Alternative estimation methods for survival models

John William Trabert

University of Nevada, Las Vegas

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ALTERNATIVE ESTIMATION METHODS
FOR SURVIVAL MODELS

by

John William Trabert, Jr.

A thesis submitted in partial fulfillment
of the requirements for the degree of

Masters of Science

in

Mathematics

Department of Mathematical Sciences
University of Nevada, Las Vegas
August 1996
The Thesis of John W. Trabert, Jr. for the degree of Masters of Science in Mathematics is approved.

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Graduate Faculty Representative, Laxmi Gewali, Ph.D.

Dean of the Graduate College, Ronald W. Smith, Ph.D.

University of Nevada, Las Vegas
August 1996
ABSTRACT

In this thesis, we will address the problem of estimating the parameters of two types of three parameter survival models for the force of mortality, the Makeham model and the Weibull model. The existing methods for estimating these parameters by the method of least-squares include using a log-transformation on the force of mortality data. We will propose another method of estimating these parameters that uses linear regression, and finding the least-squares estimates of the parameters by using the golden section search procedure for computer minimization.
ACKNOWLEDGMENTS

I would like to thank Dr. Rohan Dalpatadu and Dr. Peter Shiue. Their guidance and patience has proved to be invaluable.

I would also like to thank John W. Trabert, Sr., Dave and Judy Lathrop, the Pack, Barley Pops II and Bryn, Buck R. Rambo, Michael Palmieri, and Marie Palmieri. To leave it behind.

This thesis is dedicated to the memory of

Cecilia DeVonne Trabert
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CHAPTER 1
INTRODUCTION

In the field of actuarial mathematics, the force of mortality is defined as,

\[ \mu(t) = \frac{f(t)}{1 - F(t)}, \]

where \( f(t) \) is the probability density function and \( F(t) \) is the cumulative distribution function. For the popular three parameter survival models, the Makeham and the Weibull models, there are existing methods for estimating the parameters by the least-squares method. We will propose an alternative method that not only estimates these parameters by the least-squares method, but will produce a better fit than the existing methods.

Most general forms of \( \mu(t) \) involve three parameters. The two models considered in this paper are the Makeham model, with

\[ \mu(t) = A + Bc^t, \quad B > 0, \quad c > 1, \quad A > -B, \quad t \geq 0, \]

and the Weibull model, with

\[ \mu(t) = k(t - \delta)^n, \quad k > 0, \quad \delta > 0, \quad n > 0, \quad t \geq \delta. \]

Given an ordered sample, \( t_1 < t_2 < \ldots < t_n \), of size \( n \)
from a population with mortality rate $\mu(t)$, we will analyze the estimation of these parameters for both the Makeham model and the Weibull model.

The existing method for the Makeham model is known as Nesselle's Method and it involves estimating the parameters by generating a sequence of Taylor series approximations. For the Weibull model, a trial and error method is used to estimate $\delta$, a log transformation and linear regression to evaluate the remaining parameters.

In our alternative method, a procedure known as the golden section search algorithm is used to estimate the parameters. The golden section search algorithm is a method that will find the minimum of a function on an interval without the use of derivatives [6].

Suppose $g(x)$ is continuous on $[a,b]$ and has a minimum at $c \in [a,b]$. Let

$$h = b - a, \quad x_1 = b - \gamma h, \quad x_2 = a + \gamma h, \quad \gamma = \frac{\sqrt{5} - 1}{2};$$

where $\gamma$ is known as the golden ratio. If $g(x_1) < g(x_2)$, then set $b = x_2$, else set $a = x_1$. Repeat this method until $h$ is less than the given tolerance. The last computed value of $x$, $x_1$ or $x_2$, is the value $c$ at which $g(x)$ attains its minimum.
The alternative method [5] will use this algorithm by computing the sum-square error after fixing one of the parameters. For the Makeham model, \( c \) is fixed so that an equation for \( \text{SSE}(c) \) is used in the golden section search method. For the Weibull model, \( \delta \) is fixed so that an equation for \( \text{SSE}(\delta) \) is used in a similar method.

In the following chapters, a full description of the methods will be described. First, there will be detailed information on how to generate the data sets for the Makeham and Weibull models. After the estimation methods are described, some examples will show how the new method will work. An analysis of the alternative procedure will show that this new method produces better results than the existing methods. Any log that is used throughout this paper is base \( e \).
CHAPTER 2
SIMULATING THE DATA SETS

In the following section, a description of how the data sets were simulated for the examples used to test the program will be given. A common tool in simulation is the inverse transform method, and it was used only on the Weibull model. For the Makeham model, there is another method used, because the integral of this model does not have an inverse in closed form.

Weibull Model

For the Weibull model, \( \mu(t) = k(t - \delta)^n \), the inverse exists, and can be used to simulate the data. First, take \( F(t) = 1 - e^{-\int \mu(t) dt} \) [1], which is the defined as the relationship between the force of mortality and its cumulative distribution function, and solve this equation for \( \int \mu(t) dt \). Then,

\[
\log\left[ \frac{1}{1 - F(t)} \right] = \frac{k(t - \delta)^{n+1}}{n + 1}.
\]

Now, by finding the inverse to this equation, the following equation is obtained,
\[ t = \delta + \left\{ \left[ \frac{n + 1}{k} \right] \log \left( \frac{1}{1 - u} \right) \right\}, \]

where \( u \) is a random number generated by the computer that represents \( F(t) \), and the constants \( k, n, \) and \( \delta \) are entered by the user. After the user enters the number of data points to generate, a bubble sort procedure is used to order the data set. The sets are then placed into a data file. The algorithm will also report the first data point, \( t_1 \), so that when the alternative method is used, the search interval entered will not include the first data point. If this occurs, then the program will stop, due to an error in the domain of \( \log(t - \delta) \).

**Makeham Model**

For the Makeham model, \( \mu(t) = A + Bc^t \), the distribution function is defined as \( F(t) = 1 - e^{-\int \mu(t) \, dt} \). When this is solved for \( \int \mu(t) \, dt \), then

\[ \log \left[ \frac{1}{1 - F(t)} \right] = At + \frac{B(c^t - 1)}{\log c}. \]

This equation cannot be solved for \( t \), and thus the inverse transform method cannot be used here. So, Newton's method is used to simulate the data points.
By using Newton's method [2] on the cumulative distribution function, $F(t)$, the data sets for the Makeham model can be generated, given $A$, $B$, and $c$. Let

$$t_{n+1} = t_n - \frac{F(t)}{F'(t)},$$

and since,

$$F(t) = 1 - \exp \left[ - \int \mu(t) dt \right],$$

then the following results are obtained using the Makeham model,

$$F(t) = 1 - \exp \left\{ - At + \frac{B(c^t - 1)}{\log c} \right\},$$

$$F'(t) = (A + Bc^t) \exp \left\{ - At + \frac{B(c^t - 1)}{\log c} \right\}.$$

Thus, by using these equations for Newton's method, the data sets can be generated for the Makeham model. A seed must be given, $t_0$, so that each data point can be generated. For the examples that are presented later, the seed value was one. After generating the data points, the data set is sorted in ascending order. Then, the data set is placed into a file. For the Makeham model, it is not necessary to report the first data point, because the only restriction on the data set is
that it is greater than zero, unlike the Weibull model, which the simulated points for $t$ depend on the value of $\delta$. 

CHAPTER 3

ESTIMATING THE THREE PARAMETERS
FOR THE MAKEHAM MODEL

The Makeham model is used to describe human survival. In the following chapter, there will be a detailed description of the existing method for measuring the parameters for this model, given by London. Then, the new method for this model will be described, and it will make use of a procedure known as the golden section search algorithm.

