1-7-2021

Hamiltonian-Based Libration Point Orbit Control on Manifold of Constant Energy

Keum W. Lee
Catholic Kwandong University

Sahjendra N. Singh
University of Nevada, Las Vegas, sahjendra.singh@unlv.edu

Repository Citation

Lee, K., Singh, S. (2021). Hamiltonian-Based Libration Point Orbit Control on Manifold of Constant Energy. *Lecture Notes in Networks and Systems, 182* 49-60.
http://dx.doi.org/10.1007/978-3-030-65796-3_5

Hamiltonian-Based Libration Point Orbit Control on Manifold of Constant Energy

Keum W. Lee[1] and Sahjendra N. Singh[2]

[1] Department of Electronic Engineering, Catholic Kwandong University, Gangneung, Gangwon 25601, Republic of Korea
kwlee@cku.ac.kr
[2] Department of Electrical and Computer Engineering, University of Nevada Las Vegas, Las Vegas, NV 89154, USA
sahjendra.singh@unlv.edu

Abstract. The circular restricted three-body problem (CR3BP) is important from the theoretical as well as a practical viewpoint. In this paper, the objective is to control a spacecraft along stable or unstable libration point orbits. For this purpose, a manifold of constant energy is specified. Then, a Hamiltonian-based state variable feedback control law is designed for regulating the spacecraft to attain the specified level of energy. Through the Lyapunov analysis, asymptotic convergence of the the system energy to prescribed level is established. For this multi-dimensional constant energy manifold, a variety of periodic and quasi-periodic libration point orbits can exist. Interestingly, the spacecraft can be controlled using a small control force applied to the spacecraft. Simulation results are presented, which show that in the closed-loop system, the spacecraft follows libration point orbits on the manifold of prescribed energy.

1 Introduction

The study of the motion of spacecraft in the gravitational field of multiple massive bodies has been of considerable interest to researchers in the field of astrodynamics. The solution of this general problem is difficult. For this reason, the simplified circular restricted three-body Problem (CR3BP), in which it is assumed that the spacecraft has no effect on the motion of two primaries, has received much attention. Furthermore, in the simplified CR3BP, it is assumed that the primaries are in a uniform circular motion about their barycenter. For the CR3BP, Euler discovered three collinear equilibrium points (L_1, L_2, L_3), and Lagrange found two equilateral equilibrium points (L_4, L_5). These are also termed libration or Lagrangian points ($L_1, ..., L_5$). Consider, for an example, the motion of a spacecraft of relatively small mass, in the gravitation field of the Earth-Moon system. In the CR3BP, the Earth and Moon rotate about the barycenter, and the libration points remain fixed in the rotating coordinate frame of the Earth and Moon. Considerable effort has been made in the past to determine natural periodic (Halo) orbits and quasi-periodic trajectories (Lissajous trajectories) in the vicinity of the libration points. Farquhar [1, 2] proposed the use of libration points and orbits around libration points for lunar far-side communications. By placing a satellite at L_1 point and another in Halo orbit at L_2 point, a continuous communication coverage of the lunar surface would be possible. Further, effort was made to determine Halo and Lissajous trajectories [3-5]. (Readers may refer to recent survey papers on orbital dynamics and libration point orbit (LPO) control [6-7]).

For station-keeping, control system can be designed (i) by exploiting the geometric structure of the phase space around an orbit [8-11] or (ii) by applying modern control theory [12-14]. A Hamiltonian preserving control law has been designed for the CR3BP problem [11]. Although theoretically, station-keeping at an LPO, or at a quasi-periodic Lissajous trajectory, does not require any propulsion system, the controllers of [8-11] are sensitive to model uncertainties. Therefore, the authors of [11] used a combination of a Hamiltonian-based control signal with a robustifying control signal derived from a disturbance estimator (extended observer). There exist a variety of modern control techniques that can be used for uncertain systems. The state feedback controllers in Refs. [12-14] were designed based on H$^\infty$ and disturbance rejection control techniques. (Additional references can be found in [6-7].)

The Hamiltonian-based controller of [11] requires explicit expression of the reference LPO, obtained by solving the nonlinear CR3BP. To avoid any knowledge of the solution of the CR3BP, one can pose a simpler problem of steering a satellite to a manifold of constant energy. Such a formulation can achieve swinging (oscillating) control of a spacecraft on a specified manifold. Fradkov and Pogromsky [15] considered the problem of energy control of Hamiltonian systems. In this paper, the objective is to design a feedback controller for steering a spacecraft to a reference manifold in the gravitational field of two massive bodies.

