1-1-2002

A comparison of upper confidence limits for skewed distributions

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A COMPARISON OF UPPER CONFIDENCE LIMITS
FOR SKEWED DISTRIBUTIONS

by

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Bachelor of Science
Illinois Institute of Technology
1997

A thesis submitted in partial fulfillment
of the requirements for the

Master of Science Degree in Mathematical Sciences
Department of Mathematical Sciences
College of Sciences

Graduate College
University of Nevada, Las Vegas
December 2002

UMI Number: 1413499

ProQuest Information and Learning Company
300 North Zeeb Road
P.O. Box 1346
Ann Arbor, MI 48106-1346

UNLV

Thesis Approval

The Graduate College
University of Nevada, Las Vegas

November 8, 2002

The Thesis prepared by

Michael Gary Theemling

Entitled

A Comparison of Upper Confidence Limits for Skewed Distributions

is approved in partial fulfillment of the requirements for the degree of

Master of Science in Mathematics (Statistics Concentration)

Examination Committee Chair

Dean of the Graduate College

Examination Committee Member

Examination Committee Member

Graduate College Faculty Representative

PR/1017-53/1-00

ABSTRACT

A Comparison of Upper Confidence Limits for Skewed Distributions

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Remediation decisions at contaminated sites are determined by comparing the Upper Confidence Limit (UCL) of the mean contaminant to a site-specific action level. The Environmental Protection Agency (EPA) has established methods for computing this UCL, also called the Exposure Point Concentration (EPC) term.

The UCL formula depends upon the distribution of the contaminant concentrations, which are often positively skewed. The EPA frequently recommends the Lognormal distribution to model such data. However, that assumption often causes the UCL to be unreliable. What is the role of skewness in computing UCL statistics over different distributions? Computer simulation is used to test a range of skewness values over three similarly shaped distributions: Lognormal, Gamma, and Weibull. Three statistical tests will also be evaluated: Student's t-Statistic, Chebychev's Inequality, and Land's H-Statistic to compute the UCL. By modeling skewness effectively, we can determine the most appropriate distribution and statistical method to use in computing UCLs.

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CHAPTER 1

INTRODUCTION

Due to increasing concern over illegal or abandoned toxic waste sites in the United States, Congress in 1980 established a fund to locate, investigate, and clean up the worst sites nationwide known as the Superfund, which is managed by the Environmental Protection Agency (EPA, 2002).

In the Superfund applications of the US EPA, one major issue is the characterization of the site under investigation for each contaminant of concern (COC) to determine which parts of the site may require remediation. The site is typically divided into smaller areas, and if the UCL of the mean for a particular part of the site turns out to be higher than the level of concern for any one of the COCs, then that part of the site is declared contaminated and is tagged for cleanup. The distribution of the COC data collected from the various parts of the site determines which method of computing the UCL should be used.

However, at many of these sites, there is "spiked" data (a hot spot of contamination), which can often cause the data to be heavily skewed, and misleading results based upon a Lognormal distribution can occur. This can lead to unnecessary cleanup or not cleaning a contamination site at all.

In Environmental Statistics, the field of soil contamination is one that has a major problem of assured characterization. Partly because one is often limited by the number of samples and the "hot spots" which skew the data and thus makes statistical analyses difficult. Skewness is the measure of asymmetry of a distribution. The most common type of skewness, Fisher skewness, is defined as (Weisstein, 1999):

$$\gamma_1 = \frac{E[(X-\mu)]^3}{\sigma^2}$$

Where X is a random variable, μ is the mean, and σ^2 is the variance.

As mentioned, most positively skewed concentration data soil is modeled according to a Lognormal distribution. However, as shown in (Singh, 1997), the conclusions can be wrong if the variance, hence the skewness, is high for estimating the true mean. This can result in unnecessary cleanup. The EPA recognizes this problem (EPA, 2001), but offers only parametric remedies that don't address the problem directly.

Others (Iaci, 1997) have studied alternative distributions such as the Gamma or Weibull distributions for modeling contamination concentrations data. What are the effects of skewness on the determination of the UCL of the population mean?

In this paper several distributions of similar skewness have been considered to compute a UCL of the population mean. Here we will see if there are any patterns in how skewness affects the UCL statistics. We will evaluate the three different distributions that one may assume in Environmental Statistics: Lognormal, Gamma, and Weibull. In addition, we will not only look at skewness, but also see if the scale parameter (the one determining the relative height of the curve) or sample size has any significant impact on UCL statistics.

CHAPTER 2

MATHEMATICAL BACKGROUND

The Lognormal distribution is often the recommended assumption to model positively skewed data sets. Unfortunately, there are other distributions that yield better results based upon the different parameters chosen. The wrong distributional assumption can cause incorrect conclusions depending upon the test used and how skewed the data is. It has been shown (Singh, 1997) that even when data are moderately skewed, several of the UCL yield misleading results.

Probability Distributions

Perhaps the most important determination one makes when modeling the behavior of data distribution is what distribution to use. This determines vital statistics about the data such as the mean and variance. In this experiment, we will be concerned with three different distributions used in environmental statistics that have similar shapes. These are the Lognormal, Gamma, and Weibull distributions.

The Lognormal Distribution

A random variable X has a Lognormal distribution if the natural logarithm of X is Normally distributed with mean μ and variance σ^2 (i.e. $LN(X) \sim N(\mu, \sigma^2)$).

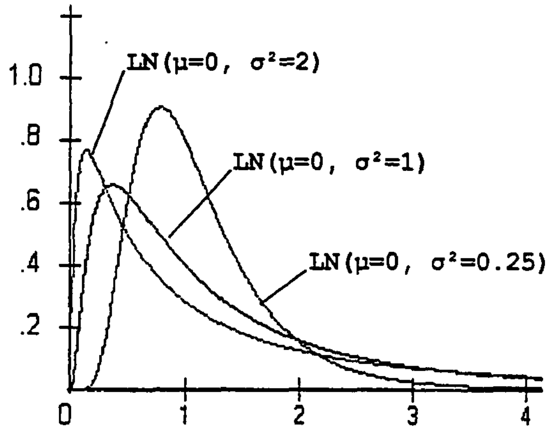


Figure 1 - Sample Lognormal PDF Graphs

The Lognormal distribution has the following relevant statistics:

$$\textbf{Mean} = \mu_1 = e^{(\mu+\frac{1}{2}\sigma^2)}$$

$$\textbf{Skewness} = \gamma_1 = \sqrt{e^{\sigma^2}-1}(2+e^{\sigma^2})$$

One important note about the mean of the Lognormal distribution is that it is not at all close towards the head of the curve as one might suspect, especially as the parameters μ and σ^2 increase, as shown in the following graph:

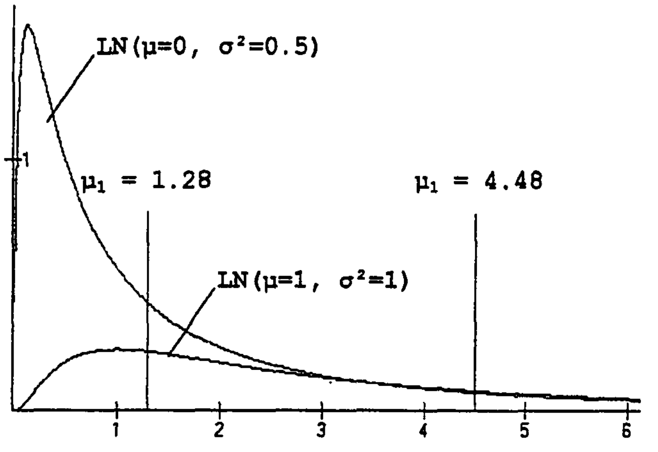


Figure 2 - Lognormal Graphs with Means

Because of this odd behavior, it is hard to find an accurate statistic to compute the UCL of the mean as seen later.

The Gamma Distribution

A random variable X has a Gamma distribution if its probability distribution function is given as follows:

$$\frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-x/\beta} \qquad 0 < x < \infty;\ \alpha,\ \beta > 0$$

Where Γ is the Gamma Function.

Given certain parameters, the shape and appearance of the Gamma distribution can mirror the Lognormal distribution.

The Gamma distribution has the following relevant statistics:

$$\textbf{Mean} = \mu_g = \alpha\beta$$

$$\textbf{Skewness} = \gamma_g = \frac{2}{\sqrt{\alpha}}$$

The Weibull Distribution

A random variable X has a Weibull distribution if its probability distribution function given is as follows:

$$\frac{\alpha}{\beta^{\alpha}} x^{\alpha-1} e^{-(x/\beta)^{\alpha}} \quad 0 < x < \infty;\ \alpha,\ \beta > 0$$

Like the Gamma distribution, it also can mirror the Lognormal.

The Weibull distribution has the following relevant statistics:

$$\textbf{Mean} = \mu_w = \beta\Gamma(1 + \alpha^{-1})$$

$$\textbf{Skewness} = \gamma_w = \frac{2\Gamma(1+\alpha^{-1})-3\Gamma(1+\alpha^{-1})\Gamma(1+2\alpha^{-1})+\Gamma(1+3\alpha^{-1})}{[\Gamma(1+2\alpha^{-1})-\Gamma^{2}(1+\alpha^{-1})]^{3/2}}$$

Statistical Tests

In addition to using the proper distribution, there are various statistical methods that can be used to compute the Confidence Interval (CI) for the mean. The three statistical methods we will use are the classical Student's t-Statistic, Chebychev's Inequality, and Land's H-Statistic. Each of these methods computes a CI based upon sample means and sample standard deviations. For each test, the UCL's are given as follows:

Student's t-Statistic

The Student's t-Statistic is used on a random sample that is assumed to be Normally distributed, but the variance is unknown. The test statistic for the UCL given a sample mean $\bar{x}$ of sample size n, and sample variance s_x is:

$$\bar{x} + t_{\alpha,n-1} s_x / \sqrt{n}$$

where t_α is the upper percentile of a t-distribution with confidence level α. Thus, for a 95% UCL, $\alpha = 0.05$.

Although we are not including any simulation of Normally distributed data, sometimes Normality for contamination data is assumed. Therefore we want to evaluate how the data perform under normality.

Chebychev Inequality

This test is based upon the well-known Chebychev Inequality (also known as Markov Inequality). Consider a positive random variable V with $0 < E(V) < \infty$, then:

$$P(V > r) \leq 1 / r, \quad r > 0$$

Now Let $I_A(v) = 1$ when $v \in A$, $0 \notin A$ be the indicator function of the set A. Then $rI_{(r,\infty)}(v) \leq V$. Taking the expected value of both sides, we set:

$$rP(V > r) \leq E(V)$$

Then taking $V = X - E(X)$ in the Chebychev Inequality we obtain:

$$P(-k\sigma_1 \leq (X - \mu_1) \leq k\sigma_1) \geq 1 - 1/k^2$$

(from Kallenberg, 1997). This would imply that if our $1 - 1/k^2$ value was 0.95 and if we solved for k it would be approximately 4.47. Then using the sample mean ($\bar{x}$ with size n) as our random variable, and replacing our known standard deviation σ_1 with the estimate, s_x, we would end up with a 95% UCL of mean by using the above inequality:

$$\bar{x}+4.47s_x/\sqrt{n}$$

It should be noted that because we are not using the true standard deviation, but rather an estimate, our result is no longer guaranteed to strictly follow the Chebychev Theorem. There has been criticism of this approach by some (Quantdec, 2001), but it is a misunderstanding. The paper interprets the test as an exact probability, whereas the test is to measure long-term coverage.

Land's H-Statistic

This Method is used specifically for Lognormally distributed data. It is relatively stable for mildly skewed data, but has complications with outliers or when the data is not Lognormal. However, we want to use it to see its impact on other distributions. The UCL statistic is given as:

$$e^{[\bar{y}+\frac{1}{2}s_y^2+s_yH_{\alpha-1}/\sqrt{n-1}]}$$

Where $\bar{y}$ is the sample mean of the log transformed data of size n, s_y is the standard deviation of the log transformed data, and H is Land's statistic (Land, 1975).

