12-2010

A Mathematical approach for optimizing the casino slot floor: A linear programming application

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A MATHEMATICAL APPROACH FOR OPTIMIZING THE CASINO SLOT FLOOR:

A LINEAR PROGRAMMING APPLICATION

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2007

A thesis submitted in partial fulfillment of
the requirements for the

Master of Science in Hotel Administration
William F. Harrah College of Hotel Administration

Graduate College
University of Nevada, Las Vegas
December 2010

THE GRADUATE COLLEGE

We recommend the thesis prepared under our supervision by

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entitled

A Mathematical Approach for Optimizing the Casino Slot Floor: A Linear Programming Application

be accepted in partial fulfillment of the requirements for the degree of

Master of Science in Hotel Administration
William F. Harrah College of Hotel Administration

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December 2010

ABSTRACT

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Linear programming is a tool that has been successfully applied to various problems across many different industries and businesses. However, it appears that casino operators may have overlooked this useful and proven method. At most casino properties the bulk of gaming revenues are derived from slot machines. It is therefore imperative for casino operators to effectively manage and cultivate the performance of this department. A primary task for the casino operator is planning and deciding the mix of slot machines in order to maximize performance.

This paper presents the task of optimizing the casino slot floor as a linear programming problem. The method has been applied to data supplied by a Las Vegas repeater market hotel casino. Two models were developed, and both produced results improving the performance of the casino slot floor. The research provides casino operators with a systematic method that will help analyze and enhance their slot operations.

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CHAPTER 1

INTRODUCTION

Purpose

The purpose of this study is to apply linear programming to the task of configuring the slot floor. Currently casino operators appear to lack a structured method for determining the optimal mix of slot machines on the floor. Amongst other factors, managers use feedback from customers, their own intuition, and limited performance data to make their decisions regarding the slot mix. Although the aforementioned are all valid factors to consider, it is proposed here that a more systematic method would yield more desirable results.

Previous authors have recognized the need for more sophisticated operations analysis in the gaming industry (Lucas & Kilby, 2008), and although slot machines have the reputation of already being a huge revenue generator, a proposal that aims to enhance the performance of the department should not be overlooked. Examining the task of configuring the casino slot floor and proposing a model that will maximize performance is the purpose of this study.

Practical Significance

All U.S. gaming markets, and in particular less established markets, rely heavily on slot revenues (Lucas, Singh, Gewali, & Singh, 2009). For the fiscal year ending July 31, 2010, Nevada statewide slot machine win was $6.6 billion, compared to $3.5 billion in table games win (Nevada Gaming Control Board, 2010). In 2009, 88% of Illinois' and 90% of Iowa's total casino win came from slot machines (Illinois Gaming Board, 2009;

Iowa Racing & Gaming Commission, 2009), and Gu (2003) has recognized that slots also provide the majority of revenues for European casinos. With this industry-wide dependence on slot machines any research aimed at enhancing the performance of this entity would be invaluable to casino executives.

Casino operators should be devoted to the development and execution of a process that provides the optimal mix of slot machines, however, as Lucas et al. (2009) state, simplistic measures are often used as the sole criterion for deciding the fate of a machine. A greater investment in empirical analysis would be favorable and previous authors have already recognized the need for scientific decision-making methods in slot operations management (Fier, 2003). This lack of rigor is characteristic of leadership and management in the gaming industry, with many still relying on dated operational methods that embrace intuition, rather than research-based policies (Bernhard, Green, & Lucas, 2008). In the future, as pointed out by Bernhard et al. (2008), gaming leaders should make efforts to take advantage of sophisticated, quantitative techniques that are available.

Without a structured, scientific, and performance driven method for deciding the slot mix operators may be limiting their casino floor's potential. Such a method may also provide a competitive edge in an industry that is growing both nationally and globally.

Academic Significance

Through the application of linear programming, two mathematical models are proposed with the objective of maximizing slot floor performance under a set of constraints. To the author's knowledge, this is the first study to investigate the task of configuring the slot floor mix. It is worth noting here that linear programming is certainly not a new field and since its invention, circa 1947 (Eiselt and Sandblom, 2007), has been successful in solving a multitude of problems across various different industries. Numerous textbooks and academic articles have been written on the subject, however, applications in casino operations have appeared to be neglected. It is the author's hope that this study will supplement current casino operations research and encourage further examination of the task at hand.

CHAPTER 2

THE EVOLUTION OF THE CASINO FLOOR

Table Games were King

The casino floor of the past looked much different from the one we are familiar with today. As Kilby, Fox, and Lucas (2005) point out “table games were king”, and in addition to being the most popular, they were also the most profitable. Now if one walks the casino floors of Las Vegas, or for that matter almost any other U.S. gaming jurisdiction, it is clear that table games no longer hold the crown. Bernhard (2007) fittingly describes this evolution as a “deforestation effect”, and still to this day casinos continue to re-evaluate their landscape to make way for more slot machines (Batt, 2010; Green, 2010).

This trend can be attributed to several various factors. For one, slot machines produce the most favorable profit margins. Table 1 below, extracted from Kilby et al. (2005), displays departmental profit margins for a standard large casino. Clearly slots are leaders in terms of profit margin percentage. Kilby et al. (2005) also highlight the fact that slots produce the greatest profit per square foot.

Table 1

Departmental Profit Margins for a Standard Large Casino

Department	Margin %
Slots	60-70
Table Games (excluding baccarat)	15-20
Keno	25-30
Race and Sports	15-25
Poker	20-30

Note. Adapted from *Casino Operations Management* (p. 179) by J. Kilby, J. Fox, and A. F. Lucas, 2005, Hoboken, NJ: John Wiley & Sons, Inc. Copyright 2005.

Technology's Role

The "deforestation effect" can also be contributed to advances in technology. While the appearance of table games has remained more or less unchanged throughout history, slots have embraced technology and today's electronic machines look very different from the mechanical machines of yesterday (Kilby et al. 2005). The most recognizable change has been the development of popular themed slot machines (Kilby et al. 2005). One can now find a variety of different themed slot machines from Wheel of Fortune to Indiana Jones, with denominations ranging anywhere from one cent to twenty-five dollars per spin.

Technology has also helped slot machines become increasingly efficient, thus more desirable to operators. There has been a move toward a cashless casino (Kilby et al. 2005). With the invention of systems such as ticket-in ticket-out (TITO), patrons are no longer paid with coins, but with a bar coded ticket that is used to redeem winnings at the cage or a ticket redemption machine. Also worth mentioning here is the innovation that is server-based gaming (SBG). SBG has been accredited with the ability to advance the player experience. For example, players will no longer have to wait in line or search for their favorite games, as SBG makes a variety of games available for download on a single machine (Binns, 2007; Bourie, 2006; Terdiman, 2009). SBG also enhances management capabilities. The technology allows for data and information to be sent to and retrieved from slots at anytime, this essentially gives operators the ability to make real time changes in response to customer demand (Terdiman 2009). As Binns (2007) points out SBG offers clear and immediate savings, as without the technology game conversions are very labor intensive. However, it should be noted that SBG has not seen the widespread implementation that had been anticipated. Many casinos are yet to implement the technology, but the industry outlook is optimistic now that certain barriers have been overcome (Terdiman, 2009). A deeper understanding and greater application of management science may help harness the power of SBG in the future.

