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Output Feedback Adaptive Variable Structure Control of a Smart Projectile Fin

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Based on the variable structure model reference adaptive control (VS-MRAC) theory, a new control system for the control of a projectile fin using a piezoelectric actuator is designed. The hollow projectile fin is rigid, within which a flexible cantilever beam with a piezoelectric active layer is mounted. The model of the fin-beam system includes the aerodynamic moment which is a function of angle of attack of the projectile. The rotation angle of the fin is controlled by deforming the flexible beam which is hinged at the tip of the rigid fin. For the derivation of the control law, it is assumed that the parameters in the model are unknown, and only the fin angle is measured for feedback. It is shown that, in the closed-loop system including the VS-MRAC system designed using bounds on uncertain functions, the fin angle tracks the reference trajectory and the vibration is suppressed. Digital simulation results show that the closed-loop system has good transient behavior and robustness to the uncertainties, unmodeled dynamics and disturbance inputs.

1. Introduction

The use of surface-mounted or bonded piezoelectric actuators for the shape control of intelligent structure has gained widespread acceptance recently. Applications can be found in many areas including the shape control of metallic or composite plates or beams^{1,2}. Applications also involve actuation of various types of aircraft structural members such as wings, fins, or rotor blade^{3,4,5,6}. Advantages of this approach are mainly due to the integration of the actuators into the structural members itself, thus saving the space required for servo motors, force transmission devices, or hydraulic systems⁶. This advantage becomes even more important when small aerial vehicles such as unmanned aircraft, small missiles, guided munitions, and projectiles are examined. Piezoelectric twist actuators used for this application are based on anisotropic straining of the host structure using directionally attached isotropic actuator⁴ or using piezoelectric fibers integrated into the composite structural members⁵. General formulation and solution procedures for an analytical model for a composite laminated plate with isotropic or anisotropic active layers is derived in Refs. 5,6. The design of active controllers using piezoelectric actuators for vibration, force and position control of systems have been considered in Refs. 7,8,9,10,11,12.

Traditionally, for the path control of missiles and projectiles, maneuvering forces and moments are generated by fin angle control using mechanical actuators which are bulky and slow. For high performance projectiles, there is a need to develop more efficient actuation mechanisms. Recently, the development of a smart fin (fin-beam model) has been considered^{13,14}. This fin has an outer hollow rigid body inside which resides a hinged flexible cantilever beam with a piezoelectric active layer. The control of the fin angle is then accomplished by deforming the beam. The design of the controller for the fin-beam model of Ref. 13

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is based on a modeling error compensation approach in which the lumped uncertainties are estimated using a high-gain observer. This requires precise measurement of the fin angle for stability in the closed-loop system. A fuzzy controller has been designed in Ref. 14 for the control of this fin. Of course, for the fuzzy controller design, the designer first has to develop a number of if-then rules which often are not easy to obtain. Ref. 15 provides an adaptive controller, based on inverse feedback linearization technique using state variable feedback.

Flexible structures are essentially infinite dimensional systems; however often finite dimensional models by neglecting the higher modes are used for analysis and design. The models of flexible structures are generally obtained by solving the eigenvalue problem resulting from finite element methods. However, it is well known that the resulting fidelity of model parameters degrades drastically for higher modes. Researchers have made considerable effort to design controllers for the control of flexible structures. Ref. 16 provides a good review of literature in which readers can find several references related to the control of elastic systems. For flexible structures, controller designs based on feedback linearization, passivity concepts and adaptive techniques have been attempted^{17–21}. Based on command generator tracker concept¹⁷, an adaptive controller for the smart projectile fin of Ref. 15 has been designed in Ref. 22. For the synthesis of the controller, the fin angle and its derivative are measured, and the adaptive loop tunes three parameters and requires sigma or dead-zone modification of the adaptation rule in order to avoid parameter divergence. Therefore, the design of a new simple controller for the smart projectile fin using only the fin angle is desirable.

The contribution of the paper lies in the design of a variable structure model reference adaptive control (VS-MRAC) for the control of a projectile fin. The projectile fin which is hollow but rigid, is controlled by deforming a cantilever flexible beam, which is mounted inside the fin. The model chosen here is similar to that reported in Ref. 15. A finite dimensional model is used for this study. The model includes the aerodynamic moment affecting the fin motion which is a function of the angle of attack of the projectile. It is assumed that only the fin angle is measured. Based on the VS-MRAC theory^{23,24}, an adaptive control law is designed. For the derivation of the control law, it is assumed that the parameters in the model are unknown, and only the fin angle is measured for feedback. It is shown that, in the closed-loop system including the VS-MRAC system designed using bounds on uncertain functions, the fin angle tracks the reference trajectory and the vibration is suppressed. Digital simulation results show that the closed-loop system has good transient behavior and robustness to the uncertainties, unmodeled dynamics and disturbance inputs. Simulation results are presented which show that the designed adaptive control system accomplishes precise fin angle control in spite of uncertainties in the fin-beam parameters and the aerodynamic moment coefficients.

