Feedback linearization and adaptive control of a nonlinear aeroelastic system

WenHong Xing

University of Nevada, Las Vegas

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FEEDBACK LINEARIZATION AND ADAPTIVE CONTROL OF A NONLINEAR AEROELASTIC SYSTEM

by

WenHong Xing

Bachelor of Engineering
Northern JiaoTong University, Beijing, China
1996

A thesis submitted in partial fulfillment of the requirements for the Master of Science Degree
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ABSTRACT

Feedback Linearization and Adaptive Control of a Nonlinear Aeroelastic System

by

WenHong Xing

Dr. Sahjendra N. Singh, Examination Committee Chair
Professor of Electrical Engineering
University of Nevada, Las Vegas

In this thesis, it is exhibited that although, the aeroelastic system is not input-state feedback linearizable, a partial linearized representation of the system in dimension three can be obtained. Based on this partially linearized representation, a new inverse controller was derived and simulation results show that control of pitch angle and plunge displacement can be accomplished.

Then, adaptive output feedback control law is examined. For the synthesis of the controller, it is assumed that only pitch angle and plunge displacement are measured. A canonical state variable representation of the system is derived for the reconstruction of the state variable. According to the new state variable form of the aeroelastic system, filters are designed and an estimate of states are constructed using a linear combination of the states of the filters. Based on a backstepping design technique, adaptive control laws for the control of pitch angle and plunge displacement are derived. Simulation results are presented to show the adaptive state regulation capability of the control system. Finally, reduced order filters are designed to obtain the unmeasured state variables and new adaptive control laws are obtained. In the close-loop system, the state vector is shown to converge asymptotically to zero.
# Table of Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF FIGURE</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td>vi</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>vii</td>
</tr>
<tr>
<td>CHAPTER 1 INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>Past work</td>
<td>1</td>
</tr>
<tr>
<td>Aeroelastic Model and Control Problem</td>
<td>2</td>
</tr>
<tr>
<td>CHAPTER 2 FEEDBACK LINEARIZABILITY AND INVERSE CONTROL</td>
<td>6</td>
</tr>
<tr>
<td>Introduction</td>
<td>6</td>
</tr>
<tr>
<td>Feedback Linearization</td>
<td>6</td>
</tr>
<tr>
<td>Inverse Control</td>
<td>10</td>
</tr>
<tr>
<td>Simulation</td>
<td>13</td>
</tr>
<tr>
<td>Summary</td>
<td>14</td>
</tr>
<tr>
<td>CHAPTER 3 ADAPTIVE OUTPUT FEEDBACK CONTROL</td>
<td>16</td>
</tr>
<tr>
<td>Introduction</td>
<td>16</td>
</tr>
<tr>
<td>A Canonical Form, Filters and State Estimation</td>
<td>17</td>
</tr>
<tr>
<td>Adaptive Control Laws</td>
<td>19</td>
</tr>
<tr>
<td>Simulation Results</td>
<td>25</td>
</tr>
<tr>
<td>Summary</td>
<td>28</td>
</tr>
<tr>
<td>Proof of Theorem 3.1</td>
<td>28</td>
</tr>
<tr>
<td>CHAPTER 4 ADAPTIVE CONTROL WITH REDUCED ORDER OBSERVER</td>
<td>37</td>
</tr>
<tr>
<td>Introduction</td>
<td>37</td>
</tr>
<tr>
<td>Reduced Order Filters and State Estimation</td>
<td>37</td>
</tr>
<tr>
<td>Adaptive Control Laws</td>
<td>40</td>
</tr>
<tr>
<td>Simulation Results</td>
<td>46</td>
</tr>
<tr>
<td>Summary</td>
<td>49</td>
</tr>
<tr>
<td>Proof of Theorem 4.1</td>
<td>49</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>58</td>
</tr>
<tr>
<td>VITA</td>
<td>62</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

1.1 Aeroelastic model ................................................................. 5
2.1 Inverse control $\alpha = -0.6 \ U = 15\text{m/s}$ ................................ 15
3.1 Pitch angle control $\alpha = -0.3 \ U = 15\text{m/s}$ ............... 31
3.2 Pitch angle control $\alpha = -0.4 \ U = 15\text{m/s}$ ............... 32
3.3 Pitch angle control $\alpha = -0.4 \ U = 20\text{m/s}$ ............... 33
3.4 Plunge motion control $\alpha = -0.85 \ U = 15\text{m/s}$ ........ 34
3.5 Plunge motion control $\alpha = -0.88 \ U = 15\text{m/s}$ ........ 35
3.6 Plunge motion control $\alpha = -0.88 \ U = 20\text{m/s}$ ........ 36
4.1 Pitch angle control $\alpha = -0.2 \ U = 15\text{m/s}$ ............... 52
4.2 Pitch angle control $\alpha = -0.2 \ U = 20\text{m/s}$ ............... 53
4.3 Pitch angle control $\alpha = -0.32 \ U = 15\text{m/s}$ ............. 54
4.4 Plunge motion control $\alpha = -0.75 \ U = 15\text{m/s}$ ........ 55
4.5 Plunge motion control $\alpha = -0.85 \ U = 15\text{m/s}$ ........ 56
4.6 Plunge motion control $\alpha = -0.85 \ U = 20\text{m/s}$ ........ 57
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>nondimensionalized distance from the midchord to the elastic axis</td>
</tr>
<tr>
<td>b_s</td>
<td>semichord of the wing</td>
</tr>
<tr>
<td>c_h</td>
<td>structural damping coefficient in plunge due to viscous damping</td>
</tr>
<tr>
<td>c_o</td>
<td>structural damping coefficient in pitch due to viscous damping</td>
</tr>
<tr>
<td>h</td>
<td>plunge displacement</td>
</tr>
<tr>
<td>I_a</td>
<td>mass moment of inertia of the wing about the elastic axis</td>
</tr>
<tr>
<td>k_h</td>
<td>structural spring constant in plunge</td>
</tr>
<tr>
<td>k_o</td>
<td>structural spring constant in pitch</td>
</tr>
<tr>
<td>m</td>
<td>mass</td>
</tr>
<tr>
<td>U</td>
<td>free stream velocity</td>
</tr>
<tr>
<td>x_a</td>
<td>nondimensionalized distance measured from the elastic axis to the center of mass</td>
</tr>
<tr>
<td>a</td>
<td>pitch angle</td>
</tr>
<tr>
<td>b</td>
<td>flap deflection</td>
</tr>
<tr>
<td>p</td>
<td>density of air</td>
</tr>
<tr>
<td>x</td>
<td>filter states</td>
</tr>
<tr>
<td>q, x</td>
<td>states of the aeroelastic system</td>
</tr>
<tr>
<td>\theta, \rho, \dot{\theta}, \dot{\rho}</td>
<td>parameters; estimate of parameters</td>
</tr>
<tr>
<td>c_i, L, L_i, d_i, \Gamma</td>
<td>design parameters</td>
</tr>
<tr>
<td>v_i, s_i</td>
<td>components of \Omega</td>
</tr>
</tbody>
</table>
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Aeroelasticity is concerned with the interaction among inertial, elastic and aerodynamic forces. Aeroelastic phenomena significantly affect the stability and control performance of aerospace vehicle. Flutter is an oscillatory aeroelastic instability caused by unsteady aerodynamic loads.

In this work, the theory that is developed for the nonlinear control of aeroelastic system are applied to two major problems. First, it’s applied to the linearizability of this system. Later, the nonlinear adaptive control law is examined and derived.

1.1 Past Work

Aeroelastic systems exhibit a variety of phenomena including instability, limit cycle, and even chaotic vibration. Active control of aeroelastic instability is an important problem. Several researchers have analyzed the stability properties of aeroelastic systems and designed controllers for the flutter suppression. Mukhopadhyay et al. and Gangsass et al. developed methods for obtaining lower order models and designed controllers. Karpel used pole placement technique to design controllers for flutter suppression and gust alleviation. Horikawa and Dowell performed flutter analysis using root-locus plots. Piezoelectric actuation has been considered for flutter control in Refs. 6 and 7. In these studies, linear control theory has been used for the design of controllers. Digital adaptive control of a linear autoregressive moving average aeroelastic model has been considered by Friedmann et al. Because
the linear design is often not adequate, researchers have developed control systems for nonlinear aeroelastic models. Although nonlinearities arising from control saturation, free play, hysteresis, and stability derivatives are encountered in aeroelastic systems, nonlinear structural stiffness may play a dominant role in causing the onset of flutter.

Recently, an aeroelastic apparatus has been developed and tests have been performed in a wind tunnel to examine the effect of nonlinear structural stiffness. In a series of interesting papers, Ko et al. have designed control systems for this aeroelastic system using a feedback linearizing technique and an adaptive control strategy. In a study by Block and Strganac, the unsteady aerodynamics are modeled with an approximation to Theodorsen's theory and linear control laws are derived for the active control of the aeroelastic model described in Ref. 12. A variable structure adaptive control of a prototypical aeroelastic wing has been also considered in Ref. 16. Since the aerodynamic and structural parameters are not known precisely, the adaptive controllers designed in Refs. 14 and 16 are useful. However, the adaptive design in Ref. 14 requires complete knowledge of the state variables and uses two control surfaces for state regulation. Although the variable structure adaptive controller of Ref. 16 uses only output feedback, it is assumed that the bounds on uncertain parameters are known for the control law derivation. Furthermore, this controller uses a high gain feedback which often leads to control saturation and may cause instability. Thus it is important to develop control systems for the active control of aeroelastic systems using output feedback in the presence of parameter uncertainty.

1.2 Aeroelastic Model and Control Problem

This section gives the equations of motion for the aeroelastic model. The prototypical aeroelastic wing section is shown in Fig. 1.1. The governing equations of
motion are provided in Ref. 13 which are given by

\[
\begin{bmatrix}
\frac{m}{mx_{ab}} & mx_{ab}h \\
mx_{ab}h & L_a
\end{bmatrix}
\begin{bmatrix}
h \\
\dot{h}
\end{bmatrix}
+ \begin{bmatrix}
c_h & 0 \\
0 & c_\alpha
\end{bmatrix}
\begin{bmatrix}
h \\
\dot{\alpha}
\end{bmatrix}
\]

\begin{equation}
+ \begin{bmatrix}
k_h & 0 \\
0 & k_\alpha(\alpha)
\end{bmatrix}
\begin{bmatrix}
h \\
\alpha
\end{bmatrix}
= \begin{bmatrix}
-L \\
M
\end{bmatrix}
\end{equation}

where \( h \) is the plunge displacement and \( \alpha \) is the pitch angle. In Eq.(1.1), \( m \) is the mass of the wing; \( c_\alpha \) and \( c_h \) are the pitch and plunge damping coefficients, respectively; and \( M \) and \( L \) are the aerodynamic lift and moment. It is assumed that the quasi-steady aerodynamic force and moment are of the form

\[
L = \rho U^2 b_s c_{\alpha} [\alpha + (\dot{h}/U) + (1/2 - a)b_s(\dot{\alpha}/U)] + \rho U^2 b_s c_{\mu a} \beta \\
M = \rho U^2 b_s^2 c_{\mu a} [\alpha + (\dot{h}/U) + (1/2 - a)b_s(\dot{\alpha}/U)] + \rho U^2 b_s^2 c_{\mu a} \beta
\]

where \( c_{\alpha} \) and \( c_{\mu a} \) are the lift and moment coefficients per angle of attack and \( c_{\mu a} \) and \( c_{\mu a} \) are lift and moment coefficients per control surface deflection. Although, other forms of nonlinear spring stiffness associated with the pitch motion can be considered, for purposes of illustration the function \( k_\alpha(\alpha) \) is considered as a polynomial nonlinearity given by

\[
k_\alpha(\alpha) = 2.82(1 - 22.1\alpha + 1315.5\alpha^2 - 8580\alpha^3 + 17.289.7\alpha^4)
\]

Defining the state vector \( q = (\alpha, \dot{\alpha}, h, \dot{h})^T \), one obtains a state variable representation of Eq.(1.1) in the form

\[
\dot{q} = \begin{bmatrix}
0_{2\times2} & I_{2\times2} \\
M_1 & M_2
\end{bmatrix} q + \begin{bmatrix}
0_{2\times1} \\
0_{2\times1}
\end{bmatrix} k_{n\alpha}(\alpha) + \begin{bmatrix}
0_{2\times1} \\
0_{2\times1}
\end{bmatrix} \beta
\]

(1.4)

where \( \alpha k_\alpha = \alpha k_{\alpha a} + k_{n\alpha} \), \( k_{n\alpha} = k_{\alpha 1} \alpha^2 + k_{\alpha 2} \alpha^3 + k_{\alpha 3} \alpha^4 + k_{\alpha 4} \alpha^5 \), \( b = (b_1, b_2)^T \), \( g = (g_1, g_2)^T \), \( 0 \) and \( I \) denote null and identity matrices of appropriate dimensions, and

\[
M_1 = \begin{bmatrix}
-(k_4 U^2 + md^{-1} k_{\alpha a}) & -k_3 \\
-(k_2 U^2 - mx_{a b s d^{-1} k_{\alpha a}}) & -k_1
\end{bmatrix}
\]
\[ M_2 = \begin{bmatrix} -c_{41} & -c_{31} \\ -c_{21} & -c_{11} \end{bmatrix} \]