Slots are King

This evolution has consequently seen slot machines become the dominant source of revenues for many casinos. Brewer and Cummings (1995) noted that slot revenues, on average, account for 50 to 80% of total casino revenues. However, given this expansion in terms of popularity and profitability, it is questionable whether or not there has been a comparable development of analytical techniques to aid in the management of this lucrative department. As Lucas, Singh, Gewali, and Singh (2009) affirm, the U.S gaming industry is rapidly maturing, and advanced analytical methods offer the opportunity for a sustainable competitive advantage.

CHAPTER 3

PREVIOUS RESEARCH IN SLOT OPERATIONS

It has been noted that the gaming industry may benefit from more advanced operations analysis, particularly with regards to slot operations given the entity's popularity and profitability. Previous research in academe has focused on slot operations and an overview of this work will now be presented. The chapter is presented in three sections. The first reviews literature regarding the indirect drivers of slot revenues. The second presents research regarding methods to increase slot revenues, specifically addressing the slot patron's experience and game interaction. Finally, research analyzing the variations in slot machine unit level performance is reviewed.

Indirect Contributors of Slot Revenue

Lucas and Brewer (2001) carried out exploratory research at a Las Vegas locals' market hotel casino that attempted to identify factors that influenced daily slot handle, or "coin-in". Coin-in is an industry term, defined as the total amount of money wagered on all slot machines on a given day. The authors built regression models using daily slot handle as the dependent variable. Independent variables included but were not limited to buy-in incentives, food covers, major holidays, bingo headcount, and slot complimentary room nights. The results of the model allowed the authors to assess the validity of commonly accepted practices and theories. For example, restaurant patronage proved to have no significant effect on slot handle, and the results brought into question the economic significance of direct mail programs and the bingo operation.

Since the aforementioned study, several other works have sought to evaluate operating and marketing variables that may or may not indirectly contribute to slot business volumes (Abarbanel, 2009; Lucas & Bowen, 2002; Lucas, Dunn, & Kharitonova, 2006; Lucas & Santos, 2003; Ollstein, 2006). Too often are underperforming departments or practices justified due to their supposed indirect contributions to slot revenues. These studies have provided valuable insight and challenged these misconceptions in the gaming industry. The focus has been at the aggregate level, helping operators recognize the often-misunderstood relationships between departments within their businesses.

Methods to Increase Slot Revenue

Lucas and Brandmeir (2005) challenged the theory that slot players were able to perceive a substantial increase in the par value of a slot machine. The term par value refers to the house advantage; the casino's expected value associated with each slot machine's pay table. The authors specifically investigated reel slot players. In games such as video poker the player is presented with the pay table, but with reel slots the outcomes are unknown to the player. This means the player is unable to calculate, or estimate, the house advantage of a reel slot game. Using a sample of $5.00 reel slots the study found that a 50% increase in the par values had no significant affect on the games' performance, that is, the players did not notice the change. The authors also concluded that the casino was winning the customers' money at an increased rate due to the increase in par value. However, this outcome was not met with an increased share of the players' wallets – players were not devoting a larger amount of their available gambling monies to the game.

Kilby et al. (2005) also addressed the gaming experience and carried out an experiment to assess the relationship between hit frequency and pulls per losing player (PPLP). Hit frequency can be defined as the number of outcomes (or spins) that result in a payout of at least one coin divided by the total number of outcomes. PPLP is the number of pulls (or spins) a player makes before they are bankrupt or their bankroll is doubled. The results of the experiment found no relationship between hit frequency and the length of time a player was at a machine (or time-on-device). The conclusion conflicted with existing assumptions regarding this relationship.

Lucas, Singh, and Gewali (2007) revisited the investigation of time-on-device by looking at the effect of variations in the standard deviation of the pay table. Lucas et al. (2007) found standard deviation to be a decisive determinant of the slot player’s experience, with an increase in standard deviation producing a decrease in time-on-device. This research along with findings from Kilby et al. (2005) disproved the mistaken belief that hit frequency had a significant effect on a player’s time-on-device, and found standard deviation of the pay table to be a far better proxy of time-on-device. Slot players have expectations regarding length of play, and this is a strong predictor of a slot patron’s satisfaction (Lucas, 2003). Via changes in specific slot machine parameters one can influence the slot patron’s experience. The authors urge casino managers to take the results into consideration when managing and positioning their slot floors. The findings are of particular importance to those managers who operate in repeater markets, where the gaming experience may be more integral to success.

Lucas and Singh (2008) via computer simulation examined the relationship between reel slot players' time-on-device and the pay table's coefficient of variation (CV). CV is defined as a game's standard deviation divided by its par. The authors found an inverse relationship between the pay table CV and PPLP. An increase in the pay table variation was associated with a decrease in a player's time-on-device. This finding once again proves that house advantage alone is not a legitimate determinant of time-on-device, and supported the previous literature discussed (Kilby et al. 2005; Lucas et al. 2007).

This body of research has given casino operators priceless information regarding unit level analysis and the effects of changes in specific game characteristics on players' slot experience.

Slot Performance-Potential Research

Lucas and Roehl (2002) also hone in on unit level analysis in a study that investigated the influence of floor location and game characteristics on video poker machine performance. Regression analysis aided in the development of a "slot performance-potential model". The authors found location to have a considerable effect on the performance of an individual machine. Machines attributed with superior access and higher traffic volumes outperformed those situated in perimeter locations. Cabinet style, house advantage, and game program (a variable which represented different pay tables) were also deterministic of a machine's performance. The model advanced existing performance evaluation techniques as it offered managers a method that considered unique parameters specific to individual machine units, thus providing an alternative to merely comparing unit results to category averages.

Lucas, Dunn, Roehl, and Wolcott (2004) extended the previously mentioned work, again proving the significance of location. They also found more complex characteristics such as “standard deviation” and “game within a game feature” to influence performance. Lucas and Dunn (2005) took it another step further developing an advanced “performance-potential model” by analyzing the effect of “micro-location variables”. These variables took into account the ceiling height at specific locations, and whether or not a machine was situated on an aisle and/or at the end of a bank of machines. More complex characteristics were also included in the model, such as whether or not a machine had advanced game technology. The results of the study provide managers with an effective and customizable method for evaluating unit level slot performance data.

Lucas et al. (2009) revisited the performance-potential research with the goal of increasing the predicting power of each of the previous models. Principal components analysis was employed which allowed for the addition of more predictor variables, increasing the power of the models. The authors also constructed a Voronoi diagram providing an overview of the results. The diagram identified over-performing and under-performing areas of the slot floor allowing casino operators to challenge the design of the floor to maximize potential.

The performance-potential research provides a valuable tool for slot operations managers and helps explain variation in unit level performance. Analysis of unit level data has been advanced, and recommendations for the slot floor layout have been presented. The research presented herein may supplement the performance-potential literature well by offering a broader analysis of the floor.