II. Dynamic Model

The model of the fin-beam system is shown in Figure 1. The flexible beam with a piezoelectric active layer bonded on the top surface, is hinged at one end to the fin and the other end is attached rigidly to the projectile body. The fin is free to rotate about an axis fixed to the projectile body. When the control voltage $u(x, t)$ is applied to the actuator, the induced strain in the actuator generates the bending moment m that is expressed (Ref. 8) as

$$m = cu(x, t) \quad (1)$$

The constant c can be obtained by considering geometrical and material properties of the beam and piezoelectric actuator. Considering the cross sectional geometry and force equilibrium along the axial direction, the constant c can be expressed as (Ref. 9)

$$c = -d_{31} \frac{h_p + h_b}{2} \frac{E_p h_b E_b}{E_p h_p + E_b h_b} b \quad (2)$$

where d_{31} is the piezoelectric strain constant and E_p and E_b are Young's modulus of the piezoelectric actuator and the beam respectively. Other geometric parameters are shown in Fig. 1.

As shown in Fig. 1, an airfoil is connected to the beam actuator using a hinge. The airfoil is assumed to be rigid and its rotation is assumed to be small and planar. A finite element approach is used to describe the dynamics of the flexible beam, which is considered as composed of finite elements satisfying

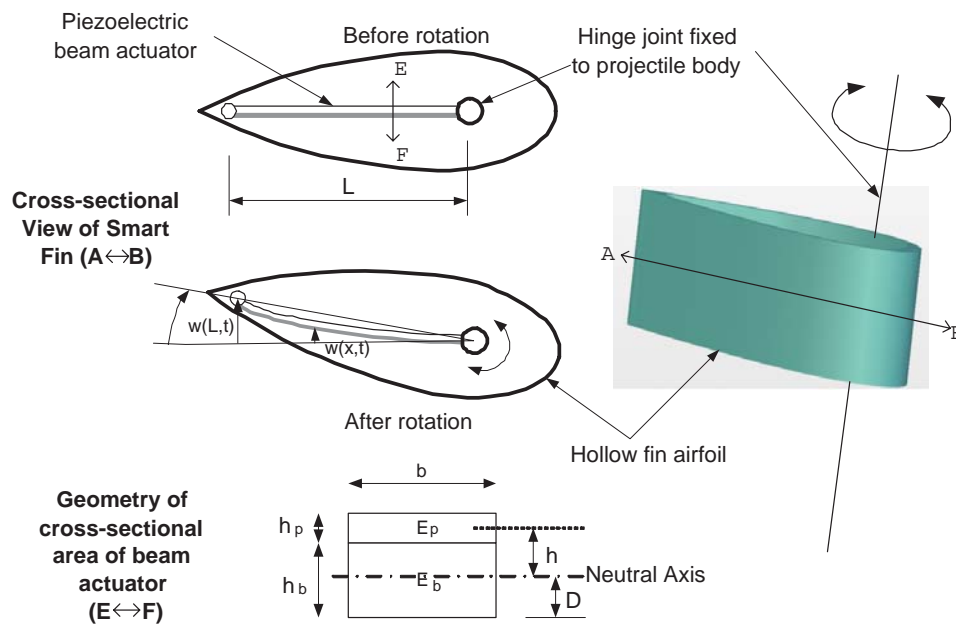


Figure 1. Configuration of Smart Fin

Euler-Bernoulli theorem. The beam is divided into n elements with equal length of L_i . The displacement w of any point on the beam element i is described in terms of nodal displacement, w_i , and slope, ϕ_i , at node i and $i + 1$, respectively and is expressed as

$$w_i = Nq_i \quad (3)$$

where $q_i = (w_i, \phi_i, w_{i+1}, \phi_{i+1})^T$ and $N = (N_1, N_2, N_3, N_4)$ is the shape function vector. The kinetic energy of an element i becomes

$$T_i = \int_0^{L_i} \rho_i \dot{w}^T \dot{w} dx_i = \frac{1}{2} \dot{q}_i^T M_i \dot{q}_i \quad (4)$$

where $M_i (= \int_0^{L_i} \rho_i N^T N dx_i)$ is a mass matrix and ρ_i is a combined density of the beam and piezoelectric actuator per unit length.

The potential energy of an element i is

$$V_i = \frac{1}{2} \int_0^{L_i} \frac{1}{E_i I_i} (E_i I_i \frac{\partial^2 w}{\partial x_i^2} + cu)^T (E_i I_i \frac{\partial^2 w}{\partial x_i^2} + cu) dx_i \quad (5)$$

where $E_i I_i$ is the product of Young's modulus of elasticity by the cross-sectional area moment of inertia for the equivalent beam for an element i .

