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Adaptive Control of a Projectile Fin Using Piezoelectric Elastic Beam

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This paper treats the question of model reference adaptive control of a projectile fin using a piezoelectric actuator. The hollow projectile fin is rigid, within which a flexible cantilever beam with a piezoelectric active layer is mounted. The model of the fin-beam system includes the aerodynamic moment which is a function of angle of attack of the projectile. The rotation angle of the fin is controlled by deforming the flexible beam which is hinged at the tip of the rigid fin. Based on the command generator tracker concept, a model reference adaptive fin angle controller is designed. For the design of the control law, a linear combination of the fin angle and fin's angular rate is chosen as the controlled output variable. In the closed-loop system, the controlled output variable tracks the reference trajectory and the fin angle asymptotically converges to the desired value and the elastic modes converges to their equilibrium values. Simulation results are presented which show that in the closed-loop system, the fin angle is precisely controlled in spite of uncertainties in the fin-beam parameters and the aerodynamic moment coefficients

I. Introduction

The use of surface-mounted or bonded piezoelectric actuators for the shape control of intelligent structure has gained widespread acceptance recently. Applications can be found in many areas including the shape control of metallic or composite plates or beams^{1,2}. Applications also involved actuation of various types of aircraft structural members such as wings, fins, or rotor blade³⁻⁶. Advantages of this approach are mainly due to the integration of the actuators into the structural members itself, thus saving the space required for servo motors, force transmission devices, or hydraulic systems⁴. This advantage becomes even more important when small aerial vehicles such as unmanned aircraft, small missiles, guided munitions, and projectiles, are examined. Piezoelectric twist actuators used for this application are based on anisotropic straining of the host structure using directionally attached isotropic actuator⁵ or using piezoelectric fibers integrated into the composite structural members⁶. General formulation and solution procedures for an analytical model for a composite laminated plate with isotropic or anisotropic active layers is derived in Refs. 3,4. The design of active controllers using piezoelectric actuators for vibration, force and position control of systems have been considered by various authors⁷⁻¹².

Traditionally, for the path control of missiles and projectiles, maneuvering forces and moments are generated by fin angle control using mechanical actuators which are bulky and slow. For high performance projectiles, there is a need to develop more efficient actuation mechanisms. Recently, the development of a smart fin (fin-beam model) has been considered^{12,13}. This fin has an outer hollow rigid body inside which resides a hinged flexible beam with a piezoelectric active layer. The control of the fin angle is then accomplished by deforming the beam. The design of the controller of

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Ref. 13 is based on a modeling error compensation approach in which the lumped uncertainties are estimated using a high-gain observer. This requires precise measurement of the fin angle for stability in the closed-loop system. A fuzzy controller has been designed in Ref. 14 for the control of this fin. Of course, for the fuzzy controller design, the designer first has to develop a number of if-then rules which often are not easy to obtain. Ref. 23 provides an adaptive controller, based on inverse feedback linearization technique. However, this requires synthesis using either the state feedback or discontinuous output feedback control.

Flexible structures are essentially infinite dimensional systems; however often finite dimensional models by neglecting the higher modes are used for analysis and design. The models of flexible structures are generally obtained by solving the eigenvalue problem resulting from finite element methods. However, it is well known that the resulting fidelity of model parameters degrades drastically for higher modes. Researchers have made considerable effort to design controllers for the control of flexible structures. Ref. 15 provides a good review of literature in which readers will find several references. For flexible structures, controller designs based on feedback linearization, passivity concepts and adaptive techniques have been attempted^{16–21}. But the methods based on passivity¹⁷ have advantage over other design techniques since the passivity approach does not rely on model truncation or higher-order models of the structure and is independent of the numerical values of the model data.

The contribution of the paper lies in the design of an adaptive control system for the control of a projectile fin. The projectile fin which is hollow but rigid, is controlled by deforming a cantilever flexible beam, which is mounted inside the fin. The model chosen here is similar to that reported in Ref. 13. A finite dimensional model is used for study, but the design is independent of the truncation order of the model obtained using finite element method. The model includes the aerodynamic moment affecting the fin motion which is a function of the angle of attack of the projectile. For the trajectory control of the fin angle, a judicious choice of a controlled output variable which is a linear combination of the tip position and velocity of the flexible beam is made. Based on the command generator tracker concept, a model reference adaptive control law is designed. In the closed-loop system the fin angle asymptotically converges to the target fin angle generated by a command generator. In the closed-loop all the elastic modes converge to their equilibrium values. Simulation results are presented which show that the designed adaptive control system accomplishes precise fin angle control in spite of uncertainties in the fin-beam parameters and the aerodynamic moment coefficients.