The Slot Floor Mix

Using advanced management science techniques, the extensive literature reviewed here has disproved misconceptions and evolved analysis in slot operations. Nevertheless, to the author's knowledge no previous research has been carried out addressing the slot floor mix. Kilby et al. (2005) defines slot mix as "the quantity, type, denomination, and strategic placement of machines that management has chosen to offer the public" and provides three variables that make up the slot mix; (1) floor configuration, (2) mechanical configuration, and (3) model mix. Floor configuration refers to where exactly machines should be placed on the casino floor; this variable has been analyzed in the discussed performance-potential research. Factors constituting mechanical configuration include pay table, par, and hit frequency; this has also been addressed in the abovementioned literature. No research has addressed the first variable - model mix - referring to how many of each type of slot machine a casino should offer. Slot machines come in various shapes and sizes, offering a variety of games with different technological capabilities. Most casinos offer both video and mechanical types, a wide range of denominations, and specialty machines including video poker, video keno, and multi-game devices that offer more than one type of game.

Kilby et al. (2005) provide general guidelines related to model mix, but offer no systematic method for the decision-making process. The authors make mix suggestions for a newly opening casino, stating that the target market should be identified and competitors' slot mix analyzed and maybe even duplicated. Mix strategies are also recommended for repeater market casinos. The authors propose that these casinos should offer more video poker machines, which have lower house advantages and involve an element of skill, because local gamblers are more sophisticated.

In light of the lack of empirical research regarding the slot machine mix this paper attempts to fill the void. Mathematical programming is considered as a technique that may lay the foundation for a more systematic procedure for determining the slot machine mix.

CHAPTER 4

LINEAR PROGRAMMING AND RELATED LITERATURE

A Brief Introduction

Linear programming (LP) was formulated in 1947 (Dantzig & Thapa, 1997). And on October 18, 1976, President Ford awarded The President's National Medal of Science to George B. Dantzig for "inventing linear programming and discovering methods that led to wide-scale scientific and technical applications to important problems in logistics, scheduling, and network optimization, and to the use of computers in making efficient use of the mathematical theory" (National Science Foundation, 2010). The description of the award provides a glimpse of just how important this invention has been. The applications of linear programming are now widespread and have helped companies and businesses throughout the world save millions of dollars (Hillier and Lieberman, 1986). In the academic world operations researchers, management scientists, mathematicians, and economists have embraced the method, and numerous textbooks and articles have been published (Dantzig & Thapa 1997).

Dantzig's contributions flourished from his experiences during World War II as Mathematical Advisor to the US Air Force Comptroller in the Pentagon (Dantzig, 2002). In this role Dantzig worked on problems involving the allocation of scarce resources. These problems would serve as a catalyst leading Dantzig to develop the simplex method, an algorithm for solving linear programming problems.

Though it was military applications that started the field, commercial applications would begin to grow in popularity soon after (Dantzig, 2002). The now famous "blending problem" was first formulated by Charnes, Cooper, and Mellon (1952) who applied linear programming to find the optimal blend of petroleum products to produce aviation gasoline. Markowitz (1952) applied mathematical programming to finance with a paper on portfolio selection. Computational capabilities were still lacking, but in 1954 William Orchard-Hays "wrote the first commercial-grade software for solving linear programs" (Dantzig, 2002, p. 45). Dantzig (2002) refers to this development as being largely responsible for the growth of the field and turning linear programming "from an interesting mathematical concept into a powerful tool that changed the way practical planning was done". The simplex method, however, is known to be of exponential time, that is, the method is not computationally efficient. Karmarkar (1984) introduced an interior point method, which came to be known as Karmarkar's Algorithm; this was the first computationally efficient algorithm for solving a linear programming in polynomial time – when the execution of the computation will take no longer than a polynomial function of the problem's complexity.

Linear programming applications continued to grow, and so too did extensions of the method. Non-linear programming, stochastic programming, goal programming, and data envelopment analysis are just a few of these extensions. Linear programming and the simplex algorithm continue to be a powerful tool with applications widespread and far too numerous to count.

Linear Programming Formulation

A delve into the mathematical theory behind linear programming is certainly out of the scope of this paper, however a short explanation of the concept will be provided here. Essentially, linear programming is the use of a mathematical model to describe and solve a specific optimization problem. The term's use of the word *linear* is clear-cut in that all mathematical functions in the model must be linear functions. The word *programming* is commonly associated with computer programming, but here it can be thought of as a synonym for planning (Hillier & Liebermann, 1986). The process attempts to arrive at an optimal solution to a given problem while obeying the requirements of the defined mathematical model.

Strayer (1989) provides a concise, more technical, definition of linear programming as "a collection of procedures for maximizing or minimizing linear functions subject to given linear constraints". Constraints can be equalities or inequalities, and the linear function to be maximized or minimized is referred to as the "objective function". A common real world example of an objective function is the maximization of profits, or the minimization of costs. Any linear programming problem can be described in the standard form. Referring to Hillier and Liebermann (1986) below is the mathematical model in standard form of a standard linear program to find values for x_1, x_2,...., x_n to maximize Z given the following data in Table 2.

Table 2

Data for Standard Linear Programming Problem

Resource	Activity				Amount of
	1	2	...	n	resource available
1	a_{11}	a_{12}	...	a_{1n}	b_1
2	a_{21}	a_{22}	...	a_{2n}	b_2
⋮			⋮		⋮
m	a_{m1}	a_{m2}	...	a_{mn}	b_m
$\Delta Z/$unit of activity	c_1	c_2	...	c_n	
Level of activity	x_1	x_2	...	x_n	

Note. Adapted from *Introduction to Operations Research: Fourth Edition* (p. 35) by F. S. Hillier, and G. J. Lieberman, Oakland, CA: Holden-Day. Inc. Copyright 1986.

The linear program in standard form can be stated as follows:

$$Maximize\ Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

subject to the restrictions

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_m$$

and

$$x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0$$

The above can be restated in vector-matrix notation as:

Maximize:	$Z = c^Tx$	
Subject to:	$A^Tx \leq b,$	$A: m \times n$
	$x \geq 0$	

Here A, b, and c represent the data for the problem. A and b are matrices indicating a set of constraints that represent specific conditions of the problem. The goal is to maximize the objective function, c^Tx, by calculating suitable values for the decision variables, x, while satisfying the constraints (Dantzig & Thapa, 1997). All mathematical programs consist of the two components: the objective function and constraints (Eiselt & Sandblom, 2007). Constraints cannot be violated and are typically predetermined and not under the control of the decision maker, whereas the objective function reflects the goal of the decision maker. Offering a real world example, a computer manufacturer's goal may be to maximize profits but he or she must realize the existence of various constraints such as financial budgets, labor hours and the supply of electrical components that may affect the ability to maximize profits.

Mathematical Programming in the Hospitality Industry

Applications of linear programming and its extensions are prevalent in fields such as engineering, manufacturing, finance, and agriculture. However, the technique does not appear to be as established in the hospitality industry.