Existing Method

The Makeham model for measuring the force of mortality is

\[ \mu(t) = A + Bc^t, \quad t > 0, \quad B > 0, \quad c > 1, \quad A > -B. \]

The existing method, known as Nesselle's Method, is described in London [3], but will be explained here as well. Given an ordered sample, \( t_1 < ... < t_n \), of size \( n \) and if \( U_i = F(t_i) \) is the \( i^{th} \) ordered observation from this sample, then \( E(U_i) = \frac{i}{n + 1} \), and \( F(t_i) \) can be
estimated by using \( \hat{F}(t_i) = \frac{i}{n + 1} \), or \( \hat{F}(t_i) = \frac{i - 0.3}{n + 0.4} \). It is also known that

\[
\hat{\mu}(t_i) = \frac{F(t_{i+1}) - F(t_i)}{(t_{i+1} - t_i)(1 - F(t_i))} = \frac{1}{(t_{i+1} - t_i)(n - l + i)}.
\]

Now that \( \mu(t_i) \) has been estimated, we can use Nesselle's Method to fit the Makeham model.

First, compute the midpoint of \( (t_i, t_{i+1}) = m_i \), by entering the initial values for \( A \), \( B \) and \( c \), the approximation of \( \mu_0(t_i) = A_0 + B_0c_0^m_i \) can be evaluated. By using a Taylor series expansion, then

\[
\mu(t_i) = \mu_0(t_i) + (A - A_0) \frac{\partial \mu}{\partial A} + (B - B_0) \frac{\partial \mu}{\partial B} + (c - c_0) \frac{\partial \mu}{\partial c},
\]

where the partial derivatives are evaluated at \( A_0 \), \( B_0 \) and \( c_0 \), and the partial derivatives are as follows,

\[
\frac{\partial \mu}{\partial A} = 1, \quad \frac{\partial \mu}{\partial B} = ct, \quad \frac{\partial \mu}{\partial c} = Btc^{t-1}.
\]

Let \( d_A = A - A_0 \), \( d_B = B - B_0 \), and \( d_c = c - c_0 \). Also, find the partial derivatives at \( A_0 \), \( B_0 \), and \( c_0 \). Then, the sum-square error is defined as,

\[
SSE = \sum_{i=1}^{n} \left[ \mu(t_i) + d_A + \left( d_B * \frac{\partial \mu}{\partial B_m} \right) + \left( d_c * \frac{\partial \mu}{\partial c_m} \right) - \hat{\mu}(t_i) \right]^2.
\]

The method of least-squares is used to find the
derivatives with respect to \( d_A, d_B, \) and \( d_C \), the normal equations are found and a 3 by 3 system of equations are created. When solved, the estimators for \( d_A, d_B, \) and \( d_C \) are found.

Then, these estimates are added to the initial estimates of \( A_0, B_0, \) and \( C_0 \), to obtain new estimators \( A_1, B_1, \) and \( C_1 \). These values are replaced back into \( \mu_0(t_i) \) so that new estimates for \( \mu(t_i) \) are found. By continuing this process, the estimates for \( A, B, \) and \( c \) will get better by giving a small sum-square error.

Then, compute the sum-square error for the new estimates. Continue to obtain improved estimates of \( A, B, \) and \( c \), until the method converges to the least square estimators, \( \hat{A}, \hat{B}, \) and \( \hat{c} \).

The Alternative Method

In the alternative method, the golden section search algorithm is used to estimate the parameters for the Makeham model, \( A, B, \) and \( c \). First, by fixing the parameter \( c \), \( \int \mu(t)dt \) is then converted into a linear form and the other parameters are found by the least-squares method. The golden section search method is then used on the fixed parameter by minimizing the sum square error.
Suppose that
\[ y(t) = \int \mu(t) dt. \]

Then,
\[ y(t_i) = \log \left( \frac{1}{1 - F(t_i)} \right), \]

where \( F(t_i) \) is replaced by
\[ \hat{F}(t_i) = \frac{i}{n + 1}, \]
or with a more robust operator,
\[ \hat{F}(t_i) = \frac{i - 0.3}{n + 0.4}. \]

Thus,
\[ \log \left( \frac{1}{1 - \hat{F}(t_i)} \right) = Ai + \frac{B(c^{t_i} - 1)}{\log c}. \]

It is easy to see that,
\[ y(t_i) = Ai + \frac{B(c^{t_i} - 1)}{\log c}, \]
is in the form of
\[ Y(t) = AX_1 + BX_2, \]

where \( X_1 = t \) and \( X_2 = \frac{c^t - 1}{\log c} \). Now, the golden section search algorithm can be used to find the estimates for the parameters \( A, B, \) and \( c. \)

First, choose an initial search interval for the
fixed parameter \( c \). Call this interval \([L, H]\) where \( L > 1 \).

Second, for a fixed \( c_1 \) and \( c_2 \), fit a linear regression line to the points \((X_1(t_i), X_2(t_i), Y(t_i))\) and obtain the corresponding least-squares estimates, \( A_c \) and \( B_c \), for \( c_1 \) and \( c_2 \). The values of \( c_1 \) and \( c_2 \) are found by taking the length of \([L, H]\) and multiplying it by the golden ratio, \( \gamma = \frac{\sqrt{5} - 1}{2} \). Then, \( c_1 \) is found by subtracting this product from \( H \), and \( c_2 \) is found by adding this product to \( L \).

If we suppress the argument \( t_i \) for the independent and dependent variables, then the sum-square error formula for the Makeham model is then defined as,

\[
SSE(c) = \sum (Y - A_c X_1 - B_c X_2)^2.
\]

To find the least-squares estimates for \( \hat{A}_c \) and \( \hat{B}_c \), take the partial derivatives of this formula with respect to \( A \) and \( B \), and obtain the following two equations,

\[
\frac{\partial SSE(c)}{\partial A} = 2 \sum [Y - A_c X_1 - B_c X_2](-X_1),
\]

\[
\frac{\partial SSE(c)}{\partial B} = 2 \sum [Y - A_c X_1 - B_c X_2](-X_2).
\]

After these equations are set equal to zero, the following system of equations are found.
\[ A_c \sum X_1^2 + B_c \sum X_1 X_2 = \sum X_1 Y \]
\[ A_c \sum X_1 X_2 + B_c \sum X_2^2 = \sum X_2 Y \]

When this system is solved, the least-square estimates are as follows.

\[ \hat{A}_c = \frac{(\sum x_2 y)(\sum x_1 x_2) - (\sum x_1 y)(\sum x_2^2)}{(\sum x_1 x_2)^2 - (\sum x_1^2)(\sum x_2^2)} \]
\[ \hat{B}_c = \frac{(\sum x_1 y)(\sum x_1 x_2) - (\sum x_2 y)(\sum x_1^2)}{(\sum x_1 x_2)^2 - (\sum x_1^2)(\sum x_2^2)} \]

Then, find the sum-square error for \( c_1 \) and \( c_2 \), where

\[ SSE(c_1) = \sum_{i=1}^{n} \left[ y(t_i) - A_c X_1(t_i) - B_c X_2(t_i) \right]^2 \]
\[ SSE(c_2) = \sum_{i=1}^{n} \left[ y(t_i) - A_c X_1(t_i) - B_c X_2(t_i) \right]^2 \]

If \( SSE(c_1) < SSE(c_2) \), then set \( H = c_2 \). If \( SSE(c_1) > SSE(c_2) \), then set \( L = c_1 \), and the process begins again with a new search interval for \( c \), either \([L, c_2]\) or \([c_1, H]\).

As the algorithm continues and the search interval decreases in size, the method will find a value for \( c \) which minimizes the sum-square error for the Makeham model. Lastly, the least-squares estimates, \( \hat{A}_c \) and \( \hat{B}_c \), are calculated when \( \hat{c} \) has been found. Notice as the search interval decreases in size, only one new value, \( c_1 \)
or \( c_2 \), is computed due to the properties of the golden ratio. There are three samples of this procedure that will follow.

