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# QUANTIZATION WITH KNOWLEDGE BASE APPLIED TO GEOMETRICAL NESTING PROBLEM

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**Keywords:** nesting, quantization, 2D allocation, knowledge base, geometry shapes, algorithm, simulation

## Abstract

Nesting algorithms deal with placing two-dimensional shapes on the given canvas. In this paper a binary way of solving the nesting problem is proposed. Geometric shapes are quantized into binary form, which is used to operate on them. After finishing nesting they are converted back into original geometrical form. Investigations showed, that there is a big influence of quantization accuracy for the nesting effect. However, greater accuracy results with longer time of computation. The proposed knowledge base system is able to strongly reduce the computational time..

## 1 Introduction

Nesting is a geometrical problem of placing two-dimensional shapes on a surface without overlap and with minimizing the surface area used. The goals of nesting algorithms can differ among minimizing wasted surface area and maximizing the amount of shapes placed on or in a specified container. This kind of problems are everyday-questions in the commercial companies, especially factories and cutting manufacturers. Also, nesting problems appear in environmental architecture planning, transport, and many other places. Nesting problems can be divided into [2]: *decision problems* (having given region and set of shapes the algorithm states whether shapes will fit in the region), *knapsack problem* (given shapes have to be placed on a given region in a way minimizing used surface), *bin packing* (there are set of shapes and set of regions, the algorithm minimizes a number of used regions needed to place set of shapes) and *strip packing problem* (with given set of shapes and a width of rectangular region, the algorithm has to minimize length of region containing all shapes placed).

Some nesting problem implementations allow overlapping shapes in specific situations [2]. Different constraints can be considered. Usually, there is no constraint on shape – it can be rectangle, also can contain roundness. More often constraints are applied to the nested region – due to technological issues, the region is often a fixed width rectangle with unlimited length (e.g. roll of material in clothing industry). According to the problem conditions, appropriate nesting algorithm should be used. There were attempts to solve this problem by using many different ways:

e.g. geometry theory, ant algorithms [1], heuristic methods [5], genetic algorithms [4] – but even for small sets of input data – nesting problem is hard to be solved in a reasonable time.

This paper describes a concept of applying quantization with knowledge base to slide the nesting algorithm. It is assumed, that there are no constraints on regions and on shapes. Each shape is converted into binary form, which is further used to pair shapes. As a pairing algorithm the *Min-Rectangle (MR)* [3] algorithm is used which is able to find a co-placement of two shapes giving smallest bounding rectangle. The proposed *QKBMR (Quantization with Knowledge Base in Minimum Rectangle)* system gives opportunities for simulations. Research done by the authors is focused on the influence of quantization and knowledge base implementation on the nesting process.

Section 2 states the problem of nesting. Section 3 contains definitions of basic terms and Section 4 presents the proposed *QKBMR* system. In Section 5 the results of investigations are discussed. Section 6 contains conclusions and perspectives of further research.

## 2 Problem statement

Nesting is a term that is used to describe several allocation problems of two- or three-dimensional cutting or placing defined set of shapes. Implementations of the problem can vary with different constraints. Constraints can be divided into the following categories:

- shapes constraints – overlapping conditions, known (or not) shapes queue, shapes queue sorting, etc.
- region constraints – shape of region, its infinity in specified dimensions, etc.
- nesting process constraints – minimizing time, minimizing usage area, etc.

In general, the nesting problem statement does not allow shape overlapping, but there are some nesting sub-problems where shapes overlapping can occur.

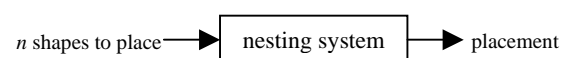


Figure 1. General nesting scheme

No matter what implementation, the scheme of nesting is always as in Fig. 1.

An example of the nesting problem – with region shape as a finite rectangle and known shape queue is shown in Fig. 2.

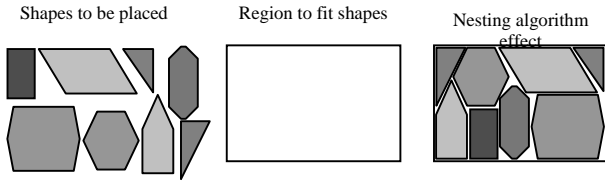


Figure 2. Example of nesting

The considered nesting problem may be stated as follows;

- *given*: the set of shapes with known geometry and the region in which shapes can be placed,
- *to find*: shape placement within the region,
- *such that*: to minimize wasted surface and the time of completing nesting process.