An extension of linear programming proposed by Charnes, Cooper, and Rhodes (1978), data-envelopment analysis (DEA), has received a notable amount of attention in the hospitality field. DEA is a linear programming technique that can be used to measure the relative efficiency of a homogenous set of decision-making units with multiple inputs and outputs (Hsieh & Lin, 2010). The reader is advised to refer to Hsieh and Lin (2010) who provide a comprehensive overview of previous works which have applied DEA in hospitality. The focus of these studies is at a more aggregate level rather than at the individual property level, measuring the efficiency of hotels in different international hotel sectors or that are part of a larger chain.

A massive body of research that has applied mathematical programming techniques, along with many other management science methods, is in the field of revenue management. Chiang, Chen, and Xu (2007) identify three major traditional applications of revenue management – the airline, hotel, and rental car industries. Of the many different problems in revenue management, one in particular has embraced linear programming methods, viz. capacity control. This problem concerns the maximization of revenue or profits by allocating different resources to different classes of demand (Chiang et al. 2007). Chiang et al. (2007), in their overview of research on revenue management provide a definitive review of the literature regarding the capacity control problem.

Another field of study that has used mathematical programming techniques is scheduling. Scheduling is an important problem that is of concern to many different industries including service organizations such as retail stores, restaurants, and hotels (Ernst, Jiang, Krishnamoorthy, & Sier, 2004). However, Choi, Hwang, and Park (2009) point out that there have been few studies exploring scheduling in hospitality services. Choi et al. (2009) propose a method for scheduling restaurant workers in order to minimize costs and meet service standards. The authors develop a model using integer programming, a linear programming problem in which the decision variables are required to take integer form. This is certainly more realistic, given the fact that it would be impossible to schedule a fraction of an employee. The results of the study bolstered the adoption of a more scientific approach to labor scheduling, noting that the proposed model reduced costs and could potentially improve employee morale.

The Retail Sector

To the author’s knowledge there are no hospitality industry specific studies that address a problem similar in nature to the task of configuring slot floor mix. A comparable problem, in an industry somewhat related to the hospitality field, is found in the retail sector, viz. the assortment selection problem. Fundamentally, this problem involves deciding how many and which products to include in a given product line

(Rajaram, 2001). Hart and Rafiq (2006) give an extensive overview of the related literature regarding this problem. They note that most interest for researchers has been at the micro-level, with many published works focusing on item level analysis. Hart and Rafiq (2006) further state that “only a handful of papers acknowledge the existence and importance of the macro-level of assortment”. Interestingly they point out that “given the retailers’ propensity to manage assortments by category, it is surprising that little attention has been given to how space should be allocated between product categories (or departments)”. As highlighted by Hart and Rafiq (2006) there appears to be only one published paper that addresses this problem, a study by Rinne, Geurts, and Kelly (1987) who address the allocation of floor space to departments in a retail store.

Rinne et al. (1987) propose a linear programming routine to decide how much floor space to give to each department in a retail store, taking into consideration the physically constrained sizes of the selling areas. The linear program’s objective function was to maximize the total gross profit margin:

$$\sum_{i=1}^{10} P_{i,t} \times D_i$$

where *P* is gross margin per square foot, *D* is the square footage allocated to one of ten departments, and *t* is the month ($t = 1, \dots, 12$). To formulate the model the authors obtained monthly sales and profits for each department, and estimated the minimum and maximum square footage required for each department on a monthly basis. The minimum and maximum bounds would serve as the constraints for the problem. Total floor capacity was also modeled as a constraint. The study made no effort to incorporate cost constraints in the model. The linear programs were then run on a monthly basis for a twelve month period to determine the size of each department for each month. The profit predictions from the model reflected a 13 per cent increase in gross margin for the year. The authors recognize that the assumption of linearity may be a limitation to the model; however the maximum and minimum limits were selected with the belief that profit growth within the ranges was linear.

It is fascinating to identify the parallels between the assortment selection research and the slot operations research. In the previous chapter it was recognized that there has been considerable academic interest in slot operations unit level analysis. As Hart and Rafiq (2006) would put it, the focus has been at the micro-level, and they too identify the same gap in the retail literature. An important similarity between casino operators and retailers can be drawn. As Hart and Rafiq (2006) point out, retailers commonly group assortments of products into categories. Casino operators take part in the same practice, grouping game types and denominations into separate categories. For example, “reel slots” is a common category that includes several denominations, games, manufacturers, and physically different units. Evidently slot operations research requires some macro-level awareness to be generated, this research attempts to do just that.

CHAPTER 5

METHODOLOGY

Data Source

A Las Vegas repeater market hotel casino provided secondary data that has been used to construct two mathematical models. These models will help define a procedure for configuring the slot floor mix. As with all Las Vegas repeater market properties, this casino's primary revenue generator is slot machines. To protect the anonymity of the benefactor the name of the property has been omitted from this paper.

The data set includes daily observations from 2,612 slot machines across a six month period, beginning October 1, 2009, and ending March 31, 2010. The data are from five main categories of slot machines – Reel Slot, Video Slot, Video Poker, Multi-Game, and Video Keno. The denomination, which refers to the minimum wager accepted by each machine, is also specified and takes one of seven values: $0.01, $0.05, $0.25, $0.50, $1.00, $5.00, $10.00. In total there were nineteen separate categories, as not every category had machines offering all denominations. Daily data for each machine included coin-in, win, base points, and promotional points. Coin-in refers to the dollar amount of wagers made on a machine. Win denotes the amount of coin-in that is retained by the casino after patron payouts are made, a value dependant on the par value of each machine. Base and promotional points are associated with a marketing practice by the casino which rewards members of the slot card club for their play. A more detailed explanation of these points is provided in the subsequent section.

The data set used includes only rated play, that is, it is information gathered from slot card club members only. There was concern that this may limit the validity of the results, however upon comparison of the total coin-in from rated play vs. aggregate play this was not an issue. Rated play accounted for approximately 99 per cent of aggregate play. Data for each machine was plotted on a time series in order to identify any outliers or discrepancies. The data proved to be sound, and therefore no adjustments were made.

The slot managers at this casino continually change the configuration of their floor; therefore not all machines were on the floor for the entire six-month period. To adjust for this instance the following comparable parameters were constructed in order to help facilitate the model development (Lucas, Dunn, Roehl, & Wolcott, 2004).

Computation of Parameters

Coin-in Per Unit Per Day (CPUPD) was calculated by dividing the total coin-in generated by a particular machine divided the number of days that machine was on the floor during the six month period. Total coin-in is defined as the dollar amount of wagers made on the machine during the sample period. *Win Per Unit Per Day* (WPUPD) was determined in the same way, but replacing total coin-in with total win. An average for CPUPD and WPUPD was then taken for each category.