Using the Lagrangian dynamics, the equations of motion are given by,

$$M\ddot{q} + Kq = B_0 u(t) \quad (6)$$

where $q = (w_2, \phi_2, \dots, w_{n+1}, \phi_{n+1})^T \in \mathbb{R}^{2n}$. (Readers may refer to Ref. 15 for the details). Considering the hinge connection between the beam actuator and the blade, the fin angle can be expressed as

$$\psi = \tan^{-1} \left(\frac{\delta_t}{l} \right) \quad (7)$$

where L is the total length of the beam and δ_t is the tip displacement of the beam. For small fin angle, it can be approximated as $\psi = \delta_t/l$.

The aerodynamic moment acting on the fin is a complicated function of the angle of attack of the projectile and the fin rotation angle. The data generated by the computational fluid dynamics show that

the aerodynamic moment can be accurately modeled as a linear function of the fin angle and a reasonable model can be expressed as

$$m_a = m_{a0}(\alpha) + p_a(\alpha)\psi = m_{a0}(\alpha) + p_a(\alpha)l^{-1}e^{*T}q \quad (8)$$

where $p_a(\alpha)$ is a polynomial in the angle of attack, α , $p_a(\alpha) = p_0 + p_1\alpha + \dots + p_k\alpha^k$ (k is a positive integer) and $e^{*T} \in \mathfrak{R}^{2n}$ is a unit vector whose $(2n-1)^{th}$ element is one and rest are zero.

The modified fin-beam model including the aerodynamic moment takes the form

$$M\ddot{q} + Kq = B_0u(t) + B_a m_a \quad (9)$$

where $B_a = [0, \dots, 0, 1, 0]^T \in \mathfrak{R}^{2n}$. Solving (9) gives

$$\ddot{q} = -M^{-1}K_m q + M^{-1}B_0u(t) + M^{-1}B_a m_{a0}(\alpha) \quad (10)$$

where $K_m = K - p_a(\alpha)l^{-1}e^{*}e^{*T}$.

The eigenvalues of $M^{-1}K_m$ are distinct positive real numbers. As such there exists a similarity transformation matrix V formed by the eigenvectors of the matrix $M^{-1}K_m$ such that

$$V^{-1}M^{-1}K_m V = \Omega^2 \quad (11)$$

where $\Omega^2 = \text{diag}(\Omega_i^2)$, $i = 1, \dots, 2n$; $\Omega_i \neq \Omega_j$, $i \neq j$.

Defining $\eta = V^{-1}q$, one obtains from (10)

$$\begin{aligned} \ddot{\eta} &= -\Omega^2\eta + V^{-1}M^{-1}B_0u(t) + V^{-1}M^{-1}B_a m_{a0}(\alpha) \\ &= -\Omega^2\eta + B_1u(t) + F_1v \end{aligned} \quad (12)$$

where $B_1 = V^{-1}M^{-1}B_0 \in \mathfrak{R}^{2n}$, $F_1 = V^{-1}M^{-1}B_a$ and $v = m_{a0}(\alpha)$. The modal form (9) has no damping. However, there is nonzero structural damping for any elastic body. As such it is common to introduce a dissipation term proportional to the rate $\dot{\eta}$. Introducing a damping term of the form $2D\dot{\eta}$, where $D = \text{diag}(\zeta_i)$, $i = 1, \dots, 2n$, $\zeta_i > 0$, one obtains the system

$$\ddot{\eta} = -2D\dot{\eta} - \Omega^2\eta + B_1u + F_1v \quad (13)$$

The fin angle in new coordinate becomes

$$\psi = l^{-1}e^{*T}q = 1^{-1}e^{*T}V\eta = C_0\eta$$

It is assumed that the system matrices D , Ω , B_1 , F_1 , v and C_0 are unknown. Furthermore, it is assumed that only the fin angle is measurable. We are interested in designing an adaptive control system such that the fin angle asymptotically tracks the reference trajectory y_m and rejects the disturbance input v . Moreover, for synthesis only the measured fin angle ψ to be used.

III. System Representation

In this section, a frequency domain representation of the system for adaptive control design is considered. Defining the state vector $x_f = (\eta^T, \dot{\eta}^T)^T$, a state variable representation takes the form

$$\begin{aligned} \dot{x}_f &= \begin{bmatrix} 0_{2n \times 2n} & I_{2n \times 2n} \\ -\Omega^2 & -2D\Omega \end{bmatrix} x_f + \begin{bmatrix} 0_{2n \times 1} \\ B_1 \end{bmatrix} u + \begin{bmatrix} 0_{2n \times 1} \\ F_1 \end{bmatrix} v \\ &\triangleq A_f x + b_f u + Fv \end{aligned} \quad (14)$$

We select the controlled output variable as

$$y = \psi = h_f x_f \quad (15)$$

Let $y_m = \psi_m$ be a smooth reference trajectory generated by a reference model. We are interested in deriving a VS-MRAC control law $u(t)$ such that the fin angle tracking error

$$e_0 = y - y_m$$

asymptotically tends to zero and the elastic modes remain bounded during maneuver. Furthermore, for a constant set point control of fin angle, it is desired that the flexible modes converge to their equilibrium values. By suitable choice of the reference trajectory y_m , desirable fin angle control is accomplished.

