Stress study in faulted tunnel models by combined photoelastic measurements and finite element analysis

Yuping Huang

University of Nevada, Las Vegas

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Stress Study in Faulted Tunnel Models by Combined Photoelastic Measurements and Finite Element Analysis

by

Yuping Huang

A thesis submitted in partial fulfillment of requirements for the degree of

Master of Science
in
Civil and Environmental Engineering

Department of Civil and Environmental Engineering
University of Nevada, Las Vegas
December 1995
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ABSTRACT

The aim of this part of the Nuclear Waste Package Project research at UNLV is to investigate the stresses in a model of a faulted mountain and the effect of the fault on the stability of drifts in a proposed High Level Nuclear Waste Repository.

An investigation was performed to develop a proper technique for analyzing the stresses in and around three adjacent scaled tunnel models, along with the stress concentration factors resulting from the existence of a fault that penetrates two of the three tunnels, at an inclined angle of 44° to the horizontal plane.

The results and experience gained from this investigation will be used in a future project in which a full-size repository drift and a penetrating fault will be modeled and analyzed.

Two parallel techniques, Photoelasticity (PhE) and the Finite Element (FE) analysis, are used to investigate the principal stress patterns in the scaled models. Principal stresses in and around the faulted adjacent rock tunnels are studied using a photoelastic plexiglass plane model with three openings and saw cuts penetrating two of the three openings. Concurrent simulations of the same plexiglass model are performed by the numerically based finite element method, using 2-D triangular elements, linear elastic analysis and the iterative H-method adaptive meshing technique, necessary for the study of stress concentration factors.

Both plexiglass photoelastic model and the finite element model represent a plate
having two adjacent parallel square openings (tunnels) with rounded corners, and one circular opening (tunnel). The circular opening, with a diameter that equals the side length of the square, is placed symmetrically under the square openings, at a clear distance of about 80% of the square side length. The actual fault in the rock is depicted as a saw cut (crack) in the model, which has the same angle of incidence with the openings as the fault incidence with the tunnels. The crack in the scaled tunnel model, which is produced by successive saw cuts, follows a line that crosses the corner of the right-hand side square opening and passes through the circular opening, close to its center. In order to investigate the influence of the faults on the tunnel under different conditions, the saw cuts, beginning at the lower corner of the right-hand side square opening, are made to grow progressively along the line. At each step, under each condition, the principal stress patterns are measured by both the PhE and the FE method, and the stress concentration factors, $K$, are calculated at predetermined points, such as the corners and sides of the openings and at the tip of the progressing saw cut.

The Adaptive H-method is used in the FE modeling. The H-method progressively improves the mesh by refining it, especially at the corners of the opening and at the tips of the crack. Results from both the PhE models and the FE models are compared to each other at each step of the analysis.

The investigation shows that, except at the tip of the fault, the principal stresses and the calculated $K$ from both techniques are within 15% from each other at the predetermined points. Since the ability to distinguish higher fringe orders by the equipment presently available in the laboratory is reached, the measured values of principal stresses from PhE method are in general slightly lower than those obtained by the FE method in the region of
the crack tip.

From this research, a conclusion can be made that, the Finite Element techniques used in this research are reliable and fully capable of representing a faulted tunnel system. Photoelastic, experimental and numerical studies of the plexiglass model will continue along with a full scale FE analysis of the faulted rock tunnel system, in which gap elements and nonlinear material behavior will be incorporated. A laser light source shall be used to observe the high photoelastic fringe orders and the effect of material plasticity on the plexiglass model.
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ACKNOWLEDGMENTS

In this endeavor, I would like to express my gratitude to Dr. Samaan G. Ladkany, for being my advisor and for serving as the director of the USDOE/UNLV Nuclear Waste Storage Project. I sincerely thank him for providing me with financial support throughout this project and for all the help and the time he has given me to make this thesis a success.

I am very grateful to Dr. Gerald Frederick, Dr. Moses Karakouzian and Dr. Brendan O'Toole for consenting to serve on my examining committee in spite of their busy schedules.

I also wish to thank Rajkumar Rajagopalan and all of the friends who worked with me on this project for their help during the last two and a half years.

A special thank you goes to my parents and my sister for the encouragement and the moral support they gave me during the course of my graduate program.
CHAPTER 1

INTRODUCTION

1.1 Research Background

The U. S. Department of Energy (DOE) is studying the suitability of a potential disposal site for high-level nuclear waste. This research is intended to give a proper reference to the stability of underground tunnels with and without major faults that penetrates them.

This thesis covers research work started in the spring of 1993, which has been supported by the USDOE/UNLV Nuclear Waste Package Program. As a benchmark, the investigation is intended for the study of stress concentration factors in the rock due to tunnel boring and due to a fault that may cross the path of an underground tunnel system in a proposed High Level Waste Repository.

1.2 Problem Description

In this research, Photoelastic (PhE) models and Finite Element (FE) models are concurrently used to determine the stress pattern in the models. Results from both methods are compared to each other and the FE mesh pattern is changed and improved accordingly.

A PhE model is first made of a plexiglass plate, 0.35 in. (8.89 mm) thick and 10.25 in. (260.35 mm) long and wide. Two parallel square openings with rounded corners are cut first and the stress analysis is performed. One circular opening underneath the square openings is cut in the plexiglass mode and the new stress patterns are investigated. A very thin saw cut
(0.007 to 0.021 in. wide, i.e., 0.178 to 0.533 mm wide) is introduced progressively to the right hand side square tunnel and to the top of the circular tunnel step by step, at an inclination of $44^\circ$ to the horizontal plane (the same angle of the fault that may penetrate the proposed tunnel system). The FE models, duplicate exactly the PhE models, using the same size of plexiglass plate, the same material constants and the same boundary conditions. 2-D triangular elements in a linear elastic analysis and an Adaptive H-method that may progressively refines the FE mesh are used in the FE modeling technique.

Under a vertical loading of 500 lbs (2245 N), principal stresses, $P_1$ and $P_2$, are determined at each step of the analysis using the two different techniques. Stress concentration factors, $K$, are then calculated by determining the relative values of the stresses in the vicinity of the tunnels, the fault and the adjacent undisturbed areas.

1.3 Photoelasticity and Finite Element Analysis

Photoelasticity is based upon the property of birefringence exhibited by some transparent isotropic solids of becoming doubly refracting when subjected to stress. Photoelasticity, as a tool for the analysis of stress distributions in solids, is a useful and powerful technique.

In the experimental part of this thesis, Photoelasticity is used to determine the principal stresses of a vertically loaded plexiglass tunnel model, using a 060 Series Transmission Polariscoppe. Please refer to Appendix A for the basic principles of Photoelasticity.

Finite Element Analysis (FEA) or the Finite Element Method (FEM) is a numerical
procedure for analyzing structures and continua. Usually the problem addressed is too complicated to be solved satisfactorily by classical analytical methods. Instead of solving a series of differential equations, the finite element procedure usually produces a large number of simultaneous algebraic equations, which can be generated from the displacements of the element nodes in the model and solved on a digital computer.

The COSMOS/M System is a FE software widely used in linear and nonlinear stress analysis. COSMOS/M version 1.6 to 1.7 installed on two 486/66Hz personal computers and a Spark Station II are used in this research work.

For details of FEA (FEM), the COSMOS/M system software and the model input sequences, please refer to Appendix B and Appendix C.
CHAPTER 2

EXPERIMENTAL STUDY AND SET UP

2.1 The Models and the Material Constants

The tunnel model used in the PhE measurements is made of plexiglass, 0.35 in. (8.89 mm) thick and 10.25 in. (260.35 mm) long and wide.

The model originally had two adjacent square openings (2-9/32 in., i.e., 57.94 mm long and wide); later a central circular opening (2-9/32 in., i.e., 57.94 mm in diameter) was placed at a clear distance approximately equal to 80% of its diameter from the bottom line of the square openings to the top of its circumference. Saw cuts (0.007 to 0.021 in., i.e., 0.533 to 0.178 mm wide), representing faults, were eventually propagated from the tunnels outward with stress measurements taken at predefined locations through the model as the saw cuts increased in length. The model configuration, along with the locations of the predefined stress measurements points (A-M, I-IV and 1-10) are shown in Figure 2-1.

The material properties of the plexiglass are given as follow:

1) Young’s modules: $E = 3.8 \times 10^5$ psi ($16.891 \times 10^5$ Pa),

2) Poisson’s ratio: $\mu = 0.3$,

3) Photoelastic stress optical constant: $f = 40$ psi/fringe/in. ($7000$ Pa/fringe/m),

4) Angle of Incidence: $\theta = 31.8^\circ$ (See section 2.4 for details of the calibration of $\theta$),

The models used in the FE analysis duplicate the PhE models, by having the same size, boundary conditions (B. C.) And material constants.
Figure 2-1  Dimension of the Photoelastic Plexiglass Model with a Thickness of 0.35 in.
boundary conditions (B.C.) and material constants.
2.2 General Outline of the Experiment

To investigate the stress pattern and stress concentrations in the plexiglass model under different conditions, the plexiglass models were developed step by step, along with duplicating FE models. In this study, six experiments were performed on six different models to measure the principal stresses at the predetermined point locations on the plexiglass model.

The six experimental steps are:

Step one - a model with two parallel square openings is created.

Step two - a circular opening is cut under the two square openings as shown in figure 2-1.

Step three - a short inclined saw cut, about 0.021 in. wide and 0.5 in. long (0.533 mm wide and 12.7 mm long), is made along a line that crosses the lower right corner of the right-hand side square opening, from point 9 to point 10, as indicated in figure 2-1.

Step four - a short saw cut about 0.021 in. wide and 0.25 in. long (0.533 mm wide and 6.35 mm long) is made starting from the upper edge of the circular opening, from point 3 to point 4, along the same line of the previous saw cut.

Step five - from the tip of the last saw cut, saw cut is lengthened to point 5 having the width of about 0.021 in. and the length of 0.5 in. (0.533 mm wide and 12.7 mm long.)

Step six - a thinner saw cut about 0.007 in. wide and 0.75 in. long (0.178 mm wide and 19.05 mm long) is made from the last tip point 5 to point 6.

All the saw cuts are colinear and inclined at an angle of 44° to the horizontal plane as shown in figure 2-1.

At each step and under a vertical constant load of 500 lbs. (2245 N), the principal
stresses, $P_1$ and $P_2$, are determined by PhE and compared to the corresponding results from the FE models at the predetermined points. Improvements on the measurement techniques of both the PhE and FE methods were tried. Details of the improved techniques can be found in chapter 3 and chapter 4.

2.3 Calibration of the Strain Indicator

In the PhE measurement, a Model P-3500 Digital Strain Indicator (Figure 2-2) is used to measure the vertical load acting on the plexiglass model through a Model 162 Loading Frame mounted in the 060 Series Transmission Polariscope. The Model 162 Loading Frame is a rigid structure that provides a constant deformation pattern to the model through a screw-operated loading system.

The Model P-3500 Strain Indicator is a portable, battery-powered precision instrument for use with resistive strain gages and transducers. To calibrate the strain indicator, zero adjustments should be done first as follows,

1. Select FULL position of BRIDGE push button,
2. Select X1 position of MULT push button,
3. Connect transducer to TRANSDUCER connector, while the load-cell on the other end is connected to the lower end of Model 162 Loading Frame,
4. Depress AMP ZERO push button. Allow instrument to warm up for minimum two minutes. Set AMP ZERO control for a read out display of $+0000$,
5. Depress GAGE FACTOR push button. Set GAGE FACTOR range switch and GAGE FACTOR control for a reading of 2.000.
6. Depress the RUN push button. Set the BALANCE switch and the BALANCE control for a reading of ±0000,

7. Depress the CAL push button and verify calibration of the instrument,

8. Depress the RUN push button again to record the reading.

At this time, a reading can be recorded exactly while the transducer is loaded. In order to calibrate the reading of the Strain Indicator to the load through the Loading Frame of the Transmission Polariscope, a compressive plexiglass model is mounted in the Loading Frame, and a scale is connected at the top portion of the Loading Frame to show the exact load, while a four digital value can be read on the panel of the Strain Indicator.

A compressive force, \( F \), was applied to the plexiglass model varying from 0.0 to 150 lbs., meanwhile, the strain reading, \( \varepsilon \), was recorded from the Strain Indicator. Values of \( F \) and \( \varepsilon \) are shown in Table 2-1.

<table>
<thead>
<tr>
<th>( F ) (lb.)</th>
<th>0.0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon )</td>
<td>0.0</td>
<td>20</td>
<td>42</td>
<td>63</td>
<td>83</td>
<td>104</td>
<td>125</td>
<td>145</td>
<td>164</td>
<td>186</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( F ) (lb.)</th>
<th>100</th>
<th>110</th>
<th>120</th>
<th>130</th>
<th>140</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon )</td>
<td>207</td>
<td>227</td>
<td>245</td>
<td>268</td>
<td>285</td>
<td>306</td>
</tr>
</tbody>
</table>

Table 2-1  The Applied Forces Versus the Relative Strain Values in Strain Indicator Calibration.
Figure 2-2  Model P-3500 Digital Strain Indicator (from Reference #6.)

Figure 2-3  Relationship between the Compressive Loading Force, $F$, and the Strain, $\varepsilon$, of the Plexiglass Model.
From table 2-1 and figure 2-3, a conclusion can be reached that the ratio of the applied force to the reading of the Strain Indication is, \( r_1 = 0.49 \text{ lb./in./in.} \).

Other two calibration experiments were made and the ratio of force to strain were calculated as:

\[
\begin{align*}
  r_2 &= 0.53 \text{ lb./in./in.}, \\
  r_3 &= 0.52 \text{ lb./in./in.}, \\
\end{align*}
\]

Based upon the above three calibration experiments, an average ratio of force to strain is obtained from \( r_1, r_2 \) and \( r_3 \),

\[
  r = \frac{r_1 + r_2 + r_3}{3} = \frac{0.49 + 0.53 + 0.52}{3} = 0.513 \text{ lb./in./in.}.
\]

In the PhE measurements of the faulted tunnel model, a 500 lb. (2245 N) vertical compressive load is applied to the top of the model, thus, a strain number of 975 (500/0.513 = 975) should be read from the Strain Indicator and kept constant during the experimental process.

2.4 Calibration of the Angle of Incidence

In the PhE analysis of the plexiglass tunnel model subjected to plane stress \( P_z = 0 \), the separation of principal stresses is achieved by the measurement of the fringe order in normal incidence \( N_n \) and oblique incidence \( N_o \) as described in Appendix A (A.5).

