A parametric study on the contained burning of solid rocket motor propellant

Jennifer Erin Politano

University of Nevada, Las Vegas

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A PARAMETRIC STUDY ON THE CONTAINED BURNING
OF SOLID ROCKET MOTOR PROPELLANT

by

Jennifer Erin Politano

Bachelor of Science
University of Nevada, Las Vegas
1994

A thesis submitted in partial fulfillment
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Dean of the Graduate College
ABSTRACT

A Parametric Study on the Contained Burning of Solid Rocket Motor Propellant

by

Jennifer Erin Politano

Dr. Robert F. Boehm, Examination Committee Chair
Professor of Mechanical Engineering
University of Nevada, Las Vegas

Outmoded solid rocket motor propellant is being destroyed by burning as part of the worldwide demilitarization process. Many of these burns have been performed in spaces where the combustion products are contained within a sealed chamber. There is currently no accepted method to predict the gas temperature and pressure that results from this process. The primary focus of this study was the prediction of the peak temperature and pressure of the gas inside an underground containment chamber as a function of the propellant consumed and the prediction of the wall surface temperature as a function of time.

A thermodynamic lumped approximation analysis was combined with the application of a semi-infinite heat conduction solution and Duhamel's integral. The result was an estimation of the peak temperature and pressure of the gas, as well as the surface temperature of the chamber wall. Parametric studies were performed using the model developed to help understand influences of various physical parameters.
Additionally, comparisons were made to data taken from three different experiments conducted at the X-Tunnel complex at the Nevada Test Site.
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CHAPTER I

INTRODUCTION

A contained burn process can be a safe, efficient, cost-effective, and environmentally and legally compliant method for the disposal of the country's growing stockpile of solid rocket motors. Containing the energy and the gases released in a controlled burn process has, therefore, become an important engineering problem.

This analytical and numerical study was an effort to explore the pressure and temperature effects that result from the contained burn of solid rocket motor propellant. The primary focus was the prediction of the peak temperature and pressure of the gas inside an underground containment chamber as a function of the propellant consumed and the prediction of the wall surface temperature as a function of time. Parametric studies were performed using the model developed to help understand the influences of the convection heat transfer coefficient and the thermal conductivity, density, and specific heat of the containment chamber walls.

Comparisons were made to three sets of experimental data obtained from a 1997 test series that took place at the X-Tunnel Complex at the Nevada Test Site. The key variables in each of the three cases were the type of propellant and the amount consumed.
Containment Chamber Model

The model developed for this analysis considers the gas to be ideal and at a uniform temperature and pressure at any time. The products of combustion are considered to be well-mixed with the existing gas in the chamber. The primary mode of heat transfer to the chamber walls is assumed to be convection. Conduction to the chamber floor is considered to be negligible. Radiation to the chamber walls is also considered to be negligible due to the opacity of the gases following combustion.

Physical Characteristics of the Containment Chamber

In this analysis, the most important features of the chamber geometry are the total volume and the total surface area of the chamber (walls and floor). The walls consist of a 6-8” layer of sprayed concrete over alluvium and the floor is a poured concrete pad, see Figure 1. The entrance to the chamber is a steel “containment barrier” with a negligible surface area compared to that of the overall chamber.
There are two important results of this study: (1) a generalized method for the prediction of peak gas temperature and pressure as a function of the propellant, and (2) prediction of the wall surface temperature variations with time. The procedure developed can now be used to aid in the engineering design of containment chambers and for estimation of the times to return to near-ambient conditions. Variations in the physical attributes of the containment chamber can be accounted for, as well.
CHAPTER II

MATHEMATICAL FORMULATION

The mathematical formulation was developed in three major sections: Prediction of the Peak Gas Temperature, Prediction of the Peak Gas Pressure, and Prediction of the Heat Transfer to the Chamber Walls. The first two sections of the analysis contain the thermodynamic development of estimates for the peak gas temperature and pressure. This gives valuable information that can be used to aid in the engineering design of containment chambers. The third section of the analysis focuses on a method of predicting the heat transfer from the hot combustion gases to the containment chamber walls as a function of time. With this information, the duration of transient conditions can be estimated.

Prediction of the Peak Gas Temperature

The thermodynamic lumped approximation analysis is based upon a uniform-state, uniform-flow model. The principles of conservation of mass and conservation of energy were applied. The following assumptions were made: (1) the containment chamber is the control volume, (2) the combustion of the propellant adds mass to the control volume, (3) the containment chamber walls are adiabatic, (4) kinetic and potential energy effects are negligible, (5) the initial and final states of the gas within the containment chamber are
equilibrium states, and (6) the gas is assumed to be air as an ideal gas with variable specific heat, see Figure 2.

![Control Volume](image)

Figure 2. Control volume for the thermodynamic analysis.