The contribution of this paper is two-fold. First, a control law is designed for controlling the trajectory of a spacecraft from its initial state to a manifold of constant energy. Here, the Hamiltonian of the CR3BP is selected to define the constant energy surface. Through the Lyapunov analysis, asymptotic convergence of the the system energy to the prescribed level of energy is established. Of course, on a multi-dimensional manifold of constant energy, there exist a variety of oscillating trajectories, which are confined on the manifold. Interestingly, regulation of the trajectories can be accomplished using propulsive force of small magnitudes. Once the trajectory evolves on the manifold, control force becomes zero. Second, simulation results for the Earth-Moon-spacecraft system are obtained. These results show the existence a variety of orbits on the manifold of constant energy. It is found that these orbits on the constant energy surface depend on the initial state of the spacecraft and the feedback parameters of the controller.

2 Dynamics of CR3BP

Figure 1 shows a spacecraft of negligible mass and two primary bodies (such as the Earth and Moon, or the Sun-Earth and Moon) of masses M_1 and M_2, respectively. A rotating coordinate frame with its basis vectors $(\hat{i}, \hat{j}, \hat{k})$ centered at the barycenter (which is at distance D_1 from M_1) is also shown in the figure. It is assumed that the primary bodies are rotating in circular orbits about the barycenter with uniform angular velocity:

$$n = \sqrt{\frac{G(M_1 + M_2)}{D^3}} \tag{1}$$

where D is the constant distance between the two primaries, and G is the universal gravitational constant. Define $M = M_1 + M_2$ and $\rho = M_2/M$. It is to use time in units of $(1/n)$, and distances in units of D. Then, the nondimensional position coordinates (x, y, z) of the spacecraft (m), with respect to the barycenter in the rotating frame, can be shown to satisfy the following equations [14]:

$$\begin{aligned}
\ddot{x} &= 2\dot{y} + x - \frac{(1-\rho)(x-\rho)}{r_1^3} - \frac{\rho(x+1-\rho)}{r_2^3} + u_1 \\
\ddot{y} &= -2\dot{x} + y - \frac{(1-\rho)y}{r_1^3} - \frac{\rho y}{r_2^3} + u_2 \\
\ddot{z} &= -\frac{(1-\rho)z}{r_1^3} - \frac{\rho z}{r_2^3} + u_3
\end{aligned} \tag{2}$$

where r_i is the distance of the spacecraft from M_i, $i = 1, 2$:

$$r_1 = \sqrt{(x-\rho)^2 + y^2 + z^2}$$

$$r_2 = \sqrt{(x+1-\rho)^2 + y^2 + z^2}$$

For the Earth-Moon-spacecraft system, the collinear points are L_1 (cislunar), L_2 (translunar), and L_3. Besides these, there are two equilateral points (L_4, L_5) (not shown in the figure). By the linear analysis, one can show that the collinear Lagrangian points are unstable, but the equilateral libration points are stable. The Lagrange points L_1 and L_2 are important for future space missions. Satellites placed at L_1 and L_2 can serve as communication platforms.

We are interested in deriving a control law for regulating the spacecraft, originating in the vicinity of the translunar libration point to a constant energy manifold in the vicinity of L_2. The manifolds of interest will be parameterized using selected constant values of Hamiltonian of the system. Of course, this derivation of the control law will be applicable to other stable or unstable libration point orbits (LPOs). It should be noted that for this multi-dimensional manifold - defined by constant a Hamiltonian - one can expect oscillating orbits of a variety of patterns.

3 Libration Point Orbit Control

The derivation of control law for the LPO control is based on the Hamiltonian of the system. For controlling the spacecraft on a manifold of constant energy, the expression of the Hamiltonian is necessary.