Promo Liability Per Unit Per Day (PLPUPD) refers to a dollar amount which is re-invested to the player. The calculation of this parameter is slightly more complex than the aforementioned. Firstly, base points per unit per day were calculated for each machine. When a player inserts their slot club membership card into a machine and begins to play, he or she earns base points for every dollar that is wagered. Sometimes players also earn promo points; these are offered as incentive to patronize the casino. Promo points are usually offered during a limited time period and their accrual rate is determined by a multiple of the base points by a pre-determined number. This marketing practice is commonplace in Las Vegas with casinos offering anywhere from 2× to 7× multipliers. Players can redeem points earned for meals, retail purchases, cash back and other offerings. For proprietary reasons, redemption rates and the actual multiplier used in the data cannot be revealed. However, an example will be provided. Let us assume that every dollar wagered earns a player one base point, i.e. $200 of coin-in is equal to 200 base points. Let us also assume that this particular day is a 7× multiplier day, i.e. $200 of coin-in is equal to 200 base points and 1,200 promo points for a total of 1,400 points. Different casinos have different rates of redemption; in this case, let us assume that 100 points is equal to a redemption value of $1.00. Assuming that this particular machine accumulated $200 of coin-in on this day, the liability to the casino would be $14.00 (1,400/100). PLPUPD is then calculated by dividing the total liability of a machine by the number of days the machine was on the floor during the six month period. This liability will vary across game type. Because each game type will not accumulate the same amount in coin-in, but also due to multiplier days being specific to certain categories.

Problem Statement

In this situation the decision maker's goal is to maximize the performance of the slot floor. The nineteen unique game categories, which incorporate differing styles and denominations, are displayed below in Table 3. As stated earlier, this casino continually makes changes to the slot floor. In order to get a representation of what the configuration looked like during the period, the average mix of slot machines was calculated. This was achieved by taking the average of the number of units of each category that were on the floor each day over the period. Table 3 below reflects the mix of the slot floor during the six month period. Also given are the values for CPUPD, WPUPD, and PLPUPD rounded to the nearest whole number for each game type.

Table 3

Average mix of slot machines during sample period

Category	Number of Machines	CPUPD	WPUPD	PLPUPD
$0.01 Reel Slots	234	908	123	10
$0.05 Reel Slots	8	374	46	4
$0.25 Reel Slots	50	915	82	9
$0.50 Reel Slots	11	865	76	9
$1.00 Reel Slots	42	1140	89	12
$5.00 Reel Slots	11	2554	205	24
$0.01 Video Slots	685	2040	316	22
$0.05 Video Slots	6	863	108	12
$0.01 Video Poker	6	4551	207	24
$0.05 Video Poker	66	3161	137	18
$0.25 Video Poker	86	4217	133	24
$1.00 Video Poker	7	2187	115	11
$0.01 Multi-Game	53	1849	117	15
$0.05 Multi-Game	275	1378	70	7
$0.25 Multi-Game	223	2410	90	12
$1.00 Multi-Game	15	9768	266	44
$5.00 Multi-Game	5	3962	114	22
$10.00 Multi-Game	2	10164	776	76
$0.05 Video Keno	27	579	47	4
Total Capacity	1812			

Stated simply, the decision maker's objective here is to maximize the performance of the slot floor by adjusting the mix of slot machines.

Model Development

There are two schools of thought with regards to measuring the performance of slot machines. Some managers swear by coin-in, where others focus more on win. Therefore two models have been proposed in this paper, one with the objective of maximizing total coin-in per day (CPD), the other maximizing total win per day (WPD) less total promo liability per day (PLD). These totals are calculated by summing the products of the number of machines and CPUPD, WPUPD, and PLPUPD respectively. Constraints for both models were identical, and were constructed based on the literature review and through discussions with management.

The first is a capacity constraint. Management had no intention of increasing the number of units on the floor. Therefore the first constraint requires that the total number of slot machines be less than or equal to 1,812.

The second set of constraints deal with the decision variables. Each category can be characterized as one decision variable, and the number of units in each category is the value of these variables. The model will attempt to maximize the objective function by finding optimal values for each decision variable. The casino cannot offer just one type of slot machine, as their patrons have different tastes, preferences, and discretionary income. An upper and lower bound for each game type was proposed by allowing for a 10% change from the current value. Management agreed this was appropriate. In cases where 10% would only alter the current number by a fraction, management was consulted and a greater upper and lower bound was determined. It is assumed that growth of coin-in and win within these upper and lower limits is linear (Rinne, Geurts, & Kelly, 1987). Table 4 presents the upper and lower bound constraints for the decision variables.

Table 4

Upper and Lower Bound Constraints for Decision Variables

Category	Lower Bound	Current Number of Machines	Upper Bound
$0.01 Reel Slots	211	234	257
$0.05 Reel Slots	6	8	10
$0.25 Reel Slots	45	50	55
$0.50 Reel Slots	9	11	13
$1.00 Reel Slots	38	42	46
$5.00 Reel Slots	9	11	13
$0.01 Video Slots	617	685	754
$0.05 Video Slots	5	6	7
$0.01 Video Poker	5	6	7
$0.05 Video Poker	59	66	73
$0.25 Video Poker	77	86	95
$1.00 Video Poker	6	7	8
$0.01 Multi-Game	48	53	58
$0.05 Multi-Game	248	275	303
$0.25 Multi-Game	201	223	245
$1.00 Multi-Game	14	15	17
$5.00 Multi-Game	3	5	7
$10.00 Multi-Game	2	2	4
$0.05 Video Keno	24	27	30

In an attempt to advance the model proposed by Rinne et al. (1987) and incorporate a cost variable, the third constraint involved promo liability. Management stipulated that this liability can be no greater than 30% of total win. In other words the casino did not want to reinvest any more than 30% of the total win generated by slot machines to their players. Non-negativity constraints were also included in the model which required the decision variables to take values greater than zero. The final constraint stipulates that each of the decision variables' values must be an integer. Obviously a fraction of a slot machine cannot be assigned on the casino floor. The integer constraint is discussed further in the following assumptions section.

The mathematical models are presented below, (1) the "Coin-in Model", and (2) the "Win Model".

(1) $$Max \ \sum_{i=1}^{19} c_i \times x_i$$

(2) $$Max \ \sum_{i=1}^{19} (w_i - p_i) \times x_i$$

subject to

$$\sum_{i=1}^{19} x_i \le M$$

$$L_i \le x_i \le U_i$$

$$\sum_{i=1}^{19} p_i x_i \le \left(\sum_{i=1}^{19} w_i x_i\right) \times 0.3$$

$$x_i > 0$$

x_i must be integers

where

x_i = the number of machines for category *i*, *i* = 1, 19

c_i = CIPUPD for category *i*

w_i= WPUPD for category *i*

p_i = PLPUPD for category *i*

M = the maximum number of total machines allowed on the floor

L_i = the minimum number of machines of category *i*

U_i = the maximum number of machines of category *i*

Assumptions

It is important to understand the basic assumptions of linear programming and how these relate to the task of configuring the slot floor mix. There are three assumptions of linear programming, (1) deterministic property, (2) divisibility, and (3) proportionality (Eiselt & Sandblom, 2007).