Unfortunately, it is a very sensitive test. First of all, the exact values for most of the s_y do not exist, so extrapolation is necessary. First, a single linear regression equation was developed based upon the chart. Then a piecewise regression equation was obtained simply based upon the 2 successive increments of s_y. For values between 0 and the s_y value, an estimated slope was calculated based upon the trends of the next few slope values. For example consider sample values at n = 10:

Table 1 - Example of H-Statistic Interpolation

s_y	**H-value**	**slope**	**y-intercept**	**slope difference**
0.1	1.802	?	?	N/A
0.2	1.881	0.79	1.723	?
0.3	1.977	0.96	1.689	0.17
0.4	2.089	1.12	1.641	0.16
0.5	2.22	1.31	1.565	0.19
0.6	2.368	1.48	1.48	0.17
0.7	2.532	1.64	1.384	0.16

To figure out the equation of the line between the point (s_y = 0.1, H-value = 1.881) and (s_y = 0.2, H-value = 1.997), we simply apply the elementary rise / run formula for the slope of the line, then pick a point to arrive at the y-intercept to get our equation in the form y = mx + b. For example, for s_y values between 0.1 and 0.2, our H-Statistic value would be $H_{0.95} = 0.79\ s_y + 1.723$.

A problem arises for finding the equation of the line with s_y values between 0 and 0.1. Simply using 0 for an H-value and applying the 2-point slope method will produce a huge slope many times larger than any of the others and inconsistent with the model. Based upon the above chart, the calculated slope would be 18.02, which produced wildly incorrect results in our initial tests. Therefore, this final extrapolation must be based upon an " educated guess" of the differences in slopes between the given s_y values near that point. Looking at the differences in slope values, an educated guess would place the slope around 0.16 (the value decided upon). Thus our slope value used would simply be 0.79 - 0.16 = 0.63. This gives us a reasonable value of the H-Statistic no matter what the s_y value is.

Another problem that arose is that with the exception of n = 10, the sample sizes used in the experiment are 1 less than the value in the chart (50 compared to an actual 51 in the chart). Despite this uncertainty, there is a fair amount of confidence the estimated H-value is nearly exact to its true value.

Finally, we are relying upon the accuracy of the table itself. As shown later in the graphs, even when the distribution is Lognormal, the average coverage is lower than the predicted 95% as sample size increases. This raises some suspicion about the H-values or precision of the numbers.

The full table for the values for the H-Statistic line equation is given below:

Table 2 - Interpolated H-Statistic Lines for 95% UCL

n=10				n=21			
s_y	**H-value**	**slope**	**y-int**	**s_y**	**H-value**	**slope**	**y-int**
0.1	1.802	0.63	1.739	0.1	1.722	0.37	1.685
0.2	1.881	0.79	1.723	0.2	1.771	0.49	1.673
0.3	1.977	0.96	1.689	0.3	1.833	0.62	1.647
0.4	2.089	1.12	1.641	0.4	1.905	0.72	1.617
0.5	2.22	1.31	1.565	0.5	1.989	0.84	1.569
0.6	2.368	1.48	1.48	0.6	2.085	0.96	1.509
0.7	2.532	1.64	1.384	0.7	2.191	1.06	1.449
0.8	2.71	1.78	1.286	0.8	2.307	1.16	1.379
0.9	2.902	1.92	1.174	0.9	2.432	1.25	1.307
1	3.103	2.01	1.093	1	2.564	1.32	1.244
1.25	3.639	2.144	0.959	1.25	2.923	1.436	1.128
1.5	4.207	2.272	0.799	1.5	3.311	1.552	0.983
1.75	4.795	2.352	0.679	1.75	3.719	1.632	0.863
2	5.396	2.404	0.588	2	4.141	1.688	0.765
2.5	6.621	2.45	0.496	2.5	5.013	1.744	0.653
3	7.864	2.486	0.406	3	5.907	1.788	0.543
3.5	9.118	2.508	0.34	3.5	6.815	1.816	0.459
4	10.38	2.524	0.284	4	7.731	1.832	0.403
4.5	11.64	2.52	0.3	4.5	8.652	1.842	0.363
5	12.91	2.54	0.21	5	9.579	1.854	0.309
6	15.45	2.54	0.21	6	11.44	1.861	0.274
7	18	2.55	0.15	7	13.31	1.87	0.22
8	20.55	2.55	0.15	8	15.18	1.87	0.22
9	23.1	2.55	0.15	9	17.05	1.87	0.22
10	25.66	2.56	0.06	10	18.93	1.88	0.13
n=31				**n=51**			
s_y	**H-value**	**slope**	**y-int**	**s_y**	**H-value**	**slope**	**y-int**
0.1	1.701	0.31	1.67	0.1	1.684	0.25	1.659
0.2	1.742	0.41	1.66	0.2	1.718	0.34	1.65
0.3	1.793	0.51	1.64	0.3	1.761	0.43	1.632
0.4	1.856	0.63	1.604	0.4	1.813	0.52	1.605
0.5	1.928	0.72	1.568	0.5	1.876	0.63	1.561
0.6	2.01	0.82	1.518	0.6	1.946	0.7	1.526
0.7	2.102	0.92	1.458	0.7	2.025	0.79	1.472
0.8	2.202	1	1.402	0.8	2.112	0.87	1.416
0.9	2.31	1.08	1.338	0.9	2.206	0.94	1.36
1	2.423	1.13	1.293	1	2.306	1	1.306
1.25	2.737	1.256	1.167	1.25	2.58	1.096	1.21
1.5	3.077	1.36	1.037	1.5	2.881	1.204	1.075
1.75	3.437	1.44	0.917	1.75	3.2	1.276	0.967
2	3.812	1.5	0.812	2	3.533	1.332	0.869

2.5	4.588	1.552	0.708	2.5	4.228	1.39	0.753
3	5.388	1.6	0.588	3	4.947	1.438	0.633
3.5	6.201	1.626	0.51	3.5	5.681	1.468	0.543
4	7.024	1.646	0.44	4	6.424	1.486	0.48
4.5	7.854	1.66	0.384	4.5	7.174	1.5	0.424
5	8.688	1.668	0.348	5	7.929	1.51	0.379
6	10.36	1.672	0.328	6	9.449	1.52	0.329
7	12.05	1.69	0.22	7	10.98	1.531	0.263
8	13.74	1.69	0.22	8	12.51	1.53	0.27
9	15.43	1.69	0.22	9	14.05	1.54	0.19
10	17.13	1.7	0.13	10	15.59	1.54	0.19

n=101			
s_y	**H-value**	**slope**	**y-int**
0.1	1.67	0.19	1.651
0.2	1.697	0.27	1.643
0.3	1.733	0.36	1.625
0.4	1.777	0.44	1.601
0.5	1.83	0.53	1.565
0.6	1.891	0.61	1.525
0.7	1.96	0.69	1.477
0.8	2.035	0.75	1.435
0.9	2.117	0.82	1.379
1	2.205	0.88	1.325
1.25	2.447	0.968	1.237
1.5	2.713	1.064	1.117
1.75	2.997	1.136	1.009
2	3.295	1.192	0.911
2.5	3.92	1.25	0.795
3	4.569	1.298	0.675
3.5	5.233	1.328	0.585
4	5.908	1.35	0.508
4.5	6.59	1.364	0.452
5	7.277	1.374	0.407
6	8.661	1.384	0.357
7	10.05	1.389	0.327
8	11.45	1.4	0.25
9	12.85	1.4	0.25
10	14.26	1.41	0.16

CHAPTER 3

TEST SETUP

The following Monte Carlo simulation will rigorously model the three distributions over varying parameters. Since the primary objective is to focus on skewness, we must choose parameters that generate the same or similar skewness for each distribution. We will choose skewness varying from 0.1 to 3 in increments of 0.1.

The following chart shows the values of skewness used to compare the parameter value using the equations for skewness to determine the parameters needed. Note that in most cases, the values are not exact, but still within minimum acceptable limits.

Table 3 - Skewness Values for Distributions

Gamma		Weibull		Lognormal	
alpha	skewness	alpha	skewness	var (for N)	skewness
400	0.100	3.22	0.101	0.001	0.095
100	0.200	2.9	0.202	0.004	0.190
44.4	0.300	2.64	0.299	0.01	0.302
25	0.400	2.41	0.400	0.018	0.407
16	0.500	2.22	0.498	0.027	0.501
11.1	0.600	2.05	0.598	0.038	0.598
8.2	0.698	1.9	0.701	0.051	0.698
6.2	0.803	1.77	0.804	0.066	0.801
4.9	0.904	1.66	0.902	0.082	0.902
4	1.000	1.56	1.004	0.099	1.001
3.3	1.101	1.48	1.096	0.117	1.101
2.8	1.195	1.4	1.198	0.136	1.201
2.4	1.291	1.33	1.299	0.156	1.302
2	1.414	1.27	1.395	0.176	1.400
1.8	1.491	1.21	1.502	0.197	1.501
1.6	1.581	1.16	1.601	0.218	1.601
1.4	1.690	1.11	1.711	0.239	1.699
1.2	1.826	1.07	1.807	0.261	1.801
1.1	1.907	1.03	1.913	0.282	1.898
1	2.000	1	2.000	0.304	2.000
0.91	2.097	0.97	2.094	0.326	2.102
0.82	2.209	0.94	2.195	0.347	2.199
0.76	2.294	0.91	2.306	0.369	2.302
0.69	2.408	0.89	2.385	0.39	2.401
0.64	2.500	0.86	2.514	0.411	2.501
0.59	2.604	0.84	2.608	0.432	2.602
0.55	2.697	0.82	2.707	0.452	2.700
0.51	2.801	0.8	2.815	0.472	2.798
0.47	2.917	0.79	2.871	0.492	2.898
0.44	3.015	0.77	2.991	0.512	3.000

The other parameter we will vary is the scale parameter, which determines how "tall" the curve becomes. For the Lognormal distribution this is the mean of the Normal distribution, μ. For the Gamma and Weibull distribution, this is β. The values will vary from 0.1 to 5 in increments of 0.1. We will see if the scale parameter has any significant impact on the UCL results.

Random samples of size n = 10, 20, 30, 50 and 100 are used to compute these statistics.

Each test will be run at 100, 1,000, and 10,000 trials per parameter and sample size. This will give us the long run coverage probabilities of the number of times the UCL is greater than the true mean. Over the varying parameters, we will produce 50 different UCL coverage probabilities for each skewness value.

The algorithm for each distribution will be as follows:

```
For each sample size n
   For each μ / β
      For each σ² / α
         Get true mean
         Do number of test run times
            Get random vector of size n
            Find each test statistic
            Compare test statistic with true mean
         End Do
         (Calculate num times UCL > true mean) / (number of
          test runs)
      End For
   End For
End For
```

Figure 3 - Computer Algorithm for the Experiment

The programming language used was Visual Basic 6.0. The code is in Appendix A. The algorithms used for generating random deviates for the distributions are in Appendix B.

CHAPTER 4

RESULTS

The following are sample descriptive statistics from results of 10,000 test runs. They are categorized first according to distribution, then test statistic, then sample size. The descriptive statistics included are the scale parameter, shape parameter, the true mean, the skewness, the sample mean, the sample standard deviation, the maximum value, the minimum value the median, the first quartile and the third quartile respectively.

Table 4 - Gamma Distribution Sample Descriptive Statistics

t-test										
n=10										
Beta	**Alpha**	**Truemean**	**Skewness**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	400.00	40.00	0.10	41.24	0.79	43.41	38.87	41.27	40.78	41.77
2.50	400.00	1000.00	0.10	1032.22	20.85	1110.02	969.60	1033.54	1018.18	1045.68
5.00	400.00	2000.00	0.10	2070.21	39.44	2199.43	1937.84	2071.76	2043.60	2094.88
0.10	1.80	0.18	1.49	0.27	0.07	0.59	0.10	0.26	0.22	0.31
2.50	1.80	4.50	1.49	6.53	1.78	14.27	2.58	6.41	5.19	7.59
5.00	1.80	9.00	1.49	12.82	3.68	28.38	5.09	12.28	9.98	15.17
0.10	0.44	0.04	3.02	0.09	0.04	0.29	0.01	0.08	0.06	0.11
2.50	0.44	1.10	3.02	2.21	1.06	6.47	0.23	2.04	1.42	2.83
5.00	0.44	2.20	3.02	4.30	2.27	20.45	0.21	3.77	2.64	5.49
n=20										
Beta	**Alpha**	**Truemean**	**Skewness**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	400.00	40.00	0.10	40.82	0.60	42.69	38.95	40.84	40.42	41.22
2.50	400.00	1000.00	0.10	1019.96	12.35	1063.48	975.53	1019.84	1011.48	1027.73
5.00	400.00	2000.00	0.10	2040.39	27.42	2130.05	1950.81	2040.51	2022.38	2058.44
0.10	1.80	0.18	1.49	0.24	0.05	0.42	0.11	0.24	0.21	0.27
2.50	1.80	4.50	1.49	5.95	1.20	10.06	2.75	5.90	5.11	6.68
5.00	1.80	9.00	1.49	12.07	2.13	20.20	6.74	12.01	10.51	13.48
0.10	0.44	0.04	3.02	0.07	0.03	0.22	0.01	0.07	0.05	0.09
2.50	0.44	1.10	3.02	1.89	0.67	4.37	0.35	1.82	1.42	2.31
5.00	0.44	2.20	3.02	3.58	1.30	8.83	0.74	3.35	2.64	4.48
n=30										
Beta	**Alpha**	**Truemean**	**Skewness**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	400.00	40.00	0.10	40.60	0.44	42.18	39.24	40.58	40.30	40.88
2.50	400.00	1000.00	0.10	1016.85	11.69	1053.16	983.53	1016.50	1008.56	1024.20
5.00	400.00	2000.00	0.10	2032.44	21.57	2102.44	1957.04	2031.40	2017.56	2046.59