3.1 Hamiltonian for the CR3BP

Define $x_s = (x, y, z)^T \in R^3$ as the position coordinate vector of the spacecraft. The kinetic energy of a spacecraft in the gravitational field of the two primaries is:

$$T(x_s, \dot{x}_s) = \frac{1}{2}\dot{x}_s^T\dot{x}_s + b^T\dot{x}_s + \frac{1}{2}(x^2 + y^2) \tag{3}$$

where $b(x_s) = (-y, x, 0)^T$, and the potential energy P is:

$$P(x_s) = -\frac{1-\rho}{r_1} - \frac{\rho}{r_2} \tag{4}$$

The Lagrangian has the expression:

$$L(x_s, \dot{x}_s) = T - P = \frac{1}{2}\dot{x}_s^T\dot{x}_s + b^T\dot{x}_s + \frac{1}{2}(x^2 + y^2) - P \tag{5}$$

The generalized momenta $p = (p_1, p_2, p_3)^T \in R^3$ is defined by:

$$p = \frac{\partial L}{\partial \dot{x}_s} = \dot{x}_s + b \tag{6}$$

Next, solving for $\dot{x}_s$, Eq. (6) gives

$$\dot{x}_s = p - b = p + [y, -x, 0]^T \tag{7}$$

Now, eliminating $\dot{x}_s$ in (5) by using Eq. (7), and noting that $||b||^2 = (x^2 + y^2)$, one obtains Lagrangian of the form:

$$\begin{aligned} L(x_s, p) &= \frac{1}{2}(p-b)^T(p-b) + b^T(p-b) + \frac{1}{2}(x^2+y^2) - P \\ &= \frac{1}{2}||p||^2 - \frac{1}{2}||b||^2 + \frac{1}{2}(x^2+y^2) - P = \frac{1}{2}||p||^2 - P \end{aligned} \tag{8}$$

The Hamiltonian of the system is defined as:

$$\begin{aligned} H &= \left[\frac{\partial L}{\partial \dot{x}_s}\right]^T \dot{x}_s - L \\ &= p^T\dot{x}_s - L \end{aligned} \tag{9}$$

Then, H as a function of p and x_s takes the form:

$$\begin{aligned} H(p, x_s) &= p^T(p-b) - L = p^T(p-b) - \frac{1}{2}||p||^2 + P \\ &= \frac{1}{2}||p||^2 - p^Tb + P \end{aligned} \tag{10}$$

Alternatively, the generalized momenta p can be eliminated using Eq. (6) to obtain H as :

$$\begin{aligned} H &= \frac{1}{2}||\dot{x}_s + b||^2 - (\dot{x}_s + b)^Tb + P \\ &= \frac{1}{2}(\dot{x}_s + b)^T(\dot{x}_s - b) + P \\ &= \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - \frac{1}{2}(x^2 + y^2) - \frac{(1-\rho)}{r_1} - \frac{\rho}{r_2} \end{aligned} \tag{11}$$

By taking the partial derivative of H in Eq. (10), with respect to x_s and p, it can be shown that:

$$\dot{x}_s = \frac{\partial H}{\partial p} = p + [y, -x, 0]^T \tag{12}$$

$$\dot{p} = -\frac{\partial H}{\partial x_s} = [p_2, -p_1, 0]^T - \frac{\partial P}{\partial x_s} \tag{13}$$

By differentiating Eq. (12), substituting $\dot{p}$ from Eq. (13), and eliminating p using Eq. (6), one can obtain the second-order equation for $\ddot{x}_s$ as given in Eq. (2). Thus, Eqs. (12) and (13) provide a state variable representation of Eq. (2). For a given constant H^*, a constant, $H(x_s, p) = H^*$ defines a manifold of constant energy in the state space with coordinates (x_s, p).

Next, a control law will be derived such that $H(p(t), x_s(t))$ asymptotically converges to H^*. For a given H^*, define a manifold as:

$$\mathcal{M} = \{(x_s^T, p^T)^T \in R^6 : H(x_s, p) = H^*\} \tag{14}$$

Of course, in general, convergence of $(H - H^*)$ to zero, does not necessarily imply that the trajectory $(x_s(t), p(t))$ will converge to $\mathcal{M}$, unless for the chosen H^*, $\mathcal{M}$ is a compact set [15]. It should be noted that $H(x_s, p)$ is not a positive definite function. However, if the trajectory happens to be bounded, then it will tend to the set $\mathcal{M}$.