A mass rate balance on the control volume takes the form:

\[ \Delta m_p = m_p \quad \text{or} \quad m_2 = m_1 + m_p \quad (1) \]

where, \( m_p \) is the known mass of the propellant combusted, \( m_1 \) is the mass of the gas initially in the chamber, and \( m_2 \) is the total mass of the gas in the chamber after the burn process is complete. In equation (2), the mass \( m_1 \) is calculated as the density of air (at ambient temperature) multiplied by the total volume of the chamber, as calculated in Appendix II.

\[ m_1 = \rho \times V = (1.2 \frac{kg}{m^3}) \times (4644 m^3) \quad (2) \]

An energy rate balance on the control volume is:

\[ \Delta E_{cv} = Q_{cv} - W_{cv} + m_p \left( h_p + KE + PE \right) \quad (3) \]

Based on the assumptions listed above, equations (1) and (3) reduce to the following form of the 1\textsuperscript{st} Law:
\[ Q_{\text{gen}} + m_p h_p = m_2 u_2 - m_1 u_1 \] (4)

\( Q_{\text{gen}} \) is the total energy generated inside the containment chamber as a result of the propellant consumed and was calculated based upon the known heat of combustion of the propellant, \( H \).

\[ Q_{\text{gen}} = H \left( \frac{\text{cal}}{gm} \right) \times \left( \frac{1000 \text{cal}}{gm} \right) \times \left( \frac{4.1868 J}{\text{cal}} \right) \times m_p (\text{kg}) \] (5)

The enthalpy of the combustion gases, \( h_p \), is based upon the known flame temperature of the propellant. The internal energy, \( u_1 \), is based upon the ambient gas temperature in the chamber. Both of the above values were extracted from a table of the properties of air at low pressures [Irvine and Hartnett, 1976].

The internal energy, \( u_2 \) was solved for in equation (4) and the corresponding temperature \( T_{\text{max}} \) was extracted from a table of the properties of air at low pressures [Irvine and Hartnett, 1976]. Thus, symbolically,

\[ T_{\text{max}} = f(u_2) \] (6)

\( T_{\text{max}} \) is assumed to be the peak temperature of the gas in the chamber after the burn process is complete. This is regarded as a time-dependent but spatially averaged value.

Prediction of the Peak Gas Pressure

The ideal gas law was used to calculate the peak pressure, \( P_2 \), in the chamber after the completion of the burn process. The mass of the existing gas in the chamber is assumed to be well-mixed with the mass of the propellant (combustion gases), as
calculated in equations (1) and (2). For simplicity, the “mixture” is assumed to be air as an ideal gas with the constant $R = 287 \frac{J}{kgK}$.

$$P_2 = \frac{m_2RT_{\text{max}}}{V}$$  \hfill (7)

$P_2$ is the peak pressure inside the containment chamber corresponding to the peak temperature, $T_{\text{max}}$ from equation (6). $V$ is the volume of the containment chamber, as calculated in Appendix II.

**Prediction of Heat Transfer to the Chamber Walls**

The heat transfer between the hot combustion gases and the surface of the chamber walls is assumed to occur only by convection. Convection is believed to be fully established at the completion of the burn process at which time the peak temperature and pressure of the gas have been reached. For this analysis, time ($t = 0$) begins at the completion of the burn process, which was experimentally reported to last approximately 20 seconds [Stubbs and Heinle, 1997 a, b, and c]. The peak temperature $T_{\text{max}}$, from equation (6), is substituted into equation (8) as the initial value for $T_a(t)$.

$$\dot{Q}(t) = hA(T_a(t) - T_w(t))$$  \hfill (8)

In equation (8), the convection heat transfer coefficient, $h$, is varied from 15 – 24 $\frac{W}{m^2K}$. These values fall in the mid to upper range of typical values for free convection of gases [Incropera and DeWitt, 1996]. Further discussion of the selection of the convection heat transfer coefficient is provided in Chapter III. $A$ is the surface area of the chamber walls, as calculated in Appendix II. The wall surface temperature $T_w(t)$ is
initially taken as an ambient temperature of 300 K. $\dot{Q}(t)$ is solved for in equation (8) as the maximum amount of heat that could theoretically be transferred to the chamber walls by convection.

Next, an energy balance on the tunnel gas produces:

$$-\dot{Q}(t) = (m_c) \frac{du_{gas}}{dt} \quad \text{where, } du_{gas} = u_2(T_{a2}) - u_2(T_{a1})$$

(9)

The heat generated, $\dot{Q}(t)$ from equation (8) is found for each time step and substituted into equation (9) to find the unknown $u_2(T_{a2})$. A curve fit of data extracted from a table of the properties of air at low pressures [Irvine and Hartnett, 1976] produces a new gas temperature $T_g(t)$ at each time step ($u_2(T_{a2})$ is the new value and $u_2(T_{a1})$ is the previous value).

