Dc and Ac modeling of heterostructure bipolar transistors (Hbts)

Roxana Arvandi
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DC AND AC MODELING OF HETEROSTRUCTURE
BIPOLAR TRANSISTORS (HBTs)

by

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Sharif University of Technology, Iran
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A thesis submitted in partial fulfillment
of the requirements for the degree of

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ABSTRACT

DC and AC Modeling of Heterostructure Bipolar Transistors (HBTs)

by

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Starting with a homojunction or graded heterojunction bipolar transistor, a complete model for the dc and ac performances of the device is developed based on the Ebers-Moll methodology. The formulation is modified to include the abrupt single HBT, by introducing the effects of the conduction band discontinuity at the base-emitter junction as an energy barrier for electrons flow resulting in transmission and reflection processes. Finally, using the same approach, the formulation is extended to the abrupt double HBT. The collector current as a function of collector-emitter voltage, the collector and base currents as a function of base-emitter voltage, the dc current gain as a function of collector current, the $h$ parameters as a function of collector current, the ac current gain as a function of frequency, and finally the unity current gain cut-off frequency as a function of the collector current are calculated for various graded and abrupt, single and double AlGaAs/GaAs HBTs. The results are in good agreement with the reported data.
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CHAPTER 1

INTRODUCTION

1.1 BJT Design Limitations

Once the semiconductor material is chosen for a bipolar junction transistor (BJT) (say, Si, Ge, GaAs, etc.), the only flexibilities in the device design are the doping levels and the device dimensions. This is a serious handicap for the realization of high performance transistors, as discussed below. To begin, let us examine the parameters controlling the transistor performance. The emitter injection efficiency of the device, $\gamma_E$, is given by:

$$\gamma_E = \frac{I_E^e}{I_E^e + I_E^p} \bigg|_{V_{CB}=0}$$

$$\approx \frac{1}{1 + \frac{p_E^0 D_E W_B}{n_B^2 D_B L_E}}.$$  \hspace{1cm} (1.1)

where $I_E^e$ is the emitter electron current, $I_E^p$ is the emitter hole current, $V_{CB}$ is the collector-base applied voltage, $n_B^0$ is the equilibrium electron concentration in the base, $D_B$ is the electron diffusion coefficient in the base, $W_B$ is the neutral base width, $p_E^0$ is the equilibrium hole concentration in the emitter, $D_E$ is the hole diffusion coefficient in the emitter, and $L_E$ is the hole diffusion length in the emitter, given by $L_E = (D_E \tau_E)^{\frac{1}{2}}$ where $\tau_E$ is the hole recombination lifetime in the emitter. The transistor dc current gain, $\beta$, is defined as $\beta = \frac{I_C}{I_B}$ where $I_C$ and $I_B$ are the collector and base currents respectively. In the forward active mode (for-
ward biased base-emitter junction and reverse biased collector-base junction), $\beta$ can be approximated in terms of the emitter injection efficiency, as follows:

$$ \beta \approx \frac{1}{\gamma_E - 1} \approx \frac{n_B^o D_B L_E}{p_E^o D_E W_B}. $$

For $\gamma_E$ to be close to unity and $\beta$ to be high, it is essential that the emitter doping, $N_E^d$, be much higher than the base doping, $N_B^a$, and the base width, $W_B$, be as small as possible. A small base width along with relatively low doping introduces a large base series resistance which degrades the device performance. However, a far greater problem arises from the band-gap shrinking at the emitter region due to heavy doping. If the emitter band-gap shrinks by $\Delta E_E^p$, the intrinsic carrier concentration of the emitter, $n_E^i$, increases by a factor of $\exp \left( \frac{\Delta E_E^p}{k_B T} \right)$ where $k_B T$ is the thermal energy with $k_B$ the Boltzman constant and $T$ the absolute temperature. Hence, the emitter hole concentration, $p_E^o$, which is:

$$ p_E^o = \frac{(n_E^i)^2}{N_E^d}, $$

increases by a factor of $\exp \left( \frac{\Delta E_E^p}{k_B T} \right)$. As a result of the exponential increase of $p_E^o$, the current gain, $\beta$, given by Equation 1.2, decreases by a factor of $\exp \left( -\frac{\Delta E_E^p}{k_B T} \right)$. The above discussion shows that the high performance requisite poses a set of conflicting requirements on the doping and geometry of the BJT.

1.2 Band Engineering: HBT

In the 50s, the limitations in BJT design led Shockley and Kroemer to propose a new bipolar transistor with different band-gaps in the emitter, base, and collector called the heterojunction bipolar transistor (HBT) [1,2]. Typically, in these devi-
ces, the emitter is fabricated from a wide band-gap material. This dramatically suppresses the injection of holes from the base to the emitter. If the emitter band-gap is increased by \( \Delta E^g_E \), the current gain, \( \beta \), increases by a factor of \( \exp\left(\frac{\Delta E^g_E}{kT}\right) \). Typically, \( \frac{\Delta E^g_E}{kT} \) is chosen to be \( \approx 10 \), so that an improvement of \( 10^4 \) in \( \beta \) results.

With an HBT, the base can be doped heavily without the loss of current gain. Due to the high base doping, the base can be made narrow without resulting a large base series resistance. The emitter doping can also be reduced so that the emitter depletion region can increase, thus reducing the depletion capacitance of the base-emitter junction. A narrow base along with reduced base-emitter depletion capacitance result in reduction of the device transit time, thus increasing the transistor bandwidth. Additionally, the Early effect and the danger of punch-through are greatly reduced, since the neutral base width \( W_B \) is essentially independent of the collector-base reverse bias, \( V_{CB} \), due to the heavy doping of the base. The possibility for the base pushout is also minimized, due to the high built-in voltage in the collector-base junction. From the above discussion, it is clear that the HBT is not prone to any of the problems associated with optimizing the BJT.

In a typical III-V compound semiconductor HBT, the base and collector are fabricated with GaAs and the emitter is made of a material, e.g. AlGaAs, that has a wider band-gap than GaAs and a lattice constant and thermal coefficient very similar to that of GaAs. To avoid abrupt discontinuities, a thin graded layer is inserted at the base-emitter junction in which the Al mole fraction is graded linearly to zero. The device structure along with its energy band diagram are illustrated in Figure 1. For the rest of the discussion, this device is referred as the graded single HBT. If the graded layer is not inserted at the base-emitter junction, the Al mole fraction changes abruptly from the emitter to the base. In this case, the device has sharp discontinuities in the material parameters at the base-emitter.
junction. Specifically, there are discontinuities in the conduction band minimum and valence band maximum at the junction. The band-gap difference between the emitter and the base, $\Delta E_{EB}^p$, consists of a conduction band energy step, $\Delta E_{EB}^c$, and a valence band energy step, $\Delta E_{EB}^v$, with

$$\Delta E_{EB}^p = \Delta E_{EB}^c + \Delta E_{EB}^v.$$ 

The device structure along with its energy band diagram are illustrated in Figure 2. An energy barrier appears in the conduction band at the base-emitter junction, which tends to retard the flow of electrons, as shown in Figure 2. On the other hand, the same barrier provides the benefits for high speed operation, because the electrons that surmount the barrier are injected into the base with high forward velocities (hot electrons). For the rest of the discussion, this device is referred as the abrupt single HBT. Another possibility offered by band-gap engineering is to increase the collector band-gap of the graded single HBT, as is illustrated in Figure 3. In this device, the GaAs collector of the graded single HBT has been replaced by an AlGaAs one along with a graded layer inserted between the base and collector to suppress any discontinuity. For the rest of the discussion, this device is referred as the graded double HBT. This structure is very much like a symmetrical transistor, and consequently the roles of collector and emitter can be exchanged. Such an exchange of roles allows the architectures of digital integrated circuits to be optimized, as illustrated in Figure 4.a in the cross sectional view of an emitter-coupled logic (ECL) gate envisaged by Kroemer. The graded double HBT offers additional advantages associated with the suppression of hole injection in the collector working in a saturated mode. This property results in significant improvement of the dynamical performances of some saturated logic gates, owing to the dramatic reduction in the transistor turn-off time. Figure 4.b illustrates such a gate. Additional advantages of these devices are the substantial decrease of the offset voltage and the increase of the breakdown voltage.
Although the idea of utilizing the wide band-gap emitter to improve device performance was first proposed in 1951, most of the developments on the HBT occurred during the early 1980s when the technologies for the metal organic chemical vapor deposition (MOCVD) and molecular beam epitaxy (MBE) became feasible. HBTs are now well established as leading contenders for high-gain and high-speed applications, particularly for digital circuits. The material combinations commonly used for these devices include AlGaAs/GaAs, InGaP/GaAs, InP/InGaAs, and AlInAs/InGaAs. Each of these materials presents a different band structure with different discontinuities in the conduction and valence bands. Since details of the band structure have a profound effect on the carrier transport in the HBT, it is of crucial importance to have both an accurate theoretical description and physical interpretation of transport in the device. Several research papers focusing on various aspects of HBTs have appeared in the literature. In these papers, due to their complexity, it is usually difficult to maintain a physical picture of the device operation which is a fundamental requirement for innovation in design. A model should be as simple as possible to maintain the transparency of the conduction mechanisms. At the same time, it should be as general as possible to cover the complete range of materials and geometries. In the present work, such a model for the HBT is developed and employed to investigate its dc and ac performances.

1.3 Thesis Organization

A brief literature survey, discussing several experimental and theoretical studies of various types of HBTs is presented in Chapter 2. In Chapter 3, a unified and concise theoretical formulation of the device dc and ac characteristics is developed. The graded single HBT, the abrupt single HBT, and the graded double HBT are modeled in details, followed by a brief discussion on various abrupt double HBT
structures. Although the npn HBT is studied in this work, many concepts and modeling approaches presented are applicable to the pnp HBT, also. In Chapter 4, a set of typical HBTs reported in the literature are simulated and the results are compared with the reported experimental and theoretical data. Conclusions along with the proposal for future work are presented in Chapter 5.
CHAPTER 2

LITERATURE SURVEY

2.1 Experimental Reports

The availability of high quality heterojunction structures produced by MBE and MOCVD techniques has made it possible to fabricate HBTs offering high gain and high frequency performances. The first attempts were mainly focused on the AlGaAs/GaAs system. Ito et al. [4] were among the pioneers who fabricated MBE-grown AlGaAs/GaAs abrupt single HBT. They achieved a cut-off frequency of 25 GHz for a collector current density of $10^4$ A/cm$^2$ and a collector-emitter voltage of 3 V. In their device, the cut-off frequency increased in the high current density region with neither the base pushout nor the emitter crowding effects. They found the limitation on the cut-off frequency to be caused mainly by the emitter series resistance.

In order to fully exploit the high-speed potential of the AlGaAs/GaAs HBT, the device must be scaled down to minimize the emitter, base, and collector series resistances and the base and collector transit times. However, in a small-geometry device for which the area-to-perimeter ratio is small, the base surface recombination current can be a major problem leading to significant degradation of the current gain, the so-called “emitter size effect”. Liu et al. [5] fabricated AlGaAs/GaAs graded single HBTs of various emitter areas to examine the diode ideality factor.
for surface recombination. For bulk recombination, the ideality factor is usually 1, for space-charge recombination, the factor is 2. According to Liu et al. [5], the ideality factor for surface recombination is between 1 and 1.33. Several sub-micron processes have been developed to suppress the surface recombination in AlGaAs/GaAs HBTs. Lee et al. [6] utilized a self-aligned process with the major feature of incorporation of a thin depleted AlGaAs layer as a surface passivation structure around the entire base-emitter junction periphery to reduce the surface recombination, as a result of which dc current gains of more than 30 and cut-off frequencies of more than 40 GHz were obtained at a collector current density of $10^5$ A/cm$^2$. By using a self-aligned technique, Yang et al. [7] fabricated sub-micron graded single HBTs of different emitter areas with very heavily carbon doped ($10^{20}$ cm$^{-3}$) base layers. The heavily carbon doped base was intended to minimize the influence of surface recombination in the base region and provide low base series resistance. Their devices exhibited dc current gains and cut-off frequencies essentially independent of the emitter size.

Another important issue in the fabrication of AlGaAs/GaAs HBTs is the growth of low resistance base contacts. The predominant factor in the base resistance of AlGaAs/GaAs HBTs is the contact resistance, resulting from the relatively high contact resistivity of p-GaAs. Increasing the doping density in the base is a simple and effective approach for lowering the contact resistance, but it often causes other problems, such as dopant redistribution during epitaxial growth and reduced reliability. Kusano et al. [8] developed a process for fabricating base electrodes with very low ohmic contact resistivity of $10^{-7}$Ω.cm$^2$, using a AuZn/Mo/Au alloy. The fabricated AlGaAs/GaAs HBT was shown to have a cut-off frequency of 45 GHz for a collector current density $10^4$ A/cm$^2$. Shimawaki et al. [9] presented a new approach to fabricating high performance HBTs with low base contact re-
sistance, using metalorganic molecular beam epitaxy (MOMBE) selective growth process.

Besides the AlGaAs/GaAs system, many other material combinations have been investigated for the realization of optimum HBT structures. InGaP/GaAs HBTs have attracted much attention as an alternative to AlGaAs/GaAs HBTs. Some advantages of the InGaP/GaAs system have been demonstrated, such as high etching selectivity between GaAs and InGaP for high process yield and formation of a sharp p-n junction at the InGaP/GaAs interface for low base leakage current [10]. In addition, the InGaP/GaAs heterostructure is believed to have an energy band alignment more favorable for the optimization of npn transistors than that of AlGaAs/GaAs heterostructure [11]. Song et al. [11] reported on the fabrication of an MOCVD-grown InGaP/GaAs double HBT with heavily carbon-doped base. The device exhibited a cut-off frequency of 53 GHz while maintaining a high breakdown voltage of 20 V which is highly favorable for high-power microwave applications.

InP/InGaAs HBTs grown on InP substrates are other possibilities for the HBT technology. Due to the excellent transport properties of InGaAs and the low surface recombination velocity associated with p⁺-InGaAs, InP/InGaAs HBTs are emerging as key devices for high speed electronic and optoelectronic applications. Hong et al. [12] reported on the fabrication of InP/InGaAs HBTs with almost ideal dc characteristics: a gain independent of collector current, a near unity ideality factor, a very small offset voltage and a high breakdown voltage. The devices exhibited a maximum cut-off frequency of 115 GHz at a collector current density of 10⁵ A/cm².

Si/SiGe HBTs are considered to be serious contenders for the next generation of high speed, high frequency devices based on Si technology. These devices have
an edge over III-V compound HBTs from the processing and cost points of view, because Si is much less expensive than any other semiconductor material and Si/SiGe processing is compatible with the existing Si processing technology. Over a relatively short period of time, Si/SiGe HBTs have progressed from a laboratory curiosity to the fastest Si-based bipolar transistors, as reported by Patton et al. [13] with the realization of 75 GHz cut-off frequency Si/SiGe HBTs.

Although InP/InGaAs and AlInAs/InGaAs HBTs have exhibited excellent high frequency performances, their immature InP-based process technology prevents these HBTs on InP substrates from being candidates for future applications. At present, AlGaAs/GaAs and Si/SiGe HBTs are the prominent devices for high speed analog and digital circuit applications. In a comparison made by Liou [14], the AlGaAs/GaAs HBT has been indicated to possess less uniform, but higher peak current gain and cut-off frequency than its Si/SiGe counterpart. Furthermore, it has been shown that at high current densities the thermal effects become important and degrade the performance of AlGaAs/GaAs HBT more significantly than that of Si/SiGe HBT, due to the poorer thermal conductivity of GaAs than Si.

2.2 Theoretical Reports

There have been several recent publications relating to analytical descriptions of HBT characteristics where one of the Ebers-Moll or Gummel-Poon methodologies [15] has been utilized. Teeter et al. [16] investigated several models for the graded single HBT to determine their usefulness at millimeter wave frequencies. The most detailed model involved numerically solving the moments of the Boltzmann transport equation. Two analytical models, the conventional Gummel-Poon and a modified Ebers-Moll model, were also employed. They found that the commonly used Gummel-Poon model exhibits poor agreement with the numerical
and experimental data at millimeter wave frequency, due to the neglect of transit times. They also found that with Ebers-Moll model improved agreement between measured and modeled data results in by the inclusion of transit time effects.

The high current handling capability of HBTs makes them very attractive for high power, high frequency applications. Such a high power level together with the poor thermal conductivities of the usual HBT materials inevitably generates a large amount of heat in the device and therefore results in a much higher temperature in the HBT than that of the ambient. Liou et al. [17] presented a detailed analytical model to predict the dc and ac performances of the graded single HBT. Their results suggested that the assumption of the HBT having the same temperature as that of the ambient, can overestimate the current gain and cut-off frequency considerably when the collector current is high. Their model correctly explained the rapid fall-off of the current gain and cut-off frequency at high collector currents.

Yang et al. [18] studied the injection performance of the single abrupt HBT and related effects on the device characteristics. They took into account the coupled transport phenomena of drift-diffusion and tunneling-emission across the abrupt heterojunction in a single coupled formulation by employing numerical techniques. In their results, the tunneling current through the spike was shown to be small. They also found a small displacement of the p-n junction into the narrow band-gap semiconductor very attractive for the performance optimization of the single abrupt HBT, a fact which was previously reported by Zhang et al. [19] for the symmetrical graded double HBT. Zhang et al. [19] simulated the device behavior by using the conventional drift-diffusion approach which demonstrated enhanced performance with p-n junctions that are not coincident with the heterojunctions.

Besides the conventional Gummel-Poon model which is a charge-control model based on the charge stored in the base, there have been some other charge-control
models developed to describe the HBT behavior. Marty et al. [20] derived a charge-control model in which they assumed that the current flow across an abrupt heterojunction is by drift and diffusion only. Lundstrom [21] proposed that the carrier flow across the heterojunction interface can be modeled by a generalized interface transport velocity (S) which was later utilized by Ryum et al. [22] to derive a Gummel-Poon-like model for the device. The major advantages of their model were that it took the Early effect into account and was amenable to quasi-saturation phenomena. Unlike the Gummel-Poon model for the homojunction transistor, their model was strictly valid only for low injection and constant doping density in the base. Parikh et al. [23] generalized the model of Ryum et al. [22] by deriving a new charge-control relation, which was valid for arbitrary doping profiles and for all levels of injection in the base. The model was applicable to any heterojunction system, graded or abrupt, single or double. All these charge-control models were intended to study only the dc behavior of the device. As mentioned by Teeter et al. [16], the Gummel-Poon methodology fails to describe correctly the ac behavior of the device, due to the neglect of transit times.