**Three Examples**

After simulating the data sets for one of the three examples, Nesselle's method was run with the data set and then the alternative method was run with the same data set. This was necessary before another simulated data set was run so that no information was lost. The data sets are read from left to right. The results for the examples using the alternative method will appear in Appendix I.

1. The first example simulates 45 data points with the given parameters \( A = 0.01, B = 0.01, \) and \( c = 1.0265 \).

\[
\begin{align*}
0.4095, & \quad 0.5274, & \quad 1.1136, & \quad 3.4926, & \quad 5.2796, & \quad 9.5114, \\
10.6255, & \quad 11.6233, & \quad 13.2095, & \quad 15.2599, & \quad 15.7811, & \quad 17.7177, \\
18.3986, & \quad 18.5886, & \quad 20.4102, & \quad 20.7241, & \quad 20.9972, & \quad 25.0851, \\
25.6236, & \quad 26.3535, & \quad 29.0354, & \quad 29.4645, & \quad 30.8951, & \quad 33.9568, \\
36.1071, & \quad 36.5943, & \quad 37.1334, & \quad 38.6673, & \quad 39.1088, & \quad 41.1252, \\
41.2528, & \quad 45.3060, & \quad 47.1332, & \quad 47.2031, & \quad 48.1147, & \quad 50.0118, \\
57.0683, & \quad 58.2514, & \quad 63.2000, & \quad 66.2405, & \quad 67.8248, & \quad 69.8141, \\
77.5625, & \quad 86.2595, & \quad 89.6539.
\end{align*}
\]

With the less robust operator:

<table>
<thead>
<tr>
<th></th>
<th>Nesselle's Method</th>
<th>Alternative Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated A:</td>
<td>0.05264</td>
<td>-0.03136</td>
</tr>
<tr>
<td>Estimated B:</td>
<td>0.02610</td>
<td>0.04674</td>
</tr>
<tr>
<td>Estimated c:</td>
<td>1.00039</td>
<td>1.00894</td>
</tr>
<tr>
<td>Estimated SSE:</td>
<td>1.31905</td>
<td>0.22331</td>
</tr>
<tr>
<td>Estimated R-Squared:</td>
<td>0.99827</td>
<td>0.99950</td>
</tr>
</tbody>
</table>
With the more robust operator:

<table>
<thead>
<tr>
<th></th>
<th>Nesselle's Method</th>
<th>Alternative Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated A:</td>
<td>0.08782</td>
<td>0.01540</td>
</tr>
<tr>
<td>Estimated B:</td>
<td>0.00216</td>
<td>0.00444</td>
</tr>
<tr>
<td>Estimated c:</td>
<td>1.00164</td>
<td>1.03654</td>
</tr>
<tr>
<td>Estimated SSE:</td>
<td>1.38177</td>
<td>0.55333</td>
</tr>
<tr>
<td>Estimated R-Squared</td>
<td>0.99769</td>
<td>0.99945</td>
</tr>
</tbody>
</table>

Nesselle's Method used the initial estimates of A = 0.015, B = 0.005, and c = 1.03. The initial interval for c in the alternative method is [1.0001, 1.05].

2. The second example simulates 50 data points with the given parameters A = 0.009, B = 0.006, and c = 1.0623.


With the less robust operator:

<table>
<thead>
<tr>
<th></th>
<th>Nesselle's Method</th>
<th>Alternative Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated A:</td>
<td>0.09102</td>
<td>0.00138</td>
</tr>
<tr>
<td>Estimated B:</td>
<td>0.02368</td>
<td>0.00862</td>
</tr>
<tr>
<td>Estimated c:</td>
<td>1.00595</td>
<td>1.05191</td>
</tr>
<tr>
<td>Estimated SSE:</td>
<td>2.75025</td>
<td>0.18420</td>
</tr>
<tr>
<td>Estimated R-Squared:</td>
<td>0.99040</td>
<td>0.99966</td>
</tr>
</tbody>
</table>

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With the more robust operator:

<table>
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<tr>
<th></th>
<th>Nesselle's Method</th>
<th>Alternative Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated A:</td>
<td>0.09632</td>
<td>0.01218</td>
</tr>
<tr>
<td>Estimated B:</td>
<td>0.02251</td>
<td>0.00259</td>
</tr>
<tr>
<td>Estimated c:</td>
<td>1.00766</td>
<td>1.07942</td>
</tr>
<tr>
<td>Estimated SSE:</td>
<td>2.88711</td>
<td>0.54408</td>
</tr>
<tr>
<td>Estimated R-squared:</td>
<td>0.98827</td>
<td>0.99956</td>
</tr>
</tbody>
</table>

Nesselle's Method used the initial estimates of $A = 0.01$, $B = 0.01$, and $c = 1.02$. The initial search interval for $c$ in the alternative method was $[1.004, 1.08]$.

3. The third example will simulate 70 data points with the parameters $A = 0.004$, $B = 0.008$, and $c = 1.0413$.


With a less robust operator:

<table>
<thead>
<tr>
<th></th>
<th>Nesselle's Method</th>
<th>Alternative Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated A:</td>
<td>0.07945</td>
<td>0.01137</td>
</tr>
<tr>
<td>Estimated B:</td>
<td>0.03538</td>
<td>0.00543</td>
</tr>
<tr>
<td>Estimated c:</td>
<td>1.00335</td>
<td>1.04104</td>
</tr>
<tr>
<td>Estimated SSE:</td>
<td>1.60695</td>
<td>0.42955</td>
</tr>
<tr>
<td>Estimated R-Squared:</td>
<td>0.99504</td>
<td>0.99950</td>
</tr>
</tbody>
</table>

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<tr>
<th></th>
<th>Nesselle's Method</th>
<th>Alternative Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated A:</td>
<td>0.08479</td>
<td>0.01626</td>
</tr>
<tr>
<td>Estimated B:</td>
<td>0.03181</td>
<td>0.00243</td>
</tr>
<tr>
<td>Estimated c:</td>
<td>1.00485</td>
<td>1.05641</td>
</tr>
<tr>
<td>Estimated SSE:</td>
<td>1.64245</td>
<td>0.41393</td>
</tr>
<tr>
<td>Estimated R-Squared:</td>
<td>0.99326</td>
<td>0.99978</td>
</tr>
</tbody>
</table>

Nesselle's Method used the initial estimates of

A = 0.005, B = 0.01, and c = 1.03. The initial search interval for c in the alternative method is [1.001, 1.1].
CHAPTER 4

THE ALTERNATIVE METHOD AND

THE WEIBULL MODEL

Existing Method

For the Weibull model, the existing method for estimating the parameters \( k, n, \) and \( \delta \), involves choosing a value for \( \delta \), and then finding the least-squares estimates for the remaining parameters, and \( \delta \) is varied until a reasonable fit is found.

The Alternative Method

The alternative procedure will work for the Weibull model by using the golden section search method, as was done with the Makeham model. By entering an initial search interval for \( \delta \), two interior points are chosen by the golden section search method and the least-squares estimates can be found for the remaining parameters, \( k \) and \( n \). After the sum square error is found for those two interior points, the procedure will choose the one with the smaller error. Then, the search interval is reduced so that two new interior points can be chosen by the algorithm. This is repeated until the length of the
search interval is less than the tolerance, which is entered by the user. The procedure for the Weibull model is as follows:

Fix the parameter $\delta$, convert $\int \mu(t) dt$ into a linear form and find the least-squares estimates for the other two parameters, $k$ and $n$.