### 3 Nomenclature

To widely describe the nesting problem issues the following nomenclature is introduced:

**Shape** – a geometric closed form defined by geometric characteristic. Any shape can be described by set of points and arcs. Shape combined with  $n$  amount of  $e$  elements, where each  $e$  is a point  $P$  (described by position  $x,y$ ) or an arc  $A$  (described by two  $P$  and radius) is denoted as

$$G = \{e_n \in P:(x_n,y_n) \vee A:(x_{n1},y_{n1}, x_{n2},y_{n2}, r_n)\}.$$

**Region** – an area of potential placement of shapes. For the considered nesting problem the region is rectangular with infinite length and width. Region containing points  $P(x_n,y_n)$  where  $x_n,y_n \in R$  is denoted as

$$R = \{P(x_n,y_n): -\infty < x_n < +\infty \wedge -\infty < y_n < +\infty\}.$$

**Quantum** – a discrete part of geometric area, is characterized by size and logical binary state assigned. Quantum having  $a$  size of square side length in geometric interpretation and  $(x,y)$  discrete position in mesh  $M$  is denoted as  $Q_{(a)}(x,y)=\{0,1\}$ .

**Quantum size** – denoted as  $a$ , describes the size of quantum, what determines mesh structure and quantization accuracy.

**Intersection function** – denoted as  $INT(A,B)$  – returns logical true/false result: if area  $A$  and area  $B$  intersect, logical  $1$  is returned, otherwise  $INT(A,B)$  returns logical  $0$ .

**Bounding rectangle  $B_G(x,y)$**  – minimum size rectangle of width= $x$  and height= $y$  that  $G$  can fit inside without intersecting  $B_G$  boundaries.  $B_G$  assigned to  $G$  will satisfy following formula:

$$B_G = \{P(x_b,y_b): x_b,y_b \in R \wedge x_b < x_m,x_n < x_m \wedge y_b < y_m,y_n < y_m : (\forall P(x_p,y_p) \in G, x_m \leq x_p \wedge x_n \geq x_p \wedge y_m \leq y_p \wedge y_n \geq y_p)\}.$$

**Mesh** –  $R$  is divided (Fig. 3) into  $Q_{(a)}$  parts, that composes two dimensional mesh on nesting area. According to  $R$ , mesh having quantum size of  $a$  is denoted as  $M_{(a)}$  and has infinite length and width:

$$M_{(a)} = \{Q_{(a)}(x_n,y_n) \in R: -\infty < x_n < +\infty \wedge -\infty < y_n < +\infty\}$$

**Quantization process  $QP$**  – process of changing geometry shape into its binary representation.  $QP$  is performed using quantization function:

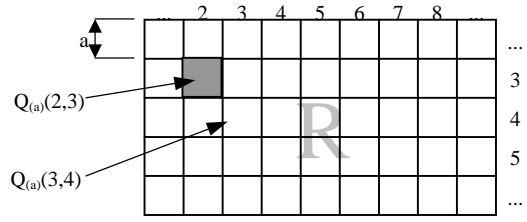


Figure 3. Region divided into  $M_{(a)}$  mesh

**Quantization process  $QP$**  – process of changing geometry shape into its binary representation.  $QP$  is performed using quantization function:

$$QP(G) = (\forall Q_{(a)}(x_m,y_n) \in B_G, Q_{(a)}(x_m,y_n) = INT(Q_{(a)}(x_m,y_n), G))$$

**Binary shape** – denoted as  $S(w,h)$  is quantized representation of geometric shape  $G$ .  $S(w,h)$  having width= $w$  and height= $h$  (measured in  $Q_{(a)}$  width/height which means multiplying  $w$  and  $h$  by  $a$ ), consist of  $Q_{(a)}(x_m,y_n) \in B_G$  and is the result of  $QP(G)$  function.

$$S(w,h) = \{Q_{(a)}(x_m,y_n) \in B_G : Q_{(a)}(x_m,y_n) = INT(Q_{(a)}(x_m,y_n), G) \wedge x_n \in \langle 0, w \rangle, y_n \in \langle 0, h \rangle; B_G(x_b,y_b): x_b = a * w, y_b = a * h\}$$

**Binary Shape Set BSS** – a finite set of  $S$  containing  $BSSC$  ( $BSSC$  Capacity) elements.  $BSSC$  can be variable during nesting process, which allows interactive adding of new shapes to  $BSS$ , where  $BSS = \{S_1, S_2, \dots, S_{BSSC}\}$ .