3.2 A Control Law for Swinging Control on $\mathcal{M}$

The derivation of control law is based of the Lyapunov method. For the purpose of control, a modified Hamiltonian:

$$H_m(x_s, p, u) = H - H_1 = H - x_s^T u \tag{15}$$

is considered, where $u = (u_1, u_2, u_3)^T \in R^3$ is the control force vector, H_1 is called interaction Hamiltonian [15]. Then, one obtains:

$$\dot{x}_s = \frac{\partial H_m}{\partial p} = \frac{\partial H}{\partial p}$$

$$\dot{p} = -\frac{\partial H_m}{\partial x_s} = -\frac{\partial H}{\partial x_s} + u \tag{16}$$

Consider a Lyapunov function:

$$V(x_s, p) = \frac{1}{2}(H(p, x_s) - H^*)^2 \tag{17}$$

Differentiating V along the solution of the system (16) gives:

$$\dot{V} = (H - H^*)\left[\left(\frac{\partial H}{\partial p}\right)^T \dot{p} + \left(\frac{\partial H}{\partial x_s}\right)^T \dot{x}_s\right]$$

$$= (H - H^*)\left[\left(\frac{\partial H}{\partial p}\right)^T \left(-\frac{\partial H}{\partial x_s} + u\right) + \left(\frac{\partial H}{\partial x_s}\right)^T \frac{\partial H}{\partial p}\right]$$

$$= (H - H^*)\dot{x}_s^T u \tag{18}$$

For regulating V to zero, one selects u_i as:

$$u_i = -\gamma_i \dot{x}_{si}^{\nu_1}(H - H^*)^{\nu_2}, i = 1, 2, 3 \tag{19}$$

where γ_i is positive, and ν_1 and ν_2 are positive odd integers. Then, Eq. (18) gives:

$$\dot{V} = -\sum_{i=1}^{3}[\gamma_i \dot{x}_{si}^{\nu_1+1}(H - H^*)^{\nu_2+1} \leq 0 \tag{20}$$

because $\nu_j + 1$ are even integers.

Here $V(x_s, p) \geq 0$, and $\dot{V} \leq 0$. This implies that V approaches a finite value $V(\infty) \leq V(x_s(0), p(0))$. Suppose the trajectory $(x_s(t), p(t))^T$ remains bounded, then its positive limit set Γ^+ will be bounded, and V will tend to a constant value on Γ^+ [14]. Then $\dot{V}$ will be zero on Γ^+. This will imply that on Γ^+, either $\dot{x}_s$=0 or $H - H^*$=0. If $\dot{x}_s = 0$, then $\ddot{x}_s = 0$, which will imply the converge of the trajectory to an equilibrium point. However, if $H - H^*$ is zero, its derivative will be zero. This will imply the convergence of trajectory on $\mathcal{M}$.

3.3 A Saturating Control Law

One can alternatively choose a smooth saturating control law as:

$$u_i = -\gamma_i \tanh(\delta_i (H - H^*) \dot{x}_{si}), i = 1, 2, 3 \tag{21}$$

where $\gamma_i > 0$, $\delta_i > 0$, and $\dot{x}_s = (\dot{x}_{si}, \dot{y}_{si}, \dot{z}_{si})^T$. Then, substituting this control input in Eq. (18) gives:

$$\dot{V} = -\sum_{i=1}^{3} [(H - H^*) \dot{x}_{si} \gamma_i \tanh(\delta_i (H - H^*) \dot{x}_{si})] \leq 0 \tag{22}$$

Because $\xi \tanh(\delta_i \xi) \geq 0$ for all $\xi \in R$. Equation (21) shows that control magnitude does not exceed the value γ_i. This completes the control law design.

Remark 1: A sufficient condition under different assumptions has been presented in Ref. [15] for controlling the energy of Hamiltonian systems. Here, it was assumed that the trajectory remained bounded. It will be seen in next section that the controller does accomplish regulation of the energy, but requires proper selection of H^* for station-keeping.

4 Simulation Results

This section presents simulated responses of the closed-loop system (Eq. (2) with unconstrained control law Eq. (19)), as well as the system (Eq. (2) with saturating control law (21)). For the Earth (M_1)-Moon (M_2) system, the parameters given in [14] are: $\rho = 0.01215$, $D_1 = 4674D$, $D_2 = 380073$ [km], $D = R_{12} = D_1 + D_2$ [km], and $n = 2.661699 \times 10^6$ [rad/s]. The nondimensional coordinate vector of L_2 point, with respect to the barycenter, is $(x_u = -1.15568, y_u = 0, z_u = 0)$. The design parameters in the control law are $\gamma_i = 10^{-6}/(D * n^2) = 0.3669$, $\nu_i = 1$ $(i = 1, 2)$, $\delta_1 = 0.01, \delta_2 = 0.1$, and $\delta_3 = 1$. A Halo orbit given in [14] based on linearization has coordinates (with respect to the L_2 point):