The abrupt single HBT has been studied by Monte Carlo (MC) simulation, also. Dodd et al. [24] examined the electron transport across the base by using MC simulation. They studied the base transit time and electron distribution as a function of base width. Clear ballistic behavior (no scattering) was found to be the case only for extremely thin bases (much less than 100 Å). They concluded that over the range of base widths of interest for devices, base transport appears to be diffusive, but the electrons are very far from thermal equilibrium. The diffusive behavior was shown to arise from the sensitivity of the steady state carrier population to small amounts of large angle scattering. Kumar et al. [25] used a rigorous quantum mechanical treatment of electron tunneling and thermionic
emission across the base-emitter junction to determine the flux injected into the base region; the flux was used as the input to an MC simulator to model the electron transport from the base to the collector and estimate the base and collector transit times.

There have been some studies on the recombination processes in HBTs. Also, Parikh et al. [26] derived a model for space-charge recombination current in the abrupt and graded single HBTs. They showed that if an energy barrier is present in the conduction band at the heterojunction, the space-charge recombination current becomes interrelated with the collector current.

Most of the previously reported work fail to give a clear physical picture of the transport mechanisms under a wide range of structures and people often resort to approximations or special cases to explain their analysis physically. However, Lee et al. [27] introduced a new approach in modeling the dc behavior of the HBT, based on the concept of "transmission and reflection" of diffusing electrons in the base as they encounter the energy barrier made by the conduction band discontinuity. The model is applicable to any heterojunction system, graded or abrupt, single or double. By using this model, Lee et al. [27] successfully generated the output characteristics of HBTs fabricated in their laboratory. They also described quantitatively the physical effects resulting in the collector-emitter offset voltage in the HBTs output characteristics. The present work uses the same approach of Lee et al. [27] to develop a generalized comprehensive formulation of the dc and ac characteristics of the HBT.

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CHAPTER 3

THEORETICAL FORMULATION

3.1 General Formulation

In this section, the dc and ac modeling of a general bipolar junction transistor is discussed. The energy band diagram of the device is illustrated in Figure 5. The emitter, base, and collector regions may have different band-gaps; however, no band discontinuity is allowed in the structure. Hence, in the device under study, the base-emitter and collector-base junctions are either homojunctions or graded heterojunctions.

3.1.1 DC Modeling

The emitter and collector dc currents, $I_E$ and $I_C$, can be formulated based on Ebers-Moll equations, as follows:

$$I_E = (I_{B}^{m} + I_{E}^{p})[\exp(\frac{qV_{BE}}{k_BT}) - 1] - \alpha I_B^{m}[\exp(-\frac{qV_{BE}}{k_BT}) - 1], \quad (3.1)$$

and

$$I_C = \alpha I_B^{m}[\exp(\frac{qV_{BE}}{k_BT}) - 1] - (I_{B}^{m} + I_{C}^{p})[\exp(-\frac{qV_{CB}}{k_BT}) - 1], \quad (3.2)$$

where $V_{BE}$ and $V_{CB}$ are the base-emitter and collector-base applied voltages, respectively, $k_BT$ is the thermal energy with $k_B$ the Boltzmann constant and $T$ the absolute temperature, and $I_{B}^{m}$, $I_{E}^{p}$, $I_{C}^{p}$, and $\alpha$ are given by:
In Equations (3.3) to (3.6), $q$ is the electronic charge, $n_g$ is the equilibrium electron density in the base, $D_B$ is the electron diffusion coefficient in the base, and $L_B$ is the electron diffusion length in the base, defined as $L_B = (D_B \tau_B)^{\frac{1}{2}}$ where $\tau_B$ is the electron recombination lifetime in the base. Similarly, $p_E$ is the equilibrium hole density in the emitter, $D_E$ is the hole diffusion coefficient in the emitter, and $L_E$ is the hole diffusion length in the emitter, defined as $L_E = (D_E \tau_E)^{\frac{1}{2}}$ where $\tau_E$ is the hole recombination lifetime in the emitter. Finally, $p_C$, $D_C$, $L_C$, and $\tau_C$ are, respectively, the same parameters as $p_E$, $D_E$, $L_E$, and $\tau_E$ defined for the collector region. Note that, for the sake of simplicity, the current carried by electrons flowing from the emitter to the collector has been considered as a positive current.

The neutral base width, $W_B$, is given by:

$$ W_B = W_B^m - x_{B,\text{dep,E}} - x_{B,\text{dep,C}}, $$

where $W_B^m$ is the metallurgical base width, $x_{B,\text{dep,E}}$ is the base depletion width at the base-emitter junction, and $x_{B,\text{dep,C}}$ is the base depletion width at the collector-base junction (Figure 5). The expressions for $x_{B,\text{dep,E}}$ and $x_{B,\text{dep,C}}$ are obtained by solving the Poisson equation for the base-emitter and collector-base depletion layers, respectively. These expressions depend on the composition and doping profiles of the junctions, and hence, they should be derived for each specific device, separately.
The built-in voltages at the base-emitter and collector-base junctions, $V_{BE}^{bi}$ and $V_{CB}^{bi}$, are given by:

$$V_{BE}^{bi} = \frac{k_B T}{q} \ln\left[\left(\frac{m_E^2}{m_E^*}\right)^{\frac{1}{2}} \frac{N_E^d}{n_E^b}\right], \quad (3.8)$$

and

$$V_{CB}^{bi} = \frac{k_B T}{q} \ln\left[\left(\frac{m_B^2}{m_B^*}\right)^{\frac{1}{2}} \frac{N_C^d}{n_C^b}\right], \quad (3.9)$$

where $N_E^d$ and $N_C^d$ are the emitter and collector donor densities, respectively, and $m_E^*$, $m_B^*$, and $m_C^*$ are the electron effective masses in the emitter, base, and collector regions, respectively.

### 3.1.2 AC Modeling

Consider a device that operates at dc voltages $V_{BE}$ and $V_{CB}$ and is subjected to small ac voltages $v_{be}$ and $v_{cb}$ with frequency $\omega$. The instantaneous applied voltages, $v_{BE}(t)$ and $v_{CB}(t)$, and resultant currents, $i_E(t)$ and $i_C(t)$, are given by:

$$v_{BE}(t) = V_{BE} + v_{be} \exp(j\omega t), \quad (3.10)$$

$$v_{CB}(t) = V_{CB} + v_{cb} \exp(j\omega t), \quad (3.11)$$

$$i_E(t) = I_E + i_e \exp(j\omega t), \quad (3.12)$$

and

$$i_C(t) = I_C + i_c \exp(j\omega t), \quad (3.13)$$

where $v_{be} \ll V_{BE}$, $v_{cb} \ll V_{CB}$, $i_e \ll I_E$, and $i_c \ll I_C$. 

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3.1.2.1 $Y_\omega$ Parameters

The $y_\omega$ parameters which are used to describe the transistor ac characteristics are defined as:

\[
\begin{align*}
y_{ee,\omega} &= \frac{i_c}{v_{be}} \bigg|_{v_{cb}=0}, \\
y_{ec,\omega} &= \frac{i_c}{v_{ce}} \bigg|_{v_{be}=0}, \\
y_{ce,\omega} &= \frac{i_c}{v_{be}} \bigg|_{v_{cb}=0}, \\
y_{ee,\omega} &= \frac{i_c}{v_{ce}} \bigg|_{v_{be}=0}.
\end{align*}
\tag{3.14}
\]

At low frequencies, the above definitions for $y_\omega$ parameters are equivalent to:

\[
\begin{align*}
y_{EE} &= \frac{\partial I_E}{\partial V_{BE}} \bigg|_{V_{CB}=\text{constant}}, \\
y_{EC} &= \frac{\partial I_E}{\partial V_{CE}} \bigg|_{V_{BE}=\text{constant}}, \\
y_{CE} &= \frac{\partial I_C}{\partial V_{BE}} \bigg|_{V_{CB}=\text{constant}}, \\
y_{CC} &= \frac{\partial I_C}{\partial V_{CE}} \bigg|_{V_{BE}=\text{constant}},
\end{align*}
\tag{3.15}
\]

where the use of upper case subscripts and the omission of $\omega$ in the parameters notation are to emphasize the low frequency nature. By substituting the expressions for $I_E$ and $I_C$ given by Equations (3.1) to (3.6), into Equation (3.15), the low frequency $y$ parameters are obtained as:

\[
y_{EE} = \frac{q}{k_B T} (I_B^m + I_E^m) \exp\left(\frac{q V_{BE}}{k_B T}\right) + y_{BE}\left[\exp\left(\frac{q V_{BE}}{k_B T}\right) - 1\right] - (\lambda E I_B^m + \alpha y_{BE}^m)\left[\exp\left(-\frac{q V_{CB}}{k_B T}\right) - 1\right],
\tag{3.16}
\]

\[
y_{EC} = \frac{q}{k_B T} \alpha I_B^m \exp\left(-\frac{q V_{CB}}{k_B T}\right) + y_{BC}\left[\exp\left(\frac{q V_{BE}}{k_B T}\right) - 1\right] - (\lambda C I_B^m + \alpha y_{BC}^m)\left[\exp\left(-\frac{q V_{CB}}{k_B T}\right) - 1\right],
\tag{3.17}
\]

\[
y_{CE} = \frac{q}{k_B T} \alpha I_B^m \exp\left(\frac{q V_{BE}}{k_B T}\right) + (\lambda E I_B^m + \alpha y_{BE}^m)\left[\exp\left(\frac{q V_{BE}}{k_B T}\right) - 1\right] - y_{BE}\left[\exp\left(-\frac{q V_{CB}}{k_B T}\right) - 1\right],
\tag{3.18}
\]

and
\[ y_{CC} = \frac{q}{k_B T} (I_B^m + I_C^m) \exp\left(-\frac{qV_{CB}}{k_B T}\right) + \]
\[ (\lambda_C I_B^m + \alpha y_{BC}^m) [\exp\left(\frac{qV_{BE}}{k_B T}\right) - 1] - y_{BC}^m [\exp\left(-\frac{qV_{CB}}{k_B T}\right) - 1] , \quad (3.19) \]

where \( y_{BE}^m, y_{BC}^m, \lambda_E \), and \( \lambda_C \) are given by:

\[ y_{BE}^m = \left. \frac{\partial I_B^m}{\partial V_{BE}} \right|_{V_{CB}=constant} \]
\[ = A \frac{qD_B}{L_B} \left[ 1 - \coth^2\left(\frac{W_B}{L_B}\right) \right] \nu_{BE} \]  \quad (3.20)

\[ y_{BC}^m = \left. \frac{\partial I_B^m}{\partial V_{CB}} \right|_{V_{BE}=constant} \]
\[ = A \frac{qD_B}{L_B} \left[ 1 - \coth^2\left(\frac{W_B}{L_B}\right) \right] \nu_{BC} \]  \quad (3.21)

\[ \lambda_E = \left. \frac{\partial \alpha}{\partial V_{BE}} \right|_{V_{CB}=constant} \]
\[ = - \frac{\sinh\left(\frac{W_B}{L_B}\right) \nu_{BE}}{\cosh^2\left(\frac{W_B}{L_B}\right) L_B} \]  \quad (3.22)

and

\[ \lambda_C = \left. \frac{\partial \alpha}{\partial V_{CB}} \right|_{V_{BE}=constant} \]
\[ = - \frac{\sinh\left(\frac{W_B}{L_B}\right) \nu_{BC}}{\cosh^2\left(\frac{W_B}{L_B}\right) L_B} \]  \quad (3.23)

with \( \nu_{BE} \) and \( \nu_{BC} \) defined as:

\[ \nu_{BE} = \left. \frac{\partial W_B}{\partial V_{BE}} \right|_{V_{CB}=constant} \]
\[ = - \frac{dx_{dep.E}^{BE}}{dV_{BE}} \]  \quad (3.24)

and

\[ \nu_{BC} = \left. \frac{\partial W_B}{\partial V_{CB}} \right|_{V_{BE}=constant} \]
\[ = - \frac{dx_{dep.C}^{BC}}{dV_{CB}} \]  \quad (3.25)
Similar to $x_{B}^{\text{dep,E}}$ and $x_{B}^{\text{dep,C}}$, the expressions for $\frac{dx_{B}^{\text{dep,E}}}{dv_{BE}}$ and $\frac{dx_{B}^{\text{dep,C}}}{dv_{CB}}$ should be derived for each specific device, separately. If the base doping density, $N_{B}^{0}$, is high compared to the emitter and collector doping densities, $N_{E}^{0}$ and $N_{C}^{0}$, then $\frac{dx_{B}^{\text{dep,E}}}{dv_{BE}}$ and $\frac{dx_{B}^{\text{dep,C}}}{dv_{CB}}$ are negligible and so are $\nu_{BE}$ and $\nu_{BC}$. In this case, all the terms including $y_{bc}^{\text{mn}}$, $y_{bc}^{\text{m}}$, $\lambda_{e}$, and $\lambda_{c}$ in Equations (3.16)-(3.19) can be neglected. Resulting the following simplified formulae for the low frequency $y$ parameters:

\[
y_{EE} = \frac{q}{kBT}(I_{B}^{m} + I_{E}^{p})\exp\left(\frac{qV_{BE}}{kBT}\right), \tag{3.26}
\]

\[
y_{EC} = \frac{q}{kBT}\alpha I_{B}^{m}\exp\left(-\frac{qV_{CB}}{kBT}\right), \tag{3.27}
\]

\[
y_{CE} = \frac{q}{kBT}\alpha I_{B}^{m}\exp\left(\frac{qV_{BE}}{kBT}\right). \tag{3.28}
\]

and

\[
y_{CC} = \frac{q}{kBT}(I_{B}^{m} + I_{C}^{p})\exp\left(-\frac{qV_{CB}}{kBT}\right). \tag{3.29}
\]

3.1.2.2 $H_{\omega}$ Parameters

The $h_{\omega}$ parameters which are used to describe the transistor ac characteristics are defined as:

\[
\begin{align*}
    h_{fe,\omega} &= \left. \frac{v_{ce}}{i_{b}} \right|_{v_{ce}=0}, & h_{ie,\omega} &= \left. \frac{v_{be}}{i_{b}} \right|_{v_{ce}=0}, \\
    h_{ce,\omega} &= \left. \frac{i_{c}}{v_{ce}} \right|_{i_{b}=0}, & h_{re,\omega} &= \left. \frac{v_{be}}{v_{ce}} \right|_{i_{b}=0}.
\end{align*} \tag{3.30}
\]

where $i_{b}$ is the base ac current and is equal to $i_{c} - i_{e}$, and $v_{ce}$ is the collector-emitter ac voltage and is equal to $v_{be} + v_{cb}$. At low frequencies, the above definitions for $h_{\omega}$ parameters are equivalent to:

\[
\begin{align*}
    h_{FE} &= \left. \frac{\partial I_{C}}{\partial I_{B}} \right|_{V_{CE}=\text{constant}}, & h_{IE} &= \left. \frac{\partial V_{BE}}{\partial I_{B}} \right|_{V_{CE}=\text{constant}}.
\end{align*}
\]
\[ h_{OE} = \left. \frac{\partial I_C}{\partial V_{CE}} \right|_{I_B=\text{constant}}, \quad h_{RE} = \left. \frac{\partial V_{BE}}{\partial V_{CE}} \right|_{I_B=\text{constant}}. \] (3.31)

The \( h \) and \( y \) parameters are related as:

\[
\begin{align*}
    h_{fe} &= \frac{y_{ee} - y_{cc}}{y_{ee} - y_{cc} + y_{ee}}, \quad h_{te} = \frac{1}{y_{ee} - y_{cc} + y_{ee} + y_{ee}}. \\
    h_{oe} &= \frac{y_{ee} y_{cc} - y_{ee} y_{ee}}{y_{ee} - y_{ee} + y_{ee} + y_{ee}}, \quad h_{re} = -\frac{y_{ee} - y_{ee}}{y_{ee} - y_{ee} + y_{ee} + y_{ee}}. \quad (3.32)
\end{align*}
\]

The low frequency \( h \) and \( y \) parameters are related as:

\[
\begin{align*}
    h_{FE} &= \frac{y_{CE} - y_{CC}}{y_{OE} - y_{EC} - y_{CE} + y_{CC}}, \quad h_{IE} = \frac{1}{y_{OE} - y_{EC} - y_{CE} + y_{CC}}, \\
    h_{OE} &= \frac{y_{EE} y_{CC} - y_{EC} y_{CE}}{y_{EE} - y_{EC} - y_{CE} + y_{CC}}, \quad h_{RE} = -\frac{y_{EC} - y_{CC}}{y_{EE} - y_{EC} - y_{CE} + y_{CC}}. \quad (3.33)
\end{align*}
\]

3.1.2.3 Small Signal Equivalent Circuit

The small signal equivalent circuit of a transistor is shown in Figure 6. The expressions for the circuit parameters, \( g_m, Z_e, Z_\mu, \) and \( Z_o \) can be derived in terms of the \( y \) parameters, as follows.