**AND( $S_1, S_2$ )** – binary AND operation for bit sequences  $S_1, S_2$ .

**AND( $S_1, S_2, \dots, S_n$ )** – binary AND operation for bit sequences  $S_1, S_2, \dots, S_n$

**OR( $S_1, S_2$ )** – binary OR operation for bit sequences  $S_1$  and  $S_2$ .

**XOR( $S_1, S_2$ )** – binary XOR operation for bit sequences  $S_1, S_2$ .

**NEG( $S$ )** – binary negation operation for bit sequence  $S$ .

**ROR1( $S$ )** – binary rotating right of binary sequence. Rotates by one position:

$$ROR1(S_1) = S_2: S[i] = S[i+1], S[n] = S[1]; i \in \langle 2, n \rangle,$$

where  $S_1$  is  $S$  before  $ROR1$ ,  $S_2$  is  $S$  after  $ROR1$ ,  $i$  is a position in a binary sequence,  $n$  is amount of bits in sequence.

**ROR( $S, t$ )** – binary rotating right of binary sequence ( $S$  shape) by  $t$  bits. Performs  $ROR1$  operation  $t$  times.

**ROL1( $S$ )** – binary rotating left of binary sequence. Rotates by one position:

$$ROL1(S_1) = S_2: S[i] = S[i-1], S[1] = S[n]; i \in \langle 2, n \rangle$$

where  $S_1$  is  $S$  before  $ROL1$ ,  $S_2$  is  $S$  after  $ROL1$ ,  $i$  is a position in a binary sequence,  $n$  is amount of bits in sequence.

**ROL( $S, t$ )** – binary rotating left of binary sequence ( $S$  shape) by  $t$  bits. Performs  $ROL1$  operation  $t$  times.

**Knowledge Base Element KBE** – a data set containing information about two  $S$  and their best combination.  $KBE$  containing knowledge about  $S_1, S_2$ , and the best combination  $C$  is denoted as  $KBE(S_1, S_2) = C$ .

**Knowledge Base KB** – finite set containing *KBC* (*KB Capacity*) number of *KBE*-s. *KB* expands self during nesting.

$$KB = \{KBE_1, KBE_2, \dots, KBE_{KBC}\}.$$

In the paper, the following assumptions are taken into consideration:

- *R* has no specified shape,
- resulting area with nested shapes can be of any shape,
- the system tries to afford the best speed of nesting, and next allocating efficiency,
- the shape queue is known to the system and it is able to preprocess it before starting *QP* and nesting process,
- all shapes are quantized using the same *a* in  $Q_{(a)}$ .
- the shapes can be rotated to obtain better efficiency (*UseRotating* parameter),
- rotating of *S* is performed by angle

$$\gamma = \frac{k\pi}{2}, k \in \{0,1,2,3\}.$$

## 4 Nesting system

The core of the proposed nesting system is *QKBMR* algorithm which reads a list of *G*, quantizes them and then tries to nest them in *R*. *QKBMR* transforms *R* in such a way, that  $(0,0)$  point can be defined, but it is not related to any *G* – no matter what shape of a *G* (or *G* combination) is – the algorithm places it starting from  $(0,0)$  point of *R*.

During quantization *a* parameter is used, which has a large influence on precision of quantizing – if *a* is smaller, more  $Q_{(a)}$  elements are used for quantizing *G*, which also changes the final nesting effect. *KB* is a very important part of nesting system.