$$x_H^T(t) = [-A_x \sin \omega_{xy} t, -A_y \cos \omega_{xy} t, A_z \sin \omega_z t] \tag{23}$$

where $\omega_{xy} = 1.86265, \omega_z = 1.78618, k_{uu} = 2.91261$, $A_x = 0.00312, A_y = k_{uu} A_x$, and $A_z = A_y$. The initial conditions of spacecraft close to the Halo orbit (expressed relative to the barycenter) were assumed to be of the form $x_s(0) \approx ((x_u, 0, 0)^T + x_H(0))$ and $\dot{x_s}(0) \approx \dot{x}_H(0)$. The selected two initial conditions were:

$$X_{ica} = (x_s(0)^T, \dot{x}_s^T(0)) = [-1.1557, -0.0091, 0, -0.0058, 0, 0.0162] \tag{24}$$

$$X_{icb} = (x_s(0)^T, \dot{x}_s^T(0)) = X_{ica} + [-0.003, 0.002, 0.004, 0, 0, 0] \tag{25}$$

The value of the Hamiltonian at the L_2 point, denoted as H_{L2}^*, can be evaluating by setting $x = x_u$, $y = z = 0$, and $\dot{x}_s = 0$ in Eq. (11). It should be noted that the coordinates $(x - x_u, y, z)$ plotted in all of the figures are shown relative to the libration point L_2.

***Case A.* Swinging control:** $H^* = H_{L2}^*$, $(x_s^T(0), \dot{x}_s^T(0)) = X_{ica}$

The responses of the closed-loop system (Eq. (2) and (19)) for $H^* = H_{L2}^*$ and $(x_s^T(0), \dot{x}_s^T(0)) = X_{ica}$ are shown in Figs. 2 and 3. One observes quasi-periodic orbit. After initial transients, H converges to H^*. The maximum control acceleration is within 5×10^{-5} [km/s^2].

***Case B.* Swinging control:** $H^* = H_{L2}^* - 0.0005$, $(x_s^T(0), \dot{x}_s^T(0)) = X_{ica}$

Simulation was done to steer the spacecraft to a lower energy manifold by setting $H^* = H_{L_2}^* - 0.0005$. It is seen that control law (19) is effective in regulating the energy (see Figs. 4 and 5), but the orbits are of different pattern.

***Case C.* Swinging control with saturating control law:** $H^* = H^*_{L2}$, $(x_s^T(0), \dot{x}_s^T(0)) = X_{ica}$
Next, the saturating control law (21) was implemented. It was assumed that $H^* = H^*_{L2}$ and $(x_s^T(0), \dot{x}_s^T(0)) = X_{ica}$. The remaining parameters of Case A were retained. One can observe in Fig. 6 that compared to case A, regulation is achieved with smaller control acceleration and controller saturates briefly.

***Case D.* Swinging control:** $H^* = H^*_{L2}$, $(x_s^T(0), \dot{x}_s^T(0)) = X_{icb}$

Finally, simulation was done similar to Case A (with control law Eq. (19)), but the initial state was assumed to be X_{icb}. It is interesting to see that now, for the choice of a different initial condition, one obtains an entirely different orbit (see Figs. 7 and 8).

5 Conclusion

In this paper, the control of a spacecraft of the restricted circular three-body problem along stable or unstable libration point orbits confined to a manifold was considered. The Hamiltonian of the system was chosen to defined a manifold of constant energy. Then, a Hamiltonian-based state variable feedback control law was designed for regulating the spacecraft to attain the specified level of energy. Through the Lyapunov stability analysis, asymptotic convergence of the system energy to prescribed level was established. Interestingly, it is possible to control the spacecraft using small control forces. Simulation results showed that in the closed-loop system, the spacecraft asymptotically followed quasi-periodic orbits on the chosen manifold. It was seen that a variety of quasi-periodic orbits (Lissajous trajectories) exist on the manifold.