Let \( v_{cb} = 0 \); by applying Kirchhoff current law (KCL) at the emitter and collector nodes in Figure 6, and noting that now \( v_{be} = v_{ce} \), \( i_e \) and \( i_c \) are obtained as:

\[
    i_eigg|_{v_{cb}=0} = g_m v_{be} + \frac{v_{be}}{Z_\mu} + \frac{v_{be}}{Z_\pi}, \quad (3.34)
\]

and

\[
    i_cigg|_{v_{cb}=0} = g_m v_{be} + \frac{v_{be}}{Z_\mu}. \quad (3.35)
\]

Thus, \( y_{ee} \) and \( y_{cc} \) are resulted as:

\[
    y_{ee} = g_m + \frac{1}{Z_\mu} + \frac{1}{Z_\pi}, \quad (3.36)
\]
and

\[ y_{ce,\omega} = g_m + \frac{1}{Z_\mu} \]  \hspace{1cm} (3.37)

Now, let \( v_{be} = 0 \); by applying the KCL at the emitter and collector nodes in Figure 6, and noting that now \( v_{cb} = v_{ce} \), \( i_e \) and \( i_c \) are obtained as:

\[ i_c \bigg|_{v_{be}=0} = \frac{v_{cb}}{Z_o} \]  \hspace{1cm} (3.38)

and

\[ i_c \bigg|_{v_{be}=0} = \frac{v_{cb}}{Z_o} + \frac{v_{cb}}{Z_\mu} \]  \hspace{1cm} (3.39)

Thus, \( y_{ec,\omega} \) and \( y_{cc,\omega} \) are resulted as:

\[ y_{ec,\omega} = \frac{1}{Z_o} \]  \hspace{1cm} (3.40)

and

\[ y_{cc,\omega} = \frac{1}{Z_o} + \frac{1}{Z_\mu} \]  \hspace{1cm} (3.41)

Equations (3.36), (3.37), (3.40), and (3.41) can be solved to obtain the expressions for the circuit parameters in terms of \( y_\omega \) parameters, as follows:

\[ g_m = y_{ce,\omega} - y_{cc,\omega} \]

\[ = \frac{i_C}{v_{BE}} \bigg|_{v_{CB}=0} - \frac{i_E}{v_{CB}} \bigg|_{v_{BE}=0} \]  \hspace{1cm} (3.42)

\[ \frac{1}{Z_\pi} = y_{ce,\omega} - y_{cc,\omega} \]

\[ = \frac{i_B}{v_{BE}} \bigg|_{v_{CB}=0} \]  \hspace{1cm} (3.43)

\[ \frac{1}{Z_\mu} = -(y_{ec,\omega} - y_{cc,\omega}) \]

\[ = -\frac{i_B}{v_{CB}} \bigg|_{v_{BE}=0} \]  \hspace{1cm} (3.44)
and

\[ \frac{1}{Z_0} = y_{ec,\omega} \]

\[ = \frac{i_E}{v_{CB} v_{BE}=0} . \]  

(3.45)

Separating the real and imaginary parts, Equations (3.43) and (3.42) can be written as:

\[ \frac{1}{r_\pi} = \frac{i_B}{v_{BE} v_{CB}=0} \]

\[ = y_{ee,\omega} - y_{ce,\omega} . \]  

(3.46)

\[ \frac{1}{C_\pi} = \frac{i_B}{\omega v_{BE} v_{CB}=0} \]

\[ = \frac{1}{\omega} (y_{ee,\omega} - y_{ce,\omega}) . \]  

(3.47)

\[ \frac{1}{r_\mu} = -\frac{i_B}{v_{CB} v_{BE}=0} \]

\[ = -(y_{ee,\omega} - y_{ce,\omega}) . \]  

(3.48)

and

\[ \frac{1}{C_\mu} = -\frac{i_B}{\omega v_{CB} v_{BE}=0} \]

\[ = -\frac{1}{\omega} (y_{ee,\omega} - y_{ce,\omega}) . \]  

(3.49)

where the superscripts \( r \) and \( i \) stand for real and imaginary, respectively.

Although the circuit parameters have been introduced in a purely mathematical framework, each of them corresponds to a specific physical phenomenon in the device. \( r_\pi \) is the differential resistance associated with the base-emitter junction and is dependent upon the value of the injected emitter current. \( C_\pi \) is the result of two parallel capacitances at the base-emitter junction, the depletion layer capacitance which is related to the majority carriers response to the junction voltage, and the diffusion capacitance which is related to the minority carriers response to
the junction voltage. \( r_\mu \) and \( C_\mu \) play the same roles as \( r_\pi \) and \( C_\pi \), respectively, for the collector-base junction. \( r_o \) is the device output resistance and is due to the Early effect. Finally, \( g_m v_{BE} \) represents the transistor main action as a voltage controlled current source realized through the collector current modulation by the base-emitter voltage. Since the base-emitter bias does not have significant capacitive (charging) effect on the collector-base junction and vice versa, the imaginary part of \( y_{ec,\omega} \) and \( y_{ce,\omega} \) can be ignored; therefore, the transconductance \( g_m \) and output impedance \( Z_o \) can be considered as real numbers and approximated with their real parts in Equations (3.42) and (3.45), respectively. Besides its use in simulating the small signal behavior of the transistor, the equivalent circuit clearly separates the complex resistive and capacitive effects in the device and relates them to the material parameters. Note that although the \( y_\omega \) or \( h_\omega \) parameters are defined for any mode of device operation, the equivalent circuit as illustrated in Figure 6 is applicable only for the forward active mode.

The short-circuit small signal current gain of the transistor is defined as \( \left| \frac{i_c}{i_b} \right|_{v_{ce}=0} \), which is indeed the definition for \( h_{fe,\omega} \) and can be formulated as:

\[
h_{fe,\omega} = \left| \frac{i_c}{i_b} \right|_{v_{ce}=0} = \frac{g_m - \frac{1}{Z_e}}{\frac{1}{Z_e} + \frac{1}{Z_\mu}}.
\]

A schematic Bode plot for \( h_{fe,\omega} \) is shown in Figure 7. The magnitude of \( h_{fe,\omega} \) decreases with frequency; the frequency at which \( |h_{fe,\omega}| \) drops to unity is called the transistor cut-off frequency \( f_T \). \( f_T \) is a measure of transistor gain and bandwidth, and hence is the most important figure for the device performance as an amplifier.

### 3.1.2.4 Derivation of AC Characteristics

Consider the base region of the device (Figure 5). The time and spatial evolu-
tion of the instantaneous excess electron density in the base, \( \Delta n_B(x,t) \), is governed by the minority carrier continuity equation (MCCE), given by:

\[
D_B \frac{\partial^2 \Delta n_B(x,t)}{\partial x^2} - \frac{\Delta n_B(x,t)}{\tau_B} = \frac{\partial \Delta n_B(x,t)}{\partial t}.
\]  
(3.51)

The solution to Equation (3.51) has the general form of:

\[
\Delta n_B(x,t) = \Delta n_B(x) + \delta n_B(x) \exp(j \omega t),
\]  
(3.52)

where \( \Delta n_B(x) \) is the excess electron density in the base under steady state and \( \delta n_B(x) \) is the excess electron density in the base due to the ac components of the applied voltages, \( v_{be} \) and \( v_{cb} \). Substituting Equation (3.52) into (3.51), results in:

\[
D_B \frac{d^2 \delta n_B(x)}{dx^2} - \frac{\delta n_B(x)}{\tau_B} = j \omega \delta n_B(x),
\]  
(3.53)

or

\[
\frac{d^2 \delta n_B(x)}{dx^2} - \frac{\delta n_B(x)}{\frac{\tau_B^2}{1+j \omega \tau_B}} = 0.
\]  
(3.54)

The boundary conditions at the edges of the base depletion layers, i.e. \( x = 0 \) and \( x = W_B \), are:

\[
\delta n_B(x = 0) = \frac{q v_{be}}{k_B T} n_B^0 \exp\left(\frac{q V_{BE}}{k_B T}\right),
\]  
(3.55)

and

\[
\delta n_B(x = W_B) = -\frac{q v_{cb}}{k_B T} n_B^0 \exp\left(-\frac{q V_{CB}}{k_B T}\right).
\]  
(3.56)

The solution for the second order differential equation given by Equation (3.54) with boundary conditions given by Equations (3.55) and (3.56), is:

\[
\delta n_B(x) = \frac{n_B^0}{\sinh\left[\frac{W_B}{L_B} (1+j \omega \tau_B)^{\frac{1}{2}}\right]} \times
\left\{ \frac{q v_{be}}{k_B T} \sinh\left[\frac{W_B - x}{L_B} (1+j \omega \tau_B)^{\frac{1}{2}}\right] \exp\left(\frac{q V_{BE}}{k_B T}\right) - \frac{q v_{cb}}{k_B T} \sinh\left[\frac{x}{L_B} (1+j \omega \tau_B)^{\frac{1}{2}}\right] \exp\left(-\frac{q V_{CB}}{k_B T}\right) \right\}.
\]  
(3.57)
The ac electron current at the emitter depletion layer edge, i.e. \( x = -W_{BE}^{\text{dep}} \), is obtained from Equation (3.57) as:

\[
\begin{align*}
    i_e^n(-W_{BE}^{\text{dep}}) &= i_b^n(0) \\
    &= - AqD_B \left. \frac{d\delta n_B(x)}{dx} \right|_{x=0} \\
    &= \frac{qV_{be}}{k_BT} A \frac{qD_B n_B^0}{L_B} (1 + j\omega \tau_B)^{\frac{1}{2}} \coth \left[ \frac{W_B}{L_B} (1 + j\omega \tau_B)^{\frac{1}{2}} \right] \exp \left( \frac{qV_{BE}}{k_BT} \right) + \\
    &+ \frac{qV_{cb}}{k_BT} A \frac{qD_B n_B^0}{L_B} (1 + j\omega \tau_B)^{\frac{1}{2}} \frac{1}{\sinh \left[ \frac{W_B}{L_B} (1 + j\omega \tau_B)^{\frac{1}{2}} \right]} \exp \left( - \frac{qV_{CB}}{k_BT} \right).
\end{align*}
\]

Equation (3.58) can be rearranged as:

\[
\begin{align*}
    i_e^n(-W_{EB}^{\text{dep}}) &= \frac{qV_B E}{k_BT} I_{B,\omega}^m \exp \left( \frac{qV_{BE}}{k_BT} \right) + \frac{qV_{CB}}{k_BT} \alpha \omega I_{B,\omega}^m \exp \left( - \frac{qV_{CB}}{k_BT} \right), \quad (3.59)
\end{align*}
\]

where

\[
I_{B,\omega}^m = A \frac{qD_B n_B^0}{L_B} (1 + j\omega \tau_B)^{\frac{1}{2}} \coth \left[ \frac{W_B}{L_B} (1 + j\omega \tau_B)^{\frac{1}{2}} \right], \quad (3.60)
\]

and

\[
\alpha = \frac{1}{\cosh \left[ \frac{W_B}{L_B} (1 + j\omega \tau_B)^{\frac{1}{2}} \right]} \quad (3.61)
\]

Similarly, the MCCE for instantaneous excess hole densities in the emitter and collector can be solved to obtain the corresponding expressions for the ac hole currents. Adding up the electron and hole components of the currents appropriately, the ac version of Ebers-Moll equations for \( i_e \) and \( i_c \) are obtained as:

\[
\begin{align*}
    i_e &= \frac{qV_{be}}{k_BT} (I_{B,\omega}^m + I_{E,\omega}^p) \exp \left( \frac{qV_{BE}}{k_BT} \right) + \frac{qV_{cb}}{k_BT} \alpha \omega I_{B,\omega}^m \exp \left( - \frac{qV_{CB}}{k_BT} \right), \quad (3.62)
\end{align*}
\]

and

\[
\begin{align*}
    i_c &= \frac{qV_{be}}{k_BT} \alpha \omega I_{B,\omega}^m \exp \left( \frac{qV_{BE}}{k_BT} \right) + \frac{qV_{cb}}{k_BT} (I_{B,\omega}^m + I_{C,\omega}^p) \exp \left( - \frac{qV_{CB}}{k_BT} \right), \quad (3.63)
\end{align*}
\]
where

\[ I_{B,\omega}^{m} = A \frac{qD_{B}n_{B}}{L_{B}} (1 + j\omega\tau_{B})^{\frac{1}{2}} \coth \left[ \frac{W_{B}}{L_{B}} (1 + j\omega\tau_{B})^{\frac{1}{2}} \right] . \]  

(3.64)

\[ I_{E,\omega}^{p} = A \frac{qD_{E}n_{E}}{L_{E}} (1 + j\omega\tau_{E})^{\frac{1}{2}} . \]  

(3.65)

\[ I_{C,\omega}^{p} = A \frac{qD_{C}n_{C}}{L_{C}} (1 + j\omega\tau_{C})^{\frac{1}{2}} . \]  

(3.66)

and

\[ \alpha_{\omega} = \frac{1}{\cosh \left[ \frac{W_{B}}{L_{B}} (1 + j\omega\tau_{B})^{\frac{1}{2}} \right]} . \]  

(3.67)

By substituting Equations (3.62) and (3.63) into Equation (3.14), the \( y_{\omega} \) parameters are obtained as:

\[ y_{ee,\omega} = \frac{q}{k_{B}T} (I_{B,\omega}^{m} + I_{E,\omega}^{p}) \exp \left( \frac{qV_{BE}}{k_{B}T} \right) . \]  

(3.68)

\[ y_{ee,\omega} = \frac{q}{k_{B}T} \alpha_{\omega} I_{B,\omega}^{m} \exp \left( -\frac{qV_{CB}}{k_{B}T} \right) . \]  

(3.69)

\[ y_{ee,\omega} = \frac{q}{k_{B}T} \alpha_{\omega} I_{B,\omega}^{m} \exp \left( \frac{qV_{BE}}{k_{B}T} \right) . \]  

(3.70)

and

\[ y_{ee,\omega} = \frac{q}{k_{B}T} (I_{B,\omega}^{m} + I_{C,\omega}^{p}) \exp \left( -\frac{qV_{CB}}{k_{B}T} \right) . \]  

(3.71)

By substituting Equations (3.68)-(3.71) into Equations (3.42)-(3.45), the circuit parameters are obtained as:

\[ g_{m} = \frac{q}{k_{B}T} \alpha_{\omega} I_{B,\omega}^{m} \exp \left( \frac{qV_{BE}}{k_{B}T} \right) - \exp \left( -\frac{qV_{CB}}{k_{B}T} \right) \]  

(3.72)

\[ \frac{1}{Z_{\pi}} = \frac{q}{k_{B}T} [(1 - \alpha_{\omega}) I_{B,\omega}^{m} + I_{E,\omega}^{p}] \exp \left( \frac{qV_{BE}}{k_{B}T} \right) , \]  

(3.73)

\[ \frac{1}{Z_{\mu}} = \frac{q}{k_{B}T} [(1 - \alpha_{\omega}) I_{B,\omega}^{m} + I_{E,\omega}^{p}] \exp \left( -\frac{qV_{CB}}{k_{B}T} \right) , \]  

(3.74)

and

\[ \frac{1}{Z_{o}} = \frac{q}{k_{B}T} \alpha_{\omega} I_{B,\omega}^{m} \exp \left( -\frac{qV_{CB}}{k_{B}T} \right) . \]  

(3.75)
While deriving the ac currents $i_e$ and $i_c$ given by Equations (3.62) and (3.63), the following simplifications were made. Firstly, the ac changes of the neutral base width, $W_B$, due to the ac voltages, $v_{be}$ and $v_{cb}$, are neglected. Secondly, the time needed for the injected carriers to cross the base-emitter and collector-base depletion layers is neglected. The first simplification results in the exclusion of the Early effect which is reasonable for a heavily doped base, as $W_B$ does not change much with the device biases. Additionally, it results in the exclusion of the depletion layers capacitances which is incorrect specially in low level of injection where the depletion capacitances are the dominant ones in the device. In order to avoid these problems, the MCCE must be solved along with moving boundaries, which is not feasible by analytical methods. Instead, the dc expressions for the depletion capacitances, $C_{\pi}^{\text{dep}}$ and $C_{\mu}^{\text{dep}}$, will be used to correct $C_{\pi}$ and $C_{\mu}$ and hence, the $y_{\omega}$ parameters. $C_{\pi}^{\text{dep}}$ and $C_{\mu}^{\text{dep}}$ are given by:

\begin{equation}
C_{\pi}^{\text{dep}} = Aq N_B \left| \frac{d x_{\pi}^{\text{dep},E}}{d V_{BE}} \right| ,
\end{equation}

and

\begin{equation}
C_{\mu}^{\text{dep}} = Aq N_B \frac{d x_{\mu}^{\text{dep},C}}{d V_{CB}} ,
\end{equation}

and similar to $\frac{d x_{\pi}^{\text{dep},E}}{d V_{BE}}$ and $\frac{d x_{\mu}^{\text{dep},C}}{d V_{CB}}$, they should be derived for each specific device separately. The second simplification is equivalent to the exclusion of the time delays in the currents, resulted while flowing through the depletion layers. This is reasonable for the relatively narrow base-emitter depletion layer, but not so for the collector-base junction, as the collector region is usually lightly doped and hence, the collector-base depletion width is large specially in forward active mode of operation. To correct for this effect, the MCCE for the electrons must be solved over an extended region including the collector-base depletion layer, which is out
of the scope of this work. Instead, the time delay, \( \tau_{\text{delay}} \), resulted while electrons cross the collector-base depletion layer, is simulated in the equivalent circuit by adding a second capacitance, \( C_{\mu}^{\text{delay}} \), between the base and collector terminals in parallel with \( C_{\mu} \), given by:

\[
C_{\mu}^{\text{delay}} = \tau_{\mu}^{\text{delay}} g_m = \frac{W_{\text{CB}}^{\text{dep}}}{v_{\text{sat}}} g_m ,
\]  

(3.78)

where \( W_{\text{CB}}^{\text{dep}} \) is the collector-base depletion layer width and \( v_{\text{sat}} \) is the saturation velocity with which the electrons travel through this layer [28]. After these corrections, the \( y_{\omega} \) parameters given by Equations (3.68)-(3.71), are modified to:

\[
y_{\text{ee}, \omega} = \frac{q}{k_B T} \left( I_{B_{\omega}}^{m} + I_{E_{\omega}}^{p} \right) \exp \left( \frac{qV_{BE}}{k_B T} \right) + j\omega C_{\mu}^{\text{dep}} ,
\]  

(3.79)

\[
y_{\text{ec}, \omega} = \frac{q}{k_B T} \alpha_{\omega} I_{B_{\omega}}^{m} \exp \left( -\frac{qV_{CB}}{k_B T} \right) ,
\]  

(3.80)

\[
y_{\text{ce}, \omega} = \frac{q}{k_B T} \alpha_{\omega} I_{E_{\omega}}^{m} \exp \left( \frac{qV_{BE}}{k_B T} \right) ,
\]  

(3.81)

and

\[
y_{\text{cc}, \omega} = \frac{q}{k_B T} \left( I_{B_{\omega}}^{m} + I_{E_{\omega}}^{p} \right) \exp \left( -\frac{qV_{CB}}{k_B T} \right) + j\omega C_{\mu}^{\text{dep}} + j\omega C_{\mu}^{\text{delay}} .
\]  

(3.82)

Similarly, the circuit parameters given by Equations (3.72)-(3.75), are modified to:

\[
g_m = \frac{q}{k_B T} \alpha_{\omega} I_{B_{\omega}}^{m} \left[ \exp \left( \frac{qV_{BE}}{k_B T} \right) - \exp \left( -\frac{qV_{CB}}{k_B T} \right) \right] ,
\]  

(3.83)

\[
\frac{1}{Z_{\pi}} = \frac{q}{k_B T} \left[ (1 - \alpha_{\omega}) I_{B_{\omega}}^{m} + I_{E_{\omega}}^{p} \right] \exp \left( \frac{qV_{BE}}{k_B T} \right) + j\omega C_{\mu}^{\text{dep}} ,
\]  

(3.84)

\[
\frac{1}{Z_{\mu}} = \frac{q}{k_B T} \left[ (1 - \alpha_{\omega}) I_{B_{\omega}}^{m} + I_{E_{\omega}}^{p} \right] \exp \left( -\frac{qV_{CB}}{k_B T} \right) + j\omega C_{\mu}^{\text{dep}} + j\omega C_{\mu}^{\text{delay}} , (3.85)
\]

and

\[
\frac{1}{Z_{\omega}} = \frac{q}{k_B T} \alpha_{\omega} I_{B_{\omega}}^{m} \exp \left( -\frac{qV_{CB}}{k_B T} \right) .
\]  

(3.86)
Due to the complex nature of the ac characteristics, further simplification of the expressions for the circuit parameters is not possible. Equations (3.83)-(3.86) can be used along with Equation (3.50) to analyze the frequency behavior of $h_{fe\omega}$ and thus determine the cut-off frequency, $f_T$. 