The system parameters are given by

\[ \begin{align*}
 b_s &= 0.135 \text{ m} & k_h &= 2844.4 \text{ N/m} & c_h &= 27.43 \text{ Ns/m} \\
 c_a &= 0.036 \text{ Ns} & \rho &= 1.225 \text{ kg/m}^3 & c_{\alpha} &= 6.28 \\
 c_{ld} &= 3.358 & c_{\alpha a} &= (0.5 + a) & c_{m3} &= -0.635 \\
m &= 12.387 \text{ kg} & I_a &= 0.065 \text{ kgm}^2 & x_a &= [0.0873 - (b_s + ab_s)]/b_s
\end{align*} \]

The system variables are given by

\[ \begin{align*}
 d &= m(I_a - mx_a^2 b_s^2) \\
k_1 &= I_a k_h / d \\
k_2 &= (I_a \rho b_s c_{\alpha} + m x_a b_s^3 \rho c_{ma}) / d \\
k_3 &= -m x_a b_s k_h / d \\
k_4 &= (-m x_a b_s^2 \rho c_{\alpha} - m \rho b_s^2 c_{ma}) / d \\
c_{11} &= [I_a (c_h + \rho U b_s c_{\alpha}) + m x_a \rho U b_s^3 c_{ma}] / d \\
c_{21} &= [I_a \rho U b_s^2 c_{\alpha} (1/2 - a) - m x_a b_s c_{\alpha} + m x_a \rho U b_s^4 c_{ma} (1/2 - a)] / d \\
c_{31} &= (-m x_a b_s c_h - m x_a \rho U b_s^2 c_{\alpha} - m \rho U b_s^3 c_{ma}) / d \\
c_{41} &= [mc_a - m x_a \rho U b_s^3 c_{\alpha} (1/2 - a) - m \rho U b_s^3 c_{ma} (1/2 - a)] / d \\
b_1 &= U^2 (m x_a b_s^2 \rho c_{\beta} + m \rho b_s^2 c_{m3}) / d \\
b_2 &= U^2 (-I_a \rho b_s c_{\beta} - m x_a b_s^3 \rho c_{m3}) / d \\
g &= \begin{bmatrix} -m/d \\ mx_a b_s / d \end{bmatrix}
\]
Figure 1.1: Aeroelastic model
CHAPTER 2

FEEDBACK LINEARIZABILITY AND INVERSE CONTROL

2.1 Introduction

This chapter treats the question of feedback linearization and design of a new inverse control law for the control of a nonlinear aeroelastic system. A single trailing-edge control surface is used for the control of the pitch and plunge motion of the system. Based on geometric control theory, it is shown that the system is not feedback linearizable, but a partial feedback linearization of index 3 is possible. Then using a partially linearized representation of index three, an inverse control law for the control of the plunge displacement and the pitch angle is derived. It is shown that in the closed-loop system, the state vector of the aeroelastic system asymptotically converges to the origin.

2.2 Feedback Linearization

This aeroelastic system of Eq.1.1 can be represented

\[
\dot{x} = \begin{bmatrix} x_3 \\ x_4 \\ f_3 \\ f_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ g_3 \\ g_4 \end{bmatrix} \beta \overset{\Delta}{=} f(x) + gu
\]  

(2.1)

where

\[
x = (h, \alpha, \dot{h}, \dot{\alpha})^T
\]  

(2.2)
We are interested in deriving an inverse control law so that in the closed-loop system the state vector \( x = (h, \alpha, \dot{h}, \dot{\alpha})^T \) converges to zero as \( t \to \infty \).

Nonlinear geometric control theory provides useful techniques for the control of nonlinear systems. Often, nonlinear coordinate transformation and feedback are used to obtain a linear or partial linear representation. Such a linear form of system allows derivation of control laws easily. With this point of view, first, feedback linearizability of the aeroelastic system is examined. Now, we compute the vector fields \( \text{ad}_f g, \text{ad}^2 f g, \text{ad}^3 f g \). Using the definition of Lie bracket, one obtains

\[
\text{ad}_f g = [f, g] = \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g = \begin{bmatrix}
-g_3 \\
-g_4 \\
c_1 g_3 + c_2 g_4 \\
c_3 g_3 + c_4 g_4 
\end{bmatrix}
\] (2.5)

\[
\text{ad}^2 f g = [f, \text{ad}_f g] = -\frac{\partial f}{\partial x} \text{ad}_f g = -\begin{bmatrix}
c_1 g_3 + c_2 g_4 \\
c_3 g_3 + c_4 g_4 \\
k_1 g_3 + P' g_4 - c_1^2 g_3 - c_1 c_2 g_4 - c_2 c_3 g_3 - c_2 c_4 g_4 \\
k_3 g_3 + Q' g_4 - c_3 c_1 g_3 - c_3 c_2 g_4 - c_3 c_4 g_3 - c_4^2 g_4 
\end{bmatrix}
\] (2.6)

\[
\text{ad}^3 f g = [f, \text{ad}^2 f g] = \frac{\partial \text{ad}^2 f g}{\partial x} f - \frac{\partial f}{\partial x} \text{ad}^2 f g = \begin{bmatrix}
k_1 g_3 + P' g_4 - c_1^2 g_3 - c_1 c_2 g_4 - c_2 c_3 g_3 - c_2 c_4 g_4 \\
k_3 g_3 + Q' g_4 - c_3 c_1 g_3 - c_3 c_2 g_4 - c_3 c_4 g_3 - c_4^2 g_4 \\
k_1 r_1 + P' r_2 + c_1 r_3 + c_2 r_4 \\
k_3 r_1 + Q' r_2 + c_3 r_3 + c_4 r_4 
\end{bmatrix}
\] (2.7)

where

\[
\text{ad}^2 f g = (r_1, r_2, r_3, r_4)^T
\]
\[ P' = k_2 U^2 + p(x_2) + x_2 \frac{\partial p}{\partial x_2} \]
\[ Q' = k_3 U^2 + q(x_2) + x_2 \frac{\partial q}{\partial x_2} \]
\[ P'' = \frac{\partial p'}{\partial x_2} \]
\[ Q'' = \frac{\partial Q'}{\partial x_2} \]

Lemma 1:

For the aeroelastic model

(i) there exists an open set \( \Omega \in \mathbb{R}^4 \) such that at each \( x \in \Omega \), \( \text{rank}\{g, adjg, adj^2g, adj^3g\} = 4\).

(ii) the distribution \( G_2 = \text{span}\{g, adjg, adj^2g\} \) is not involutive unless \( I_\alpha - (mx_\alpha b)^2 = 0 \).

Proof: Computing the determinant of \([g, adjg, adj^2g(0), adj^3g(0)]\) one can show that it is nonsingular. and. therefore. there exists an open neighborhood \( \Omega \) of \( \mathbb{R}^4 \) surrounding the origin in which the vector fields \( adj^k g(k = 0, 1, 2, 3) \) are independent. This establishes the first part of the lemma.

One easily shows that

\[ [g, adjg] = 0 \]
\[ [g, adj^2g] = 0 \]
\[ [adjg, adj^2g] = (\frac{\partial}{\partial x} adj^2g)adjg = \begin{bmatrix} 0 \\ 0 \\ P'' g_4^2 \\ Q'' g_4^2 \end{bmatrix} \]

The distribution \( G_2 \) is involutive if

\[ [adjg, adj^2g] \in G_2 \quad (2.8) \]
In order to verify this, we compute the determinant $\triangle$ of

$$(g, adfg, ad^2fg, [adfg, ad^2fg]) = \begin{bmatrix}
0 & -g_3 & -c_1g_3 - c_2g_4 & 0 \\
0 & -g_4 & -c_3g_3 - c_4g_4 & 0 \\
g_3 & c_1g_3 + c_2g_4 & r_3 & P''g_4^2 \\
g_4 & c_3g_3 + c_4g_4 & r_4 & Q''g_4^2
\end{bmatrix} \quad (2.9)$$

It follows that

$$\triangle = [g_4(c_1g_3 + c_2g_4) - g_3(c_3g_3 + c_4g_4)](g_3Q'' - g_4P'') \quad (2.10)$$

For the given parameters of the aeroelastic model, $\triangle = 0$ if

$$g_3Q'' - g_4P'' = (\frac{\partial^2}{\partial x_2^2}(x_2k_\alpha(x_2))(g_3 \frac{m}{d} + g_4 \frac{mx_\alpha b}{d}) = 0 \quad (2.11)$$

Since $\frac{\partial^2}{\partial x_2^2}(x_2k_\alpha(x_2)) \neq 0$, $\triangle$ is zero if $g_3 + g_4x_\alpha b = 0$ which using the values of $g_i$ gives

$$mx_\alpha^2 b^2 - I_\alpha = 0 \quad (2.12)$$

However, for the given parameters of the aeroelastic model $mx_\alpha^2 b^2 - I_\alpha \neq 0$, and, therefore, Eq.(2.8) is not valid. This establishes that the distribution $G_2$ is not involutive.

According to Lemma 1, using a result of Ref. 24, it follows that the aeroelastic model is not feedback linearizable in general unless condition $I_\alpha - (mx_\alpha b)^2 = 0$ is satisfied.

Since it is not feedback linearizable, we now consider partial feedback linearization.

Lemma 2:

For the aerelastic model the involutive closure $\bar{G}_1$ of $G_1$ has rank two, and $adfg(x)$ does not belong to $\bar{G}_1$, where $G_1(x) = \text{span}\{g, adfg\}$. Therefore, the aeroelastic model is partially feedback linearizable with index 3.

Proof: It is easily seen that the vector fields $g$ and $adfg$ are independent. Furthermore, the vector fields $g$, $adfg$, and $ad^2fg$ are independent which implies that $ad^2fg \notin G_1(x)$.

Then following result of Refs. 24 and 25, one establishes that the aeroelastic model is partially feedback linearizable with index 3.
2.3 Inverse Control

According to Lemma 2, one can obtain a partially linearized system of dimension 3. Define a new variable \( y \) as

\[
y = \alpha + \lambda h = C x \tag{2.13}
\]

where \( \lambda \neq 0 \) and \( C = [\lambda, 1, 0, 0] \). It will be shown that for a suitable value of \( \lambda \) one obtains a partially feedback linearizable system. Differentially \( y \) along the solution of system (1), gives

\[
y = \dot{\alpha} + \lambda \dot{h} = L_f(C x) \tag{2.14}
\]

where \( L_f(\cdot) = \left[ \frac{\partial}{\partial x} (\cdot) \right] f(x) \). The second derivative of \( y \) is

\[
\ddot{y} = \ddot{\alpha} + \lambda \ddot{h} = L_f^2(C x) + L_g L_f(C x) u = f_4 + g_4 u + \lambda [f_3 + g_3 u] \tag{2.15}
\]

where \( L_f L_f^k(\cdot) = L_f^{k+1}(\cdot) \) and \( L_g L_f(C x) = \left( \frac{\partial L_f(C x)}{\partial x} \right) g \). In view of Eq. (2.15), \( \ddot{y} \) will not depend on the input \( u \) if one has \( \lambda = -\frac{a_4}{g_3} \). For this value of \( \lambda \), Eq. (2.15) gives

\[
\ddot{y} = f_4 + \lambda f_3 = -(k_3 + k_1 \lambda)x_1 - [k_4 U^2 + q(x_2) + \lambda (k_4 U^2 + p(x_2))]x_2 - (c_3 + c_1 \lambda)x_3
\]

\[-(c_4 + c_2 \lambda)x_4 = L_f^2(C x) \tag{2.16}
\]

Taking the derivative of \( y \) one more time, gives

\[
\dddot{y} = L_f^3(C x) + \left[ \frac{\partial}{\partial x} L_f^2(C x) \right] g \beta \overset{\Delta}{=} a^* + b^* \beta \tag{2.17}
\]

where \( a^* = L_f^3(C x) \) and \( b^* = \left[ \frac{\partial}{\partial x} L_f^2(C x) \right] g \).

We can choose the control input \( \beta \) as

\[
\beta = \frac{1}{b^*} [-a^* + \ddot{y}_r - p_3 (\ddot{y} - \ddot{y}_r) - p_2 (\dot{y} - \dot{y}_r) - p_1 (y - y_r)] \tag{2.18}
\]

where \( p_i \) are real numbers and \( y_r \) is the reference trajectory to be tracked by the output \( y \). Substituting control law (2.18) in (2.17) gives

\[
\dddot{y} + p_3 \ddot{y} + p_2 \dot{y} + p_1 y = 0 \tag{2.19}
\]
where \( \tilde{y} = y - y_r \) is the tracking error. The characteristic polynomial associated with Eq.(2.19) is

\[
\Pi(S) = (S^3 + p_3 S^2 + p_2 S + p_1)(y - y_r) = 0
\]  

(2.20)

where \( S \) denotes the Laplace variable. The parameters \( p_i \) are selected such that \( \Pi(s) \) is a hurwitz polynomial and thus \( y \rightarrow y_r \) as \( t \rightarrow \infty \). For the purpose of regulation, consider smooth reference trajectory \( y_r \) exponentially converging to zero. In view of Eq.(2.19), it follows that as \( y_r \) and its derivatives tend to zero, \( y(t) \) and its derivatives also converge to zero.

