrary rotating NSs (i.e., $M_b = M_g + 0.073 \times M_g^2$), with the transformation uncertainty as $[-0.12M_\odot, +0.14M_\odot]$. Putting together all the uncertainties in the transformation and the ejecta mass, the gravitational remnant mass should be estimated as $M_{g,\rm rem} = 2.52^{+0.25}_{-0.18}$. If we assume the initial spin period of the rigid rotating NS produced by NS-NS merger is always the Kepler period corresponding to its baryonic mass, the EOS-independent universal quadratic formula for maximally rotating NSs (i.e., $M_b = M_g + 0.064 \times M_g^2$) should be adopted. The gravitational remnant mass is thus estimated as $2.56M_\odot$, with transformation uncertainty as $[-0.08M_\odot, 0.08M_\odot]$. Putting together all the uncertainties, the remnant mass should be estimated as $M_{g,\rm rem} = 2.56^{+0.19}_{-0.14}$. Recently, Radice *et al.* [30] proposed that given an EOS, the initial spin period of the merger remnant could be estimated based on the value of its baryonic mass. In this case, if we have some strong prior for the NS EOS, the transformation uncertainty between M_b and M_g could be reduced to the order of $0.01M_\odot$, if one applies the cubic transformation formula with the best fit values for a specific EOS with a specific initial spin period. The overall uncertainty

2)The total baryonic mass is highly insensitive to the binary mass ratio. For instance, the total baryonic mass for GW170817 is $3.04 \pm 0.10M_\odot$ for $m_{g,1}/m_{g,2} = 1$ and $3.05 \pm 0.10M_\odot$ for $m_{g,1}/m_{g,2} = 0.7$.

for this case is mainly contributed by the ejecta mass uncertainty, which might be further reduced in the future when a larger sample of EM counterparts for binary NS mergers are observed.

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