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3.2 Graded Single HBT

The graded single HBT (Figure 1) can be modeled by using the general formulation developed in Section 3.1, along with the appropriate expressions for \( x_{B}^{\text{dep,E}} \), \( x_{B}^{\text{dep,C}} \), \( \frac{dx_{B}^{\text{dep,E}}}{dV_{BE}} \), \( \frac{dx_{B}^{\text{dep,C}}}{dV_{CB}} \), \( C_{\mu}^{\text{dep}} \), and \( C_{\mu}^{\text{dep}} \) presented in the following. The expression for \( x_{B}^{\text{dep,E}} \) is obtained by solving the Poisson equation for the base-emitter depletion layer including the graded layer with a position dependent permittivity as [29]:

\[
x_{B}^{\text{dep,E}} = \frac{A_{2} + (A_{2}^{2} + 4A_{1}A_{3})^{1/2}}{2A_{1}} ,
\]

with

\[
A_{1} = \frac{1}{2} \frac{N_{B}^{2}}{\varepsilon_{B}} (1 + \frac{\varepsilon_{B}N_{B}^{2}}{\varepsilon_{E}N_{E}^{2}}) ,
\]

\[
A_{2} = \frac{qN_{B}^{2}}{\varepsilon_{B}V_{BE}}(\frac{\varepsilon_{B}}{\varepsilon_{E}} - 1) ,
\]

and

\[
A_{3} = V_{BE}^{bi} - V_{BE} - \frac{1}{2} q \frac{N_{E}^{2}}{\varepsilon_{E}} (W_{BE}^{g})^{2}(1 - \frac{\varepsilon_{E}}{\varepsilon_{B}})(1 - \frac{1}{6} \frac{\varepsilon_{E}}{\varepsilon_{B}}) .
\]

The expression for \( x_{B}^{\text{dep,C}} \) is the one for a homo junction [28], given by:

\[
x_{B}^{\text{dep,C}} = \frac{1}{qN_{B}^{2}} \left[ \frac{2\varepsilon_{E}N_{B}^{2}(qV_{CB}^{bi} + qV_{CB})}{1 + \frac{N_{B}^{2}}{N_{C}^{2}}} \right]^{1/2} ,
\]

In Equations (3.87)-(3.91), \( \varepsilon_{E} \), \( \varepsilon_{B} \), and \( \varepsilon_{C} \) are the emitter, base, and collector permittivities, respectively, and \( W_{BE}^{g} \) is the base-emitter junction grading width. The expressions for \( \frac{dx_{B}^{\text{dep,E}}}{dV_{BE}} \) and \( \frac{dx_{B}^{\text{dep,C}}}{dV_{CB}} \) are obtained from Equations (3.87)-(3.91) as:

\[
\frac{dx_{B}^{\text{dep,E}}}{dV_{BE}} = -(A_{2}^{2} + 4A_{1}A_{3})^{-1/2} ,
\]

and

\[
\frac{dx_{B}^{\text{dep,C}}}{dV_{CB}} = \frac{q}{2} \frac{1}{qN_{B}^{2}} \left( \frac{2\varepsilon_{E}N_{B}^{2}}{1 + \frac{N_{B}^{2}}{N_{C}^{2}}} \right)^{1/2} (qV_{CB}^{bi} + qV_{CB})^{-1/2} .
\]
Substituting Equations (3.92) and (3.93) into Equations (3.76) and (3.77), respectively, the expressions for $C_{\pi}^{\text{dep}}$ and $C_{\mu}^{\text{dep}}$ are obtained as:

\[
C_{\pi}^{\text{dep}} = A q N_B^2 (A_2^2 + 4 A_1 A_3)^{-\frac{1}{2}} . \tag{3.94}
\]

and

\[
C_{\mu}^{\text{dep}} = A q \frac{2\varepsilon B N_B}{2} (q V_{CB}^V + q V_{CB})^{-\frac{1}{2}} . \tag{3.95}
\]
3.3 Abrupt Single HBT

In the abrupt single HBT (Figure 2), the presence of the energy barrier made by the conduction band discontinuity retards the electron flow, resulting in a few modifications in the general formulation developed in Section 3.1, as discussed in this section.

3.3.1 DC Modeling

3.3.1.1 Derivation of DC Characteristics

The emitter and collector dc electron currents of the abrupt device, $I_{E}^{a,n}$ and $I_{C}^{a,n}$, can be modeled based on their counterparts in the general formulation of Section 3.1, including transmission and reflection processes occurring at the energy barrier [27], as follows. Consider the electron injection from the emitter (Figure 8.a). The electrons incident to the barrier get transmitted and reflected by it. The incident and reflected electrons, together, form that component of $I_{E}^{a,n}$ which is injected from the emitter, $I_{E}^{a,n}|_{Einj}$. This process can be mathematically described as:

$$I_{E}^{a,n}|_{Einj} = I_{E}^{n}|_{Einj} - (1 - T_{E}) I_{E}^{n}|_{Einj}$$

$$= T_{E} I_{E}^{n}|_{Einj} .$$

(3.96)

where

$$I_{E}^{n}|_{Einj} = I_{B}^{n}[\exp(\frac{qV_{BE}}{k_{B}T_{C}}) - 1] .$$

(3.97)

according to Equation (3.1). $T_{E}$ is the transmission coefficient of the barrier. The transmitted electrons diffuse through the base and form that component of $I_{C}^{a,n}$ which is injected from the emitter, $I_{C}^{a,n}|_{Einj}$. This process can be mathematically
described as:

\[ I_{C}|_{\text{E inj}}^{a,n} = \alpha T_E I_{E}|_{\text{E inj}}^{n} \]

where \( \alpha \) is the base transport factor given by Equation (3.6). Now, consider the electron injection from the collector (Figure 8.b). After diffusing through the base, the incident electrons get transmitted and reflected by the barrier. The transmitted electrons form that component of \( I_{E}|_{\text{E inj}}^{a,n} \) which is injected from the collector, \( I_{C}|_{\text{C inj}}^{a,n} \).

The reflected electrons diffuse back through the base back to the collector, and together with the incident electrons form that component of \( I_{C}|_{\text{C inj}}^{a,n} \) which is injected from the collector, \( I_{C}|_{\text{C inj}}^{a,n} \). These processes can be mathematically described as:

\[ I_{E}|_{\text{C inj}}^{a,n} = \alpha T_E I_{C}|_{\text{C inj}}^{a,n} \]

and

\[ I_{C}|_{\text{C inj}}^{a,n} = I_{C}|_{\text{C inj}}^{n} - \alpha^2(1 - T_E) I_{C}|_{\text{C inj}}^{n} \]

\[ = [1 - \alpha^2(1 - T_E)] I_{C}|_{\text{C inj}}^{n} \]

where

\[ I_{C}|_{\text{C inj}}^{n} = -I_{B}|_{\text{E inj}}^{n}\exp\left(-\frac{q V_{C B}}{k_B T}\right) - 1 \]

according to Equation (3.2). The hole transport in the abrupt device is not affected by the valence band discontinuity, since no energy barrier is created in the valence band as in the conduction band at the base-emitter heterojunction. Hence, the emitter and collector hole currents of the abrupt device, \( I_{E}|_{\text{E inj}}^{a,p} \) and \( I_{C}|_{\text{C inj}}^{a,p} \), are given by:

\[ I_{E}|_{\text{E inj}}^{a,p} = I_{E}^{p} \]

and

\[ I_{C}|_{\text{C inj}}^{a,p} = I_{C}^{p} \]
where

\[ I^p_E = I_E^{sp}[\exp\left(\frac{qV_{BE}}{k_BT}\right) - 1] , \]  

(3.104)

and

\[ I^p_C = -I_C^{sp}[\exp\left(-\frac{qV_{CB}}{k_BT}\right) - 1] . \]  

(3.105)

according to Equations (3.1) and (3.2). The total emitter and collector currents of the abrupt device, \( I_E^a \) and \( I_C^a \), are obtained by adding, appropriately, the components given by Equation (3.96), (3.98), (3.99), (3.100), (3.102), and (3.103), as follows:

\[ I_E^a = T_E I_E^{Einj} + \alpha T_E I_C^{Cinj} + I_E^p , \]  

(3.106)

and

\[ I_C^a = \alpha T_E I_E^{Einj} + T_3 I_C^{Cinj} + I_C^p . \]  

(3.107)

where

\[ T_3 = 1 - \alpha^2(1 - T_E) . \]  

(3.108)

Substituting Equations (3.97), (3.101), (3.104), and (3.105) into Equations (3.106) and (3.107), the dc characteristics of the abrupt single HBT are obtained as:

\[ I_E^a = (T_E I_B^m + I_E^{sp})[\exp\left(\frac{qV_{BE}}{k_BT}\right) - 1] - \alpha T_E I_B^m[\exp\left(-\frac{qV_{CB}}{k_BT}\right) - 1] . \]  

(3.109)

and

\[ I_C^a = \alpha T_E I_B^m[\exp\left(\frac{qV_{BE}}{k_BT}\right) - 1] - (T_3 I_B^m + I_C^{sp})[\exp\left(-\frac{qV_{CB}}{k_BT}\right) - 1] . \]  

(3.110)

The expressions for \( x_B^{dep.E} \) and \( x_B^{dep.C} \) are given by:

\[ x_B^{dep.E} = \frac{1}{qN_B^a} \frac{2\varepsilon_B N_B^a (\Delta E_{EB}^c + qV_{BB}^{hi} - qV_{BE})}{1 + \frac{e_B N_B^a}{\varepsilon E N_E^a}} \]  

(3.111)
and

\[ x_{\text{dep},C}^d = \frac{1}{q N_B^2} \left[ \frac{2e_B N_B^2 (q V^b_{CB} + q V^c_{CB})}{1 + \frac{N_A^2}{N_C^2}} \right]^{1/2} . \]  

(3.112)

where \( \Delta E^c_{EB} \) is the conduction band discontinuity at the base-emitter junction.

### 3.3.1.2 Transmission Coefficient

The physical mechanisms of electron transport across the conduction band barrier at the base-emitter junction are quantum tunneling and thermionic emission. Between these two, the tunneling contribution to the electron current is shown to be negligible for a wide range of heterojunction configurations, due to the relatively wide energy barrier at the junction [18]. Considering the thermionic emission as the effective transport mechanism, only the electrons with thermal energies exceeding the barrier height are transmitted and the rest are reflected (Figure 9). Assuming a Maxwellian distribution for the electrons, the transmission coefficient through the barrier, \( T_E \), is given by:

\[ T_E = \exp\left(-\frac{q V^\text{barrier}_{BE}}{k_B T} \right) . \]  

(3.113)

where

\[ V^\text{barrier}_{BE} = \frac{\Delta E^c_{EB} - \Delta E^c_{EB} + V^b_{BE} - V^c_{BE}}{q} \]  

(3.114)

The electrons that surmount the energy barrier are injected into the base with high kinetic energy, and hence, they behave as a thermodynamic system with a temperature much higher than the lattice temperature. These so-called "hot electrons" enter the base with high forward velocity and high diffusion coefficient, \( D_B \), but they undergo scattering events while crossing the base region and loose
part of their initial kinetic energy before reaching the collector-base junction. Their rate of thermalization depends on the base width. In a narrow base device, the electrons undergo very few scattering events and hence, are expected to cross the base without significant decrease in their kinetic energy or temperature. In a wide base device, the electrons undergo many scattering events and hence, are expected to lose most of their extra kinetic energy and come into thermal equilibrium with the lattice before reaching the collector-base junction. To derive the expressions for the abrupt device dc currents given by Equations (3.109) and (3.110), it is implicitly assumed that the base is narrow enough to let the electrons traverse the whole base width with the initial high kinetic energy, forward velocity, and diffusion coefficient, $D_B$. This assumption is well-justified, because wide base devices are not of interest for HBT design. For wide base devices where the assumption is not valid anymore, the diffusion of hot electrons through the base takes place with a position dependent diffusion coefficient, $D_B(x)$, determined by the scattering mechanisms of the hot electrons. In this case, the transport problem can not be formulated by the Ebers-Moll equations anymore; instead, it requires the solution of the Boltzman transport equation [28].

3.3.2 AC Modeling

3.3.2.1 Derivation of AC Characteristics

Consider a device that operates at dc voltages $V_{BE}$ and $V_{CB}$ and is subjected to small ac voltages $v_{be}$ and $v_{cb}$ with frequency $\omega$. The instantaneous applied voltages, $v_{BE}(t)$ and $v_{CB}(t)$, and resultant currents, $i_E^a(t)$ and $i_C^a(t)$, are given by:

$$v_{BE}(t) = V_{BE} + v_{be}\exp(j\omega t), \quad (3.115)$$

$$v_{CB}(t) = V_{CB} + v_{cb}\exp(j\omega t), \quad (3.116)$$

$$i_E^a(t) = I_E^a + i_E^a\exp(j\omega t), \quad (3.117)$$

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\[ i_c^a(t) = I_c^a + i_c^a \exp(j\omega t) \, . \quad (3.118) \]

The transmission coefficient has an ac term due to its bias dependency, as follows:
\[ t_E(t) = T_E + t_e \exp(j\omega t) \, , \quad (3.119) \]

where
\[ t_e = \frac{qV_{BE}}{k_BT(1 + \frac{eB_{N_2}}{eB_{N_2}})} T_E \, . \quad (3.120) \]

The emitter and collector ac electron currents of the abrupt device, \( i_e^{a,n} \) and \( i_c^{a,n} \), can be modeled based on their counterparts in the general formulation, including transmission and reflection processes occurring at the energy barrier. Using a similar procedure to that employed in Subsection 3.3.1 and including the ac current components resulting from the ac part of the transmission coefficient, the ac electron currents of the abrupt device injected from the emitter, \( i_e^{a,n}|_{Einj} \) and \( i_c^{a,n}|_{Einj} \), and the ones injected from the collector, \( i_e^{a,n}|_{Cinj} \) and \( i_c^{a,n}|_{Cinj} \), are obtained as:
\[ i_e^{a,n}|_{Einj} = T_E i_e^{n}|_{Einj} + t_e I_E^{n}|_{Einj} \, , \quad (3.121) \]
\[ i_c^{a,n}|_{Einj} = \alpha_\omega T_E i_c^{n}|_{Einj} + \alpha_\omega t_e I_E^{n}|_{Einj} \, , \quad (3.122) \]
\[ i_e^{a,n}|_{Cinj} = \alpha_\omega T_E i_c^{n}|_{Cinj} + \alpha t_e I_C^{n}|_{Cinj} \, , \quad (3.123) \]

and
\[ i_c^{a,n}|_{Cinj} = [1 - \alpha_\omega^2(1 - T_E)] i_c^{n}|_{Cinj} + \alpha_\omega t_e I_C^{n}|_{Cinj} \, , \quad (3.124) \]

where
\[ i_e^{n}|_{Einj} = \frac{qV_{BE}}{k_BT} I_B^{n}\exp\left(\frac{qV_{BE}}{k_BT}\right) \, , \quad (3.125) \]
and

\[ i_{c|\text{inj}}^n = \frac{q\nu_{cb}}{k_BT} f_{B,\omega}^m \exp(-\frac{qV_{CB}}{k_BT}), \quad (3.126) \]

according to Equations (3.62) and (3.63), and \( \alpha_s \), \( I_{E|\text{inj}}^m \), and \( I_{C|\text{inj}}^n \) are given by Equations (3.67), (3.97), and (3.101), respectively. The emitter and collector ac hole currents of the abrupt device, \( i_{e|\text{p}}^s \) and \( i_{c|\text{p}}^s \), are not affected by the valence band discontinuity and hence, are given by:

\[ i_{e|\text{p}}^s = i_{e|\text{p}}^p, \quad (3.127) \]

and

\[ i_{c|\text{p}}^s = i_{c|\text{p}}^p, \quad (3.128) \]

where

\[ i_{e|\text{p}}^p = \frac{q\nu_{be}}{k_BT} f_{E,\omega}^p \exp(\frac{qV_{BE}}{k_BT}), \quad (3.129) \]

and

\[ i_{c|\text{p}}^p = \frac{q\nu_{cb}}{k_BT} f_{C,\omega}^p \exp(-\frac{qV_{CB}}{k_BT}), \quad (3.130) \]

according to Equations (3.62) and (3.63). Using Equations (3.121)-(3.130), the emitter and collector ac currents of the abrupt single HBT, \( i_{e}^s \) and \( i_{c}^s \), are obtained as:

\[ i_{e}^s = \frac{q\nu_{be}}{k_BT} (T_E f_{B,\omega}^m + f_{E,\omega}^p) \exp(\frac{qV_{BE}}{k_BT}) + \frac{q\nu_{cb}}{k_BT} \alpha_s T_E f_{B,\omega}^m \exp(-\frac{qV_{CB}}{k_BT}) + t_E f_{B,\omega}^m [\exp(\frac{qV_{BE}}{k_BT}) - 1] - \alpha t_E f_{B,\omega}^m [\exp(-\frac{qV_{CB}}{k_BT}) - 1], \quad (3.131) \]

and

\[ i_{c}^s = \frac{q\nu_{cb}}{k_BT} \alpha_s T_E f_{B,\omega}^m \exp(\frac{qV_{BE}}{k_BT}) + \frac{q\nu_{cb}}{k_BT} (T_{C,\omega} f_{B,\omega}^m + f_{C,\omega}^p) \exp(-\frac{qV_{CB}}{k_BT}) + \alpha_s t_E f_{B,\omega}^m [\exp(\frac{qV_{BE}}{k_BT}) - 1] - \alpha t_E f_{B,\omega}^m [\exp(-\frac{qV_{CB}}{k_BT}) - 1]. \quad (3.132) \]