The organization of the paper is as follows. The mathematical model is presented in Section II. Section III presents the control law and simulation results are given in Section IV.

II. Dynamic Model

The model of the fin-beam system is shown in Figure 1. The flexible beam with a piezoelectric active layer bonded on the top surface, is hinged at one end to the fin and the other end is attached rigidly to the projectile body. The fin is free to rotate about an axis fixed to the projectile body. When the control voltage $u(x, t)$ is applied to the actuator, the induced strain in the actuator generates the bending moment m that is expressed⁸ as

$$m = cu(x, t) \quad (1)$$

The constant c can be obtained by considering geometrical and material properties of the beam and piezoelectric actuator. Considering the cross sectional geometry and force equilibrium along the axial direction, the constant c can be expressed as⁹

$$c = -d_{31} \frac{h_p + h_b}{2} \frac{E_p h_b E_b}{E_p h_p + E_b h_b} b \quad (2)$$

where d_{31} is the piezoelectric strain constant and E_p and E_b are Young's modulus of the piezoelectric actuator and the beam respectively. Other geometric parameters are shown in Fig. 1.

As shown in Fig. 1, an airfoil is connected to the beam actuator using a hinge. The airfoil is assumed to be rigid and its rotation is assumed to be small and planar. A finite element approach is used to describe the dynamics of the flexible beam, which is considered as composed of finite elements

satisfying Euler-Bernoulli theorem. The beam is divided into n elements with equal length of L_i . The displacement w of any point on the beam element i is described in terms of nodal displacement, w_i , and slope, ϕ_i , at node i and $i + 1$, respectively and is expressed as

$$w_i = Nq_i \quad (3)$$

where $q_i = (w_i, \phi_i, w_{i+1}, \phi_{i+1})^T$ and $N = (N_1, N_2, N_3, N_4)$ is the shape function vector with

$$\begin{aligned} N_1 &= \frac{1}{L_i^3}(2x_i^3 - 3x_i^2L_i + L_i^3) \\ N_2 &= \frac{1}{L_i^3}(x_i^3L_i - 2x_i^2L_i^2 + x_iL_i^3) \\ N_3 &= \frac{1}{L_i^3}(-2x_i^3 + 3x_i^2L_i) \\ N_4 &= \frac{1}{L_i^3}(x_i^3L_i - x_i^2L_i^2) \end{aligned} \quad (4)$$

where x_i is the element local coordinate variable defined along the beam neutral axis. The kinetic energy of an element i becomes

$$T_i = \int_0^{L_i} \rho_i \dot{w}^T \dot{w} dx_i \doteq \frac{1}{2} \dot{q}_i^T M_i \dot{q}_i \quad (5)$$

where $M_i (= \int_0^{L_i} \rho_i N^T N dx_i)$ is a mass matrix and ρ_i is a combined density of the beam and piezoelectric actuator per unit length.

The potential energy of an element i is

$$V_i = \frac{1}{2} \int_0^{L_i} \frac{1}{E_i I_i} (E_i I_i \frac{\partial^2 w}{\partial x_i^2} + cu)^T (E_i I_i \frac{\partial^2 w}{\partial x_i^2} + cu) dx_i \quad (6)$$

where $E_i I_i$ is the product of Young's modulus of elasticity by the cross-sectional area moment of inertia for the equivalent beam for an element i . If the piezoelectric actuator has a uniform geometry and that a uniform voltage is applied along its length, u can be assumed to be function of time only. The potential energy of an element can be further expressed as,

$$V_i = \frac{1}{2} q_i^T K_i q_i + q_i^T \left(\int_0^{L_i} \frac{\partial^2 N}{\partial x_i^2} dx_i \right) cu(t) + \frac{1}{2E_i I_i} c^2 u^2(t) \quad (7)$$

where the stiffness matrix K_i becomes

$$K_i = \int_0^{L_i} E_i I_i \left(\frac{\partial^2 N}{\partial x_i^2} \right)^T \left(\frac{\partial^2 N}{\partial x_i^2} \right) dx_i \quad (8)$$

The kinetic energy of the rigid fin is

$$T_f = \frac{1}{2} \dot{w}_{n+1}^T \frac{J_f}{L^2} \dot{w}_{n+1} \quad (9)$$

where J_f is the mass moment of inertia of the fin.