Consider the input-output representation of the system Eq. (14) given by

$$y(s) = h_f(sI - A_f)^{-1}b_f u(s) + h_f(sI - A_f)^{-1}Fv(s) \quad (16)$$

$$\triangleq \frac{k_p n_p(s)u(s) + n_v(s)v(s)}{d_p(s)} \quad (17)$$

where s denotes the differential operator or the Laplace variable.

From Eq. (17), one has

$$\begin{aligned} y &= W(s)[u + W_v v] \\ &= W(s)[u + g(v, t)] \end{aligned} \quad (18)$$

where $W(s) = k_p n_p(s)d_p^{-1}(s)$, $W_v = n_v(s)(k_p n_p)^{-1}(s)$, and $g(v, t)$ is the filtered signal $W_v v$.

For the projectile fin model, the transfer function $W(s)$ has the following properties.

- (P1) The relative degree (n^*) of $W(s)$ is 2;
- (P2) $W(s)$ is minimum phase.

The property (P1) follows easily since the second derivative of y explicitly depends on the control input u . Since the $W(s)$ is minimum phase, $n_p(s)$ is Hurwitz and it follows that $W_v(s)$ is a stable transfer matrix. Thus the function $g(v, t)$ is bounded since v is bounded.

Consider a reference model of relative degree 2 with input r and output y_m given by

$$y_m = W_m(s)r \quad (19)$$

$$W_m(s) = \frac{k_m}{s^2 + \alpha_{m1}s + \alpha_{m2}} = \frac{k_m}{d_m(s)}$$

where the poles of W_m are assumed to be stable. Now a control law will be derived for tracking the reference trajectory y_m .

IV. VS-MRAC Control Law Design

In this section, the design of control system following Refs. 23, 24 is considered. For the design of a variable structure adaptive controller, consider the input-output representation of the system given in Eq. (18). Then a controllable and observable representation of Eq. (18) is given by

$$\begin{aligned} \dot{x} &= Ax + b(u + g(v, t)) \\ y &= h^T x \end{aligned} \quad (20)$$

It is pointed out that the knowledge of matrices A , b , h and g are not required for the design of the control system.

For the synthesis of the controller, now the following filters are introduced.

$$\begin{aligned} \dot{\omega}_1 &= F\omega_1 + \nu u \\ \dot{\omega}_2 &= F\omega_1 + \nu y \end{aligned} \quad (21)$$

where $\omega_1, \omega_2 \in \mathfrak{R}^{N-1}$, $N = 4n$,

$$F = \begin{bmatrix} -\lambda_{N-2} & -\lambda_{N-1} & \dots & -\lambda_0 \\ & I & & \dots \\ & & & \dots \\ & & & 0 \end{bmatrix}$$

$$\nu = \begin{bmatrix} 1 & 0^T \end{bmatrix}$$

and λ_i are coefficients of the polynomial

$$\Lambda(s) = s^{N-1} + \lambda_{N-2}s^{N-2} + \dots + \lambda_1s + \lambda_0 = \det(sI - F) \quad (22)$$

Define $\omega = [\omega_1^T, y, \omega_2^T, r]^T \in \mathfrak{R}^{2N}$. The control law is to be synthesized using only the regressor vector ω .

Since the relative degree of $W(s)$ and $W_m(s)$ are equal ($n^* = 2$), for $g(v) = 0$, there exists a unique constant vector $\theta^* = [\theta_{\omega_1}^*, \theta_y, \theta_{\omega_2}^*, \theta_r^*]^T \in \mathfrak{R}^{2N}$, such that the transfer function of the closed-loop system with the control input

$$u^* = \theta^{*T} \omega = (\theta_{\omega_1}^*)^T \omega_1 + \theta_y^* y + (\theta_{\omega_2}^*)^T \omega_2 + \theta_r^* r$$

matches $W_m(s)$ exactly, i.e.,

$$y = W(s)u = W(s)\theta^{*T} \omega = W_m(s)r \quad (23)$$

For model matching, the parameter vector θ^* should satisfy [25]

$$\theta_{2N}^* = k_m/k_p \quad (24)$$

$$\theta_{\omega_1}^{*T} \alpha(s) d_p(s) + k_p (\theta_{\omega_2}^{*T} \alpha(s) + \theta_y^* \Lambda(s)) n_p(s) = \Lambda(s) d_p(s) - n_p(s) \Lambda(s) d_m(s) \quad (25)$$

where $\alpha(s) = [s^{N-2}, \dots, s, 1]$. Solving Eq. (25) gives the parameter vector θ^* .