Finally, the wall surface temperature $T_{wall}(t)$ is estimated from a semi-infinite heat conduction solution and Duhamel's integral [Carslaw and Jaeger, 1959]:

$$T_{wall}(t) - T_{wall}(t = 0) = \frac{\alpha^{1/2}}{k_{wall} \pi^{1/2}} \int_0^t \frac{\dot{Q}(t - \tau)}{A} d\tau$$

(10)

Equation (10) assumes that the containment chamber is a semi-infinite solid and that the heat flux at $x = 0$ is a prescribed function of time, see Figure 3. While this equation assumes a boundary condition of the second kind (specified heat flux), this problem actually involves a boundary condition of the third kind (convective heat transfer). A boundary condition of the second kind is, however, valid because of the numerical approach used for the solution.
In equation (10) the thermal conductivity was initially chosen as $k = 0.81 \frac{W}{mK}$ which is a value for concrete extracted from a table of the thermo-physical properties of non-metals [Kakac and Yener, 1993]. The thermal diffusivity, $\alpha$, was solved for as:

$$\alpha = \frac{k}{\rho c_p} = 4.383 \times 10^{-7} \left(\frac{m^2}{sec}\right)$$

Equations (8), (9), and (10) were solved simultaneously in an algorithm written in “Visual Basic”, see appendix III. The value $\dot{Q}(t - \tau)$ in equation (10) was taken outside...
the integral and held constant for each time step. This allowed for equation (10) to be solved as a definite integral as shown in equation (12).

\[ \int_{0}^{\sqrt{2}} \tau \, d\tau = 2\sqrt{b} - 2\sqrt{a} \]  

Equation (10) can then be rewritten as equation (13) with the constant \( c = \frac{\alpha^{1/2}}{A k_{\text{wall}} \pi^{1/2}} \).

\[ T_{\text{wall}}(t) - T_{\text{wall}}(t = 0) = c \dot{Q}(A \tau) \int_{0}^{t} \tau^{-1/2} \, d\tau \]  

Equation (13) was solved in discrete time intervals of \( \Delta \tau \). The first three iterations are shown in equations (14), (15), and (16).

At the first time step, \( t = \Delta \tau \) and the right hand side of equation (13) reduces to:

\[ \dot{Q}(0) c \times (2\sqrt{\Delta \tau} - 2\sqrt{0}) \]  

At the second time step, \( t = 2\Delta \tau \) and the right hand side of equation (13) becomes:

\[ \dot{Q}(0) c \times (2\sqrt{2\Delta \tau} - 2\sqrt{\Delta \tau}) + \dot{Q}(\Delta \tau) c \times (2\sqrt{\Delta \tau} - 2\sqrt{0}) \]  

At the third time step, \( t = 3\Delta \tau \) and the right hand side of equation (13) becomes:

\[ \dot{Q}(0) c \times (2\sqrt{3\Delta \tau} - 2\sqrt{2\Delta \tau}) + \dot{Q}(\Delta \tau) c \times (2\sqrt{2\Delta \tau} - 2\sqrt{\Delta \tau}) + \dot{Q}(2\Delta \tau) c \times (2\sqrt{\Delta \tau} - 2\sqrt{0}) \]  

\( \dot{Q}(0) \) is from equation (8) at time \( t = 0 \), \( \dot{Q}(\Delta \tau) \) is from equation (8) evaluated at the first time step \( t = \Delta \tau \), \( \dot{Q}(2\Delta \tau) \) is from equation (8) evaluated at the second time step \( t = 2\Delta \tau \), etc. As the solution moves out in time, the right hand side of equation (13) approaches zero. The left hand side of equation (13), the wall surface temperature, starts at ambient, rises to a peak, and then gradually falls back toward ambient.
Because an explicit-finite difference method was used, the selection of the time increment $\Delta \tau$ was important to the stability of the solution. In an explicit method of solution, unknown nodal temperatures for the new time are determined exclusively by the known nodal temperatures at the previous time. The initial temperature was known at time ($t = 0$) from the prescribed initial conditions and the calculations began at $t = \Delta \tau$.

When the solution was carried out for long time periods and large $\Delta \tau$ increases in the convection coefficient, for instance, began to produce oscillations in the solution. These oscillations are numerically induced and physically impossible. In order to improve the accuracy of the finite-difference solution and eliminate the oscillations, the time increments were selected with a consideration for the time period of interest. That is to say, small time increments were chosen for small time periods of interest and somewhat larger time increments were chosen for longer time periods of interest.
CHAPTER III

ANALYSIS AND RESULTS

Three sets of experimental data were considered for comparison to the results of the mathematical model that was developed. All three data sets were obtained from an underground test series conducted at the X-Tunnel Complex at the Nevada Test Site in May and June of 1997. The purpose of these tests was to "evaluate, demonstrate, and validate new demilitarization technologies for conventional munitions and rocket motors" [Boehm and Chen, 1997]. Case I data was from "Sunspot" in which two NIKE rocket motors were burned [Stubbs and Heinle, 1997 a]. Case II data was from "Thunderbird" in which 4 NIKE rocket motors were burned [Stubbs and Heinle, 1997 b]. Case III data was from "Dazzler" in which two HAWK rocket motors were burned [Stubbs and Heinle, 1997 c]. In each case, the key variables were the type and amount of solid rocket motor propellant consumed. The NIKE rocket motors contain double base propellant and the HAWK rocket motors contain composite propellant. These are representative of the two most common propellant formulations found in the demilitarization inventory. Table 1 is a summary of the propellant characteristics that were important to this study. The last column, "Total Energy Generated" is as calculated in equation (5).
Table 1. Propellant Characteristics