6 References

1. Farquhar, R. M.: The Control and Use of Libration-Point Satellite. NASA Tech. Report TR R-346, 1970.
2. Farquhar, R. M.: The Utilization of Halo Orbits in Advanced Lunar Operations. NASA TN D-6345, 1971.
3. Farquhar, E. W., Kamel, A. A.: Quasi-Periodibc Orbits About the Translunar Libration Point. Celestial Mechanics, Vol. 7, 458-473 (1973)
4. Richardson, D. L.: Halo Orbit Determination for the ISEE-3 Mission. J. Guid. Contr. Dyn. 3, 543-548 (1980)
5. Howell, K. C., Pernicka, H. J. Numerical Determination of Lissajous Trajectories in the Restricted Three-Body Problem. Celestial Mechanics, V. 41, 1988, pp. 107-124.
6. Xu, M., Liang, Y., Ren, K. Survey on Advances in Orbital Dynamics and Control for Libration Points. Progress in Aerospace Sciences. V. 82, 2016, pp. 24-35.
7. Shirobokov, M.,Trofimov, S., and Ovchinnikov, M.: Survey of Station- Keeping Techniques for Libration Point Orbits, Journal of Guidance, Control, and Dynamics, Vol. 40, No. 5, 2017, pp. 1085-1105.
8. Simo, C., Gomez, G., Llibre, J., Martinez, R., and Rodriguez, J.: On the Optimal Station Keeping Control of Halo Orbits. Acta Astronautica. 15 (6-7), 391-397, 1897
9. Farres, A., and Jorba, A.: Station-Keeping of a Solar Sail Around a Halo Orbit. Acta Astronautica. 94, 527-539, 2014.
10. Scheeres, D. J., Hsiao, F. Y., and Vinh, N. X. Stabilizing Motion Relative to an Unstable Orbit: Applications to Spacecraft Formation Flight. Journal of Guidance, Control, and Dynamics. 26, 62-73, 2003
11. Jung,S., Kim, Y.: Hamiltonian Structure-Based Robust Station-Keeping for Unstable Libration Point Orbits. J Guid. Contr. Dyn. 42(12), 1-18 (2019)
12. Kulkarni, J. E., Campbell, M. E., and Dullerud, G. E.: Stabilization of Spacecraft Flight in Halo Orbits: An H^∞ Approach. IEEE Transactions on Control Systems Technology. 14(3), 572-578 (2006).
13. Lou, Z., Zhang, K., and Wang, Y.: Active Disturbance Rejection Station-Keeping Control for Solar-Sail Libration-Point Orbits. Journal of Guidance, Control, and Dynamics 39(8), 1917-1921 (2016) 1921.
14. Wie, B. Space Vehicle Dynamics and Control. AIAA., Alexander Bell Dr., VA. (2008)
15. Fradkov, A. L., Pogromsky, A. Y. Introduction to Control of Oscillations and Chaos. World Scientific, N. J., (1998)
16. Khalil, H. K.: Nonlinear Systems. Prentice Hall, N.J. (2002)