Define $\kappa^* = (\theta_r^*)^{-1} = k_p/k_m$, and $\tilde{u} = u - u^*$. For the fin-beam model, $k_p > 0$ and the chosen parameter k_m is positive.

Defining the vector $X^T = (x^T, \omega_1^T, \omega_2^T)^T \in \mathfrak{R}^{3N-2}$, the system Eqs. (20) and (21) can be written as

$$\dot{X} = A_a X + b_a u + \tilde{b}_g g \quad (26)$$

$$y = h_c^T X$$

$$\text{where } A_a = \begin{bmatrix} A & 0 & 0 \\ 0 & F & 0 \\ \nu h^T & 0 & F \end{bmatrix}, b_a = \begin{bmatrix} b \\ \nu \\ 0 \end{bmatrix}, \tilde{b}_g = \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix}$$

$$h_c^T = \begin{bmatrix} h^T & 0 \end{bmatrix}.$$

Define

$$\begin{bmatrix} \omega_1 \\ y \\ \omega_2 \end{bmatrix} = \begin{bmatrix} 0 & I & 0 \\ h^T & 0 & 0 \\ 0 & 0 & I \end{bmatrix} X \triangleq NX \quad (27)$$

Now adding and subtracting $b_a u^*$ in Eq. (26) and using Eq. (27), gives

$$\dot{X} = A_c X + b_c \kappa^* \tilde{u} + b_c r + \tilde{b}_g g(v, t) \quad (28)$$

$$y = h_c^T X$$

where $A_c = A_a + b_a [\theta_{\omega_1}^{*T}, \theta_y^*, \theta_{\omega_2}^{*T}] N$ and $b_c = \theta_r^* b_a$. For $u = u^*$ (i.e. $\tilde{u} = 0$) and $g = 0$, one has $W_m = h_c^T (sI - A_c)^{-1} b_c$. Therefore, the output of Eq. (28), ignoring the exponentially decaying signals due to initial conditions, which is not essential for derivation, can be written as

$$y = W_m(s)r + \kappa^* W_m(s)\tilde{u} + g_c(v) \quad (29)$$

where $g_c = \bar{W}_m(s)g$, $\bar{W}_m(s) = h_c^T (sI - A_c)^{-1} \tilde{b}_g$. Here $\bar{W}_m(s)$ is a stable transfer function, and therefore, $g_c(v, t)$ is bounded for any bounded function $g(v, t)$.

A non-minimal realization of the reference model (Eq. (19)) is

$$\dot{X}_m = A_c X_m + b_c r \quad (30)$$

$$y_m = h_c^T X_m$$

where $X_m \in \mathfrak{R}^{3N-2}$.

Let the state vector error be $e = X - X_m$. Subtracting Eq. (30) from Eq. (28) gives the error equation and the tracking error ($y - y_m$) given by

$$\dot{e} = A_c e + b_c \kappa^* \tilde{u} + \bar{b}_g g \quad (31)$$

$$e_0 = h_c^T e$$

Using Eq. (31), the output tracking error can be written as $e_0 = \kappa^* W_m(s) \tilde{u} + g_c(v)$.

For the synthesis of the controller, it is essential to introduce a chain of auxiliary errors (e'_i). Since the relative degree n^* of the reference model is two, $W_m(s)$ cannot be chosen SPR (strictly positive real). In order to overcome this difficulty, a polynomial $L(s)$ of degree $(n^* - 1)$ is chosen so that $W_m(s)L(s)$ is SPR. We select $L(s)$ of the form

$$L(s) = \frac{s + \delta}{\delta}, \delta > 0$$

Now, we introduce the following set of filtered signals:

$$\chi_0 = L^{-1} \chi_1$$

$$\xi_0 = L^{-1} \xi_1$$

where $\chi_1 = u$, $\xi_1 = \omega$ and $\xi_i = (\xi_{i1}, \dots, \xi_{i2N})^T \in \mathfrak{R}^{2N}$, ($i = 0, 1$). These signals are used to generate a chain of auxiliary error signals e'_i ($i = 0, 1$). Based on the results of Refs. 23, 24, the complete algorithm for the fin angle control is given in Table 1.

Table 1: VS-MRAC Algorithm

Auxiliary Errors	$y_a = \kappa_{nom} W_m L[u_0 - L^{-1} u_1]$ $e_0 = y - y_m; e'_0 = e_0 - y_a$ $e'_1 = (u_0)_{eq} - L^{-1}(u_1)$
Modulation Functions	$f_0 \geq \bar{\kappa} \chi_0 - \theta_{nom}^T \xi_0 + \sum_{j=1}^{2N} \bar{\theta}_{0j} \xi_{0j} + \bar{g}_0 + \epsilon_0$ $f_1 \geq \sum_{j=1}^{2N} \bar{\theta}_{1j} \xi_{1j} + \bar{g}_1 + \epsilon_1$
Control Laws	$u_i = f_i \text{sgn}(e'_i), i = 0, 1$ $u = -u_1 + u_{nom}$ $u_{nom} = \theta_{nom}^T \omega$