In order to examine the stability in the closed-loop system, we obtain a new representation of the system. Define new coordinates

\[
(x^T, \alpha) = (y, \dot{y}, \ddot{y}, \alpha)^T = T(x)
\]

Then

\[
\begin{pmatrix}
\xi \\
\alpha
\end{pmatrix} =
\begin{bmatrix}
\alpha + \lambda h \\
\dot{\alpha} + \lambda \dot{h} \\
L_f(Cx) \\
\alpha
\end{bmatrix}
\]

(2.22)

The Jacobian matrix \( \frac{\partial T}{\partial x} \) is

\[
\frac{\partial T}{\partial x} =
\begin{bmatrix}
\lambda & 1 & 0 & 0 \\
0 & 0 & \lambda & 1 \\
-(k_3 + k_1 \lambda) & -Q' - \lambda P' & -(c_3 + c_1 \lambda) & -(c_4 + c_2 \lambda) \\
0 & 1 & 0 & 0
\end{bmatrix}
\]

(2.23)

Since determinant of \( \frac{\partial T}{\partial x} \) is \( (c_3 + c_1 \lambda) + \lambda(c_4 + c_2 \lambda) \) which is nonzero for all \( x \in R^4 \). \( T(x) \) is a global diffeomorphism. Using \( L_f(Cx) \) from Eq.(2.16) and solving Eq.(2.22) gives

\[
h = (\xi_1 - \alpha)/\lambda
\]

(2.24)

\[
\dot{\alpha} = \xi_2 - \lambda \dot{h}
\]

(2.25)

\[
\dot{h} = -[\xi_3 + (k_3 + k_1 \lambda)h + [k_4 U^2 + q(x_2) + \lambda(k_4 U^2 + p)]x_2 + (c_4 + c_2 \lambda)\dot{\alpha}]/(c_3 + c_1 \lambda)
\]

(2.26)
Now using $\dot{h}$ and solving for $\dot{a}$ gives
\[
\dot{a} = \frac{1}{(c_3 + c_1\lambda) - \lambda(c_4 + c_2\lambda)} \left[ \xi_2(c_3 + c_1\lambda) + \lambda\xi_3 + (k_3 + k_1\lambda)(\xi_1 - \alpha) \right. \\
\left. + (k_4 + U^2(1 + \lambda) + q + \lambda p)\lambda \alpha \right] \triangleq f_a(\alpha) + a^T \xi
\] (2.27)

where
\[
a^T = \left[ \lambda(k_3 + k_1\lambda), (c_3 + c_1\lambda), \lambda \right] \left[ (c_3 + c_1\lambda) - \lambda(c_4 + c_2\lambda) \right]^{-1}
\] (2.28)
\[
f_a(\alpha) = \left[ -(k_3 + k_1\lambda)\alpha + \lambda(k_4 + U^2(1 + \lambda) + q(\alpha) + \lambda p(\alpha))\alpha \right] \left[ (c_3 + c_1\lambda) - \lambda(c_4 + c_2\lambda) \right]^{-1}
\] (2.29)

For stability analysis, we assume that the reference trajectory $y_r = 0$, then in view of Eq.(2.22) the differential equations describing the closed-loop system are given by
\[
\dot{\xi}_1 = \xi_2 \\
\dot{\xi}_2 = \xi_3 \\
\dot{\xi}_3 = p_1\xi_1 + p_2\xi_2 + p_3\xi_3
\] (2.30)
\[
\dot{\alpha} = f_a(\alpha) + a^T \xi
\]

Since $\Pi(S)$ is Hurwitz, in view of Eqs(2.27)-(2.30), one has that $\xi_i \to 0$ as $t \to \infty$, $i = 1, 2, 3$. The internal dynamics of the system is described by the last equation in Eq.(2.30). As $\xi \to 0$, Eq.(2.30) gives
\[
\dot{\alpha} = f_a(\alpha)
\] (2.31)
which describes the zero dynamics of the system. For stability in the closed-loop system, the residual dynamics Eq.(2.31) must be stable. One notices that Eq.(2.31) has multiple equilibrium points. Since we are interested in regulating $x$ to the origin, consider the linearized system about the equilibrium point $\alpha = 0$ obtained from Eq.(2.31) given by
\[
\dot{\alpha} = \frac{\partial f_a(0)}{\partial \alpha} \alpha \triangleq a_{\alpha 0} \alpha
\] (2.32)
The element $a_{\alpha_0}$ is a nonlinear function of the parameters $a, U,$ and $x_{\alpha}$ etc., and it is negative only for certain values of these parameters. The inverse controller gives stable responses converging to the origin only if $a_{\alpha_0}$ is negative. Since $T(x)$ is a diffeomorphism. as $(\xi, \alpha) \to 0$, the state vector $x \to 0$, as $t \to \infty$ if $a_{\alpha_0} < 0$. It is seen in the next section that the zero dynamics are indeed stable for a range of values of the parameters of the aeroelastic model.

2.4 Simulation

In this section, the results of simulation are presented. The parameters of the model are provided in the appendix. A fourth order command generator

$$(S^4 + \mu_4 S^3 + \mu_3 S^2 + \mu_2 S + \mu_1) y_r = 0$$

is chosen to generate smooth command trajectory $y_r(t)$ converging to zero. The parameters of $\mu_i$ are $\mu_4 = 4, \mu_3 = 6, \mu_2 = 4, \mu_1 = 1$. The feedback gains of the inverse controller are selected as $p_3 = 3, p_2 = 3, p_1 = 1$. The initial condition are $\alpha(0) = 5.73(deg), \dot{\alpha}(0) = 2(deg), h(0) = 0.1(m)$, and all other initial values are set to be zero. It is assumed that control input $\beta$ satisfy $|\beta| \leq 30(deg)$. Thus for simulation, control surface is clamped to its maximum or minimum value, whenever it exceeds the prescribed limit. The model is simulated for $a = -0.6$ and $U = 15$. The linearized zero dynamics has a pole at $s = -69.4642$ which is negative, therefore, the zero dynamics are asymptotically stable. Figure 2.1. shows the simulated responses.

We observed smooth trajectory tracking and $y$ converges to $y_r$ in about 5 seconds. As predicted, the state vector $x(t)$ also converges to zero as $t \to \infty$ since the zero dynamics are stable.

It is pointed out that in Ref. 13, inverse control of $\alpha$ or $h$ has been considered. Since in Ref. 13 the output has relative degree two, zero dynamics of dimension 2 are obtained in each case.
we observe that unlike Ref. 13, improved responses of $\alpha$ and $h$ are obtained since the output variable $y$ is a linear combination of $\alpha$ and $h$ and zero dynamics have dimension only one.

2.5 Summary

In this chapter, feedback linearizability of an aeroelastic system was considered. It was shown that although, the model is not input-state feedback linearizable, a partial linearized representation of the system of dimension three can be obtained. Based on this partially linearized representation, a new inverse controller for the trajectory control of the derived output was presented. In the closed-loop system, asymptotic regulation of the state vector to the origin was accomplished. Simulation results were presented which show that control of the pitch angle and the plunge displacement can be accomplished using the designed controller. It is noted that unlike the published works on the inverse control of the pitch angle or the plunge displacement, here a possibility of simultaneous shaping of the transient responses of both the pitch angle and the plunge displacement exist since the controlled output variable is a linear combination of the pitch angle and the plunge displacement.
Figure 2.1: Inverse control: $\alpha = -0.6$ $U=15\text{m/s}$
Based on a backstepping design technique, a new adaptive controller for the control of an aeroelastic system using output feedback is derived. The chosen dynamic model describes the nonlinear plunge and pitch motion of a wing. The parameters of the system are assumed to be completely unknown and only the plunge displacement and the pitch angle measurements are used for the synthesis of the controller. A canonical state variable representation of the system is derived and filters are designed to obtain the estimates of the derivatives of the pitch angle and the plunge displacement. Then adaptive control laws for the trajectory control of the pitch angle and the plunge displacement are derived. It is shown that in the closed-loop system, the state vector asymptotically converges to the origin. Simulation results are presented which show that regulation of the state vector to the equilibrium state and trajectory following are accomplished using a single control surface in spite of the uncertainty in the aerodynamic and structural parameters.

Consider a reference trajectory $y_r$ that represents either a prescribed pitch angle trajectory $\alpha_r$ for pitch angle control or a plunge displacement trajectory $h_r$ for the plunge motion control. Appropriate reference trajectories are generated by a second-order command generator. We are interested in deriving output feedback adaptive control systems so that $\alpha$ tracks $\alpha_r$ or $h$ tracks $h_r$ asymptotically, and in the closed-
loop system the state vector \((h, \alpha, \dot{h}, \dot{\alpha})^T\) converges to zero as \(t \to \infty\).

3.2 A Canonical Form, Filters, and State Estimation

Since \(\dot{\alpha}\) and \(\dot{h}\) are not measured, it is essential to obtain an estimate of these variables so that control synthesis can be accomplished. In order to obtain a state estimator, a representation of the system in a canonical form is obtained which is useful in designing certain filters. Then a linear combination of filter states provides an estimate of the state vector.

Consider a state transformation \(x = Tq\), where

\[
T = \begin{bmatrix}
I_{2\times2} & 0_{2\times2} \\
-M_2 & I_{2\times2}
\end{bmatrix}
\]  (3.1)

Then it is easily seen that a new state variable representation is given by

\[
\dot{x} = \begin{bmatrix}
M_2 & I_{2\times2} \\
M_1 & 0_{2\times2}
\end{bmatrix} x + \begin{bmatrix}
0_{2\times1} \\
g
\end{bmatrix} k(x) + \begin{bmatrix}
0_{2\times1} \\
b
\end{bmatrix} \beta
\]  (3.2)

where \(x = (x_1, x_2, x_3, x_4)^T\). \(x_1 = \alpha\) and \(x_2 = h\). Define

\[
\left(\begin{array}{c}
M_2 \\
M_1
\end{array}\right) \left(\begin{array}{c}
\alpha \\
h
\end{array}\right) + \left(\begin{array}{c}
g \\
b
\end{array}\right) k(x) + \left(\begin{array}{c}
g \\
b
\end{array}\right) \beta = F^T(\alpha, h, \beta)\theta
\]  (3.3)

where \(\theta = (b^T, M_{2(1)}^T, M_{2(2)}^T, M_{1(2)}^T, M_{1(1)}^T, g_1p_a, g_2p_a)^T\). The superscript \(T\) denotes matrix transposition. \(M_{i(k)}\) denotes the \(k\)th row of \(M_i\), \(p_a = (k_{a1}, k_{a2}, k_{a3}, k_{a4})\), and the \(4 \times 18\) matrix \(F^T\) is

\[
F^T = (\beta e_3, \beta e_4, \alpha e_1, he_1, \alpha e_2, he_2, \alpha e_3, he_3, \alpha e_4, he_4, \alpha^2 e_3, \alpha^3 e_3,
\alpha^4 e_3, \alpha^5 e_3, \alpha^2 e_4, \alpha^3 e_4, \alpha^4 e_4, \alpha^5 e_4)
\]  (3.4)

Here \(e_k\) denotes a vector of appropriate dimension whose \(k\)th element is one and the remaining elements are zero.

Using the definition of matrix \(F\), Eq. (3.2) can be written as

\[
\dot{x} = Ax + L(\alpha, h)^T + F^T(\alpha, h, \beta)\theta
\]  (3.5)

where \(L = (L_1^T, L_2^T)^T\),

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The matrices $L_1, L_2$ are chosen so that $A$ is a stable matrix. Eq. (3.5) is a canonical representation of system Eq. (1.1) in which $A$ is in a special form. The regressor matrix $F$ is a function of the measured variables and the input $\beta$, and all the unknown parameters of the system are included in the vector $\theta$. Now based on Eq.(3.5), certain filters are designed.

In view of Eq. (3.5), following Ref.19, consider filters given by

$$
\dot{\xi} = A\xi + L(\alpha, h)^T \\
\dot{\Omega}^T = A\Omega^T + F^T(\alpha, h, \beta)
$$

where $\xi \in \mathbb{R}^4$ and $\Omega^T \in \mathbb{R}^{4 \times 18}$. Define a state estimate as

$$
\hat{x} = \xi + \Omega^T \theta
$$

and let the state error be $\bar{x} = (x - \hat{x})$. Using Eq.(3.5) - (3.7), it easily follows that the error $\bar{x}$ is governed by

$$
\dot{\bar{x}} = A\bar{x}
$$

Since $A$ is a Hurwitz matrix, $\bar{x}(t) \to 0$ as $t \to \infty$ and, therefore, $\bar{x}(t)$ asymptotically converges to $x(t)$. Of course, $\theta$ is not known, and Eq.(3.7) cannot be used to construct $\hat{x}(t)$. However, it will be seen that it is useful in the derivation of an adaptive control law.