<table>
<thead>
<tr>
<th>Case</th>
<th>Propellant (kg)</th>
<th>Heat of Combustion (cal/gm)</th>
<th>Total Energy Generated (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>680</td>
<td>764</td>
<td>$2.182 \times 10^9$</td>
</tr>
<tr>
<td>II</td>
<td>1360</td>
<td>764</td>
<td>$4.364 \times 10^9$</td>
</tr>
<tr>
<td>III</td>
<td>544</td>
<td>600</td>
<td>$1.371 \times 10^9$</td>
</tr>
</tbody>
</table>

In the three tests discussed here, "linear shaped charges and explosive cutting tape were used to split the steel rocket motor cases longitudinally and partially around the circumference. These explosives also served to initiate the low-pressure (i.e., non-propulsive) propellant burns" [Department of Defense “Executive Summary Report”, 1998].

Lawrence Livermore National Laboratory (LLNL) and Bechtel Nevada “were responsible for measuring and recording a variety of physical parameters associated with each rocket motor burn test” [Department of Defense “Executive Summary Report”, 1998]. “The data they obtained for each test included temperature measurements from an array of K-type thermocouples located in the test chamber and the pressure history of the test chamber volume” [Department of Defense “Executive Summary Report”, 1998]. These data were recorded prior to test initiation in order to establish a baseline and continued for approximately two hours after the initiation. The gas temperature was measured as a function of height in the test chamber by seven or eight thermocouples in each test. In “Sunspot” the thermocouples were located between the invert (floor) and the back (top) of the test chamber. In “Thunderbird” and “Dazzler” the thermocouples were located between the invert and the mid-height of the test chamber.
Comparison of Peak Chamber Temperatures and Pressures

Table 2 is a summary of the results of the calculated and measured temperature data for all three cases. Figure 4 shows the experimental results plotted against the results of the mathematical model. A best fit line is shown for reference. The Case III calculated temperature is about 10.5% higher than the measured. This was the closest approximation of all three cases. The Case I temperature was calculated to be about 35% lower than the measured and the Case II temperature was calculated to be about 60% higher than the measured. Since the amount of propellant in Case II is double the amount in Case I, intuitively, the peak chamber temperature should be much greater than was measured.

Table 2. Summary of Peak Temperatures (Kelvin)

<table>
<thead>
<tr>
<th>Case</th>
<th>Calculated</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>1131</td>
<td>1523</td>
</tr>
<tr>
<td>Case II</td>
<td>1714</td>
<td>1073</td>
</tr>
<tr>
<td>Case III</td>
<td>920</td>
<td>833</td>
</tr>
</tbody>
</table>

The large discrepancies between the measured and calculated data seen for Cases I and II could be attributed to the method by which the data was measured and recorded. The mathematical model was developed to predict a peak global average of the gas temperature for the entire chamber, whereas the experimental data was recorded by thermocouples in specific locations within the chamber.
Table 3 is a summary of the results of the calculated and measured pressure data for all three cases. Figure 5 shows the experimental results plotted against the results of the mathematical model. A best fit line is shown for reference. Fair correlations generally result between the experimental and calculated data, although the calculated pressures are higher than the measured pressures in all three cases. For example, the Case I calculated pressure is about 15% higher than the measured, the Case II calculated pressure is about 25% higher than the measured, and the Case III calculated pressure is about 20% higher than the measured. This could be attributed to some sizeable leaks around the containment barrier that were noted in the data analysis reports [Stubbs and Heinle, 1997 a, b, and c] and some degree of porosity in the chamber walls. Although an effort was made to tightly seal the chamber prior to each burn, it was not a perfectly closed system.
Table 3. Summary of Peak Pressures (psia)

<table>
<thead>
<tr>
<th>Case</th>
<th>Calculated</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>63</td>
<td>55</td>
</tr>
<tr>
<td>Case II</td>
<td>107</td>
<td>85</td>
</tr>
<tr>
<td>Case III</td>
<td>50</td>
<td>42</td>
</tr>
</tbody>
</table>

Figure 5. Peak measured chamber pressure vs. peak calculated chamber pressure. Measured values are shown.

Figure 6 shows the peak calculated temperature plotted against the total energy generated by the propellant. As expected, the temperature increases with increased energy generation. Similarly, Figure 7 shows the peak calculated pressure plotted against the total energy generated by the propellant. Again, as expected, the pressure increases with increased energy generation.
Figure 6. Peak calculated temperature vs. total energy generated. Calculated data values are shown.

Figure 7. Peak calculated pressure vs. total energy generated. Calculated data values are shown.
In Figure 8, the gas temperature decay with time is shown for Case I. The solution to the mathematical model was carried out in time increments of $\Delta \tau = 20$ seconds for a time period of 1000 seconds. The first 300 seconds are shown to correspond to experimental data that was available. Note that time ($t = 0$) starts at the completion of the burn process when the peak gas temperature has been reached. The convection heat transfer coefficient was varied from $15 - 24 \frac{W}{m^2 K}$. Values of $h = 15$ and $h = 18$ appeared to be a fairly good match. A value of $h = 24$ showed a definite under prediction.