Using the Lagrangian dynamics, the equations of motion for an element i becomes

$$M_i \ddot{q}_i + K_i q_i = B_i (-cu(t)), \quad i = 1, \dots, n-1 \quad (10)$$

where $B_i = (0, -1, 0, -1)^T$ which represent two concentrated moments at two nodes of the element i . For the last element or $i = n$, the equation of motion including the mass of the rigid fin becomes

$$M_n \ddot{q}_n + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{J_f}{L^2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \ddot{q}_n + K_n q_n = B_n (-cu(t)) \quad (11)$$

The equations derived for each element can be assembled after expansion and matrix reduction from the boundary conditions of the cantilever beam as follows,

$$M\ddot{q} + Kq = B_0u(t) \quad (12)$$

where $q = (w_2, \phi_2, \dots, w_{n+1}, \phi_{n+1})^T \in \mathfrak{R}^{2n}$. Considering the hinge connection between the beam actuator and the blade, the fin angle can be expressed as

$$\theta = \tan^{-1}\left(\frac{\delta_t}{L}\right) \quad (13)$$

where L is the total length of the beam and δ_t is the tip displacement of the beam. For small fin angle, it can be approximated as $\theta = \delta_t/L$.

The aerodynamic moment acting on the fin is a complicated function of the angle of attack of the projectile and the fin rotation angle. The data generated by the computational fluid dynamics show that the aerodynamic moment can be accurately modeled as a linear function of the fin angle and a reasonable model can be expressed as

$$m_a = m_{a0}(\alpha) + p_a(\alpha)\theta = m_{a0}(\alpha) + p_a(\alpha)L^{-1}e^{*T}q \quad (14)$$

where $p_a(\alpha)$ is a polynomial in the angle of attack, α , $p_a(\alpha) = p_0 + p_1\alpha + \dots + p_k\alpha^k$ (k is a positive integer) and $e^{*T} \in \mathfrak{R}^{2n}$ is a unit vector whose $(2n-1)^{th}$ element is one and rest are zero.

The modified fin-beam model including the aerodynamic force takes the form

$$M\ddot{q} + Kq = B_0u(t) + B_a m_a \quad (15)$$

where $B_a = L^{-1}[0, \dots, 0, 1, 0]^T \in \mathfrak{R}^{2n}$. Solving (15) gives

$$\ddot{q} = -M^{-1}K_m q + M^{-1}B_0u(t) + M^{-1}B_a m_{a0}(\alpha) \quad (16)$$

where $K_m = K - p_a(\alpha)L^{-1}B_a e^{*T}$.

The eigenvalues of $M^{-1}K_m$ are distinct positive real numbers. As such there exists a similarity transformation matrix V formed by the eigenvectors of the matrix $M^{-1}K_m$ such that

$$V^{-1}M^{-1}K_m V = \Omega^2 \quad (17)$$

where $\Omega^2 = \text{diag}(\Omega_i^2)$, $i = 1, \dots, 2n$; $\Omega_i \neq \Omega_j$, $i \neq j$.

Defining $\eta = V^{-1}q$, one obtains from (16)

$$\begin{aligned} \ddot{\eta} &= -\Omega^2\eta + V^{-1}M^{-1}B_0u(t) + V^{-1}M^{-1}B_a m_{a0}(\alpha) \\ &= -\Omega^2\eta + B_1u(t) + d \end{aligned} \quad (18)$$

where $B_1 = V^{-1}M^{-1}B_0 \in \mathfrak{R}^{2n}$ and $d = V^{-1}M^{-1}B_a m_{a0}(\alpha)$. The modal form (15) has no damping. However, there is nonzero structural damping for any elastic body. As such it is common to introduce a dissipation term proportional to the rate $\dot{\eta}$. Introducing a damping term of the form $2D\Omega$, where $D = \text{diag}(\zeta_i)$, $i = 1, \dots, 2n$, $\zeta_i > 0$, one obtains the system

$$\ddot{\eta} = -2D\Omega\dot{\eta} - \Omega^2\eta + B_1u \quad (19)$$

The fin angle in new coordinate becomes

$$\theta = L^{-1}e^{*T}q = L^{-1}e^{*T}V\eta = C_0\eta$$

It is assumed that the system matrices D , Ω , B_1 and C_0 are unknown. Furthermore, it is assumed that only the fin angle and the angular rate is measurable. Suppose that reference model of the form

$$\dot{x}_m = A_m x_m + B_m u_m \quad (20)$$

$$y_m = C_m x_m \quad (21)$$

is given. We are interested in designing an adaptive control system such that the fin angle asymptotically tracks the reference trajectory y_m . Moreover, for synthesis only the measured angles θ and $\dot{\theta}$ are to be used.