Assuming that \( x_1 \) and \( x_2 \) are directions of principal stresses \( P \) and \( P_z \) (Figure 2-4), the fringe orders observed in normal and oblique incidence are correlated to stresses by the expressions:
Where \( N_n \) is the fringe order in normal incidence, \( N_o \) is the fringe order in oblique incidence, \( t \) is the thickness of the plexiglass model, and \( f \) is the stress optical constant, which was provided by the manufacturer. In order to obtain accurate measurements, the model material must be calibrated to determine the angle of incidence \( \theta \), which depends on the index of refraction of the plexiglass material.

The calibration of angle of incidence \( \theta \) is performed as follows:

1. Prepare a simple tensile specimen. Place the specimen in the loading frame and apply a tensile load (Figure 2-5). The tensile specimen has the same material constant \( f = 40 \) psi/fringe/in (7000 Pa/fringe/m) as that of the tunnel model, and the thickness of \( t = 0.35 \) in (8.89 mm).

2. Since \( P_2 = P_z = 0 \), equations 2-1 and 2-2 become,

\[
N_n = \frac{t}{f} P_1 \tag{2-3}
\]

\[
N_o = \frac{t}{f \cos \theta} P_1 \tag{2-4}
\]
Figure 2-4  Front and Top View of the Prism within the Model 163 Oblique Incidence Prism Adaptor.

Figure 2-5  Configuration of the Tensile Specimen and the Loading Condition.
Thus, from the above two equations,

\[
\cos \theta = \frac{N_n}{N_o}
\]  

(2-5)

Since \( \cos \theta \) and \( f \) should be known as accurately as possible, particular care should be executed to measure \( N_n \) and \( N_o \) for calibration purposes. Four calibration tests are performed and the measured results are tabulated as follow:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_n )</td>
<td>1.552</td>
<td>2.750</td>
<td>1.925</td>
<td>3.300</td>
</tr>
<tr>
<td>( N_o )</td>
<td>1.852</td>
<td>3.250</td>
<td>2.248</td>
<td>3.840</td>
</tr>
<tr>
<td>( \cos \theta )</td>
<td>0.838</td>
<td>0.846</td>
<td>0.856</td>
<td>0.859</td>
</tr>
<tr>
<td>( \theta (^\circ) )</td>
<td>33.1</td>
<td>32.2</td>
<td>31.1</td>
<td>30.8</td>
</tr>
<tr>
<td>Average of ( \theta (^\circ) )</td>
<td></td>
<td></td>
<td></td>
<td>31.8</td>
</tr>
</tbody>
</table>

Table 2-2 Measurement values of \( N_n \) \( N_o \) and the relative value of the Angle of Incidence, \( \theta \), in the calibration experiments.

From the calibration experiments, an average angle of incidence of 31.8\(^\circ\) is obtained from the measurements of \( N_n \) and \( N_o \). This value, as well as a material constant of the plexiglass model, the thickness \( t \) and the stress optical constant \( f \), were used in the measurement processes and in the calculations throughout the entire PhE experimental process described in chapter 3.
CHAPTER 3

OPERATION OF PHOTOELASTIC EXPERIMENT

3.1 The Photoelastic Instrumentation

After creating the plexiglass tunnel model and setting up (calibrating) the instrument of Strain Indicator and the Angle of Incidence, \( \theta \), of the plexiglass model, the PhE measurements were determined by using a 060 Series Transmission Polariscope manufactured by Measurements Group, Inc..

The basic model 061 instrument of the 060 Series Transmission Polariscope consists of polarizing assembly, analyzing assembly, mechanical drive coupling system for remote control of all four filters, and a built-in X-Y traversing rack for camera and microscope support. All components are mounted on a common base frame (Figure 3-1). The polarizer, analyzer, and quarter-wave plates are of glass-laminated construction. They are mounted in aluminum rings, and rotate on ball bearings. The rotation is indicated on a precision-engraved dial with a color-coded measuring scale.

In use, a plexiglass model is placed in the polarscope, and when forces are applied, a colorful fringe pattern results. This pattern reveals a visible picture of the stress distribution over the whole area of the model.

In the PhE measurement, a couple of accessories were used as well. They were,

1) Diffused While Light Source (model 361) - A bright, uniform-intensity light source.
Figure 3-1  060 Transmission Polariscope with Diffused Light System (from Reference #7.)

Figure 3-2  Frontal View of Polariscope Showing controls for Operation of the Instrument (from Reference #7.)
2) Articulated Null-Balance Compensator (model 067) - A compensator that provides accurate measurements of fringe order, full or fractional, at any arbitrary point in the field of view by "compensation" methods.

3) Oblique Incidence Prism Adaptor (model 163) - An attachment that consists of two specially manufactured prisms which are fixed to rotatable metal housings. It provides capability of separating principal stresses.

4) Loading Frame (model 162) - A rigid structure that provides constant deformation of the model by incorporating a screw-operated loading system.

5) Telemicroscope (model 065) - A precision optical instrument for enlargement of fringe patterns observed with the 060 Series Polariscope.

6) Strain Indicator (model P-3500) - An instrument that provides digital readout of strain with resistive strain gages and transducers connected to the Loading Frame.

7) Monochromator (model 068) - An interferential filter that produces a dense black fringe at each point in the PE pattern where a tint-of-passage or integral fringe order occurs in white light. It is used in those cases when high fringe orders are encountered.

The 060 Series Polariscope is a precision optical instrument for performing full-field interpretation of fringe patterns and quantitative measurements. The full-field interpretation of fringe patterns provides an overall assessment of nominal stress magnitudes and gradients. The quantitative measurements give the capabilities to measure: (1) the direction of the principal stresses, (2) the magnitude and sign of the tangential stress along free (unloaded) boundaries, and in regions where the state of stress is uniaxial, (3) the magnitude of the difference in principal stresses in a biaxial stress state.
The most significant feature of the 060 Series Transmission Polariscope is the common rotation of the polarizer/analyzer assembly, and quarter-wave plates from the instrument's Observation and Control Station (figure 3-2).

The position of knob “B” at the filter control station determines whether the polariscope is set up for measuring the directions of the principal stresses, or for measuring stress magnitudes. Placing knob “B” in the “D” (direction) position aligns the optical axes of the quarter-wave plates with those of the polarizer and analyzer. This has the effect of optically removing the quarter-wave plates from the system, and converts the unit to a plane polariscope for stress direction measurement. When knob “B” is in the “M” (magnitude) position, the quarter-wave plates are oriented with their optical axes at 45° to the polarizer/analyzer axes, and the instrument is restored to a circular polariscope condition for stress magnitude measurement. The position of knob “B” should always be verified before making a measurement.

When handle “H” is rotated, the orientation of the outer ring, and thus the polarizer/analyzer axes, can be read from the lower scale graduations opposite the DIRECTION index arrow “A” (graduated ±0 to 90°). Tightening lever “I” will lock the polarizer/analyzer assemblies in any desired position.

Knobs “C” are used to rotate the inner ring with respect to the outer engraved dial ring. The analyzer is attached to the inner ring, and its rotation can be used to measure fractional fringe orders by the Tardy compensation method (not used in this experiment). For all operation other than Tardy compensation, the inner ring must always be set so that index arrows “G” are aligned with the 0 and 100 marks on the engraved dial.
3.2 Operation Process of Photoelastic Measurement

3.2.1 Full-Field Interpretation of Fringe Patterns

One of the major measurement capabilities of the 060 Series Polariscope is to perform full-field interpretation of fringe patterns. This is the facility for immediate recognition of nominal stress magnitudes, stress gradients, and overall stress distribution - including identification of over-stressed and understressed areas. Its successful application depends only on the recognition of isochromatic fringe orders by color, and an understanding of the relationship between fringe order and stress magnitude (Appendix A).

When the plexiglass tunnel model is subjected to a load, the resulting stresses produce proportional optical effects which appear as isochromatic fringes when viewed with a polarscope.

The 060 Series Polariscope is normally used as a dark-field instrument, which means that with no stress in the model all light is extinguished and the model appears uniformly black. At the beginning of the experiment, the plexiglass model is free of loading. At this time, a "0000" read out display can be seen on the panel of the Strain Indicator, and black color usually show everywhere on the plexiglass model. When the applied load is increased from zero, fringes will appear first at the most highly stressed points, such as the corner points of the two square openings and the tip point(s) of the saw cut(s). And these points are the stress concentration points to be concerned about in the initial study of the PhE experiment, before saw cuts are introduced into the model. As the load is increased and new fringes appear, the earlier fringes are pushed toward the areas of the lower stress. While further load
is added to the model, additional fringes are generated in the highly stressed regions while the initial fringes move toward regions of zero or low stress until the maximum load is reached. In this experiment, a maximum load of 500 lbs, i.e., 2245 N (a read out display of 975 is shown on the panel of the Strain Indicator at this time), is applied to the plexiglass tunnel model through the Loading Frame.

The fringes can be assigned ordinal numbers (first, second, third, etc.) as they appear, and they will retain their individual identities (orders) throughout the loading sequence. They are continuous and never cross or merge with one another. They always maintain their respective number in the ordered sequence. Therefore, the fringe order and stress level is uniform at every point on a fringe. Furthermore, the fringe always exist in a continuous sequence by both number and color.

It is also easy to see from this experiment that high fringe orders appear at the two edges, upper and lower edges of the plexiglass model, which directly contact the Loading Frame. That is, stress concentration also occurs at the boundaries because of the rough contact between the plexiglass model and the Loading Frame. Stress concentration at the boundaries may cause boundary disturbance in the fringe pattern, so that fringe orders (principal stresses) at points 1, 2 and 10 in the model (Figure 2-1), may not be measured accurately.

3.2.2 Operation Process of Quantitative Measurement at a Point

Accurate measurement of fringe orders requires aligning certain elements in the polariscope with the principal stress directions, of the test specimen. In general, the directions
of the principal stresses vary from point to point over the surface, depending upon the shape of the model and mode of the loading; but they are not affected by load magnitudes if all loads change proportionally.

The directions of the principal stresses can be obtained very easily with the 060 Series Polariscope by utilizing the properties of the *isoclinics*. (An isoclinic is a locus along which the directions of the principal stresses are the same at every point.) Isoclinics appear as black lines, bands, or areas in plane polarized light, and are superimposed on the isochromatic fringe pattern. As shown in figure 3-3, the directions of the principal stresses at every point on the isoclinic coincide exactly with the axes of the polarizer and analyzer. Thus, the directions of the polarizer/analyzer axes define the directions of the principal stresses everywhere along the isoclinic. For each angular position there is a different isoclinic along which the directions of the principal stresses coincide with those of the crossed polarizer/analyzer. By rotating the polarizer/ analyzer together through a $90^\circ$ angle, every point in the field of view will have been swept over by an isoclinic.

Several critical points are marked on the plexiglass model (Figure 2-1). They are: points A to point L located around the area of the two square openings; points 1 to point 10 located along the line crossing the right-hand side square and the lower circular openings (saw cuts will be introduced along this line from step two to step six in the investigation); points I to point IV are located at the quadrant of the circular opening.

In the following part of this section, a critical point, *point B* in the first step of the experiment (refer to section 2.2 for the experiment schedule), is chosen to illustrate a step-by-step procedure for the accurate measurement of the direction of the principal stresses and
the fringe orders, from which the difference between the two principal stresses (or the maximum shear stress in the model) is obtained. The procedure for separation of principal stresses is also illustrated.

Figure 3-3  Isoclinics Seen on the Model at the 20° Position of Crossed Polarizer/Analyzer. At this Position the Principal Stress Directions are the Same Everywhere along the Isoclinics (from Reference #7.)
A. Determination of Principal Stress Directions

At first, directions of the principal stresses $P_1$ and $P_2$ at point $B$ on the plexiglass model are measured as follow:

1. Turn on the power of the Strain Indicator box and wait for a minimum of two minutes to warm up. Following the procedures shown in section 2.3, perform zero adjustments for the Strain Indicator.

2. Place the plexiglass model in the Loading Frame of the Polariscope and apply a vertical load of 500 lbs (2275 N.) Take care when loading the model to ensure there is no torsion or load eccentricity.

3. Unlock level "I" (figure 3-2), and rotate the polarizer/analyzer assembly by its handle "H" until the direction index "A" is at zero degrees. Place level "I" in the locking position and engage knob "B" in the "D" (direction) position.

4. Rotate the analyzer using knob "C" until indexes "G" are positioned at zero and 100.

5. Unlock level "I", rotate handle "H" and observe the fringe patterns. The colored fringes and some of the black fringe will remain fixed during rotation. However, other black fringes will be observed moving. The black lines or areas which move are isoclinics.

6. Rotate the polarizer/analyzer assembly by handle the "H" until an isoclinic crosses over the marked point $B$. When the isoclinic crosses the point, the axes of the polarizer and analyzer are respectively parallel and perpendicular to the direction of the principal stresses, and their position with respect to the direction of the vertical instrument axis is shown in
degrees on the scale by pointer "A". The clockwise rotation represents a positive angle and
counterclockwise rotation, a negative angle.

Principal stress directions of point B in the first step of the experiment is, \(+3.5^\circ\) with respect to the direction of the vertical instrument axis.

**B. Determination of Difference in Principal Stresses**

Before the determination of the two separated principal stresses \(P_1\) and \(P_2\) is possible, the difference in principal stresses, \(\Delta P = P_1 - P_2\), must be determined first, using a Model 067 Compensator.

The process of PhE stress measurements consists of first determining the fringe order at any point of interest, and then multiplying the observed fringe order by an appropriate constant to obtain the difference in principal stresses at that point. Again, point B is chosen to illustrate the process.

As the fringe order increases, the capability for resolution by color decreases; and fringe order above 4 and 5 are virtually indistinguishable by color. Furthermore, there may not be a recognizable fringe color present at a specific preselected test point. These limitations are readily overcome; and accurate measurements of fringe order, full or fractional, can be made at any arbitrary point in the field of view by the "compensation" methods. The Null-balance compensation method is one of the two "compensation" methods and will be used in this PhE experiment.

Null-balance compensation operates on the principle of introducing into the light path of the polariscope a calibrated variable birefringence of opposite sign to that induced in the
PhE model under load. When the opposite-sign variable birefringence is adjusted to precisely match the magnitude of the stress-induced birefringence in the model, complete cancellation will occur, and the net birefringence in the light path will be zero. The condition of zero net birefringence is easily recognized because it produces a black fringe in the isochromatic pattern where, before introducing the compensation birefringence, a colored fringe existed. The manner in which a null-balance compensator operates is illustrated schematically in figure 3-4 by analogy with the common knife-edge balance.

