Energy flow in nonlinear circuits

Jianzhong Jiang

University of Nevada, Las Vegas

Follow this and additional works at: https://digitalscholarship.unlv.edu/rtds

Repository Citation
https://digitalscholarship.unlv.edu/rtds/537
INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.
ENERGY FLOW IN NONLINEAR CIRCUITS

by

Jianzhong Jiang

A thesis submitted in partial fulfillment of the requirements for the degree of

Master of Science
in
Electrical Engineering

Department of Electrical Engineering
University of Nevada, Las Vegas

December, 1995
The thesis of Jianzhong Jiang for the degree of Master of Science in Electrical Engineering is approved.

Chairperson, Yahia Baghzouz, Ph.D

Examining Committee Member, Shahram Latifi, Ph.D

Examining Committee Member, William Brogan, Ph.D

Graduate Faculty Representative, Mohamed Trabia, Ph.D

Interim Graduate Dean, Cheryl L. Bowles, Ed.D.

University of Nevada, Las Vegas

December, 1995
Apparent power and reactive power are two critical quantities in energy flow studies in electrical power systems. Existing definitions for these terms work well when both voltages and currents are sinusoidal with respect to time. However, both apparent and reactive power have no physical meaning. When these physically unclarified quantities are applied to non-sinusoidal systems, the following questions related to power flow remain not fully answered to date. What do these quantities really mean? Is it fair to bill customers based on the measurement of physically not meaningful quantities? What is the efficient way, both economical and technical, to compensate non-active power in power networks?

To answer the above mentioned questions, this thesis analyzes energy flow in nonlinear circuits, clarifies and proposes new power quantities with physical interpretations that are practical and effective when voltages and/or currents are non-sinusoidal. The suggested definitions are measurable quantities based on time-domain approach, and are useful in evaluating the power quality and efficiency in electrical systems. The measurement method and compensation with active filters are also discussed.
ACKNOWLEDGMENTS

First and foremost, I would like to express my special thanks to Dr. Yahia Baghzouz, under whose direction this work was carried out. I am grateful to Dr. Baghzouz for the help rendered by him during the graduate course work and this thesis. His vast knowledge in electric power engineering, critical judgment, continuing encouragement and warmhearted instructions have always promoted me to accomplish the work one step beyond.

My sincere thanks also goes to Dr. Shahram Latifi, Dr. William Brogan and Dr. Mohamed Trabia for agreeing to be in my thesis committee and providing valuable advise on this thesis.

Finally, I wish to express my great appreciation to my wife, my son, my parents and my host-family for their love, encouragement, patience and understanding.
Contents

ABSTRACT ................................................................. iii
ACKNOWLEDGMENTS .............................................. iv

1 Introduction .......................................................... 1

2 Review of Power Definitions in Sinusoidal Circuits .............. 4
  2.1 Instantaneous Power ............................................. 5
  2.2 Active or Average Power ....................................... 7
  2.3 Reactive Power .................................................... 8
  2.4 Apparent Power .................................................... 9
  2.5 Power Factor ....................................................... 9

3 Power Definitions in Non-sinusoidal Circuits - Frequency Domain ............. 10
  3.1 System with Non-sinusoidal Voltage and Current ............ 11
    3.1.1 Instantaneous Power ....................................... 12
    3.1.2 Active or Average Power ................................ 13
    3.1.3 Reactive Power ............................................ 13
    3.1.4 Apparent Power ............................................ 14
    3.1.5 Distortion Power .......................................... 14
    3.1.6 Power Factor ............................................... 15
    3.1.7 Numerical Example ........................................ 15
  3.2 Systems with Sinusoidal Voltage and Nonlinear Load .......... 18
  3.3 Comments on Frequency Domain Approach .................... 20

4 Power Definitions in Non-sinusoidal Circuits - Time Domain .......... 21
  4.1 Existing Definitions .............................................. 22
  4.2 Proposed Terms for Physical Interpretations of Apparent and Reactive Powers 24
  4.3 Numerical Examples ............................................. 26
    4.3.1 Linear Circuit with Sinusoidal Supply Voltage ........ 26
    4.3.2 Nonlinear Circuit with Sinusoidal Supply Voltage ..... 30

5 Power Measurement and Compensation in Non-sinusoidal Systems .................... 35
  5.1 Measurement of Proposed Currents and Powers ................ 35
5.2 Compensation of Nonlinear Loads with Active Filters ................. 37

6 Conclusions .................................................. 41
BIBLIOGRAPHY .................................................. 42
# List of Figures

2.1 Series R-L circuit with sinusoidal voltage supply ............................................... 4  
2.2 Waveforms of voltage and current in sinusoidal circuit ................................. 6  
2.3 Instantaneous active, reactive and total powers in sinusoidal circuit ............. 7  
3.1 Series R-L circuit with non-sinusoidal voltage supply ................................. 16  
3.2 Waveforms of voltage and current in non-sinusoidal circuit ..................... 16  
3.3 Waveforms of instantaneous active, reactive, distortion and total power .... 17  
4.1 Waveforms of voltage and active, reactive, apparent and total current in linear sinusoidal circuit ................................................................. 29  
4.2 Waveforms of active, reactive, apparent and total powers in linear sinusoidal circuit ................................................................. 29  
4.3 Single-phase AC chopper with sinusoidal voltage supply. ....................... 30  
4.4 Waveforms of voltage and active, reactive, apparent and total current in chopper circuit ................................................................. 32  
4.5 Waveforms of active, reactive, apparent and total powers in non-sinusoidal circuit ................................................................. 34  
5.1 Block diagram of instantaneous power and current measurement ........... 36  
5.2 System diagram of nonlinear circuit with current-type APF ................. 39  
5.3 Time domain compensation using hysteresis method with three-state switching function ................................................................. 40
Chapter 1

Introduction

With the increasing use of different types of static power electronic converters, waveform distortion in both voltage and current is becoming a major concern to electric power utilities. These nonlinear loads inject harmonic currents into the power grid system and induce extra power loss on transmission lines, overheating power apparatus, and interfering with communication systems [1]. The resulting electrical “pollution”, whether it is produced by large single-source or by the cumulative effect of many small loads, often propagates for miles along distribution feeders and causes problems at points remote from the source. Moreover, polluting sources are frequently very sensitive to pollution from other sources. Power electronic loads are both villains and victims from a power quality point of view [2].

In addition to the traditional need of power factor improvement with regard to the efficiency of power transmission, a new need for better quality of power emerged from the wide-spread use of nonlinear loads. In order to resolve the power quality issue, one
needs to verify various power components through decomposition. The essential difficulty is in the comprehension of power flow in electric circuits when the voltage and current are non-sinusoidal [3].