III. Adaptive Control Law

In the following derivation, adaptive controller will be designed assuming that the disturbance input d is zero. Defining the state vector $x = (\eta^T, \dot{\eta}^T)^T$, a state variable representation of (20) with $d = 0$ takes the form

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0_{2n \times 2n} & I_{2n \times 2n} \\ -\Omega^2 & -2D\Omega \end{bmatrix} x + \begin{bmatrix} 0_{2n \times 1} \\ B_1 \end{bmatrix} u \\ &\triangleq Ax + Bu \end{aligned} \quad (22)$$

We associate with the system (22), an output variable

$$y = Cx \quad (23)$$

where C is yet to be chosen.

For the design of controller based on CGT method^{16,17}, almost strictly positive real (ASPR) condition for the system (19) and (20) is required²⁵.

Definition: A system $\{A, B, C\}$ is ASPR if there exists a gain K_e and symmetric positive definite matrices $P, Q \in \mathbb{R}^{4n \times 4n}$ such that

$$P(A - K_e BC) + (A - K_e BC)^T P = -Q \quad (24)$$

$$PB = C^T \quad (25)$$

and if $K_e = 0$, then $\{A, B, C\}$ is said to be strictly positive real (SPR).

The model with the choice of the output $y \triangleq \theta = C_0 \eta$ cannot be ASPR because the associated transfer function is of relative degree two. The ASPR condition can be satisfied if we make a judicious choice of the output variable of the form

$$\begin{aligned} y &= (\dot{\theta} + \mu\theta) = C_0 \dot{\eta} + \mu C_0 \eta \\ &= \begin{pmatrix} \mu C_0 & C_0 \end{pmatrix} x \triangleq Cx \end{aligned} \quad (26)$$

where μ is positive real number.

Mufti¹⁷ has shown that for a choice of

$$\mu < \mu^* = \min\{2\zeta\omega_i, i = 1, \dots, 2n\}$$

there exists matrices P and Q which satisfy (21) for $K_e = 0$ and therefore, the system is in fact, strictly positive real. This is also easily verified by computing the transfer function relating y and u , which is

$$\begin{aligned} \frac{\hat{y}(s)}{\hat{u}(s)} &= \sum_{i=1}^{2n} \frac{c_{0i} b_{1i} (s + \mu)}{s^2 + 2\zeta\omega_i s + \omega_i^2} \\ &= \sum_{i=1}^{2n} H_i(s) \end{aligned}$$

where $C_0 = (c_{01}, \dots, c_{0,2n})$ and $B_1 = (b_{11}, \dots, b_{1,2n})$. For the model under consideration, computing the matrices C_0 and B_1 , one finds that

$$c_{0i} b_{1i} > 0, i = 1, \dots, 2n$$

and therefore, each of the transfer functions $H_i(s)$ with $\mu < 2\delta\Omega_i$ is SPR. Apparently, the parallel combination of SPR transfer functions is also SPR.

Later, even though (A, B, C) is SPR, we shall introduce additional output feedback for modifying the transient characteristics. It may be noted that in fact, for any real number $K_e > 0$, the system

$\{(A - K_e BC), B, C\}$ remain SPR. This is easily verified by noting that SPR system $\{A, B, C\}$ implies that there exist $P > 0$ and $Q > 0$, such that

$$\begin{aligned} A^T P + PA &= -Q \\ PB &= C^T \end{aligned} \quad (27)$$

Subtracting both sides by $KPBC$ and $K_e C^T B^T P$ in (24) and noting that $PB = C^T$, one obtains

$$P(A - K_e BC) + (A - K_e BC)^T P = -Q - 2K_e C^T C < 0 \quad (28)$$

and therefore, same matrix P solves (28). However, a good value of K_e is not known. Therefore, we intend to design a controller which will adaptively seek a good output feedback gain K_e for control.