In the above table, θ_{nom} and κ_{nom} are nominal values of the parameters θ^* and κ^* , respectively, obtained from some nominal model of the plant, $\epsilon_0 > 0$, $\epsilon_1 > 0$, and the upper bounds $\bar{\theta}_{ij}$ ($i = 0, 1$ and $j = 1, \dots, 2N$), $\bar{\kappa}$, and \bar{g}_i ($i = 0, 1$) for any bounded disturbance input satisfying $g(v)$ are defined as

$$\bar{\theta}_{0j} > \rho | \theta_j^* - \theta_{j,nom} |, \bar{\theta}_{1j} > | \theta_j^* - \theta_{j,nom} |, \bar{\kappa} > | \rho - 1 |$$

$$\bar{g}_0 > \sup_{t \geq 0} (k_{nom} W_m L)^{-1} g_c = \sup_{t \geq 0} (k_{nom} W_m L)^{-1} \bar{W}_m g \quad (32)$$

$$\bar{g}_1 > \sup_{t \geq 0} \theta_r^* W_m^{-1} g_c = \sup_{t \geq 0} \theta_r^* W_m^{-1} \bar{W}_m g$$

where $\rho = \kappa^*/\kappa_{nom}$. Note that one must choose $\kappa_{nom} \neq 0$. The $(u_0)_{eq}$ is the equivalent control which is approximately obtained from u_0 by means of a low-pass filter with high cut-off frequency. The block diagram of Figure 2 gives the complete closed-loop system. (In Figure 2, d denotes the disturbance signal $g(v, t)$).

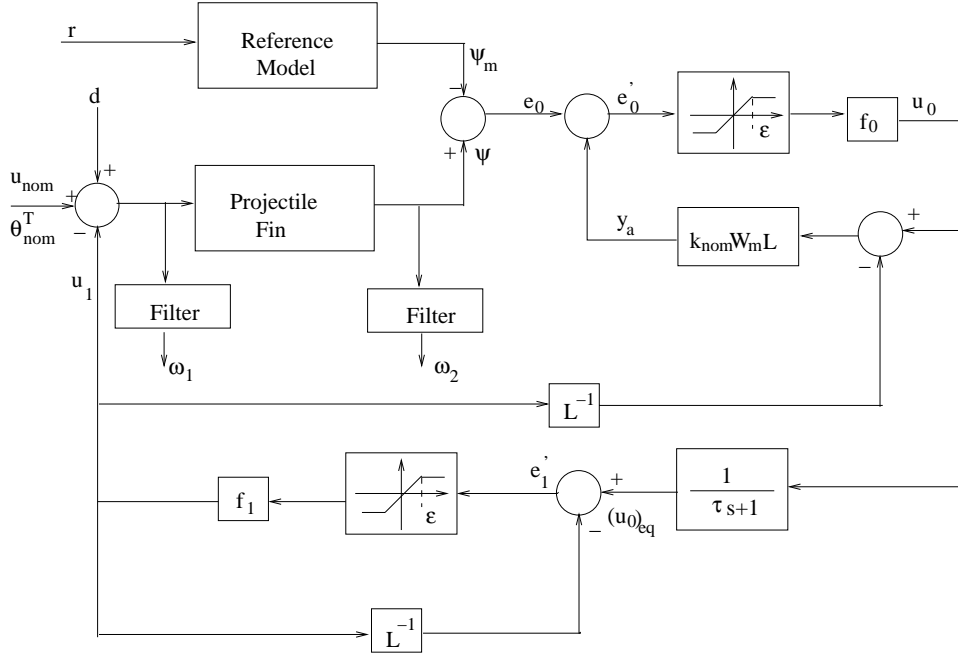


Figure 2. VSMRAC System

Now consider the **VS-MRAC** law of Table 1. Then for any trajectory the closed-loop system, following Ref. 23, 24, it can be shown that the fin angle tracking error e_0 converge exponentially to zero.

The control law is discontinuous which can cause control chattering. In order to obtain smooth control signals for **VSC** systems and to avoid undesirable chattering phenomenon, one uses a continuous approximation of the switching functions. For this, one replaces $u = \text{sgn}(e'_i)$ by $u = \text{sat}(e'_i)$, where $\text{sat}(\eta)$ is defined as $\text{sat}(\eta) = \text{sign}(\eta)$ if $|\eta| > \Delta$, and $\text{sat}(\eta) = (\eta/\delta)$ if $|\eta| \leq \Delta$. Here Δ is the bounded layer thickness.

V. Simulation Results

In this section, simulation results for the closed-loop system with the control law derived in Table 1 are presented. The closed-loop system is shown in Figure 2. The mechanical properties of the simulated model are the same as given in [22]. Using the finite element method (with $n=5$ elements), a state-variable representation of the fin-beam model of dimension of 20 is obtained for simulation.