0.10	1.80	0.18	1.49	0.23	0.04	0.35	0.13	0.23	0.20	0.25
2.50	1.80	4.50	1.49	5.73	0.88	8.91	3.13	5.70	5.10	6.32
5.00	1.80	9.00	1.49	11.13	1.59	16.95	7.05	11.16	10.05	12.19
0.10	0.44	0.04	3.02	0.07	0.02	0.16	0.02	0.07	0.05	0.08
2.50	0.44	1.10	3.02	1.80	0.48	3.79	0.56	1.78	1.46	2.08
5.00	0.44	2.20	3.02	3.40	0.98	7.27	1.05	3.36	2.68	4.10
n=50										
Beta	**Alpha**	**Truemean**	**Skewness**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	400.00	40.00	0.10	40.46	0.39	41.91	39.35	40.46	40.19	40.73
2.50	400.00	1000.00	0.10	1011.88	8.93	1047.59	981.94	1011.65	1006.01	1017.54
5.00	400.00	2000.00	0.10	2023.62	20.37	2098.33	1967.01	2023.02	2010.15	2036.90
0.10	1.80	0.18	1.49	0.22	0.03	0.33	0.12	0.21	0.19	0.24
2.50	1.80	4.50	1.49	5.39	0.78	8.06	3.41	5.37	4.80	5.91
5.00	1.80	9.00	1.49	10.64	1.63	16.38	6.70	10.55	9.49	11.72
0.10	0.44	0.04	3.02	0.07	0.02	0.14	0.03	0.06	0.05	0.08
2.50	0.44	1.10	3.02	1.58	0.46	3.90	0.57	1.53	1.24	1.88
5.00	0.44	2.20	3.02	3.30	0.84	6.60	1.24	3.29	2.71	3.86
n=100										
Beta	**Alpha**	**Truemean**	**Skewness**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	400.00	40.00	0.10	40.31	0.36	41.55	39.25	40.29	40.06	40.55
2.50	400.00	1000.00	0.10	1008.06	8.66	1038.00	982.35	1007.60	1001.92	1013.91
5.00	400.00	2000.00	0.10	2016.04	17.47	2082.50	1960.70	2015.03	2004.69	2027.03
0.10	1.80	0.18	1.49	0.21	0.03	0.30	0.13	0.21	0.19	0.22
2.50	1.80	4.50	1.49	5.12	0.71	7.29	3.42	5.10	4.63	5.58
5.00	1.80	9.00	1.49	10.09	1.38	14.44	5.98	10.08	9.18	10.97
0.10	0.44	0.04	3.02	0.06	0.01	0.12	0.02	0.06	0.05	0.07
2.50	0.44	1.10	3.02	1.44	0.37	3.03	0.57	1.42	1.17	1.69
5.00	0.44	2.20	3.02	2.90	0.71	5.55	1.21	2.86	2.38	3.36
Chebychev										

n=10										
Beta	**Alpha**	**Truemean**	**Skewness**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	400.00	40.00	0.10	42.59	1.04	45.49	39.52	42.62	41.96	43.29
2.50	400.00	1000.00	0.10	1065.18	27.37	1158.06	987.35	1066.82	1047.20	1083.59
5.00	400.00	2000.00	0.10	2139.04	52.37	2317.55	1973.29	2139.04	2105.87	2170.25
0.10	1.80	0.18	1.49	0.36	0.10	0.87	0.13	0.34	0.28	0.41
2.50	1.80	4.50	1.49	8.65	2.50	18.81	3.18	8.41	6.79	10.10
5.00	1.80	9.00	1.49	17.00	5.19	37.99	6.70	16.39	13.01	20.22
0.10	0.44	0.04	3.02	0.13	0.06	0.44	0.01	0.11	0.08	0.16
2.50	0.44	1.10	3.02	3.25	1.62	9.53	0.32	3.03	2.03	4.15
5.00	0.44	2.20	3.02	6.34	3.48	32.46	0.32	5.48	3.89	8.17
n=20										
Beta	**Alpha**	**Truemean**	**Skewness**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	400.00	40.00	0.10	41.87	0.71	44.13	39.53	41.90	41.39	42.35
2.50	400.00	1000.00	0.10	1045.32	14.95	1099.04	994.52	1044.66	1034.48	1055.18
5.00	400.00	2000.00	0.10	2090.84	31.99	2208.91	1988.83	2090.15	2068.55	2111.79
0.10	1.80	0.18	1.49	0.31	0.06	0.54	0.13	0.30	0.26	0.35
2.50	1.80	4.50	1.49	7.69	1.64	13.12	3.48	7.59	6.50	8.67
5.00	1.80	9.00	1.49	15.59	2.88	27.64	8.19	15.53	13.51	17.46
0.10	0.44	0.04	3.02	0.11	0.04	0.35	0.02	0.10	0.08	0.14
2.50	0.44	1.10	3.02	2.74	1.00	6.30	0.46	2.61	2.03	3.35
5.00	0.44	2.20	3.02	5.14	1.92	13.00	1.05	4.78	3.75	6.41
n=30										
Beta	**Alpha**	**Truemean**	**Skewness**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	400.00	40.00	0.10	41.47	0.49	43.21	39.96	41.44	41.13	41.79
2.50	400.00	1000.00	0.10	1038.88	13.11	1080.74	1000.64	1038.68	1029.42	1047.67
5.00	400.00	2000.00	0.10	2076.43	24.67	2151.20	1990.34	2075.15	2059.17	2093.26
0.10	1.80	0.18	1.49	0.28	0.05	0.46	0.15	0.28	0.25	0.32
2.50	1.80	4.50	1.49	7.18	1.16	12.20	3.85	7.18	6.31	7.90
5.00	1.80	9.00	1.49	14.00	2.09	21.69	8.61	14.05	12.61	15.38

0.10	0.44	0.04	3.02	0.10	0.03	0.21	0.03	0.10	0.08	0.12
2.50	0.44	1.10	3.02	2.54	0.70	5.42	0.77	2.51	2.05	2.93
5.00	0.44	2.20	3.02	4.81	1.43	10.45	1.50	4.73	3.75	5.80
n=50										
Beta	**Alpha**	**Truemean**	**Skewness**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	400.00	40.00	0.10	41.14	0.43	42.60	39.94	41.14	40.84	41.44
2.50	400.00	1000.00	0.10	1028.86	9.67	1067.05	999.35	1028.72	1022.37	1035.19
5.00	400.00	2000.00	0.10	2057.83	22.22	2135.96	1996.97	2057.44	2042.74	2072.99
0.10	1.80	0.18	1.49	0.26	0.04	0.39	0.15	0.26	0.23	0.29
2.50	1.80	4.50	1.49	6.51	0.98	9.95	4.16	6.48	5.79	7.16
5.00	1.80	9.00	1.49	12.85	2.06	19.98	8.14	12.75	11.39	14.20
0.10	0.44	0.04	3.02	0.09	0.02	0.20	0.04	0.09	0.07	0.10
2.50	0.44	1.10	3.02	2.14	0.64	5.76	0.78	2.08	1.67	2.55
5.00	0.44	2.20	3.02	4.47	1.18	10.08	1.62	4.45	3.65	5.23
n=100										
Beta	**Alpha**	**Truemean**	**Skewness**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	400.00	40.00	0.10	40.79	0.38	42.02	39.70	40.78	40.52	41.04
2.50	400.00	1000.00	0.10	1020.06	9.28	1052.06	993.67	1019.68	1013.27	1026.54
5.00	400.00	2000.00	0.10	2040.14	18.24	2105.60	1984.07	2039.09	2027.92	2052.14
0.10	1.80	0.18	1.49	0.24	0.03	0.34	0.15	0.24	0.22	0.26
2.50	1.80	4.50	1.49	5.91	0.84	8.85	3.87	5.89	5.33	6.44
5.00	1.80	9.00	1.49	11.65	1.63	16.54	6.83	11.64	10.62	12.66
0.10	0.44	0.04	3.02	0.08	0.02	0.15	0.03	0.08	0.06	0.09
2.50	0.44	1.10	3.02	1.83	0.48	4.25	0.72	1.81	1.48	2.15
5.00	0.44	2.20	3.02	3.69	0.94	7.38	1.54	3.65	3.02	4.30
H-Stat										
n=10										
Beta	**Alpha**	**Truemean**	**Skewness**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	400.00	40.00	0.10	41.02	1.02	43.14	38.76	41.06	40.58	41.53

2.50	400.00	1000.00	0.10	1026.72	1.02	1102.07	966.09	1027.96	1013.52	1040.09
5.00	400.00	2000.00	0.10	2058.53	1.02	2183.62	1931.58	2060.73	2032.63	2083.33
0.10	1.80	0.18	1.49	0.41	1.72	14.39	0.11	0.38	0.30	0.52
2.50	1.80	4.50	1.49	10.00	1.75	641.23	2.61	9.25	7.16	12.23
5.00	1.80	9.00	1.49	20.32	1.80	483.67	6.18	18.51	13.50	27.47
0.10	0.44	0.04	3.02	46.58	354.10	7.49E+24	0.02	9.09	1.00	287.98
2.50	0.44	1.10	3.02	978.55	469.13	3.50E+33	0.60	159.17	24.79	4386.68
5.00	0.44	2.20	3.02	2133.34	477.68	1.45E+25	1.05	362.14	50.29	8725.22
n=20										
Beta	**Alpha**	**Truemean**	**Skewness**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	400.00	40.00	0.10	40.67	1.01	42.49	38.85	40.70	40.28	41.07
2.50	400.00	1000.00	0.10	1016.48	1.01	1058.49	972.66	1016.49	1008.44	1024.18
5.00	400.00	2000.00	0.10	2033.38	1.01	2119.64	1945.25	2033.92	2015.82	2051.19
0.10	1.80	0.18	1.49	0.28	1.27	1.68	0.11	0.28	0.24	0.33
2.50	1.80	4.50	1.49	7.11	1.32	30.45	3.26	7.05	5.93	8.23
5.00	1.80	9.00	1.49	15.02	1.30	102.82	7.23	14.75	12.80	17.15
0.10	0.44	0.04	3.02	3.20	18.21	1.23E+11	0.03	1.66	0.46	10.21
2.50	0.44	1.10	3.02	97.26	19.06	8.88E+10	0.44	48.47	14.07	305.51
5.00	0.44	2.20	3.02	146.39	15.90	7.44E+08	0.83	66.48	20.83	626.01
n=30										
Beta	**Alpha**	**Truemean**	**Skewness**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	400.00	40.00	0.10	40.48	1.01	42.04	39.14	40.47	40.19	40.77
2.50	400.00	1000.00	0.10	1013.99	1.01	1049.91	981.06	1013.57	1005.73	1021.31
5.00	400.00	2000.00	0.10	2026.66	1.01	2097.46	1952.47	2025.82	2012.06	2040.66
0.10	1.80	0.18	1.49	0.27	1.23	0.97	0.13	0.27	0.23	0.30
2.50	1.80	4.50	1.49	6.68	1.24	17.01	3.38	6.58	5.79	7.56
5.00	1.80	9.00	1.49	13.40	1.23	35.64	7.22	13.14	11.70	14.89
0.10	0.44	0.04	3.02	1.69	7.26	9.59E+06	0.05	1.13	0.43	4.33
2.50	0.44	1.10	3.02	47.34	5.90	1.67E+06	1.69	32.97	13.70	111.18
5.00	0.44	2.20	3.02	84.00	7.57	6.26E+10	2.01	60.78	23.34	180.89