Attempts of a quantitative description of power flow in non-sinusoidal circuits have a long history and many different power quantities have been suggested [4] - [6]. Efforts have been made to define the “reactive” and “distortion” powers so that they can be measured and controlled to improve the power factor and quality of power. Fundamental approaches for defining power terms under non-sinusoidal condition are presented in two different domains: frequency-domain and time-domain.

The frequency-domain approach to non-sinusoidal systems is characteristic of the work by Budeanu [7], Rissik [8], Shepherd and Zand [9], Sharon [10] and many others [12] - [14]. In here, the periodic but non-sinusoidal variables are expanded in terms of Fourier series. The active, reactive, apparent and distortion power are then defined. But the physical meaning of these quantities under these definitions are not as clear as those demonstrated in sinusoidal circuits [9]. The main disadvantage of the Fourier analysis approach is that the method is conceptually and mathematically indirect and complicated. Often, the technique requires a large sum of functions to represent a waveform that is not possible in practice. From the measurement point of view, the method requires sophisticated instruments with built-in Fast Fourier Transform (FFT) capability.

The time-domain approach, on the other hand, is pioneered by Fryze [11], Kusters and Moore [15], and Page [16]. In here, the general current waveform is decomposed into the “in-phase” and “quadrature” components with reference to the source voltage. Articles [15] and [16] demonstrate that the time-domain approach is conceptually straightforward
and all quantities discussed are readily measured by simple instrumentation. However, the interpretation of the introduced quantities in [16] need to be re-examined to reflect the effects of non-sinusoidal voltage source.

The objective of this thesis is to undertake a basic study in the time-domain for active and non-active power flow in non-sinusoidal systems. The study includes an appraisal of existing definitions of active, reactive and distortion powers, and introduces new power terms with physical interpretations. The purpose is to establish a basic understanding of reactive and apparent power so that a consensus can be reached on power definitions. These definitions can then be implemented by meter and control manufacturers for legal and economic issues with respect to billing consumers who pollute the network and for developing efficient compensation methods.

This thesis is divided into 6 chapters. The Introduction in Chapter 1 gives an overview of the thesis. A review of existing concepts of active, reactive, instantaneous and apparent powers in sinusoidal system with their practical usage will be presented in Chapter 2. Chapter 3 discusses power definitions in non-sinusoidal circuits based on the frequency-domain approach. The existing time-domain approach is presented in Chapter 4 with an appraisal. A new alternative description of reactive and apparent powers with physical interpretation is then proposed and illustrated by examples. Chapter 5 discusses the measurement and compensation methods of the proposed quantities. Chapter 6 ends with a conclusion on the accomplishments of this thesis.
Chapter 2

Review of Power Definitions in Sinusoidal Circuits

This chapter reviews the concept of power flow in linear and sinusoidal systems. The circuit used for illustration is an ideal single-phase, sinusoidal voltage source of constant frequency which supplies a series load of resistance $R$ and inductance $L$, as shown in Figure 2.1.

![Series R-L circuit with sinusoidal voltage supply](image)

Figure 2.1: Series R-L circuit with sinusoidal voltage supply
The instantaneous load voltage $v(t)$ and current $i(t)$ are given by

$$v(t) = i(t) R + L \frac{d i(t)}{dt} = \sqrt{2} V \sin \omega t,$$  

and

$$i(t) = \sqrt{2} \frac{V}{Z} \sin(w t - \phi) = \sqrt{2} I \sin(w t - \phi),$$  

where

$$|Z| = \sqrt{R^2 + (wL)^2},$$  

$$\phi = \tan^{-1}\left(\frac{wL}{R}\right),$$  

$\omega$ is the source angular frequency in rad/s. Note that $\phi$ represents the phase shift between the voltage and current due to the energy storage element of the circuit. Obviously, for an inductive load, $0^\circ < \phi < 90^\circ$ (i.e., the current lags the voltage). $V$ and $I$, represent the rms values of the voltage and current, respectively. The rms value $M$ of any function of time, $m(t)$, is defined as

$$M = \sqrt{\frac{1}{T} \int_0^T m^2(t) \, dt}.$$  

Figure 2.2 shows the voltage and current as given by equations (2.1) and (2.2) for $\phi = 60^\circ$.

### 2.1 Instantaneous Power

The instantaneous power is defined as the product of instantaneous voltage and current:

$$p(t) = v(t) i(t),$$
\[ = 2 VI \sin wt \sin (wt - \phi), \]
\[ = VI \cos \phi (1 - \cos 2wt) - VI \sin \phi \sin 2wt, \]
\[ = p_a + p_r. \quad (2.6) \]

Figure 2.2: Waveforms of voltage and current in sinusoidal circuit

Instantaneous power \( p \) is the instantaneous rate of energy transfer or energy utilization. This quantity satisfies the principle of Conservation of Energy, which is stated as follows: *In any electric circuit, at any instant, the instantaneous rate of energy transfer at the input terminals is equal to the sum of the instantaneous rates of energy transfer in the various load components.*

Note that the instantaneous power has two terms as shown in equation (2.6). The first term is interpreted as the instantaneous active power \( p_a \), which has a mean value of \( VI \cos \phi \) and an alternating component with twice the line frequency. The term \( p_a \) never becomes negative and therefore, represents the unidirectional part of \( p \). The second term, \( p_r \), may be interpreted as the instantaneous reactive part of \( p \). This term represents the
bidirectional power at twice the line frequency and zero average value. Figure 2.3 shows the instantaneous active, reactive and total power corresponding to $v(t)$ and $i(t)$ in Figure 2.2.

![Graph showing instantaneous active, reactive and total powers in sinusoidal circuit]

Figure 2.3: Instantaneous active, reactive and total powers in sinusoidal circuit

### 2.2 Active or Average Power

Active power is the net rate of instantaneous energy transfer. For any circuit or element having instantaneous current $i(t)$ and voltage drop $v(t)$, the average power $P$ is defined as

$$P = \frac{1}{T} \int_{0}^{T} p(t) \, dt = \frac{1}{T} \int_{0}^{T} v(t) \, i(t) \, dt.$$  \hspace{1cm} (2.7)

It should be noted that the definition of average power is independent of the waveforms of $v(t)$ and $i(t)$. For a linear circuit with sinusoidal voltage supply and current the active power is

$$P = VI \cos \phi = \bar{p} = \bar{p}_a.$$  \hspace{1cm} (2.8)
\( P \) is also the average value of instantaneous active power \( p_a \). In any linear circuit, since \(-90^0 < \phi < +90^0\), the sign of active power is always positive, which indicates that the active power always flows in one direction, i.e., from source to load. The active (average) power is also a real physical quantity and satisfies the Principle of Conservation of Energy.