The first assumes that the problem's structure and all parameters in the model are known with certainty. The parameters that have been constructed, CPUPD, WPUPD, and PLPUPD, are not known with absolute certainty. This is the very nature of the casino business. There is variance in the performance of individual games due to the probabilities inherent in casino games. As Eiselt and Sandblom (2007) point out, by definition and with very few exceptions models deal with future events and hence include parameters that also relate to future events. The parameters included in the model serve as a proxy and attempt to account for the ambiguity of the slot floor's future performance. Certainly this problem possesses stochastic characteristics, but this does not mean that a deterministic model will not be beneficial.

The second assumption of divisibility means that each variable can be expressed as any real number, rather than solely integers. Clearly this assumption does not hold for this problem as it is impossible to assign a fraction of a slot machine. In these instances integer programming is applied (Choi, Hwang, & Park, 2009).

The final assumption requires that all functions in the model are linear. In this case it is assumed that the coin-in, win, and promo liability are proportional to the quantity of slot machines assigned. This relationship is thought to be a reasonable approximation of the dynamics of the slot floor within the ranges of the upper and lower bounds. Referring to Table 3, if one $0.01 Reel Slot machine is added to the floor we assume the CPUPD to increase by $908.00.

CHAPTER 6

RESULTS

Solving the Models

With the problem formulated and the mathematical models defined, solutions to the models were calculated. Excel 2007's Solver Add-in, a tool for optimization and equation solving, was used to solve the problems. There is a huge amount of software available for solving linear programs, and Excel Solver was selected as most suitable due to its wide use and availability in the hotel casino industry.

Solutions

Excel Solver found optimal solutions for both models while satisfying assumptions and constraints. Table 5 presents the solutions from both models compared to the original mix. Both solutions obey the maximum capacity constraint; therefore each solution assigns a total of 1,812 machines to the slot floor. If the coin-in model were adopted by the casino the new mix could potentially produce 3.91% more coin-in per day (CPD) than the original mix. With the win model, the casino can expect a total win per day (WPD) of $320,975 vs. a total win per day of $303,799 from the original mix (a 5.65% increase). Both solutions perform better than the original mix on all performance measures (Total CPD, Total WPD, and Total WPD – Total PLPD). Total PLPD was actually increased by the coin-in and win models, 2.99% and 3.58% respectively.

Table 5

Model Mix Solutions vs. Original Mix

	Original Mix	Coin-in Model Mix		Win Model Mix	
Category	Number of Machines	Number of Machines	% Change	Number of Machines	% Change
$0.01 Reel Slots	234	211	-9.83%	234	0.00%
$0.05 Reel Slots	8	6	-25.00%	6	-25.00%
$0.25 Reel Slots	50	45	-10.00%	45	-10.00%
$0.50 Reel Slots	11	9	-18.18%	9	-18.18%
$1.00 Reel Slots	42	38	-9.52%	38	-9.52%
$5.00 Reel Slots	11	13	18.18%	13	18.18%
$0.01 Video Slots	685	709	3.50%	754	10.07%
$0.05 Video Slots	6	5	-16.67%	5	-16.67%
$0.01 Video Poker	6	7	16.67%	7	16.67%
$0.05 Video Poker	66	73	10.61%	73	10.61%
$0.25 Video Poker	86	95	10.47%	77	-10.47%
$1.00 Video Poker	7	8	14.29%	6	-14.29%
$0.01 Multi-Game	53	48	-9.43%	48	-9.43%
$0.05 Multi-Game	275	248	-9.82%	248	-9.82%
$0.25 Multi-Game	223	245	9.87%	201	-9.87%
$1.00 Multi-Game	15	17	13.33%	17	13.33%
$5.00 Multi-Game	5	7	40.00%	3	-40.00%
$10.00 Multi-Game	2	4	100.00%	4	100.00%
$0.05 Video Keno	27	24	-11.11%	24	-11.11%
Total Machines	1812	1812	0.00%	1812	0.00%
Total CPD	$3,580,197	$3,720,334*	3.91%	$3,630,868	1.42%
Total WPD	$332,900	$341,093	2.46%	$351,118	5.47%
Total PLPD	$29,101	$29,970	2.99%	$30,144	3.58%
Total WPD - PLPD	$303,799	$311,123	2.41%	$320,975*	5.65%

Note. Values marked with an * denote objective functions.

Answer Analysis

Excel Solver provides an answer report when an optimal solution is achieved. The answer report provides the value for the objective function and the values for the decision variables; this information is already provided in Table 5. The answer report also provides status information and slack values for the constraints. The status classifies each constraint in the model as “binding” or “not binding”. A binding status means that the solution value is equal to that of the upper or lower limit of the constraint. A not binding status indicates that the solution value is not equal to its bound. The slack value is the difference between the decision variable’s solution and its bound; hence a constraint with a binding status will have a slack value of zero. A breakdown of each of the constraints will now be provided. The maximum capacity constraint will first be addressed, followed by the lower and upper bound category constraints. Finally, the promo liability constraint will be discussed.

The capacity constraint was binding for both models; each model utilized the capacity that was available. If the value of this constraint was increased to something more than 1,812 the models could potentially produce more favorable objective function values. This should be straightforward, as if we made the maximum capacity higher each additional machine would contribute more to the objective function.

Each category was assigned the constraint of an upper and lower bound. In the mathematical presentation of the models this appeared as one constraint; however Excel Solver only allows for the upper and lower bounds to be entered as two separate constraints. Therefore if the upper bound constraint was found to be binding for a particular category, the lower bound portion would obviously be non binding, and vice versa. Taking this into consideration, Table 6 presents a status as “Upper Binding” if the upper bound constraint is binding, and “Lower Binding” if the lower bound constraint is binding. This will help identify which categories have had machines added and which have had their floor space reduced. The models could produce higher objective functions if the values for these constraints were loosened.

Table 6

Status and Slack Values of Constraints

	Coin-in Model		Win Model	
Constraint	Status	Slack	Status	Slack
$0.01 Reel Slots	Lower Binding	0	**Not Binding**	**23, 23**
$0.05 Reel Slots	Lower Binding	0	Lower Binding	0
$0.25 Reel Slots	Lower Binding	0	Lower Binding	0
$0.50 Reel Slots	Lower Binding	0	Lower Binding	0
$1.00 Reel Slots	Lower Binding	0	Lower Binding	0
$5.00 Reel Slots	Upper Binding	0	Upper Binding	0
$0.01 Video Slots	**Not Binding**	**92, 45**	Upper Binding	0
$0.05 Video Slots	Lower Binding	0	Lower Binding	0
$0.01 Video Poker	Upper Binding	0	Upper Binding	0
$0.05 Video Poker	Upper Binding	0	Upper Binding	0
$0.25 Video Poker	Upper Binding	0	Lower Binding	0
$1.00 Video Poker	Upper Binding	0	Lower Binding	0
$0.01 Multi-Game	Lower Binding	0	Lower Binding	0
$0.05 Multi-Game	Lower Binding	0	Lower Binding	0
$0.25 Multi-Game	Upper Binding	0	Lower Binding	0
$1.00 Multi-Game	Upper Binding	0	Upper Binding	0
$5.00 Multi-Game	Upper Binding	0	Lower Binding	0
$10.00 Multi-Game	Upper Binding	0	Upper Binding	0
$0.05 Video Keno	Lower Binding	0	Lower Binding	0

The final constraint stipulated that the total PLPD could be no larger than 30% of the total expected win generated by the proposed mix. The constraint was not binding for both models. The coin-in model produced a slack value of $72,358, and the win model $75,192. We can interpret these slack values as a remaining budget for the casino. The casino had stated that they would be willing to reinvest up to 30% of total WPD back to their players, however both models produce reinvestment rates substantially below 30%. In fact, the coin-in model reinvests only 8.79% of WPD, and the win model only 8.59%. Essentially management can expect to have a significant surplus in their marketing budget, whichever model is adopted.