n=50										
Beta	Alpha	Truemean	Skewness	Mean	StdDev	Max	Min	Median	Q1	Q3
0.10	400.00	40.00	0.10	40.36	1.01	41.82	39.27	40.36	40.09	40.63
2.50	400.00	1000.00	0.10	1009.48	1.01	1044.86	979.41	1009.28	1003.68	1015.10
5.00	400.00	2000.00	0.10	2018.87	1.01	2092.55	1961.49	2018.15	2005.52	2031.92
0.10	1.80	0.18	1.49	0.24	1.22	0.77	0.12	0.24	0.21	0.28
2.50	1.80	4.50	1.49	6.11	1.20	15.47	3.43	6.03	5.38	6.81
5.00	1.80	9.00	1.49	11.95	1.21	27.28	6.82	11.73	10.50	13.36
0.10	0.44	0.04	3.02	0.92	3.93	1.28E+04	0.06	0.74	0.35	1.92
2.50	0.44	1.10	3.02	20.50	4.25	3.79E+06	0.85	16.42	7.08	45.18
5.00	0.44	2.20	3.02	48.03	4.37	1.79E+09	2.96	35.75	18.30	90.35
n=100										
Beta	Alpha	Truemean	Skewness	Mean	StdDev	Max	Min	Median	Q1	Q3
0.10	400.00	40.00	0.10	40.23	1.01	41.47	39.17	40.22	39.99	40.47
2.50	400.00	1000.00	0.10	1006.18	1.01	1035.74	980.31	1005.79	1000.20	1012.02
5.00	400.00	2000.00	0.10	2012.24	1.01	2079.00	1956.85	2011.46	2001.20	2023.29
0.10	1.80	0.18	1.49	0.23	1.17	0.47	0.13	0.22	0.20	0.25
2.50	1.80	4.50	1.49	5.60	1.19	16.61	3.62	5.54	4.98	6.17
5.00	1.80	9.00	1.49	11.14	1.19	21.40	6.15	11.03	10.00	12.25
0.10	0.44	0.04	3.02	0.55	3.44	3.34E+05	0.03	0.45	0.24	0.97
2.50	0.44	1.10	3.02	12.74	3.26	1.72E+06	1.01	10.79	5.71	22.37
5.00	0.44	2.20	3.02	26.59	3.45	1.59E+06	2.53	22.47	10.64	51.45

Table 5 - Lognormal Distribution Sample Descriptive Statistics

t-test										
n=10										
Mu	**Sigma**	**Truemean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	0.001	1.11	0.09	1.13	0.01	1.18	1.09	1.13	1.12	1.14
2.50	0.001	12.19	0.09	12.45	0.15	13.09	12.02	12.44	12.35	12.55
5.00	0.001	148.49	0.09	151.70	1.82	158.40	146.40	151.67	150.43	152.92
0.10	0.197	1.22	1.50	1.57	0.29	3.05	1.00	1.52	1.36	1.74
2.50	0.197	13.44	1.50	17.58	2.98	29.66	11.15	17.34	15.32	19.45
5.00	0.197	163.78	1.50	192.47	35.57	378.98	113.07	188.09	166.37	213.27
0.10	0.512	1.43	3.00	2.15	0.72	6.45	0.87	2.02	1.64	2.50
2.50	0.512	15.74	3.00	24.37	7.45	63.53	8.80	23.31	19.43	27.79
5.00	0.512	191.71	3.00	301.36	104.97	781.62	105.55	280.33	227.27	348.45
n=20										
Mu	**Sigma**	**Truemean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	0.001	1.11	0.09	1.12	0.01	1.15	1.10	1.12	1.12	1.13
2.50	0.001	12.19	0.09	12.37	0.11	12.71	12.08	12.37	12.30	12.45
5.00	0.001	148.49	0.09	150.70	1.21	155.29	147.32	150.66	149.86	151.47
0.10	0.197	1.22	1.50	1.45	0.17	2.30	1.01	1.45	1.34	1.56
2.50	0.197	13.44	1.50	16.19	2.09	26.97	11.36	16.20	14.71	17.49
5.00	0.197	163.78	1.50	176.83	20.21	256.97	126.64	175.10	162.43	188.96
0.10	0.512	1.43	3.00	1.92	0.46	3.75	0.95	1.84	1.60	2.16
2.50	0.512	15.74	3.00	21.29	5.25	50.96	11.01	20.60	17.55	23.78
5.00	0.512	191.71	3.00	258.0289	57.5983	562.135	134.26	249.07	215.96	294.19
n=30										
Mu	**Sigma**	**Truemean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	0.001	1.11	0.09	1.12	0.01	1.14	1.09	1.12	1.11	1.12
2.50	0.001	12.19	0.09	12.33	0.08	12.64	12.07	12.33	12.28	12.38
5.00	0.001	148.49	0.09	150.20	0.91	153.39	146.82	150.17	149.58	150.79

0.10	0.197	1.22	1.50	1.42	0.13	1.93	1.01	1.41	1.34	1.50
2.50	0.197	13.44	1.50	15.51	1.55	20.50	10.84	15.54	14.64	16.47
5.00	0.197	163.78	1.50	173.60	17.71	233.25	118.99	173.21	161.62	184.86
0.10	0.512	1.43	3.00	1.85	0.30	3.00	1.10	1.83	1.62	2.05
2.50	0.512	15.74	3.00	19.74	3.64	41.24	10.62	19.52	16.98	22.12
5.00	0.512	191.71	3.00	248.56	46.27	430.23	142.13	244.74	214.69	278.76
n=50										
Mu	**Sigma**	**Truemean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	0.001	1.11	0.09	1.12	0.01	1.14	1.09	1.12	1.11	1.12
2.50	0.001	12.19	0.09	12.29	0.07	12.53	12.07	12.29	12.24	12.34
5.00	0.001	148.49	0.09	149.77	0.82	152.76	146.93	149.76	149.23	150.30
0.10	0.197	1.22	1.50	1.35	0.14	1.87	0.97	1.36	1.28	1.44
2.50	0.197	13.44	1.50	15.21	1.34	21.97	11.38	15.10	14.36	15.94
5.00	0.197	163.78	1.50	167.18	17.03	243.47	118.39	167.67	158.64	177.10
0.10	0.512	1.43	3.00	1.76	0.26	3.47	1.14	1.74	1.57	1.91
2.50	0.512	15.74	3.00	19.50	3.00	34.98	12.09	19.12	17.48	21.15
5.00	0.512	191.71	3.00	230.71	35.74	402.73	144.88	226.71	205.50	250.65
n=100										
Mu	**Sigma**	**Truemean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	0.001	1.11	0.09	1.11	0.01	1.13	1.09	1.11	1.11	1.12
2.50	0.001	12.19	0.09	12.27	0.06	12.46	12.09	12.26	12.22	12.31
5.00	0.001	148.49	0.09	149.49	0.86	152.44	146.50	149.46	148.87	150.11
0.10	0.197	1.22	1.50	1.32	0.11	1.78	1.00	1.32	1.25	1.39
2.50	0.197	13.44	1.50	14.45	1.24	23.40	10.98	14.50	13.79	15.18
5.00	0.197	163.78	1.50	161.64	14.18	217.73	116.50	161.34	154.67	168.78
0.10	0.512	1.43	3.00	1.64	0.24	3.41	1.06	1.62	1.47	1.78
2.50	0.512	15.74	3.00	18.09	2.65	32.11	11.73	17.71	16.27	19.50
5.00	0.512	191.71	3.00	219.65	31.88	348.52	142.56	216.03	197.07	239.64
Chebychev										
n=10										

Mu	Sigma	Truemean	Skew	Mean	StdDev	Max	Min	Median	Q1	Q3
0.10	0.001	1.11	0.09	1.15	0.02	1.21	1.11	1.15	1.14	1.16
2.50	0.001	12.19	0.09	12.71	0.18	13.41	12.17	12.70	12.58	12.83
5.00	0.001	148.49	0.09	154.80	2.25	162.58	148.00	154.80	153.36	156.25
0.10	0.197	1.22	1.50	1.94	0.42	4.23	1.11	1.85	1.63	2.15
2.50	0.197	13.44	1.50	21.75	4.21	37.53	13.03	21.26	18.63	24.38
5.00	0.197	163.78	1.50	237.30	50.18	543.28	129.86	229.47	200.57	264.16
0.10	0.512	1.43	3.00	2.87	1.07	9.52	1.06	2.64	2.12	3.29
2.50	0.512	15.74	3.00	32.55	11.09	96.88	10.22	30.79	25.24	37.20
5.00	0.512	191.71	3.00	402.72	157.94	1192.00	131.58	370.54	294.60	465.82
n=20										
Mu	**Sigma**	**Truemean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	0.001	1.11	0.09	1.14	0.01	1.17	1.11	1.14	1.13	1.15
2.50	0.001	12.19	0.09	12.58	0.12	12.91	12.20	12.58	12.49	12.66
5.00	0.001	148.49	0.09	153.20	1.40	158.23	149.57	153.16	152.26	154.11
0.10	0.197	1.22	1.50	1.74	0.23	2.76	1.13	1.72	1.59	1.87
2.50	0.197	13.44	1.50	19.39	2.88	35.70	13.29	19.29	17.39	21.07
5.00	0.197	163.78	1.50	211.77	27.27	324.37	144.77	209.19	192.52	227.33
0.10	0.512	1.43	3.00	2.49	0.67	5.35	1.14	2.37	2.03	2.79
2.50	0.512	15.74	3.00	27.70	7.88	74.45	13.41	26.62	22.16	30.95
5.00	0.512	191.71	3.00	333.56	84.33	794.39	160.50	319.72	274.39	382.72
n=30										
Mu	**Sigma**	**Truemean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	0.001	1.11	0.09	1.13	0.01	1.16	1.11	1.13	1.13	1.14
2.50	0.001	12.19	0.09	12.50	0.08	12.83	12.20	12.50	12.44	12.56
5.00	0.001	148.49	0.09	152.24	0.98	155.64	148.44	152.21	151.59	152.88
0.10	0.197	1.22	1.50	1.66	0.17	2.33	1.15	1.65	1.55	1.77
2.50	0.197	13.44	1.50	18.23	2.09	24.56	12.29	18.25	17.03	19.49
5.00	0.197	163.78	1.50	203.92	23.53	284.29	134.60	203.00	187.96	218.34
0.10	0.512	1.43	3.00	2.35	0.43	4.12	1.37	2.31	2.02	2.61

2.50	0.512	15.74	3.00	24.91	5.11	59.91	12.76	24.60	21.02	28.12
5.00	0.512	191.71	3.00	314.39	65.67	638.79	171.22	308.30	267.53	352.49
n=50										
Mu	**Sigma**	**Truemean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	0.001	1.11	0.09	1.13	0.01	1.15	1.10	1.13	1.12	1.13
2.50	0.001	12.19	0.09	12.42	0.07	12.67	12.19	12.42	12.37	12.48
5.00	0.001	148.49	0.09	151.35	0.87	154.37	148.31	151.32	150.80	151.92
0.10	0.197	1.22	1.50	1.54	0.18	2.26	1.08	1.54	1.45	1.64
2.50	0.197	13.44	1.50	17.30	1.69	27.05	12.66	17.10	16.26	18.15
5.00	0.197	163.78	1.50	190.62	21.24	291.64	131.10	190.82	179.30	202.96
0.10	0.512	1.43	3.00	2.14	0.36	4.63	1.36	2.11	1.89	2.35
2.50	0.512	15.74	3.00	23.82	4.08	47.27	14.61	23.25	21.03	26.07
5.00	0.512	191.71	3.00	280.89	49.00	517.56	169.34	274.41	246.36	306.29
n=100										
Mu	**Sigma**	**Truemean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	0.001	1.11	0.09	1.12	0.01	1.14	1.10	1.12	1.12	1.13
2.50	0.001	12.19	0.09	12.36	0.06	12.57	12.18	12.35	12.31	12.40
5.00	0.001	148.49	0.09	150.61	0.89	153.56	147.39	150.59	149.99	151.25
0.10	0.197	1.22	1.50	1.45	0.13	2.02	1.08	1.45	1.37	1.53
2.50	0.197	13.44	1.50	15.91	1.47	28.56	11.87	15.94	15.14	16.72
5.00	0.197	163.78	1.50	178.01	16.99	248.79	125.72	177.16	169.44	186.14
0.10	0.512	1.43	3.00	1.91	0.31	4.55	1.20	1.87	1.70	2.08
2.50	0.512	15.74	3.00	21.02	3.40	41.91	13.13	20.52	18.66	22.85
5.00	0.512	191.71	3.00	255.28	40.57	444.73	162.09	250.44	225.98	280.09
H-Stat										
n=10										
Mu	**Sigma**	**Truemean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	0.001	1.11	0.09	1.13	0.01	1.17	1.09	1.13	1.12	1.13
2.50	0.001	12.19	0.09	12.40	0.14	13.04	11.98	12.40	12.30	12.50
5.00	0.001	148.49	0.09	151.15	1.77	157.82	145.97	151.11	149.92	152.31