The reference model is chosen as

$$W_m = \frac{\lambda_m^2}{(s + \lambda_m)^2} \quad (33)$$

where λ_m is 0.1. We choose $L = (s + \lambda_m)/\lambda_m$ so that $W_m L$ is SPR. For the computation of $u_{nom} = \theta_{nom}^T \omega$, the nominal value of θ was arbitrarily chosen as $\theta_{nom} = (0, \dots, 0, 1)^T \in R^{2N}$, giving $u_{nom} = r$. This is rather an unfavorable choice of estimate of θ^* . In the saturation function, the boundary layer thickness set

to $\Delta = 0.5$. A simplified relay type controller was synthesized by using constant modulation functions as $f_0 = 1000000$ and $f_1 = 1000000$.

Adaptive Control: $\psi = 5^\circ$, $\alpha = -5^\circ$, 5°

Simulation results for a fin angle command of 5° for angle of attack $\alpha = -5^\circ$ are shown in Figure 3. Figure 4 shows the simulation results for fin angle command of 5° with angle of attack, $\alpha = 5^\circ$. It is observed that the fin angle asymptotically converges to the desired value in less than 1 second in both the cases. In the steady state, the control input needed to deflect the fin to an angle of 5° for $\alpha = -5^\circ$ is around 1000 volts. The deflections at other points on the beam remain bounded during the maneuver and converge to constant values. The tracking error is of the order of 10^{-3} . We note that there is no overshoot in the fin angle trajectory and the control input never exceeds u^* , the voltage required to maintain $\psi = 5^\circ$ in the equilibrium condition.

VI. Conclusions

Based on the variable structure model reference adaptive control theory, a new control law for the control of fin angle of projectile fin was presented. In the fin-beam model, unmodeled dynamics and disturbance input were assumed to be present. A variable structure model reference adaptive control system was synthesized using only measurement on the fin angle and measurement of other flexible modes was not needed for control. Interesting, unlike usual adaptive controller, the derived VS-MRAC system does not have integral type adaptation law for updating the parameters of controller. This structure of adaptive controller has significant advantage over other adaptive schemes, since in this case controller parameter divergence cannot occur. In the closed-loop system, fin angle tracked the reference trajectory, and stabilization of flexible modes was accomplished. Extensive simulation results were presented which showed good transient characteristics of the designed controller in spite of the presence of UN-modeled dynamics, uncertainty in system parameters, and disturbance input.

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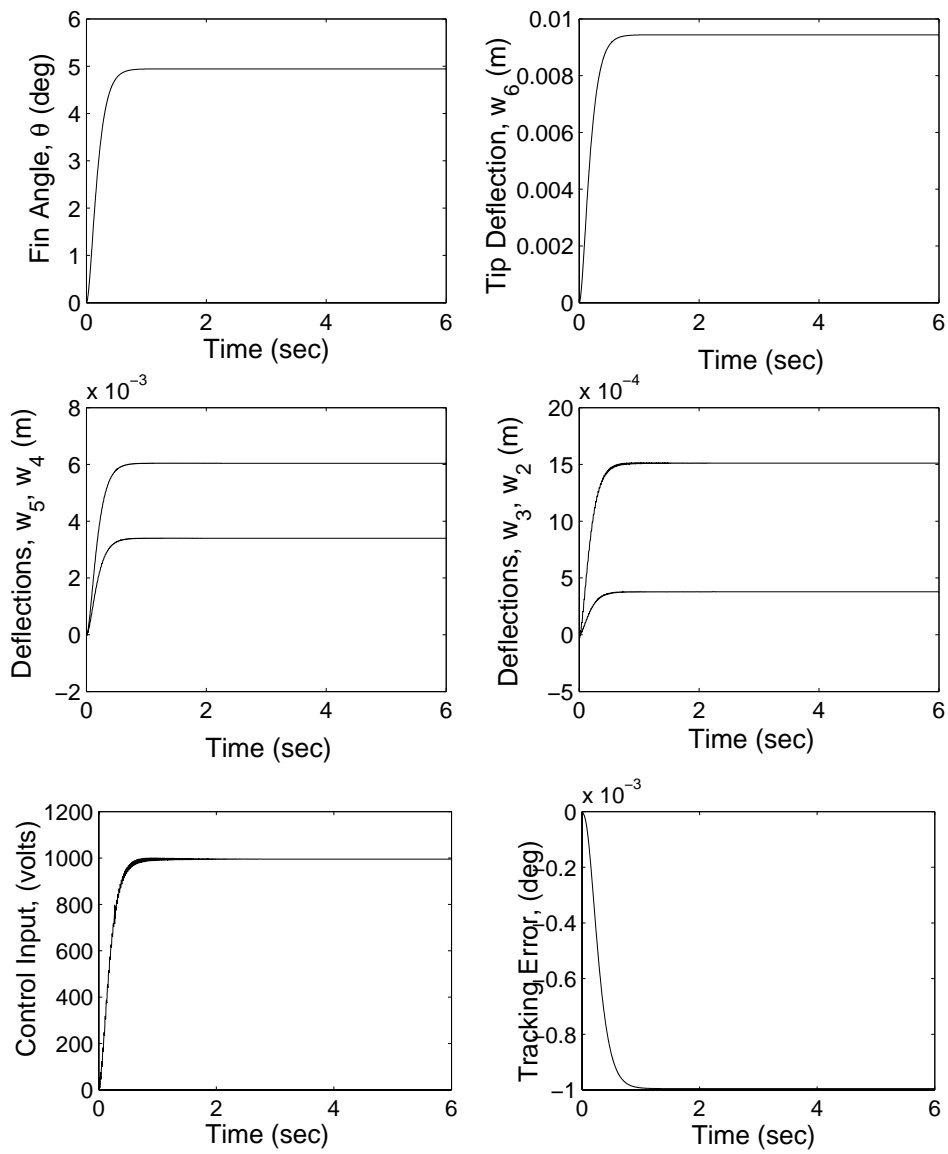