For simplicity in synthesis, in view of the special form of the matrix $F$, one can reduce the dimension of the $\Omega$-filter. Define

$$
\Omega^T = [v_1, v_0, s_1, s_2, \ldots, s_{16}] = [v_1, v_0, S]
$$

where each column of $\Omega^T$ is a 4-vector. Due to the special structure of $F^T$ in Eq. (3.4), it follows from Eq.(3.6) that $v_i$ and $s_i$ satisfy
\[ \dot{v}_1 = Av_1 + e_3 \beta \quad \dot{v}_0 = Av_0 + e_4 \beta \]
\[ \dot{s}_1 = As_1 + e_1 \alpha \quad \dot{s}_2 = As_2 + e_1 h \]
\[ \dot{s}_3 = As_3 + e_2 \alpha \quad \dot{s}_4 = As_4 + e_2 h \]
\[ \dot{s}_5 = As_5 + e_3 \alpha \quad \dot{s}_6 = As_6 + e_3 h \]
\[ \dot{s}_7 = As_7 + e_4 \alpha \quad \dot{s}_8 = As_8 + e_4 h \]
\[ \dot{s}_9 = As_9 + e_3 \alpha^2 \quad \dot{s}_{10} = As_{10} + e_3 \alpha^3 \]
\[ \dot{s}_{11} = As_{11} + e_3 \alpha^4 \quad \dot{s}_{12} = As_{12} + e_3 \alpha^5 \]
\[ \dot{s}_{13} = As_{13} + e_4 \alpha^2 \quad \dot{s}_{14} = As_{14} + e_4 \alpha^3 \]
\[ \dot{s}_{15} = As_{15} + e_4 \alpha^4 \quad \dot{s}_{16} = As_{16} + e_4 \alpha^5 \]

Noting that \( e_2 = Ae_4 \) and \( e_1 = Ae_3 \), in view of Eq.(3.10), one finds that
\[ s_1 = As_5 \quad s_2 = As_6 \]
\[ s_3 = As_7 \quad s_4 = As_8 \]

That is, \( s_i \ (i = 1, \ldots, 4) \) can be simply obtained by using Eq. (3.11) for synthesis. However, for the purpose of analysis Eq.(3.10) will be used.

### 3.3 Adaptive Control Laws

First the derivation of the control law for the trajectory control of the pitch angle is considered.

**Pitch angle control**

Let \( y_r = \alpha_r \) be a smooth trajectory which is to be tracked by \( \alpha \). In view of Eqs.(3.2) and (3.7), the derivative of the controlled output variable \( \alpha \) is given by
\[ \dot{\alpha} = x_3 + M_{2(1)}(\alpha, h)^T = \xi_3 + \Omega_{(3)}^T \theta + \ddot{x}_3 + M_{2(1)}(\alpha, h)^T \]
where $\Omega^T_{(k)}$, $\xi_k$, and $\bar{x}_k$ denote kth rows of $\Omega^T$, $\xi_k$, and $\bar{x}_i$, respectively. Using the definitions of $\theta$ and $\Omega^T$, Eq.(3.12) gives

$$\dot{\alpha} = b_1 v_{13} + \xi_3 + \bar{x}_3 + \bar{\omega}^T \theta$$  \hspace{1cm} (3.13)

where $\bar{\omega}^T = (0, v_0, S_{(3)} + e_3^T \alpha + e_4^T h)$. $M_{2(1)} = (\theta_3, \theta_4)$, and $v_{ik}$ and $S_{(k)}$ denote the kth rows of $v_i$ and $S$, respectively. Since we are interested in the trajectory control of $y = \alpha$, consider the tracking error $z_1$ defined as

$$z_1 = y - y_r$$  \hspace{1cm} (3.14)

Now the controller design is performed in two steps following a backstepping technique of Ref. 19.

Step 1:
The derivative of $z_1$ is

$$\dot{z}_1 = b_1 v_{13} + \xi_3 + \bar{x}_3 + \bar{\omega}^T \theta - \dot{y}_r$$  \hspace{1cm} (3.15)

Since $v_{13}$ is treated as a virtual control for controlling $z_1$, define

$$z_2 = v_{13} - \hat{\rho} \dot{y}_r - \alpha_1$$  \hspace{1cm} (3.16)

where $\hat{\rho}$ is an estimate of $\rho = b_1^{-1}$ and $\alpha_1$ is the stabilizing function yet to be chosen.

Using Eq. (3.16) in Eq. (3.15) gives

$$\dot{z}_1 = \bar{x}_3 + \xi_3 + b_1 [z_2 + \alpha_1 + \hat{\rho} \dot{y}_r] + \bar{\omega}^T \theta - \dot{y}_r$$  \hspace{1cm} (3.17)

The stabilizing function $\alpha_1$ is chosen as

$$\alpha_1 = \hat{\rho} \bar{\alpha}_1$$
\[ \hat{\alpha}_1 = -c_1 z_1 - \xi_3 - \hat{\omega}^T \hat{\theta} - d_1 z_1 \]  

(3.18)

where \( c_i, d_i > 0 \) and \( \hat{\theta} \) is an estimate of \( \theta \).

Noting that \( b_1 \hat{\rho} = b_1 (\rho - \hat{\rho}) = 1 - b_1 \hat{\rho} \), it follows from Eqs. (3.17) and (3.18) that

\[ \hat{z}_1 = b_1 z_2 - b_1 \hat{\rho} \tilde{\alpha}_1 + \hat{\omega}^T \hat{\theta} - d_1 z_1 + \tilde{x}_3 - b_1 \hat{\rho} \tilde{y}_r - c_1 z_1 \]  

(3.19)

Now consider a Lyapunov function of the form

\[ V_1 = d_1^{-1} \hat{x}^T \hat{P} \hat{x} + (z_1^2 + |b_1| \gamma^{-1} \rho^2)/2 \]  

(3.20)

where \( \gamma > 0 \) and the positive definite symmetric matrix \( P \) satisfies the Lyapunov equation

\[ PA + A^T P = -I_{4 \times 4} \]  

(3.21)

Since \( A \) is a stable matrix, \( P \) is the unique solution of Eq. (3.21). The derivative of \( V_1 \) is given by

\[ \dot{V}_1 = d_1^{-1}(\hat{x}^T \hat{P} \hat{x} + \hat{x}^T \hat{P} \hat{x}) + z_1 \hat{z}_1 - |b_1| \gamma^{-1} \rho \hat{\rho} \]  

(3.22)

Substituting Eqs. (3.8), (3.19), and (3.21) in Eq. (3.22) gives

\[ \dot{V}_1 = b_1 z_1 z_2 + \hat{\omega}^T \hat{\theta} \tilde{z}_1 - c_1 z_1^2 - \frac{|\tilde{x}|^2}{d_1} - b_1 \hat{\rho} \tilde{\alpha}_1 z_1 - d_1 z_1^2 + \tilde{x}_3 z_1 - \frac{\hat{\rho} |b_1|}{\gamma} \rho - b_1 \hat{\rho} \tilde{y}_r z_1 \]  

(3.23)

where \(|.|\) denotes the Euclidean norm of a vector. Using Young's inequality, one has

\[ \tilde{x}_3 z_1 \leq |\tilde{x}_3| |z_1| \leq d_1 z_1^2 + \tilde{x}_3^2/(4d_1) \leq d_1 z_1^2 + |\tilde{x}|^2/(4d_1) \]  

(3.24)

Using Eq. (3.24) in (3.23) gives

\[ \dot{V}_1 \leq b_1 z_1 z_2 + \hat{\omega}^T \hat{\theta} \tilde{z}_1 - c_1 z_1^2 - \frac{3|\tilde{x}|^2}{4d_1} + z_1 \hat{\rho}[-b_1 \tilde{\alpha}_1 - b_1 \tilde{y}_r] - \frac{\hat{\rho} |b_1| \rho}{\gamma} \]  

(3.25)

Since \( \hat{\rho} \) is unknown, this can be eliminated from Eq. (3.25) by choosing an update law of the form

\[ \hat{\rho} = -\gamma \text{sign}(b_1)[z_1 (\tilde{\alpha}_1 + \tilde{y}_r)] \]  

(3.26)
Now substituting the update law in Eq. (3.25) gives

$$
\dot{V}_1 \leq -c_1 z_1^2 - \frac{3}{4d_1} |\tilde{x}|^2 + \omega^T \tilde{\theta} z_1 + b_1 z_1 z_2
$$

(3.27)

The unknown $\theta$-dependent term in Eq. (3.27) will be compensated in the second step.

Step 2:

The derivative of $z_2$ is given by

$$
\dot{z}_2 = \dot{v}_{13} - \hat{\dot{\rho}} y_r - \hat{\dot{\rho}} \dot{y}_r - \dot{\alpha}_1
$$

(3.28)

Since $\alpha_1$ is a function of $\hat{\rho}, \xi_3, S_{(3)}, v_{03}, y_r, \hat{\theta}, x_1, x_2$, its derivative is given by

$$
\dot{\alpha}_1 = \frac{\partial \alpha_1}{\partial v_{03}} \dot{v}_{03} + \frac{\partial \alpha_1}{\partial \hat{\rho}} \dot{\hat{\rho}} + \frac{\partial \alpha_1}{\partial \xi_3} \dot{\xi}_3 + \frac{\partial \alpha_1}{\partial S_{(3)}} \dot{S}_{(3)} + \frac{\partial \alpha_1}{\partial y_r} \dot{y}_r + \frac{\partial \alpha_1}{\partial \hat{\theta}} \dot{\hat{\theta}} + \frac{\partial \alpha_1}{\partial x_1} \dot{x}_1 + \frac{\partial \alpha_1}{\partial x_2} \dot{x}_2
$$

\[= a_0 + (\frac{\partial \alpha_1}{\partial x_1}) \dot{x}_1 + (\frac{\partial \alpha_1}{\partial x_2}) \dot{x}_2 \tag{3.29}\]

where $a_0$ is obtained by comparing terms in Eq. (3.29). For the computation of $a_0$, the derivatives of various signals are substituted in Eq. (3.29), but $\dot{\hat{\theta}}$ is yet to be determined. Using Eqs. (3.2) and (3.7), the derivative of $x_2$ is given by

$$
\dot{x}_2 = M_{2(2)}(\alpha, h)^T + x_4 = M_{2(2)}(\alpha, h)^T + \bar{x}_4 + \xi_4 + \Omega_{(4)}^T \theta
$$

(3.30)

Noting that $M_{2(1)} = (\theta_3, \theta_4)$ and $M_{2(2)} = (\theta_5, \theta_6)$, and adding and subtracting appropriate $\hat{\theta}$-dependent terms, and using Eqs. (3.12) and (3.30) in Eq. (3.29) gives

$$
\dot{\alpha}_1 = a_0 + \left( \frac{\partial \alpha_1}{\partial x_1} \right) (\xi_3 + \Omega_{(3)}^T \theta + \bar{x}_3 + \theta_3 \alpha + \theta_4 h) + \left( \frac{\partial \alpha_2}{\partial x_2} \right) (\xi_4 + \Omega_{(4)}^T \theta + \bar{x}_4 + \theta_5 \alpha + \theta_6 h)
$$

\[= a_1 + a_2^T \tilde{x}_\alpha + a_3^T \tilde{\theta} \tag{3.31}\]

where

$$
a_1 = a_0 + \frac{\partial \alpha_1}{\partial x_1} (\xi_3 + \Omega_{(3)}^T \hat{\theta} + \hat{\theta}_3 \alpha + \hat{\theta}_4 h) + \frac{\partial \alpha_1}{\partial x_2} (\xi_4 + \Omega_{(4)}^T \hat{\theta} + \hat{\theta}_5 \alpha + \hat{\theta}_6 h)
$$

$$
a_2^T = \left( \frac{\partial \alpha_1}{\partial x_1}, \frac{\partial \alpha_1}{\partial x_2} \right)
$$
\[
\ddot{x}_a = (\ddot{x}_3, \ddot{x}_4)^T
\]

\[
a_3^T = \frac{\partial \alpha_1}{\partial x_1}(\Omega_3^T + e_3^T \alpha + e_3^T h) + \frac{\partial \alpha_1}{\partial x_2}(\Omega_4^T + e_4^T \alpha + e_4^T h)
\]

Substituting Eq. (3.31) in Eq. (3.28) gives

\[
\dot{z}_2 = -L_{21} v_{11} + \beta - \dot{\hat{y}}_r - \dot{\hat{y}}_r - a_1 - a_3 \ddot{x}_a - a_3 \ddot{\theta} \dot{\theta} = a^* - a_2^T \ddot{x}_a - a_3 \ddot{\theta} + \beta \quad (3.32)
\]

where \(a^* = -L_{21} v_{11} - \dot{\hat{y}}_r - \dot{\hat{y}}_r - a_1\)

In view of Eq. (3.32), we choose control \(\beta\) as

\[
\beta = -a^* - c_2 z_2 - d_2 |a_2|^2 z_2 - \hat{b}_1 z_1 \quad (3.33)
\]

Now consider a Lyapunov function

\[
V_2 = V_1 + d_2^{-1} \ddot{x}^T P \ddot{x} + (z_2^2 + \dot{\theta}^T \Gamma^{-1} \dot{\theta})/2
\]

where \(\Gamma\) is a positive definite symmetric matrix. In view of Eq. (3.21), the derivative of \(V_2\) is given by

\[
\dot{V}_2 = \dot{V}_1 - \frac{|\ddot{x}|^2}{d_2} + z_2 \dot{z}_2 - \dot{\theta}^T \Gamma^{-1} \dot{\theta}
\]

Using Eq. (3.32) in Eq. (3.35) and noting that \(b_1 = \theta_1\) gives

\[
\dot{V}_2 \leq -c_1 z_1^2 - \frac{3|\ddot{x}|^2}{4d_1} + \omega^T \dot{\theta} z_1 + (\dot{\theta}_1 + \dot{\theta}_1) z_1 z_2 - c_2 z_2^2 - d_2 z_2^2 |a_2|^2 - \hat{b}_1 z_1 z_2
\]

\[
- z_2 a_2^T \ddot{x}_a - a_3 \ddot{\theta} z_2 - \dot{\theta}^T \Gamma^{-1} \dot{\theta} - \frac{|\ddot{x}|^2}{d_2}
\]

Define

\[
\tau = (\omega z_1 + e_1 z_1 z_2 - a_3 z_2)
\]

Using Young’s inequality, one has

\[
|z_2 a_2^T \ddot{x}_a| \leq |z_2| |a_2| |\ddot{x}_a| \leq d_2 z_2^2 |a_2|^2 + \frac{|\ddot{x}_a|^2}{4d_2}
\]

(3.38)
Substituting Eqs. (3.37) and (3.38) in Eq. (3.36) and noting that $|\dot{x}_d|^2 \leq |\dot{x}|^2$, one has

$$\dot{V}_2 \leq -c_1 z_1^2 - c_2 z_2^2 - \frac{3}{4}(d_1^{-1} + d_2^{-1})|\ddot{x}|^2 + \dot{\theta}^T (\tau - \Gamma^{-1} \dot{\theta})$$

(3.39)

Now one chooses the adaptation law for $\dot{\theta}$ as

$$\dot{\theta} = \Gamma \tau$$

(3.40)

which yields

$$\dot{V}_2 \leq -c_1 z_1^2 - c_2 z_2^2 - \frac{3}{4}(d_1^{-1} + d_2^{-1})|\ddot{x}|^2$$

(3.41)

Theorem 3.1: Consider the closed-loop system Eqs. (3.26),(3.33), and (3.40). Suppose that $y_r$ is a bounded and smooth trajectory converging to zero, and the zero dynamics of the system are stable. Then the solution of Eq. (1.1) beginning from any initial condition $q(0) \in \mathbb{R}^4$ is such that the tracking error $(\alpha - \alpha_r)$ and $h$ tend to be zero as $t \to \infty$. Furthermore, if $y_r = 0$, then the state vector $q(t)$ tends to the origin as $t \to \infty$.