### 2.3 Reactive Power

In linear circuits driven by sinusoidal sources, reactive power \( Q \) is defined as the peak value of the oscillating power component with no net transfer of energy and is caused entirely by energy storage components. Although \( Q \) does not contribute to the transfer of energy, it does constitute just as much a loading of the equipment as if it did.

\[
Q = V I \sin \phi = \max \{p_r\}. \tag{2.9}
\]

Reactive power, by this definition, is a bidirectional quantity. For an inductive load, \( 0^0 < \phi < 90^0 \); hence, \( Q \) is positive. For a capacitive load, \(-90^0 < \phi < 0^0\); hence, \( Q \) is negative. \( Q \) reflects that part of the power transferred back and forth between the energy storage device and the power source. The travel of reactive power through transmission lines and power transformers will cause extra loading and heating. Hence, reactive power should be minimized for energy flow efficiency. Reactive power flow is measured by a basic watt meter with a phase-shifting circuit.

With above defined quantities, the relationships for instantaneous, active and reactive powers may be represented as

\[
p = P (1 - \cos 2wt) - Q \sin 2wt. \tag{2.10}
\]
2.4 Apparent Power

The apparent power $S$ was introduced to give the same power value if alternating current and voltage could be treated as if they were constant as in a DC circuit. For this reason, $S$ is defined as the product of rms voltage and rms current. Mathematically,

$$S = \sqrt{\frac{1}{T} \int_0^T v(t)^2 \, dt} \sqrt{\frac{1}{T} \int_0^T i(t)^2 \, dt} = V I. \quad (2.11)$$

$S$ is also independent of the waveforms of $v(t)$ and $i(t)$, and may be written as a function of $P$ and $Q$ as follows:

$$S = \sqrt{P^2 + Q^2}. \quad (2.12)$$

Notice that $S = P$ is valid only if the load in the AC circuit is a passive linear resistor, (i.e., $\phi = 0$ and $Q = 0$). Therefore, $S$ is a figure of merit representing the maximum energy transfer capability of a system. In practice, $S$ is also used for equipment rating.

2.5 Power Factor

Power factor is defined to represent the energy transfer efficiency in electrical networks and is used by power companies as a penalty index for billing purposes. Mathematically the power factor is defined as the ratio of the average power entering the circuit to the apparent power, that is

$$PF = \frac{P}{S} = \frac{V I \cos \phi}{V I} = \cos \phi. \quad (2.13)$$

Since both $P$ and $S$ are independent of current and voltage waveforms, power factor is also independent of current and voltage waveforms.
Chapter 3

Power Definitions in

Non-sinusoidal Circuits —

Frequency Domain

Power studies in frequency domain are based on Fourier analysis and are developments of the early conceptions [7], [8], [10]. The main results of this approach are summarized in a book by Shepherd and Zand [9]. Fourier analysis has become a dominant tool for non-sinusoidal circuits studies. The definitions of various types of powers found in the current IEEE Standard Dictionary of Electrical and Electronics Terms (IEEE Std. 100-88) are the results of Fourier analysis. However, with the exceptions of instantaneous power and active power, all remaining kinds of powers are debatable to date [5].

This chapter discusses the basic definitions which have been established in nonlinear circuit analysis. Comments on the frequency-domain approach are given at the end of this
3.1 System with Non-sinusoidal Voltage and Current

In general, any periodic, non-sinusoidal function that is absolutely integrable can be resolved into Fourier Series. Therefore, a non-sinusoidal source of voltage of instantaneous value \( v(t) \) may be written as:

\[
v(t) = \sum_{n=1}^{N} v_n(t) = \sqrt{2} \sum_{n=1}^{N} V_n \sin(nwt + \theta_n)
\]  

(3.1)

where \( V_n \) is the rms value of the nth harmonic voltage and \( \theta_n \) is the phase angle of nth harmonic frequency with respect to the fundamental frequency. The order \( n \) is made up of odd numbers only due to the symmetry of the voltage generated. \( N \) denotes the upper limit of the harmonic order under considerations.

The physical meaning of such representation is that a non-sinusoidal supply voltage is equivalent to a linear sum of a series of sinusoidal voltages, each with an integer multiple of the fundamental frequency.

The general expression of current may also be written as:

\[
i(t) = \sum_{n=1}^{N} i_n(t) = \sqrt{2} \sum_{n=1}^{N} I_n \sin(nwt + \theta_n - \phi_n)
\]  

(3.2)

where \( I_n \) is the rms value of the nth harmonic current, \( \phi_n \) is the phase angle between the nth harmonic current and corresponding harmonic voltage. In nonlinear loads, \( \phi_n \) spans over the entire range \((0^\circ - 360^\circ)\), while in linear loads, \(-90^\circ \leq \phi_n \leq 90^\circ\).
3.1.1 **Instantaneous Power**

At any instant, the rate of energy flow in a circuit with a periodic non-sinusoidal supply voltage and current is given by:

\[
p(t) = v(t) i(t) = \sum_{n=1}^{N} v_n(t) \sum_{n=1}^{N} i_n(t)
\]

\[
= 2 \sum_{n=1}^{N} \sum_{n=1}^{N} V_n \sin(n \omega t + \theta_n) I_n \sin(n \omega t + \theta_n + \phi_n)
\]

\[
= \sum_{n=1}^{N} V_n I_n \cos\phi_n [1 - \cos(2n \omega t + 2\theta_n)] - \sum_{n=1}^{N} V_n I_n \sin\phi_n \sin(2n \omega t + 2\theta_n)
\]

\[
+ \sum_{n \neq m} V_n I_m \{\cos[(n - m) \omega t + (\theta_n - \theta_m) - \phi_n] - \cos[(n + m) \omega t + (\theta_n + \theta_m) + \phi_n]\}
\]

\[
= p_a + p_r + p_d.
\] (3.3)

Equation (3.3) indicates the following: (i) The Principle of Superposition does not apply to the instantaneous power. This is because the instantaneous power is proportional to the square of the current and, therefore, not a linear function. (ii) The instantaneous power fluctuates with frequencies equal to the sum and difference of the frequencies of the current and voltage components. Thus, while a voltage component of frequency \(n\) times the supply frequency causes only a current of the same frequency to flow, the instantaneous transfer of energy is not limited to the products of like frequency voltage and current components, but contains cross-frequency products as well.