Model 067 Compensator, employs a pair of linearly birefringent plates arranged in tandem so that the total birefringence introduced into the light path is proportional to the displacement of one plate with respect to the other, and is uniform over the field through the window of the unit. Adjustment of the control knob on the compensator displaces the screw-driven movable plate and operates the digital turns-counter to register the displacement.

The procedure of measuring fringe order at point B, is shown as follows,

1. Engage handle “H” of the polariscope in the “D” (direction) position. Viewing point B on the plexiglass model through the analyzer, rotate the polarizer and analyzer together using knob “H” until an isoclinic crosses point B (same as the last step of the procedure of determining principal stress directions shown above).

2. Return knob “B” to the “M” (magnitude) position. The isoclinics are now eliminated and only the colored isochromatic pattern is seen.

3. Insert the compensator into the field of view of the polariscope and align its axes with one of the principal stress directions marked at point B. In order that the compensating
birefringence be opposite in sign to that of the stress-induced birefringence in the test specimen, the long axis of the compensator must always be aligned with the algebraically maximum principal stress ($P_1$).

4. Turn the compensator control knob counterclockwise while viewing the test point $B$ through the compensator window, and continue turning until a black fringe is centered over the test point $B$. If no black fringe comes to the test point and, instead, the fringe appearance there becomes ever paler with added birefringence from the compensator, it is because the long axis of the compensator is aligned with the algebraically minimum principal stress ($P_2$). In this case, the compensator is adding birefringence of the same sign as that in the specimen and null-balance is impossible. To correct this situation, simply realign the long axis of the
compensator with the other maximum principal stress (90° rotation).

5. Read the digital counter on the compensator and record the setting. Referring to the calibration graph which accompanies the compensator (Figure 3-5), enter the abscissa at the count setting and read the fringe order, \( N \), from the ordinate.

6. Calculate the principal stress differences (maximum shear stress) for normal incidence at the test point \( B \) using equation A-13 (Appendix A) as follows:

\[
\tau_{\text{max}} = P_1 - P_2 = \frac{Nf}{t}
\]  

(3-1)

where: \( P_1, P_2 \) = algebraically maximum and minimum principal stresses respectively,

\( N \) = measured fringe order in normal incidence,

\( f \) = stress optical constant, 40 psi/ fringe/ in. (7000 Pa/fringe/m),

\( t \) = thickness of the model, 0.35 in. (8.89 mm).

At the first step of the experiment, the digital read out of the compensator at point \( B \) is, \( C_N = 146.5 \). Referring to the calibration graph in Figure 3-5, it is easy to be read that, \( N = 3.04 \). Thus,

\[
\Delta P = P_1 - P_2 = \frac{3.04 \times 40}{0.35} = 347 \text{ lb/\text{inch}^2} (1542 \text{ Pa}),
\]

which is the maximum shear stress \( \tau_{\text{max}} \) at point \( B \).
Figure 3-5  Model 285 Null-balance Compensator Calibration Chart (from Reference #7.)
C. Determination of Individual Principal Stresses

The information obtained from a normal-incidence fringe-order measurement is sufficient to determine the difference in the principal stresses. To obtain the signs and magnitudes of the individual principal stresses, an additional measurement using a Model 163 Oblique Prism Adaptor (Figure 2-4) can be used.

With the oblique-incidence attachment, the polarized light is directed through the model at an angle to the normal surface, and thus traverses the model at an oblique angle. Under these conditions, the measured birefringence corresponds to the difference in the secondary principal stresses in the plane perpendicular to the light ray. Combining the oblique and the normal-incidence measurements at a point provides the necessary information for determining the separate values of the principal stresses in the plane of the model.

Model 163 Oblique Prism Adaptor, essentially consists of two specially manufactured prisms which are fixed to rotatable metal housings. The prism rotation is indicated by a dial graduated in degrees, $0^\circ$ to $360^\circ$ (Figure 3-6). The mounting hardware, supplied with the adapter, provides for adjustment of the prism in the X, Y, and Z directions for accurate placement over the model test point.

To separate the two principal stresses, $P_1$ and $P_2$ (in X-Y plane) at point B, procedure is shown as follows:

1. Place the prisms in the desired field of view with the plexiglass model located between them. The prisms should be as close as possible to the model with the center of each prism coinciding with the point of measurement.

2. Set the polariscope up for observation of isoclinics by engaging knob “B” in the
Figure 3-6  Photo Diagram Showing X-Y-Z Adjustments for Positioning Prism around Model (from Reference #7.)

Figure 3-7  Fringe Pattern Observed through Oblique Incidence Adapter (from Reference #7.)
"D" (direction) position. With the prisms removed from the field of view, rotate the polarizer and analyzer (crossed) together until an isoclinic is observed at the point on the model under investigation. Lock the crossed polarizers in this position with lever I.

3. Place the prisms in the field of view and rotate them so that they are angularly aligned with the angular position of crossed polarizer/analyzer. Use the scales provided on the oblique incidence adapter for accurate alignment.

4. Next restore the polariscope to a circular light condition by engaging knob "B" in the "M" magnitude position.

Three zones will now be observed in the prism (Figure 3-7). In zone I and III, the light is transmitted in oblique-incidence (angle $\theta$ and $-\theta$). In zone II, the observation is made in normal incidence of light.

5. Follow the procedures in part B of this section above, combined with using Model 067 Compensator, record the setting separately from the compensator of $C_N$ (normal-incidence) and $C_O$ (oblique-incidence).

At the first step of the experiment, the digital read out of the compensator at point B are, $C_N = 146.5$ (zone II), $C_Ol = 127$ (zone I) and $C_{oil} = 134$ (zone III). The average of $C_O$ is 130.5. From the calibration graph in figure 3-5, it is easy to obtain that, $N_N = 3.04$, $N_O = 2.7$.

Referring to equation A-16 and A-17 in Appendix A.4, the separated principal stresses of $\sigma_x$ and $\sigma_y$ will be,

$$P_1 = \frac{\int}{\int} \frac{1}{1 - \cos^2 \theta} \left( N_o \cos \theta - N_n \cos^2 \theta \right)$$  \hspace{1cm} (3-2)
where: \( P_1, P_2 \) = algebraically maximum and minimum principal stresses respectively,

\( N_n \) and \( N_o \) = measured fringe orders in normal and oblique incidence,

\( f = \) stress optical constant, 40 psi/fringe/in. (7000 Pa/fringe/m),

\( t = \) thickness of the model, 0.35 in. (8.89 mm),

\( \theta = \) angle of incidence, which has been calibrated in section 2.4, 31.8°.

Substituting the above known values into equation 3-2 and 3-3, the principal stresses at point B are,

\[ P_1 = 40.7 \text{ psi (180.9 Pa)}, \]

\[ P_2 = -306.7 \text{ psi (-1363 Pa)}. \]

The directions, \( P_1 \) and \( P_2 \) are perpendicular to each other, are at an angle of +3.5° with respect to the directions of vertical and horizontal instrument axes as shown in part B of this section.

Also, it is obvious that, \( \Delta P = 347 \text{ psi (from part B), approximately equal to, } P_1 - P_2 = 40.7 - (-306.7) = 347.4 \text{ psi (from part C), which provides a check on the principal stress calculations.} \)

### 3.3 Improved Techniques of Photoelastic Measurement

Since the experiments performed in this thesis study use two totally different methods-
the PhE method and the FE method, differences (errors) in the measured results of principal stresses and their directions from the two methods might be found in the process of the experiments, while they are compared to each other to verify accuracy. The differences would mostly come out at some typical points and areas where stress concentration occurs. Careless measurements in PE method and mistaken modeling and running in FE method would probably cause these differences. Thus, to avoid such mistakes, measurement process will be performed very carefully and patiently.

Many standard techniques have been shown in the previous sections to avoid mistakes, like accurate calibrations of the Strain Indicator and the Angle of Incidence of the plexiglass, zero adjustments of the Strain Indicator before the measurement, properly adding the load to the model to avoid torsion, and, placing the Oblique Prism as close as possible to the model with the center of each prism coinciding with the point of measurement, etc..

In this section, some improved techniques of the PhE measurements are developed to reach a high degree measurement accuracy.

First of all, mechanical screw-driven movement would cause errors in the output display by the instruments because of the gap distance between gears. Isoclinic lines and areas between the lines are very difficult to be located at a point while polarizer is being rotated because the isoclinics are dim (not clear enough) on a colorful background, when a Diffused Light System is used in the measurement. Thus, taking the average of a series of readouts is necessary. In the experiment, the angle should be marked down from both directions while rotating the polarizer/analyzer assembly clockwise and counterclockwise. The setting should be recorded from the digital counter on the compensator also in two ways.
- turning the compensator in both the clockwise and counterclockwise directions. Of course, average values should also be taken from a series of measurements several times at the same step. This will also help to avoid unnecessary errors.

In the process of determining principal stress directions, the Null-Balance compensation method is used, instead of the Tardy Compensation method, because it is more difficult to mark down the reading on the Tardy Compensation Scale exactly than on the Null-Balance Compensation digital counter.

Since the fringe pattern observed from the analyzer is colorful and relatively dim in some areas when using a Diffused Light System, a Telemicroscope and a Monochromator are used in the experiment. The Telemicroscope helps enlarge the area where high orders of the fringe must be identified because of the stress concentration. The Monochromator produces a dense black fringe at each point in the fringe pattern where integral fringe order occurs in while light. It helps identify high fringe orders (more than 5 orders) easier at the crack tip points and the corner points where the compensator is out of scale at this time.

Even though more fringe orders can be counted with the help of the Monochromator, the fringe pattern at the very close vicinity of the crack tip is still too dense to be identified by the bare eyes when a total load on the model is 500 lbs.. In the experiment, since the deformation of the plexiglass under a load is still in the linear elastic zone and proportional to the load, a fraction of the total load is added to the model, like 50 lb., 100 lb., and 200 lbs. at the same measurement. The highest orders of the fringes that can be counted are measured from each loading condition and then converted to the orders at the 500 lbs. loading condition by linear interpolation. This technique can help count higher fringe orders at the critical
Finally, it is very important to mention that, since the Diffused Light System used in the 060 series Polariscope provides bright light emission, color fringes appear on the model surface when the white light is polarized through the system and are very difficult to be individually identified. To get more accurate measurements, another lighting system, a Collimated Light System, should be used in future measurements.
CHAPTER 4

FINITE ELEMENT ANALYSIS

4.1 Finite Element Modeling

Another technique for analyzing the stresses of the tunnel model in this research is performed by the numerically based Finite Element (FE) method. The FE method used is a computer analysis of a model which exactly duplicates the PhE model, using the same size plate openings, the same material properties of the plexiglass plate and the same boundary and loading conditions used in the PhE measurements at each step of the investigation. Six different models, associated with the six different PhE experimental steps (section 2.2), have been created using COSMOS/M, the Finite Element System software. The geometry of the model, corner shape and position of the tunnels, thickness, length and inclination of the cracks (saw cuts), are exactly duplicated by the FE models developed.

Six node, two-dimensional elements for plane stress analysis, are used in the FE modeling. Two translational degrees of freedom (d.o.f.) per node are available in this element. In addition, the Adaptive H-Method (see section 4.2) is used in the last four models when saw cuts are introduced. Element sizes are reduced greatly around the crack tips and the tunnel corners.

At the boundaries, FE models duplicate exactly those of the plexiglass model. Two vertical sides are free; the lower edge is fixed in both $x$ and $y$ directions at every node; and the upper edge is compressively loaded in the $y$ direction at the nodes (force at node). To
duplicate exactly the boundary conditions of the plexiglass model used in PhE experiments, the forces applied to the boundary nodes are zero at the outer corner nodes and become larger at the adjacent nodes. The nodal forces remain constant at the main middle part of the upper edge (about 75% of the total length). The sum of the total load on the FE model is the same as the load applied to the plexiglass model. The boundary conditions of the FE model are illustrated in Figure 4-1.

Because no plastic deformation is assumed to occur, the FE models developed are analyzed using linear static analysis combined with a repetitive iterations to accommodate a mandated tolerance (percent error), when the Adaptive H-method is used (Figure 4-2).

**4.2 Adaptive H-Method**

The Adaptive command specifies parameters for adaptive meshing. The parameters of this command are used by the R_STATIC command to progressively improve the mesh until a desired accuracy level is reached. The improvement is accomplished by either refining the mesh (H-Method), or increasing the polynomial order (P-Method). The HP-Method is another Adaptive method that refines the mesh first and then increases the polynomial order. Only the H-Method is used in this analysis.

Adaptive meshing using the H-Method provides the user with automatic mesh refinement at the stress concentration areas to evaluate and improve the accuracy of results for linear static analysis problems.

In this particular faulted tunnel model, stress concentration will occur in the vicinity of the crack tips and the square tunnel corners. Stresses at these areas will be very much
Figure 4-1  Boundary Conditions of the Finite Element Model.

Figure 4-2  Strain - Stress Curve in Linear Elastic Zone, which the Slope $E$ is the Modulus of Elasticity.
higher (more than ten times at the tip areas) than those far away from the tips and the corners.

Errors will occur in the results of principal stresses from the COSMOS/M calculation while the ratio of the tip and corner size to the element size around them is too small. When using the Adaptive H-Method, the automatic mesh refinement will progressively subdivide the elements with high relative error into smaller elements until calculated average error becomes less than the specified allowable error level, such as 4%, 8%, 10% etc. Mesh refinement will also occur at the upper and lower edges of the model because of the point-loaded and point-fixed conditions at the nodes.

Illustrations of the tunnel model before and after the mesh refinement are shown in figure 4-3 and 4-4. A close look at the saw cuts with refined mesh is shown in figure 4-5. Figure 4-6 presents a plot out of the principal stress pattern using Adaptive H-method in step six. The principal stress at the lower corner of the tip point (point 6) in this stress pattern is 3800 psi in compression.

4.3 COSMOS/M Command Codes

The analysis of the principal stresses in the tunnel models is performed by the COSMOS/M system software through a series of command codes (See Appendix C for details). These command codes are inputted through computer keyboard and saved in a file with an extension file name of .SES. This file contains the material properties, geometry, boundary conditions, solution techniques etc. It is called after activating a program run by the R_STATIC command.

COSMOS/M requires the element type to be defined first. For that purpose, EGROUP
command is used to define the element type (TRIANG). The RCONST command (Real Constant) is used to define the thickness of the 2-D tunnel model. The modules of elasticity, $E_x$ and $E_y$, and Poisson's ratio, $\mu$, are defined under MPROP command (Material Properties).