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3.3.2.2 \( Y_\omega \) Parameters

Substituting the expressions for the dc currents given by Equations (3.109) and (3.110) into Equation (3.15), the low frequency \( y \) parameters for the abrupt single HBT are obtained as:

\[
y_{EE}^a = \frac{q}{k_BT} (T_E I_B^m + I_C^m) \exp\left(\frac{qV_{BE}}{k_BT}\right) +
\]

\[
(u_E I_B^m + T_E y_{BE}^m)\left[\exp\left(\frac{qV_{BE}}{k_BT}\right) - 1\right] -
\]

\[
(\lambda_E T_E I_B^m + \alpha u_E I_B^m + \alpha T_E y_{BE}^m)\left[\exp\left(-\frac{qV_{CB}}{k_BT}\right) - 1\right],
\]

(3.133)

\[
y_{EC}^a = \frac{q}{k_BT} \alpha T_E I_B^m \exp\left(-\frac{qV_{CB}}{k_BT}\right) +
\]

\[
T_E y_{BC}^m\left[\exp\left(\frac{qV_{BE}}{k_BT}\right) - 1\right] -
\]

\[
(\lambda_C T_E I_B^m + \alpha T_E y_{BE}^m)\left[\exp\left(-\frac{qV_{CB}}{k_BT}\right) - 1\right],
\]

(3.134)

\[
y_{CE}^a = \frac{q}{k_BT} \alpha T_E I_B^m \exp\left(\frac{qV_{BE}}{k_BT}\right) +
\]

\[
(\lambda_E T_E I_B^m + \alpha u_E I_B^m + \alpha T_E y_{BE}^m)\left[\exp\left(\frac{qV_{BE}}{k_BT}\right) - 1\right] -
\]

\[
(u_3E I_B^m + T_3 y_{BE}^m)\left[\exp\left(-\frac{qV_{CB}}{k_BT}\right) - 1\right],
\]

(3.135)

and

\[
y_{CC}^a = \frac{q}{k_BT} (T_3 I_B^m + I_C^m) \exp\left(-\frac{qV_{CB}}{k_BT}\right) +
\]

\[
(\lambda_C T_E I_B^m + \alpha T_E y_{BC}^m)\left[\exp\left(\frac{qV_{BE}}{k_BT}\right) - 1\right] -
\]

\[
(u_3C I_B^m + T_3 y_{BC}^m)\left[\exp\left(-\frac{qV_{CB}}{k_BT}\right) - 1\right],
\]

(3.136)

where

\[
u_E = \frac{dT_E}{dV_{BE}}
\]

\[
= \frac{-q}{k_BT(1 + \frac{\epsilon_B N_s}{\epsilon_E N_E})} T_E,
\]

(3.137)
\[ u_{3E} = \frac{\partial T_3}{\partial V_{BE}} \bigg|_{V_{CB}=\text{constant}} = \alpha^2 u_E - 2\alpha(1 - T_E)\lambda_E , \quad (3.138) \]

and

\[ u_{3C} = \frac{\partial T_3}{\partial V_{CB}} \bigg|_{V_{BE}=\text{constant}} = -2\alpha(1 - T_E)\lambda_C . \quad (3.139) \]

The expressions for \( \frac{dE_{dep,E}}{dV_{BE}} \) and \( \frac{dE_{dep,C}}{dV_{CB}} \) are obtained from Equations (3.111) and (3.112) as:

\[ \frac{dE_{dep,E}}{dV_{BE}} = -q \frac{1}{2} \frac{1}{qN_B^a} \left( \frac{2\epsilon_B N_B^a}{1 + \frac{\epsilon_B N_B^a}{N_E^a}} \right)^{\frac{1}{2}} (\Delta E_{BE}^C + qV_{BE}^b - qV_{BE})^{-\frac{1}{2}} . \quad (3.140) \]

and

\[ \frac{dE_{dep,C}}{dV_{CB}} = q \frac{1}{2} \frac{1}{qN_B^a} \left( \frac{2\epsilon_B N_B^a}{1 + \frac{\epsilon_B N_B^a}{N_E^a}} \right)^{\frac{1}{2}} (qV_{CB}^b + qV_{CB})^{-\frac{1}{2}} . \quad (3.141) \]

If the base doping density, \( N_B^a \), is high compared to the emitter and collector doping densities, \( N_E^a \) and \( N_C^a \), then \( \frac{dE_{dep,E}}{dV_{BE}} \) and \( \frac{dE_{dep,C}}{dV_{CB}} \) are negligible. In this case, the expressions for the low frequency \( \gamma \) parameters given by Equations (3.133)-(3.136), reduce to:

\[ y_{\gamma EE} = \frac{q}{k_BT} (T_E I_B^{sm} + I_B^{sp}) \exp\left(\frac{qV_{BE}}{k_BT}\right) , \quad (3.142) \]

\[ y_{\gamma EC} = \frac{q}{k_BT} \alpha T_E I_B^{sm} \exp\left(-\frac{qV_{CB}}{k_BT}\right) , \quad (3.143) \]

\[ y_{\gamma CE} = \frac{q}{k_BT} \alpha T_E I_B^{sm} \exp\left(\frac{qV_{BE}}{k_BT}\right) , \quad (3.144) \]

and

\[ y_{\gamma CC} = \frac{q}{k_BT} (T_C I_B^{sm} + I_C^{sp}) \exp\left(-\frac{qV_{CB}}{k_BT}\right) . \quad (3.145) \]
Substituting the expressions for the ac currents given by Equations (3.131) and (3.132) into Equation (3.14), and modifying the results to include the depletion capacitances of the base-emitter and collector-base junctions and the collector-base depletion layer transit time, the $y_{\omega}$ parameters for the abrupt double HBT are obtained as:

\[
y_{ee,\omega}^a = \frac{q}{k_B T} (T_B I_{B,\omega}^m + I_{E,\omega}^p) \exp(\frac{q V_{BE}}{k_B T}) + \alpha T_B I_{B,\omega}^m \exp(\frac{q V_{CB}}{k_B T}) - 1\alpha u_B I_{B,\omega}^m \exp(-\frac{q V_{CB}}{k_B T}) - 1 \right) + j\omega C_{\mu}^{dep} \quad (3.146)
\]

\[
y_{ee,\omega} = \frac{q}{k_B T} \alpha \omega T_B I_{B,\omega}^m \exp(-\frac{q V_{CB}}{k_B T}) \quad (3.147)
\]

\[
y_{cc,\omega}^a = \frac{q}{k_B T} \alpha \omega T_B I_{B,\omega}^m \exp(\frac{q V_{BE}}{k_B T}) + \alpha u_B I_{B,\omega}^m \exp(\frac{q V_{BE}}{k_B T}) - 1 \right) + j\omega C_{\mu}^{delay} \quad (3.148)
\]

Substituting Equations (3.140) and (3.141) into Equations (3.76) and (3.75), respectively, the expressions for $C_{\pi}^{dep}$ and $C_{\mu}^{dep}$ are obtained as:

\[
C_{\pi}^{dep} = A_2 \frac{2\varepsilon_B N_B^a}{1 + \frac{\varepsilon_B N_B^a}{\varepsilon_C N_E^a}} \left( E_{EB}^c + qV_{BE} - qV_{BE} \right)^{-\frac{1}{2}} \quad (3.150)
\]

and

\[
C_{\mu}^{dep} = A_2 \frac{2\varepsilon_B N_B^a}{1 + \frac{\varepsilon_B N_B^a}{\varepsilon_C N_E^a}} \left( qV_{CB} + qV_{CB} \right)^{-\frac{1}{2}} \quad (3.151)
\]

and the expression for $C_{\mu}^{delay}$ is given by Equation (3.78).

3.3.2.3 $H_{\omega}$ Parameters

The low frequency $h$ parameters of the abrupt single HBT are calculated by
substituting the expressions for the low frequency $y$ parameters given by Equations (3.133)-(3.136) into Equation (3.31). The $h_\omega$ parameters are calculated by substituting the expressions for the $y_\omega$ parameters given by Equations (3.146)-(3.149) into Equation (3.30).

3.3.2.4 Small Signal Equivalent Circuit

The small signal equivalent circuit parameters of the abrupt single HBT are obtained by substituting the expressions for the $y_\omega$ parameters given by Equations (3.146)-(3.149) into Equations (3.42)-(3.45), as follows:

$$g_m^a = \frac{q}{k_B T} \alpha_\omega T_E I_B^m \left[ \exp\left(\frac{qV_{BE}^E}{k_B T}\right) - \exp\left(-\frac{qV_{CB}}{k_B T}\right) \right] + \alpha_\omega u_E I_B^m \left[ \exp\left(\frac{qV_{BE}^E}{k_B T}\right) - 1 \right] \left[ \exp\left(-\frac{qV_{CB}}{k_B T}\right) - 1 \right],$$

$$\frac{1}{Z_\alpha^a} = \frac{q}{k_B T} \left[ (1 - \alpha_\omega) T_E I_B^m + I_{E^m} \right] \exp\left(\frac{qV_{BE}^E}{k_B T}\right) + (1 - \alpha_\omega) u_E I_B^m \left[ \exp\left(\frac{qV_{BE}^E}{k_B T}\right) - 1 \right] \left[ \exp\left(-\frac{qV_{CB}}{k_B T}\right) - 1 \right] + j\omega C_{dep^a},$$

$$\frac{1}{Z_\mu^a} = \frac{q}{k_B T} \left[ (T_E - \alpha_\omega T_E) I_B^m + I_{C^m} \right] \exp\left(-\frac{qV_{CB}}{k_B T}\right) + j\omega C_{dep^a} + j\omega C_{delay^a},$$

and

$$\frac{1}{Z_\delta^a} = \frac{q}{k_B T} \alpha_\omega T_E I_B^m \exp\left(-\frac{qV_{CB}}{k_B T}\right).$$

To obtain the transistor cut-off frequency, $f_T$, Equation (3.50) along with Equations (3.152)-(3.155) can be used to analyze the frequency dependency of $h_{fe,\omega}$ and thus determine the frequency for which it drops to unity.
3.4 Graded Double HBT

The graded double HBT (Figure 3) can be modeled by using the general formulation developed in Section 3.1, along with the appropriate expressions for $x_{B}^{\text{dep,E}}$. $x_{B}^{\text{dep,C}}$, $\frac{dz_{B}^{\text{dep,E}}}{dV_{BE}}$, $\frac{dz_{B}^{\text{dep,C}}}{dV_{CB}}$, $C_{s}^{\text{dep}}$, and $C_{\mu}^{\text{dep}}$ presented in the following. The expressions for $x_{B}^{\text{dep,E}}$ and $x_{B}^{\text{dep,C}}$ are obtained by solving the Poisson equation for the base-emitter and collector-base depletion layers, respectively, including the graded layers with position dependent permittivities as [29]:

\[ x_{B}^{\text{dep,E}} = \frac{A_{2} + (A_{2}^{2} + 4A_{1}A_{3})^{\frac{1}{2}}}{2A_{1}} \]  
(3.156)

and

\[ x_{B}^{\text{dep,C}} = \frac{B_{2} + (B_{2}^{2} + 4B_{1}B_{3})^{\frac{1}{2}}}{2A_{1}} \]  
(3.157)

with

\[ A_{1} = \frac{1}{2}q\frac{N_{B}^{a}}{\epsilon_{B}}(1 + \frac{\epsilon_{B}N_{B}^{a}}{\epsilon_{E}N_{E}^{a}}) \]  
(3.158)

\[ A_{2} = q\frac{N_{B}^{a}}{\epsilon_{B}}W_{BE}^{q}(\frac{\epsilon_{B}}{\epsilon_{E}} - 1) \]  
(3.159)

\[ A_{3} = V_{BE}^{b} - \frac{1}{2}q\frac{N_{E}^{d}}{\epsilon_{E}}(W_{BE}^{q})^{2}(1 - \frac{\epsilon_{E}}{\epsilon_{B}})(1 - \frac{1}{6} \frac{\epsilon_{E}}{\epsilon_{B}}) \]  
(3.160)

\[ B_{1} = \frac{1}{2}q\frac{N_{B}^{a}}{\epsilon_{B}}(1 + \frac{\epsilon_{B}N_{B}^{a}}{\epsilon_{E}N_{E}^{a}}) \]  
(3.161)

\[ B_{2} = q\frac{N_{B}^{a}}{\epsilon_{B}}W_{CB}^{q}(\frac{\epsilon_{B}}{\epsilon_{C}} - 1) \]  
(3.162)

and

\[ B_{3} = V_{CB}^{b} + V_{CB} - \frac{1}{2}q\frac{N_{C}^{d}}{\epsilon_{C}}(W_{CB}^{q})^{2}(1 - \frac{\epsilon_{C}}{\epsilon_{B}})(1 - \frac{1}{6} \frac{\epsilon_{C}}{\epsilon_{B}}) \]  
(3.163)

where $W_{BE}^{q}$ and $W_{CB}^{q}$ are the base-emitter and collector-base junction grading widths, respectively. The expressions for $\frac{dz_{B}^{\text{dep,E}}}{dV_{BE}}$ and $\frac{dz_{B}^{\text{dep,C}}}{dV_{CB}}$ are obtained from Equations (3.156)-(3.163) as:
\[
\frac{d\epsilon_B^{dep,E}}{dV_{BE}} = -(A_2^2 + 4A_1A_3)^{-\frac{1}{2}}, \quad (3.164)
\]

and

\[
\frac{d\epsilon_B^{dep,C}}{dV_{CB}} = -(B_2^2 + 4B_1B_3)^{-\frac{1}{2}}. \quad (3.165)
\]

Substituting Equations (3.164) and (3.165) into Equations (3.76) and (3.77), respectively, the expressions for \(C_x^{dep}\) and \(C_\mu^{dep}\) are obtained as:

\[
C_x^{dep} = AqN_B^a(A_2^2 + 4A_1A_3)^{-\frac{1}{2}}, \quad (3.166)
\]

and

\[
C_\mu^{dep} = AqN_B^a(B_2^2 + 4B_1B_3)^{-\frac{1}{2}}. \quad (3.167)
\]
3.5 Abrupt Double HBT

Three possible structures for the abrupt double HBT along with their energy band diagrams are shown in Figures 10, 11, and 12. The presence of the energy barrier(s) made by the conduction band discontinuity(s) retards the electron flow, resulting in a few modifications in the general formulation developed in Section 3.1. In the present section, these modifications are discussed for the abrupt double HBT shown in Figure 12. The results are directly applicable to the two other abrupt devices shown in Figures 10 and 11.

3.5.1 DC Modeling

3.5.1.1 Derivation of DC Characteristics

The emitter and collector dc electron currents of the abrupt device, \( I_{E}^{a,n} \) and \( I_{C}^{a,n} \), can be modeled based on their counterparts in the general formulation, including multiple transmission and reflection processes occurring at the energy barriers [27], as follows. Consider the electron injection from the emitter (Figure 13). The electron current incident to the base-emitter junction barrier, \( I_{E}^{a,n} \), gets transmitted and reflected by it. The incident and reflected electron currents form the first component of \( I_{E}^{a,n} \) injected from the emitter, \( I_{E}^{a,n} \). The transmitted electron current diffuses through the base and forms the first component of \( I_{C}^{a,n} \) injected from the emitter, \( I_{C}^{a,n} \). These processes can be mathematically described as:

\[
I_{E}^{a,n} \big|_{Einj} = T_{E} I_{E}^{a,n} \big|_{Einj}, \quad (3.168)
\]

and

\[
I_{C}^{a,n} \big|_{Einj} = \alpha I_{E}^{a,n} \big|_{Einj}, \quad (3.169)
\]

where \( T_{E} \) is the transmission coefficient for the base-emitter junction barrier. Next,
$I_{E}^{a,n}|_{Einj}$ gets reflected by the collector-base junction barrier. The reflected electron current forms the second component of $I_{E}^{a,n}$ injected from the emitter, $I_{E}^{a,n}|_{Einj}$. The reflected electron current diffuses through the base and forms the second component of $I_{E}^{a,n}$ injected from the emitter, $I_{E}^{a,n}|_{Einj}$. These processes can be mathematically described as:

$$I_{E}^{a,n}|_{Einj} = -(1 - T_{C}) I_{E}^{a,n}|_{Einj}, \quad (3.170)$$

and

$$I_{C}^{a,n}|_{Einj} = \alpha I_{E}^{a,n}|_{Einj}, \quad (3.171)$$

where $T_{C}$ is the transmission coefficient for the collector-base junction barrier.