Let

$$y(t) = \int \mu(t) dt = \log \left[ \frac{1}{1 - F(t)} \right],$$

with

$$\mu(t) = k(t - \delta)^n, \quad k > 0, \quad n > 0, \quad \delta > 0, \quad t > \delta,$$

and an estimator for $F(t_i)$,

$$\hat{F}(t_i) = \frac{i}{n + 1},$$

or with a more robust estimator for $F(t_i)$,

$$\tilde{F}(t_i) = \frac{i - 0.3}{n + 0.4}.$$

Integrate $\int \mu(t) dt$ to obtain

$$y(t) = \frac{k(t - \delta)^n}{n + 1},$$

and take the logarithm of both sides,

$$\log[y(t)] = \log \left( \frac{k}{n + 1} \right) + (n + 1) \log(t - \delta).$$
This equation is the linear form, \( Y = A + BX \), where

\[
Y(t_i) = \log\left(\log\left(\frac{1}{1 - F(t_i)}\right)\right),
\]

\[
A = \log\left(\frac{k}{n + 1}\right),
\]

\[
B = n + 1
\]

\[
X(t_i) = \log(t_i - \delta).
\]

The following algorithm is used to find the estimators for the parameters, \( k, n, \) and \( \delta \). First, select an initial search interval for \( \delta \). When the samples are entered into the procedure, the simulated value for \( \delta \) is inside this interval. This interval is known as \([L, H]\).

For a fixed \( \delta_1 \) and \( \delta_2 \), fit a linear regression line to the points \((X(t_i), Y(t_i))\), and obtain the least-square estimates, \( A_\delta \) and \( B_\delta \). The values for \( \delta_1 \) and \( \delta_2 \) are found by taking the length of \([L, H]\) and multiplying it by the golden ratio, \( \gamma = \frac{\sqrt{5} - 1}{2} = 0.6180339 \). Then, \( \delta_1 \) is found by subtracting this product from \( H \), and \( \delta_2 \) is found by adding this product to \( L \).

Next, calculate the sum-square error for \( \delta_1 \) and \( \delta_2 \),
by using the following formula [4].

\[
SSE(\delta) = \sum_{i=1}^{n} [Y(t_i) - A_0 - B_0 X(t_i)]^2
\]

If \(SSE(\delta_1) < SSE(\delta_2)\), then set \(H = \delta_2\). If not, then set \(L = \delta_1\). This will choose the value of \(\delta_1\) or \(\delta_2\) that has the smaller sum-square error.

By continuing this procedure, the search interval decreases in size until the length of the interval is less than the tolerance. Then, the last iteration will give the value for \(\delta_1\) or \(\delta_2\) that has the smallest sum-square error. Call this value \(\delta_0\).

The estimators for the three parameters are as follows:

\[
\hat{\delta} = \delta_0, \quad \hat{k} = B_0 e^{A_0}, \quad \hat{n} = B_0 - 1.
\]

The results from this method are then placed into a file. It is important to note that the method will run twice, once for the less robust operator and once for the more robust operator.

**Three Examples**

For the Weibull model, the alternative method is run to show how quickly the estimates for the three
parameters are evaluated, and to compute the sum square error and its coefficient of determination for these data sets. It is important to note that when this method is implemented, the initial search interval for \( \delta \) includes the simulated point for \( \delta \) and the value for \( H \) must be less than the first data point for each set.

1. The first example simulates 50 data points with the parameters, \( k = 0.8 \), \( n = 2.0 \), and \( \delta = 3.5 \).

\[
\]

The results for the alternative method appear on page 30.

The initial search interval for \( \delta \) is \([0.1, 3.75]\).

2. The second example simulates 50 data points with the parameters \( k = 1.3 \), \( n = 1.3 \), and \( \delta = 6.1 \).

\[
\]

The results for the alternative method appear on page 32.

The initial search interval for \( \delta \) is \([0.2, 6.15]\).
3. The third example simulates 70 data points with parameters $k = 1.7$, $n = 0.6$, and $\delta = 8.9$. 

$9.9344, 9.9395, 9.9621, 9.9710, 10.0411, 10.0488, 10.1140, 10.1658, 10.2022, 10.2663, 10.2704, 10.2937,$ 
$10.2966, 10.3817, 10.4124, 10.4165, 10.4265, 10.4525, 10.4607, 10.4725, 10.4937, 10.8264, 11.1178, 11.2698.$ 

The results with the alternative method appear on page 34. The initial search interval for $\delta$ is $[0.3, 8.92]$. 

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CHAPTER 5
ANALYSIS AND CONCLUSIONS

It is important to state that the tolerance level for each sample was 0.0001, and that includes simulating the data sets and using Nesselle’s method as well. The comparison of the two methods will be observed with the Makeham model, the more difficult model to fit. The Weibull model just gives more support to the reason why the proposed method gives excellent results. For the first three samples, the Makeham samples, Nesselle’s method was executed first, once for each robust operator for $F(t)$. This is done so that when the alternative method is executed, the initial search interval for $c$ will include the estimates for $c$ obtained by Nesselle’s method.

When comparing the results for the three examples using the Makeham model, it is clear to see that the new method give better results than Nesselle’s method. The sum-square error and the coefficient of determination, $R^2$, for each sample gives a better fit to the linear regression. But, there are some interesting aspects of the samples themselves.
The first example using the Makeham model proves to be eye catching because the best results occurred when the less robust operator was used in the alternative method and it gives an estimated value for $A$ that is less than zero. The results still fit to the restrictions of the Makeham model, but it is somewhat interesting to see this happen.

In the second example, it is seen that for Nesselle's method converged to an estimate for $c$ that is close to 1, considering the fact that the data was simulated at $c = 1.0623$. When the proposed method was executed with this sample, the estimates for parameter $c$, stayed closer to the simulated value for $c$. Also, notice that even the method will converge to an estimate for $c$ that is higher than the simulated value, as is shown for the more robust operator, even though it was not the better estimate.

The most important quality to the third Makeham example is the fact that the more robust operator gave better results, and it is important to show that this can occur. Also, this example has the best reported value for $R^2$.

The advantage in showing the three Weibull samples is the manner in which the method converges. The initial
search intervals for \( \delta \) were chosen carefully so that there can be no domain error and so that when the method is completed, the estimated value for \( \delta \) could possibly be at an endpoint. The only example that this nearly occurred in was the last example, when the method converged to a point near the value of \( \delta \) that was used to simulate the data points. It is also interesting to point out that first two Weibull samples gave the best results when the more robust operator was used to estimate the three parameters.

Overall, the methods themselves are more direct than the existing methods. Nesselle's method is much more involving when it came to evaluating all the partial derivatives. Also, the fact that these models now can be fit by using a log-transformation and then simple linear regression is also appealing. For the Makeham model, it is nice to see that there is a method that only requires an interval for \( c \), whereas the existing method requires initial estimates for all three parameters. For the Weibull model, there is now an efficient way for estimating the three parameters and there will be no more need for a trial and error method. For both models, this makes the alternative method even more desirable to use.
APPENDIX I

RESULTS FROM THE EXAMPLES

TABLE 1 - RESULTS FROM MAKEHAM EXAMPLE 1

FOR THE LESS ROBUST OPERATOR:

<table>
<thead>
<tr>
<th>L-VALUE</th>
<th>H-VALUE</th>
<th>C1</th>
<th>C2</th>
<th>SSE1</th>
<th>SSE2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00010</td>
<td>1.05000</td>
<td>1.01916</td>
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ESTIMATED A: -.03136
ESTIMATED B: .04674
ESTIMATED C: 1.00894
SSE: .22331
R-SQUARED: .99950

FOR THE MORE ROBUST OPERATOR:

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<th>C2</th>
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<th>SSE2</th>
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ESTIMATED A: .01540
ESTIMATED B: .00444
ESTIMATED C: 1.03654
SSE: .55333
R-SQUARED: .99945
### TABLE 2 - RESULTS FROM MAKEHAM EXAMPLE 2