Now the design of the controller for the asymptotic tracking of the reference fin angle y_m is considered. When perfect tracking occurs, i.e, $y = y_m$ for $t \geq 0$, let x^* , u^* and y^* , respectively denote the corresponding plant state, input and output trajectories of the ideal model (22) for perfect tracking satisfying

$$\dot{x}^* = Ax^* + Bu^* \quad (29)$$

$$y^* = Cx^* = C_m x_m = y_m \quad (30)$$

In the command generator tracker theory^{16,17}, it is assumed that the starred signals satisfy

$$\begin{aligned} x^* &= S_{11}x_m + S_{12}u_m \\ u^* &= S_{21}x_m + S_{22}u_m \end{aligned} \quad (31)$$

where S_{ij} are constant matrices. We assume here that u_m is a step function. Indeed for the existence of matrices S_{ij} , one needs to satisfy only a mild (CGT) condition. For the solvability of (31), it is sufficient that, the transfer function $C(SI - A)^{-1}B$ has no zeros at the origin or in common with any eigenvalue of A_m . For the model under consideration, this CGT condition holds.

Defining $\tilde{x} = x^* - x$, one obtains the state error equation

$$\begin{aligned} \dot{\tilde{x}} &= A\tilde{x} + B(u^* - u) \\ &= (A - BK_e^*C)\tilde{x} + B[K_e^*\tilde{y} + S_{21}x_m + S_{22}u_m - u] \end{aligned} \quad (32)$$

where $\tilde{y} = y_m - y = C_m x_m - Cx = Cx^* - Cx = C\tilde{x}$ and $K_e^* > 0$. S_{21} , S_{22} are treated as unknown. The feedback gain K_e^* denotes an unknown gain which gives good performance.

Define $\nu = \begin{bmatrix} K_e^* & S_{21} & S_{22} \end{bmatrix}^T \in \mathfrak{R}^{m+2}$. Then the error equation (32) can be written as

$$\dot{\tilde{x}} = A_a \tilde{x} + B(\nu^T \Phi - u) \quad (33)$$

where $\Phi = \begin{bmatrix} \tilde{y} & x_m^T & u_m \end{bmatrix} \in \mathfrak{R}^{m+2}$ and $A_a = (A - BK_e^*C)$.

The control law is chosen as

$$u = \hat{\nu}^T \Phi + \nu_p^T \Phi \quad (34)$$

Here $\hat{\nu}$ denotes an estimate of the actual parameter vector ν and ν_p is a proportional gain.

Substituting the control law in (33) gives

$$\dot{\tilde{x}} = A_a \tilde{x} + B[\tilde{\nu}^T \Phi - \nu_p^T \Phi] \quad (35)$$

where $\tilde{\nu} = \nu - \hat{\nu}$ is the parameter estimation error.

For the derivation of the adaptation law, consider a quadratic Lyapunov function

$$V = \tilde{x}^T P \tilde{x} + \tilde{\nu}^T \Gamma \tilde{\nu} \quad (36)$$

where Γ is a positive definite symmetric matrix (denoted as $\Gamma > 0$). The derivative of V along the solution of (22) is given by

$$\dot{V} = \tilde{x}^T (PA_a + A_a^T P) + 2\tilde{x}^T PB[\tilde{\nu}^T \Phi - \nu_p^T \Phi] + 2\tilde{\nu}^T \Gamma \dot{\tilde{\nu}} \quad (37)$$

Note that $\{A_a, B, C\}$ is SPR and $\tilde{x}^T P B = \tilde{x}^T C^T = \tilde{y}$. For making \dot{V} negative, select the update law and gain ν_p of the form

$$\dot{\hat{\nu}} = -\dot{\nu} = \Gamma^{-1} \Phi \tilde{y} \quad (38)$$

$$\nu_p = \Gamma_p \Phi \tilde{y} \quad (39)$$

where the matrix $\Gamma_p \geq 0$.

Substituting the adaptation laws (38) and (39) in (37) and using (28), one obtains

$$\dot{V} \leq -\tilde{x}^T Q \tilde{x} - 2\lambda_{\min}(\Gamma_p) \|\Phi\|^2 \tilde{y}^2 \leq 0 \quad (40)$$

where $\lambda_{\min}(\Gamma_p)$ is the smallest eigenvalue of Γ_p . In view of (40), \dot{V} is negative semidefinite. According to Lyapunov stability results^{22,24}, it follows that $\tilde{x}(t)$ tends to zero as $t \rightarrow \infty$ and therefore, the target pitch angle is attained.