Next, $I_{E}^{a,n}|_{Einj}$ gets reflected by the base-emitter junction barrier. The reflected electron current forms the third component of $I_{E}^{a,n}$ injected from the emitter, $I_{E}^{a,n}|_{Einj}$. The reflected electron current diffuses through the base and forms the third component of $I_{C}^{a,n}$ injected from the emitter, $I_{C}^{a,n}|_{Einj}$. These processes can be mathematically described as:

$$I_{E}^{a,n}|_{Einj} = -(1 - T_{E}) I_{E}^{a,n}|_{Einj}, \quad (3.172)$$

and

$$I_{C}^{a,n}|_{Einj} = \alpha I_{E}^{a,n}|_{Einj}. \quad (3.173)$$

Similar expressions can be written for higher order transmission and reflection processes. The total emitter and collector electron currents of the abrupt device, injected from the emitter, $I_{E}^{a,n}|_{Einj}$ and $I_{C}^{a,n}|_{Einj}$, are the sum of all the components $I_{E}^{a,n}|_{Einj}$ and $I_{C}^{a,n}|_{Einj}$, and can be formulated as:

$$I_{E}^{a,n}|_{Einj} = \sum_{i} I_{E}^{a,n}|_{Einj}^{i} = T_{1} I_{E}^{a,n}|_{Einj}. \quad (3.174)$$
and

\[ I_{C}^{n,n} \big|_{E_{inj}} = \sum_{i} I_{C}^{n,n} \big|_{E_{inj}} = \alpha T_{2} I_{E}^{n} \big|_{E_{inj}}, \tag{3.175} \]

where

\[ T_{1} = \frac{T_{E}[1 - \alpha^{2}(1 - T_{C})]}{1 - \alpha^{2}(1 - T_{E})(1 - T_{C})}, \tag{3.176} \]

and

\[ T_{2} = \frac{T_{E}T_{C}}{1 - \alpha^{2}(1 - T_{E})(1 - T_{C})}. \tag{3.177} \]

According to Equation (3.1), \( I_{E}^{n} \big|_{E_{inj}} \) is given by:

\[ I_{E}^{n} \big|_{E_{inj}} = I_{B}^{n} \left\{ \exp\left(\frac{qV_{BE}}{k_{B}T}\right) - 1 \right\}. \tag{3.178} \]

Now, consider the electron injection from the collector. Using a similar procedure, the emitter and collector electron currents of the abrupt device, injected from the collector, \( I_{E}^{n,n} \big|_{C_{inj}} \) and \( I_{C}^{n,n} \big|_{C_{inj}} \), are obtained as:

\[ I_{E}^{n,n} \big|_{C_{inj}} = \alpha T_{2} I_{C}^{n} \big|_{C_{inj}}, \tag{3.179} \]

and

\[ I_{C}^{n,n} \big|_{C_{inj}} = T_{3} I_{C}^{n} \big|_{C_{inj}}, \tag{3.180} \]

where

\[ T_{3} = \frac{T_{C}[1 - \alpha^{2}(1 - T_{E})]}{1 - \alpha^{2}(1 - T_{E})(1 - T_{C})}. \tag{3.181} \]

According to Equation (3.2), \( I_{C}^{n} \big|_{C_{inj}} \) is given by:

\[ I_{C}^{n} \big|_{C_{inj}} = -I_{B}^{n} \left\{ \exp\left(-\frac{qV_{CB}}{k_{B}T}\right) - 1 \right\}. \tag{3.182} \]
The hole transport in the abrupt device is not affected by the valence band discontinuities at the abrupt junctions. Hence, the emitter and collector hole currents of the abrupt device, $I_{E}^{a,p}$ and $I_{C}^{a,p}$, are given by:

$$I_{E}^{a,p} = I_{E}^{p}$$

(3.183)

and

$$I_{C}^{a,p} = I_{C}^{p}$$

(3.184)

where

$$I_{E}^{p} = I_{E}^{p}[\exp\left(\frac{qV_{BE}}{k_{B}T}\right) - 1]$$

(3.185)

and

$$I_{C}^{p} = -I_{C}^{p}[\exp\left(-\frac{qV_{CB}}{k_{B}T}\right) - 1]$$

(3.186)

according to Equations (3.1) and (3.2). The total emitter and collector currents of the abrupt device, $I_{E}^{a}$ and $I_{C}^{a}$, are given by:

$$I_{E}^{a} = T_{1}I_{E}^{a}|_{E_{inj}} + \alpha T_{2}I_{C}^{a}|_{C_{inj}} + I_{E}^{p}$$

(3.187)

and

$$I_{C}^{a} = \alpha T_{2}I_{E}^{a}|_{E_{inj}} + T_{3}I_{C}^{a}|_{C_{inj}} + I_{C}^{p}$$

(3.188)

Substituting Equations (3.178), (3.182), (3.185), and (3.186) into Equations (3.187) and (3.188), the dc characteristics of the abrupt single HBT are obtained as:

$$I_{E}^{a} = (T_{1}I_{E}^{m} + I_{E}^{sp})[\exp\left(\frac{qV_{BE}}{k_{B}T}\right) - 1] - \alpha T_{2}I_{B}^{m}[\exp\left(-\frac{qV_{CB}}{k_{B}T}\right) - 1]$$

(3.189)

and

$$I_{C}^{a} = \alpha T_{2}I_{B}^{m}[\exp\left(\frac{qV_{BE}}{k_{B}T}\right) - 1] - (T_{3}I_{B}^{m} + I_{C}^{sp})[\exp\left(-\frac{qV_{CB}}{k_{B}T}\right) - 1]$$

(3.190)
The expressions for $x_{B}^{\text{dep,E}}$ and $x_{B}^{\text{dep,C}}$ are given by:

$$x_{B}^{\text{dep,E}} = \frac{1}{qN_{B}^{2}} \left( \frac{2\epsilon_{B}N_{B}^{2}(\Delta E_{EB}^{c} + qV_{BE}^{bi} - qV_{BE})}{1 + \frac{\epsilon_{B}N_{B}^{2}}{\epsilon_{E}N_{E}^{2}}} \right)^{1/2},$$

(3.191)

and

$$x_{B}^{\text{dep,C}} = \frac{1}{qN_{B}^{2}} \left( \frac{2\epsilon_{B}N_{B}^{2}(\Delta E_{CB}^{c} + qV_{CB}^{bi} + qV_{CB})}{1 + \frac{\epsilon_{B}N_{B}^{2}}{\epsilon_{C}N_{C}^{2}}} \right)^{1/2},$$

(3.192)

where $\Delta E_{EB}^{c}$ and $\Delta E_{CB}^{c}$ are the conduction band discontinuities at the base-emitter and collector-base junctions, respectively.

3.5.1.2 Transmission Coefficients

Since the tunneling currents through the energy barriers are negligible, the mechanism of electron transport across the barriers is essentially the thermionic emission. Assuming a Maxwellian energy distribution for the electrons, the expressions for the transmission coefficients, $T_{E}$ and $T_{C}$, are given by:

$$T_{E} = \exp\left(-\frac{qV_{\text{barrier}}^{BE}}{k_{B}T}\right),$$

(3.193)

and

$$T_{C} = \exp\left(-\frac{qV_{\text{barrier}}^{CB}}{k_{B}T}\right),$$

(3.194)

where

$$V_{\text{barrier}}^{BE} = \frac{\Delta E_{EB}^{c}}{q} - \frac{\Delta E_{EB}^{c} + V_{BE}^{bi} - V_{BE}}{1 + \frac{\epsilon_{B}N_{B}^{2}}{\epsilon_{E}N_{E}^{2}}},$$

(3.195)

and

$$V_{\text{barrier}}^{CB} = \frac{\Delta E_{CB}^{c}}{q} - \frac{\Delta E_{CB}^{c} + V_{CB}^{bi} + V_{CB}}{1 + \frac{\epsilon_{B}N_{B}^{2}}{\epsilon_{C}N_{C}^{2}}}.$$  

(3.196)
To derive the expressions for the abrupt device dc currents given by Equations (3.189) and (1.190), it is implicitly assumed that the collector-base barrier transmission coefficient, $T_C$, is the same for the electrons incident from the base side and the ones incident from the collector side. However, the barrier transmission coefficient depends on the energy distribution of the incident electrons, in addition to the shape of the barrier. The assumption of same $T_C$ for electrons incident from both directions is justified only if the electrons have identical energy distributions. The electrons incident from the collector side are in thermal equilibrium with the lattice. On the other hand, the electrons incident from the base side are initially hot electrons and not in thermal equilibrium with the lattice. For the assumption to be valid, the electrons incident from the base side should come into equilibrium with the lattice before reaching the collector-base junction. This will be the case if the base is wide enough to allow sufficient scattering for the electrons to equilibrate with the lattice. In this case, the electrons trapped in the well formed by the two energy barriers at the base-emitter and collector-base junctions, get recombined with the holes (the majority carriers in the base), resulting in a large base current and a small collector current, a significant degradation of the transistor performance. If the base is narrow, the electrons injected from the emitter are still hot when reaching the collector-base barrier. In this case, depending on the energy distribution of the electrons, two scenarios of transport across the collector-base barrier are possible as illustrated in Figure 14. If the lowest energy in the electrons energy spectrum exceeds the collector-base energy barrier height, then the electrons surpass it without getting influenced by its presence at all (Figure 14.a). If the barrier height exceeds the lowest energy in the electrons energy spectrum, then the electrons get partly transmitted and reflected while crossing the barrier (Figure 14.b). Similar arguments hold for the the base-emitter barrier transmis-
sion coefficient, $T_E$, and the electrons incident to the barrier from the base and collector sides. For the situation shown in Figure 14.4, the device currents, $I_E^0$ and $I_C^0$, are formulated as:

$$I_E^0 = (T_E I_B^{in} + I_E^{in})[\exp\left(\frac{V_{BE}}{k_BT}\right) - 1] - \alpha T_E I_B^{in}[\exp\left(-\frac{V_{CB}}{k_BT}\right) - 1] \quad (3.197)$$

and

$$I_C^0 = \alpha T_E I_B^{in}[\exp\left(\frac{V_{BE}}{k_BT}\right) - 1] - (T_3 I_B^{in} + I_C^{in})[\exp\left(-\frac{V_{CB}}{k_BT}\right) - 1] \quad (3.198)$$

where

$$T_3 = T_C[1 - \alpha^2(1 - \frac{T_E}{T_C})] \quad (3.199)$$

### 3.5.2 AC Modeling

#### 3.5.2.1 Derivation of AC Characteristics

Consider a device that operates at dc voltages $V_{BE}$ and $V_{CB}$ and is subjected to small ac voltages $v_{be}$ and $v_{cb}$ with frequency $\omega$. The instantaneous applied voltages, $v_{BE}(t)$ and $v_{CB}(t)$, and resultant currents, $i_E^0(t)$ and $i_C^0(t)$, are given by:

$$v_{BE}(t) = V_{BE} + v_{be} \exp(j\omega t) \quad (3.200)$$

$$v_{CB}(t) = V_{CB} + v_{cb} \exp(j\omega t) \quad (3.201)$$

$$i_E^0(t) = I_E^0 + i_E^0 \exp(j\omega t) \quad (3.202)$$

and

$$i_C^0(t) = I_C^0 + i_C^0 \exp(j\omega t) \quad (3.203)$$

Using a similar procedure to that employed in Subsection 3.5.1, the emitter and
collector ac currents of the abrupt double HBT, \( i_e^a \) and \( i_c^a \), are obtained as:

\[
\begin{align*}
    i_e^a &= \frac{qV_{BE}}{k_BT}(T_{1,\omega}I_{B,\omega}^m + I_{E,\omega}^n) \exp\left(\frac{qV_{BE}}{k_BT}\right) + \frac{qV_{CB}}{k_BT} \alpha_\omega T_{2,\omega}I_{B,\omega}^m \exp\left(-\frac{qV_{CB}}{k_BT}\right), \\
    i_c^a &= \frac{qV_{BE}}{k_BT} \alpha_\omega T_{2,\omega}I_{B,\omega}^m \exp\left(\frac{qV_{BE}}{k_BT}\right) + \frac{qV_{CB}}{k_BT} (T_{3,\omega}I_{B,\omega}^m + I_{C,\omega}^n) \exp\left(-\frac{qV_{CB}}{k_BT}\right),
\end{align*}
\]

(3.204) and

\[
\begin{align*}
    T_{1,\omega} &= \frac{T_E[1 - \alpha_c^2(1 - T_C)]}{1 - \alpha_c^2(1 - T_E)(1 - T_C)}, \\
    T_{2,\omega} &= \frac{T_E T_C}{1 - \alpha_c^2(1 - T_E)(1 - T_C)}, \\
    T_{3,\omega} &= \frac{T_C[1 - \alpha_c^2(1 - T_E)]}{1 - \alpha_c^2(1 - T_E)(1 - T_C)}.
\end{align*}
\]

(3.206) (3.207) (3.208)

The inclusion of the ac changes of the transmission coefficients given by:

\[
\begin{align*}
    t_E(t) &= T_E + t_e \exp(j\omega t), \\
    t_C(t) &= T_C + t_c \exp(j\omega t),
\end{align*}
\]

(3.209) (3.210)

where

\[
\begin{align*}
    t_e &= -\frac{qV_{BE}}{k_BT(1 + \frac{e_B N_B^a}{\epsilon_E N_E^a})} T_E, \\
    t_c &= \frac{qV_{Ceb}}{k_BT(1 + \frac{e_B N_B^a}{\epsilon_C N_C^a})} T_C.
\end{align*}
\]

(3.211) (3.212)

brings considerable mathematical complexity in the analysis of the ac currents. For designs of practical interest, the base doping density, \( N_B^a \), is much more heavier
than that of the emitter and collector, \( N_E^d \) and \( N_C^d \). In this case, according to Equations (3.210) and (3.211), the ac changes of the transmission coefficients are negligible, and so are their contribution to the ac currents. The expressions for the ac currents given in Equations (3.203) and (3.204) are derived based on such an assumption of heavily doped base.

### 3.5.2.2 \( y_\omega \) Parameters

By substituting the expressions for the dc currents given by Equations (3.203) and (3.204) into Equation (3.15), the low frequency \( y \) parameters for the abrupt double HBT are obtained as:

\[
y_{EE}^a = \frac{q}{k_B T} (T_1 I_B^m + I_E^m) \exp\left(\frac{qV_{BE}}{k_B T}\right) + \\
\left( u_{1E} I_B^m + T_1 y_{BE}^m \right) \left\{ \exp\left(\frac{qV_{BE}}{k_B T}\right) - 1 \right\} - \\
\left( \lambda_E T_2 I_B^m + \alpha u_{2E} I_B^m + \alpha T_2 y_{BE}^m \right) \left\{ \exp\left(-\frac{qV_{CB}}{k_B T}\right) - 1 \right\}, \quad (3.213)
\]

\[
y_{EC}^a = \frac{q}{k_B T} \alpha T_2 I_B^m \exp\left(-\frac{qV_{CB}}{k_B T}\right) + \\
\left( u_{1C} I_B^m + T_1 y_{BC}^m \right) \left\{ \exp\left(\frac{qV_{BE}}{k_B T}\right) - 1 \right\} - \\
\left( \lambda_C T_2 I_B^m + \alpha u_{2E} I_B^m + \alpha T_2 y_{BC}^m \right) \left\{ \exp\left(-\frac{qV_{CB}}{k_B T}\right) - 1 \right\}, \quad (3.214)
\]

\[
y_{CE}^a = \frac{q}{k_B T} \alpha T_2 I_B^m \exp\left(\frac{qV_{BE}}{k_B T}\right) + \\
\left( \lambda_E T_2 I_B^m + \alpha u_{2E} I_B^m + \alpha T_2 y_{BE}^m \right) \left\{ \exp\left(\frac{qV_{BE}}{k_B T}\right) - 1 \right\} - \\
\left( u_{3E} I_B^m + T_3 y_{BE}^m \right) \left\{ \exp\left(-\frac{qV_{CB}}{k_B T}\right) - 1 \right\}, \quad (3.215)
\]

and

\[
y_{CC}^a = \frac{q}{k_B T} (T_3 I_B^m + I_C^m) \exp\left(-\frac{qV_{CB}}{k_B T}\right) + \\
\left( \lambda_C T_2 I_B^m + \alpha u_{2C} I_B^m + \alpha T_2 y_{BC}^m \right) \left\{ \exp\left(\frac{qV_{BE}}{k_B T}\right) - 1 \right\} - \\
\left( u_{3C} I_B^m + T_3 y_{BC}^m \right) \left\{ \exp\left(-\frac{qV_{CB}}{k_B T}\right) - 1 \right\}, \quad (3.216)
\]
where

\[
\begin{align*}
\quad u_{1E} &= \frac{\partial T_1}{\partial V_{BE}}\bigg|_{V_{CB}=\text{constant}} \frac{[1 - \alpha^2(1 - T_c)]^2}{[1 - \alpha^2(1 - T_e)(1 - T_c)]^2} u_E - \frac{2\alpha T_E^2(1 - T_c)}{[1 - \alpha^2(1 - T_e)(1 - T_c)]^2} \lambda_E, \quad (3.217) \\
\quad u_{1C} &= \frac{\partial T_1}{\partial V_{CB}}\bigg|_{V_{CB}=\text{constant}} \frac{\alpha^2 T_E^2}{[1 - \alpha^2(1 - T_e)(1 - T_c)]^2} u_C - \frac{2\alpha T_E^2(1 - T_c)}{[1 - \alpha^2(1 - T_e)(1 - T_c)]^2} \lambda_C, \quad (3.218) \\
\quad u_{2E} &= \frac{\partial T_2}{\partial V_{CB}}\bigg|_{V_{CB}=\text{constant}} \frac{T_c [1 - \alpha^2(1 - T_c)]}{[1 - \alpha^2(1 - T_e)(1 - T_c)]^2} u_E + \frac{2\alpha T_E T_c (1 - T_e)(1 - T_c)}{[1 - \alpha^2(1 - T_e)(1 - T_c)]^2} \lambda_E, \quad (3.219) \\
\quad u_{2C} &= \frac{\partial T_2}{\partial V_{CB}}\bigg|_{V_{CB}=\text{constant}} \frac{T_e [1 - \alpha^2(1 - T_c)]}{[1 - \alpha^2(1 - T_e)(1 - T_c)]^2} u_C + \frac{2\alpha T_E T_c (1 - T_e)(1 - T_c)}{[1 - \alpha^2(1 - T_e)(1 - T_c)]^2} \lambda_C, \quad (3.220) \\
\quad u_{3E} &= \frac{\partial T_3}{\partial V_{CB}}\bigg|_{V_{CB}=\text{constant}} \frac{\alpha^2 T_E^2}{[1 - \alpha^2(1 - T_e)(1 - T_c)]^2} u_E - \frac{2\alpha T_E^2(1 - T_e)}{[1 - \alpha^2(1 - T_e)(1 - T_c)]^2} \lambda_E, \quad (3.221) \\
\quad u_{3C} &= \frac{\partial T_3}{\partial V_{CB}}\bigg|_{V_{CB}=\text{constant}} \frac{[1 - \alpha^2(1 - T_e)]^2}{[1 - \alpha^2(1 - T_e)(1 - T_c)]^2} u_C - \frac{2\alpha T_E^2(1 - T_e)}{[1 - \alpha^2(1 - T_e)(1 - T_c)]^2} \lambda_C, \quad (3.222)
\end{align*}
\]

with

\[
\begin{align*}
\quad u_E &= \frac{dT_E}{dV_{BE}} \\
&= \frac{-q}{k_B T(1 + \frac{e_B N_k}{e_{BE} N_E^T})} T_E, \quad (3.223) \\
\quad u_C &= \frac{dT_C}{dV_{CB}} \\
&= \frac{q}{k_B T(1 + \frac{e_B N_k}{e_{CB} N_C^T})} T_C. \quad (3.224)
\end{align*}
\]
The expressions for \( \frac{d\tau_{BE}^{\text{dep,E}}}{dV_{BE}} \) and \( \frac{d\tau_{CB}^{\text{dep,C}}}{dV_{CB}} \) are obtained from Equations (3.191) and (3.192) as:

\[
\frac{d\tau_{BE}^{\text{dep,E}}}{dV_{BE}} = \frac{q}{2} \frac{1}{N_{B}^{1/2}} \left( \frac{2\varepsilon_{B}N_{B}}{1 + \frac{N_{B}^{1/2}}{N_{C}^{1/2}}} \right)^{1/2} (\Delta E_{CB}^{E} + qV_{CB}^{E} - qV_{BE})^{-1/2}.
\] (3.225)

and

\[
\frac{d\tau_{CB}^{\text{dep,C}}}{dV_{CB}} = \frac{q}{2} \frac{1}{N_{B}^{1/2}} \left( \frac{2\varepsilon_{B}N_{B}}{1 + \frac{N_{B}^{1/2}}{N_{C}^{1/2}}} \right)^{1/2} (\Delta E_{CB}^{C} + qV_{CB}^{C} + qV_{CB})^{-1/2}.
\] (3.226)