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**ESTIMATED A:** 0.00138  
**ESTIMATED B:** 0.00862  
**ESTIMATED C:** 1.05191  
**SSE:** 1.18420  
**R-SQUARED:** 0.99966

**FOR THE MORE ROBUST OPERATOR:**

<table>
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<th>C2</th>
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**ESTIMATED A:** 0.01218  
**ESTIMATED B:** 0.00259  
**ESTIMATED C:** 1.07942  
**SSE:** 0.54408  
**R-SQUARED:** 0.99956

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### Table 3 - Results from Makeham Example 3

#### For the Less Robust Operator:

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<th>H-VALUE</th>
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<th>SSE2</th>
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Estimated A: .01137
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Estimated C: 1.04104
SSE: .42955
R-Squared: .99950

#### For the More Robust Operator:

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Estimated A: .01626
Estimated B: .00243
Estimated C: 1.05641
SSE: .41393
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### TABLE 4 - RESULTS FROM WEIBULL EXAMPLE 1

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ESTIMATED K: .25129  
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TABLE 4 - CONTINUED

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TABLE 5 - RESULTS FROM WEIBULL EXAMPLE 2

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TABLE 5 - CONTINUED

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ESTIMATED DELTA: 5.97669
ESTIMATED N : 1.03416
ESTIMATED K : 1.09577
SSE : 1.34207
R-SQUARED : .97953
### TABLE 6 - RESULTS FROM WEIBULL EXAMPLE 3

**FOR THE LESS ROBUST OPERATOR:**

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**ESTIMATED DELTA:** 3.86475  
**ESTIMATED N:** .43864  
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**R-SQUARED:** .98144
TABLE 6 - CONTINUED

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ESTIMATED DELTA: 3.84126
ESTIMATED N: .50528
ESTIMATED K: 1.43912
SSE: 1.63975
R-SQUARED: .97968
APPENDIX II

FORTRAN PROGRAM: SIMULATING THE DATA SETS

This program will simulate data points for the three parameter models, Makeham and Weibull. The user will enter the number of data points and then the model to simulate. The program will go out to the hundreth-second on the clock to use as a seed for the generation of the random numbers. After this the user will enter the parameters for the model chosen, the data points will be simulated.

```
INTEGER I, J, CHOICE, SWITCH
REAL A, B, C, K, D, N
REAL IG(IOOO), HT(IOOO), G(IOOO), GP(IOOO), X(IOOO)
REAL RCN, TEMP, DUMMY
REAL T(IOOO), TOL, ER
INTEGER*2 HR, MIN, SEC, HSEC

This part will get the seed for the random number generator.

CALL GETTIM(HR, MIN, SEC, HSEC)
CALL SEED(HSEC)

This part will select the type of model to use, and the number of points to generate.

WRITE(*,*), 'ENTER THE TYPE OF MODEL TO BE SIMULATED.'
WRITE(*,*) '1. MAKEHAM MODEL'
WRITE(*,*) '2. WEIBULL MODEL WITH K, D, AND N'
READ (*,*) CHOICE
IF (CHOICE .EQ. 1 ) THEN

This part will simulate the points for the Makeham model. Since this model does not have an inverse, as described in Chapter 2, Newton's method is used

36
C to create the data points. In the first part, the user will enter the parameter values for this model, so that the data points can be simulated.

```
WRITE(*,*) 'MAKEHAM MODEL ACTIVATED.'
WRITE(*,*)
20 WRITE(*,*) 'ENTER THE POSITIVE PARAMETER B:'
READ (*,*) B
IF (B .LE. 0.0) THEN
  GOTO 20
ENDIF
WRITE(*,*)
30 WRITE(*,*) 'ENTER THE PARAMETER A >= -B :' 
WRITE(*,*) 'FOR CONVENIENCE, TRY A < 1.0'
WRITE(*,*)
READ(*,*) A
IF ((A + B) .LE. 0.0) THEN
  GOTO 30
ENDIF
WRITE(*,*)
40 WRITE(*,*) 'ENTER THE PARAMETER C > 1 :
WRITE(*,*)
READ (*,*) C
IF (C .LE. 1.0) THEN
  GOTO 40
ENDIF
```

This part contains Newton's Method for generating the data points for the Makeham model.

```
WRITE(*,*) 'ENTER AN INITIAL T > 0'
WRITE(*,*)
READ(*,*) T(1)
WRITE(*,*) 'ENTER THE TOLERANCE'
WRITE(*,*)
READ (*,*) TOL
DO 140 I = 1, J
  CALL RANDOM(U)
  DO 145 K = 1, 20
    IG(K) = (A*T(K)+((B*((C**T(K))-1.0))/LOG(C))
    HT(K) = EXP(-1.0*IG(K))
    G(K) = U - 1 + HT(K)
    GP(K) = (A + (B * (C**T(K)))))*(HT(K))
    ER = G(K)/GP(K)
    IF (GP(K) .GT. TOL) THEN
      T(K+1) = T(K) + ER
    ELSE
      GOTO 146
```

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This part will sort the data set in ascending order.

```
SWITCH = 1
190 IF (SWITCH .EQ. 1) THEN
    SWITCH = 0
    DO 195 I = 2, J
        IF (X(I) .LT. X(I-1)) THEN
            TEMP = X(I)
            X(I) = X(I-1)
            X(I-1) = TEMP
            SWITCH = 1
        END IF
    CONTINUE
GOTO 190
END IF
COUNTER = 0
END IF
```

This part will write the data set to a file.

```
WRITE(*,*)
WRITE(*,'# 'THE DATA FILE IS DATA.TXT')
OPEN (3, FILE = 'DATA.TXT')
WRITE(3,*) J
DO 165 I = 1, J
    WRITE(3,163) X(I)
    FORMAT('# F7.4')
163 CONTINUE
GOTO 500
```

This part will generate the data set for the Weibull model.

```
IF (CHOICE .EQ. 2) THEN
    WRITE(*,'# 'WEIBULL MODEL #1 ACTIVATED.'
    WRITE(*,'# 'THE ONLY RESTRICTION IS THAT ALL THE'
    WRITE(*,'# 'PARAMETERS ARE DECLARED POSITIVE.'
    WRITE(*,'# 'ENTER THE PARAMETER K:'
    READ (*,*) K
```
IF (K .LE. 0.0) THEN
    GOTO 50
ENDIF
WRITE(*,*)
WRITE(*,*) 'ENTER THE PARAMETER D:'
WRITE(*,*)
READ (*,*) D
IF (D .LE. 0.0) THEN
    GOTO 60
ENDIF
WRITE(*,*)
WRITE(*,*) 'ENTER THE PARAMETER N:'
WRITE(*,*)
READ (*,*) N
WRITE(*,*)
IF (N .LE. 0.0) THEN
    GOTO 70
ENDIF
RCN = 1.0 / (N + 1)
DO 110 I = 1, J
    CALL RANDOM(U)
    DUMMY = ((N+1)/K)*LOG(1/(1-U))
    T(I) = D + (DUMMY**RCN)
110 CONTINUE
END IF
C This part will sort the data set in ascending order
C for the Weibull model.
SWITCH = 1
210 IF (SWITCH .EQ. 1) THEN
    SWITCH = 0
    DO 200 I = 2, J
        IF (T(I) .LT. T(I-1)) THEN
            TEMP = T(I)
            T(I) = T(I-1)
            T(I-1) = TEMP
            SWITCH = 1
        ENDIF
200 CONTINUE
GOTO 210
ENDIF
WRITE(*,*) 'THE DATA FILE IS DATA.TXT'
OPEN (3, FILE = 'DATA.TXT')
WRITE(3,*) J
APPENDIX III
FORTRAN PROGRAM: NESSELLE'S METHOD

This program will estimate the parameters for the Makeham model. This method is known as Nesselle's method, and it is fully described in Chapter 3. The program will enter the Makeham values from a file, DATA.TXT, and then the user will enter the type of estimator to use, the initial parameter estimates, and the tolerance. After the method is completed, the output is put into a file called RESULTS2.TXT.