It is well known that the presence of disturbance input d can cause parameter divergence. For obtaining robust stability, one modifies the update law as

$$\dot{\hat{\nu}} = 0 \quad (41)$$

when the error is small. With this dead-zone modification, one can show that all the signals and the tracking error are bounded in the closed loop system if $d \neq 0$ ²⁶.

IV. Simulation Results

This section presents the simulation results for the smart fin (fin-beam model) including the model reference adaptive control law. The mechanical properties of the simulated model are taken from Yim et al.¹³. These are: $\rho_b = 8,300(kg/m^3)$, $\rho_p = 7,500(kg/m^3)$, $E_b = 70GPa$, $E_p = 63GPa$, $L = 140mm$, $b = 25mm$, $h_b = 0.5mm$, $h_p = 0.127mm$, $d_{31} = 1.8E-10(m/volt)$. Using finite element method (with $n=5$ elements), a state-variable representation of the fin-beam model of dimension 20 is obtained for simulation. The aerodynamic moment (14) is chosen for different angles of attack of the projectile based on the CFD analysis. By a linear approximation of the data obtained by the CFD analysis, the parameters of the aerodynamic moment are found to be $m_{a0} = -0.005$, $p_a = +0.0263$ for $\alpha = -10^\circ$, and $m_{a0} = 0.005$, $p_a = -0.0286$ for $\alpha = +10^\circ$. The value of $\Gamma = 5000000(I_{2n \times 2n})$ and Γ_p is chosen to be zero and therefore ν_p becomes zero. The minimum value of μ obtained is 0.5. The initial values of $\hat{\nu}$ are taken as $[-10, 0, 0]^T$. The damping coefficient is taken as $\zeta = 0.005$. The aerodynamic moment is chosen as

$$m_a = a(\alpha) + b(\alpha)\theta$$

where α is the angle of attack and θ is the fin angle. Simulation results are shown for different fin angle commands. The reference command trajectory is obtained using a first order command generator, with values $A_m = -6$, $B_m = 6$, $C_m = 1$.

A. Adaptive Control: Fin Angle = 10° , Angle of attack = -10° , $+10^\circ$

Figure. 2 and Figure. 3 show the simulation results for fin angle of 10° , with angles of attack -10° and $+10^\circ$. It is observed that the fin angle asymptotically converge to the desired value less than 6 seconds. The control input needed to deflect the fin to an angle of 10° with angle of attack -10° is around 1800 volts while that for $+10^\circ$ is just above 2000 volts. We observe that for larger fin angle command the control input needed is much larger. It is also interesting to see that there is no overshoot for the flexible modes and they reach their equilibrium values at the steady state, in both the cases.

V. Conclusion

This paper considered the control of rotation angle of a smart projectile fin. A flexible beam with a piezoelectric active layer was used for rotating the fin. The model of the fin-beam system includes the aerodynamic moment which is a function of angle of attack of the projectile. A state variable model using finite element method was obtained. A model reference adaptive controller, based on the command generator tracker concept is designed. Simulation results show that the

designed adaptive control system accomplishes precise fin angle control in spite of uncertainties in the fin-beam parameters and the aerodynamic moment coefficients.

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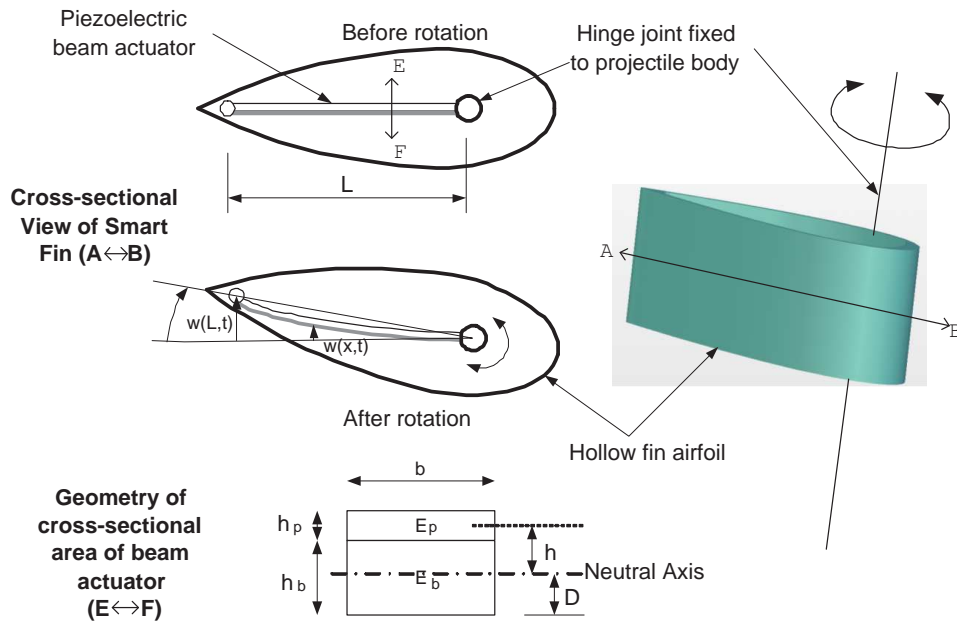