Proof: A proof is given in the 3.6.

Zero dynamics describe the internal dynamics of the system when the output $y = \alpha$ is identically zero. For the control of $\alpha$, Theorem 3.1 assumes that the zero dynamics are stable. The stability properties of zero dynamics have been extensively examined in Refs. 13, 14, and 16. It is noted that stability of the zero dynamics is essential even in the nonadaptive output trajectory control systems.

Adaptive Control of Plunge Motion

In the previous section, an adaptive control law for the trajectory control of $\alpha$ has been presented. Following, a similar approach, one can derive a control law for the trajectory control of the plunge displacement. Define the tracking error

$$z_1 = h - h_r$$

(3.42)
Using Eq. (3.30) the differential equation for \( h \) is given by

\[
\dot{h} = b_2 v_{04} + \xi_4 + [v_{14}, 0, S_{(4)}] \theta + \ddot{x}_4 + M_{(2)}(\alpha, h)^T
\]  

(3.43)

Apparently, for controlling \( h \), one treats \( v_{04} \) as the virtual control since in the derivative of \( v_{04} \), control input \( \beta \) appears. In this case \( \rho = b_2^{-1} \) and

\[
z_2 = v_{04} - \rho \dot{h}_r - \alpha_1
\]

Following the steps of the previous section, one obtains a virtual control \( \alpha_1 \) and the adaptation law for \( \dot{\rho} \) which is an estimate of \( \rho = b_2^{-1} \) in the first step of derivation, and the control law \( \beta \) and the update law for \( \dot{\theta} \) is obtained in the second step. Since the control law for \( h \)-control can be similarly derived, the details are not presented here.

Similar to \( \alpha \)-control, for the stability in the closed-loop system, it is assumed that the parameters of the aeroelastic system are such that the zero dynamics are stable. It is pointed out that unlike \( \alpha \)-control, the zero dynamics associated with the output \( h \) are nonlinear and exhibit complex dynamic behavior. In this case one has only local stability in the closed-loop system. Since a proof of stability can be established following the steps in the proof of Theorem 3.1, it is not presented here.

3.4 Simulation Results

In this section, numerical results for the pitch angle control and plunge motion control are presented. The parameters of the system are given in the appendix. Simulation is performed for different values of \( \alpha \) and \( U \). The transfer function of the command generator is chosen as

\[
W_m = \frac{\lambda^2}{(s + \lambda)^2}
\]

to obtain exponentially decaying command trajectories to zero where \( \lambda > 0 \). For the pitch angle control, the initial conditions selected are \( \alpha(0) = 5.75(\text{deg}), h(0) = \ldots \)
The initial conditions of the command generator are set as $y_r(0) = 5.73 \text{deg}$, $\dot{y}_r(0) = 0$.

The initial conditions for the parameters are $\hat{b}_1(0) = -0.1$, $\hat{b}_2(0) = -0.03$, and the remaining components of $\hat{\theta}$ and $\hat{\rho}$ have initial values zero. The initial states of the filters are set as $\Omega(0) = 0$ and $\xi(0) = 0$. The design parameters are selected as $\lambda = 1$, $c_1 = c_2 = d_3 = d_4 = 100$, $\gamma = 1$, $\Gamma = I_{18 \times 18}$, $L_{11} = L_{12} = 20$, $L_{21} = L_{22} = 100$. These design parameters are chosen after several trials by observing simulated responses.

Case 1: The closed-loop system Eq. (1.1) with the control law Eq. (3.33) and the update law Eqs. (3.26) and (3.40) for $\alpha = -0.3$ and $U = 15 \text{ m/s}$ is simulated. For the chosen value of $\alpha$ and $U$, one has $b_1 = -0.282, b_2 = -0.047$. Selected responses are shown in Fig. 3.1. We observe that after an initial transient, the pitch angle asymptotically tracks the command trajectory. The response time is of the order of 7-8 seconds. Only a small control magnitude (less than $10 \text{ (deg)}$) is required for control. Since for $\alpha = -0.3$ and $U = 15 \text{ m/s}$ the zero dynamics are stable, the plunge displacement also converges to zero as predicted. Here only parameter $b_1$ is shown in Fig. 3(d), but it is found that all other components of $\hat{\theta}$ and $\hat{\rho}$ also converge to constant values.

Case 2: In order to examine the sensitivity of the controller with respect to parameter $\alpha$, the closed-loop system for a different value of $\alpha = -0.4$, but with the same value of $U = 15 \text{ m/s}$ is simulated. We observe that although the pitch angle asymptotically tracks the command trajectory, larger control magnitude (less than 30 (deg)) is required (Fig.3.2). Moreover, larger plunge displacement is observed in this case. The response time is of the order of 6-7 seconds. In this case, increase in control magnitude can be attributed to reduced degree of stability of the zero dynamics, since as $\alpha \to -0.55$, the poles of the zero dynamics move to the right in the complex plane (Ref.13).
Case 3: The closed-loop system for $a = -0.4$ and $U = 20 \text{ m/s}$ is simulated. Selected responses are shown in Fig. 3.3. The response time of the same order as in case 2 is observed, but due to enhanced control effectiveness at higher air speed $U$, smaller control magnitude (about $18 \text{ (deg)}$) compared to case 2 is required. The plunge displacement and control input are somewhat similar to those of case 2 and, therefore, these plots are not shown here.

Case 4: Now simulation results for plunge motion control are presented. The parameters $c_i = d_i = 3000$ and $\Gamma = 300I_{18 \times 18}$ are chosen. The initial conditions are $\alpha(0) = 5.75(\text{deg}), h(0) = 0.01(\text{m})$, and $\dot{\alpha}(0) = \dot{h}(0) = 0$. The remaining control parameters and initial conditions of case 1 are retained. The aeroelastic model for $a = -0.85$ and $U = 15 \text{m/s}$ is considered for simulation. In this case $b_1 = 2.47, b_2 = -0.21$. It is seen that $h$ tracks $y_r$ and the pitch angle tends to zero (Fig.3.4). Since the zero dynamics are nonlinear, we observe high frequency oscillations in control $\beta$. The response time is of the order of 5-6 seconds. The maximum control magnitude is about $18 \text{ (deg)}$. Similar to case 1, all the parameter estimates converge to constant values.

Case 5: Simulation for the aeroelastic model with $a = -0.88$ and $U = 15$ is performed. Selected responses are shown in Fig.3.5. It is seen that the state vector is regulated to zero, but compared to case 4, smoother response for $h$ is obtained and transient in $\alpha$ decays relatively faster (in about 4 seconds). The control magnitude (about $17 \text{ (deg)}$) is also slightly smaller than case 4. For $h$-control, the degree of stability of the zero dynamics improves as the parameter $a \to -1$ (Ref. 13). For this reason, the transient responses of $h$ and $\alpha$ are better than those of case 4.

Case 6: To examine the sensitivity of the controller with respect to the air speed, simulation is performed for the model with $U = 20$, but the remaining parameters of case 5 are retained. In this case, one has $b_1 = 10, b_2 = -0.8$. It is seen that $h$
follows \( y_r \) and \( \alpha \) is regulated to zero. In this case at higher air speed, similar to case 3, smaller control magnitude (about 9.5 deg) compared to case 5 is required. The response time is of the order of 6-7 seconds.

Extensive simulation has been performed. Based on these results, it is found that by a suitable choice of parameters \( c_i, d_i, L, \gamma, \Gamma \) desirable responses in the closed-loop system can be obtained. For smaller values of \( c_i \), the control magnitude is reduced. Furthermore, the command trajectory \( y_r \) can be properly chosen to shape the transient responses.

3.5 Summary

In this chapter, a new controller for the control of an aeroelastic system based on a backstepping design technique was presented. Adaptive control laws for the trajectory control of \( \alpha \) and \( h \) were derived. For the derivation of the controller, a canonical representation of the aeroelastic model was obtained. Filters were designed to obtain the estimate of the state vector. In the closed-loop system, asymptotic regulation of the state vector to the origin was accomplished. Simulation results were presented which show that control of the pitch angle and the plunge displacement can be accomplished using output feedback in spite of the uncertainties in the system parameters with reasonable control magnitude. The adaptive controller has several design parameters which can be adjusted to obtain desirable response characteristics.

3.6 Proof of Theorem 3.1

First it will be shown that all the signals in the closed-loop system are bounded. \( V_2 \) is a positive definite function of \( z_1, z_2, \bar{\theta}, \bar{\rho}, \bar{x} \) and \( \dot{V}_2 \) is negative semidefinite. Thus, it follows that \( (z_1, z_2, \bar{\theta}, \bar{\rho}, \bar{x}) \in \mathcal{L}_\infty \), where \( \mathcal{L}_\infty \) denotes the set of bounded functions. Since \( z_1 \in \mathcal{L}_\infty \), one has \( \alpha \in \mathcal{L}_\infty \).
Now the differential equations associated with the zero dynamics are derived. The output \( \alpha \) has relative degree 2 since the control \( \beta \) appears in its second derivative. Thus the zero dynamics has dimension 2. Define

\[
\eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} b_2 \alpha - b_1 h \\ b_2 \dot{\alpha} - b_1 \dot{h} \end{bmatrix}
\]  

(3.44)

Then, using Eq. (1.1), it follows that \( \eta \) satisfies

\[
i = A_\eta \eta + (0.1)^T f_\eta(\alpha, \dot{\alpha})
\]  

(3.45)

where

\[
A_\eta = \begin{bmatrix} 0 & 1 \\ -a_{\eta 1} & -a_{\eta 2} \end{bmatrix}
\]

\( c_{\eta i} = b_2 M_i(1) - b_1 M_i(2), \ i = 1, 2 \)

\( a_{\eta i} = c_{\eta i}(0, b_i^{-1})^T \)

\( f_{\eta} = (b_2, b_1)g k_n(\alpha) \)

\( f_\eta = (\alpha c_{\eta 1} + \dot{\alpha} c_{\eta 2})(1, b_2 b_i^{-1})^T + f_{\eta} \)

The zero dynamics are obtained when \( \alpha = 0 \) and \( \dot{\alpha} = 0 \), i.e., \( f_\eta = 0 \) in Eq. (3.45). Thus, for the stability of the zero dynamics, \( A_\eta \) must be a Hurwitz matrix. Solving for \( \eta_1 \) from Eq. (3.45), one obtains

\[
\tilde{\eta}_1(s) = \left[ \frac{(c_{\eta 1} + sc_{\eta 2})(1, b_2 b_i^{-1})^T}{H_\eta(s)} \right] \ddot{\alpha}(s) + \frac{1}{H_\eta^{-1}(s)} f_{\eta}(\alpha)
\]  

(3.46)

where in this section, \( s \) denotes the Laplace variable, functions with overbar denote Laplace transforms, and \( H_\eta = s^2 + a_{\eta 2}s + a_{\eta 1} \). Since \((c_{\eta 1} + sc_{\eta 2})H_\eta^{-1}\) and \(H_\eta^{-1}\) are stable transfer functions and \( \alpha \) is bounded, from Eq. (3.46) it follows that \( \eta_1 \) is bounded. But \( \eta_1 = b_2 \alpha - b_1 h \), therefore, one has that \( h \in L_\infty \). Since \((\alpha, h) \in L_\infty \), \( S \) and \( \xi \) are bounded. In view of the differential equations for \( v_1 \) and \( v_0 \) in Eq. (3.10), one has for \( i = 0, 1 \)

\[
\dot{v}_{i 1} = -L_{11} v_{i 1} + v_{i 3}
\]

\[
\dot{v}_{i 2} = -L_{12} v_{i 2} + v_{i 4}
\]
\[ \dot{v}_{13} = -L_{21}v_{11} + \delta_{i1}\beta \quad (3.47) \]
\[ \dot{v}_{14} = -L_{22}v_{12} + \delta_{i0}\beta \]

where \( \delta_{ik} = 1 \) if \( i = k \), and it is zero otherwise. In view of Eq. (3.47), it is easily seen that \( v_{01}, v_{03}, v_{12}, v_{14} \in L_\infty \) and this implies that \( \bar{\omega} \) and \( \alpha_1 \) are bounded. Since \( z_2 \) and \( \alpha_1 \) are bounded. Eq.(3.16) implies that \( v_{13} \) is bounded. But in view of Eq.(3.47), \( v_{13} \in L_\infty \) implies that \( v_{11} \in L_\infty \). This shows that \( v_1 \in L_\infty \) and \( \Omega_{(1)}^T \in L_\infty \), and in view of Eq.(3.10), \( \dot{\alpha} \in L_\infty \). Since \( \alpha, \dot{\alpha} \in L_\infty \), using Eq.(3.45) one concludes that \( \eta \in L_\infty \).