![Gas Temperature Data (Case I)](image)

Figure 8. Measured and calculated gas temperature data for Case I. The measured data was reproduced from [Department of Defense “Executive Summary Report”, 1998].

In Figure 9, the gas temperature decay with time is shown for Case II. The solution to the mathematical model was carried out in time increments of $\Delta \tau = 20$ seconds for a
time period of 1000 seconds. The first 300 seconds are shown to correspond to experimental data that was available. Note that time ($t = 0$) starts at the completion of the burn process when the peak gas temperature has been reached. As previously stated, the calculated temperatures for Case II suggest that higher temperatures should be expected for the measured values. Nonetheless, variations in the convection heat transfer coefficient from $15 - 24 \frac{W}{m^2 K}$ produce similar results to those seen in Figure 8.

![Gas Temperature Data (Case II)](image)

Figure 9. Measured and calculated gas temperature data for Case II. The measured data was reproduced from [Department of Defense “Executive Summary Report”, 1998].
In Figure 10, the gas temperature decay with time is shown for Case III. The solution to the mathematical model was carried out in time increments of $\Delta t = 20$ seconds for a time period of 1000 seconds. The first 300 seconds are shown to correspond to experimental data that was available. Note that time ($t = 0$) starts at the completion of the burn process when the peak gas temperature has been reached.

Variations in the convection heat transfer coefficient from $15 - 24 \frac{W}{m^2 K}$ again produce results similar to those seen in Figure 8. Values of $h = 15$ and $h = 18$ appeared to be a fairly good match. A value of $h = 24$ showed a definite under prediction.

![Gas Temperature Data](image)

Figure 10. Measured and calculated gas temperature data for Case III. The measured data was reproduced from [Department of Defense “Executive Summary Report”, 1998].

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In Figures 11, 12, and 13 the gas pressure decay with time is shown for Cases I, II, and III, respectively. The solution to the mathematical model in all three cases was carried out in time increments of $\Delta \tau = 20$ seconds for a time period of 1000 seconds. The first 500 seconds are shown to correspond with experimental data that was available. Note that time $(t = 0)$ begins at the completion of the burn process when the peak gas pressure has been reached.

![Gas Pressure Data (Case I)](image)

Figure 11. Measured and calculated gas pressure data for Case I. The measured data was reproduced from [Department of Defense “Executive Summary Report”, 1998].
Figure 12. Measured and calculated gas pressure data for Case II. The measured data was reproduced from [Department of Defense "Executive Summary Report", 1998].
Figure 13. Measured and calculated gas pressure data for Case III. The measured data was reproduced from [Department of Defense “Executive Summary Report”, 1998].

The calculated values are consistently higher than the measured values in all three cases. It appears that raising the convection heat transfer coefficient lowers the gas pressure, however, from the results of figures 8, 9, and 10 values of $h > 18$ show a pronounced under prediction in the gas temperature data.
Containment Chamber Wall Surface Temperatures

The figures and discussion that follow are the results of the mathematical formulation of the heat transfer from the hot combustion gases to the containment chamber walls. Figure 14 shows the results of the gas temperature $T_g$ and the wall surface temperature $T_w$, for the mathematical formulation for Case I. The solution to the model was carried out in time increments of $\Delta \tau = 20$ seconds for a time period of 1000 seconds. The first 500 seconds are shown. Although experimental data of the wall surface temperature was unavailable for comparison, variations in the convection heat transfer coefficient from $15 - 24 \frac{W}{m^2 K}$ are shown. The thermal conductivity, density, and specific heat are held constant at the values stated in Chapter II.

Figure 14 shows that the gas temperature falls off more quickly with the higher convection coefficient value but only a slight increase in the wall surface temperature is seen within the first 200 seconds. It is hypothesized that a convection coefficient of $h = 18$ is a probable choice based upon the comparisons of the gas temperatures and pressures previously presented.
Figure 14. Wall surface temperature decay with time for Case I. Effects of variations in the heat transfer coefficient are shown.
Figure 15 shows the heat transfer from the hot combustion gases to the containment chamber walls for Case I. The solution to the mathematical model was carried out in time increments of $\Delta t = 20$ seconds for a time period of 1000 seconds. The first 500 seconds are shown. Variations in the heat transfer coefficient from $15 - 24 \frac{W}{m^2K}$ show that $h = 18$ is an intermediate value and based upon the results previously presented, this remains a probable choice.