Figure 1. Configuration of Smart Fin

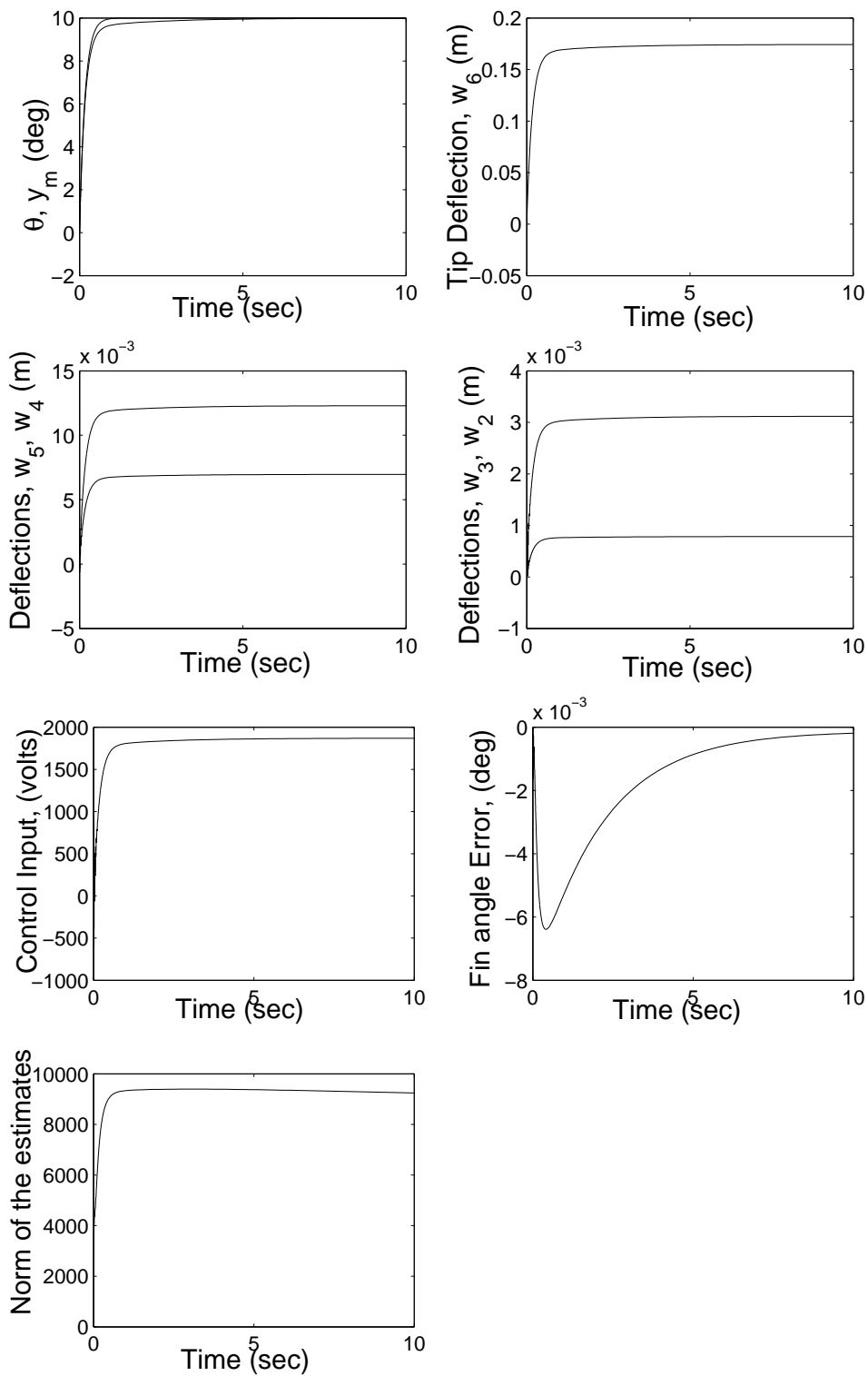


Figure 2. Adaptive Control: Fin Angle = 10^0 , Angle