The three terms in equation (3.3), are referred to as instantaneous active power \(p_a\), reactive power \(p_r\), and distortion power \(p_d\).

where

\[
p_a = \sum_{n=1}^{N} V_n I_n \cos\phi_n [1 - \cos(2n \omega t + 2\theta_n)],
\] (3.4)
\[ p_r = - \sum_{n=1}^{N} V_n I_n \sin \phi_n \sin(2nwt + 2\theta_n), \] (3.5)

and

\[ p_d = \sum_{n \neq m} V_n I_m \{ \cos[(n - m)wt + (\theta_n - \theta_m) - \phi_n] - \cos[(n + m)wt + (\theta_n + \theta_m) + \phi_n] \}. \] (3.6)

### 3.1.2 Active or Average Power

By definition, the average power \( P \) is the net rate of energy flow. \( P \) is expressed as follows:

\[ P = \frac{1}{T} \int_0^T p(t) \, dt = \sum_{n=1}^{N} I_n V_n \cos \phi_n = \sum_{n=1}^{N} P_n = \bar{P}_a, \] (3.7)

where

\[ P_n = I_n V_n \cos \phi_n, \] (3.8)

is the \( n \)th harmonic active power. When compared with equation (3.3), one can see that of all terms of instantaneous power with different frequencies, only the terms with the same frequencies contribute to the net energy flow. This means that a nonzero time-average power can only be transferred by the combination of voltage and current components of the same frequency. By the definitions given to the instantaneous power (3.3), the active power is also the average value of the instantaneous active power.

### 3.1.3 Reactive Power

The reactive power in non-sinusoidal system is defined as \([7]-[9]:\)

\[ Q = \sum_{n=0}^{N} V_n I_n \sin \phi_n. \] (3.9)
Here, the definition of reactive power only gives the mathematically symmetric form with reference to the active power, and it seems to be a logical extension from the sinusoidal case. However, $Q$ can not be interpreted as the peak value of the power component that flows back and forth between the source and load.

### 3.1.4 Apparent Power

Apparent power is the figure of merit defining the maximum energy transfer capability of a system. It is defined the same as the definition given in Chapter 2,

\[
S = \sqrt{\left( \sum_{n=1}^{N} V_n^2 \right) \left( \sum_{n=1}^{N} I_n^2 \right)} = V I. \tag{3.10}
\]

Equation (3.10) indicates that every frequency component of the supply voltage is separately combined with every frequency component of the supply current. It is obvious that

\[
S \neq \sqrt{\sum_{n=1}^{N} V_n^2 I_n^2}, \tag{3.11}
\]

and it is also true that

\[
S \neq \sqrt{P^2 + Q^2}. \tag{3.12}
\]

### 3.1.5 Distortion Power

In order to balance equation (3.12), a new quantity called "distortion power" is introduced:

\[
D = \sqrt{S^2 - P^2 - Q^2} = \sqrt{\sum_{m \neq n} V_n^2 I_m^2 + V_m^2 I_n^2 - 2V_n V_m I_n I_m \cos(\phi_n - \phi_m)}. \tag{3.13}
\]
It is important to note that, when the supply voltage is non-sinusoidal, instantaneous power, apparent power and distortion power all contain terms that involve cross-frequency products. Whereas, the average power and reactive power contains terms that only involve like-frequency products of voltage and current.

3.1.6 Power Factor

Power factor $PF$ is the ratio between the average power $P$ and apparent power $S$.

For nonlinear systems, this is given by

$$PF = \frac{P}{S} = \frac{\sum_{n=1}^{N} I_n V_n \cos \phi_n}{\sqrt{(\sum_{n=1}^{N} V_n^2)(\sum_{n=1}^{N} I_n^2)}}.$$  \hspace{1cm} (3.14)

The expression for load power factor cannot be conveniently split into smaller terms. Furthermore, both reactive and distortion power need to be compensated in non-sinusoidal systems to improve power factor.

3.1.7 Numerical Example

The following example illustrates the above definitions through a system with RL load supplied by a non-sinusoidal voltage source. As shown in Figure 3.1, where the non-sinusoidal voltage contains only two harmonic components for simplicity.

$$v(t) = \sqrt{2}V_1 \sin wt + \sqrt{2}V_5 \sin(5wt + \theta).$$

The circuit current is

$$i(t) = \sqrt{2}I_1 \sin(wt - \phi_1) + \sqrt{2}I_5 \sin(5wt + \theta - \phi_5),$$
Figure 3.1: Series R-L circuit with non-sinusoidal voltage supply

where $\phi_1$ and $\phi_5$ are positive ($< 90^\circ$) due to the inductive load. The instantaneous voltage and current are shown in Figure 3.2 for $V_1 = 200 \, V$, $V_5 = 100 \, V$, $R = 4 \, \Omega$, and $L = 0.0265 \, H$. The instantaneous power is given by

$$
p(t) = v(t)i(t) = v_1(t)i_1(t) + v_1(t)i_5(t) + v_5(t)i_1(t) + v_5(t)i_5(t)
$$

Figure 3.2: Waveforms of voltage and current in non-sinusoidal circuit
\[ P = P_1 + P_5 = V_1 I_1 \cos \phi_1 + V_5 I_5 \cos \phi_5. \]

\[ Q = Q_1 + Q_5 = V_1 I_1 \sin \phi_1 + V_5 I_5 \sin \phi_5. \]
Therefore, the apparent and distortion powers are

\[ S = \sqrt{V^2 I^2} = \sqrt{V_1^2 I_1^2 + V_2^2 I_2^2 + V_5^2 I_5^2 + V_6^2 I_6^2}, \]

and

\[ D = \sqrt{S^2 - P^2 - Q^2} \]

\[ = \sqrt{V_1^2 I_1^2 + V_5^2 I_5^2 - 2V_1 V_5 I_1 I_5 \cos(\phi_1 - \phi_5)}. \] (3.16)

The relative contributions of the like-frequency and cross frequency terms cannot be determined without a knowledge of the rms voltages and currents.

### 3.2 Systems with Sinusoidal Voltage and Nonlinear Load

A system with nonlinear load and a sinusoidal voltage source is considered as a special case of the above section. The definitions are the same while their mathematical representations are simplified.

When the voltage is sinusoidal, it contains only the fundamental frequency as in equation (2.1). However, due to the nonlinear load, the current is non-sinusoidal and its expression in terms of Fourier series is the same as equation (3.2).