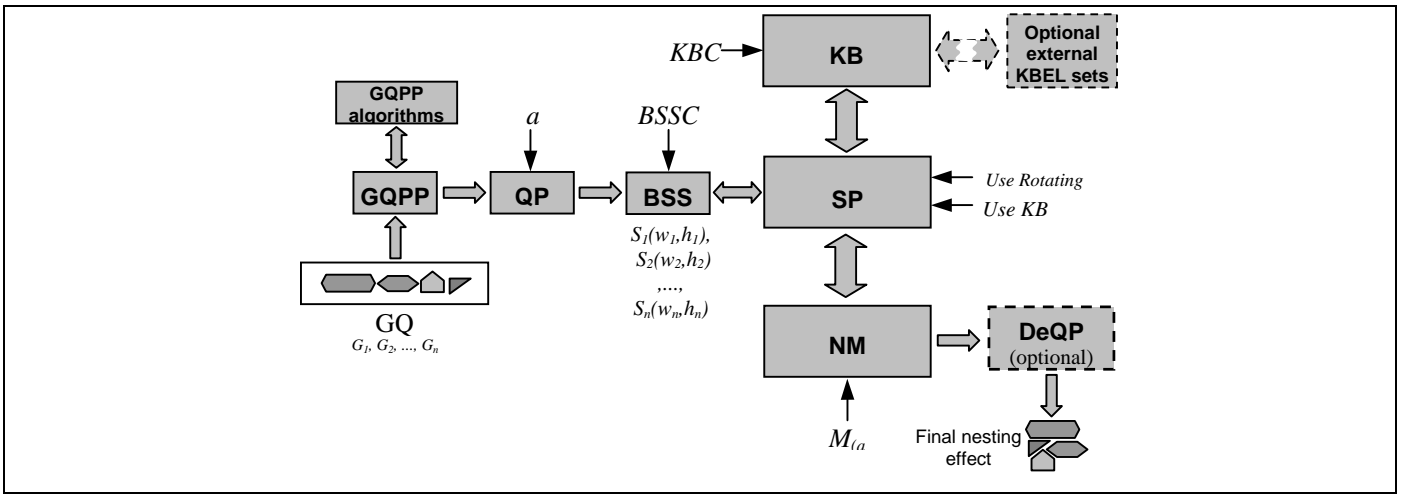


Figure 4. The block-diagram of the proposed nesting system

It may save a lot of time when nesting a shape that the system already processed. The *QKBMR* system with cooperating elements is presented in Fig.4.

The components of the system are:

**GQ** – *G Queue* – input queue of geometrical shapes *G*.

**GQPP** – *GQ Preprocessor* – a module that is able to operate on input *GQ* before *QP*. GQPP uses *GQPP algorithm set* to choose method of processing *GQ*.

**QP** – *Quantization Process* – converting *GQ* containing *G* elements to *BSS*. Parameter *a* determines precision of conversion process.

**BSS** – *Binary Shape Set* – set of *BSSC* number of *S* elements. *BSS* is the result of *QP*.

**SP** – *Shape pairing* – module trying to find the best co-placement of  $S_a$  and  $S_b$ . *UseRotating* parameter determines whether *QKBMR* is allowed to use rotating of *S* elements, *Use KB* parameters enables to use *KB*.

**KB** – *Knowledge Base*, containing *KBC* number of *KBEL*.

**Optional external KBEL sets** – *KB* can use knowledge that was generated by another instance of *KBE*. External sets could also be implemented as global network database, used by many *QKBMR* systems to share knowledge about shape pairing.

**NM** (*Nesting Module*) – main part of system that manages *S* elements and finally places them in *R*.  $M_{(a)}$  parameter is used to describe mesh in *R*.

**DeQP** – *DeQuantization Process* – converting *S* elements into *G* elements. *DeQP* is an optional module, can be omitted.

### 4.1 Quantization process

Quantization process (*QP*) outputs with *S* having *G* object as an input. *QP* is performed for each *G* separately. The size of *S* and accuracy of *QP* depends on *a* parameter. The time needed to perform *QP* on *G* can be expressed with a following relation:

$$T_{QP} = kWHla \quad (1)$$

where: *W* – geometric width of *G*, *H* – geometric height of *G*, *k, l* – coefficients related to the machine used.

The *QP* algorithm for a given *G* works as follows:

1. Find  $B_G(x_m, y_m)$ .
2. Divide  $B_G(x_m, y_m)$  into  $Q_a(x_a, y_b)$  elements. The result is a mesh of  $Q_a$  elements with *w* amount of  $Q_a$  in each row of mesh and *h* amount of  $Q_a$  elements in each column.

- Assign state to all  $Q_a(x_a, y_b)$  elements:  $\forall Q_a(x_c, y_d) \in B_G$ ,  $Q_a(x_c, y_d) = INT(Q_a(x_c, y_d), G)$ ,  $c \in \langle 0, w \rangle$ ,  $d \in \langle 0, h \rangle$ .