Using Eq.(3.44) now one has that \( \dot{h} \in L_\infty \). Using boundedness of \( \dot{h} \), one concludes from Eq.(3.30) that \( v_{04} \in L_\infty \). Since \( \dot{v}_{02} = -L_{12}v_{02} + v_{04}, v_{02} \in L_\infty \). Thus, \( v_0 \in L_\infty \).

This establishes the boundedness of all the signals. Now using LaSalle-Yoshikawa theorem (Ref. 19. page 489-492), one has that \((z_1, z_2) \to 0 \) as \( t \to \infty \). Therefore, \( \alpha \to 0 \) as \( t \to \infty \). Now in view of Eqs.(3.46), \( \eta \) converges to zero which according to Eq. (3.44) implies that \( h \) converges to zero.

For the case when \( y_r = 0 \), according to the LaSalle invariance theorem (Ref. 19. page 25), the state vector converges to the largest invariant set \( \mathcal{V}_i \) contained in the set \( \mathcal{V} = \{z_1 = 0, z_2 = 0, \bar{x} = 0\} \). But in \( \mathcal{V}_i \), \( \dot{z}_1 = \dot{\alpha} = 0 \). Now convergence of \( (\dot{\alpha}, \eta_2) \) to zero implies convergence of \( \dot{h} \) to zero. Therefore, \( q(t) \) converges to zero.

This completes the proof of Theorem 3.1.
Figure 3.1: Pitch angle control: $\alpha = -0.3 \ U=15\text{m/s}$
Figure 3.2: Pitch angle control: $\alpha = -0.4 \ U=15m/s$
Figure 3.3: Pitch angle control: $\alpha = -0.4 \ U=20\text{m/s}$
Figure 3.4: Plunge motion control: $\alpha = -0.85 \ U = 15 \text{m/s}$
Figure 3.5: Plunge motion control: $\alpha = -0.88 \ U = 15 \text{m/s}$
Figure 3.6: Plunge motion control: $\alpha = -0.88 \, U=20\text{m/s}$
CHAPTER 4

ADAPTIVE CONTROL WITH REDUCED ORDER OBSERVER

4.1 Introduction

Although in chapter 3, nonlinear adaptive control based on backstepping design technique is implemented, filters of large dimension are needed for the synthesis of controller. In this chapter, for the simplicity in implementation, filters of reduced dimensions are judiciously constructed. Furthermore, the control law of this chapter differs from previous one. Interestingly, simulation results show that the closed-loop system including the reduced order observer gives improved response characteristics compared to those obtained in chapter 3 which uses high order filters.

4.2 Reduced Order Filters and State Estimation

In this section, design of filters for state estimation is considered. Since $\dot{x}$ and $\dot{h}$ are not measured, it is essential to obtain an estimate of these variables so that control system can be synthesized. In order to obtain a state estimator, a representation of the system in a canonical form is obtained which is useful in designing certain filters. Then a linear combination of filter states provides an estimate of the state vector.

Consider a state transformation $x = Tq$, where

$$ T = \begin{bmatrix} I_{2\times2} & 0_{2\times2} \\ -M_2 & I_{2\times2} \end{bmatrix} \quad (4.1) $$

Then it is easily seen that a new state variable representation of Eq. (1.1) is given by

$$ \dot{x} = \begin{bmatrix} M_2 & I_{2\times2} \\ M_1 & 0_{2\times2} \end{bmatrix} x + \begin{bmatrix} 0_{2\times1} \\ g \end{bmatrix} k_n(\alpha) + \begin{bmatrix} 0_{2\times1} \\ b \end{bmatrix} \beta \quad (4.2) $$

37
where \( x = (x_1, x_2, x_3, x_4)^T, x_1 = \alpha, \) and \( x_2 = h. \) Define \( y = (\alpha, h)^T, p = (x_3, x_4)^T, \) one has

\[
\begin{bmatrix}
\dot{y} \\
\dot{p}
\end{bmatrix} =
\begin{bmatrix}
p \\
0
\end{bmatrix} + \Phi \theta_a +
\begin{bmatrix}
0 \\
b
\end{bmatrix} \beta
\]  

(4.3)

where \( \theta_a = (M_{2(1)}, M_{2(2)}, M_{1(1)}, M_{1(2)}, g_1 p_a, g_2 p_a)^T \in \mathbb{R}^{16}, \) the superscript \( T \) denotes matrix transposition, \( M_{i(k)} \) denotes the \( k \)th row of \( M_i, \)

\[
p_a = (k_{a1}, k_{a2}, k_{a3}, k_{a4})
\]

\[
\Phi \theta_a =
\begin{bmatrix}
M_{2y} \\
M_{1y} + g k_{n_a}(\alpha)
\end{bmatrix}
\]

\[
b \beta = \Psi(\beta) b
\]

\[
\Psi(\beta) =
\begin{bmatrix}
\beta & 0 \\
0 & \beta
\end{bmatrix}, \Phi =
\begin{bmatrix}
\Phi_1 \\
\Phi_2
\end{bmatrix}, \Phi \in \mathbb{R}^{4 \times 16}
\]

\[
\Phi = [\alpha e_1, h e_1, \alpha e_2, h e_2, \alpha e_3, h e_3, \alpha e_4, h e_4, \alpha^2 e_3, \alpha^3 e_3, \alpha^4 e_3, \alpha^5 e_3, \alpha^2 e_4, \alpha^3 e_4, \alpha^4 e_4, \alpha^5 e_4]
\]

where \( e_i \) denotes a vector of appropriate dimension (here \( e_i \in \mathbb{R}^4 \)) whose \( i \)th element is 1 and the remaining elements are zero. Then Eq. (4.3) can be written as

\[
\dot{y} = p + M_2 y = p + \Phi_1(y) \theta_a
\]

(4.4)

\[
\dot{p} = M_1 y + g k_{n_a}(\alpha) + b \beta \overset{\Delta}{=} \Phi_2(y) \theta_a + \Psi(\beta) b
\]

(4.5)

Note that \( y \) is measured but the subvector \( p \) is not available for feedback. In view of Eqs.(4.4) and (4.5), for obtaining an estimate of \( p, \) consider filters given by

\[
\dot{\xi} = -K \xi - K^2 y
\]

\[
\dot{v} = -K v + \Psi(\beta)
\]

\[
\dot{\Omega}^T = -K \Omega^T - K \Phi_1(y) + \Phi_2(y)
\]

(4.6)

where \( \xi \in \mathbb{R}^2, v \in \mathbb{R}^{2 \times 2}, \)

\[
v =
\begin{bmatrix}
v^{(1)} \\
v^{(2)}
\end{bmatrix} =
\begin{bmatrix}
v_{11} & v_{10} \\
v_{12} & v_{20}
\end{bmatrix} = [v_1, v_0]
\]
\[ K = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \]

and \( \Omega^T \in \mathbb{R}^{2 \times 16} \). It is noted that the filters in Eq. (4.6) are independent of the parameters of the model. This is essential since the model parameters are unknown.

Define a state estimate \( \hat{p} \) of \( p \) as

\[ \hat{p} = \xi + \nu b + \Omega^T \theta_a + k y \tag{4.7} \]

and let the state error be \( \bar{p} = (p - \hat{p}) \). Using Eqs.(4.5) -(4.7), it easily follows that the error \( \bar{p} \) is governed by

\[ \dot{\bar{p}} = -K \bar{p} \tag{4.8} \]

It is interesting to note that the structure of the filters has been judiciously chosen such that the state estimation error satisfies a linear differential equation which is independent of the parameters of the model. Since -K is a Hurwitz matrix, \( \bar{p}(t) \to 0 \) as \( t \to \infty \) and, therefore, \( \bar{p}(t) \) asymptotically converges to \( p(t) \). Of course, \( \theta \) is not known, and Eq.(4.7) cannot be used to construct \( \hat{p}(t) \). However, it will be seen that it is useful in the derivation of an adaptive control law.

Define

\[ \Omega^T = [s_1, s_2, \ldots, s_{16}] \tag{4.9} \]

where each column of \( \Omega \) is a 2-vector. Due to the special structure of \( \Phi \) and \( \Psi \) in Eq. (4.6), it follows from Eq.(4.6) that \( s_i \) satisfy

\[
\begin{align*}
\dot{v}_1 &= -K v_1 + e_1 \beta \\
\dot{s}_1 &= -K s_1 - e_1 k_1 \alpha \\
\dot{s}_3 &= -K s_3 - e_2 k_2 \alpha \\
\dot{s}_5 &= -K s_5 + e_1 k_1 \alpha \\
\dot{s}_7 &= -K s_7 + e_2 k_2 \alpha \\
\dot{v}_0 &= -K v_0 + e_2 \beta \\
\dot{s}_2 &= -K s_2 - e_1 k_1 h \\
\dot{s}_4 &= -K s_4 - e_2 k_2 h \\
\dot{s}_6 &= -K s_6 + e_1 k_1 h \\
\dot{s}_8 &= -K s_8 + e_2 k_2 h
\end{align*}
\tag{4.10} \]
\[ \dot{s}_9 = -K s_9 + e_1 k_1 \alpha^2 \quad \dot{s}_{10} = -K s_{10} + e_1 k_1 \alpha^3 \]
\[ \dot{s}_{11} = -K s_{11} + e_1 k_1 \alpha^4 \quad \dot{s}_{12} = -K s_{12} + e_1 k_1 \alpha^5 \]
\[ \dot{s}_{13} = -K s_{13} + e_2 k_2 \alpha^2 \quad \dot{s}_{14} = -K s_{14} + e_2 k_2 \alpha^3 \]
\[ \dot{s}_{15} = -K s_{15} + e_2 k_2 \alpha^4 \quad \dot{s}_{16} = -K s_{16} + e_4 k_2 \alpha^5 \]

For simplicity in synthesis, in view of the special form of the matrix \( \Phi_1(y), \Phi_2(y) \). one can reduce the dimension of the \( \Omega \)-filter. In view of Eq.(4.10), one finds that it is possible to reduce the dimension of the filter if \( k_1 = k_2 = k \). Defining
\[ A_0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \]
and noting that \( e_1 = A_0 e_2 \), it is easily seen that
\[ v_1 = A_0 v_0 \]
\[ s_1 = s_5 = A_0 v_0 \quad s_7 = s_3 \]
\[ s_2 = s_6 = A_0 s_4 \quad s_8 = s_4 \]
\[ s_9 = A_0 s_{13} \quad s_{10} = A_0 s_{14} \quad (4.11) \]
\[ s_{11} = A_0 s_{15} \quad s_{12} = A_0 s_{16} \]

Thus, one needs to construct the filter only for \( v_0 \) and \( s_i, (i = 3, 4, 13, 14, 15, 16) \). and \( v_1 \) and \( s_i, (i = 1, 2, 5, 6, 7, 8, 9, 10, 11, 12) \) are obtained by Eq.(4.11).

### 4.3 Adaptive Control Laws

First the derivation of the control law for the trajectory control of the pitch angle is considered.

Pitch angle control
Let $y_r = \alpha_r$ be a smooth trajectory which is to be tracked by $\alpha$. In view of Eqs. (4.2) and (4.7), the derivative of the controlled output variable $\alpha$ is given by

$$\dot{\alpha} = x_3 + M_{2(1)}(\alpha, h)^T = \xi_1 + \Omega_{(1)}^T \theta_a + \tilde{p}_1 + M_{2(1)}(\alpha, h)^T + v_{(1)} b + k_1 \alpha$$  \hspace{1cm} (4.12)

where $\Omega_{(k)}$, $\xi_k, v_k$ and $\tilde{p}_k$ denote kth rows of $\Omega^T$, $\xi, v$ and $\tilde{p}$, respectively, $\theta = (b^T, \theta_a^T) \in R^{18}$. Using the definitions of $\theta$ and $\Omega^T$, Eq. (4.12) gives

$$\dot{\alpha} = b_1 v_{11} + \xi_1 + \tilde{p}_1 + \bar{\omega}^T \theta + k_1 \alpha$$  \hspace{1cm} (4.13)

where $\bar{\omega}^T = (v_{10} e_3^T, \Omega_{(1)}^T) + e_3^T \alpha + e_4^T h$, $M_{2(1)} = (\theta_3, \theta_4)$, $\theta_i$ is the ith component of $\theta$, $e_i \in R^{18}$ ($i = 3, 4$) and $e_2 \in R^2$. Since we are interested in the trajectory control of $y = \alpha$. Consider the tracking error $z_1$ defined as

$$z_1 = y - y_r$$  \hspace{1cm} (4.14)

Now the controller design is performed in two steps following a backstepping technique of Ref. 19.