```
INTEGER J, I, R
REAL T(70), MU(70), TOL, N

Start of program.

WRITE(*,*), 'THIS PROGRAM CONTAINES THE EXISTING METHODS FOR ESTIMATING THE THREE PARAMETERS FOR THE MAKEHAM MODEL.'
WRITE(*,*), 'DO YOU WISH TO USE A MORE ROBUST ESTIMATOR?'
WRITE(*,*), 'ENTER 1 FOR YES'  ENTER 2 FOR NO
READ(*,*), R
WRITE(*,*), 'ENTER THE DESIRED ACCURACY LEVEL:'
WRITE(*,*), TOL
READ(*,*), TOL

IF (R .EQ. 1) THEN
  WRITE(*,*), 'MORE ROBUST ESTIMATOR ACTIVATED.'
  WRITE(*,*)
```

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DO 230 I = 1, J
    WRITE(3,228) T(I)
228 FORMAT(' 'F7.4)
230 CONTINUE
    WRITE(*,*) 'THE FIRST DATA POINT IS:', T(1)
500 WRITE(*,*)

END
DO 20 I = 1, J-1
   MU(I) = 1.0 / ((T(I+1)- T(I))*(N+0.7-I))
20 CONTINUE
CALL MAKEHAM(MU, T, J, TOL)
ELSE
   WRITE(*,*), 'LESS ROBUST ESTIMATOR ACTIVATED.'
   WRITE(*,*)
   DO 30 I = 1, J-1
      MU(I) = 1.0 / ((T(I+1)- T(I))*(N+1-I))
30 CONTINUE
   CALL MAKEHAM(MU, T, J, TOL)
END IF

C End of program.
END

C Subroutine to find the estimates of the parameters
C for the Makeham model.

SUBROUTINE MAKEHAM(MU, T, J, TOL)
INTEGER J, I, FLAG
REAL MU(70), T(70), TOL
REAL IA, IB, IC, CHECK
REAL MP(70), OMU(70)
REAL DUMMY, DUMMY2, MUH

C These are the partial derivatives needed for
C summing.
REAL DU(70), PUB(70), PUC(70)

C These are the counters for the normal equations.
REAL DA, DB, DC, A1, B1, C1

C This part will ask the user for the initial
C estimates.
WRITE(*,*), 'PLEASE ENTER THE INITIAL ESTIMATE A'
WRITE(*,*), 'MAKE SURE THAT A > -B'
READ(*,*) IA
WRITE(*,*), 'PLEASE ENTER THE INITIAL ESTIMATE B'
READ(*,*) IB
WRITE(*,*), 'PLEASE ENTER THE INITIAL ESTIMATE C'
READ(*,*) IC
WRITE(*,*)

C Initial work to find the sum-square error.

DUMMY = 0.0
DO 195 I = 1, J
   DUMMY = DUMMY + MU(I)
195 CONTINUE
MUH = DUMMY / J

C Calculate the midpoint and the initial values for
C the force of mortality.

DO 200 I = 1, J-1
   MP(I) = (T(I) + T(I-1))/2
   OMU(I) = IA + (IB * (IC ** MP(I)))
200 CONTINUE

C Nesselle's method starts here.

FLAG = 1
DO WHILE (FLAG .EQ. 1)

C Calculate the initial estimates for the force of
C mortality and its partial derivatives. Mainly
C used after the initial calculations.

DO 220 I = 1, J-1
   OMU(I) = IA + (IB * (IC ** MP(I)))
   DU(I) = MU(I) - OMU(I)
   PUB(I) = (IC ** MP(I))
   PUC(I) = (IB * MP(I)) * (IC ** (MP(I-1)))
220 CONTINUE

C Initialize the variables that will hold all the
C sums needed to calculate the sum-square error.

N = J - 1
K = 0
L = 0
M = 0
P = 0
Q = 0
R = 0
S = 0
U = 0
V = 0
\begin{verbatim}
W = 0
X = 0
Y = 0
Z = 0
DA = 0
DB = 0
DC = 0

C Calculate the variables needed for the normal
equations. This is fully described in Chapter 3.

DO 240 I = 1, J-1
    K = DU(I) + K
    L = (DU(I) * PUB(I)) + L
    M = (DU(I) * PUC(I)) + M
    P = PUB(I) + P
    Q = PUC(I) + Q
    R = (PUB(I) ** 2.0) + R
    S = (PUC(I) ** 2.0) + S
    U = (PUB(I) * PUC(I)) + U
240 CONTINUE

W = (Q*K) - (N*L)
X = (P*P) - (N*R)
Y = (P*Q) - (N*U)
Z = (Q*Q) - (N*S)

C This is the solution to the normal equations.

DC = (((V*Y) - (W*X))/(((Y*Y) - (X*Z))*N)
DB = (((W*Y) - (V*Z))/(((Y*Y) - (X*Z))*N)
DA = (K - ((DB*P) + (DC*Q)))/N

C The check needed to stop the procedure.

IF ((ABS(DC).LT.TOL).OR.((IC + DC) .LT. 1.0)) THEN
    FLAG = 2
ELSE
    IA = IA + DA
    IB = IB + DB
    IC = IC + DC
END IF

C Calculate the sum-square error, and the
C coefficient of determination, also known as
C R-squared.

SSE = 0
\end{verbatim}
SSM = 0
DO 330 I = 1, J-1
DUMMY2 = (MU(I) - MUH)**2
SSM = SSM + DUMMY2
DUMMY3=(OMU(I)+DA+(DB*PUB(I))+(DC*PUC(I))-MU(I))**2
SSE = SSE + DUMMY3
CONTINUE
330
R2 = SSE/SSM
C  End the do - while loop
END DO
C  Write the results to the file.
OPEN(3, FILE = 'RESULTS2.TXT')
WRITE(3, 300) IA
300 FORMAT ('  ESTIMATED A: ',F12.6)
WRITE(3, 310) IB
310 FORMAT ('  ESTIMATED B: ',F12.6)
WRITE(3, 320) IC
320 FORMAT ('  ESTIMATED C: ',F12.6)
WRITE(3, 340) SSE
340 FORMAT ('  ESTIMATED SSE: ',F12.6)
WRITE(3, 350) R2
350 FORMAT ('  ESTIMATED R-SQUARED: ',F12.6)
RETURN
C  End the procedure for the Makeham model.
END
APPENDIX IV

FORTRAN PROGRAM: ALTERNATIVE METHOD

This program will take in a set of points, t(i), from a file called DATA.TXT and run the new method described in Chapters 3 and 4. The user will enter the type of model to use, the tolerance, on the fixed parameter, and the initial search interval for the fixed parameter. Then, the results are put into a file called RESULTS.TXT.

INTEGER J, I, M, R
REAL T(IOOO), Y(IOOO), YR(IOOO), TOL, N
REAL MY(IOOO), MYR(IOOO)

This part will read in the values from the file DATA.TXT.

WRITE(*,*)
OPEN (3, FILE = 'DATA.TXT')
READ(3,*) N
J = INT(N)
DO 10 I = 1, J
   READ(3,*) T(I)
10 CONTINUE

This section asks the user for the type of model, the tolerance, and then moves onto the corresponding subroutine.

WRITE(*,*)
WRITE(*,*) 'ENTER THE TYPE OF SURVIVAL MODEL: '