The instantaneous power and its active, reactive, and distortion components are

\[ p(t) = v(t) i(t) \]

\[ = p_a + p_r + p_d \]

\[ = V I_1 \cos \phi_1 (1 - \cos 2wt) - V I_1 \sin \phi_1 \sin 2wt \]
\[ + V \sum_{n>1}^{N} I_n \{ \cos[(n-1)\omega t + \phi_n] - \cos[(n+1)\omega t + \phi_n] \}, \]  

(3.17)

where \( p_a \) and \( p_r \) contain only the fundamental component, what they have in sinusoidal system.

Similarly, the active, reactive, distortion and apparent powers are simplified as follows:

\[ P = \frac{1}{T} \int_{0}^{T} i(t) v(t) \, dt, \]
\[ = V I_1 \cos \phi_1, \]
\[ = P_1, \]

(3.18)

\[ Q = V I_1 \sin \phi_1, \]

(3.19)

\[ S = V I = V \sqrt{\sum_{n=1}^{N} I_n^2}, \]

(3.20)

\[ D^2 = S^2 - P^2 - Q^2 = V^2 \sum_{n=2}^{N} I_n^2, \]

(3.21)

and the power factor is

\[ PF = \frac{P}{S} = \frac{I_1 \cos \phi_1}{I}. \]

(3.22)

although the reactive power has the same format as in linear circuits, the compensation of distortion power \( D \) must be considered to improve the power factor.
3.3 Comments on Frequency Domain Approach

Although Fourier analysis can be applied to any periodic waveforms this approach has some basic disadvantages. First of all, this method is conceptually and mathematically indirect and complicated [16]. Decomposing waveforms into harmonics requires a large sum of terms to accurately describe the waveform. This in turn requires a rather burdensome computation. Second, from the measurement point of view, Fourier analysis requires sophisticated instruments with built-in Fast Fourier Transform (FFT) capability. Third, the way Fourier analysis describes the power terms does not give clear physical meanings for the reactive and distortion powers. As discussed in this chapter, the definition of reactive and distortion power in frequency domain only gives the mathematical convenience and it does not lead to a physical explanation nor to optimal compensation. From the compensation point of view, since both reactive and distortion power have to be compensated, it is not necessary to separate these two physically meaningless terms. Trying to separate reactive and/or distortion power for measurement and compensation is neither efficient nor economically right since an active power filter (APF) is able to cancel the whole non-active power as long as it is mathematically defined and measured.

For years, further work on this approach has been focused on the decomposition of “non-active power”, which is the residual term remaining in the apparent power after the active power components have been extracted [3], [17], [18]. Various considerations and measurement methods are proposed. However, besides their complexity due to its indirect conception and mathematical description, there is still no acceptable definition of reactive and/or distortion power [5].
Chapter 4

Power Definitions in
Non-sinusoidal Circuits - Time Domain

Time-domain analysis is an alternative approach for defining power terms in non-sinusoidal situations. In this approach, the concepts of "in-phase" and "quadrature" current components, which have been used in sinusoidal case with success, are generalized. Unlike frequency-domain analysis, this method is conceptually and mathematically straightforward, and often leads to simple instrumentation. Therefore, it is very attractive. However, the theory itself is not fully established due to the lack of physical meaning for reactive and apparent power. This chapter reviews and re-examines the existing definitions of power terms, and proposes new quantities to enhance the physical interpretations and measurements for reactive and apparent powers in the time-domain.
4.1 Existing Definitions

The basic concepts of time-domain analysis in non-sinusoidal situations can be summarized as follows: Any current $i(t)$ can be divided into two mutually orthogonal components; an active component $i_a(t)$ which is in time-phase with the supply voltage $v(t)$, and a reactive component $i_r(t)$ which is in quadrature with the voltage. In mathematical terms:

$$i(t) = i_a(t) + i_r(t), \quad (4.1)$$

where

$$i_a(t) = G v(t) \quad (4.2)$$

where $G$ is a constant coefficient representing the equivalent conductivity of the circuit. This is equal to state that the active component $i_a(t)$ flows through a resistance which consumes the entire active power $P$ supplied. The mathematical form of this statement is

$$P = \frac{1}{T} \int_0^T v(t) i_a(t) dt = \frac{G}{T} \int_0^T v(t)^2 dt = GV^2, \quad (4.3)$$

where $V$ is the rms value of the voltage. Equation (4.3) yields

$$G = \frac{P}{V^2}. \quad (4.4)$$

On the other hand, the general definition of active power

$$P = \frac{1}{T} \int_0^T v(t) i(t) dt \quad (4.5)$$
when combined with equations (4.1) and (4.2) gives

\[ \int_0^T i_a(t)i_r(t)dt = 0. \]  

(4.6)

Hence, \( i_a(t) \) and \( i_r(t) \) are orthogonal, and this orthogonality yields

\[ I^2 = I_a^2 + I_r^2 \]  

(4.7)

where \( I, I_a, I_r \) are rms values of total, active and reactive current, respectively.

The instantaneous power can then be expressed as

\[ p(t) = v(t) i(t) = v(t) (i_a(t) + i_r(t)) = p_a(t) + p_r(t), \]  

(4.8)

and the average power can be rewritten as

\[ P = V I_a. \]  

(4.9)

Reactive power is defined as [16]

\[ Q = V I_r, \]  

(4.10)

but since

\[ \frac{1}{T} \int_0^T v(t) i_r(t) dt = 0, \]  

(4.11)

\( Q \) is interpreted as the reactive power component with no net energy transfer, just as in sinusoidal systems. Further more, apparent power, active and reactive power have the same
relationship as in the sinusoidal case, i.e.,

\[ S^2 = V^2 I^2 = P^2 + Q^2. \]  \hspace{1cm} (4.12)

Due to its straightforward conception and easy instrumentation this approach is very attractive. However, unlike the sinusoidal case, the definition of reactive power in (4.10) is not the peak value of instantaneous reactive power \( p_r \). As a consequence, questions regarding the meaning of \( Q \), and hence, \( S \) emerge. The next section of this chapter proposes new quantities to explain these unknowns.

4.2 Proposed Terms for Physical Interpretations of Apparent and Reactive Powers

The concepts of electrical power originated in DC circuits. When a DC voltage is applied across a resistor \( R \), a current \( I \) passes through \( R \) and its quantity is defined according to Ohm's law \( V = RI \). The power is defined by the rate at which the resistor \( R \) consumes energy,

\[ P = VI = \frac{V^2}{R} = I^2 R \]  \hspace{1cm} (4.13)

There exists no reactive power in a DC circuit, \( Q = 0 \), and \( S = P \).