After declaring the element type and the material properties, the geometry of the model is defined step by step. Different commands by COSMOS/M are to be used for this purpose. Since the geometry of the model, including three tunnels and an inclined crack, is irregular, the model has to be divided into several small regions (Command RG) and to be meshed separately.

When meshing is done, DND command (Displacement of Node) and FND command (Force on Node) are used to define the boundary and the loading conditions. The A_STATIC command is used to specify details of the linear static analysis to be performed by R_STATIC command.

Finally, the ADAPTIVE command is used to specify the parameters for adaptive meshing (H-Method), allowable error, maximum loops of running, etc.

The R_CHACK command can be used to double check the .SES file before running the program.

The final forms of COSMOS/M codes (.SES file) of the six different experimental steps are listed in details in Appendix C.
Figure 4-3  The Element Mesh of the FE Tunnel Model before Using Adaptive H-method.
Figure 4-4  The Element Mesh of the FE Tunnel Model after Using Adaptive H-method.
A Close Look at the Mesh around (a) the Saw Cut next to the Square Opening; (b) the Saw cut next to the Circular Opening.

Figure 4-5
Figure 4-6 The Principal Stress Pattern Using Adaptive H-method in Step Six. The Principal Stress at the Lower Corner of the Tip Point (Point 6) in this Stress Pattern is 3800 psi in Compression.
CHAPTER 5

DISCUSSION OF RESULTS

The principal stresses, \( P_1 \) and \( P_2 \), at predetermined critical test points, are determined by both the PhE and the FE methods. The stress concentration factors, \( K \), are then calculated from the principal stresses. Results \( (P_1 \text{ and } K) \) from both PhE and FE methods are compared to each other in this chapter.

To be clearly described the locations of the predetermined points on the model, an illustration of the model and the predetermined points on it, are shown again in figure 5-1, same as in figure 2-1.

5.1 Stress Results at Critical Points before the Introduction of Saw Cuts

In order to verify the accuracy of the two different techniques - the PhE and the FE methods, stresses are compared at critical points before the saw cuts (faults) are introduced.

5.1.1 Comparison between PhE and FE Results at Step Two

Principal stress results obtained by both the PhE and FE methods \( (P_1 \text{ and } P_2) \) of 22 points are shown in table 5-1. These principal stresses are determined by both PhE and FE methods at step two (Section 2-2), in which only two square openings and one circular opening has been cut in the plexiglass model. In step two of the analysis, point 1 to point 10 (figure 5-1) do not have any stress concentrations.

Notice that, for the principal stresses listed in table 5-1, a positive value means tensile
stress and a negative value means compressive stress. \( P_1 \) is always the algebraically larger value of principal stress and \( P_2 \) is the algebraically smaller one. The compared differences between the two methods are based on the PhE values. The principal stresses \((P_1 \text{ or } P_2)\) to be compared are the absolute larger value.

Figure 5-1 The Photoelastic Plexiglass Model and the Predetermined Test Points on the Model.
<table>
<thead>
<tr>
<th>Principal Stresses</th>
<th>PE Method (psi)</th>
<th>FE Method (psi)</th>
<th>Differences (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point A</td>
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Table 5-1  Comparison of the Principal Stresses, $P_1$ and $P_2$, at Selected Critical Points

Obtained by both PhE and FE Methods at Step Two (Continued).
<table>
<thead>
<tr>
<th>Principal Stresses</th>
<th>PE Method (psi)</th>
<th>FE Method (psi)</th>
<th>Differences (%)</th>
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</tr>
<tr>
<td>P2</td>
<td>-343.5</td>
<td>-268.6</td>
<td>-21.8 *</td>
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<tr>
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<td></td>
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<tr>
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<td>0</td>
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<td>Point II</td>
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<td>0</td>
<td></td>
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<tr>
<td>P2</td>
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<td>-19.6 *</td>
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<tr>
<td>Point III</td>
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<tr>
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<td>119.9</td>
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<tr>
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<td>Point IV</td>
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<td>0</td>
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<td>P2</td>
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<td>-482.2</td>
<td>-20.4 *</td>
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</table>

Table 5-1 (Continued)
From table 5-1, it is clear to see that, principal stresses from both PhE and FE methods have very good correlation with each other, except at points next to the loaded ends and at the edge of the circular opening.

Because of the boundary influence and the measurement difficulties at edge points, the principal stresses obtained by PhE measurements are relatively larger than those obtained by FE measurements at points I, 2, 3, I, II, III, IV (marked "*").

5.1.2 Comparison between Results from Step One and Step Two

At step two, a circular opening is cut under the two square openings previously cut in step one, with a clear distance of about 80% of the side length of the square. Principal stresses at points around the circular opening, as well as at points around the square openings, are affected by the circular opening. A comparison of the principal stresses before and after the circular opening is cut (step one and step two) is given in table 5-2. Examination of the results leads to the following conclusions:

1) Great changes occur at the edge points of the circular opening (points 2, 3, I, II, III, IV). The stress concentration factors of these points vary from 0.5 at point 3 and 2 to 11 at all other points around the circumference. At point I and point III, $P_2$ changes from compression to 0 (marked **) and at point III, $P_1$ is changed from compression to tension (marked ***).

2) The principal compressive stresses at points above the circular (point B, C, H, K) are greatly reduced, since the compressive stress contour lines have to bypass the open region.
<table>
<thead>
<tr>
<th>Principal Stresses</th>
<th>Step One (psi)</th>
<th>Step Two (psi)</th>
<th>Concentration Factor K</th>
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</thead>
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<td>29.51</td>
<td>-1.322</td>
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<td>P2</td>
<td>-274.9</td>
<td>-236.4</td>
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<td>P1</td>
<td>43.85</td>
<td>30.64</td>
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<td></td>
<td>P2</td>
<td>-233.1</td>
<td>-144.3</td>
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<td>Point E</td>
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<td>-119.6</td>
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<td>P2</td>
<td>-658.2</td>
<td>-975.9</td>
</tr>
<tr>
<td>Point F</td>
<td>P1</td>
<td>163.9</td>
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</tr>
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<td>P2</td>
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<td>0</td>
</tr>
<tr>
<td>Point G</td>
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<td>0</td>
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<tr>
<td></td>
<td>P2</td>
<td>-340.2</td>
<td>-314.3</td>
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<td>-19.26</td>
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<td>P2</td>
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Table 5-2  Comparison of Principal Stress Values, $P_1$ and $P_2$, Obtained from the FE Method, before and after the Circular Opening is Cut Out (Continued).
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<th>Concentration Factor K</th>
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<td>119.9</td>
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<td>-127.4</td>
<td>-482.2</td>
<td>3.78</td>
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</table>

Table 5-2 (Continued)
3) At points J and L (corner points at the lower boundary of the squares) and at points A and E (the corner points of the squares next to the boundary), the principal stresses increase from 48% to 69% in compression. At the other corner point M, which is far away from the circular opening, the principal stresses decrease by 25% in compression.

4) At points between the circular opening and the right-hand side square, point 4 and point 5, the principal stresses increase in compression as well.

5) At the lower side of the square, the principal stresses increase in tension by about 24% to 45% (point F and 8).

6) The effect of the circular opening on the principal stress pattern in the model fades away rapidly. At points H and 9, which are almost two diameters away from the center of the opening, principal stress values remain stable.

5.2 Effects of the Fault (Saw Cuts) on the Stress Pattern in the Tunnel Model

Saw cuts are introduced in the tunnel model to simulate the effect of natural faults. In steps three to six, four different saw cuts are introduced to both the PhE plexiglass models and the FE models (section 2.2). Due to these saw cuts, the principal stresses at the tip of saw cut and in its vicinity, as well as in other critical areas around the three tunnels, experience major transformations. High stress concentration occurs at the tip points of the saw cuts. Starting at step three, the Adaptive H-method (section 4.2 and appendix B) is used to refine the mesh in the vicinity of the crack tip and at the corner of the squares in FE analysis, in order to accommodate the sharp rise in the principal stresses observed by PhE.

Stress concentration factors, $K$, are calculated by comparing of the principal stresses
obtained after the introduction of the saw cuts to those stresses which had been obtained in step two.

For the critical points shown in figure 5-1 and for step two to six, the absolute larger principal stresses, $|P|$, and the relative stress concentration factors, $K$, are tabulated with results shown from both the PhE and the FE methods, followed by a plot of the stress concentration value, $K$, for different models (at different steps) shown at that point with values obtained from both the PhE and FE methods.

The values of $|P|$ and $K$ at points 10, 4, 5, 6, F, J and M, are shown in table 5-3 to table 5-9. The plots for $K$ are shown in figure 5-1 to figure 5-7. In these tables and figures, the FE results are obtained from models using the Adaptive H-method with an allowable tolerance of less than 20%.

In the PhE experiments, thicker saw cuts (0.021 in. or 0.533 mm wide) are first introduced to the model. The stress concentration factors, $K$, at tip points 10, 4 and 5, range from 5.4 to 9.0, while the principal stresses obtained from step three, step four and step five are compared to those obtained from step two before the introduction of saw cuts. A thinner saw cut (0.007 in. or 0.178 mm wide) is introduced to the model up to point 6 at step six, the stress concentration factor, $K$, at this time rises up to 31.6.

The FE models are then created step by step using the same geometry, the same material properties, the same loading and boundary conditions of the PhE models. The Adaptive H-method is used in the FE analysis with an allowable tolerance of 20%. The stress concentration factors, $K$, calculated at the same steps as in the PhE experiments, range from 7.0 to 10.1, when the three thicker saw cuts that represent the faults are measured. At step
Table 5-3 Comparison of the Principal Stresses, $P$, and the Calculated Relative Stress Concentration Factor, $K$, Obtained from both the PhE and the FE Method at Point 10.

<table>
<thead>
<tr>
<th></th>
<th>Step Two</th>
<th>Step Three</th>
<th>Step Four</th>
<th>Step Five</th>
<th>Step Six</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Photoelastic</strong></td>
<td>$P$ (psi)</td>
<td>-300.1</td>
<td>-1560</td>
<td>-1640</td>
<td>-1580</td>
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<td><strong>Method (PE)</strong></td>
<td>$K$</td>
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<td>5.5</td>
<td>5.3</td>
</tr>
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<td><strong>Finite Element</strong></td>
<td>$P$ (psi)</td>
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<td>-2060</td>
<td>-2130</td>
<td>-2100</td>
</tr>
<tr>
<td><strong>Method (FE)</strong></td>
<td>$K$</td>
<td>1.0</td>
<td>6.5</td>
<td>6.7</td>
<td>6.6</td>
</tr>
</tbody>
</table>

Figure 5-2 Stress Concentration Factors, $K$, at Step Two to Step Six by both the PhE and the FE Method at Point 10.
Table 5-4  Comparison of the Principal Stresses, $P$, and the Calculated Relative Stress Concentration Factor, $K$, Obtained from both the PhE and the FE Method at Point 4.

<table>
<thead>
<tr>
<th></th>
<th>Step Two</th>
<th>Step Three</th>
<th>Step Four</th>
<th>Step Five</th>
<th>Step Six</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photoelastic</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P (psi)</td>
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<td>0.99</td>
<td>7.3</td>
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<td>Finite Element</td>
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</tr>
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<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>K</td>
<td>1.0</td>
<td>1.0</td>
<td>8.1</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Figure 5-3  Stress Concentration Factors, $K$, at Step Two to Step Six by both the PhE and the FE Method at Point 4.
<table>
<thead>
<tr>
<th>Method</th>
<th>Step Two</th>
<th>Step Three</th>
<th>Step Four</th>
<th>Step Five</th>
<th>Step Six</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photoelastic Method (PE)</td>
<td>P (psi)</td>
<td>-202.0</td>
<td>-210.5</td>
<td>-250.4</td>
<td>-1824</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>1.0</td>
<td>1.0</td>
<td>1.2</td>
<td>9.0</td>
</tr>
<tr>
<td>Finite Element Method (FE)</td>
<td>P (psi)</td>
<td>-207.2</td>
<td>-211.7</td>
<td>-224.1</td>
<td>-2100</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>1.0</td>
<td>1.0</td>
<td>1.1</td>
<td>10.1</td>
</tr>
</tbody>
</table>

Table 5-5 Comparison of the Principal Stresses, $P$, and the Calculated Relative Stress Concentration Factor, $K$, Obtained from both the PhE and the FE Method at Point 5.

Figure 5-4 Stress Concentration Factors, $K$, at Step Two to Step Six by both the PhE and the FE Method at Point 5.
Table 5-6  Comparison of the Principal Stresses, $P$, and the Calculated Relative Stress Concentration Factor, $K$, Obtained from both the PhE and the FE Methods at Point 6.

<table>
<thead>
<tr>
<th>Method</th>
<th>Step Two</th>
<th>Step Three</th>
<th>Step Four</th>
<th>Step Five</th>
<th>Step Six</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photoelastic</td>
<td>P (psi)</td>
<td>-108.6</td>
<td>-110.5</td>
<td>-115.2</td>
<td>-120.0</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>1.0</td>
<td>1.0</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>Finite Element</td>
<td>P (psi)</td>
<td>-102.5</td>
<td>-111.6</td>
<td>-112.2</td>
<td>-122.0</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>1.0</td>
<td>1.1</td>
<td>1.1</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Figure 5-5  Stress Concentration Factors, $K$, at Step Two to Step Six by both the PhE and the FE Method at Point 6.
## Table 5-7

Comparison of the Principal Stresses, $P$, and the Calculated Relative Stress Concentration Factor, $K$, Obtained from both the PhE and the FE Method at Point $F$.

<table>
<thead>
<tr>
<th>Method</th>
<th>Step Two</th>
<th>Step Three</th>
<th>Step Four</th>
<th>Step Five</th>
<th>Step Six</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photoelastic</td>
<td>$P$ (psi)</td>
<td>234.7</td>
<td>219.7</td>
<td>222.5</td>
<td>250.4</td>
</tr>
<tr>
<td></td>
<td>$K$</td>
<td>1.0</td>
<td>0.94</td>
<td>0.95</td>
<td>1.07</td>
</tr>
<tr>
<td>Finite Element</td>
<td>$P$ (psi)</td>
<td>202.6</td>
<td>202.1</td>
<td>209.6</td>
<td>238.0</td>
</tr>
<tr>
<td></td>
<td>$K$</td>
<td>1.0</td>
<td>1.0</td>
<td>1.03</td>
<td>1.17</td>
</tr>
</tbody>
</table>

**Figure 5-6** Stress Concentration Factors, $K$, at Step Two to Step Six by both the PhE and the FE Method at Point $F$. 
Table 5-8  
Comparison of the Principal Stresses, $P$, and the Calculated Relative Stress Concentration Factor, $K$, Obtained from both the PhE and the FE Method at Point $J$.