Sensitivity Analysis

In addition to the answer report, Excel Solver also produces a sensitivity analysis. This report supplies information regarding the effects of changes in the objective function coefficients and constraints. For example the coin-in model proposed that $0.01 Reel Slots be reduced from 234 machines to the lower bound of 211 machines. We may be interested in finding out how much this category's CPUPD needs to increase before we begin to add these machines to the floor. However, this report is meaningless for integer programming problems. This is due to a concept known as *duality*, and fails in integer programming (Williams, 1999). In this case, however, dropping the integer constraint, thus formulating a traditional linear program, has no effect on the solution. We can therefore formulate both problems, dropping the integer constraints, as linear programs and perform sensitivity analysis.

The integer program solution is equal to the linear program solution because all the corner points of the set of feasible solutions are integer valued. Looking at the upper and lower limit constraints for each category, we can see that all but one category in each model has been driven to the upper or lower bound during the optimization process. Due to the upper and lower bound values being integers, it should be clear why the linear program solution is equivalent to the integer program solution. Hypothetically, if there were another constraint in the model which prevented the decision variables being driven to the upper and lower bounds the integer program solution would most likely differ from the linear program solution.

The sensitivity analysis for the coin-in model is presented in Table 7, all numbers have been rounded to the nearest whole number. 1E+30 denotes infinity.

Table 7

Coin-in Model Sensitivity Analysis

Decision Variables

Category	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$0.01 Reel Slots	211	-1132	908	1132	1E+30
$0.05 Reel Slots	6	-1666	374	1666	1E+30
$0.25 Reel Slots	45	-1125	915	1125	1E+30
$0.50 Reel Slots	9	-1175	865	1175	1E+30
$1.00 Reel Slots	38	-900	1140	900	1E+30
$5.00 Reel Slots	13	513	2554	1E+30	513
$0.01 Video Slots	709	0	2040	147	191
$0.05 Video Slots	5	-1178	863	1178	1E+30
$0.01 Video Poker	7	2510	4551	1E+30	2510
$0.05 Video Poker	73	1121	3161	1E+30	1121
$0.25 Video Poker	95	2177	4217	1E+30	2177
$1.00 Video Poker	8	147	2187	1E+30	147
$0.01 Multi-Game	48	-191	1849	191	1E+30
$0.05 Multi-Game	248	-662	1378	662	1E+30
$0.25 Multi-Game	245	370	2410	1E+30	370
$1.00 Multi-Game	17	7728	9768	1E+30	7728
$5.00 Multi-Game	7	1921	3962	1E+30	1921
$10.00 Multi-Game	4	8124	10164	1E+30	8124
$0.05 Video Keno	24	-1461	579	1461	1E+30

Constraints

Name	Final Value	Shadow Price	Constraint R. H. Side	Allowable Increase	Allowable Decrease
Total PLPD	29970	0	102328	1E+30	72358
Total Capacity	1812	2040	1812	45	92

The reduced cost column presents values which are non zero for those decision variables whose values were driven to the bound of the constraint during the optimization process (Williams, 1999). This means that moving the decision variable's final value away from the bound will exacerbate the objective function; whereas widening the range of the constraint will improve the objective function. With this in mind, the reduced cost represents the change in the objective function per unit increase in the decision variables' values. For example, $0.01 Reel Slots have a reduced cost of -$1,132, meaning that if a reel slot machine was added to the floor, and therefore another machine type taken off, the objective function would decrease by $1,132. The allowable increase value tells us that if the objective coefficient (CPUPD) increased by an amount more than $1,132 the model may then begin to add $0.01 Reel Slots to the floor. If the CPUPD decreased we would still continue to reduce the number of $0.01 Reel Slots, indicated by the allowable decrease of infinity (1E+30).

The shadow price for the capacity constraint tells us that if we were to allow one more machine onto the floor the model could increase total CPD (the objective function) by $2,040. This would hold true up to an additional 45 machines. In effect the model would be adding $0.01 Video Slots, as it is this category that has the next best contribution to the objective function after those that have been driven to their upper bounds.

The total PLPD constraint, could be tightened by up to $72,358 before the objective function value would change. Essentially this means that the reinvestment rate could be increased anywhere up to 30 per cent without having an effect on the objective function (total CPD).

The sensitivity analysis for the win model is now presented in Table 8, followed by a brief analysis of the report. Again, values have been rounded to the nearest whole number.

Table 8

Win Model Sensitivity Analysis

Decision Variables

Category	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$0.01 Reel Slots	234	0	113	6	4
$0.05 Reel Slots	6	-71	42	71	1E+30
$0.25 Reel Slots	45	-41	72	41	1E+30
$0.50 Reel Slots	9	-46	67	46	1E+30
$1.00 Reel Slots	38	-37	76	37	1E+30
$5.00 Reel Slots	13	68	181	1E+30	68
$0.01 Video Slots	754	181	294	1E+30	181
$0.05 Video Slots	5	-16	97	16	1E+30
$0.01 Video Poker	7	70	183	1E+30	70
$0.05 Video Poker	73	6	119	1E+30	6
$0.25 Video Poker	77	-4	110	4	1E+30
$1.00 Video Poker	6	-9	104	9	1E+30
$0.01 Multi-Game	48	-11	102	11	1E+30
$0.05 Multi-Game	248	-50	63	50	1E+30
$0.25 Multi-Game	201	-35	78	35	1E+30
$1.00 Multi-Game	17	109	222	1E+30	109
$5.00 Multi-Game	3	-20	93	20	1E+30
$10.00 Multi-Game	4	587	700	1E+30	587
$0.05 Video Keno	24	-70	43	70	1E+30

Constraints

Name	Final Value	Shadow Price	Constraint R. H. Side	Allowable Increase	Allowable Decrease
Total PLPD	30144	0	105335	1E+30	75192
Total Capacity	1812	113	1812	23	23

The win model could increase total WPD by $113 (shadow price) for every extra machine added to capacity. Effectively the model would begin to add $0.01 Reel Slots to the floor as they are the next best performing category after those which have been driven to their upper limits. Once again, there is a considerable amount by which the total PLPD constraint can be tightened without affecting the solution. If the reinvestment rate is increased anywhere up to 30 per cent, the decision variables' values remain the same, consequently so to does total WPD. However, the objective function value would show a decrease as the reinvestment rate increases. This is because the objective function is determined by subtracting total PLPD from total WPD.

