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On Implicational Dependency Families Possessing Finite Armstrong Relations

Kazem Taghva
University of Nevada, Las Vegas, kazem.taghva@unlv.edu

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KAZEM TAGHVA*

Department of Computer Science and Electrical Engineering

University of Nevada, Las Vegas
Las Vegas, Nevada 89154

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Let $X \neq \emptyset$ be a finite collection of nonempty relations over the relation scheme $R(A_1, A_2, ..., A_n)$; then the closure of X under embedding and direct product (up to isomorphism) is a *finitely generated Implicational Dependency family* (ID-family) generated by X. In this paper, we show that the class of finitely generated ID-families is identical to the class of those ID-families which possess a finite Armstrong relation.

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ON IMPLICATIONAL DEPENDENCY FAMILIES POSSESSING FINITE ARMSTRONG RELATIONS

KAZEM TAGHVA*
Department of Computer Science
The University of Nevada
Las Vegas, NV 89154

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Abstract

Let $X \neq \emptyset$ be a finite collection of nonempty relations over the relation scheme $R(A_1, A_2, ..., A_n)$; then the closure of X under embedding and direct product (up to isomorphism) is a *finitely generated Implicational Dependency family* (ID-family) generated by X. In this paper, we show that the class of finitely generated ID-families is identical to the class of those ID-families which possess a finite Armstrong relation.

1 Introduction

Data dependencies such as *functional dependencies* (FDs), *multivalued dependencies* (MVDs), and *join dependencies* (JDs) have played an important role in the design of databases[2][3]. In addition, they have been used as integrity constraints in an integrity-checking mechanism[3]. The legal databases are

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those which obey the constraints specified by the database administrator originally. Consequently, we are interested in studying families of instances characterized by a given set of dependencies such as FDs, MVDs, etc.

The class of *Implicational Dependencies* (IDs) was defined by Fagin[2] as the logical generalization of the previously defined class of full dependencies. Properties of ID-families are mainly studied in [2],[4],[5],[7], in particular, it is shown that the collection of ID-families is closed under join and projection.

In[5], it is shown that a collection of relations over scheme $R(A_1, A_2, ..., A_n)$ is axiomatizable by IDs if and only if it contains a trivial database and it is domain independent and closed under embedding and direct products.

In this paper, we use the above result to establish that the collection of ID-families with a finite Armstrong relation and the collection of finitely generated ID-families are identical.

Vardi[8] has established a finite set of IDs with no finite Armstrong relation. This, together with the above result, implies that finitely specifiable ID-families are not finitely generated.

2 Preliminaries

In this paper, we assume readers to be familiar with [2], and [5]. We will follow the notation of [2]. In addition, throughout this paper we only deal with scheme $R(A_1, A_2, ..., A_n)$.

Following Fagin[2], we define an *Implicational Dependency* (ID) to be a typed sentence σ of the form $\forall x_1 \forall x_2 ... \forall x_m (\alpha_1 \wedge \alpha_2 ... \wedge \alpha_k \rightarrow \beta)$, where each α_i is an atomic formula of the form $R(y_1, y_2, ..., y_n)$ and β is an atomic formula of the form $R(y_1, y_2, ..., y_n)$ or $y_i = y_j$, where $y_d \in \{x_1, x_2, ..., x_m\}$. We also assume that $k \geq 1$ and each x_i occurs in some α_j. For example, the formula $\forall a \forall b \forall c_1 \forall c_2 \forall d_1 \forall d_2 R(a, b, c_1, d_1) \wedge R(a, b, c_2, d_2) \rightarrow c_1 = c_2)$ represents the FD $AB \rightarrow C$ for the 4-ary relation scheme $R(A, B, C, D)$, and the formula $\forall a \forall b_1 \forall b_2 \forall c_1 \forall c_2 R(a, b_1, c_1) \wedge R(a, b_2, c_2) \rightarrow R(a, b_1, c_2)$ represents the MVD $A \rightarrow\rightarrow B$ for the 3-ary relation scheme $R(A, B, C)$.