<table>
<thead>
<tr>
<th></th>
<th>Step Two</th>
<th>Step Three</th>
<th>Step Four</th>
<th>Step Five</th>
<th>Step Six</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Photoelastic Method (PE)</strong></td>
<td>$P$ (psi)</td>
<td>-1103</td>
<td>-1085</td>
<td>-1028</td>
<td>-1306</td>
</tr>
<tr>
<td></td>
<td>$K$</td>
<td>1.0</td>
<td>0.96</td>
<td>0.93</td>
<td>1.18</td>
</tr>
<tr>
<td><strong>Finite Element Method (FE)</strong></td>
<td>$P$ (psi)</td>
<td>-1202</td>
<td>-1119</td>
<td>-1209</td>
<td>-1417</td>
</tr>
<tr>
<td></td>
<td>$K$</td>
<td>1.0</td>
<td>0.93</td>
<td>1.01</td>
<td>1.18</td>
</tr>
</tbody>
</table>

Figure 5-7  
Stress Concentration Factors, $K$, at Step Two to Step Six by both the PhE and the FE Method at Point $J$. 

---

<table>
<thead>
<tr>
<th>Step #</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress concentration factor $K$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 5-9

Comparison of the Principal Stresses, $P$, and the Calculated Relative Stress Concentration Factor, $K$, Obtained from both the PhE and the FE Method at Point $M$.

<table>
<thead>
<tr>
<th>Method (PE)</th>
<th>Step Two</th>
<th>Step Three</th>
<th>Step Four</th>
<th>Step Five</th>
<th>Step Six</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photoelastic P (psi)</td>
<td>-255.0</td>
<td>139.2</td>
<td>196.6</td>
<td>183.1</td>
<td>228.5</td>
</tr>
<tr>
<td>K</td>
<td>1.0</td>
<td>-0.55</td>
<td>-0.77</td>
<td>-0.72</td>
<td>-0.90</td>
</tr>
<tr>
<td>Finite Element P (psi)</td>
<td>-217.9</td>
<td>128.1</td>
<td>173.5</td>
<td>161.7</td>
<td>243.0</td>
</tr>
<tr>
<td>K</td>
<td>1.0</td>
<td>-0.59</td>
<td>-0.80</td>
<td>-0.74</td>
<td>-1.11</td>
</tr>
</tbody>
</table>

### Figure 5-8

Stress Concentration Factors, $K$, at Step Two to Step Six by both the PhE and the FE Method at Point $M$. 
six with a thinner saw cut, the $K$ rises up to 37.1 at the tip of the crack respectively.

From the above results, it is concluded that, once the saw cuts are present in the tunnel models, principal stresses at the boundary of the saw cut tips will experience a sharp jump in magnitude.

While using the adaptive refined meshing technique with an allowable tolerance of 20%, the values of $K$ obtained from the PhE models and the FE models are reasonably comparable. The differences between them are within 15% except at point J0, which is near the model boundary (section 4.1). The width of the thinner saw cut is about one third of the thicker cuts, and the $K$ values obtained around the thinner saw cut are about 3.5 to 6 times larger.

From the experimental results obtained at point F, point J and point M, it is easy to see that the stress values around the square tunnels are greatly influenced by the introduction of the saw cuts.

5.3 Modeling Techniques Used at the Tips of the Faults (Saw Cuts)

Since the geometrical and fringe pattern details of the saw cuts are not easy to identify even using a telemicroscope and a monochromator adaptor in PhE measurements, due to the fact that the dimensions of the saw cut tips are much smaller than those of the opening, the FE models cannot exactly duplicate the PhE models. Differences between the two techniques come out in several ways. To further study the effect of the faults on the stress patterns in the tunnel models, some changes in FE modeling around the tips of the faults have been made.

Two different shapes of the fault tip are created in the FE model in order to
investigate the influence on the stress concentration factors due to the changes in tip shape. A rounded top model and a flat top model are shown in figure 5-8. Two more allowable tolerance levels, 4% and 10%, have been used in the FE analysis with Adaptive H-method. The shape of the saw cut tip and the associated adaptive mesh in the lower fault region are shown in figure 5-8 at various allowable tolerance levels.

The results of principal stresses obtained in the previous two sections (sections 5.1 and 5.2) are measured at the lower corner boundary points of the tips. At these lower corner points, principal stresses are in compression. Principal stresses at the other boundary points around the upper corner of the tips are also measured because stress concentrations similarly occur at these points. Tensile stresses will occur at the upper corner zone and the effects of these tensile stresses are much more dangerous to the tunnel system than those of compressive stresses.

Principal stresses, obtained from the two tip shapes, at the upper and lower points of the tips are listed in table 5-10.

From table 5-10, it is clear to see that changes in the tip shape have some effect on the principal stresses values.

As the allowable tolerance decreases, principal stresses from the FE analysis change markedly. When 20% allowable tolerance is used in the adaptive mesh run, the results from both the PhE and FE methods are almost equal at the lower points of the tip. When 10% and 4% allowable tolerance are used, stress results from FE method are much larger than those from PhE method at the lower points at the tip region. At 4% allowable tolerance, the differences in the stress values are twice as high.
Figure 5-9  Rounded and Flat Faulted Tips and Adaptive Mesh Around the Lower Fault Region at Various Allowable Tolerances.
Finite Element Method, Adaptive H-method Photoelastic Measurement

<table>
<thead>
<tr>
<th></th>
<th>Finite Element Method</th>
<th>Photoelastic Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4% Allowable Tolerance</td>
<td>10% Allowable Tolerance</td>
</tr>
<tr>
<td>Rounded Top Model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper Point</td>
<td>1305 psi</td>
<td>1010 psi</td>
</tr>
<tr>
<td>Lower Point</td>
<td>-4120 psi</td>
<td>-3330 psi</td>
</tr>
<tr>
<td>Flat Top model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper Point</td>
<td>1310 psi</td>
<td>1014 psi</td>
</tr>
<tr>
<td>Lower Point</td>
<td>-4338 psi</td>
<td>-3742 psi</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upper Point:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1691</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Lower Point:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1824</td>
</tr>
</tbody>
</table>

Table 5-10  Comparison Principal Stresses Values, $P$, between the PhE Measurements and the FE Analysis, with Two Tip Shapes and Various Allowable Tolerance Values Used in the Adaptive H-method at Point 5 of Step Five. Upper and Lower Points Refer to the Top and Bottom Fillets at the Tips of the Cut.

At the upper tip points, however, the results from the adaptive method at 4% tolerance error are approximately equal to those obtained from the PhE measurements. At 10% and 20% tolerance, the stresses are lower than those obtained from the PhE measurements.

It is concluded that results obtained from the PhE and the FE techniques are in general agreement. Since the size of the cut is much smaller than that of the three openings, a slight
change in dimension and shape would cause a lot of change in stress values. Thus, the stress concentration at tip point will vary in a range.
CHAPTER 6

CONCLUSIONS AND FUTURE WORK

The investigation shows that, except at the tip of the fault, the principal stresses and the calculated $K$ from both techniques are within 15% from each other at the predetermined points. Since the ability to distinguish higher fringe orders by the available laboratory equipment is reached, the measured values of principal stresses from PhE method are, in general, slightly lower than those obtained by the FE method in the region of the crack tip.

From this research, a conclusion can be made that, the Finite Element techniques used in this research are reliable and fully capable of representing a faulted tunnel system. A full scale FE model of a faulted tunnel system shall be made in the future, in which gap elements and nonlinear material behavior will be incorporated.

In the present investigation, the 500 lbs. load applied to the model does not result in the closure of the gaps created by the thick and thin saw cuts, neither did the stress exceed the elastic stress limit of the plexiglass model. Gap elements may be used in the FE analysis to model the fault in order to resist a possible overlap between the element boundaries on the two sides of a fault, when the applied load exceeds 500 lbs.

Since plastic material behavior may be exhibited under applied loads larger than 500 lbs., in the vicinity of the crack tip, an iterative elastic-plastic Finite Element analysis, coupled with mesh patterns obtained from the Adaptive H-method, should be used. A laser light beam resource should also be used in the photoelastic measurements along with enhanced optics to get a sharper view of the fringe patterns at points of stress concentrations.

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Appendix A:

Basic Principle of Photoelasticity

A.1 History of Development

Photoelasticity was discovered well over a century ago in England by Sir David Brewster, who observed that stressed glass showed beautiful color patterns when viewed in polarized light. However, because of many limitations, very few practical applications of this phenomenon were made prior to 1900.

The process was next enhanced by the invention of Polaroid (Polaroid Corp.), which provided a method of obtaining large beams of polarized light, and by use of new plastic materials, which provided better sensitivity of measurement.

In the 1930s, two major developments transferred photoelasticity into the realm of tools capable of solving practical engineering problems. First, the "stress-freezing" process was developed by Oppel in 1936; in this process a three-dimensional model of the structure is cast or machined utilizing a stress-free transparent plastic. The second major development was the introduction of photoelastic coatings. It was proposed by Mesnager in France in 1930 that a birefringent material be bonded as a layer to an actual structure. Several additional techniques are developed and used in conjunction with the basic photoelastic concept, such as, Stroboelasticity, Thermophotoelasticity, Photoviscoelasticity and Scattered-light photoelasticity, which make it possible to analyze complex or unusual problems.
A.2 Basic Principles of Photoelasticity

The electromagnetic vibration associated with light, the electric field vector, is perpendicular to the direction of propagation. A light source emits a random train of waves containing vibrations in all possible planes (Fig. A-1). However, on the introduction of a polarizing filter, P, only one component of these vibrations is transmitted (that which is parallel to the privileged axis of the filter). If another polarizing filter, A, is placed in the beam, complete extinction of the beam can be obtained when the axes of the two polarizing filters are perpendicular to one another.

The speed of light in a transparent body, \( v \), is lower than the speed in a vacuum, \( c \), or in the air. The index of refraction, \( n \) which is equal to \( c/v \) in a homogeneous isotropic material, is independent of the orientation of the plane of vibration (plane of polarization). However, most transparent materials, notably plastics, behave homogeneously when unstressed but become heterogeneous when subjected to stresses or deformation. The index of refraction thus becomes a function of the intensity of stresses applied and of their direction.

Consider a beam of light, polarized in the plane defined by the polarizer, P, and propagating through a transparent birefringent plastic plate of thickness, \( t \). The beam will cross the plate at a point O, where the principal stresses \( \sigma_x \), \( \sigma_y \), and \( \sigma_z \) are oriented along the \( x, y, z \) directions (Fig. A-2). The assumption is made here that the principal stresses \( \sigma_x \) and \( \sigma_y \) are contained in the plane of the plate, and that their variation through the thickness \( t \) is negligible (Note that: \( P_1 \) and \( P_2 \) are used to represent the two principal stresses in the plane of the plexiglass plate in this thesis study instead of using \( \sigma_x \) and \( \sigma_y \)).

Entering the plastic, the light will split into two independent wave fronts, or two
Figure A-1  Action of Crossed Polarizer on Unpolarized Light (from Reference #1.)

Figure A-2  Principal of a Plane Polariscope: $P$, Plane of Axis of Polarizer; $\alpha$, Angle between Polarizer and Reference Direction; $\sigma$, Amplitude of Light Polarized in Plane $P$; $t$, Thickness of Sample Containing Stresses $\sigma_x$ and $\sigma_y$ in the $x$ and $y$ Directions; $\beta$, Angle between Principal Direction $x$ and Reference Direction; $\delta$, Relative Retardation of $Y$ with Respect to $X$ Wave; $A$, Plane of Axis of Analysis (from Reference #1.)
beams, X and Y, polarized in directions x and y. The speed of propagation of these waves will be \( v_x \) and \( v_y \), respectively. Emerging from the plastic, one of the two waves will be retarded with respect to the other one, and the relative retardation is easily derived as

\[
\delta = t(n_x - n_y)
\]  

(A-1)

If \( n_0 \) represents the index of refraction of the unstressed plate, the indexes \( n_x \) and \( n_y \) can be expressed as functions of stresses or strains existing at the point in question. If \( \varepsilon_x, \varepsilon_y \), and \( \varepsilon_z \) are the principal strains

\[
\begin{align*}
n_x &= n_0 + K_1 \varepsilon_x + K_2(\varepsilon_y + \varepsilon_z) \\
n_y &= n_0 + K_1 \varepsilon_y + K_2(\varepsilon_z + \varepsilon_x)
\end{align*}
\]  

(A-2) 

(A-3)

or

\[
\begin{align*}
n_x - n_y &= (K_1 - K_2)(\varepsilon_x - \varepsilon_y) = K(\varepsilon_x - \varepsilon_y)
\end{align*}
\]  

(A-4)

The dimensionless constant \( K \) is called the "strain-optical coefficient" and characterizes a physical property of the material, the photoelastic activity.

Similarly, the photoelastic effect can be expressed as a function of stresses \( \sigma_x, \sigma_y, \) and \( \sigma_z \) by

\[
\begin{align*}
n_x &= n_0 + C_1 \sigma_x + C_2(\sigma_y + \sigma_z) \\
n_y &= n_0 + C_1 \sigma_y + C_2(\sigma_x + \sigma_z)
\end{align*}
\]  

(A-5) 

(A-6)
or

\[ n_x - n_y = (C_1 - C_2)(\sigma_x - \sigma_y) = C(\sigma_x - \sigma_y) \quad \text{(A-7)} \]

The constant \( C \) is called the "stress-optical coefficient." For an elastic material, the constants \( C \) and \( K \) are related by equation A-8, where \( E \) and \( \mu \) are, respectively, modulus of elasticity and Poison's ratio. The strain- and stress-optical coefficients are functions of temperature and the wavelength used.

\[ C = \frac{K}{E(1 + \mu)} \quad \text{(A-8)} \]

In the case of plastics exhibiting inelastic behavior where stresses are not proportional to strains, such as, in the stress concentrated area where a sudden-changed shape or a crack occurs, it is necessary to consider the relative change of index of refraction as a function of both strain and acting stresses. For a purely "strain-optical material," equations A-2 to A-4 will remain valid. Such material subjected to a constant strain will exhibit a constant birefringence, \( n_x - n_y \), whereas existing stresses could relax with time. On the other hand, a purely "stress-optical material" will be better described by equations A-5 to A-7. Such material subjected to constant stress (e.g., dead-weight loaded specimen) will exhibit constant birefringence whereas the deformation will change with time.