![Heat Transfer to Chamber Walls (Case I)](image)

Figure 15. Heat transfer from the hot combustion gases to the surface of the chamber walls for Case I. Effects of variations in the heat transfer coefficient are shown.
Figure 16 shows the results of the gas temperature $T_a$, and the wall surface temperature $T_w$, for the mathematical formulation for Case II. The solution to the model was carried out in time increments of $\Delta t = 20$ seconds for a time period of 1000 seconds. The first 500 seconds are shown. Variations in the convection heat transfer coefficient from $15 - 24 \frac{W}{m^2 K}$ are shown and a value of $h = 18$ again seems the most probable choice. Figure 16 shows that the gas temperature falls off more quickly with the higher convection values but little change is seen in the wall surface temperature. The slight change that does occur happens within the first 200 seconds.

Figure 16. Wall surface temperature decay with time for Case II. Effects of variations in the heat transfer coefficient are shown.
Figure 17 shows the heat transfer from the hot combustion gases to the containment chamber walls for Case II. The solution to the mathematical model was carried out in time increments of $\Delta t = 20$ seconds for a time period of 1000 seconds. The first 500 seconds are shown. Variations in the heat transfer coefficient from $15 - 24 \frac{W}{m^2 K}$ show that $h = 18$ is an intermediate value and based upon the results previously presented, this remains a probable choice.

![Heat Transfer to Chamber Walls (Case II)](image)

Figure 17. Heat transfer from the hot combustion gases to the surface of the chamber walls for Case II. Effects of variations in the heat transfer coefficient are shown.
Figure 18 shows the results of the gas temperature $T_g$, and the wall surface temperature $T_w$, for the mathematical formulation for Case III. The solution to the model was carried out in time increments of $\Delta \tau = 20$ seconds for a time period of 1000 seconds. The results seen in Figure 18 are again very similar to those seen in Figures 14 and 16. Variations in the convection heat transfer coefficient from $15 - 24 \frac{W}{m^2 K}$ are shown and a value of $h = 18$ seems the most probable choice. Figure 18 shows that the gas temperature falls off more quickly with the higher convection values but little change is seen in the wall surface temperature. The slight change that does occur happens within the first 200 seconds.

![Wall Surface Temperature (Case III)](image)

Figure 18. Wall surface temperature decay with time for Case III. Effects of variations in the heat transfer coefficient are shown.

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Figure 19 shows the heat transfer from the hot combustion gases to the containment chamber walls for Case III. As in Figures 15 and 17, the solution to the mathematical model was carried out in time increments of $\Delta t = 20$ seconds for a time period of 1000 seconds. The first 500 seconds are shown. Variations in the heat transfer coefficient from $15 - 24 \frac{W}{m^2 K}$ show that $h = 18$ is an intermediate value and based upon the results previously presented, this remains a probable choice.

![Heat Transfer to Chamber Walls (Case III)](image)

Figure 19. Heat transfer from the hot combustion gases to the surface of the chamber walls for Case III. Effects of variations in the heat transfer coefficient are shown.

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Effects of variations in the density, specific heat, and thermal conductivity of the chamber walls is shown in Figures 20, 21, and 22. The heat transfer coefficient is held constant at \( h = 18 \) in all three cases. The values shown for \( T_{a1} \) and \( T_{w1} \) were results of the mathematical model for \( \rho = 1470 \, \frac{kg}{m^3} \), \( c_p = 616 \, \frac{J}{kgK} \), and \( k = 0.57 \, \frac{W}{mK} \). These values are approximately 30% lower than the values initially chosen. The values shown for \( T_{a2} \) and \( T_{w2} \) were results of the mathematical model for \( \rho = 2100 \, \frac{kg}{m^3} \), \( c_p = 880 \, \frac{J}{kgK} \), and \( k = 0.81 \, \frac{W}{mK} \). These are the values initially chosen as stated in Chapter II. The values shown for \( T_{a3} \) and \( T_{w3} \) were results of the mathematical model for \( \rho = 2730 \, \frac{kg}{m^3} \), \( c_p = 1144 \, \frac{J}{kgK} \), and \( k = 1.05 \, \frac{W}{mK} \). These values are approximately 30% higher than the values initially chosen.

In all three cases that the solution to the mathematical model was carried out in time increments of \( \Delta \tau = 20 \) seconds for a time period of 1000 seconds. The first 500 seconds are shown. Experimental data of the wall surface temperatures was unavailable, though all three data sets seem plausible. Further experimental testing would facilitate verification of these results.
In Figure 20 it is seen that the $-30\%$ values give a wall surface temperature about 50 K higher than the initial values. The $+30\%$ values give a wall surface temperature about 20 K lower than the initial values.

![Wall Surface Temperature Graph](image)

**Figure 20.** Wall surface temperature decay with time for Case I. The heat transfer coefficient is held constant at 18 W/m$^2$K. Effects of variations in density, specific heat, and thermal conductivity (varied simultaneously +/- 30%) of the chamber walls is shown.
In Figure 21 it is seen that the −30% values give a wall surface temperature about 100 K higher than the initial values. The +30% values give a wall surface temperature about 40 K lower than the initial values. This is a result that might be expected based upon the fact that the amount of propellant consumed in Case II is double the amount of Case I.