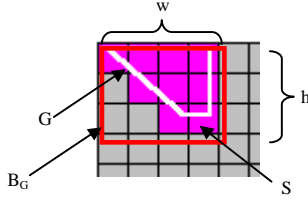


Figure 5. Shape  $G$  and its quantized representation  $S$

**Example.** The quantized form of the shape  $G$  (Fig. 5) can be denoted as

$$S(3,2)=111101110011.$$

## 4.2 Normalization and denormalization process

The *Normalization Process (NP)* is a process of extending two  $S$ -es according to their properties. Each shape ( $S_1$  and  $S_2$ ) is normalized.  $S_1$  is always a base – after normalization is centered in its own mesh.  $S_2$  is placed in left-top corner of its own mesh.  $NP$  adds columns and/or rows containing zeroes to both  $S$ , to equalize them by size.  $NP$  allows finding suitable coexistence by using  $SP$  process. The  $NP$  uses the following algorithm:

- For two received  $S$  objects:  $S_1(x_1, y_1)$  and  $S_2(x_2, y_2)$  compute:  $mx=x_1+2*y_2$ ,  $my=y_1+2*y_2$ .
- Normalize  $S_1$ :
  - Add  $c=mx-x_1$  columns to  $S_1$  (contain only zeroes).
  - $x_1=mx$
  - Add  $r=mx-x_1$  rows to  $S_1$  (contain only zeroes).
  - $y_1=my$
  - $t=c/2+r/2*y_1$ ; Perform  $ROR$  with  $t$  positions.
- Normalize  $S_2$ :
  - Add  $c=mx-x_2$  columns to  $S_2$  (contain only zeroes).
  - $x_2=mx$
  - Add  $r=mx-x_2$  rows to  $S_2$  (contain only zeroes).
  - $y_2=my$

The  $QKBMR$  algorithm also needs *Denormalization Process (DP)*, which performs removing bordering rows and columns containing only zeroes. The rule states:  $DP(NP(S))=S$ .  $DP(S(x, y))$ . It is performed using the following algorithm:

- $C_T=(2^{x+1}-1)*2^{(x+1)y}$ , If  $AND(S, C_T)=0$  - perform remove first  $x+1$  bits from binary representation.
- $C_B=NEG(2^{(x+1)(y+1)}-2^{x+1})$ , If  $AND(S, C_B)=0$  – perform remove last  $x+1$  bits from binary representation.
- $C_L=C_{x+1}$ ,  $C_k=C_{k-1}*2^{x+1}+C_0$ ,  $C_0=2^x$ ; If  $AND(S, C_L)=0$ , perform remove bits from positions  $p$ , for whose  $(p \text{ modulo } (x+1))=1$  formula is satisfied..
- $C_R=NEG(AND(C_1, C_2, C_3, \dots, C_{(y+1)}))$ , where  $C_k=(2^{k(x+1)}-2)*2^{(y+1-k)(x+1)}+2^{(y+1-k)(x+1)}-1$ . If  $AND(S, C_R)=0$ , perform remove bits from positions  $p$ , for whose  $(p \text{ modulo } (x+1))=0$  formula is satisfied.

**Example.**  $S$  normalized with  $NP$  is shown in Tab. 1.

Table 1. An example of *Normalization*

Before $NP$		After $NP$	
$S_1$	$S_2$	$S_1$	$S_2$
1111	11	00000000	11000000
0111	01	00000000	01000000
0011		00111100	00000000
		00011100	00000000
		00001100	00000000
		00000000	00000000
		00000000	00000000

## 4.3 Shape pairing

*Shape Pairing (SP)* is an algorithm, that tries to find the best coexistent  $S_2$  for a given  $S_1$ .  $S_1$  is always the first  $S$  from  $BSS$ .  $SP$  compares  $S_1$  with every unpaired  $S$  from  $BSS$  and records the actual best pair. The efficiency of a given pair is determined by the  $EFF$  (2) coefficient:

$$EFF = 1 / ((S_{1w} + 1) * (S_{1h} + 1)) \quad (2)$$

The  $EFF$  uses  $w$  and  $h$  from  $S_1$  because after  $NP$  both  $S_1$  and  $S_2$  have the same  $w$  and  $h$ . The  $S$  that is the best pair for  $S_1$  according to the  $EFF$  coefficient is marked as “paired”.  $SP$  works using the following procedure:

- $S_1$  is a base for pairing and “stationary” object (will remain unmoved in normalized mesh).  $S_2$  is a moving object.
- $GLEFF=0$
- For every unpaired  $S$  from  $BSS$  do:
  - Normalize  $S_1(x_1, y_1)$  with  $S_2(x_2, y_2)$
  - $S_R=ROR(S_2)$
  - If  $S_2[I]=0 \vee S_R[I]=1$ , go to 4.
  - If  $AND(S_2, C)=0 \vee AND(S_R, C) \neq 0$ , where  $C=C_{x_2+1}$ ,  $C_k=C_{k-1}*2^{y_1+1}+C_0$ ,  $C_0=2^{x_1}$ , perform  $ROR(S_2, x_2+1)$ , else perform  $ROR(S_2, 1)$ .
  - $M=XOR(XOR(OR(S_1, S_2), S_1), S_2)$
  - If  $M \neq 0$ , go to 3.2
  - Compute  $EFF(OR(S_1, S_2))$
  - If  $GLEFF < EFF$ ,  $GLEFF=EFF$  and  $BF=OR(S_1, S_2)$
- $BF$  contains best found co-placement of  $S_1$  and  $S_2$ ,  $GLEFF$  contains  $EFF$  for this pair.

After performing the  $SP$  algorithm, the  $QKBMR$  receives information from the  $SP$  module about the best fit found for a given  $S_1$ , the number of  $S_2$  that made the best fit with  $S_1$ , and the value of  $EFF$  coefficient for this fit.

## 4.4 Knowledge base

Every  $S$  is described by some bits and  $x$  and  $y$  coefficients. Depending on the  $a$  factor, the  $QP$  process differs in accuracy. This means, that many shapes can be quantized into the same

bit representation, and  $a$  coefficient has large influence on how many shapes from a given set will go into equal binary appearance.  $SP$  can use  $KB$  to optimize the process of searching for the best fit. Before using the internal pairing algorithm,  $SP$  can request  $KB$  for a specified pair  $S_1$  and  $S_2$ . If  $KB$  has such a record, it will reply to  $SP$  with the best fit. This best fit can be placed in  $KB$  by the same  $SP$  module, or can originate from another module. Many different *Nesting systems* can share one  $KB$ . If  $KB$  does not have such a record, according to  $S_1$  and  $S_2$ , it replies to  $SP$  with “no result” message. In that case,  $SP$  performs an internal pairing algorithm for  $S_1$  and  $S_2$  and results with the best fit for these two considered shapes. Then,  $SP$  sends the  $S_1$ ,  $S_2$  and result  $S_3$  to  $KB$ , which saves it for the future usage. When using  $KB$ , the  $SP$  works in the following way:

1.  $S_1$  is a base for pairing and “stationary” object (will remain unmoved in normalized mesh).  $S_2$  is moving object.
2.  $GLEFF=0$
3. For every unpaired  $S$  from  $BSS$  do:
  - 3.1. Ask  $KB$  for  $S_1$  and  $S_2$ . If  $KB$  replied with best fit answer, place answer to  $BF$ , count  $EFF$  and put in into  $GLEFF$ , go to 5.
  - 3.2. Normalize  $S_1(x_1, y_1)$  with  $S_2(x_2, y_2)$
  - 3.3.  $S_R=RORI(S_2)$
  - 3.4. If  $S_2[l]=0 \vee S_R[l]=1$ , go to 4.
  - 3.5. If  $AND(S_2, C)=0 \vee AND(S_R, C) \neq 0$ , where  $C=C_{x_2+1}$ ,  $C_k=C_{k-1} * 2^{x_1+1} + C_0$ ,  $C_0=2^{x_1}$ , perform  $ROR(S_2, x_2+1)$ , else perform  $ROR(S_2, 1)$ .
  - 3.6.  $M=XOR(XOR(OR(S_1, S_2), S_1), S_2)$
  - 3.7. If  $M \neq 0$ , go to 3.3.
  - 3.8. Compute  $EFF(OR(S_1, S_2))$
  - 3.9. If  $GLEFF < EFF$ ,  $GLEFF=EFF$  and  $BF=OR(S_1, S_2)$
4. Send  $S_1$ ,  $S_2$ , and  $BF$  to  $KB$ .
5.  $BF$  contains best found co-placement of  $S_1$  and  $S_2$ ,  $GLEFF$  contains  $EFF$  for this pair.