In an AC circuit, the relations among \( P \), \( Q \), \( S \), \( V \) and \( I \) are given in (4.9), (4.10) and (4.12). The meaning of the current component \( I_a \) is rather clear - it is the current in a resistive load which consumes power \( P \) when voltage \( V \) is applied to it. The "quadrature" component \( I_r \) does not contribute to the net transmission of energy, but its rms value is
non zero:

\[ I_r = \frac{Q}{V}. \]  \hspace{2cm} (4.14)

Therefore, \( Q \) may be interpreted as the amount of energy contributed by an equivalent current \( i_q(t) \), which has the same rms value as that of \( i_r(t) \), but in time phase with the supply voltage \( v(t) \). Mathematically,

\[ i_q(t) = I_r \left( \frac{v(t)}{V} \right) = \frac{Q}{V} \left( \frac{v(t)}{V} \right) = \frac{Q}{V^2} v(t). \]  \hspace{2cm} (4.15)

the corresponding "instantaneous reactive power" may be written as

\[ q(t) = v(t) i_q(t) = Q \left( \frac{v^2(t)}{V^2} \right). \]  \hspace{2cm} (4.16)

Similarly, we introduce "instantaneous apparent current" and "instantaneous apparent power" to reflect the power transmission efficiency at any instant of time \( t \). By definition of apparent power, the instantaneous apparent current \( i_s(t) \) shall have the same rms value as that the total source current \( i(t) \) and shall be in time-phase with the source voltage \( v(t) \), that is,

\[ i_s(t) = I \left( \frac{v(t)}{V} \right) = \frac{S}{V} \left( \frac{v(t)}{V} \right) = \frac{S}{V^2} v(t), \]  \hspace{2cm} (4.17)

and the corresponding instantaneous apparent power may be written as

\[ s(t) = v(t) i_s(t) = \frac{S}{V^2} v^2(t). \]  \hspace{2cm} (4.18)
Therefore, the resulting instantaneous power factor can be defined as

\[ pf(t) = \frac{p_a(t)}{s(t)} = \frac{i_a(t)}{i(t)} = \frac{P}{S} = \frac{I_a}{I} = PF \] (4.19)

where \( p_a(t) = v(t)i_a(t) \) is the instantaneous active power. By the above definitions, it is easy to show that

\[ Q = \frac{1}{T} \int_0^T q(t) dt, \] (4.20)

\[ S = \frac{1}{T} \int_0^T s(t) dt, \] (4.21)

and most importantly,

\[ s^2(t) = p_a^2(t) + q^2(t). \] (4.22)

The above quantities are introduced to enhance the physical meanings of active, reactive, and apparent power. They can be used in both sinusoidal and non-sinusoidal situations, as illustrated by the numerical examples that follow. When used in non-sinusoidal systems, these quantities give a clear physical interpretation of power flow at both steady state and transient conditions.

### 4.3 Numerical Examples

#### 4.3.1 Linear Circuit with Sinusoidal Supply Voltage

The same circuit as shown in Figure 2.1 is reused for illustration purpose. The active current component is

\[ i_a(t) = \frac{P}{V/2} v(t), \]
\[ I_a = \sqrt{\frac{P}{V}} \sin \omega t, \]
\[ = \sqrt{2} I_a \sin \omega t, \] (4.23)

where \( I_a = \frac{P}{V} \) is the rms value of the active current.

The reactive current which contribute to the power oscillation with no net transfer of energy is computed by

\[ i_r(t) = i(t) - i_a(t), \]
\[ = -\sqrt{2} \frac{Q}{V} \cos \omega t, \]
\[ = -\sqrt{2} I_r \cos \omega t. \] (4.24)

Since \( i_a(t) \) and \( i_r(t) \) are mutually orthogonal

\[ I^2 = \left( \frac{V}{|Z|} \right)^2 = I_a^2 + I_r^2. \] (4.25)

The new proposed equivalent reactive current \( i_q(t) \), which contribute to the reactive energy, is defined by

\[ i_q(t) = \sqrt{2} I_r \sin \omega t. \] (4.26)

Note that \( i_q(t) \) has the same rms value as \( i_r(t) \), and "in phase" with the supply voltage.

The newly defined apparent current \( i_s(t) \) is the quantity which has the same rms value as total current \( i(t) \) and "in phase" to the supply voltage:

\[ i_s(t) = \sqrt{2} \frac{V}{|Z|} \sin \omega t = \sqrt{2} I \sin \omega t. \] (4.27)
The instantaneous powers are then calculated as follows:

\[ p_a(t) = v(t) i_a(t) = 2 VI \cos \phi \sin^2 wt, \]  
\[ q(t) = v(t) i_q(t) = 2 VI \sin \phi \sin^2 wt, \]  
and
\[ s(t) = v(t) i_s(t) = 2 VI \sin^2 wt = \sqrt{p_a^2(t) + q^2(t)}. \]

The average values of \( p_a(t) \), \( q(t) \) and \( s(t) \) are

\[ \frac{1}{2\pi} \int_0^{2\pi} p_a(t) \, dt = V I \cos \phi = P, \] 
\[ \frac{1}{2\pi} \int_0^{2\pi} q(t) \, dt = V I \sin \phi = Q, \] 
and
\[ \frac{1}{2\pi} \int_0^{2\pi} s(t) \, dt = V I = \sqrt{P^2 + Q^2} = S. \]

Note that these quantities are the same as the conventional definition, but now \( Q \) and \( S \) are attached to a physical meaning.

The instantaneous values of voltage, currents and powers as discussed above, are shown in Figure 4.1 and Figure 4.2 for the following values: \( V = 120(V) \), \( w = 377(rad/s) \), \( L = 0.2226(H) \), and \( R = 100(\Omega) \).

The corresponding power factor can be corrected to unity by using a shunt capacitor.
Figure 4.1: Waveforms of voltage and active, reactive, apparent and total current in linear sinusoidal circuit

Figure 4.2: Waveforms of active, reactive, apparent and total powers in linear sinusoidal circuit
The value of this capacitance is determined by

$$C = \frac{V \sin \phi}{w|Z|},$$  \hspace{1cm} (4.34)

where $\phi$, $w$, and $Z$ are circuit parameters independent of $C$.