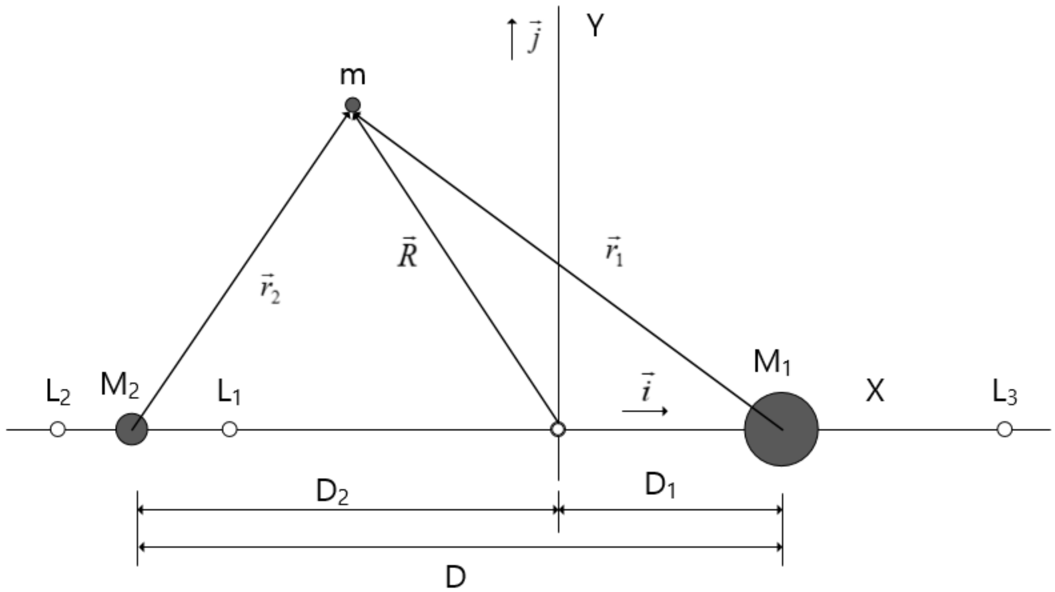

Fig. 1. Circular restricted three-body problem.

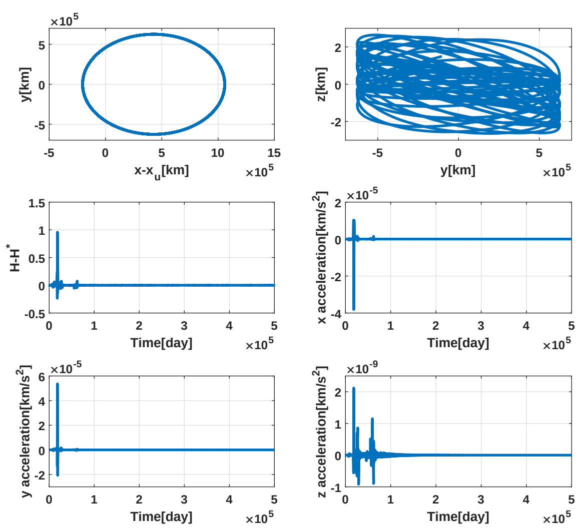

Fig. 2. Swinging control: $H^* = H^*_{L2}$, $(x_s^T(0), \dot{x}_s^T(0)) = X_{ica}$

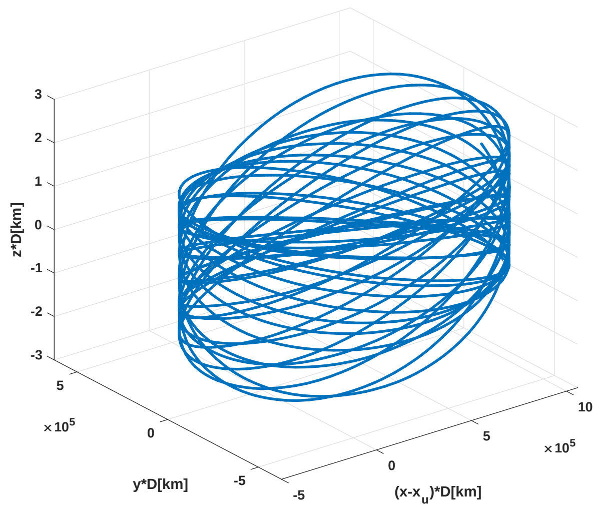

Fig. 3. Continued

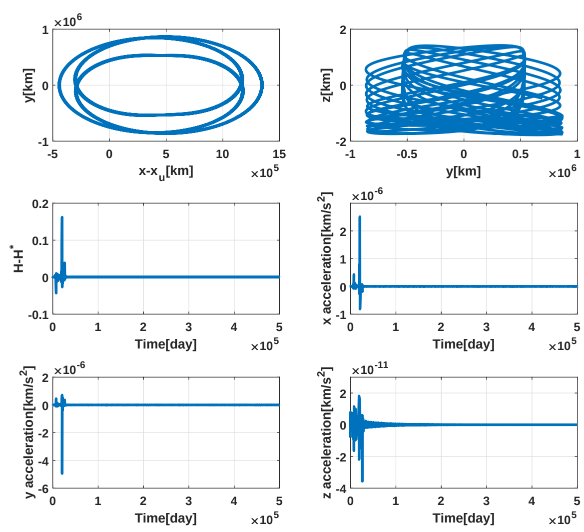

Fig. 4. Swinging control: $H^* = H^*_{L2} - 0.0005$, $(x_s^T(0), \dot{x}_s^T(0)) = X_{ica}$