## 4.5 Rotating mechanism

The nesting problem, stated in this paper, allows  $G$  objects to be rotated. This means, that also  $S$  objects can be rotated.  $QKBMR$  algorithm uses  $RM$  (*Rotating Mechanism*) to rotate binary representations of  $G$ , to afford better fit of two shapes. Rotating is performed when searching for the best fit, the rotated figure is also used while nesting. The  $BSS$  always contains non-rotated objects – also objects that are nested in rotated form, remain in original non-rotated figure. Rotating binary shapes is not easy when rotating angle is other than  $\gamma=k\pi/2$ ,  $k \in \{0, 1, 2, 3\}$ , so  $QKBMR$  algorithm uses only these four values while rotating  $S$  objects. Rotating can increase the time needed for finding the best fit, but in many cases it is able to find much better fit. When using  $KB$ , performing  $RM$  is suggested.  $RM$  gives better results of pairing, these results will be added to  $KB$ , so it is good to add better fits because pairs recorded in  $KB$  will not be paired anymore.  $SP$  with  $RM$  (and also  $KB$ ) works using the following procedure:

1.  $S_1$  is base of pairing and “stationary” object (will remain unmoved in normalized mesh).  $S_2$  is moving object.
2.  $GLEFF=0$ ;  $k=-1$
3. For every unpaired  $S$  from  $BSS$  do:
  - 3.1. Ask  $KB$  for  $S_1$  and  $S_2$ . If  $KB$  replied with best fit answer, place answer to  $BF$ , count  $EFF$  and put in into  $GLEFF$ , go to 5.
  - 3.2.  $k=k+1$ . Rotate  $S_2$  using  $\gamma=k\pi/2$  angle
  - 3.3. Normalize  $S_1(x_1, y_1)$  with  $S_2(x_2, y_2)$
  - 3.4.  $S_R=RORI(S_2)$
  - 3.5. If  $S_2[l]=0 \vee S_R[l]=1$ , go to 3.11.
  - 3.6. If  $AND(S_2, C)=0 \vee AND(S_R, C) \neq 0$ , where  $C=C_{x_2+1}$ ,  $C_k=C_{k-1} * 2^{x_1+1} + C_0$ ,  $C_0=2^{x_1}$ , perform  $ROR(S_2, x_2+1)$ , else perform  $ROR(S_2, 1)$ .
  - 3.7.  $M=XOR(XOR(OR(S_1, S_2), S_1), S_2)$
  - 3.8. If  $M \neq 0$ , go to 3.4.
  - 3.9. Compute  $EFF(OR(S_1, S_2))$
  - 3.10. If  $GLEFF < EFF$ ,  $GLEFF=EFF$  and  $BF=OR(S_1, S_2)$
  - 3.11. If  $k < 3$  go to 3.2.
4. Send  $S_1$ ,  $S_2$ , and  $BF$  to  $KB$ .
5.  $BF$  contains best found co-placement of  $S_1$  and  $S_2$ ,  $GLEFF$  contains  $EFF$  for this pair.

## 4.6 Nesting module

$NM$  is the main module of  $QKBMR$  system.  $NM$  sends requests to  $SP$ . After performing  $SP$ , resulting pair ( $S_1$  and  $S_2$ ) is merged and recorded as  $S_1$ .  $S_2$  is marked as “paired” – so it will not be taken into consideration during the next pairing processes.  $NM$  has a block, that is able to decide whether to finalize nesting of  $S_1$  and  $S_2$  (place them on  $M_{(a)}$ ) or to send a request to  $SP$  once more, but with special conditions. This can be useful, when the algorithm used in  $SP$  does not work well enough – then  $NM$  can detect that kind of pair and request to find the pair for  $S_1$  again, without using  $S_2$ , previously rated as the best fit.  $NM$  is the only module of  $QKBMR$ , that is able to operate on  $BSS$  during nesting process – so it is easy to implement some exclusions for pairing algorithms. Also, according to some rules,  $NM$  can pre-nest some shapes, before starting to request  $SP$ . These additional functions had not been researched and are categorized as future work.

## 4.7 GQPP and DeQP

Two additional mechanisms of operating on shapes, called  $GQPP$  and  $DeQP$ , are included into  $QKBMR$  algorithm.

$GQPP$  is a  $GQ$  preprocessor. For some  $G$  sets and pairing algorithms, some operations on  $G$  set can speed up the nesting process.  $GQPP$  performs some sorting or other methods of changing  $G$  order in  $GQ$ . In the presented research,  $GQPP$  was not activated.

$DeQP$  is a module that performs de-quantization: converts binary representation ( $S$ ) into geometrical figure ( $G$ ). Due to  $DeQP$ ,  $QKBMR$  is able to restore original form of shapes after the nesting process, so binary conversion is transparent for the user of  $QKBMR$  algorithm. Because  $QKBMR$  records original geometric form of shape, so there is no loss of information.  $DeQP$  also uses recorded relations of quantized form and

original form – so regardless of  $a$  parameter, after  $QP$  and  $DeQP$ , shape will be placed in the same place on the canvas.