### 4.3.2 Nonlinear Circuit with Sinusoidal Supply Voltage

When an ideal voltage supply of instantaneous value $v(t)$ is connected to a nonlinear load, the instantaneous source current $i(t)$ is periodic but non-sinusoidal. One popular nonlinear circuit is the single-phase AC chopper with a resistive load as shown in Figure 4.3. The load voltage in the circuit can be adjusted by controlling the firing of a pair of thyristors connected in inverse-parallel. For symmetrical phase angle triggering, only one of the two thyristors can conduct at any instant. Let the circuit be supplied by a sinusoidal voltage

$$v(t) = \sqrt{2} V \sin wt.$$  \hspace{1cm} (4.35)
Let the thyristor firing angle set at \( \alpha \) radians with respect to the zero crossing of \( v(t) \), then the instantaneous source current \( i(t) \) can be defined by

\[
i(t) = \begin{cases} 
\frac{\sqrt{2}V}{R} \sin \omega t, & \alpha \leq \omega t \leq \pi, \text{ or } \alpha + \pi \leq \omega t \leq 2\pi \\
0, & 0 \leq \omega t \leq \alpha, \text{ or } \pi \leq \omega t \leq \alpha + \pi.
\end{cases}
\]

The average power delivered to the load is a function of \( \alpha \), and can be written as

\[
P = \frac{1}{\pi} \int_{\alpha}^{\pi} v(t) i(t) \, d(\omega t) = \frac{V^2}{\pi R} [\pi - \alpha + \sin \alpha \cos \alpha]. \quad (4.36)
\]

The instantaneous active and reactive currents can then be defined as

\[
i_a(t) = \frac{P}{V^2} v(t) = \frac{\sqrt{2}V}{\pi R} [\pi - \alpha + \sin \alpha \cos \alpha] \sin \omega t = \sqrt{2} I_a \sin \omega t, \quad (4.37)
\]

\[
i_r(t) = i(t) - i_a(t) = \begin{cases} \sqrt{2} \left( \frac{V}{R} - \frac{P}{V^2} \right) \sin \omega t, & \alpha \leq \omega t \leq \pi, \text{ or } \alpha + \pi \leq \omega t \leq 2\pi \\
-ia(t), & 0 \leq \omega t \leq \alpha, \text{ or } \pi \leq \omega t \leq \alpha + \pi.
\end{cases}
\]

In order to define the instantaneous equivalent reactive current \( i_q(t) \), and apparent current \( i_s(t) \), the rms value of total current \( i(t) \) and non-active current \( i_r(t) \) must be found. The rms value of total current \( i(t) \) is computed by

\[
I = \sqrt{\frac{1}{\pi} \int_{\alpha}^{\pi} i^2(t) \, d\omega t} = \frac{V}{R} \sqrt{\frac{1}{\pi} \left( \pi - \alpha + \sin \alpha \cos \alpha \right)}, \quad (4.38)
\]
and due to orthogonality,

\[ I_r = \sqrt{I^2 - I_d^2} \]

\[ = \sqrt{\frac{V^2}{\pi R^2} (\pi - \alpha + \sin \alpha \cos \alpha)} - \frac{V^2}{\pi^2 R^2} (\pi - \alpha + \sin \alpha \cos \alpha)^2. \quad (4.39) \]

Finally, the instantaneous equivalent reactive current and apparent current can be calculated:

\[ i_q(t) = \sqrt{2} I_r \sin \omega t, \quad (4.40) \]

and

\[ i_s(t) = \sqrt{2} I \sin \omega t. \quad (4.41) \]

The waveforms of these currents are shown in Figure 4.4 for a firing angle \( \alpha = 60^\circ \).

Figure 4.4: Waveforms of voltage and active, reactive, apparent and total current in chopper circuit

Note that the instantaneous currents \( i_a(t) \), \( i_q(t) \), and \( i_s(t) \) are in time phase with the
supply voltage, as expected. The rms values of these currents have the following relationship:

\[ I_r^2 = I_s^2 = I_a^2 + I_q^2 = I_s^2 + I_q^2. \]  \hspace{1cm} (4.42)

In reality, the reactive current \( i_r(t) \) has to be compensated in order to improve the power quality, reduce the line loss and increase the efficiency of power transmission. This current, however, can not be compensated by introducing a shunt capacitor because the current is generally distorted. Compensation can be effective, however, by applying an active filter which injects equal-but-opposite distortion into the circuit, thereby cancelling the original distortion current. A general discussion of active filters will be given in chapter 5.

The new proposed instantaneous powers in the circuit of Figure 4.3 are:

\[ p_a(t) = v(t) i_a(t) = 2VI_a \sin^2 \omega t, \]  \hspace{1cm} (4.43)

\[ q(t) = v(t) i_q(t) = 2VI_r \sin^2 \omega t, \]  \hspace{1cm} (4.44)

and

\[ s(t) = v(t) i_s(t) = 2VI \sin^2 \omega t. \]  \hspace{1cm} (4.45)

Therefore, the relationship among \( s(t) \), \( p_a(t) \) and \( q(t) \) given by equation (4.23) still holds.

The average values of \( p_a(t) \), \( q(t) \) and \( s(t) \), namely, \( P \), \( Q \), and \( S \), are

\[ P = \frac{1}{\pi} \int_0^\pi 2VI_a \sin^2 \omega t \, d\omega t = V I_a, \]  \hspace{1cm} (4.46)

\[ Q = \frac{1}{\pi} \int_0^\pi 2VI_r \sin^2 \omega t \, d\omega t = V I_r, \]  \hspace{1cm} (4.47)
and

\[ S = \frac{1}{\pi} \int_{0}^{\pi} 2V \sin^2 \omega t \, dt = V I. \] (4.48)

Hence,

\[ S^2 = P^2 + Q^2 = V^2 I^2. \] (4.49)

By combining equations (4.43), (4.45), (4.46) and (4.48), it is easy to see that the instantaneous power factor is equivalent to the conventionally defined power factor in sinusoidal systems as well as in non-sinusoidal systems, is equal to

\[ pf(t) = PF = \frac{I_a}{I} = \sqrt{\frac{1}{\pi} (\pi - \alpha + \sin \alpha \cos \alpha)} \] (4.50)

The compensation of power factor depends on the compensation of non-active current as mentioned above. The instantaneous values of powers discussed at firing angle \( \alpha = 60^\circ \) are shown in Figure 4.5

![Figure 4.5: Waveforms of active, reactive, apparent and total powers in non-sinusoidal circuit](image)
Chapter 5

Power Measurement and Compensation in Non-sinusoidal Systems

With the new physical interpretations for reactive and apparent power, efficient measuring instruments and techniques should be developed to define the power quality and energy transfer efficiency for billing or compensation purposes. This chapter discusses the measurement of the proposed power terms and compensation by means of active filters.