### A.3 The Polariscope

The polariscope is an instrument used to measure the relative retardation or phase...
differences produced when polarized light passes through a stressed photoelastic model. It
can have a variety of forms depending on the technique best suited to the type of problem
being investigated and also to some extent on the personal preferences of the investigator.
There are two forms of polariscope, the plane polariscope and the circular polariscope. The
plane polariscope is used in our investigation.

The plane polariscope consists of a suitable light source and two polarizers. The first
polarizer converts the natural light from the source into a field of plane polarized light in the
path of which the model is placed. The second polarizer, which is called the analyzer, resolves
the component waves emerging from the model into one plane so that the effects produced
by the model can be measured from the resulting interference of the waves. As illustrated in
Figure A-2, a polarized wave is emerging from the polarizer, P, and propagating through a
stressed plate. The beam will cross the plate at a point O, where the state of stresses and
strains is described by:

\[ \sigma_\sigma, \sigma_\epsilon: \text{ principal stresses;} \]

\[ \epsilon_\sigma, \epsilon_\epsilon: \text{ principal strains;} \]

\[ \beta: \text{ angle between principal direction } x \text{ and reference direction used to measure} \]

\[ \alpha: \text{ angle between the polarizer, } P, \text{ and the reference direction.} \]

Before entering the stressed plate, the wave of amplitude \( a \) and pulsation \( \omega \) can be
represented at a point as a function of time, \( T \), by \( a \cos(\omega T) \). The stressed material will split
this wave into two independent wave fronts polarized in direction \( x \) and \( y \). The two waves are
propagating at different speeds. Substituting equations A-4 and A-7 into equation A-1, the
relative retardation that accumulates after crossing the thickness t is given by,

$\delta = K(t(e_x - e_y)) = C(t(\sigma_x - \sigma_y))$  \hspace{1cm} (A-9)

The amplitude, $A$, of the light on emerging from the analyzer can be given by

$A = a \sin 2(\beta - \alpha) \sin \phi/2$ \hspace{1cm} (A-10)

The intensity of the transmitted light, which is proportional to the square of the amplitude, is therefore a function of both the orientation of the principal stresses, given by angle $\beta$, and of the phase shift $\phi$. The light intensity will become zero whenever $a = \beta \pm \pi/2$, ie, when the polarizer is parallel to either principal stress, $x$ or $y$. Usually, this condition will be satisfied at many points at the same time. A line or a complete area will appear black. Such a line or area is called an isoclinic line. At every point of an isoclinic line the direction of a principal stress, given by an angle $\beta$, is either the same as the direction of the polarizer or perpendicular to it.

The light intensity also becomes zero if $\sin \phi/2 = 0$, ie, if $\phi/2 = N\pi (N = 0, 1, 2, ...)$. This condition can be written as

$\delta = \frac{\phi \lambda}{2\pi} = N\lambda \hspace{1cm} N = 0, 1, 2, ...$ \hspace{1cm} (A-11)

where $\lambda$ is wavelength. The light intensity thus becomes zero when the relative retardation becomes equal to an integral multiple of the wavelength of light used. At every point of such a line, which is called isochromatic, the retardation, $\delta$, is constant, ie, $\delta = N\pi$, and "$N$" is the
order of the isochromatic or simply the fringe order. From equations A-9 and A-11, we thus have

\[ N\lambda = C l (\sigma_x - \sigma_y) \]  
(A-12)

\[ \sigma_x - \sigma_y = \frac{Nf}{t} = \frac{\phi f}{2\pi t} \]  
(A-13)

where \( f = \lambda/C \) is a constant depending on the material and the wavelength of the light used.

A.4 Techniques of Measurements

Measurements, in most cases, are carried out to establish at every point: (a) The directions of principal stresses or strains, expressed by angle \( \beta \), and (b) The magnitude of the difference of principal stresses or strains, \( \sigma_x - \sigma_y \) or \( \varepsilon_x - \varepsilon_y \).

To measure directions, the plane polariscope with white light is used. As shown in Figure A-3, a black line, area, or point will appear when the polarizer and analyzer are parallel to the principal stress directions. To determine directions of stresses at a point, polarizer \( P \) and analyzer \( A \), coupled together in a crossed arrangement, are rotated together until extinction of light is achieved at the point. The common position of \( P \) and \( A \) with respect to the reference direction is indexed on a convenient scale and graduated in degrees, thereby providing the angle \( \beta \) of stress with respect to the same reference.

In order to provide a complete distribution of directions on a large model, the polarizer and analyzer are rotated to positions at which \( \beta = 0, 15^\circ, 30^\circ, 45^\circ, 60^\circ \), and \( 75^\circ \) (or in smaller increments if desired.) All isoclinal lines are then retraced on one sheet, providing
a more complete map of direction. From the isoclinal map, a set of "isostatic" lines can be traced. Isostatic lines are parallel to the direction of principal stresses at every point. Figure A-3 shows the principal of tracing of isostatic lines.

To recognize the fringes and to assign every fringe its order, a white light must be used. In a circular polariscope, only the order \( N = 0 (\delta = 0) \) then appears black. The fringe order increases or decreases without discontinuity.

For accurate stress measurements, it is necessary to measure the retardation to a fraction of the wavelength. A technique called Null-balance compensation is used in fringe order measurements. The compensator is a crystal or permanently deformed plastic exhibiting a calibrated variable retardation, \( \delta \), along its length. The compensator is superimposed so that its principal directions coincide with the directions of principal stresses in the plastic plate. When measurements are taken where the retardation in the compensator, \( \delta_c \), and the measured retardation, \( \delta \), are numerically equal but opposite in sign, the total intensity observed is zero, which is easily detectable as black in a circular crossed polariscope. There is another method called Tardy compensation in fringe order measurements, but it is not used in this experimental process.

In some instances, it may be necessary to separate the principal stresses - i.e., to determine the individual principal stress magnitudes. The procedure for doing so requires making a second fringe-order measurement with oblique-incidence lighting. With the oblique-incidence attachment, the polarized light is directed through the model at an angle to the surface normal, and thus traverses the model at an oblique angle. Under these conditions, the measured birefringence corresponds to the difference in the secondary principal stresses in the
plane perpendicular to the light ray. Combining the oblique- and normal-incidence measurements at a point provides the necessary information for determining the separate values of the principal stresses in the plane of the model.

At the condition of plane stress \((\sigma_z = 0)\), the separation of principal stresses is achieved by the measurement of the fringe order in normal incidence \(N_n\) and oblique incidence \(N_o\). Assuming that \(x\) and \(y\) are directions of principal stresses \(\sigma_x\) and \(\sigma_y\), the fringe orders observed in normal and oblique incidence are correlated to stresses by the expressions from equation A-13:

\[
N_n = \frac{t}{f} (\sigma_x - \sigma_y) \quad \text{(A-14)}
\]

\[
N_o = \frac{t}{f \cos \theta} (\sigma_x - \sigma_y \cos^2 \theta) \quad \text{(A-15)}
\]

The above equations solved in terms of \(\sigma_x\) and \(\sigma_y\) are:

\[
\sigma_x = \frac{f}{t} \frac{1}{1 - \cos^2 \theta} (N_o \cos \theta - N_n \cos^2 \theta) \quad \text{(A-16)}
\]

\[
\sigma_y = \frac{f}{t} \frac{1}{1 - \cos^2 \theta} (N_o \cos \theta - N_n) \quad \text{(A-17)}
\]
Where \( t \) is the thickness of the model, and \( k \) is the stress optical constant, which is known from the manufacturer. In order to obtain accurate measurements, the model material must be calibrated to determine the angle of incidence \( \theta \), which depends on the index of refraction of the model material.

Figure A-3  Principal of Isostatic Tracing (from Reference #1.)
Appendix B:

Finite Element Method and COSMOS/M System

B.1 Finite Element Method

The Finite Element Method (FEM) is a numerical procedure for analyzing structure and continua. Usually the problem addressed is too complicated to be solved satisfactorily by classical analytical methods. The classical approach is to write the differential equations of the model and solve them. In general, the FE method models a structure as an assemblage of small parts (elements). Each element is of simple geometry and, therefore, is much easier to analyze than the actual structure. In essence, a complicated solution is approximated by a model that consists of piecewise-continuous simple solutions. Elements are called “finite” to distinguish them from differential elements used in calculus. The FE procedure usually produces a large number of simultaneous algebraic equations, which can be generated and solved on a digital computer. The equations are of the form,

\[
[K]\{D\} = \{P\}
\]

where \([K]\) is the structure stiffness matrix, which is symmetric, \(\{D\}\) is a vector representing the generalized nodal displacement of the model and \(\{P\}\) is a vector of generalized nodal forces corresponding to the generalized nodal displacement of the model.

Results from FE method are rarely exact, except in a sudden-change area or under a concentrated loading boundary condition. Errors can be decreased by correctly using the right
type of elements, smaller size elements (i.e. using Adaptive method), more nodes in a single
element (i.e. using high order equations) and so on. Also, boundary condition (B.C.) are
important factors to be considered in FE modeling.

A FE analysis typically involves the following steps:

1. Divide the structure or continuum into finite elements. Mesh generation programs,
called preprocessors, help the user in doing this work.

2. Formulate the properties of each element.

3. Assemble elements to obtain the FE model of the structure.

4. Apply the known loads: nodal forces and/or moments.

5. Specify how the structure is supported (B.C.) by setting several nodal
displacements to known values (which often are zero).

6. Solve simultaneous linear algebraic equations to determine nodal degree of freedom
(d.o.f.).

7. Calculate element strains from the nodal d.o.f. and the element displacement field
interpolation, and finally calculate stresses from strains.

B.2 COSMOS/M System

COSMOS/M is a complete, modular, self-contained FE system (software) developed
by Structural Research and Analysis Corporation (SRAC) for personal computers and
workstations. The program includes modules to solve linear and nonlinear static and dynamic
structural problems, in addition to solving problems in the fields of heat transfer, fluid
mechanics, eletronmagnetics and structural optimization, etc.
The COSMOS/M consists of a pre- and postprocessor, various analysis modules, interfaces, translators and utilities. GEOSTAR is the basic pre- and postprocessor of the COSMOS/M finite element system. It is an interactive full three-dimensional CAD-like graphic geometric modeler, mesh generator and FE pre- and postprocessor. The user can create the model, geometry, mesh it, provide all analysis related information, perform the desired type of analysis, review, plot and print the results by using GEOSTAR.

The techniques used in the Finite Element Analysis (FEA) in this research through COSMOS/M GEOSTER are as follow:

1. 2-D plane stress model.
2. 6-node triangular elements.
3. Adaptive mesh refinement.
4. Linear static analysis.
Appendix C:

COSMOS/M Input Commands for Finite Element Analysis

C.1 Input Commands for Step One

C* COSMOS/M    Geostar V1.65
C* Problem : stpl    Date : 03-DEC-93    Time : 18:12:10
C* FILE,123,1,1,1,1,
EGROUP,1,TRIANG,0,0,0,0,0,0,0,0,
RCONST,1,1,1,2,35,0,
MPROP,1,EX,3.8E5,
MPROP,1,EY,3.8E5,
MPROP,1,NUXY,3,
PLANE,Z,0,1,
VIEW,0,0,1,0,
PT,1,0,0,0,0,0,
PTGEN,1,1,1,1,0,0,10.25,0,
PTGEN,1,1,2,1,0,10.25,0,0,
CRPLINE,1,1,2,4,3,1,
SCALE,0,
PT,5,1.9375,6.03125,0,
PTGEN,1,5,5,1,0,0,0,2.28125,0,
PTGEN, 1,5,6,1,0,2.28125,0,0,0,
PT,9,6,03125,6,03125,0,
PTGEN, 1,9,9,1,0,0,0,2.28125,0,
PTGEN, 1,9,10,1,0,2.28125,0,0,0,
PT,13,2,1,0,
PTGEN, 1,13,13,1,0,0,4.5,0,
PTGEN, 1,13,14,1,0,6.25,0,0,0,
CRPLINE, 5,5,6,8,7,5,
CRPLINE, 9,9,10,12,11,9,
CRPLINE, 13,13,14,16,15,13,
CRFILLET, 17,5,8,.07875,1,0,
CRFILLET, 18,5,6,.07875,1,0,
CRFILLET, 19,6,7,.07875,1,0,
CRFILLET, 20,7,8,.07875,1,0,
CRFILLET, 21,9,12,.07875,1,0,
CRFILLET, 22,9,10,.07875,1,0,
CRFILLET, 23,10,11,.07875,1,0,
CRFILLET, 24,11,12,.07875,1,0,
CT,1,1,56,4,1,2,3,4,
CT,2,1,36,8,5,18,6,19,7,20,8,17,
CT,3,1,36,8,9,22,10,23,11,24,12,21,
CT,4,1,120,4,13,14,15,16,
RG,1,4,1,2,3,4,
RG,2,1,4,
MA_RG,1,2,1,6,
MARGCH,2,2,1,T,6,1,1,
DCR,4,UY,0,4,1,UX,,
DCR,2,UX,0,2,1,,
FND,16,FY,-20.833,28,12,
FND,17,FY,-41.667,27,1,
NMERGE,1,4466,1,0.0001,0,1,0,
NCOMPRESS,1,4466,1, ,
A_STATIC,N,0,0,1e-06,1e+10,0,0,0,
C* R_STATIC,