0.10	0.197	1.22	1.50	1.66	0.35	3.88	0.98	1.59	1.42	1.83
2.50	0.197	13.44	1.50	18.70	3.74	34.13	11.03	18.23	16.09	20.83
5.00	0.197	163.78	1.50	203.55	40.94	431.85	114.57	197.38	174.13	224.79
0.10	0.512	1.43	3.00	2.81	1.37	17.30	0.90	2.51	1.94	3.30
2.50	0.512	15.74	3.00	31.90	14.14	207.53	8.94	28.85	22.92	37.95
5.00	0.512	191.71	3.00	391.67	195.49	1716.71	113.94	344.12	270.23	448.84
n=20										
Mu	**Sigma**	**Truemean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	0.001	1.11	0.09	1.12	0.01	1.14	1.09	1.12	1.11	1.12
2.50	0.001	12.19	0.09	12.34	0.11	12.69	12.06	12.34	12.27	12.42
5.00	0.001	148.49	0.09	150.35	1.19	154.87	146.98	150.30	149.51	151.11
0.10	0.197	1.22	1.50	1.48	0.18	2.36	1.02	1.47	1.35	1.59
2.50	0.197	13.44	1.50	16.34	2.04	26.74	11.54	16.29	14.96	17.52
5.00	0.197	163.78	1.50	179.76	21.01	257.64	127.75	178.31	164.98	191.97
0.10	0.512	1.43	3.00	2.08	0.51	4.66	0.96	1.99	1.72	2.36
2.50	0.512	15.74	3.00	22.78	5.36	44.98	11.06	22.18	18.99	25.38
5.00	0.512	191.71	3.00	281.33	66.06	623.18	137.97	270.75	234.21	321.76
n=30										
Mu	**Sigma**	**Truemean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	0.001	1.11	0.09	1.12	0.01	1.14	1.09	1.12	1.11	1.12
2.50	0.001	12.19	0.09	12.31	0.08	12.62	12.05	12.30	12.25	12.36
5.00	0.001	148.49	0.09	149.93	0.90	153.16	146.60	149.90	149.31	150.52
0.10	0.197	1.22	1.50	1.43	0.13	1.93	1.00	1.42	1.34	1.51
2.50	0.197	13.44	1.50	15.50	1.52	21.31	10.78	15.55	14.64	16.48
5.00	0.197	163.78	1.50	174.04	17.56	237.02	120.05	173.33	161.98	185.27
0.10	0.512	1.43	3.00	1.92	0.31	3.15	1.10	1.90	1.69	2.13
2.50	0.512	15.74	3.00	20.50	3.96	35.32	10.83	20.19	17.51	22.94
5.00	0.512	191.71	3.00	261.53	51.41	470.76	136.52	254.64	224.53	296.59
n=50										
Mu	**Sigma**	**Truemean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**

0.10	0.001	1.11	0.09	1.11	0.01	1.14	1.09	1.11	1.11	1.12
2.50	0.001	12.19	0.09	12.27	0.07	12.51	12.05	12.27	12.22	12.32
5.00	0.001	148.49	0.09	149.55	0.82	152.54	146.72	149.53	149.01	150.08
0.10	0.197	1.22	1.50	1.34	0.13	1.88	0.98	1.35	1.27	1.43
2.50	0.197	13.44	1.50	15.13	1.32	21.77	11.31	15.03	14.27	15.90
5.00	0.197	163.78	1.50	166.18	16.73	238.57	118.26	166.56	157.28	175.79
0.10	0.512	1.43	3.00	1.79	0.26	3.18	1.13	1.77	1.60	1.95
2.50	0.512	15.74	3.00	19.72	3.07	34.72	11.33	19.46	17.69	21.38
5.00	0.512	191.71	3.00	233.31	35.98	408.86	142.80	229.51	207.78	254.69
n=100										
Mu	**Sigma**	**Truemean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	0.001	1.11	0.09	1.11	0.01	1.13	1.09	1.11	1.11	1.12
2.50	0.001	12.19	0.09	12.25	0.06	12.45	12.08	12.25	12.21	12.29
5.00	0.001	148.49	0.09	149.32	0.86	152.27	146.37	149.29	148.70	149.94
0.10	0.197	1.22	1.50	1.31	0.10	1.72	1.00	1.31	1.24	1.37
2.50	0.197	13.44	1.50	14.34	1.21	20.51	10.96	14.41	13.66	15.06
5.00	0.197	163.78	1.50	160.26	13.76	214.09	116.88	160.14	153.10	167.36
0.10	0.512	1.43	3.00	1.63	0.23	2.65	1.02	1.61	1.47	1.78
2.50	0.512	15.74	3.00	18.03	2.55	28.51	11.34	17.72	16.28	19.43
5.00	0.512	191.71	3.00	218.62	31.09	353.90	144.54	214.87	197.03	238.40

Table 6 - Weibull Distribution Sample Descriptive Statistics

t-test										
n=10										
Beta	**Alpha**	**True Mean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	3.22	0.09	0.10	0.11	0.01	0.11	0.07	0.11	0.10	0.12
2.50	3.22	2.24	0.10	2.76	0.26	2.77	1.88	2.77	2.59	2.93
5.00	3.22	4.48	0.10	5.57	0.55	5.56	3.90	5.56	5.20	5.95
0.10	1.21	0.09	1.50	0.15	0.04	0.14	0.05	0.14	0.12	0.17
2.50	1.21	2.35	1.50	3.70	1.01	3.60	1.53	3.60	2.94	4.32
5.00	1.21	4.69	1.50	7.31	1.95	7.07	2.71	7.07	5.92	8.60
0.10	0.77	0.12	2.99	0.21	0.10	0.19	0.02	0.19	0.14	0.27
2.50	0.77	2.91	2.99	5.16	2.40	4.58	0.43	4.58	3.45	6.56
5.00	0.77	5.83	2.99	10.84	5.30	9.90	0.53	9.90	6.94	13.64
n=20										
Beta	**Alpha**	**True Mean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	3.22	0.09	0.10	0.10	0.01	0.10	0.08	0.10	0.10	0.11
2.50	3.22	2.24	0.10	2.60	0.19	2.62	1.90	2.62	2.48	2.73
5.00	3.22	4.48	0.10	5.19	0.36	5.20	3.84	5.20	4.94	5.45
0.10	1.21	0.09	1.50	0.13	0.03	0.13	0.07	0.13	0.11	0.15
2.50	1.21	2.35	1.50	3.24	0.57	3.23	1.74	3.23	2.85	3.60
5.00	1.21	4.69	1.50	6.37	1.24	6.30	3.23	6.30	5.44	7.21
0.10	0.77	0.12	2.99	0.18	0.06	0.17	0.05	0.17	0.13	0.22
2.50	0.77	2.91	2.99	4.41	1.45	4.21	1.29	4.21	3.30	5.30
5.00	0.77	5.83	2.99	8.50	2.79	8.04	2.29	8.04	6.40	10.06
n=30										
Beta	**Alpha**	**True Mean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	3.22	0.09	0.10	0.10	0.01	0.10	0.08	0.10	0.10	0.10
2.50	3.22	2.24	0.10	2.52	0.14	2.53	1.97	2.53	2.43	2.62
5.00	3.22	4.48	0.10	5.02	0.27	5.01	4.15	5.01	4.83	5.20

0.10	1.21	0.09	1.50	0.12	0.02	0.12	0.07	0.12	0.11	0.14
2.50	1.21	2.35	1.50	3.11	0.43	3.08	1.76	3.08	2.82	3.39
5.00	1.21	4.69	1.50	6.04	0.88	6.01	3.44	6.01	5.43	6.66
0.10	0.77	0.12	2.99	0.18	0.05	0.18	0.06	0.18	0.15	0.21
2.50	0.77	2.91	2.99	4.31	1.15	4.19	1.68	4.19	3.48	5.07
5.00	0.77	5.83	2.99	8.35	2.15	8.07	3.42	8.07	6.86	9.55
n=50										
Beta	**Alpha**	**True Mean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	3.22	0.09	0.10	0.10	0.01	0.10	0.08	0.10	0.09	0.10
2.50	3.22	2.24	0.10	2.45	0.11	2.45	2.08	2.45	2.38	2.53
5.00	3.22	4.48	0.10	4.90	0.23	4.89	4.18	4.89	4.73	5.05
0.10	1.21	0.09	1.50	0.12	0.02	0.12	0.07	0.12	0.11	0.13
2.50	1.21	2.35	1.50	2.93	0.41	2.93	1.76	2.93	2.65	3.20
5.00	1.21	4.69	1.50	5.72	0.73	5.71	3.67	5.71	5.20	6.21
0.10	0.77	0.12	2.99	0.16	0.04	0.15	0.08	0.15	0.13	0.18
2.50	0.77	2.91	2.99	3.96	0.97	3.79	1.78	3.79	3.26	4.56
5.00	0.77	5.83	2.99	7.74	1.65	7.52	3.79	7.52	6.60	8.72
n=100										
Beta	**Alpha**	**True Mean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	3.22	0.09	0.10	0.10	0.00	0.10	0.08	0.10	0.09	0.10
2.50	3.22	2.24	0.10	2.39	0.12	2.39	2.05	2.39	2.32	2.47
5.00	3.22	4.48	0.10	4.77	0.23	4.76	4.03	4.76	4.61	4.92
0.10	1.21	0.09	1.50	0.11	0.01	0.11	0.07	0.11	0.10	0.12
2.50	1.21	2.35	1.50	2.74	0.33	2.72	1.72	2.72	2.52	2.96
5.00	1.21	4.69	1.50	5.45	0.62	5.43	3.39	5.43	5.02	5.87
0.10	0.77	0.12	2.99	0.14	0.03	0.14	0.06	0.14	0.12	0.16
2.50	0.77	2.91	2.99	3.69	0.85	3.59	1.85	3.59	3.09	4.19
5.00	0.77	5.83	2.99	7.24	1.62	6.99	3.58	6.99	6.03	8.30
Chebychev										
n=10										

Beta	Alpha	True Mean	Skew	Mean	StdDev	Max	Min	Median	Q1	Q3
0.10	3.22	0.09	0.10	0.13	0.01	0.18	0.09	0.13	0.12	0.14
2.50	3.22	2.24	0.10	3.28	0.34	4.37	2.24	3.29	3.04	3.50
5.00	3.22	4.48	0.10	6.61	0.69	9.36	4.48	6.61	6.11	7.08
0.10	1.21	0.09	1.50	0.20	0.06	0.43	0.07	0.19	0.16	0.23
2.50	1.21	2.35	1.50	5.03	1.46	11.09	2.07	4.86	3.93	5.91
5.00	1.21	4.69	1.50	9.92	2.78	25.52	3.63	9.48	7.97	11.72
0.10	0.77	0.12	2.99	0.30	0.15	0.99	0.02	0.27	0.19	0.39
2.50	0.77	2.91	2.99	7.42	3.59	28.90	0.58	6.54	4.90	9.47
5.00	0.77	5.83	2.99	15.57	7.80	56.91	0.72	14.09	9.76	20.03
n=20										
Beta	**Alpha**	**True Mean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	3.22	0.09	0.10	0.12	0.01	0.15	0.09	0.12	0.11	0.12
2.50	3.22	2.24	0.10	3.00	0.21	3.68	2.20	3.01	2.86	3.15
5.00	3.22	4.48	0.10	5.97	0.42	7.33	4.46	5.98	5.67	6.27
0.10	1.21	0.09	1.50	0.17	0.03	0.30	0.09	0.17	0.15	0.19
2.50	1.21	2.35	1.50	4.29	0.78	7.96	2.26	4.26	3.74	4.77
5.00	1.21	4.69	1.50	8.37	1.68	15.17	4.14	8.28	7.10	9.50
0.10	0.77	0.12	2.99	0.26	0.09	0.65	0.07	0.24	0.19	0.31
2.50	0.77	2.91	2.99	6.26	2.16	18.13	1.71	5.95	4.60	7.57
5.00	0.77	5.83	2.99	12.01	4.20	35.20	2.98	11.19	8.87	14.37
n=30										
Beta	**Alpha**	**True Mean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	3.22	0.09	0.10	0.11	0.01	0.13	0.09	0.11	0.11	0.12
2.50	3.22	2.24	0.10	2.85	0.15	3.53	2.28	2.86	2.75	2.96
5.00	3.22	4.48	0.10	5.69	0.30	6.80	4.70	5.68	5.48	5.89
0.10	1.21	0.09	1.50	0.16	0.02	0.25	0.09	0.16	0.14	0.18
2.50	1.21	2.35	1.50	3.99	0.56	6.07	2.35	3.95	3.60	4.37
5.00	1.21	4.69	1.50	7.74	1.16	11.44	4.37	7.69	6.91	8.53
0.10	0.77	0.12	2.99	0.25	0.07	0.60	0.08	0.25	0.20	0.30