Let r and s be relations for R (our relations are all finite relations), then we define the direct product of r and s, in notation $r \times s$, to be the set of all tuples $t = ((t_{11}, t_{21}), (t_{12}, t_{22}), ..., (t_{1n}, t_{2n}))$ such that $t_1 = (t_{11}, t_{12}, ..., t_{1n}) \in r$ and $t_2 = (t_{21}, t_{22}, ..., t_{2n}) \in s$. For example, the direct product of the first two relations in the following diagram is the third relation.

r

A	B	C
a	b	c
a'	b'	c'

s

A	B	C
a_1	b_1	c_1
a_2	b_2	c_2

$r \times s$

(a, a_1)	(b, b_1)	(c, c_1)
(a, a_2)	(b, b_2)	(c, c_2)
(a', a_1)	(b', b_1)	(c', c_1)
(a', a_2)	(b', b_2)	(c', c_2)

The direct product of $r_1 \times r_2 \times ... \times r_m$ is defined as usual. Also, we define $Dom(r)$ to be $Dom_r(A_1) \times Dom_r(A_2) \times ... \times Dom_r(A_n)$, where each $Dom_r(A_i)$ is the set of all the ith coordinates of r. For example, the $Dom(r)$ in the above diagram is:

$Dom(r)$

A	B	C
a	b	c
a	b	c'
a	b'	c
a	b'	c'
a'	b	c
a'	b	c'
a'	b'	c
a'	b'	c'

For the relation scheme $R(A_1, ..., A_n)$, we also assume a countably in-finite underlying domain for each A_i from which A_i takes its values. Let r and s be nonempty relations for R, then $f = (f_1, f_2, ..., f_n)$ is called an *embedding* from s to r if f_i is a 1-1 function from $Dom_s(A_i)$ to $Dom_r(A_i)$ for each i and for any tuple $(a_1, ..., a_n) \in Dom(s)$, then $(a_1, ..., a_n) \in s$ iff $(f_1(a_1), f_2(a_2), ..., f_n(a_n)) \in r$. In fact, embedding is a typed 1-1 homomorphism between two structures. In case such f exists, we say s can be *embedded* into r. An embedding f is called an *isomorphism* if f is onto. We will use the notation $r \cong s$ to show that r and s are isomorphic. A subset s of r is called a *substructure* of r if $Dom(s) \cap r = s$. It is obvious that if s is a substructure of r, then the identity map from $Dom(s)$ to $Dom(r)$ is an embedding.

Let Σ be a set of IDs, then $SAT(\Sigma)$ is the set of all finite relations satisfying Σ. A nonempty collection of relations F is an ID-family if there exists a set Σ of IDs such that $F = SAT(\Sigma)$. In case Σ is finite, we say F is *finitely specifiable* ID-family.

Let Σ be a set of IDs, then $\Sigma_* = \{\sigma \mid \Sigma \models \sigma\}$, i.e. Σ_* is the set of all IDs which logically follow from Σ. A relation r is called an *Armstrong relation* if all members of Σ_* are true in r and all other IDs are false in r. Armstrong relations and their applications are extensively studied in [1], [2], and [6].

For any collection K of relations, let

$$\text{SK} = \{\ \text{r} \mid \text{r can be embedded into some member of K}\}$$
$$\text{PK} = \{\ \text{r} \mid r \cong r_1 \times r_2 \times ... \times r_n \text{ for } r_i \text{ members of K}\}$$

The next theorem gives a characterization for ID-families.

Theorem 2.1 *[5]Let F be a family of relations for R, then F is an ID-family iff:*
(1) F is closed under P.
(2) F is closed under S.
(3) F contains a singleton.

We would like to mention here that Makowsky and Vardi[5] use the term "subdatabase" instead of "substructure".

3 Main Result

Let $X = \{r_1, r_2, ..., r_n\} \neq \emptyset$ be a collection of nonempty relations for R, then theorem 2.1 implies that SPX is an ID-family generated by X (note that condition (3) is trivially satisfied as any tuple t in some r_i will form the substructure $\{t\}$ for r_i). In case X contains a single relation, we will say SPX is *singly* generated.

The next two lemmas imply that the collection of finitely generated ID-families and the collection of singly generated ID-families are identical.

Lemma 3.1 *Let s_1 and s_2 be substructures of r_1 and r_2 respectively, then $s_1 \times s_2$ is a substructure of $r_1 \times r_2$.*

Proof. Straightforward.

Lemma 3.2 *Let $X = \{r_1, r_2, ..., r_m\}$ be a collection of nonempty relations for R, then $SPX = SP\{r_1 \times r_2 \times ... \times r_m\}$.*

Proof. Let $t_2, t_3, ..., t_m$ be tuples in $r_2, r_3, ..., r_m$ respectively. By lemma 3.1, $r_1 \times \{t_2\} \times ... \times \{t_m\}$ is a substructure of $r_1 \times r_2 \times ... \times r_m$. Now since r_1 is isomorphic to $r_1 \times \{t_2\} \times ... \times \{t_m\}$, it follows that r_1 is a member of $SP\{r_1 \times r_2 \times ... \times r_m\}$. Similarly, we can show $r_i \in SP\{r_1 \times r_2 \times ... \times r_m\}$ for $i = 2, 3, ..., m$.