C.2 Input Commands for Step Two

C* COSMOS/M Geostar V1.70a
C* Problem :stp2 Date : 06-JUL-94 Time : 14:07:49
C* FILE,123,1,1,1,1,
EGROUP,1,TRIANG,0,0,0,0,0,0,0,0,
RCONST,1,1,1,2,,35,0,
MPROP,1,EX,3.8E5,
MPROP,1,EY,3.8E5,
MPROP,1,NUXY,,3,
PLANE,Z,0,1,
VIEW,0,0,1,0,
PT,1,0,0,0,0,0,
PTGEN,1,1,1,1,0,0,10.25,0,
PTGEN,1,1,2,1,0,10.25,0,0,
CRPLINE,1,1,2,4,3,1,
SCALE,0,
PT,5,1.9375,6.03125,0,
PTGEN,1,5,5,1,0,0,2.28125,0,
PTGEN,1,5,6,1,0,2.28125,0,0,0,
PT,9,6.03125,6.03125,0,
PTGEN,1,9,9,1,0,0,2.28125,0,
PTGEN,1,9,10,1,0,2.28125,0,0,0,
CRPLINE,5,5,6,8,7,5,
CRPLINE,9,9,10,12,11,9,
CRFILLET,17,5,8,.07875,1,0,1E-06,
CRFILLET,18,5,6,.07875,1,0,1E-06,
CRFILLET,19,6,7,.07875,1,0,1E-06,
CRFILLET,20,7,8,.07875,1,0,1E-06,
CRFILLET,21,9,12,.07875,1,0,1E-06,
CRFILLET,22,9,10,.07875,1,0,1E-06,
CRFILLET,23,10,11,.07875,1,0,1E-06,
CRFILLET,24,11,12,.07875,1.0,1E-06,
PT,37,5.125,3.0625,0,
CRPCIRCLE,25,37,1,1.1406,360,4,
PT,42,8.3125,6.57168,0,
PT,43,8.49234,6.74534,0,
PT,44,8.67216,6.919,0,
PT,45,7.7906,6.03125,0,
PT,46,7.61077,5.88759,0,
PT,47,7.43094,5.71393,0,
PT,48,7.25111,5.54027,0,
CRLINE,29,42,43,
CRLINE,30,43,44,
CRLINE,31,45,46,
CRLINE,32,46,47,
CRLINE,33,47,48,
PT,49,8.3125,6.60168,0,
PT,50,8.49234,6.77534,0,
PT,51,8.67216,6.949,0,
PT,52,7.75953,6.03125,0,
PT,53,7.61077,5.88759,0,
PT,54,7.43094,5.71393,0,
PT,55,7.25111,5.54027,0,
CRLINE,34,49,50,
CRLINE,35,50,51,
CRLINE,36,51,44,
CRLINE,37,52,53,
CRLINE,38,53,54,
CRLINE,39,54,55,
CRLINE,40,55,48,
CRINTCC,11,29,34,5,0,
CRINTCC,12,31,37,6,0,
PT,56,3.25,1.64644,0,
CRLINE,45,48,56,
CRINTCC,45,27,28,1,0,
PT,59,3.25,1.67644,0,
CRLINE,48,55,59,
CRINTCC,48,27,28,1,0,
PT,62,5.90248,4.20792,0,
PT,63,6.08231,4.38158,0,
PT,64,6.26214,4.55524,0,
CRLINE,51,57,62,
CRLINE,52,62,63,
CRLINE,53,63,64,
PT,65,5.90248,4.23792,0,
PT, 77, 9.35, 5.51027, 0,
CRLINE, 71, 76, 48,
CRLINE, 72, 48, 77,
CRINTCC, 67, 71, 71, 1, 0,
PT, 78, 9.35, 6.919, 0,
CRLINE, 74, 44, 78,
CRINTCC, 69, 72, 74, 2, 0,
PT, 79, 3.77349, 1, 0,
CRLINE, 77, 69, 79,
CRINTCC, 70, 77, 77, 1, 0,
PT, 80, 9.35, 4.55524, 0,
CRLINE, 79, 64, 80,
CRINTCC, 76, 79, 79, 1, 0,
CT, 1, 1, 70, 4, 1, 2, 3, 4, 0,
CT, 2, 1, 178, 9, 67, 73, 68, 69, 75, 76, 80, 70, 78, 0,
CT, 3, 1, 163, 19, 73, 68, 69, 74, 36, 35, 34, 11, 23, 10, 22, 9, 21, 12, 37, 38, 39, 40, 71, 
0, 1,
CT, 4, 1, 51, 8, 5, 18, 6, 19, 7, 20, 8, 17, 0,
CT, 5, 1, 36, 11, 72, 75, 74, 30, 29, 42, 24, 44, 31, 32, 33, 0, 1,
CRLINE, 81, 48, 64,
CT, 6, 1, 42, 4, 76, 79, 81, 72, 0, 1,
CT, 7, 1, 104, 13, 70, 77, 61, 60, 66, 25, 26, 27, 51, 52, 53, 79, 80, 0, 1,
CT, 8, 1, 12, 14, 78, 67, 71, 81, 57, 56, 55, 54, 59, 28, 62, 63, 64, 77, 0, 1,
RG, 1, 2, 1, 2, 0,
RG, 2, 2, 3, 4, 0,
RG, 3, 1, 5, 0,
RG, 4, 1, 6, 0,
RG, 5, 1, 7, 0,
RG, 6, 1, 8, 0,
MA_RG, 1, 6, 1, 6, 0,
CRLINE, 82, 43, 50,
CT, 9, 1, 4, 4, 30, 82, 35, 36, 0, 1,
RG, 7, 1, 9, 0,
CT, 10, 1, 12, 8, 31, 32, 33, 40, 39, 38, 37, 43, 0, 1,
RG, 8, 1, 10, 0,
CT, 11, 1, 11, 8, 51, 52, 53, 57, 56, 55, 54, 58, 0, 1,
RG, 9, 1, 11, 0,
CT, 12, 1, 8, 6, 61, 60, 65, 62, 63, 64, 0, 1,
RG, 10, 1, 12, 0,
MA_RG, 7, 10, 1, 6, 0,
MARGCH, 1, 10, 1, T, 6, 1, 1,
NMERGE, 1, 8151, 1, 0.0001, 0, 1, 0,
NCOMPRESS, 1, 8150,
ECOMPRESS, 1, 3653,
DND,1,UX,0,55,54,,
DND,56,UX,0,72,1,UY,,
DND,19,UX,0,37,,
FND,20,FY,-8.6207,36,16,
FND,21,FY,-17.241,35,14,
FND,22,FY,-34.483,34,1,
A_STATIC,N,0,0,1E-06,1E+10,0,0,0,0,
ADAPTIVE,1,4,3,6,1,
C* R_STATIC,

C.3 Input Commands for Step Three

C* COSMOS/M Geostar V1.70a
C* Problem : stp3 Date : 08-JUL-94 Time : 14:59:06
C* FILE,123,1,1,1,1,
EGROUP,1,TRIANG,0,0,0,0,0,0,0,0,
RCONST,1,1,1,2,35,0,
MPROP,1,EX,3.8E5,
MPROP,1,EY,3.8E5,
MPROP,1,NUXY,3,
PLANE,Z,0,1,
VIEW,0,0,1,0,
PT,1,0,0,0,0,0,
PTGEN, 1, 1, 1, 1, 0, 0, 10.25, 0,
PTGEN, 1, 1, 2, 1, 0, 10.25, 0.0,
CRPLINE, 1, 1, 2, 4, 3, 1,
SCALE, 0,
PT, 5, 1.9375, 6.03125, 0,
PTGEN, 1, 5, 5, 1, 0, 0, 2.28125, 0,
PTGEN, 1, 5, 6, 1, 0, 2.28125, 0.0, 0,
PT, 9, 6.03125, 6.03125, 0,
PTGEN, 1, 9, 9, 1, 0, 0, 2.28125, 0,
PTGEN, 1, 9, 10, 1, 0, 2.28125, 0.0, 0,
CRPLINE, 5, 5, 6, 8, 7, 5,
CRPLINE, 9, 9, 10, 12, 11, 9,
CRFILLET, 17, 5, 8, 0.07875, 1, 0, 1E-06,
CRFILLET, 18, 5, 6, 0.07875, 1, 0, 1E-06,
CRFILLET, 19, 6, 7, 0.07875, 1, 0, 1E-06,
CRFILLET, 20, 7, 8, 0.07875, 1, 0, 1E-06,
CRFILLET, 21, 9, 12, 0.07875, 1, 0, 1E-06,
CRFILLET, 22, 9, 10, 0.07875, 1, 0, 1E-06,
CRFILLET, 23, 10, 11, 0.07875, 1, 0, 1E-06,
CRFILLET, 24, 11, 12, 0.07875, 1, 0, 1E-06,
PT, 37, 5.125, 3.0625, 0,
CRPCIRCLE, 25, 37, 1, 1.1406, 360, 4,
PT,42,8.3125,6.57168,0,
PT,43,8.49234,6.74534,0,
PT,44,8.67216,6.919,0,
PT,45,7.7906,6.03125,0,
PT,46,7.61077,5.85759,0,
PT,47,7.43094,5.68393,0,
PT,48,7.25111,5.51027,0,
CRLINE,29,42,43,
CRLINE,30,43,44,
CRLINE,31,45,46,
CRLINE,32,46,47,
CRLINE,33,47,48,
PT,49,8.3125,6.60168,0,
PT,50,8.49234,6.77534,0,
PT,51,8.67216,6.949,0,
PT,52,7.75953,6.03125,0,
PT,53,7.61077,5.88759,0,
PT,54,7.43094,5.71393,0,
PT,55,7.25111,5.54027,0,
CRLINE,34,49,50,
CRLINE,35,50,51,
CRLINE,36,51,44.
CRINTCC, 67, 71, 71, 1, 0,
PT, 78, 9.35, 6.919, 0,
CRLINE, 74, 44, 78,
CRINTCC, 69, 72, 74, 2, 0,
PT, 79, 3.77349, 1, 0,
CRLINE, 77, 69, 79,
CRINTCC, 70, 77, 77, 1, 0,
PT, 80, 9.35, 4.55524, 0,
CRLINE, 79, 64, 80,
CRINTCC, 76, 79, 79, 1, 0,
CT, 1, 1, 70, 4, 1, 2, 3, 4, 0,
CT, 2, 1, 178, 9, 67, 73, 68, 69, 75, 76, 80, 70, 78, 0,
CT, 3, 1, 163, 19, 73, 68, 69, 74, 36, 35, 34, 11, 23, 10, 22, 9, 21, 12, 37, 38, 39, 40, 71, &
0, 1,
CT, 4, 1, 51, 8, 5, 18, 6, 19, 7, 20, 8, 17, 0,
CT, 5, 1, 36, 11, 72, 75, 74, 30, 29, 42, 24, 44, 31, 32, 33, 0, 1,
CRLINE, 81, 48, 64,
CT, 6, 1, 42, 4, 76, 79, 81, 72, 0, 1,
CT, 7, 1, 104, 13, 70, 77, 61, 60, 66, 25, 26, 27, 51, 52, 53, 79, 80, 0, 1,
CT, 8, 1, 112, 14, 78, 67, 71, 81, 57, 56, 55, 54, 59, 28, 28, 62, 63, 64, 87, 0, 1,
RG, 1, 2, 1, 2, 0,
RG, 2, 2, 3, 4, 0,
C.4 Input Commands for Step Four

C* COSMOS/M Geostar V1.70a

C* Problem : stp4 Date : 17-SEP-94 Time : 15:31:40

C* FILE,123,1,1,1,1,
EGROUP,1,TRIANG,0,0,0,0,0,0,0,0,
RCONST,1,1,1,2,.35,0,
MPROP,1,EX,3.8E5,
MPROP,1,EY,3.8E5,
MPROP,1,NUXY,.3,
PLANE,Z,0,1,
VIEW,0,0,1,0,
PT,1,0,0,0,0,0,
PTGEN,1,1,1,0,0,10.25,0,
PTGEN,1,1,2,1,0,10.25,0,0,
CRPLINE,1,1,2,4,3,1,
CRINTCC,27,51,54,3,0,
PT,68,3.95332,2.32563,0,
PT,69,3.77349,2.15197,0,
CRLINE,60,58,68,
CRLINE,61,68,69,
PT,70,3.95332,2.35563,0,
PT,71,3.77349,2.18197,0,
CRLINE,62,61,70,
CRLINE,63,70,71,
CRLINE,64,71,69,
CRINTCC,28,60,62,2,0,
PT,72,.9,1,0,
PTGEN,1,72,72,1,0,0,7.8,0,
PTGEN,1,72,73,1,0,8.45,0,0,
CRPLINE,67,72,73,75,74,72,
PT,76,8.62216,6.969,0,
CRLINE,71,44,76,
CRINTCC,71,35,35,1,0,
ACTDMESH,PH,1,
CRFILLET,73,35,71,0.007,1,0,1E-06,
CRFILLET,74,71,30,0.007,1,0,1E-06,
PT,85,5.85248,4.25792,0,
CRLINE, 76, 62, 85,
CRINTCC, 76, 51, 54, 3, 0,
CRFILLET, 78, 54, 76, 0.007, 1, 0, 1E-06,
CRFILLET, 79, 76, 51, 0.007, 1, 0, 1E-06,
PT, 94, 8.3125, 7.7, 0,
PT, 95, 9.35, 7.7, 0,
CRLINE, 81, 94, 95,
PT, 96, 8.3125, 6.3, 0,
PT, 97, 9.35, 6.3, 0,
CRLINE, 82, 96, 97,
PT, 98, 0.9, 4.7, 0,
PT, 99, 9.35, 4.7, 0,
CRLINE, 83, 98, 99,
PT, 100, 5.125, 4.7, 0,
CRLINE, 84, 37, 100,
PT, 101, 7.0, 4.7, 0,
PT, 102, 7.0, 3.0625, 0,
CRLINE, 85, 101, 102,
CRLINE, 86, 37, 102,
CRINTCC, 11, 81, 81, 1, 0,
CRINTCC, 42, 82, 82, 1, 0,
CRINTCC, 69, 81, 83, 1, 0,
CRINTCC, 67, 83, 83, 1, 0,
CRINTCC, 83, 84, 85, 1, 0,
CRINTCC, 84, 59, 59, 1, 2,
CRINTCC, 86, 26, 26, 1, 2,
CT, 1, 1, 70, 4, 1, 2, 3, 4, 0,
CT, 2, 1, 118, 8, 67, 92, 68, 69, 89, 90, 91, 70, 0,
CT, 3, 1, 119, 20, 92, 68, 69, 81, 11, 23, 10, 22, 9, 21, 12, 43, 44, 24, 88, 82, 90, 94, 93, 83, 0, 1,
CT, 4, 1, 31, 8, 5, 18, 6, 19, 7, 20, 8, 17, 0,
CT, 5, 1, 110, 14, 67, 83, 96, 95, 28, 65, 66, 25, 26, 98, 85, 94, 91, 70, 0, 1,
RG, 1, 2, 1, 2, 0,
RG, 2, 2, 3, 4, 0,
RG, 3, 1, 5, 0,
MA_RG, 1, 3, 1, 6, 0,
PT, 105, 9, 35, 7, 0,
CRLINE, 99, 84, 105,
ACTDMESH, PH, 1,
CRINTCC, 89, 99, 99, 1, 0,
CT, 6, 1, 10, 7, 42, 29, 30, 75, 99, 101, 82, 0, 1,
CT, 7, 1, 12, 9, 87, 81, 100, 99, 74, 71, 73, 35, 34, 0, 1,
RG, 4, 1, 6, 0,
RG, 5, 1, 7, 0,
C.5  Input Commands for Step Five