Step 1:
The derivative of $z_1$ is

$$\dot{z}_1 = b_1 v_{11} + \xi_1 + \tilde{p}_1 + \bar{\omega}^T \theta + k_1 y_1 - y_r$$  \hspace{1cm} (4.15)

Since $v_{11}$ is treated as a virtual control for controlling $z_1$, define

$$z_2 = v_{11} - \tilde{p} y_r - \alpha_1$$  \hspace{1cm} (4.16)

where $\tilde{p}$ is an estimate of $\rho = b_1^{-1}$ and $\alpha_1$ is the stabilizing function yet to be chosen. Using Eq. (4.16) in Eq. (4.15) gives

$$\dot{z}_1 = \tilde{p}_1 + \xi_1 + b_1 [z_2 + \alpha_1 + \tilde{p} y_r] + \bar{\omega}^T \theta + k_1 y_1 - y_r$$  \hspace{1cm} (4.17)
The stabilizing function $\alpha_1$ is chosen as

$$\alpha_1 = \dot{\rho}\bar{\alpha}_1$$

$$\bar{\alpha}_1 = -c_1z_1 - \xi_1 - \omega^T\hat{\theta} - d_1 z_1 - k_1y_1$$  \hspace{1cm} (4.18)

where $c_i,d_i > 0$ and $\hat{\theta}$ is an estimate of $\theta$.

Noting that $b_1\dot{\rho} = b_1(\rho - \bar{\rho}) = 1 - b_1\bar{\rho}$, it follows from Eqs. (4.17) and (4.18) that

$$\dot{z}_1 = b_1z_2 - b_1\bar{\rho}\bar{\alpha}_1 + \omega^T\hat{\theta} - d_1 z_1 + \bar{p}_1 - b_1\bar{\rho}\hat{y}_r - c_1z_1$$  \hspace{1cm} (4.19)

Now consider a Lyapunov function of the form

$$V_1 = d_1^{-1}\bar{p}^T P\bar{p} + (z_1^2 + |b_1|\gamma^{-1}\bar{\rho}^2)/2$$  \hspace{1cm} (4.20)

where $\gamma > 0$ and the positive definite symmetric matrix $P$ satisfies the Lyapunov equation

$$-(PK + K^T P) = -I_{2x2}$$  \hspace{1cm} (4.21)

It is easily seen that $P = diag(P_{11}, P_{22})$ where $P_{ii} = \frac{1}{2k_i}$. The derivative of $V_1$ is given by

$$\dot{V}_1 = d_1^{-1}(\bar{p}^T P\ddot{p} + \bar{p}^T P\ddot{p}) + z_1\dot{z}_1 - |b_1|\gamma^{-1}\ddot{\rho}$$  \hspace{1cm} (4.22)

Substituting Eqs. (4.8), (4.19), and (4.21) in Eq. (4.22) gives

$$\dot{V}_1 = b_1z_1z_2 + \omega^T\hat{\theta}z_1 - c_1z_1^2 - \frac{|\bar{p}|^2}{d_1} - b_1\bar{\rho}\bar{\alpha}_1 z_1 - d_1 z_1^2 + \bar{p}_1 z_1 - \frac{\dot{\rho}|b_1|}{\gamma}\ddot{\rho} - b_1\bar{\rho}\hat{y}_r z_1$$  \hspace{1cm} (4.23)

where $|.|$ denotes the Euclidean norm of a vector. Using Young’s inequality, one has

$$|\bar{p}_1| |z_1| \leq d_1 z_1^2 + \bar{p}_1^2/(4d_1) \leq d_1 z_1^2 + |\bar{p}|^2/(4d_1)$$  \hspace{1cm} (4.24)

Using Eq. (4.24) in (4.23) gives

$$\dot{V}_1 \leq b_1z_1z_2 + \omega^T\hat{\theta}z_1 - c_1z_1^2 - \frac{3|\bar{p}|^2}{4d_1} + z_1\bar{\rho}[ -b_1\bar{\alpha}_1 - b_1\hat{y}_r ] - \frac{\dot{\rho}|b_1|}{\gamma}\ddot{\rho}$$  \hspace{1cm} (4.25)
Since \( \hat{\rho} \) is unknown, this can be eliminated from Eq. (4.25) by choosing an update law of the form

\[ \hat{\rho} = -\gamma \text{sign}(b_1)[z_1(\alpha_1 + \gamma)] \] (4.26)

Now substituting the update law in Eq. (4.25) gives

\[ \dot{V}_1 \leq -c_1z_1^2 - \frac{3}{4d_1}||\bar{\rho}||^2 + \bar{\omega}^T \bar{\rho} z_1 + b_1 z_1 \dot{z}_2 \] (4.27)

The unknown \( \theta \)-dependent term in Eq. (4.27) will be compensated in the second step.

Step 2:

The derivative of \( z_2 \) is given by

\[ \dot{z}_2 = \dot{v}_{11} - \dot{\rho} y_1 - \dot{\rho} \bar{y}_1 - \alpha_1 \] (4.28)

Since \( \alpha_1 \) is a function of \( \dot{\rho}, \xi_1, \Omega_{(1)}, v_{10}, y_r, \bar{\theta}, \alpha_2, x_1, x_2 \), its derivative is given by

\[ \dot{\alpha}_1 = \frac{\partial \alpha_1}{\partial v_{10}} \dot{v}_{10} + \frac{\partial \alpha_1}{\partial \rho} \dot{\rho} + \frac{\partial \alpha_1}{\partial \xi_1} \dot{\xi}_1 + \frac{\partial \alpha_1}{\partial \Omega_{(1)}} \dot{\Omega}_{(1)} + \frac{\partial \alpha_1}{\partial \bar{\theta}} \dot{\bar{\theta}} + \frac{\partial \alpha_1}{\partial x_1} \dot{x}_1 + \frac{\partial \alpha_1}{\partial x_2} \dot{x}_2 \]

\[ \dot{a}_0 + (\partial \alpha_1/\partial x_1) \dot{x}_1 + (\partial \alpha_1/\partial x_2) \dot{x}_2 \] (4.29)

where \( a_0 \) is obtained by comparing terms in Eq. (4.29). For the computation of \( a_0 \), the derivatives of various signals are substituted in Eq. (4.29), but \( \dot{\bar{\theta}} \) is yet to be determined. Using Eqs. (4.2) and (4.7), the derivative of \( x_2 \) is given by

\[ \dot{x}_2 = M_{2(2)}(\alpha, h)^T + p_2 = \bar{p}_2 + \xi_2 + \Omega_{(2)}^T \theta_a + \theta_5 \alpha + \theta_6 h + v_{(2)} b + k_2 h \] (4.30)

Note that \( M_{2(2)} = (\theta_5, \theta_6) \). Adding and subtracting appropriate \( \dot{\bar{\theta}} \)-dependent terms, and using Eqs. (4.12) and (4.30) in Eq. (4.29) gives

\[ \dot{\alpha}_1 = a_0 + \left( \frac{\partial \alpha_1}{\partial x_1} \right) (\xi_1 + \Omega_{(1)}^T \theta_a + \bar{p}_1 + \theta_3 \alpha + \theta_4 h + v_{(1)} b + k_1 x_1) + \]

\[ \left( \frac{\partial \alpha_2}{\partial x_2} \right) (\xi_2 + \Omega_{(2)}^T \theta_a + \bar{p}_2 + \theta_5 \alpha + \theta_6 h + v_{(2)} b + k_2 h) \]

\[ \dot{a}_1 \triangleq a_1 + a_2^T \bar{p} + a_3^T \dot{\bar{\theta}} \] (4.31)
where

\[ a_1 = a_0 + \frac{\partial a_1}{\partial x_1}(\xi_1 + \Omega_{(1)}^T \dot{\theta}_a + \theta_3 a + \theta_4 h + k_1 x_1 + v_{11} \dot{\theta}_1 + v_{10} \dot{\theta}_2) \]

\[ + \frac{\partial a_1}{\partial x_2}(\xi_2 + \Omega_{(2)}^T \dot{\theta}_a + \theta_5 a + \theta_6 h + k_2 x_2 + v_{12} \dot{\theta}_1 + v_{20} \dot{\theta}_2) \]

\[ a_2^T = \left( \frac{\partial a_1}{\partial x_1}, \frac{\partial a_1}{\partial x_2} \right) \]

\[ \bar{p} = (\bar{p}_1, \bar{p}_2)^T \]

\[ a_3^T = \frac{\partial a_1}{\partial x_1}((0,0,\Omega_{(1)}^T) + e_1^T v_{11} + e_2^T v_{10} + e_3^T a + e_4^T h) + \]

\[ \frac{\partial a_1}{\partial x_2}((0,0,\Omega_{(2)}^T) + e_1^T v_{12} + e_2^T v_{20} + e_3^T a + e_6^T h) \]

Here \( e_i \in \mathbb{R}^{18} \). Substituting Eq. (4.31) in Eq. (4.28) gives

\[ \dot{z}_2 = -k_1 v_{11} + \beta - \dot{\theta} y_r - \dot{\theta} \bar{y}_r - a_1 - a_2^T \bar{p} - a_3^T \bar{\theta} = a^* - a_2^T \bar{p} - a_3^T \bar{\theta} + \beta \quad (4.32) \]

where \( a^* = -k_1 v_{11} - \dot{\theta} y_r - \dot{\theta} \bar{y}_r - a_1 \)

In view of Eq. (4.32), we choose control \( \beta \) as

\[ \beta = -a^* - c_2 z_2 - d_2 |a_2|^2 z_2 - \dot{b}_1 z_1 \quad (4.33) \]

Now consider a Lyapunov function

\[ V_2 = V_1 + d_2^{-1} \bar{p}^T P \bar{p} + (z_2^2 + \bar{\theta}^T \Gamma^{-1} \bar{\theta})/2 \quad (4.34) \]

where \( \Gamma \) is a positive definite symmetric matrix. In view of Eq. (4.8), the derivative of \( V_2 \) is given by

\[ \dot{V}_2 = \dot{V}_1 - \frac{|\bar{p}|^2}{d_2} + z_2 \dot{z}_2 - \bar{\theta}^T \Gamma^{-1} \dot{\bar{\theta}} \quad (4.35) \]

Using Eq. (4.32) in Eq. (4.35) and noting that \( b_1 = \theta_1 \) gives

\[ \dot{V}_2 \leq -c_1 z_1^2 - \frac{3|\bar{p}|^2}{4d_1} + \omega^T \bar{\theta} z_1 + (\dot{\theta}_1 + \dot{\theta}_1) z_1 z_2 - c_2 z_2^2 - d_2 z_2^2 |a_2|^2 - \dot{\theta}_1 z_1 z_2 \]

\[ -z_2 a_2^T \bar{p} - a_3^T \bar{\theta} z_2 - \bar{\theta}^T \Gamma^{-1} \dot{\bar{\theta}} - \frac{|\bar{p}|^2}{d_2} \quad (4.36) \]
Define
\[ \tau = (\ddot{w}z_1 + e_1z_1z_2 - a_3z_2) \] (4.37)

Using Young's inequality, one has
\[ |z_2a_2^T \tilde{p}| \leq |z_2||a_2||\tilde{p}| \leq d_2z_2^2|a_2|^2 + \frac{||\tilde{p}||^2}{4d_2} \] (4.38)

Substituting Eqs. (4.37) and (4.38) in Eq. (4.36), one has
\[ \dot{V}_2 \leq -c_1z_1^2 - c_2z_2^2 - \frac{3}{4}(d_1^{-1} + d_2^{-1})|\tilde{p}|^2 + \dot{\theta}^T(\tau - \Gamma^{-1}\dot{\theta}) \] (4.39)

Now one chooses the adaptation law for \( \dot{\theta} \) as
\[ \dot{\theta} = \Gamma \tau \] (4.40)

which yields
\[ \dot{V}_2 \leq -c_1z_1^2 - c_2z_2^2 - \frac{3}{4}(d_1^{-1} + d_2^{-1})|\tilde{p}|^2 \] (4.41)

**Theorem 4.1:** Consider the closed-loop system Eqs. (4.7),(4.26),(4.33) and (4.40). Suppose that \( y_r \) is a bounded smooth trajectory converging to zero, and the zero dynamics of the system are stable. Then the solution of Eq. (1.1) beginning from any initial condition \( q(0) \in \mathbb{R}^4 \) is such that the tracking error \( (\alpha - \alpha_r) \) and \( h \) tend to be zero as \( t \to \infty \). Furthermore, if \( y_r = 0 \), then the state vector \( q(t) \) tends to the origin as \( t \to \infty \).

Proof: A proof is given in the 4.6.

Zero dynamics describe the internal dynamics of the system when the output \( y = \alpha \) is identically zero. For the control of \( \alpha \), Theorem 4.1 assumes that the zero dynamics are stable. The stability properties of zero dynamics have been extensively examined in Refs. 13, 14, and 16. It is noted that stability of the zero dynamics is essential even in the nonadaptive output trajectory control systems.