2.50	0.77	2.91	2.99	5.95	1.69	14.77	2.22	5.78	4.71	7.04
5.00	0.77	5.83	2.99	11.55	3.17	26.52	4.72	11.09	9.28	13.31
n=50										
Beta	**Alpha**	**True Mean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	3.22	0.09	0.10	0.11	0.01	0.13	0.09	0.11	0.10	0.11
2.50	3.22	2.24	0.10	2.71	0.12	3.13	2.29	2.71	2.63	2.79
5.00	3.22	4.48	0.10	5.40	0.24	6.28	4.67	5.39	5.23	5.56
0.10	1.21	0.09	1.50	0.14	0.02	0.22	0.09	0.14	0.13	0.16
2.50	1.21	2.35	1.50	3.60	0.51	5.77	2.26	3.60	3.25	3.95
5.00	1.21	4.69	1.50	7.05	0.93	10.68	4.65	7.00	6.40	7.64
0.10	0.77	0.12	2.99	0.21	0.05	0.50	0.10	0.20	0.17	0.24
2.50	0.77	2.91	2.99	5.25	1.35	11.68	2.32	5.00	4.25	6.08
5.00	0.77	5.83	2.99	10.26	2.30	22.27	4.94	9.94	8.64	11.63
n=100										
Beta	**Alpha**	**True Mean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	3.22	0.09	0.10	0.10	0.00	0.12	0.09	0.10	0.10	0.11
2.50	3.22	2.24	0.10	2.57	0.12	2.95	2.23	2.57	2.49	2.65
5.00	3.22	4.48	0.10	5.14	0.24	5.93	4.40	5.13	4.97	5.29
0.10	1.21	0.09	1.50	0.13	0.02	0.19	0.08	0.13	0.12	0.14
2.50	1.21	2.35	1.50	3.21	0.40	5.00	1.99	3.18	2.94	3.47
5.00	1.21	4.69	1.50	6.37	0.75	9.57	4.07	6.34	5.84	6.88
0.10	0.77	0.12	2.99	0.18	0.04	0.39	0.08	0.17	0.15	0.20
2.50	0.77	2.91	2.99	4.60	1.10	11.28	2.27	4.48	3.82	5.26
5.00	0.77	5.83	2.99	9.04	2.13	18.78	4.18	8.68	7.42	10.41
H-Stat										
n=10										
Beta	**Alpha**	**True Mean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	3.22	0.09	0.10	0.12	0.02	0.35	0.08	0.12	0.11	0.13
2.50	3.22	2.24	0.10	2.97	0.45	8.96	2.04	2.92	2.69	3.16
5.00	3.22	4.48	0.10	5.98	0.85	12.36	4.02	5.89	5.42	6.37

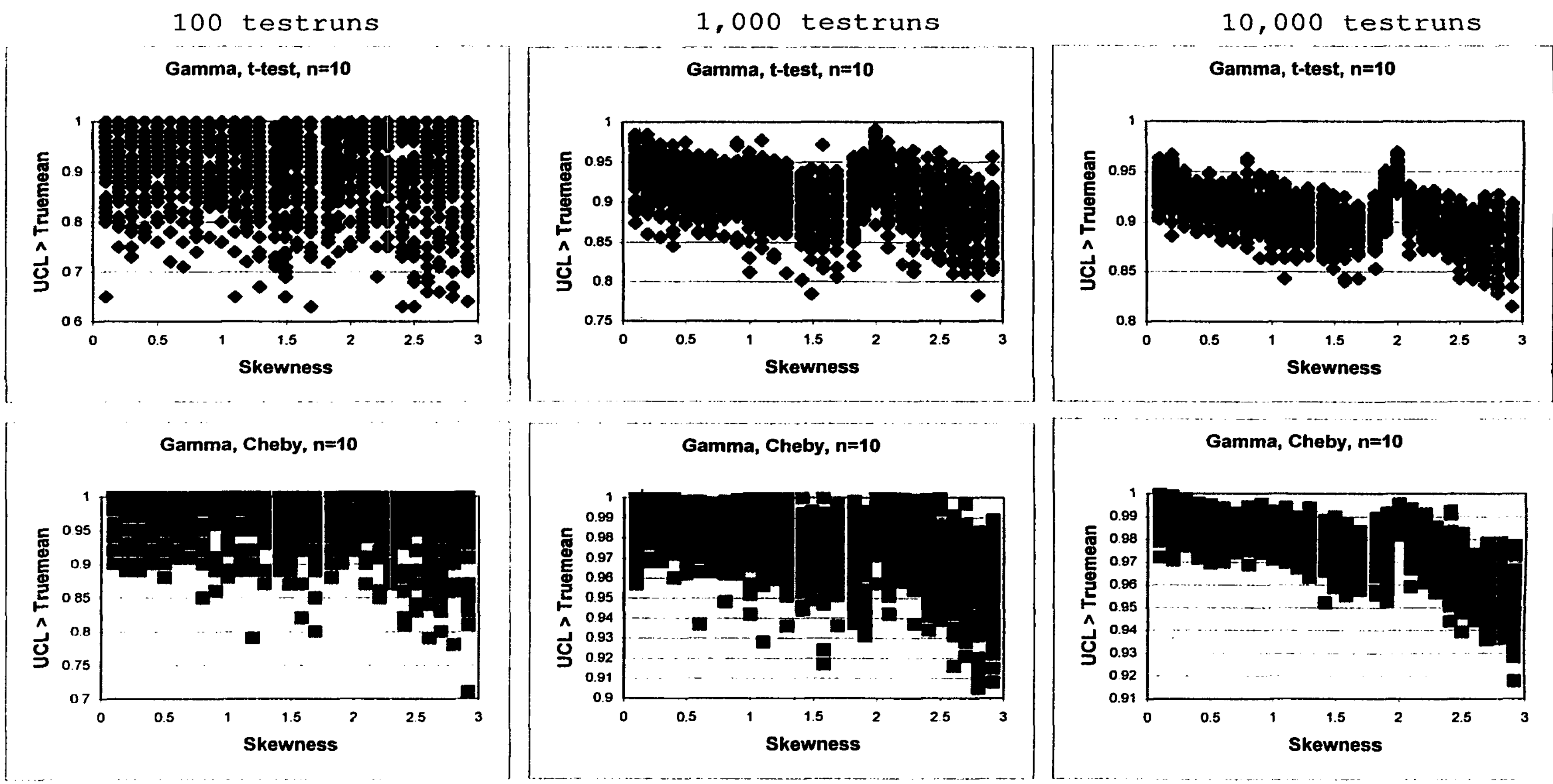
0.10	1.21	0.09	1.50	7.28	216.65	11087.86	0.07	0.30	0.20	0.51
2.50	1.21	2.35	1.50	18.39	65.09	1869.43	2.15	7.94	5.29	14.38
5.00	1.21	4.69	1.50	58.40	645.97	23538.52	3.30	14.53	9.95	22.88
0.10	0.77	0.12	2.99	2847.42	67802.99	1750679.00	0.06	1.23	0.55	4.55
2.50	0.77	2.91	2.99	1.38E+04	3.82E+05	2.09E+07	0.82	34.33	14.62	142.66
5.00	0.77	5.83	2.99	1.34E+07	4.12E+08	1.81E+10	1.36	58.45	26.58	275.87
n=20										
Beta	**Alpha**	**True Mean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	3.22	0.09	0.10	0.11	0.01	0.17	0.08	0.11	0.10	0.11
2.50	3.22	2.24	0.10	2.69	0.21	3.76	1.98	2.69	2.55	2.82
5.00	3.22	4.48	0.10	5.36	0.44	7.81	3.98	5.35	5.06	5.62
0.10	1.21	0.09	1.50	0.22	0.18	3.58	0.08	0.18	0.15	0.23
2.50	1.21	2.35	1.50	5.31	2.93	36.31	2.16	4.41	3.71	5.55
5.00	1.21	4.69	1.50	10.88	9.97	209.94	3.99	8.55	7.05	10.86
0.10	0.77	0.12	2.99	3.05	24.80	910.90	0.08	0.57	0.32	1.25
2.50	0.77	2.91	2.99	76.28	1396.76	82982.85	1.83	13.04	7.52	25.46
5.00	0.77	5.83	2.99	83.45	500.53	17595.18	3.36	23.91	13.58	50.55
n=30										
Beta	**Alpha**	**True Mean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	3.22	0.09	0.10	0.10	0.01	0.14	0.08	0.10	0.10	0.11
2.50	3.22	2.24	0.10	2.59	0.16	3.35	2.05	2.59	2.49	2.69
5.00	3.22	4.48	0.10	5.17	0.31	6.86	4.26	5.15	4.95	5.36
0.10	1.21	0.09	1.50	0.20	0.08	1.05	0.09	0.18	0.15	0.22
2.50	1.21	2.35	1.50	4.90	1.78	26.80	2.41	4.47	3.81	5.43
5.00	1.21	4.69	1.50	9.34	3.52	54.44	3.97	8.49	7.09	10.66
0.10	0.77	0.12	2.99	1.05	1.91	52.71	0.10	0.58	0.36	1.01
2.50	0.77	2.91	2.99	18.75	26.44	292.14	2.40	11.64	7.53	19.25
5.00	0.77	5.83	2.99	39.50	71.57	1760.09	5.78	22.12	15.05	38.97
n=50										
Beta	**Alpha**	**True Mean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**