Figure 21. Wall surface temperature decay with time for Case II. The heat transfer coefficient is held constant at 18 W/m²K. Effects of variations in density, specific heat, and thermal conductivity (varied simultaneously +/− 30%) of the chamber walls is shown.
In Figure 22 it is seen that the -30% values give a wall surface temperature about 50 K higher than the initial values. The +30% values give a wall surface temperature about 20 K lower than the initial values.

Figure 22. Wall surface temperature decay with time for Case III. The heat transfer coefficient is held constant at 18 W/m²K. Effects of variations in density, specific heat, and thermal conductivity (varied simultaneously +/- 30%) of the chamber walls is shown.

Figure 23 shows the wall surface temperature decay to near-ambient for Case I and Figure 24 shows the corresponding heat transfer decay with time. The results shown here are from the mathematical model carried out in time increments of Δτ = 120 seconds for a time period of 10,000 seconds. The first 5000 seconds are shown. It is seen that the wall surface temperature is approximately equal to the gas temperature after about 1000 seconds at which time the heat transfer to the chamber walls is almost zero. The chamber environment reaches near-ambient conditions, gas temperature and wall surface...
temperature of 320 K, after approximately 3000 seconds. After 5000 seconds, the gas temperature and wall surface temperature have dropped to about 315 K. After 10,000 seconds, the gas temperature and wall surface temperature are about 310 K.

Figure 23. Wall surface temperature decay to near-ambient for Case I.
Figure 24. Heat transfer decay with time for Case I.

Figure 25 shows the wall surface temperature decay to near-ambient for Case II and Figure 26 shows the corresponding heat transfer decay with time. The results shown here are from the mathematical model carried out in time increments of $\Delta \tau = 120$ seconds for a time period of 10,000 seconds. The first 5000 seconds are shown. It is seen that the wall surface temperature is approximately equal to the gas temperature after about 2000 seconds at which time the heat transfer to the chamber walls is almost zero. The chamber environment reaches near-ambient conditions, gas temperature and wall surface temperature of 330 K, after approximately 5000 seconds. After 10,000 seconds, the gas temperature and wall surface temperature have dropped to about 321 K.
Wall Surface Temperature
(Case II)

Figure 25. Wall surface temperature decay to near-ambient for Case II.
Figure 26. Heat transfer decay with time for Case II.

Figure 27 shows the wall surface temperature decay to near-ambient for Case III and Figure 28 shows the corresponding heat transfer decay with time. The results shown here are from the mathematical model carried out in time increments of $\Delta t = 120$ seconds for a time period of 10,000 seconds. The first 5000 seconds are shown. It is seen that the wall surface temperature is approximately equal to the gas temperature after about 1000 seconds at which time the heat transfer to the chamber walls is almost zero. The chamber environment reaches near-ambient conditions, gas temperature and wall surface temperature of 317 K, after approximately 2000 seconds. After 5000 seconds, the gas temperature and wall surface temperature have dropped to about 311 K. After 10,000 seconds, the gas temperature and wall surface temperature are about 308 K.
Figure 27. Wall surface temperature decay to near-ambient for Case III.
Figure 28. Heat transfer decay with time for Case III.

Figures 23 through 28 are solutions to the mathematical model with the convection heat transfer coefficient held constant at $h = 18$. The values of density, specific heat, and thermal conductivity are held constant at the initial values chosen, as stated in Chapter II.
CHAPTER IV

CONCLUSIONS

The primary focus of this study was the prediction of the peak temperature and pressure of the gas inside an underground containment chamber as a function of the propellant consumed and the prediction of the wall surface temperature as a function of time. A mathematical model has successfully been developed that gives a reasonable approximation of the gas temperatures and pressures and offers an estimate of the wall surface temperature variations with time.

Parametric studies that were performed with the model provided some insights as to the influences of the convection heat transfer coefficient and the thermal conductivity, density, and specific heat of the containment chamber walls.

Satisfactory results were obtained that will aid in the engineering design of containment chambers for future contained burn processes involving different types and amounts of solid rocket motor propellant. The properties of the propellant that are necessary to carry out the solution are the mass, heat of combustion, and the flame temperature. The physical attributes of the containment chamber such as the volume, surface area of the walls, and thermo-physical properties can be varied with relative ease.

Additionally, satisfactory results were obtained that provide an estimate of the duration of the transient conditions. The time for the temperatures and pressures to
return to near ambient is important information, which will aid personnel in planning of a
safe re-entry into the chamber.