Figure 3. Variable Structure Control: $\psi = 5^0, \alpha = -5^0$

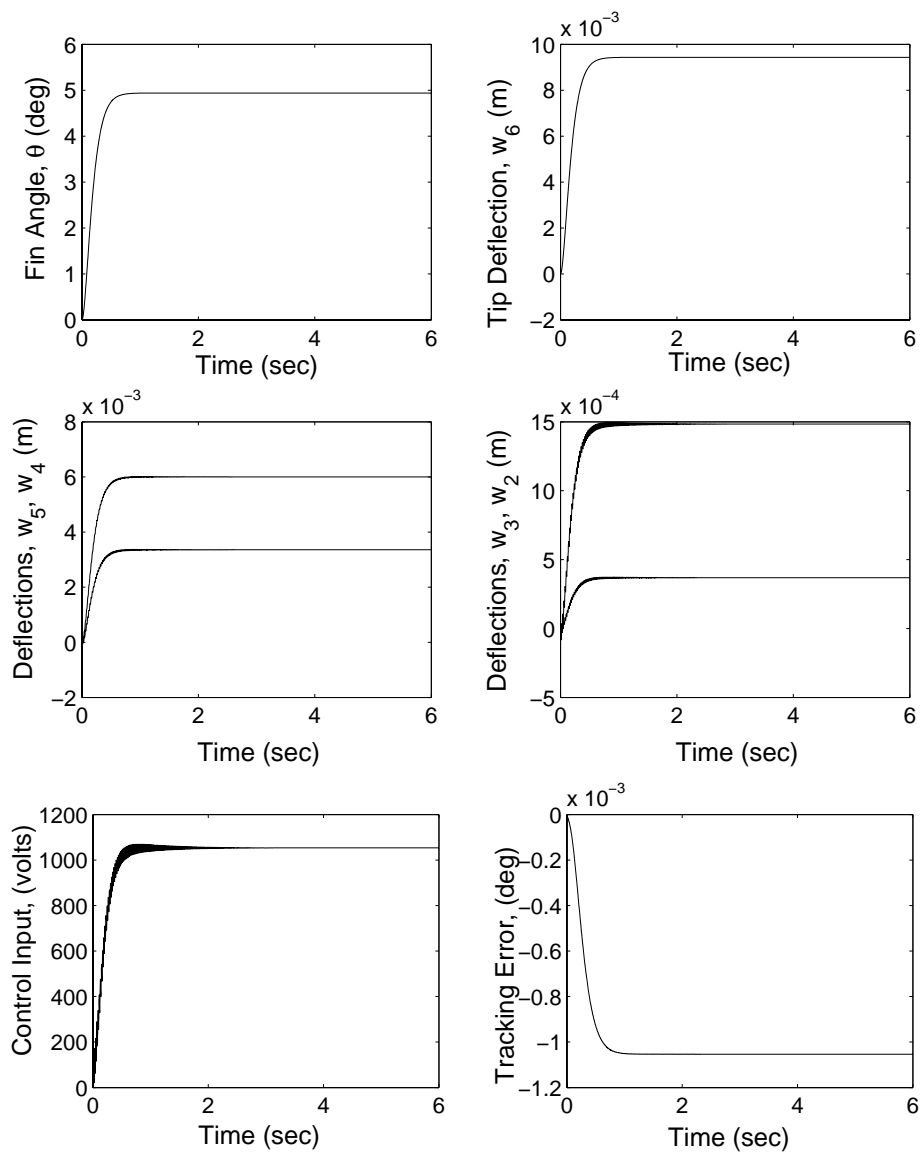


Figure 4. Variable Structure Control: $\psi = 5^0$, $\alpha = 5^0$