of attack = -10^0

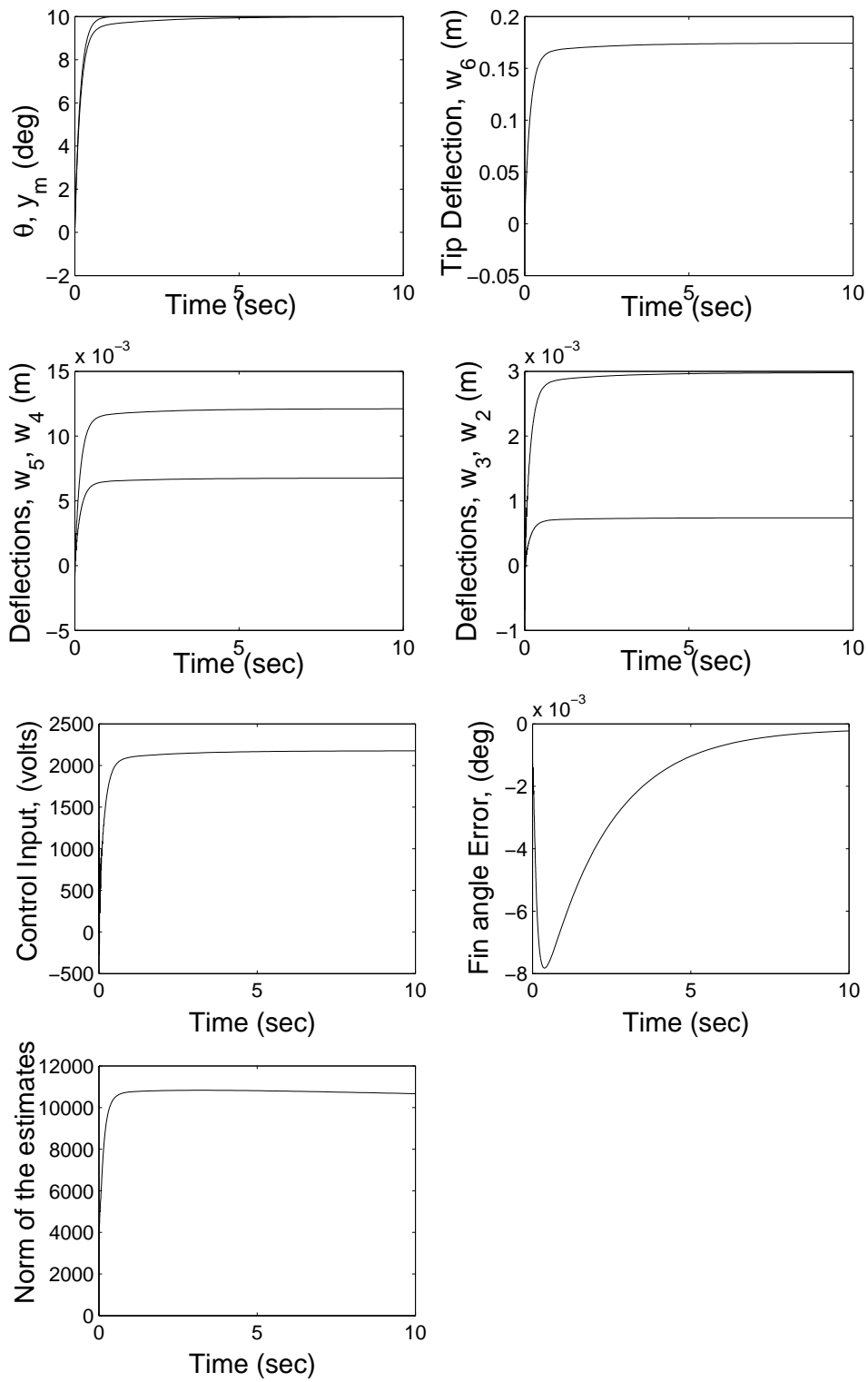


Figure 3. Adaptive Control: Fin Angle = 10^0 , Angle of attack = 10^0