WRITE(*,*) ' 1. MAKEHAM MODEL'
WRITE(*,*) ' 2. WEIBULL MODEL WITH PARA. K'
WRITE(*,*) ' 3. EXIT'
WRITE(*,*)
READ(*,*) M
WRITE(*,*)
WRITE(*,*) 'ENTER THE DESIRED ACCURACY LEVEL:'
WRITE(*,*)
READ(*,*) TOL
WRITE(*,*)
DO 37 I = 1, N
   Y(I) = LOG(LOG((N+1)/(N+1-I)))
   YR(I) = LOG(LOG((N+0.7)/(N+0.4-I)))
   MY(I) = LOG((N+1)/(N+1-I))
   MYR(I) = LOG((N+0.7)/(N+0.4-I))
37 CONTINUE

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IF (M .EQ. 1) THEN
    WRITE(*,*) 'MAKEHAM MODEL ACTIVATED.'
    R = 1
    CALL MAKEHAM(T, MY, J, TOL, R)
    R = 2
    CALL MAKEHAM(T, MYR, J, TOL, R)
ELSEIF (M .EQ. 2) THEN
    WRITE(*,*) 'WEIBULL MODEL W/PARA. K ACTIVATED.'
    WRITE(*,*)
    R = 1
    CALL WEIBT(T, Y, J, TOL, M, R)
    R = 2
    CALL WEIBT(T, YR, J, TOL, M, R)
    WRITE(*,*)
ELSEIF (M .EQ. 3) THEN
    GOTO 500
ELSE
    GOTO 400
END IF
500 WRITE(*,*)

C End of the program

END

C This subroutine will find the estimates least square estimates of the parameters for the Makeham model.
C The user will enter the search interval for c and the method is performed. All dummy variables are
C used to compute the sums needed for calculating the sum-square error and the coefficient of
C determination, also known as R-squared.

SUBROUTINE MAKEHAM(XI, Y, J, TOL, R)

REAL XI(1000), Y(1000), TOL
INTEGER J, I, FLAG, R
REAL L, U, GR, H, C1, C2
REAL XC1(1000), XC2(1000)
REAL DUMMY1, DUMMY2, DUMMY3, DUMMY4
REAL DUMMY5, DUMMY6, DUMMY7, DUMMY8
REAL SUM1, SUM2, SUM3, SUM4
REAL SUM5, SUM6, SUM7, SUM8
REAL A1, B1, A2, B2
REAL YH1(1000), YH2(1000), SUMY, YBAR
REAL SSE1, SSE2, SS, DUMMY9, DUMMY10, DUMMY11
REAL R1, R2
This is where the user enters the search interval for c.

\[
\text{WRITE}(\ast,\ast) \ 'PLEASE ENTER LOWER BOUND FOR C' \\
\text{READ}(\ast,\ast) \ L \\
\text{WRITE}(\ast,\ast) \ 'PLEASE ENTER UPPER BOUND FOR C' \\
\text{READ}(\ast,\ast) \ U \\
\text{GR} = 0.6180339887
\]

This part labels the output file.

\[
\text{OPEN}(3, \text{FILE} = \ 'RESULTS.TXT') \\
\text{IF (R .EQ. 1) THEN} \\
\hspace{1cm} \text{WRITE}(3, \ast) \ 'FOR THE LESS ROBUST OPERATOR:' \\
\hspace{1cm} \text{WRITE}(3, \ast) \\
\text{END IF} \\
\text{IF (R .EQ. 2) THEN} \\
\hspace{1cm} \text{WRITE}(3, \ast) \ 'FOR THE MORE ROBUST OPERATOR:' \\
\hspace{1cm} \text{WRITE}(3, \ast) \\
\text{END IF} \\
\text{WRITE}(3, \ast) \ 'L-VALUE H-VALUE C1 C2 SSE1 SSE2' \\
\text{WRITE}(3, \ast) \ '----------------------------------------'
\]

The method starts here. The first part finds the two internal points needed for the golden section search, known as C1 and C2.

\[
\text{DO WHILE}((U-L) \ .GT. \ TOL) \\
\hspace{1cm} H = U - L \\
\hspace{1cm} C1 = U - (GR \ast H) \\
\hspace{1cm} C2 = L + (GR \ast H) \\
\hspace{1cm} \text{DO 300 I = 1, J} \\
\hspace{2cm} \text{XC1(I) = (((C1**X1(I))-1)/LOG(C1))} \\
\hspace{2cm} \text{XC2(I) = (((C2**X1(I))-1)/LOG(C2))} \\
300 \ \text{CONTINUE}
\]

This part computed the sum-square error and the coefficient of determination for the two points, C1 and C2.

\[
\text{DO 310 I = 1, J} \\
\hspace{1cm} \text{DUMMY1 = X1(I)*X1(I)} \\
\hspace{1cm} \text{DUMMY2 = XC1(I)*XC1(I)} \\
\hspace{1cm} \text{DUMMY3 = XC2(I)*XC2(I)} \\
\hspace{1cm} \text{DUMMY4 = X1(I)*XC1(I)} \\
\hspace{1cm} \text{DUMMY5 = X1(I)*XC2(I)} \\
\hspace{1cm} \text{DUMMY6 = X1(I)*Y(I)}
\]
DUMMY7 = XC1(I)*Y(I)
DUMMY8 = XC2(I)*Y(I)
SUM1 = SUM1 + DUMMY1
SUM2 = SUM2 + DUMMY2
SUM3 = SUM3 + DUMMY3
SUM4 = SUM4 + DUMMY4
SUM5 = SUM5 + DUMMY5
SUM6 = SUM6 + DUMMY6
SUM7 = SUM7 + DUMMY7
SUM8 = SUM8 + DUMMY8

310 CONTINUE

C This part is adjusted to fit nicely on one line.

A1=((-SUM4*SUM7)-(SUM2*SUM6))/((-SUM4*SUM4)-(SUM1*SUM2))
B1=((-SUM4*SUM6)-(SUM1*SUM7))/((-SUM4*SUM4)-(SUM1*SUM2))
A2=((-SUM5*SUM8)-(SUM3*SUM6))/((-SUM5*SUM5)-(SUM1*SUM3))
B2=((-SUM5*SUM6)-(SUM1*SUM8))/((-SUM5*SUM5)-(SUM1*SUM3))

SUMY = 0.0
DO 320 I = 1, J
   YH1(I) = (A1 * X1(I)) + (B1 * XC1(I))
   YH2(I) = (A2 * X1(I)) + (B2 * XC2(I))
   SUMY = SUMY + Y(I)
320 CONTINUE

YBAR = SUMY / J
SSE1 = 0.0
SSE2 = 0.0
DO 330 I = 1, J
   DUMMY9 = (Y(I) - YH1(I))**2.0
   DUMMY10 = (Y(I) - YH2(I))**2.0
   DUMMY11 = (Y(I) - YBAR)**2.0
   SSE1 = SSE1 + DUMMY9
   SSE2 = SSE2 + DUMMY10
   SSM = SSM + DUMMY11
330 CONTINUE

R1 = (SSM - SSE1)/SSM
R2 = (SSM - SSE2)/SSM

C Print the SSE and the coefficient of determination to
C the output file for the current values, C1 and C2.

WRITE(3,400) L, U, C1, C2, SSE1, SSE2
400 FORMAT (F8.5,' ', F8.5, F8.5, F8.5, F8.5, F8.5, F8.5)

C This part will choose the value, C1 or C2, that has
C the smaller sum-square error.

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IF (SSE1 .LT. SSE2) THEN
    U = C2
    FLAG = 1
ELSE
    L = C1
    FLAG = 2
END IF
DO 410 I = 1, J
    XC1(I) = 0.0
    XC2(I) = 0.0
410 CONTINUE
SUM1 = 0.0
SUM2 = 0.0
SUM3 = 0.0
SUM4 = 0.0
SUM5 = 0.0
SUM6 = 0.0
SUM7 = 0.0
SUM8 = 0.0
END DO