C*  COSMOS/M     Geostar V1.71

C*  Problem :stp5     Date : 10-31-94  Time : 17:32:23
C* FILE, 123, 1, 1, 1, 1

EGROUP, 1, TRIANG, 0, 0, 0, 0, 0, 0, 0, 0

RCONST, 1, 1, 1, 2, 35, 0

MPROP, 1, EX, 3.8E5

MPROP, 1, EY, 3.8E5

MPROP, 1, NUXY, .3

PLANE, Z, 0, 1

VIEW, 0, 0, 1, 0

PT, 1, 0, 0, 0, 0, 0

PTGEN, 1, 1, 1, 1, 0, 0, 10.25, 0

PTGEN, 1, 1, 2, 1, 0, 10.25, 0, 0

CRPLINE, 1, 1, 2, 4, 3, 1

SCALE, 0

PT, 5, 1.9375, 6.03125, 0

PTGEN, 1, 5, 5, 1, 0, 0, 0.2, 2.8125, 0

PTGEN, 1, 5, 6, 1, 0, 2.8125, 0, 0, 0

PT, 9, 6.03125, 6.03125, 0

PTGEN, 1, 9, 9, 1, 0, 0, 0.2, 2.8125, 0

PTGEN, 1, 9, 10, 1, 0, 2.8125, 0, 0, 0

CRPLINE, 5, 5, 6, 8, 7, 5

CRPLINE, 9, 9, 10, 12, 11, 9

CRFILLET, 17, 5, 8, .07875, 1, 0, 1E-06.
CRFILLET, 18, 5, 6, 0.07875, 1.0, 1E-06,
CRFILLET, 19, 6, 7, 0.07875, 1.0, 1E-06,
CRFILLET, 20, 7, 8, 0.07875, 1.0, 1E-06,
CRFILLET, 21, 9, 12, 0.07875, 1.0, 1E-06,
CRFILLET, 22, 9, 10, 0.07875, 1.0, 1E-06,
CRFILLET, 23, 10, 11, 0.07875, 1.0, 1E-06,
CRFILLET, 24, 11, 12, 0.07875, 1.0, 1E-06,
PT, 37, 5.125, 3.0625, 0,
CRPCIRCLE, 25, 37, 1, 1.1406, 360, 4,
PT, 42, 8.3125, 6.57168, 0,
PT, 43, 8.49234, 6.74534, 0,
PT, 44, 8.67216, 6.919, 0,
PT, 45, 7.7906, 6.03125, 0,
PT, 46, 7.61077, 5.85759, 0,
PT, 47, 7.43094, 5.68393, 0,
PT, 48, 7.25111, 5.51027, 0,
CRLINE, 29, 42, 43,
CRLINE, 30, 43, 44,
CRLINE, 31, 45, 46,
CRLINE, 32, 46, 47,
CRLINE, 33, 47, 48,
PT, 49, 8.3125, 6.60168, 0,
PT, 50, 8.49234, 6.77534, 0, 
PT, 51, 8.67216, 6.9490, 
PT, 52, 7.75953, 6.03125, 0, 
PT, 53, 7.61077, 5.88759, 0, 
PT, 54, 7.43094, 5.71393, 0, 
PT, 55, 7.25111, 5.54027, 0, 
CRLINE, 34, 49, 50, 
CRLINE, 35, 50, 51, 
CRLINE, 36, 51, 44, 
CRLINE, 37, 52, 53, 
CRLINE, 38, 53, 54, 
CRLINE, 39, 54, 55, 
CRLINE, 40, 55, 48, 
CRINTCC, 11, 29, 34, 5, 0, 0.00005, 
CRINTCC, 12, 31, 37, 6, 0, 0.00005, 
PT, 56, 3.25, 1.64644, 0, 
CRLINE, 45, 48, 56, 
CRINTCC, 45, 27, 28, 1, 0, 0.00005, 
PT, 59, 3.25, 1.67644, 0, 
CRLINE, 48, 55, 59, 
CRINTCC, 48, 27, 28, 1, 0, 0.00005, 
PT, 62, 5.90248, 4.20792, 0,
PT,63,6.08231,4.38158,0,
PT,64,6.26214,4.55524,0,
CRLINE,51,57,62,
CRLINE,52,62,63,
CRLINE,53,63,64,
PT,65,5.90248,4.23792,0,
PT,66,6.08231,4.41158,0,
PT,67,6.26214,4.58524,0,
CRLINE,54,60,65,
CRLINE,55,65,66,
CRLINE,56,66,67,
CRLINE,57,67,64,
CRINTCC,27,51,54,3,0,0.00005,
PT,68,3.95332,2.32563,0,
PT,69,3.77349,2.15197,0,
CRLINE,60,58,68,
CRLINE,61,68,69,
PT,70,3.95332,2.35563,0,
PT,71,3.77349,2.18197,0,
CRLINE,62,61,70,
CRLINE,63,70,71,
CRLINE,64,71,69,
CRINTCC, 28, 60, 62, 2, 0, 0.00005,
PT, 72, 9, 1, 0,
PTGEN, 1, 72, 72, 1, 0, 0.78.0,
PTGEN, 1, 72, 73, 1, 0, 0.845, 0, 0,
CRPLINE, 67, 72, 73, 75, 74, 72,
PT, 76, 8.6216, 6.969, 0,
CRLINE, 71, 44, 76,
CRINTCC, 71, 35, 35, 1, 0, 0.00005,
ACTDMESH, PH, 1,
ACTDMESH, PH, 1,
CRFILLET, 73, 35, 71, 0.01, 1, 0, 1E-06,
CRFILLET, 74, 71, 30, 0.01, 1, 0, 1E-06,
ACTDMESH, PH, 1,
PT, 85, 6.21214, 4.60524, 0,
ACTDMESH, PH, 1,
CRLINE, 76, 64, 85,
CRINTCC, 76, 56, 56, 1, 0, 0.00005,
CRFILLET, 78, 56, 76, 0.01, 1, 0, 1E-06,
ACTDMESH, PH, 1,
ACTDMESH, PH, 1,
CRFILLET, 79, 76, 53, 0.01, 1, 0, 1E-06,
PT, 94, 8.3125, 7.7, 0,
CT, 1, 1, 70, 4, 1, 2, 3, 4, 0,
CT, 2, 1, 118, 8, 67, 89, 69, 84, 85, 86, 70, 0,
CT, 3, 1, 117, 20, 82, 88, 24, 44, 43, 12, 21, 9, 22, 10, 23, 11, 81, 69, 68, 89, 83, 93, 94, &
85, 0, 1,
CT, 4, 1, 31, 8, 5, 18, 6, 19, 7, 20, 8, 17, 0,
CT, 5, 1, 115, 14, 96, 95, 28, 65, 66, 25, 26, 98, 91, 94, 86, 70, 67, 83, 0, 1,
RG, 1, 2, 1, 2, 0,
RG, 2, 2, 3, 4, 0,
RG, 3, 1, 5, 0,
MA_RG, 1, 3, 1, 6, 0,
CT, 6, 1, 22, 13, 34, 35, 73, 71, 74, 75, 30, 29, 42, 82, 84, 81, 87, 0, 1,
RG, 4, 1, 6, 0,
MA_PTRG, 4, 84, 0, 1, 5, 0,
PT, 105, 7, 4, 44643, 0,
CRLINE, 99, 93, 105,
ACTDMESH, PH, 1,
CRINTCC, 91, 99, 99, 1, 0, 0.00005,
CT, 7, 1, 17, 9, 51, 52, 53, 80, 99, 101, 98, 97, 27, 0, 1,
RG, 5, 1, 7, 0,
CT, 8, 1, 18, 11, 54, 55, 56, 78, 76, 79, 99, 100, 93, 96, 59, 0, 1,
RG, 6, 1, 8, 0,
MA_PTRG, 5, 93, 0, 1, 5, 0,
MA_PTRG, 6, 93, 0.1, 5, 0,
MARGCH, 1, 6, 1, T, 6, 1, 1,
NMERGE, 1, 4619, 1, 0.0001, 0, 1, 0,
NCOMPRESS, 1, 4619,
DND, 1, UX, 0, 55, 54,,
DND, 56, UX, 0, 72, 1, UY,,
DND, 19, UX, 0, 37, 1,,
FND, 20, FY, -8.6207, 36, 16,
FND, 21, FY, -17.241, 35, 14,
FND, 22, FY, -34.483, 34, 1,
A_STATIC, N, 0, 0, 1E-06, 1E+10, 0, 0, 0, 0, 0,
ADAPTIVE, 1, 4, 3, 10, 1,
C* R_STATIC,

C.6  Imput Commands for Step Six
C* COSMOS/M  Geostar V1.71
C* Problem : stp6  Date : 1-30-95  Time : 13:54:59
C* FILE, 123, 1, 1, 1, 1,
EGROUP, 1, TRIANG, 0, 0, 0, 0, 0, 0, 0,
RCONST, 1, 1, 1, 2, 35, 0,
MPROP, 1, EX, 3.8E5,
MPROP, 1, EY, 3.8E5,
CRFILLET,23,10,11,.07875,1,0,1E-06,
CRFILLET,24,11,12,.07875,1,0,1E-06,
PT,37,5.125,3.0625,0,
CRPCIRCLE,25,37,1,1.1406,360,4,
PT,42,8.3125,6.57168,0,
PT,43,8.49234,6.74534,0,
PT,44,8.67216,6.919,0,
PT,45,7.7906,6.03125,0,
PT,46,8.80164,5.07623,0,
PT,47,6.2614,6.55524,0,
PT,48,3.25,1.64644,0,
CRLINE,29,42,43,
CRLINE,30,43,44,
CRLINE,31,45,46,
CRLINE,32,46,47,
CRLINE,33,47,48,
ACTDMESH,PH,1,
CRINTCC,33,27,28,1,0,0.00005,
PT,51,8.3125,6.60168,0,
PT,52,8.49234,6.77534,0,
PT,53,8.67216,6.949,0,
PT,54,7.75953,6.03125,0,
PT,55,7.78024,6.03125,0,
PT,56,6.79655,5.08132,0,
PT,57,6.25705,4.56033,0,
PT,58,6.24688,4.5705,0,
PT,59,3.25,1.67644,0,
ACTDMESH,PH,1,
CRLINE,36,51,52,
CRLINE,37,52,53,
CRLINE,38,53,44,
CRLINE,39,55,56,
CRLINE,40,56,57,
ACTDMESH,PH,1,
CRINTCC,11,29,36,7,0,0.00005,
CRINTCC,12,31,39,8,0,0.00005,
CRLINE,45,46,56,
CRLINE,46,57,58,
CRLINE,47,58,59,
CRINTCC,47,27,28,1,0,0.00005,
CRINTCC,27,33,47,14,0,0.00005,
CRINTCC,28,35,49,14,0,0.00005,
ACTDMESH,PH,1,
PT,62,0.5,6.0,
PT,63,5.125,5.6,0,
PT,64,7.5,5.6,0,
PT,65,10.25,5.6,0,
PT,66,7.5,3.0625,0,
CRLINE,54,62,63,
CRLINE,55,63,64,
CRLINE,56,64,65,
CRLINE,57,64,65,
CRLINE,57,63,37,
CRLINE,58,37,66,
CRLINE,59,66,64,
PT,67,8.3125,7.7,0,
PT,68,8.3125,6.3,0,
PT,69,10.25,7.7,0,
PT,70,10.25,6.3,0,
CRLINE,60,67,69,
CRLINE,61,68,70,
CRINTCC,11,60,60,1,0,0.00005,
CRINTCC,42,61,61,1,0,0.00005,
CRINTCC,3,60,61,1,0,0.00005,
CRINTCC,65,56,56,1,0,0.00005,
CRINTCC,57,51,51,1,2,0.00005,
CRINTCC,58,26,26,1,2,0.00005,
CRINTCC, 1.54, 54, 1, 0, 0.00005,
CT, 1, 0, 0.3, 14, 1.54, 57, 67, 28, 52, 25, 26, 70, 59, 56, 66, 4, 0,
CT, 2, 0, 0.3, 20, 71, 2, 3, 60, 11, 23, 10, 22, 9, 21, 12, 43, 44, 24, 63, 61, 65, 61, 56, 55, 54&
, 0, 1,
CT, 3, 0, 0.3, 8, 5, 18, 6, 19, 7, 20, 8, 17, 0,
PT, 73, 8, 62216, 6, 969, 0,
CRLINE, 72, 73, 44,
CRINTCC, 72, 37, 37, 1, 2, 0.00005,
CRFILLET, 75, 37, 74, 0.01, 1, 0, 1E-006,
CRFILLET, 76, 30, 74, 0.01, 1, 0, 1E-006,
CT, 4, 0, 0.3, 13, 36, 37, 75, 74, 77, 76, 30, 29, 42, 61, 64, 60, 62, 0, 1,
CRFILLET, 78, 46, 40, 0.007, 1, 0, 1E-006,
CRFILLET, 79, 46, 47, 0.007, 1, 0, 1E-006,
CRFILLET, 80, 40, 45, 0.0025, 1, 0, 1E-006,
ACTDMESH, PH, 1,
CRFILLET, 81, 32, 45, 0.0025, 1, 0, 1E-006,
CT, 5, 0, 0.3, 18, 47, 79, 46, 78, 40, 80, 45, 82, 81, 32, 33, 27, 69, 70, 59, 55, 57, 51, 0,&
1,
RG, 1, 1, 1, 0,
RG, 2, 2, 2, 3, 0,
RG, 3, 1, 4, 0,
RG, 4, 1, 5, 0,
MA_RG, 1.2, 1.6, 0,
MA_PTRG, 3.77, 0.1.5, 0,
MA_PTRG, 4.89, 0.056, 0,
ACTDMESH, PH, 1,
NMERGE, 1.1539, 10.0001, 0.0, 0,
NCOMPRESS, 1.1539,
ECOMPRESS, 1.2662,
DND, 1, UX, 0, 106, 105,
DND, 107, UX, 0, 140, 1, UY,
DND, 631, UX, 0, 666, 1,
FND, 632, FY, -4.4643633, 1,
FND, 664, FY, -4.4643665, 1,
FND, 634, FY, -8.9286626, 1,
FND, 661, FY, -8.9286631, 1,
FND, 637, FY, -17.8576601, 1,
A_STATIC, N, 0, 0, 1E-006, 1E+010, 0, 0, 0, 0, 0,
ADAPTIVE, 1.4, 3, 10, 1,
C*R_STATIC,
REFERENCES


12. N. Kandalaft-Ladkany, R. V. Wyman and S. G. Ladkany, *Investigation in 3-D of Stress Distribution in a Circular Tunnel and Vertical Emplacement Holes due to Thermal and*