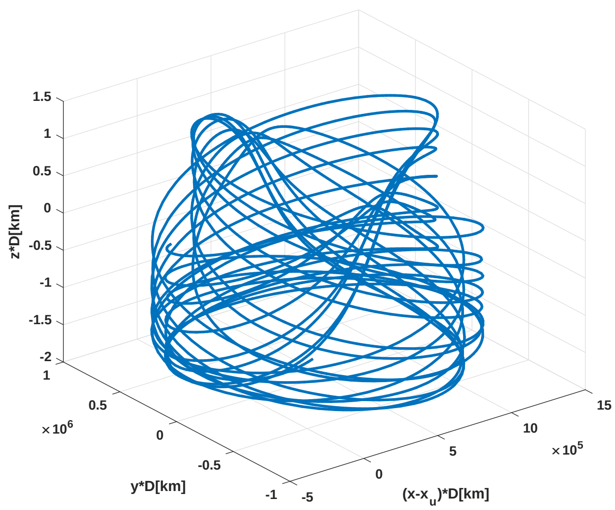

Fig. 5. Continued

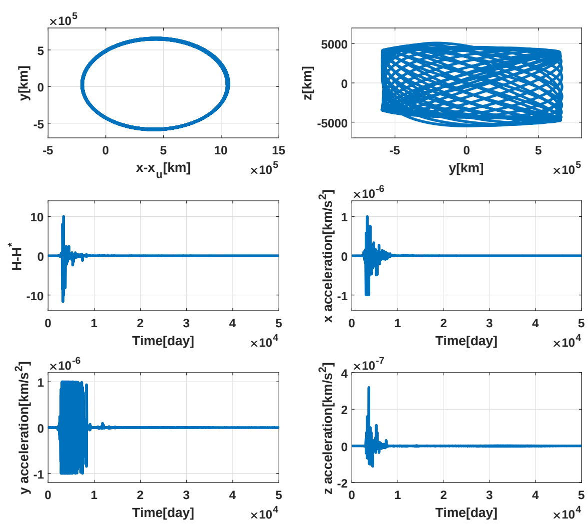

Fig. 6. Saturating control: $H^* = H^*_{L2}$, $(x_s^T(0), \dot{x}_s^T(0)) = X_{ica}$

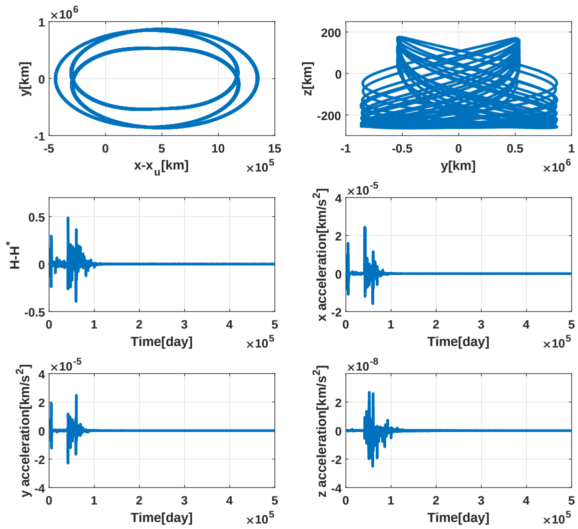

Fig. 7. Swinging control: $H^* = H^*_{L2}$, $(x_s^T(0), \dot{x}_s^T(0)) = X_{icb}$

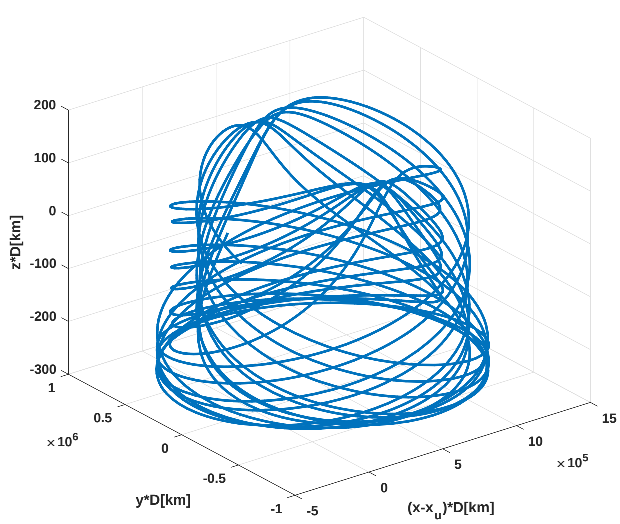

Fig. 8. Continued