5.1 Measurement of Proposed Currents and Powers

In non-sinusoidal systems, traditional induction-type meters introduce considerable errors [19], [24]; hence, new measurement methods need to be considered. The measurement of
the proposed instantaneous powers can be illustrated with a block diagram as shown in Figure 5.1.

![Block diagram of instantaneous power and current measurement](image)

Figure 5.1: Block diagram of instantaneous power and current measurement

The instantaneous values of voltage \( v(t) \) and current \( i(t) \) are picked by a potential transformer (VT) and a current transformer (CT). These instantaneous values are then fed into a current analyzer where the current \( i(t) \) is decomposed into its active component \( i_a(t) \) and reactive component \( i_r(t) \) according to equations (4.1) and (4.2). The rms values of both components are then used to calculate the equivalent reactive current \( i_q(t) \) and apparent current \( i_s(t) \) as defined by equations (4.15) and (4.17). There is a total of four instantaneous current outputs from the current analyzer: the active, equivalent reactive and apparent current \( i_a(t), i_q(t), \) and \( i_s(t) \) respectively) are sent to a multiplier where these currents are multiplied by the instantaneous voltage \( v(t) \) to create the instantaneous active,
reactive and apparent powers $p_a(t)$, $q(t)$, and $s(t)$ as defined by equations (4.8), (4.16) and (4.18). The instantaneous powers can be sent to an integrator to calculate $P$, $Q$, $S$, and $PF$, i.e., the average active, reactive, apparent power, and power factor according to equations (4.20), (4.21), (4.22) and (4.19).

For compensation purposes, the non-active current $i_r$ is sent back to the circuit through an inverter for $180^\circ$ phase shifting. Further discussion of compensation by means of active filters is given in the next section.

The proposed measurement method enables the identification of two power properties of a system at a specified terminal if only the instantaneous values of the voltage and current at this terminal are accessible for measurement.

5.2 Compensation of Nonlinear Loads with Active Filters

The concept of active compensators for power factor improvement in systems with nonlinear loads was presented in the late 1960's [20], [21]. In the time-domain approach, the instantaneous line current supplying a nonlinear load can be decomposed to active and reactive components as given by (4.1). If a compensating device connected in parallel with the nonlinear load supplies an instantaneous current

$$i_c(t) = -i_r(t)$$

(5.1)

then the total line current, according to the Kirchhoff's current law, is equal to the active component:

$$i_t(t) = i(t) + i_c(t)$$
Technically, compensation in the time-domain is based on the principle of holding the instantaneous current within some reasonable tolerance to a desired reference sine wave. An instantaneous error function is computed by subtracting the actual waveform from the reference waveform. The error function is then sent to an active filter. The basic components of an active filter include an inverter and a DC source which receives its power from the AC power system.

The function of an APF is to produce a precisely chopped waveform to correct a distorted current. Depending upon the APF power ratings and switching frequencies, transistors such as bipolar junction transistors (BJT), silicon-controlled rectifiers (SCR) or gate-turnoff thyristors (GTO) are frequently used [22].

The most commonly proposed time domain compensation technique is the hysteresis method [22]. Figure 5.2 shows a current type APF connected in parallel with a nonlinear load. In order to provide a virtually constant current, \( I_L \), the inductance value is chosen to be large. Once the current \( I_L \) is built up, its small fluctuations can be corrected by gating the appropriate switches during the positive or negative half cycle of the source voltage, \( v(t) \). Because of the inductance in the power line and load, a capacitor is required to provide a path for the current, \( I_L \), immediately after gating a pair of switches. The switching action of the APF gives three cases for the currents \( I_{in} \) and \( I_{out} \). If switch pair 1 and 2 are ON, then \( I_{in} = I_L \), and \( I_{out} = 0 \). If switch pair 3 and 4 are ON, then \( I_{out} = I_L \), and \( I_{in} = 0 \). If switch pair 2 and 4 are ON, then \( I_{in} = I_{out} = 0 \), and freewheeling of \( I_L \) occurs.
The control of these gating cases uses the hysteresis method with a three-state switching function. Under this method, preset upper and lower tolerance limits are compared to the extracted error signal, i.e., the

\[ i_r(t) = i(t) - i_a(t). \]  \hspace{1cm} (5.3)

First, the load current is sensed for one cycle and the active component, \( i_a(t) \), is calculated. The active current is the reference current that the source current, \( i(t) \), should follow. During the next cycle, the source current is sampled, and at each sampling instant, it is subtracted from the active current. The resulting instantaneous error current, \( i_r(t) \), is compared to the preset upper and lower tolerance limits (TL). The hysteresis control illustrated by Figure 5.3 chooses one of the following switching modes:

1) If \( i_r(t) > TL \), gate switch pair 1 and 2 to ON.

2) If \( i_r(t) < TL \), gate switch pair 3 and 4 to ON.

3) If \( -TL < i_r(t) < TL \), gate switch pair 2 and 4 to ON.
Figure 5.3: Time domain compensation using hysteresis method with three-state switching function.
Chapter 6

Conclusions

This thesis reviewed power definitions and their physical interpretations in sinusoidal situations. It also reviewed and discussed existing results of power studies in both frequency-domain and time-domain approaches in non-sinusoidal systems. The main disadvantage of the frequency-domain approach is its complexity and indirectness in concepts and mathematics. The application of this method requires sophisticated instruments. The time-domain approach, on the other hand, is characterized as a method which is not only conceptually straightforward but also practically simple for measurement. Unfortunately, the theory itself is not fully established. Both methods do not give clear physical interpretations to reactive and apparent powers which are key quantities in power quality studies and power transfer efficiency studies.

Through analyzing energy flow in non-sinusoidal circuits, it is found that:
1) In order to give physical interpretations to apparent and reactive powers, new quantities: $i_q$, the instantaneous reactive current; $q$, the instantaneous reactive power; $i_s$, the instan-
taneous apparent current; \( s_i \), the instantaneous apparent power; and \( pf \), the instantaneous power factor are introduced with mathematical representations and physical interpretations. These quantities can be used not only in steady state but transient analysis as well.

2) All instantaneous components of current and power are measurable with relatively simple instrumentation. With the physical explanation of reactive and apparent powers, these quantities should be used for billing purposes with fairness to both parties involved. Compensation of network distortion caused by load nonlinearity can be done by using an APF which injects equal-but-opposite distortion.

3) From both the detection and compensation point of view, the time domain approach is effective, efficient and economical.
Bibliography