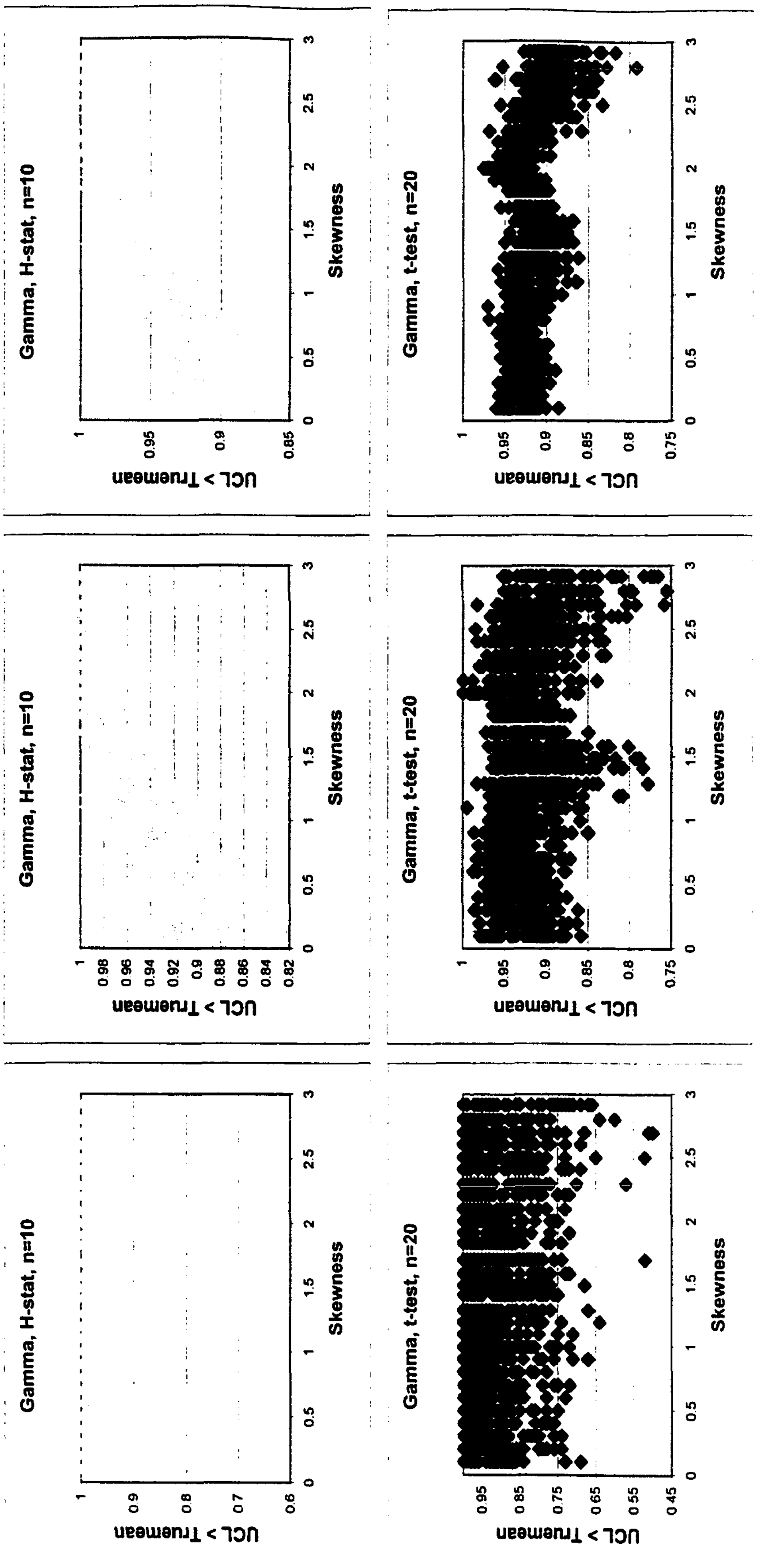
0.10	3.22	0.09	0.10	0.10	0.01	0.16	0.08	0.10	0.10	0.10
2.50	3.22	2.24	0.10	2.50	0.12	2.96	2.09	2.49	2.42	2.58
5.00	3.22	4.48	0.10	4.98	0.24	6.03	4.21	4.97	4.81	5.14
0.10	1.21	0.09	1.50	0.16	0.05	1.03	0.08	0.15	0.13	0.18
2.50	1.21	2.35	1.50	4.06	1.11	18.79	2.02	3.92	3.35	4.53
5.00	1.21	4.69	1.50	7.88	2.41	59.10	4.39	7.38	6.52	8.66
0.10	0.77	0.12	2.99	0.55	1.32	43.20	0.11	0.35	0.24	0.54
2.50	0.77	2.91	2.99	12.75	18.22	631.42	2.02	9.03	6.67	13.48
5.00	0.77	5.83	2.99	25.92	69.30	2525.89	5.24	16.90	12.29	25.89
n=100										
Beta	**Alpha**	**True Mean**	**Skew**	**Mean**	**StdDev**	**Max**	**Min**	**Median**	**Q1**	**Q3**
0.10	3.22	0.09	0.10	0.10	0.00	0.13	0.08	0.10	0.09	0.10
2.50	3.22	2.24	0.10	2.42	0.12	2.94	2.06	2.42	2.34	2.49
5.00	3.22	4.48	0.10	4.83	0.24	6.12	4.13	4.82	4.67	4.98
0.10	1.21	0.09	1.50	0.14	0.03	0.47	0.08	0.14	0.12	0.16
2.50	1.21	2.35	1.50	3.50	0.78	17.71	2.01	3.41	3.00	3.86
5.00	1.21	4.69	1.50	7.02	1.60	46.78	3.75	6.79	6.07	7.68
0.10	0.77	0.12	2.99	0.34	0.68	35.44	0.09	0.26	0.20	0.37
2.50	0.77	2.91	2.99	9.56	16.98	1035.50	2.02	7.06	5.17	10.27
5.00	0.77	5.83	2.99	18.42	30.02	1121.62	5.14	13.78	9.98	20.10

From this we can see that as the scale parameter increases (either μ or β), the overall standard deviation tends to increase regardless of distribution or test. Another observation is that for the H-Statistic, when the distribution is not Lognormal, the test tremendously overshoots the true mean, especially when skewness is high.

Next we will look at the graphs of the results which will tell us how often the UCL is greater than the true mean (but does not tell us by how much). We will be plotting the skewness vs. the coverage percentage, i.e. the number of times the UCL > true mean / number of test runs. The following graphs give the raw data points:

Figure 4 - Gamma Coverage Probabilities




Gamma, H-stat, n=10
UCL > Truemean
Skewness
Gamma, H-stat, n=10
UCL > Truemean
Skewness
Gamma, H-stat, n=10
UCL > Truemean
Skewness
Gamma, t-test, n=20
UCL > Truemean
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UCL > Truemean
Skewness
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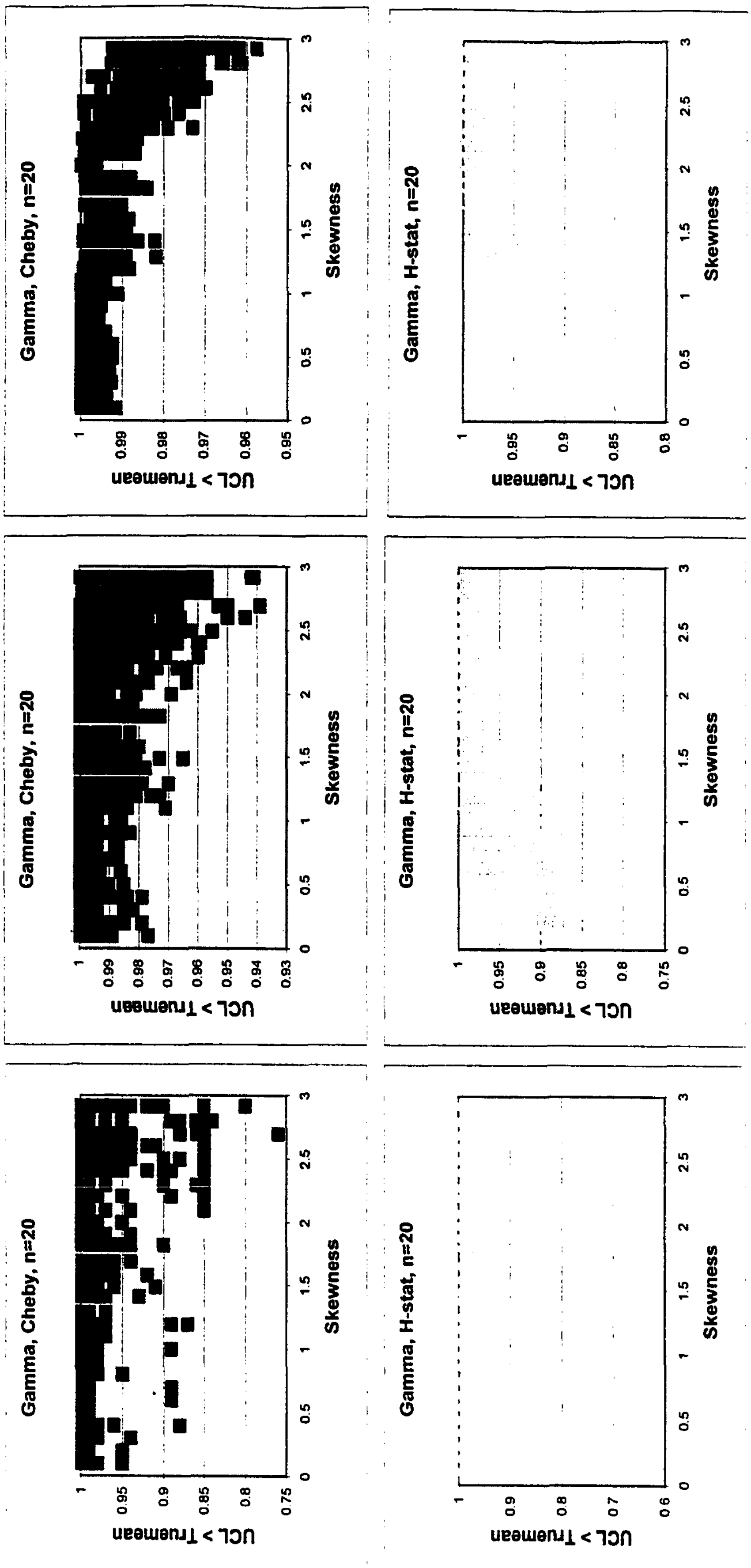
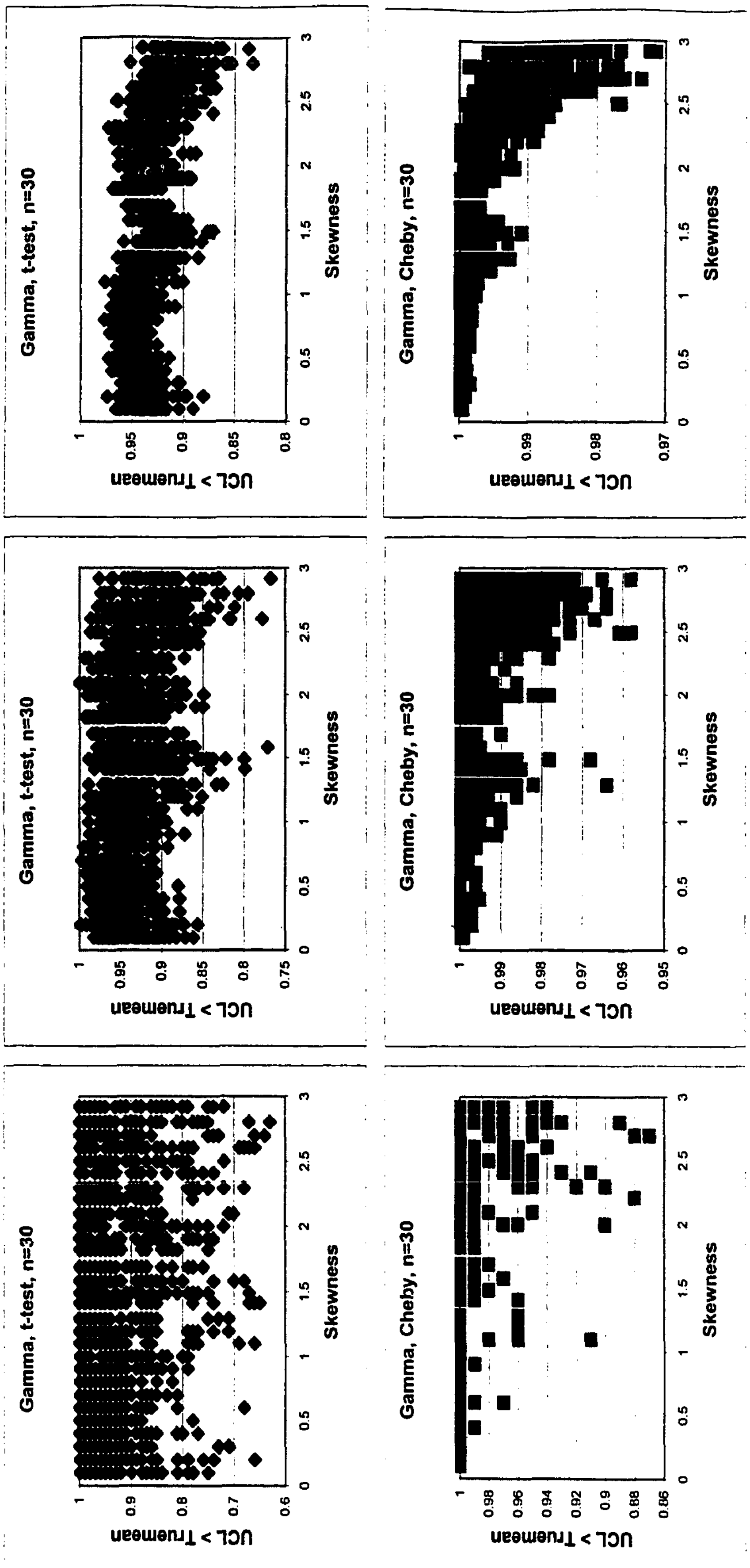
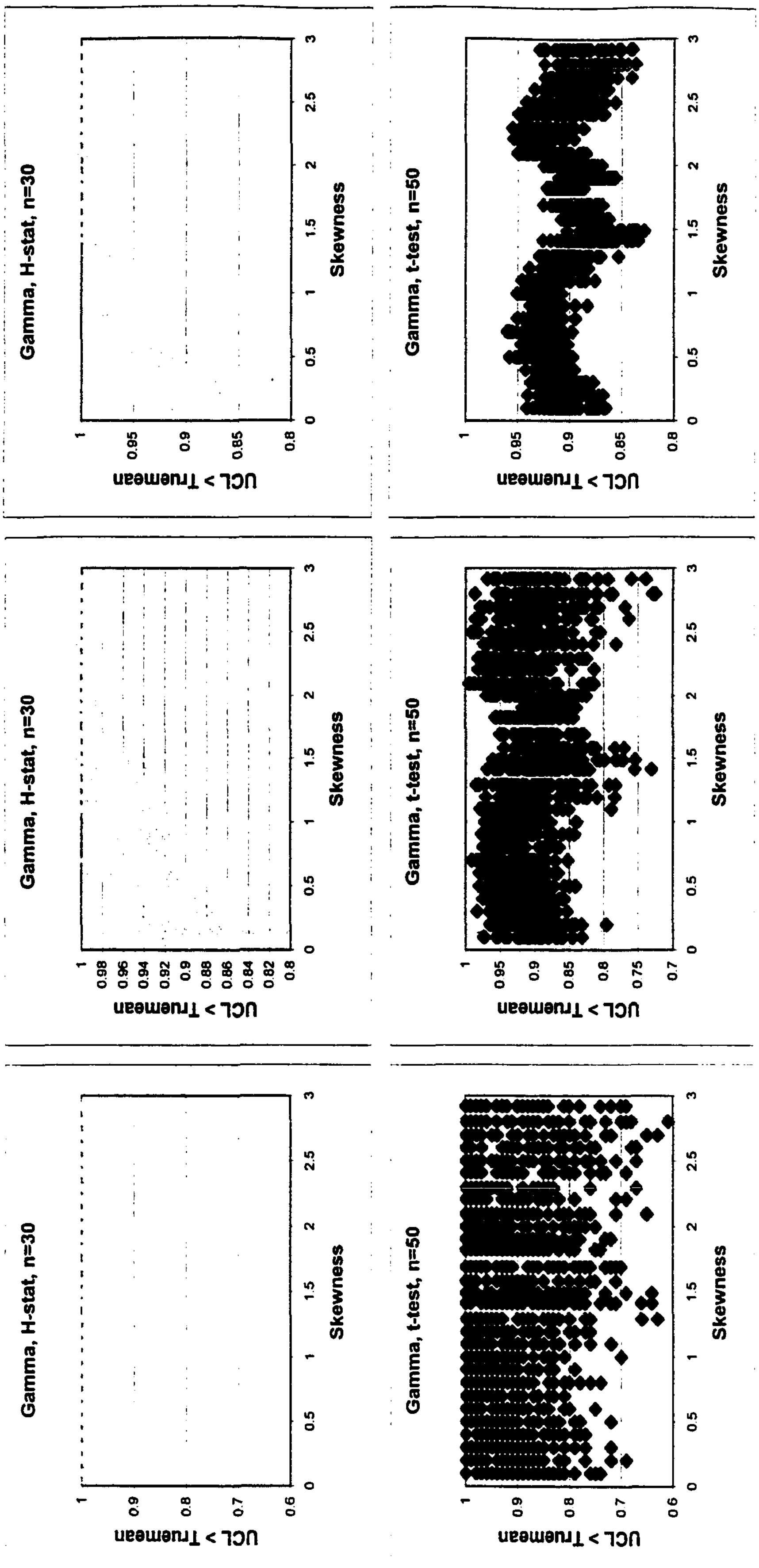
UCL > Truemean
Skewness