Suggestions for further study of this problem include a more accurate representation
of the actual gaseous products of combustion and consideration of radiation as one of the
mechanisms of heat transfer to the chamber walls. Improvements may also be made to
the method of numerical solution. For example, use of an implicit method rather than an
explicit method. The stability of the explicit method employed here was influenced by
the selection of the time increment and time interval. An implicit method has the
advantage of unconditionally stability for all space and time intervals.
APPENDIX I

NOMENCLATURE

$A = $ Surface area of the containment chamber walls

$H = $ Heat of combustion of the propellant

$L = $ Length of the chamber

$P_2 = $ Peak pressure of the gas

$Q_{gen} = $ Total energy generated

$Q = $ Heat transfer to the chamber wall

$R = $ Specific gas constant

$S = $ Surface area of the chamber walls

$T_a = $ Temperature of the gas

$T_{max} = $ Peak temperature of the gas

$T_w = $ Temperature of the wall at the surface

$V = $ Volume of the containment chamber

$h = $ Convection heat transfer coefficient

$h_p = $ Enthalpy of the propellant products of combustion at the flame temperature

$k = $ Thermal conductivity

$m_1 = $ Mass of gas existing in the chamber

$m_2 = $ Mass of gas in the chamber plus mass of the propellant
\( m_p = \text{Mass of the propellant} \)

\( t = \text{Time} \)

\( u = \text{Ideal gas internal energy} \)

\( \alpha = \text{Thermal diffusivity} \)

\( \rho = \text{Density} \)

\( \tau = \text{Variable of integration} \)
The chamber is idealized as a half cylinder.

Total Surface Area (half cylinder + floor + ends):

\[ S = \frac{2\pi L}{2} + 2rL + 2\left(\frac{\pi r^2}{2}\right) = 943 + 601 + 305 = 1849 \, \text{m}^2 \]

Total Chamber Volume: \[ V = \frac{\pi r^2 h}{2} = 4644 \, \text{m}^3 \]
APPENDIX III

COMPUTER CODE

‘Definition of Variables’
‘SOne = Time’
‘Ee = Duration’
‘lOne = Time Increment’
‘data_count = Number of Data Points’
‘twall = Wall Temperature’
‘Ma = Mass of the Gas + Propellant’
‘Ta = Gas Temperature
‘U = Internal Energy’
‘X() = Variable Array’
‘Q = Energy Generated’

Public SOne, Ee, lOne, data_count, i, j As Integer
Public twall, Ma, Ta, U As Double
Public X() As Variant

Sub AddTerm(X1() As Variant, j, cc)
    If j = 0 Then
        twall = 300
        Exit Sub
    End If
    tsub = 0
    twall = 0
    tj = X1(j, 3)
    tjl = tj + lOne
    tj2 = tj1 + lOne
    Qj = X1(j, 2)
    If j > data_count + 1 Then Qj2 = X1(j + 1, 2) Else Qj2 = 0
    If j > 0 Then Qj1 = X1(j - 1, 2) Else Qj1 = Q0
    Q0 = X1(0, 2)
    SqO1 = (2 * Sqr(lOne) - 2 * Sqr(SOne))
    m = j

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For L = 1 To j
    Qm = X1(L - 1, 2)
    Sq1 = (2 * Sqr(X1(m, 3)) - 2 * Sqr(X1(m - 1, 3)))
    tsub = tsub + Qm * Sq1
    m = m - 1
Next L
twall = cc * tsub + 300
End Sub

'Definition of Constants'
'h = Convection Heat Transfer Coefficient'
'asv = Surface Area of the Chamber Walls'
'alpha = Thermal Diffusivity'
'k = Thermal Conductivity'

Sub Calculate()
    h = 18
    asv = 1849
    alpha = 0.0000004383
    k = 0.81
    c = (Sqr(alpha) / (asv * k * (Sqr(3.1459)))) '6x10E-7
    SOne = 0
    Ma = Val(Form1.Text5.Text)
    Ee = Val(Form1.Text2.Text)
    IOne = Val(Form1.Text3.Text)
    data_count = Fix((Ee - SOne) / IOne)
    t = SOne
    Tw0 = 300
    Tw = Tw0
    Ta = Val(Form1.Text1.Text)
    Ta0 = Ta
    U = Val(Form1.Text4.Text)
    Qw0 = h * asv * (Ta - Tw0)
    Qw = Qw0
ReDim X(0 To data_count, 0 To data_count) As Variant
For i = SOne To data_count Step 1
   U1 = (-Qw * IOne / Ma) + U
   T1al = (U1 / 279) ^ 0.874
   Ta = T1al
   X(i, 0) = Ta
   X(i, 3) = t
   X(i, 2) = Qw
   Call AddTerm(X(), i, c)
   Tw1 = twall
   Tw = Tw1
   X(i, 1) = Tw
   Qw = h * asv * (Ta - Tw)
   U = U1
   t = t + IOne
Next i
X(0, 2) = Qw0
X(0, 0) = Ta0
End Sub
REFERENCES


VITA

Graduate College
University of Nevada, Las Vegas

Jennifer Erin Politano

Home Address:
10669 Oak Apple Avenue
Las Vegas, Nevada 89134

Degrees:
Bachelor of Science, Mechanical Engineering, 1994
University of Nevada, Las Vegas

Thesis Title: A Parametric Study on the Contained Burning of Solid Rocket Motor Propellant

Thesis Examination Committee:
Chairperson, Dr. Robert F. Boehm, Ph.D.
Committee Member, Dr. William Graebel, Ph.D.
Committee Member, Dr. Darrell W. Pepper, Ph.D.
Graduate Faculty Representative, Dr. Gerald Frederick, Ph.D.