We now establish a sequence of results to prove our main result.

Lemma 3.3 *Let $\{F_i \mid i \in I\}$ be a collection of ID-families, then $G = \bigcap\{F_i \mid i \in I\}$ is an ID-family.*

Proof. Since singleton relations satisfy all IDs, it is clear that $G \neq \emptyset$. To prove the lemma, we will use theorem 2.1. Let $r_1, r_2 \in G$, then $r_1, r_2 \in F_i$ for each i. Therefore, $r_1 \times r_2 \in F_i$ for each i. Hence, $r_1 \times r_2 \in G$ and G is closed under products. Similarly we can prove that G is closed under substructure.

Definition 3.1 *Let X be a collection of relations over R, then the smallest ID-family containing X is defined to be:*

$$G(X) = \bigcap\{F \mid X \subseteq F \text{ and F is an ID-family }\}$$

Lemma 3.3 together with the fact that $X \subseteq SAT(\emptyset)$ implies that $G(X)$ always exists.

Theorem 3.1 *Let $X = \{r\}$, then SPX is an ID-family and r is an Armstrong relation.*

Proof. Since SPX is closed under S and P, then by theorem 2.1, $SPX = SAT(\Sigma)$ for some set of IDs Σ. The definition of G(X), smallest ID-family containing X, and theorem 2.1 together imply that SPX = G(X).

Let $\Gamma = \{\gamma \mid r \models \gamma$ and γ is an ID $\}$, then by definition of G(X), we have $SPX \subseteq SAT(\Gamma)$. Also, since every member of Σ is true in r, we have $\Sigma \subseteq \Gamma$ which implies $SAT(\Gamma) \subseteq SAT(\Sigma)$. This shows that

$$G(X) = SPX = SAT(\Gamma) = SAT(\Sigma)$$

Now we show that r is an Armstrong relation for Σ. It is obvious that any σ which is the logical consequence of Σ is true in r. Suppose σ is not the logical consequence of Σ, then there exists a relation $s \in SAT(\Sigma)$ such that σ is false in s. Now, if σ is true in r, then σ will be a member of Γ. But this is a contradiction since $s \in SAT(\Sigma) = SAT(\Gamma)$.

Finally, we show that the collection of finitely generated ID-families is the same as the collection of ID-families possessing a finite Armstrong relation.

Theorem 3.2 *The collection of finitely generated ID-families and the collection of ID-families possessing a finite Armstrong relation are identical.*

Proof. By theorem 3.1 and lemma 3.2, finitely generated ID-families possess finite Armstrong relations. On the other hand, suppose F possesses a finite Armstrong relation r. Since $SP\{r\} = SAT(\Sigma)$ is the smallest ID-family containing r, it follows that $SAT(\Sigma) \subseteq F$. Now let $s \in F$ and suppose s is not a member of $SAT(\Sigma)$, then there exists a $\sigma \in \Sigma$ which is false in s. Since r is an Armstrong relation for F, it follows that σ is false in r. But this is a contradiction as $r \in SAT(\Sigma)$. This shows that $F \subseteq SAT(\Sigma)$.

4 Final remarks

Let $r = \{t\}$, then $F = SP\{r\}$ is the collection of all singletons together with $\emptyset$. F can be axiomatized by the set of all IDs. In addition, F can be axiomatized by the following finite set of IDs:

$$\forall x_1 ... \forall x_n \forall y_1 ... \forall y_n (R(x_1, x_2, ..., x_n) \wedge R(y_1, y_2, ..., y_n) \rightarrow x_1 = y_1)$$
$$\forall x_1 ... \forall x_n \forall y_1 ... \forall y_n (R(x_1, x_2, ..., x_n) \wedge R(y_1, y_2, ..., y_n) \rightarrow x_2 = y_2)$$

...

...

...

...

$$\forall x_1 ... \forall x_n \forall y_1 ... \forall y_n (R(x_1, x_2, ..., x_n) \wedge R(y_1, y_2, ..., y_n) \rightarrow x_n = y_n)$$

This example motivates one to investigate the relationship between finitely generated and finitely specifiable ID-families. Vardi[8] has constructed a finite set of IDs with no finite Armstrong relation. This together with theorem 3.2 shows that finitely specifiable ID-families are not finitely generated. We do not know whether finitely generated ID-families are finitely specifiable.

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