For a device with heavily doped base, \( \frac{d\tau_{BE}^{\text{dep,E}}}{dV_{BE}} \), \( \frac{d\tau_{CB}^{\text{dep,C}}}{dV_{CB}} \), \( u_{E} \), and \( u_{C} \) are negligible. In this case, the expressions for the low frequency \( y \) parameters given by Equations (3.212) to (3.215) reduce to:

\[
y_{EE} = \frac{q}{k_{B}T}(T_{1}I_{B}^{m} + I_{E}^{sp}) \exp(qV_{BE}/k_{B}T),
\] (3.227)

\[
y_{EC} = \frac{q}{k_{B}T}\alpha T_{2}I_{B}^{m} \exp(-qV_{CB}/k_{B}T),
\] (3.228)

\[
y_{CE} = \frac{q}{k_{B}T}\alpha T_{2}I_{B}^{m} \exp(qV_{BE}/k_{B}T),
\] (3.229)

and

\[
y_{CC} = \frac{q}{k_{B}T}(T_{3}I_{B}^{m} + I_{C}^{sp}) \exp(-qV_{CB}/k_{B}T).
\] (3.230)

By substituting the expressions for the ac currents given by Equations (3.203) and (3.204) into Equation (3.14) and modifying the results to include the depletion capacitances and the collector depletion layer transit time, the \( y_{\omega} \) parameters for the abrupt double HBT are obtained as:

\[
y_{ee,\omega} = \frac{q}{k_{B}T}T_{1,\omega}I_{B,\omega}^{m} \exp(qV_{BE}/k_{B}T) + j\omega C_{\omega}\tau_{\omega}^{\text{dep}},
\] (3.231)

\[
y_{ee,\omega} = \frac{q}{k_{B}T}\alpha_{\omega}T_{2,\omega}I_{B,\omega}^{m} \exp(-qV_{CB}/k_{B}T),
\] (3.232)

\[
y_{ee,\omega} = \frac{q}{k_{B}T}\alpha_{\omega}T_{2,\omega}I_{B,\omega}^{m} \exp(qV_{BE}/k_{B}T),
\] (3.233)
Substituting Equations (3.224) and (3.225) into Equations (3.76) and (3.77), respectively, the expressions for $C_{\pi}^{\text{dep}}$ and $C_{\mu}^{\text{dep}}$ are obtained as:

$$C_{\pi}^{\text{dep}} = A \frac{q}{2} \left( \frac{2\varepsilon_B N_B^a}{\varepsilon_B N_B^a} \right)^{\frac{1}{2}} (\Delta E_{CB}^c + qV_{CB}^{bi} + qV_{CB})^{-\frac{1}{2}},$$

and

$$C_{\mu}^{\text{dep}} = A \frac{q}{2} \left( \frac{2\varepsilon_B N_B^a}{\varepsilon_B N_B^a} \right)^{\frac{1}{2}} (\Delta E_{CB}^c + qV_{CB}^{bi} + qV_{CB})^{-\frac{1}{2}}.$$

and the expression for $C_{\mu}^{\text{delay}}$ is given by Equation (3.78).
In this chapter, the dc and ac characteristics of various HBTs reported in the literature [4,7,17,18,19,23], are studied using the formulation developed in Chapter 3 and the results are compared with the reported theoretical and experimental data. All the examples presented in this work belong to the AlGaAs/GaAs material system.

4.1 Simulation procedure

The device characteristics simulation requires the knowledge of several material parameters. These parameters include the composition, width, dopant concentration, minority carrier diffusion coefficient, and minority carrier recombination lifetime of the emitter, base, and collector regions. Additionally, the knowledge of electron saturation velocity in the collector-base depletion layer, $v_{sat}$, is required. Except for the compositions, widths, and dopant concentrations which can be precisely determined, accurate values for the rest of these parameters are not available. For all the simulations in this work, the hole diffusion coefficients in the emitter and collector, $D_E$ and $D_C$, are roughly chosen to be 10 cm$^2$/s and the hole recombination lifetimes in the emitter and collector, $\tau_E$ and $\tau_C$, are chosen to be 10 ns. Systematic studies have shown that even two orders of magnitude variation in the
values for $D_E$, $D_C$, $r_E$, and $r_C$ does not affect the device characteristics. The values for $D_B$ and $r_B$ for each and every device are extracted from the reported plots on the study of the same device, using the following procedure.

Considering a typical plot of the collector current density, $J_C (= I_C/A)$, versus the base-emitter voltage, $V_{BE}$, as shown in Figure 15.a, two distinct regions can be identified, the linear region of low and medium $J_C$s and the saturated region of high $J_C$s. The deviation from the linear behavior at high $J_C$s is due to the base pushout and thermal effects. When the density of the electrons injected into the collector, $n$, approaches the background donor density, $N_D^B$, the collector space-charge density, $N_C^d = n$, starts to decrease, leading to an increase in the collector depletion width. Eventually, the depletion layer covers the entire collector region. Further injection allows the base region to move out towards the collector, thus effectively increasing the base width, and decreasing the device gain. This is known as the base pushout effect. In an HBT operating in high $J_C$s regime, considerable amount of heat is generated, which along with the poor thermal conductivity of usual HBT materials, leads to a significant increase in the device temperature. This so-called "self-heating" forces new thermal conditions for the carrier energy distributions, resulting in a significant degradation of the device gain. The formulation developed in Chapter 3 does not account for these effects. Hence, the validity of its results is restricted to the linear region for which the slope is proportional to $D_B$. For all the examples presented, the linear region slope of the reported plots of $J_C$ versus $V_{BE}$ is used to calculate $D_B$.

Now consider a typical plot of the dc current gain, $h_{FE}$, versus the collector current density, $J_C$, as shown in Figure 15.b. Three distinct regions can be identified. The first region is the region of low $J_C$s in which $h_{FE}$ increases with $J_C$. In this region, the space-charge recombination and surface recombination currents
are comparable to the bulk recombination current, adding up to a relatively large base current, thus resulting in a small current gain. The second region is the region of medium $J_C$s for which $h_{FE}$ saturates. The third region is the region of high $J_C$s for which $h_{FE}$ falls off due to the base pushout and thermal effects. However, the formulation developed in Chapter 3, predicts a saturated $h_{FE}$, almost constant over the whole range of $J_C$s, for which the value is dictated by $\tau_B$. For all the examples presented, the values for the saturated $h_{FE}$ in the reported plots of $h_{FE}$ versus $J_C$, is used to calculate $\tau_B$. To account for the space-charge recombination and surface recombination effects in low $J_C$s, an ideality factor $n$ is introduced in the expression for the collector current given by Equation (3.2), as follows:

$$I_C = \alpha I_B^m \left[ \exp\left(\frac{qV_{BE}}{nk_BT}\right) - 1 \right] - (I_B^m + I_C^p) \left[ \exp\left(-\frac{qV_{CB}}{k_BT}\right) - 1 \right].$$

(4.1)

For small $J_C$s, $n$ is larger than unity and as $J_C$ increases, $n$ tends to 1. For all the examples presented, the dependency of $n$ on the injection level is extracted from the curvature of the reported plots of $h_{FE}$ versus $J_C$. All the values thus extracted for $D_B$, $\tau_B$, and $n$ are found to be in the acceptable range. A detailed discussion on the extraction results is given in Section 4.3.

The electron saturation velocity in the collector-base depletion layer, $v_{sat}$, is chosen to be in the range of $10^6 - 10^7$ cm/s [28]. Note that the formulation developed in Chapter 3, implicitly considers the largest possible base-emitter voltage for calculations, $V_{BE}^{max}$, to be the voltage for which the conduction band minimum of the base-emitter junction is flat. In other words,

$$V_{BE}^{max} = V_{BE}^{bi},$$

(4.2)

where $V_{BE}^{bi}$ is the base-emitter junction built-in voltage given by Equation (3.8).
4.2 Examples

The abrupt single HBT of Yang et al. [18]

Yang et al. [18] studied the injection performance of the abrupt single HBT (Figure 2, Section 3.3) by applying a numerical technique which took into account the coupled transport processes of drift-diffusion and tunneling-emission across the abrupt heterojunction. The device material parameters are listed in Table 4.1. For this device, \( J_C \) is calculated as a function of the collector-emitter voltage, \( V_{CE} = V_{BE} + V_{CB} \), by using Equation (3.110), and the results are plotted in Figure 16. The plot shows a typical HBT output characteristics where no Early effect is present. In Figure 17, \( J_C \) is plotted versus \( V_{BE} \) along with the simulation results of Yang et al. [18]. For \( J_C \)s up to \( 10^4 \) A/cm\(^2\), the agreement between our model predictions and the simulation results of Yang et al. [18] is excellent. However, our model is not able to cover the range of \( J_C \)s above \( 10^4 \) A/cm\(^2\), due to the upper limit imposed by Equation (4.2). The low frequency \( h \) parameters (\( h_{FE}, h_{IE}, h_{OE}, h_{RE} \)) are calculated as a function of \( J_C \) by using Equations (3.31) along with Equations (3.133)-(3.136), and the results are plotted in Figures 18 to 21. The plots exhibit the typical \( J_C \) dependency of the \( h \) parameters similar to the ones observed in a BJT. In Figure 22, the dc current gain, \( h_{FE} \), is replotted over a smaller range of \( J_C \)s along with the simulation results of Yang et al. [18]. For the whole range of

<table>
<thead>
<tr>
<th>Table 4.1: The HBT material parameters of Yang et al. [18]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>Emitter</td>
</tr>
<tr>
<td>Base</td>
</tr>
<tr>
<td>Collector</td>
</tr>
</tbody>
</table>
$J_C$s predicted by our model, the agreement between our predictions and the simulation results of Yang et al. [18] is excellent. By analyzing the frequency spectrum of the ac current gain, $h_{fe,\omega}$, using Equation (3.50) along with Equations (3.152)-(3.155) and identifying the frequency at which $|h_{fe,\omega}|$ drops to unity, the cut-off frequency, $f_T$, is determined as a function of $J_C$, and the results are shown in Figure 23 along with the simulation results of Yang et al. [18]. Up to the largest $J_C$ allowed by our model, i.e. $10^4$ A/cm$^2$, the agreement between our predictions and the simulation results of Yang et al. [18] is excellent. According to the simulation results of Yang et al. [18] shown in Figures 16, 21, and 22, the high current effects become significant for $J_C$s above $10^4$ A/cm$^2$.

The abrupt single HBT of Ito et al. [4]

Ito et al. [4] fabricated an abrupt single HBT (Figure 2, Section 3.3) and measured its high-frequency characteristics. The device material parameters are listed in Table 4.2. The device which had an emitter area of $9 \times 10^{-7}$ cm$^2$, exhibited a maximum $h_{FE}$ of 90 at $I_C$ of 10 mA. For this device, the squared magnitude of the ac current gain, $|h_{fe,\omega}|^2$, is calculated as a function of frequency, $f$, by using Equation (3.32) along with Equations (3.146)-(3.149), and the results are shown in Figure 24 along with the experimental results of Ito et al. [4]. The theoretical

<table>
<thead>
<tr>
<th>Layer</th>
<th>Type</th>
<th>Thickness (Å)</th>
<th>Doping (cm$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emitter</td>
<td>n Al$<em>{0.3}$Ga$</em>{0.7}$As</td>
<td>1000</td>
<td>$5 \times 10^{17}$</td>
</tr>
<tr>
<td>Base</td>
<td>p GaAs</td>
<td>1000</td>
<td>$1 \times 10^{19}$</td>
</tr>
<tr>
<td>Collector</td>
<td>n GaAs</td>
<td>3000</td>
<td>$1 \times 10^{17}$</td>
</tr>
</tbody>
</table>

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and experimental results are in good agreement with each other. Next, \( f_T \) is calculated as a function of \( I_C \) by analyzing the frequency spectrum of \( h_{fe,\omega} \) using Equation (3.50) along with Equations (3.152)-(3.155), and the results are shown in Figure 25 along with the experimental results of Ito et al. [4]. The condition of maximum possible \( V_{BE} \) given by Equation (4.2) limits the largest predicted \( I_C \) to 30 mA, up to which the theoretical and experimental results are in very good agreement with each other.

The graded single HBT of Liou et al. [17]

Liou et al. [17] presented a detailed analytical model to predict the dc and high-frequency performance of the graded single HBT (Figure 1, Section 3.2). The model included the base pushout and thermal effects of the high injection regime. The device material parameters are listed in Table 4.3. For this device, \( J_C \) and \( J_B \) are calculated as a function of \( V_{BE} \) by using Equations (3.1) and (3.2), and the results are shown in Figure 26 along with the simulation results of Liou et al. [17]. As long as the high injection effects are absent, the agreement between our model predictions and the simulation results of Liou et al. [17] is good. The disagreement become significant for \( J_C \)s above \( 2 \times 10^4 \) A/cm². Next, \( h_{FE} \) is calculated as a function of \( J_C \) by using Equation (3.31) along with Equations (3.16)-(3.19), and

<table>
<thead>
<tr>
<th>Layer</th>
<th>Type</th>
<th>Thickness (Å)</th>
<th>Doping (cm⁻³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emitter</td>
<td>n Al₀.₃Ga₀.₇As</td>
<td>1700</td>
<td>5×10¹⁷</td>
</tr>
<tr>
<td>Grading</td>
<td>n 0.3→0.0</td>
<td>300</td>
<td>×10¹⁷</td>
</tr>
<tr>
<td>Base</td>
<td>p GaAs</td>
<td>1000</td>
<td>1×10¹⁹</td>
</tr>
<tr>
<td>Collector</td>
<td>n GaAs</td>
<td>3000</td>
<td>1×10¹⁷</td>
</tr>
</tbody>
</table>

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the results are shown in Figure 27 along with the simulation results of Liou et al. [17]. For $J_{CS}$ up to $2 \times 10^4$ A/cm$^2$, the agreement between our model predictions and the simulation results of Liou et al. [17] is good. However, for $J_{CS}$ above $2 \times 10^4$ A/cm$^2$, Liou et al. [17] correctly predicted a rapid fall-off in $h_{FE}$ which is missing in our results. This rapid fall-off in $h_{FE}$ is mainly due to the self-heating effects in the device at high $J_{CS}$. At last, $f_T$ is calculated as a function of $J_C$ by analyzing the frequency spectrum of $h_{fe,\omega}$ using Equation (3.50) along with Equations (3.83)-(3.86), and the results are shown in Figure 28 along with the simulation results of Liou et al. [17]. For $J_{CS}$ up to $2 \times 10^4$ A/cm$^2$, the agreement between our model predictions and the simulation results of Liou et al. [17] is good. However, for $J_{CS}$ above $2 \times 10^4$ A/cm$^2$, the exclusion of the high injection effects in our model results in overestimation of $f_T$.

The graded single HBT of Yang et al. [7]

By using a self-aligned technique, Yang et al. [7] fabricated a submicron graded single HBT (Figure 1, Section 3.2) with a very heavily carbon doped ($10^{20}$ cm$^{-3}$) base layer. The heavily carbon doped base was intended to suppress the so-called "emitter size effect", the degradation of the device current gain for small emitter area. The device material parameters are listed in Table 4.4. For this device, $J_C$

<table>
<thead>
<tr>
<th>Layer</th>
<th>Type</th>
<th>Thickness (Å)</th>
<th>Doping (cm$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emitter</td>
<td>n Al$<em>{0.3}$Ga$</em>{0.7}$As</td>
<td>1900</td>
<td>$5 \times 10^{17}$</td>
</tr>
<tr>
<td>Grading</td>
<td>n 0.3$\rightarrow$0.0</td>
<td>200</td>
<td>$5 \times 10^{17}$</td>
</tr>
<tr>
<td>Base</td>
<td>p GaAs</td>
<td>900</td>
<td>$1 \times 10^{20}$</td>
</tr>
<tr>
<td>Collector</td>
<td>n GaAs</td>
<td>5000</td>
<td>$2 \times 10^{15}$</td>
</tr>
</tbody>
</table>
and $J_B$ are calculated as a function of $V_{BE}$ by using Equations (3.1) and (3.2), and results are shown in Figure 29 along with the experimental results of Yang et al. [7]. The present model accurately predicts the experiment results up to $J_{CS}$ of about $10^4$ A/cm$^2$ above which the high injection effects start to reveal themselves.

The graded single HBT of Yang et al. [18]

Yang et al. [18] studied the injection performance of the graded single HBT (Figure 1, Section 3.2). The device material parameters are listed in Table 4.5. For this device, $J_C$ is calculated as a function of $V_{BE}$ by using Equation (3.2), and the results are shown in Figure 30 along with the numerical simulation results of Yang et al. [18]. For $J_{CS}$ up to $2 \times 10^4$ A/cm$^2$, the agreement between our model predictions and the simulation results of Yang et al. [18] is excellent. Next, $h_{FE}$ is calculated as a function of $J_C$ by using Equation (3.31) along with Equations (3.16)-(3.19), and the results are shown in Figure 31 along with the simulation results of Yang et al. [18]. Again, the agreement between the results is excellent.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Type</th>
<th>Thickness (Å)</th>
<th>Doping (cm$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emitter</td>
<td>n Al$<em>{0.25}$Ga$</em>{0.75}$As</td>
<td>1500</td>
<td>$5 \times 10^{17}$</td>
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<td>n 0.25$\rightarrow$0.00</td>
<td>300</td>
<td>$5 \times 10^{17}$</td>
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<tr>
<td>Base</td>
<td>p GaAs</td>
<td>1000</td>
<td>$1 \times 10^{19}$</td>
</tr>
<tr>
<td>Collector</td>
<td>n GaAs</td>
<td>4000</td>
<td>$5 \times 10^{17}$</td>
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</tbody>
</table>

The graded double HBT of Zhang et al. [19]

Zhang et al. [19] studied the effects of displacements of the p-n junctions from the heterojunctions in the symmetrical graded double HBT (Figure 3, Section 3.5)
by using two-dimensional simulation techniques. The device material parameters are listed in Table 4.6. For this device, $J_C$ and $J_B$ are calculated as a function of $V_{BE}$ by using Equations (3.1) and (3.2), and the results are shown in Figure 32 along with the simulation results of Zhang et al. [19]. Although Zhang et al. [19] performed their simulation for $V_{BES}$ up to 1.80 V, in our model $V_{BE}$ should not exceed $V_{BE}^{bi} = 1.40$ V. For $V_{BES}$ up to 1.40 V corresponding to $J_C$s up to $10^4$ A/cm², the agreement between our model predictions and the simulation results of Zhang et al. [19] is good. Next, $h_{FE}$ is calculated as a function of $J_C$ by using Equation (3.31) along with Equations (3.16)-(3.19), and the results are shown in Figure 33 along with the simulation results of Zhang et al. [19]. Over the whole range of $J_C$s predicted by our model, the agreement between our predictions and the simulation results of Zhang et al. [19] is very good. At last, $f_T$ is calculated as a function of $J_C$ by analyzing $h_{f_e \omega}$'s spectrum using Equation (3.50) along with Equations (3.83)-(3.86), and the results are shown in Figure 34 along with the simulation results of Zhang et al. [19]. This time, the agreement between the results is poor. Our estimations of $f_T$ is about 3-4 times larger than the ones of Zhang et al. [19]. The reason is believed to lie in the approximate nature of the Gummel formula [15] used by Zhang et al. [19] to evaluate $f_T$.