## 5 Investigations

$QKBMR$  algorithm had been implemented in the experimentation system to research the efficiency of the proposed mechanisms.  $QKBMR$  algorithm is a complex system, containing many modules presented in Section 4 and requiring the usage of several algorithms.

$QP$  is a process that is required to be performed before the other nesting modules.  $QP$  effect depends on  $a$  coefficient in an exponential way. The relationship between  $a$  and  $QP$ , found on the basis of results of simulations is shown in Fig. 6.

A small  $a$  coefficient gives (more accurate) mapping of  $G$  to  $S$ , but requires more time. When performing the nesting process using algorithms that use quantized shapes, it is possible to quantize them once and store in that form, so no quantization would be needed during next nesting processes. For every nesting case, an appropriate  $a$  coefficient should be taken. It also affects the nesting accuracy. Many random-generated sets of shapes were taken, processed with  $QP$ , and then results averaged.

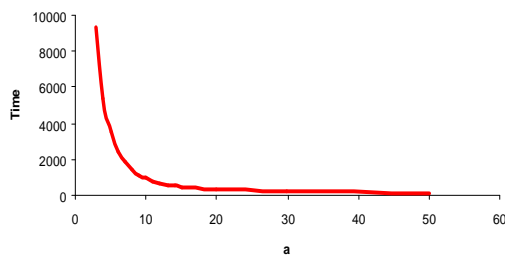


Figure 6. Influence of  $a$  coefficient to time of  $QP$

The quantization coefficient  $a$  also affects to the time of nesting process. When  $G$  is transformed into a larger set of bits,  $SP$  module has more data to process. This is an exponential relation, shown in Fig. 7. in logarithmic scale. As is visible on the graph, there is an  $a$  value, for which time decreases much, and for larger  $a$  values, its influence to time is smaller. If results of nesting for this edge value are satisfactory, this value should be used for processing shape sets. The time and  $a$  impact can differ according to the algorithm used by  $SP$  module.

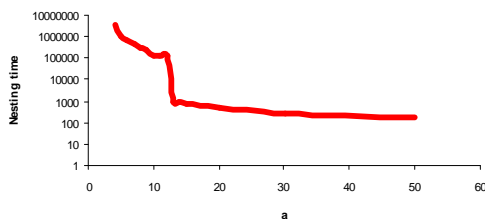


Figure 7. Influence of  $a$  coefficient to time of nesting process

During the nesting process, in module  $SP$  the same  $S$  can form a pair with other shapes many times. Because of the fact that the nesting process is very time-consuming it is worth trying to save the time by recording already computed best-fits for shapes. A knowledge base system used in  $QKBMR$  algorithm saves all the computed cases for future use, not only by the

given system but also by other systems that share the given  $KB$ . There is some time needed by  $KB$ , but it is very small compared to the pairing process. Investigations showed, that in the standard averaged case taking information about pairs from  $KB$  is approx 2% of the time-consumed for computing their best fit (which strongly depends on the  $a$  coefficient).

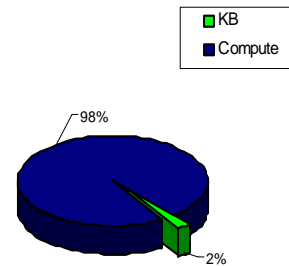


Figure 8. Time of pairing and time time of acquiring info from  $KB$

The implementation of  $RM$  may result with a better  $EFF$  coefficient, but requires more computations, because of the fact that every  $S$  is processed 4 times (0, 90, 180 and 270 rotating angle). Investigations showed, that depending on type of  $S$  (its geometrical shape),  $RM$  can highly increase the  $EFF$  coefficient. Table 2 shows the results - the averaged values of  $EFF$  computed with (2) for  $RM$  (turned on) and  $RM$  (turned off) for the same data sets.

Table 2. Efficiency and using  $RM$ .

	RM OFF	RM ON
Average EFF	0.021	0.038

## 6 Conclusions

The proposed  $QKBMR$  algorithm seems to be promising for sets of shapes with many objects of the same categories, as in the most industrial nesting processes. Thus, the proposed approach can strongly decrease the time of nesting.

The further work in the area of nesting systems will be concentrated on finding more effective shape pairing algorithms as it is the most time consuming module.

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