**Adaptive Control of Plunge Motion**
In the previous section, an adaptive control law for the trajectory control of \( \alpha \) has been presented. Following, a similar approach, one can derive a control law for the trajectory control of the plunge displacement. Define the tracking error

\[
z_1 = h - h_r
\]  

Using Eq. (4.30) the differential equation for \( h \) is given by

\[
\dot{h} = M_2(\alpha, h)^T + \xi_2 + \Omega(2)\beta + u(2)b + k_2h + \ddot{\rho}_2
\]  (4.43)

Apparently, for controlling \( h \), one treats \( v_2 \) as the virtual control since in its derivative, control input \( \beta \) appears. In this case \( \rho = b_2^{-1} \) and

\[
z_2 = v_2 - \dot{\rho}\dot{h}_r - \alpha_1
\]

Following the steps of the previous section, one obtains a virtual control \( \alpha_1 \) and the adaptation law for \( \dot{\beta} \) which is an estimate of \( \rho = b_2^{-1} \) in the first step of derivation, and the control law \( \beta \) and the update law for \( \dot{\beta} \) are obtained in the second step. Since the control law for \( h \)-control can be similarly derived, the details are not presented here.

Similar to \( \alpha \)-control, for the stability in the closed-loop system, it is assumed that the parameters of the aeroelastic system are such that the zero dynamics are stable. It is pointed out that unlike \( \alpha \)-control, the zero dynamics associated with the output \( h \) are nonlinear and exhibit complex dynamic behavior. In this case one has only local stability in the closed-loop system. Since a proof of stability can be established following the steps in the proof of Theorem 4.1, it is not presented here.

4.4 Simulation Results

In this section, numerical results for the pitch angle control and plunge motion control are presented. The parameters of the system are given in the appendix. Simulation
is performed for different values of \( a \) and \( U \). The transfer function of the command generator is chosen as

\[
W_m \left( \frac{\lambda^2}{(s + \lambda)^2} \right)
\]

to obtain exponentially decaying command trajectories to zero where \( \lambda > 0 \). For the pitch angle control, the initial conditions selected are \( \alpha(0) = 5.75(\text{deg}), h(0) = 0.01(m), \dot{h}(0) = 0, \text{ and } \dot{\alpha}(0) = 2(\text{deg/s}). \) The initial conditions of the command generator are set as \( y_r(0) = 5.73(\text{deg}), \dot{y}_r(0) = 0. \) The initial conditions for the parameters are \( \dot{b}_1(0) = -0.1, \dot{b}_2(0) = -0.03, \) and the remaining components of \( \dot{\theta} \) and \( \dot{\rho} \) have initial values zero. The initial states of the filters are set as \( \Omega(0) = 0, v(0) = 0 \) and \( \xi(0) = 0. \) The design parameters are selected as \( \lambda = 1, c_1 = c_2 = d_3 = d_4 = 30, \gamma = 1, \Gamma = I_{18 \times 18}, k_1 = k_2 = 10. \) These design parameters are chosen after several trials by observing simulated responses.

Case 1: The closed-loop system Eq. (1.1) with the control law Eq. (4.33) and the update law Eqs. (4.26) and (4.40) for \( a = -0.2 \) and \( U = 15 \text{ m/s} \) is simulated. For the chosen value of \( a \) and \( U \), one has \( b_1 = -0.4301, b_2 = -0.0537. \) Selected responses are shown in Fig. 4.1. We observe that after an initial transient, the pitch angle asymptotically tracks the command trajectory. The response time is of the order of 6 seconds. Only a small control magnitude (less than 18 (deg)) is required for control. Since for \( a = -0.2 \) and \( U = 15 \text{ m/s} \) the zero dynamics are stable, the plunge displacement also converges to zero as predicted.

Case 2: The closed-loop system for \( a = -0.2 \) and \( U = 20 \text{ m/s} \) is simulated. Selected responses are shown in Fig. 4.2. The response time of the same order as in case 1 is observed, but due to enhanced control effectiveness at higher air speed \( U \), smaller control magnitude (about 8 (deg)) compared to case 1 is required. The plunge displacement and control input are somewhat similar to those of case 1.
Case 3: In order to examine the sensitivity of the controller with respect to parameter $a$, the closed-loop system for a different value of $a = -0.32$, but with the same value of $U = 15$ (m/s) is simulated. We observe that although the pitch angle asymptotically tracks the command trajectory, larger control magnitude (less than 35 (deg)) is required (Fig.4.3). Moreover, larger plunge displacement is observed in this case. The response time is of the order of 6 seconds. In this case, increase in control magnitude can be attributed to reduced degree of stability of the zero dynamics, since as $a \rightarrow -0.55$, the poles of the zero dynamics move to the right in the complex plane (Ref.13).

Case 4: Now simulation results for plunge motion control are presented. The parameters $c_i = d_i = 500$, $k_1 = k_2 = 100$ and $\Gamma = 100I_{18 \times 18}$ are chosen. The initial conditions are $\alpha(0) = 5.75$(deg), $h(0) = 0.01$(m), and $\dot{\alpha}(0) = 2$(deg), $\dot{h}(0) = 0$. The remaining control parameters and initial conditions of case 1 are retained. The aeroelastic model for $a = -0.75$ and $U = 15$m/s is considered for simulation. In this case $b_1 = 0.5279$, $b_0 = -0.0731$. It is seen that $h$ tracks $y_r$ and the pitch angle tends to zero (Fig.4.4). Since the zero dynamics are nonlinear, we observe high frequency oscillations in control $\beta$. The response time is of the order of 7-8 seconds. The maximum control magnitude is about 28 (deg). Similar to case 1, all the parameter estimates converge to constant values.

Case 5: Simulation for the aeroelastic model with $a = -0.85$ and $U = 15$ is performed. Selected responses are shown in Fig.4.5. It is seen that the state vector is regulated to zero, but compared to case 4, smoother response for $h$ is obtained and transient in $\alpha$ decays relatively faster (in about 5 seconds). The control magnitude (about 25 (deg)) is also slightly smaller than case 4. For $h$-control, the degree of stability of the zero dynamics improves as the parameter $a \rightarrow -1$ (Ref. 16). For this reason, the transient responses of $h$ and $\alpha$ are better than those of case 4.
Case 6: To examine the sensitivity of the controller with respect to the air speed, simulation is performed for the model with $U = 20$, but the remaining parameters of case 5 are retained. In this case, one has $b_1 = 2.4762, b_2 = -0.2109$. It is seen that $h$ follows $y_r$ and $\alpha$ is regulated to zero (Fig. 4.6). In this case at higher air speed, similar to case 3, smaller control magnitude (about 18 deg) compared to case 5 is required. The response time is of the order of 6-7 seconds.

Extensive simulation has been performed. Based on these results, it is found that by a suitable choice of parameters $c_i, d_i, L, \gamma$ and $\Gamma$, desirable responses in the closed-loop system can be obtained. For smaller values of $c_i$, the control magnitude is reduced. Furthermore, the command trajectory $y_r$ can be properly chosen to shape the transient responses.

4.5 Summary

In this chapter, a new controller for the control of an aeroelastic system based on a backstepping design technique was presented. Adaptive control laws for the trajectory control of $\alpha$ and $h$ were derived. For the synthesis of the controller, only the measured variables (plunge displacement and pitch angle) were used. For the derivation of the controller, a canonical representation of the aeroelastic model was used. Reduced order filters were designed to obtain the estimate of the state variables. In the closed-loop system, asymptotic regulation of the state vector to the origin was accomplished. Simulation results were presented which show that control of the pitch angle and the plunge displacement can be accomplished using output feedback in spite of the uncertainties in the system parameters with reasonable control magnitude. The adaptive controller has several design parameters which can be adjusted to obtain desirable response characteristics. Simulation results showed that the control systems with reduced order observer yield smooth response.

4.6 Proof of Theorem 4.1
First it will be shown that all the signals in the closed-loop system are bounded. $V_2$ is a positive definite function of $z_1, z_2, \ddot{\theta}, \ddot{\rho}, \ddot{p}$ and $\dot{V}_2$ is negative semidefinite. Thus, it follows that $(z_1, z_2, \ddot{\theta}, \ddot{\rho}, \ddot{p}) \in \mathcal{L}_\infty$, where $\mathcal{L}_\infty$ denotes the set of bounded functions. Since $z_1 \in \mathcal{L}_\infty$, one has $\alpha \in \mathcal{L}_\infty$.

Now the differential equations associated with the zero dynamics are derived. The output $\alpha$ has relative degree 2 since the control $\beta$ appears in its second derivative. Thus the zero dynamics has dimension 2. Define

$$\eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} b_2 \alpha - b_1 \dot{h} \\ b_2 \dot{\alpha} - b_1 \dot{h} \end{bmatrix}$$

(4.44)

Then, using Eq. (1.1), it follows that $\eta$ satisfies

$$\dot{\eta} = A_\eta \eta + (0, 1)^T f_\eta(\alpha, \dot{\alpha})$$

(4.45)

where

$$A_\eta = \begin{bmatrix} 0 & 1 \\ -a_{\eta 1} & -a_{\eta 2} \end{bmatrix}$$

$$c_{\eta i} = b_2 M_{ii(1)} - b_1 M_{ii(2)}, i = 1, 2$$

$$a_{\eta i} = c_{\eta i}(0, b_1^{-1})^T$$

$$f_{\eta \eta} = (b_2, b_1) g k_{\eta \alpha}(\alpha)$$

$$f_\eta = (\alpha c_{\eta 1} + \dot{\alpha} c_{\eta 2})(1, b_2 b_1^{-1})^T + f_{\eta \eta}$$

The zero dynamics are obtained when $\alpha = 0$ and $\dot{\alpha} = 0$, i.e., $f_\eta = 0$ in Eq. (4.45). Thus, for the stability of the zero dynamics, $A_\eta$ must be a Hurwitz matrix. Solving for $\eta_1$ from Eq. (4.45), one obtains

$$\tilde{\eta}_1(s) = \left[\frac{(c_{\eta 1} + s c_{\eta 2})(1, b_2 b_1^{-1})^T}{H_\eta(s)}\right] \ddot{\alpha}(s) + H_{\eta^{-1}}(s) \ddot{f}_{\eta \eta}(\alpha)$$

(4.46)

where in this section, $s$ denotes the Laplace variable, functions with overbar denote Laplace transforms, and $H_\eta = s^2 + a_{\eta 2} s + a_{\eta 1}$. Since $(c_{\eta 1} + s c_{\eta 2}) H_{\eta^{-1}}$ and $H_{\eta^{-1}}$ are stable transfer functions and $\alpha$ is bounded, from Eq. (4.46) it follows that $\eta_1$ is bounded. But $\eta_1 = b_2 \alpha - b_1 h$, therefore, one has that $h \in \mathcal{L}_\infty$. Since $(\alpha, h) \in \mathcal{L}_\infty$, $\Omega$ and $\xi$ are
bounded. In view of the differential equations in Eq.(4.10), one has

\[
\begin{align*}
\dot{v}_{11} &= -k_1 v_{11} + \beta \\
\dot{v}_{10} &= -k_1 v_{10} \\
\dot{v}_{12} &= -k_2 v_{12} \\
\dot{v}_{20} &= -k_2 v_{20} + \beta
\end{align*}
\]

(4.47)

In view of Eq.(4.47), \( v_{10}, v_{12} \in \mathcal{L}_\infty \). This implies that \( \bar{\omega} \) and \( \alpha_1 \) are bounded. Since \( z_2 \in \mathcal{L}_\infty \), from Eq.(4.16) one has that \( v_{11} \in \mathcal{L}_\infty \) which implies that \( v_{(1)} \in \mathcal{L}_\infty \). From Eq.(4.12), it follows that \( \dot{\alpha} \in \mathcal{L}_\infty \). Since \( (\alpha, \dot{\alpha}) \in \mathcal{L}_\infty \), using Eq.(4.45), one concludes that \( \eta \in \mathcal{L}_\infty \). Now Eq.(4.44) implies that \( \dot{h} \in \mathcal{L}_\infty \) and \( (\alpha, \dot{\alpha}, h, \eta) \in \mathcal{L}_\infty \). Since \( \dot{h} \in \mathcal{L}_\infty \), Eq.(4.30) implies that \( v_{20} \in \mathcal{L}_\infty \), thus \( v \in \mathcal{L}_\infty \). This establishes the the boundness of all the signals. Now using LaSalle-Yoshikawa theorem (Ref.19, page489-492), one has that \( (z_1, z_2) \to 0 \) as \( t \to \infty \). Therefore , \( \alpha \to 0 \) as \( t \to \infty \).

Now in view of Eq.(4.45), \( \eta \) converges to zero which according to Eq.(4.44) implies that \( h \) converges to zero.

For the case when \( y_r = 0 \), according to the LaSalle invariance theorem (Ref. 19, page 25), the state vector converges to the largest invariant set \( \mathcal{V}_i \) contained in the set \( \mathcal{V} = \{ z_1 = 0, z_2 = 0, \bar{x} = 0 \} \). But in \( \mathcal{V}_i \), \( \dot{z}_1 = \dot{\alpha} = 0 \). Now convergence of \( (\dot{\alpha}, \eta_2) \) to zero implies convergence of \( \dot{h} \) to zero. Therefore, \( q(t) \) converges to zero. This completes the proof of Theorem 4.1.
Figure 4.1: Pitch angle control: $\alpha = -0.2$ $U=15\text{m/s}$
Figure 4.2: Pitch angle control: $\alpha = -0.2$ U=20m/s
Figure 4.3: Pitch angle control: $\alpha = -0.32 \ U = 15 \text{m/s}$
Figure 4.4: Plunge motion control: $\alpha = -0.75 \ U=15\text{m/s}$

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Figure 4.5: Plunge motion control: $\alpha = -0.85 \text{ U}=15\text{m/s}$
Figure 4.6: Plunge motion control: $\alpha = -0.85 \text{ U}=20\text{m/s}$


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