Table 4.6: The HBT material parameters of Zhang et al. [19]

<table>
<thead>
<tr>
<th>Layer</th>
<th>Type</th>
<th>Thickness (Å)</th>
<th>Doping (cm⁻³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emitter</td>
<td>n Al₀.2₈Ga₀.₇₂As</td>
<td>2000</td>
<td>$3 \times 10^{17}$</td>
</tr>
<tr>
<td>Grading</td>
<td>n 0.2₈—&gt;0.00</td>
<td>300</td>
<td>$3 \times 10^{17}$</td>
</tr>
<tr>
<td>Base</td>
<td>p GaAs</td>
<td>1000</td>
<td>$1 \times 10^{19}$</td>
</tr>
<tr>
<td>Grading</td>
<td>n 0.00—&gt;0.2₈</td>
<td>300</td>
<td>$3 \times 10^{17}$</td>
</tr>
<tr>
<td>Collector</td>
<td>n Al₀.2₈Ga₀.₇₂As</td>
<td>2000</td>
<td>$3 \times 10^{17}$</td>
</tr>
</tbody>
</table>
The graded double HBT of Parikh et al. [23]

Parikh et al. [23] developed a charge-control model to formulate the dc characteristics of the HBT. They applied the model to study a symmetrical graded double HBT (Figure 3, Section 3.5) fabricated by Marty et al. [30]. The device material parameters are given in Table 4.7. For this device, $J_C$ is calculated as a function of $V_{BE}$ by using Equation (3.2), and the results are shown in Figure 35 along with the theoretical results of Parikh et al. [23] and the experimental results of Marty et al. [30]. Our results and the results of Parikh et al. [23], both show an equally accurate agreement with the experimental results of Marty et al. [30].

<table>
<thead>
<tr>
<th>Layer</th>
<th>Type</th>
<th>Thickness (Å)</th>
<th>Doping (cm$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emitter</td>
<td>n Al$<em>{0.4}$Ga$</em>{0.6}$As</td>
<td>2500</td>
<td>1×10$^{16}$</td>
</tr>
<tr>
<td>Grading</td>
<td>n 0.4→0.0</td>
<td>800</td>
<td>1×10$^{16}$</td>
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<tr>
<td>Base</td>
<td>p GaAs</td>
<td>800</td>
<td>5×10$^{18}$</td>
</tr>
<tr>
<td>Grading</td>
<td>n 0.0→0.4</td>
<td>800</td>
<td>1×10$^{16}$</td>
</tr>
<tr>
<td>Collector</td>
<td>n Al$<em>{0.4}$Ga$</em>{0.6}$As</td>
<td>1500</td>
<td>1×10$^{16}$</td>
</tr>
</tbody>
</table>

4.3 Discussion

The extracted values of $D_B$ for the GaAs base of the graded HBTs are shown as a function of the base acceptor density, $N_B^a$, in Figure 36. Harmon et al. [31] reported on the measurement of $D_B$ for the GaAs base of homojunction BJTs, the results of which are shown in Figure 36, also. The reported data on $D_B$ in various sources show a large variability [31,33]. In light of this fact, the agreement between the extracted values for $D_B$ and the measured values reported by Harmon.
et al. [31] is fair. Note that no matter how smooth the base-emitter heterojunction grading is, still it consists of many layers of small, finite abrupt heterojunctions. Thus, the conduction band of the heterojunction grading includes many small finite discontinuities. These small discontinuities in the conduction band can give rise to a slightly larger electron forward velocity and diffusion coefficient, \( D_B \), for the graded HBT in comparison to the homojunction BJT. If the base-emitter heterojunction is an abrupt one, the conduction band discontinuity is large enough to provide significant rise in \( D_B \). In this case, the relatively large \( D_B \) of the abrupt HBT is not comparable anymore with the small \( D_B \)s of the graded HBT or the homojunction BJT. This is specifically illustrated by the values of \( D_B \) extracted for the GaAs base of the abrupt single HBTs of Yang et al. [18] and Ito et al. [4]. For the former device, \( D_B \) is found to be 800 cm\(^2\)/s, and for the later to be 12000 cm\(^2\)/s. An approximate proportionality relation between \( D_B \) and the height of the barrier formed by the conduction band discontinuity at the base-emitter heterojunction (Figure 9), \( V_{BE}^{\text{barrier}} \), can be established, as follows. For carriers at temperature \( T \), the diffusion coefficient, \( D(T) \), and the mobility, \( \mu(T) \), obey the Einstein relation, given by:

\[
D(T) = \frac{k_B T}{q} \mu(T) .
\]  

(4.3)

For hot electrons at temperature \( T_e \), \( \mu(T_e) \) increases with \( T_e \) as:

\[
\mu(T_e) \propto \left( \frac{T_e}{T_l} \right)^{x-1} ,
\]  

(4.4)

where \( T_l \) is the lattice temperature and \( x \) is a number larger than 1, determined by the details of the scattering mechanisms of the hot electrons [32]. Combining Equation (4.3) with Relation (4.4), the following relation results:

\[
D(T_e) \propto \left( \frac{T_e}{T_l} \right)^x .
\]  

(4.5)
In the abrupt single HBT, the temperature of the hot electrons injected into the base is determined by the conduction band barrier height, $V_{BE}^{\text{barrier}}$, as:

$$\frac{T_e}{T_i} = \frac{qV_{BE}^{\text{barrier}} + \frac{3}{2}k_BT_i}{\frac{3}{2}k_BT_i}.$$  (4.6)

Combining Equation (4.6) with Relation (4.5), the following proportionality relation between $D_B$ and $V_{BE}^{\text{barrier}}$ results:

$$D_B \propto \left(\frac{qV_{BE}^{\text{barrier}} + \frac{3}{2}k_BT_i}{\frac{3}{2}k_BT_i}\right)^x.$$  (4.7)

According to the measurements of Harmon et al. [31], the GaAs bulk electrons have a $D_B$ of 35 cm$^2$/s at a base acceptor density of $1 \times 10^{19}$cm$^{-3}$. Using this value along with Relation (4.7), $x$ is found to be 1.79 and 1.51 for the abrupt single HBTs of Yang et al. [18] and Ito et al. [4], respectively. This is in general agreement with the energy dependence of the dominant scattering mechanisms in the heavily doped GaAs, i.e. the polar optical phonon scattering and the ionized impurity scattering [32].

The extracted values of $\tau_B$ for the GaAs base of the graded HBTs are plotted as a function of the base acceptor density, $N_B^s$, in Figure 37. Tiwari et al. [33] reported on the measurement of $\tau_B$ for doped bulk GaAs material, the results of which are shown in Figure 36, also. The reported data on $\tau_B$ in various sources show a large variability [31,33]. In light of this fact, the agreement between the extracted values for $\tau_B$ and the measured values reported by Tiwari et al. [33] is fair. It is observed that for the same base acceptor density of $1 \times 10^{19}$cm$^{-3}$, the graded HBT with larger $D_B$ exhibits shorter $\tau_B$. Same trend is observed for the single abrupt HBTs of Yang et al. [18] and Ito et al. [4], for which $\tau_B$ is extracted to be $1.25 \times 10^{-11}$ s and $4.00 \times 10^{-13}$ s, respectively. The inverse relation between $D_B$ and $\tau_B$ can be explained as follows. The electrons with high $D_B$ are hot elec-
trons and belong to high energy levels. Electrons belonging to higher energy levels are less stable, a fact mathematically formulated by Fermi's Golden rule [32]. Hence, these electrons recombine faster, resulting shorter $\tau_B$.

For each example in which $h_{FE}$ is simulated, the ideality factor $n$ is extracted and the results are plotted as a function of $V_{BE}$ in Figure 38. In all cases, $n$ remains close to unity in the forward active region which is in agreement with the measurements of the diode ideality factor for AlGaAs/GaAs HBTs, performed by Liu et al. [5]. The bias dependency of $n$ is found to follow the relation:

$$n = 1 + \delta n \exp\left(-\frac{qV_{BE}}{mk_BT}\right),$$

(4.8)

where $m$ and $\delta n$ are constants, specified for each device. The value of $m$ ranges between 2.0 to 2.5. Combining Equations (4.1) and (4.8), $n$ is found to have the following injection level dependency:

$$n - 1 \propto \frac{1}{\sqrt{I_C}}.$$

(4.9)

Relation (4.9) indicates that for low level of injections, the space-charge recombination and the surface recombination processes significantly influence the ideal behavior of the HBT, leading to an ideality factor $n$ greater than unity. However, as the injection level increases, the space-charge recombination and the surface recombination processes become less pronounced compared to the bulk recombination process. At high injection levels, the HBT exhibits an ideal behavior where the dominant recombination process is the one of base bulk recombination, leading to an ideality factor $n$ of unity.
CHAPTER 5

CONCLUSION

Starting with a homojunction or graded heterojunction bipolar transistor, a complete model for the dc and ac performances of the device was developed based on the Ebers-Moll methodology. Then, the formulation was modified to include the abrupt single HBT, by introducing the effects of the conduction band discontinuity at the base-emitter junction as an energy barrier for electrons flow, resulting in transmission and reflection processes. Finally, using the same approach, the formulation was extended to the abrupt double HBT. Only the npn HBT was studied in this work; however, the same concepts and modeling approaches presented can be used to study the pnp HBT, also.

The model correctly predicted the dc and ac characteristics of various graded and abrupt, single and double AlGaAs/GaAs HBTs reported in the literature [4,7,17,18,19,23]. The collector current as a function of collector-emitter voltage, the collector and base currents as a function of base-emitter voltage, the dc current gain as a function of collector current, the $h$ parameters as a function of collector current, the ac current gain as a function of frequency, and finally the unity current gain cut-off frequency as a function of the collector current were calculated and the results were compared with the reported data. In all cases, the agreement was found to be good.
Only AlGaAs/GaAs HBTs were investigated; however, the model is applicable to any material system HBTs. Throughout the work, the quantum tunneling at the conduction band discontinuity was neglected. The space-charge recombination and surface recombination processes were not included in the original formulation; but, their net effect was taken into account for the simulation of the devices by introducing an ideality factor $n$ in the expression for the collector current. Also, the high injection effects of base pushout and self-heating were not taken into consideration. This places an upper limit for the model predictions validity. Finally, the modeling was limited to the intrinsic device, neglecting the effects of any series resistances at the emitter, base, and collector.

The development of the model was such that for both steady state and high frequency performances, the physical mechanisms of transport remained transparent, leaving the original model flexible enough to be able to include any neglected phenomenon in the future. For instance, the effects of tunneling can be incorporated through the expression for the transmission coefficient for the energy barrier formed by the conduction band discontinuity, while space-charge recombination and surface recombination processes can be taken into account in the initial Ebers-Moll equations. The extrinsic resistances can be calculated from the device geometry and added to the present formulation to complete the modeling.
FIGURES
Figure 1: a) Schematic illustration of the graded single HBT, b) Energy band diagram at equilibrium, c) Energy band diagram under forward active mode.
Figure 2: a) Schematic illustration of the abrupt single HBT, b) Energy band diagram at equilibrium, c) Energy band diagram under forward active mode.
Figure 3: a) Schematic illustration of the graded double HBT, b) Energy band diagram at equilibrium, c) Energy band diagram under forward active mode.
Figure 4: a) ECL gate and b) $I^2L$ gate designed with double HBTs [3].
Figure 5: Energy band diagram of a homojunction or graded heterojunction bipolar transistor along with interesting widths in the device.
Figure 6: Small signal equivalent circuit.

Figure 7: Plots of magnitude and phase of $h_{fe,\omega}$ versus frequency.

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Figure 8: Current components in the abrupt single HBT: a) Electron injection from the emitter to the base, b) Electron injection from the collector to the base.
Figure 9: The conduction band of the base-emitter junction of the abrupt single HBT compared with that of a graded single HBT (dotted).
Figure 10: Possible structure for the abrupt double HBT with one energy barrier in the conduction band at the base-emitter junction.

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Figure 11: Possible structure for the abrupt double HBT with one energy barrier in the conduction band at the collector-base junction.
Figure 12: a) Schematic illustration of the abrupt double HBT, b) Energy band diagram at equilibrium, c) Energy band diagram under forward active mode.
First transmission and reflection at the base-emitter junction barrier

Second transmission and reflection at the collector-base junction barrier

Third transmission and reflection at the base-emitter junction barrier

Figure 13: Illustration of multiple transmission and reflection processes at the base-emitter and collector-base junction barriers.
Figure 14: Two possibilities of transport for the narrow base abrupt double HBT under forward bias.
Figure 15: a) Typical plot of the collector current density, $J_C$, versus the base-emitter voltage, $V_{BE}$, b) Typical plot of the dc current gain, $h_{FE}$, versus the collector current density, $J_C$. 

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Figure 16: Plot of collector current density, $J_C$, versus collector-emitter voltage, $V_{CE}$, with base-emitter voltage, $V_{BE}$, kept constant. The curves are obtained for equal steps of $\Delta V_{BE}=0.01$ V.
Figure 17: Plot of collector current density, $J_C$, versus base-emitter voltage, $V_{BE}$, for $V_{CE}=2$ V along with simulation results of Yang et al. [18].
Figure 18: Plot of dc current gain, $h_{FE}$, versus collector current density, $J_C$, for $V_{CE}=2$ V.
Figure 19: Plot of input impedance, $h_{IE}$, versus collector current density, $J_C$, for $V_{CE}=2$ V.
Figure 20: Plot of output admittance, $h_{OE}$, versus collector current density, $J_C$, for $V_{CE}=2$ V.
Figure 21: Plot of voltage feedback ratio, $h_{RE}$, versus collector current density, $J_C$, for $V_{CE}=2$ V.
Figure 22: Plot of dc current gain, $h_{FE}$, versus collector current density, $J_C$, for $V_{CE}=2$ V along with simulation results of Yang et al. [18].
Figure 23: Plot of cut-off frequency, $f_T$, versus collector current density, $J_C$, for $V_{CE}=2$ V along with simulation results of Yang et al. [18].
Figure 24: Plot of the squared magnitude of the ac current gain, $|h_{fe\omega}|^2$, versus frequency, $f$, for $I_C=10$ mA and $V_{CE}=3$ V along with measurement results of Ito et al. [4].
Figure 25: Plot of cut-off frequency, $f_T$, versus collector current, $I_C$, for $V_{CE}=2$ V along with measurement results of Ito et al. [4].
Figure 26: Plots of collector and base current densities, $J_C$ and $J_B$, versus base-emitter voltage, $V_{BE}$, for $V_{CE} = 2.5$ V along with predictions of Liou et al. [17].

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Figure 27: Plot of dc current gain, $h_{FE}$, versus collector current density, $J_C$, for $V_{CE} = 2.5$ V along with predictions of Liou et al. [17].
Figure 28: Plot of cut-off frequency, $f_T$, versus collector current density, $J_C$, for $V_{CE} = 2.5$ V along with predictions of Liou et al. [17].

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Figure 29: Plots of collector and base current densities, $J_C$ and $J_B$, versus base-emitter voltage, $V_{BE}$, for $V_{CE} = 2$ V along with measurements results of Yang et al. [7].

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Figure 30: Plot of collector current density, $J_C$, versus base-emitter voltage, $V_{BE}$, for $V_{CE} = 2$ V along with numerical simulation results of Yang et al. [18].
Figure 31: Plot of dc current gain, $h_{FE}$, versus collector current density, $J_C$, for $V_{CE} = 2$ V along with numerical simulation results of Yang et al. [18].
Figure 32: Plots of logarithm of collector and base current densities, \( \log(J_C) \) and \( \log(J_B) \), versus base-emitter voltage, \( V_{BE} \), for \( V_{CE} = 2 \text{ V} \) along with simulation results of Zhang et al. [19].
Figure 33: Plot of dc current gain, $h_{FE}$, versus logarithm of collector current density, $\log(J_C)$, for $V_{CE} = 2$ V along with simulation results of Zhang et al. [19].
Figure 34: Plot of cut-off frequency, $f_T$, versus logarithm of collector current density, $\log(J_C)$, for $V_{CE} = 2$ V along with simulation results of Zhang et al. [19].

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Figure 35: Plot of collector current density, $J_C$, versus base-emitter voltage, $V_{BE}$, for $V_{CE} = 2$ V along with predictions of Parikh et al. [23] and measurement results of Marty et al. [30].
Figure 36: Plot of the extracted electron diffusion coefficient, $D_B$, for GaAs base of the graded HBTs, as a function of the base acceptor density, $N_g^b$, along with $D_B$ measurement results of Harmon et al. [31], obtained for GaAs base of homo junction BJT.
Figure 37: Plot of the extracted electron recombination lifetime, $\tau_B$, for GaAs base of the graded HBTs, as a function of the base acceptor density, $N_B^0$, along with $\tau_B$ measurement results of Tiwari et al. [33], obtained for doped bulk GaAs material.
Figure 38: Plots of the extracted ideality factors, $n$, as a function of the base-emitter voltage, $V_{BE}$. 

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Graduate Faculty Representative. Dr. Tao Pang, Ph.D.