This part will print the final results to the output file. It includes the least square estimates for the parameters A, B, and c, the sum-square error, and the coefficient of determination.

WRITE(*,*)
WRITE(*,*) 'THE RESULTS ARE IN THE FILE RESULTS.TXT'
WRITE(3,* )
IF (FLAG .EQ. 1) THEN
    WRITE(3,600) A1
    FORMAT(' ESTIMATED A: 'F12.5)
WRITE(3,610) B1
    FORMAT(' ESTIMATED B: 'F12.5)
WRITE(3,620) C1
    FORMAT(' ESTIMATED C: 'F12.5)
WRITE(3,630) SSE1
    FORMAT(' SSE: 'F12.5)
WRITE(3,640) R1
    FORMAT(' R-SQUARED: 'F12.5)
WRITE(3,* )
END IF
IF (FLAG .EQ. 2) THEN
    WRITE(3,650) A2
    FORMAT(' ESTIMATED A: 'F12.5)
WRITE(3,660) B2
SUBROUTINE WEIBT(T, Y, N, TOL, M, R)

C This routine will find the least square estimates
C for the parameters of the Weibull model. The user
C will enter the search interval for the fixed
C parameter, delta, and the method is performed.
C All dummy variables are used to compute the sums
C needed for the sum-square error and the coefficient
C of determination, also known as R-squared.

INTEGER K, J, FLAG, M, N, R
REAL T(1000), Y(1000), TOL, L, U, GR
REAL X1(1000), X2(1000)
REAL YH1(1000), YH2(1000)
REAL SUMY, SSY
REAL SUMX1, SYX1, SSX1
REAL SUMX2, SYX2, SSX2
REAL DUMMY1, DUMMY2, DUMMY3, DUMMY4, DUMMY5
REAL ABOVE1, ABOVE2, BELOW1, BELOW2
REAL YBAR, XBAR1, XBAR2, A1, B1, A2, B2
REAL SSE1, SSE2
REAL DUMMY6, DUMMY7, DUMMY8, SSM
REAL R1, R2
REAL WK, WN
REAL TEST1, TEST2

C This where the user will enter the search interval
C for delta.

WRITE(*,*) 'PLEASE ENTER L'
READ(*,*) L
WRITE(*,*) 'PLEASE ENTER U'
READ(*,*) U

GR = 0.6180339887

C This part will label the output file.

OPEN (3, FILE = 'RESULTS.TXT')
IF (R .EQ. 1) THEN
WRITE(3,*) 'FOR THE LESS ROBUST OPERATOR:'
WRITE(3,*)
END IF
IF (R .EQ. 2) THEN
WRITE(3,*) 'FOR THE MORE ROBUST OPERATOR:'
WRITE(3,*)
END IF

WRITE(3,*) 'L-VALUE  H-VALUE  DEL1  DEL2  SSE1  SSE2'
WRITE(3,*) '------------------------------------------'

C The method starts here. The first part finds the two internal points needed to operate the golden section search algorithm. The second part will calculate the sum-square error and R-squared for each one of those points, called D1 and D2.

DO WHILE ((U-L) .GT. TOL)
H = U - L
D1 = U - (GR * H)
D2 = L + (GR * H)
SUMY = 0.0
SSY = 0.0
DO 50 J = 1, N
X1(J) = LOG(T(J) - D1)
X2(J) = LOG(T(J) - D2)
SUMY = SUMY + Y(J)
DUMMY = Y(J) * Y(J)
SSY = SSY + DUMMY
DUMMY = 0.0
50 CONTINUE

C This computed the sum-square error and R-squared for each of the points, D1 and D2.

SUMX1 = 0.0
SUMX2 = 0.0
SSX1 = 0.0
SSX2 = 0.0
SYX1 = 0.0
SYX2 = 0.0
DO 60 J = 1, N
  SUMX1 = SUMX1 + X1(J)
  SUMX2 = SUMX2 + X2(J)
  DUMMY2 = X1(J) * X1(J)
  DUMMY3 = X2(J) * X2(J)
  DUMMY4 = X1(J) * Y(J)
  DUMMY5 = X2(J) * Y(J)
  SSX1 = SSX1 + DUMMY2
  SSX2 = SSX2 + DUMMY3
  SYX1 = SYX1 + DUMMY4
  SYX2 = SYX2 + DUMMY5
  DUMMY2 = 0.0
  DUMMY3 = 0.0
  DUMMY4 = 0.0
  DUMMY5 = 0.0
CONTINUE

60 YBAR = SUMY / N
XBAR1 = SUMX1 / N
XBAR2 = SUMX2 / N
ABOVE1 = SYX1 - ((SUMX1 * SUMY) / N)
BELOW1 = SSX1 - ((SUMX1 * SUMX1) / N)
B1 = ABOVE1 / BELOW1
A1 = YBAR - (XBAR1 * B1)
ABOVE2 = SYX2 - ((SUMX2 * SUMY) / N)
BELOW2 = SSX2 - ((SUMX2 * SUMX2) / N)
B2 = ABOVE2 / BELOW2
A2 = YBAR - (XBAR2 * B2)
TEST1 = YBAR - (A1 + (B1 * XBAR1))
TEST2 = YBAR - (A2 + (B2 * XBAR2))
IF (TEST1 .GT. TOL) THEN
  GOTO 1300
END IF
IF (TEST2 .GT. TOL) THEN
  GOTO 1300
END IF
DO 70 K = 1, N
  YH1(K) = A1 + (B1 * X1(K))
  YH2(K) = A2 + (B2 * X2(K))
CONTINUE

70 SSM = 0.0
SSE1 = 0.0
SSE2 = 0.0
DO 80 K = 1, N
  DUMMY6 = (Y(K) - YH1(K))**2
  DUMMY7 = (Y(K) - YH2(K))**2
  DUMMY8 = (Y(K) - YBAR)**2
CONTINUE
54

$$SSM = SSM + DUMMY8$$
$$SSE1 = SSE1 + DUMMY6$$
$$SSE2 = SSE2 + DUMMY7$$
$$DUMMY6 = 0.0$$
$$DUMMY7 = 0.0$$

80 CONTINUE

$$R1 = (SSM - SSE1)/SSM$$
$$R2 = (SSM - SSE2)/SSM$$

C This part will print the sum-square error and R-squared to the output file.

WRITE(3, 1000) L, U, D1, D2, SSE1, SSE2

1000 FORMAT(F8.5, F8.5, F8.5, F8.5, F8.5, F8.5)

C This part will choose the value, D1 or D2, that has the smaller sum-square error.

IF (SSE1 .LT. SSE2) THEN
  U = D2
  FLAG = 1
ELSE
  L = D1
  FLAG = 2
END IF

DO 40 J = 1, N
  X1(J) = 0.0
  X2(J) = 0.0
40 CONTINUE

END DO

C This part will print the final results to the output file. It includes the least-square estimates for the parameters k, n, and delta. It also includes the sum-square error and R-squared for those estimates.

WRITE(*,*)
WRITE(*,*)
WRITE(3,*)

!'THE RESULTS ARE IN RESULTS.TXT'

WRITE(*,*)

IF ((FLAG .EQ. 1) .AND. (M .EQ. 2)) THEN
  WN = Bl - 1
  WK = (Bl * (EXP(A1)))
END IF

WRITE(3, 1110) D1

1110 FORMAT(' ESTIMATED DELTA: ' F8.5)
WRITE(3, 1120) WN

1120 FORMAT(' ESTIMATED N : ' F8.5)
WRITE(3, 1130) WK
WRITE(3, 1140) SSE1
WRITE(3, 1150) R1
WRITE(3, 1160) D2
WRITE(3, 1170) WN
WRITE(3, 1180) WK
WRITE(3, 1190) SSE2
WRITE(3, 1200) R2
WRITE(*,*)
RETURN
C This ends the subroutine for the Weibull model.
END
REFERENCES