CHAPTER 7

DISCUSSION

Theoretically, if either of the two proposed models were adopted, the casino can expect to improve the performance of their slot operations. The linear programming routine evaluates the expected contributions from each game category, and proposes a machine mix to maximize the slot floor's potential. The research offers a more scientific approach to the task at hand, and lays the foundation for more macro-level analysis of slot operations.

Managerial Implications

Both the coin-in and win model outperform the original mix. Management must be cautious when deciding which model to adopt. The decision can be related to the argument regarding revenue vs. profit. Coin-in is an important performance measure; however management must read this data with caution. Only a portion of coin-in is actually retained by the casino. The win model's objective function (total WPD – total PLPD) can be read with somewhat more confidence, as this number takes into account the machines' par value and also promotional liabilities. Rather than adopting just one of the models, management may consider comparing the proposals made. The results show that the models' mix recommendations differ in only six out of the nineteen categories. A deeper analysis into these discrepancies is advised. For example, the $5.00 Multi Game category ranks high in terms of coin-in and was consequently increased by the coin-in model. But the win model did not consider this a top performing category and subsequently reduced its share of the slot floor.

Promo liability, as discussed, had little effect on the optimal solution. However, upon further analysis this constraint does highlight areas for consideration. The $0.25 Video Poker category was ranked highly in terms of win (seventh out of nineteen), but the model reduced the number of these machines. In fact, even if the total floor capacity was increased, the model would still not add $0.25 Video Poker machines to the floor, as the reduced cost is -$4.00. The PLPUPD may be of some concern here. The reinvestment rate for this category is around 18 per cent, roughly twice as high as the floor average. This is also important to consider given the fact the coin-in model added machines to this category. Either solution should not be taken at face value, but deeper analysis and comparisons between models is desirable.

Although in the results it was stated that the dual values (reduced costs and shadow prices) represent opportunities for the casino to increase performance, these values should be read with vigilance. Reckless loosening of constraints in an attempt to produce more favorable objective functions should be avoided. The upper and lower constraints on the decision variables have been constructed with linearity in mind. The per unit increase (or decrease) is expected to be constant, within the constraint's range. For instance, $10.00 Multi Games have a significant reduced cost; $8,124 per unit increase in CPD, and $587 per unit increase in WPD (less PLPD). Management cannot simply continue adding these machines to the floor, as supply will more than likely offset demand. This is especially true at Las Vegas repeater market casinos, which attract fewer high limit players than their competitors on the Strip.

The shadow prices present some opportunity for growth. The casino may consider increasing the capacity of its slot floor. Assuming linearity, the win model suggests adding 23 $0.01 Reel Slots, and the coin-in model 92 $0.01 Video Slots. The win model solution drove $0.01 Video Slots to the upper bound, where as the coin-in model reduced the number of $0.01 Reel Slots. This once again highlights the importance of careful comparison and analysis of the solutions. Management must also consider if there is sufficient demand to account for any increase in capacity.

The deterministic nature of the models should be addressed. The solutions do not take into account future variations. It is therefore recommended that this process be carried out on a regular basis. For example, management may adopt the mix proposed by the win model. After four to twelve weeks, management should compute new comparable performance parameters (CPUPD, WPUPD, and PLPUPD) for the period and repeat the linear programming routine. These results may help shed light on which categories have become over-supplied and under-supplied. This system may also help to affirm (or challenge) the assumption of linearity.

The results also point out that the reinvestment rate is substantially lower than what management is willing to permit. Solid recommendations cannot be made without detailed financial data and targets for the property. However, this is certainly an area of further consideration for the casino.

With the outlook of server-based gaming (SBG) promising, the routine proposed here will certainly help exploit the technology’s potential. SBG is labeled with the ability to more efficiently manage the slot floor. However, as Lucas and Kilby (2008) recognize, technological innovations and solutions are abundant but are not synonymous with analytical techniques. In the future, the research may be utilized in collaboration with SBG to develop the routine and exploit the technology.

Limitations

This research has been carried out at one Las Vegas repeater market hotel casino. Clearly this exact routine cannot be replicated. Different casinos offer different categories of slot machines, and operate in different markets. However, the general procedure is transferable and can certainly be tailored to the specific needs of other properties.

The time period for which data was gathered limited the model development. Rinne, Geurts, & Kelly (1987) were able to gain access to monthly data for a twelve month period, and were therefore able to produce solutions that accounted for monthly variations. Although Rinne et al. (1987) did not account for the uncertainty in future variables, they were able to produce somewhat of a dynamic system (using deterministic means) for allocating floor space in a retail setting.

A more detailed data set would have been desirable and allowed for the development of a more complex model with additional parameters. Although successful and useful, the small number of constraints could limit the strength of the models.

Important limitations can also be drawn from the assumptions of linear programming. The models are deterministic (the antonym for deterministic is probabilistic or stochastic). As stated in the assumptions, linear programming assumes all the parameters of a problem to be known with certainty. Future demand is uncertain, therefore so too are the parameters included in the models. The assumption of linearity also poses an important limitation, as it is not known whether functions in the problems are in fact linear or if linearity is a reasonable assumption to make. This assumption was in fact considered to be realistic based on the constraints constructed with management. Nevertheless, the use of managerial judgment is in itself a limitation of the study.

Recommendations for Future Research

Reproduction of the research at a different property would test the robustness of the proposed system, and could help advance the formulation of a more generalizable programming routine. Research carried out at different properties and in different markets may also help identify the dissimilarity in casino patrons' slot machine preferences.

Any further research on the problem should also attempt to obtain a richer data set. A more detailed data set will allow for the formulation of a more complex problem that may generate stronger results. Particularly, variables which have been identified in the performance potential research (Lucas & Dunn, 2005; Lucas, Dunn, Roehl, & Wolcott, 2004; Lucas & Roehl, 2002; Lucas, Singh, Gewali, & Singh, 2009), that have been proven to influence unit level performance variation would be desirable additions. Also, a data set whereby seasonal variations could be identified would be beneficial. This would allow for a more dynamic solution, analogous to that proposed by Rinne et al. (1987).

Future studies should attempt to validate the assumptions of linear programming as they pertain to the slot mix problem. This future research may prove the assumptions to be unrealistic. Mathematical programming techniques that take into account uncertainty and non-linearity should then be pursued; namely stochastic programming and non-linear programming. Another pitfall of linear programming, as it pertains to this problem, is the single objective. Slot floor managers may have to consider several objectives and targets to meet their goals. A research opportunity may exist investigating the multi-objective technique known as goal programming.

The relationships between slot machine categories is another area of research which should be addressed. For example, what is the effect of adding machines to a certain category on the performance of an individual machine within that category? Conclusions regarding these relationships would be of great use when formulating the slot mix problem, particularly when constructing upper and lower bound constraints for each category.

In light of the lack of academic interest regarding macro-level analysis of slot operations, any research with this focus in mind would also be valuable.

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