Gamma, Cheby, n=20
UCL > Truemean
Skewness
Gamma, Cheby, n=20
UCL > Truemean
Skewness
Gamma, Cheby, n=20
UCL > Truemean
Skewness
Gamma, H-stat, n=20
UCL > Truemean
Skewness
Gamma, H-stat, n=20
UCL > Truemean
Skewness
Gamma, H-stat, n=20
UCL > Truemean
Skewness


Gamma, t-test, n=30
Gamma, Cheby, n=30
Skewness
UCL > Truemean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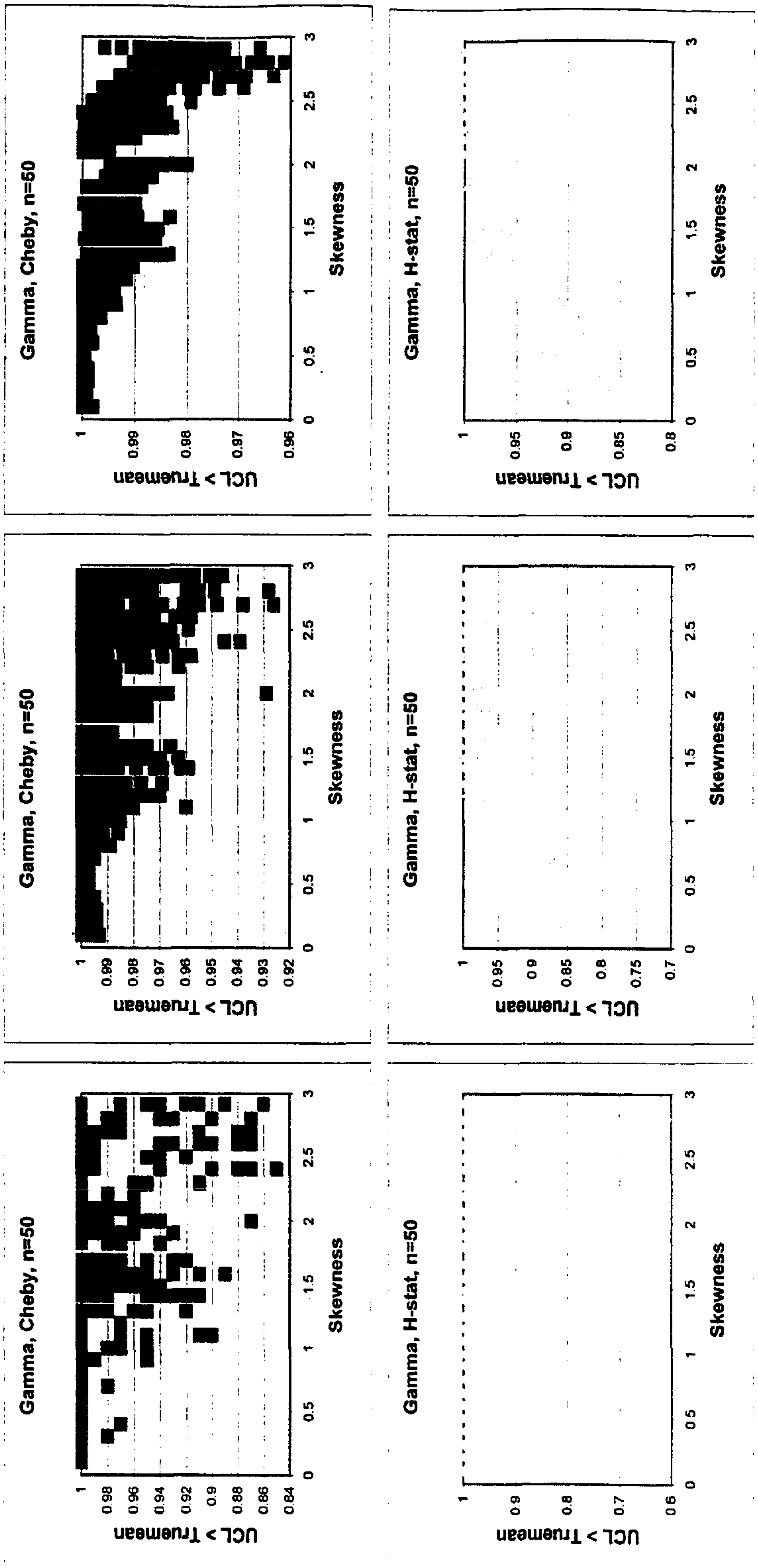
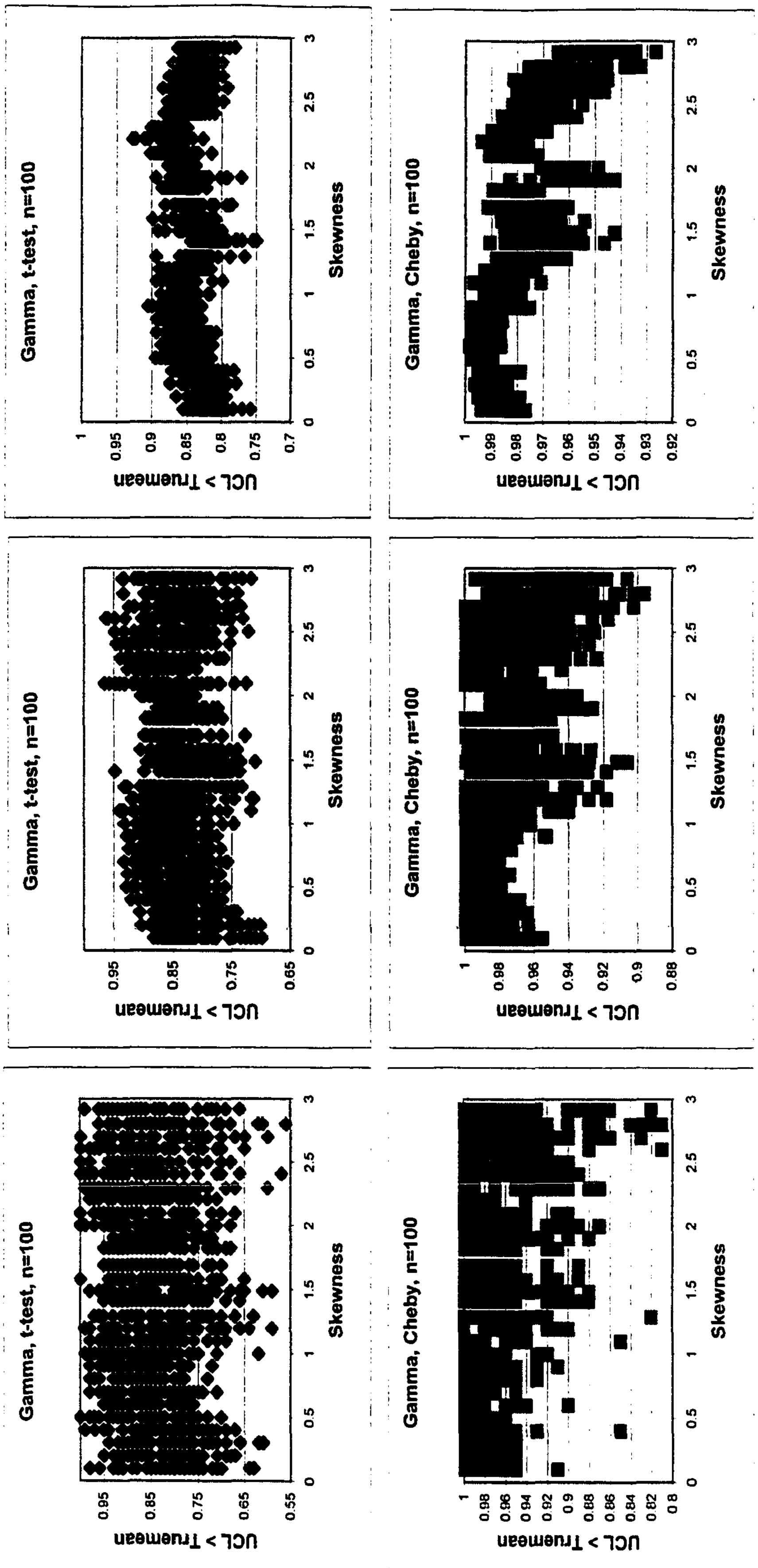
Gamma, H-stat, n=30
Gamma, H-stat, n=30
Gamma, t-test, n=50
Gamma, t-test, n=50
Gamma, t-test, n=50
UCL > Truemean
Skewness


Gamma, Cheby, n=50
UCL > Truemean
Skewness
Gamma, Cheby, n=50
UCL > Truemean
Skewness
Gamma, Cheby, n=50
UCL > Truemean
Skewness
Gamma, H-stat, n=50
UCL > Truemean
Skewness
Gamma, H-stat, n=50
UCL > Truemean
Skewness
Gamma, H-stat, n=50
UCL > Truemean
Skewness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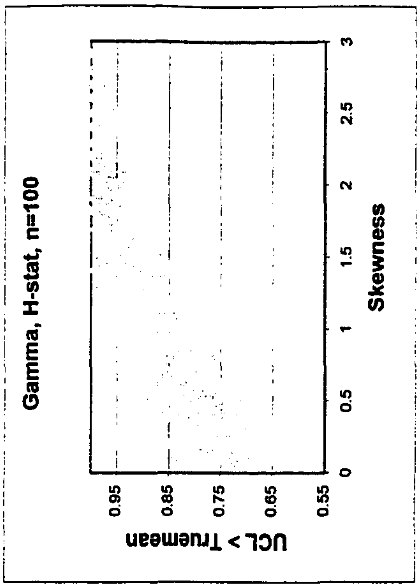
Skewness
Gamma, t-test, n=100
UCL > Truemean
Skewness
Gamma, t-test, n=100
UCL > Truemean
Skewness
Gamma, Cheby, n=100
UCL > Truemean
Skewness
Gamma, Cheby, n=100
UCL > Truemean
Skewness
Gamma, Cheby, n=100
UCL > Truemean
Skewness


Gamma, H-stat, n=100
UCL > Truemean
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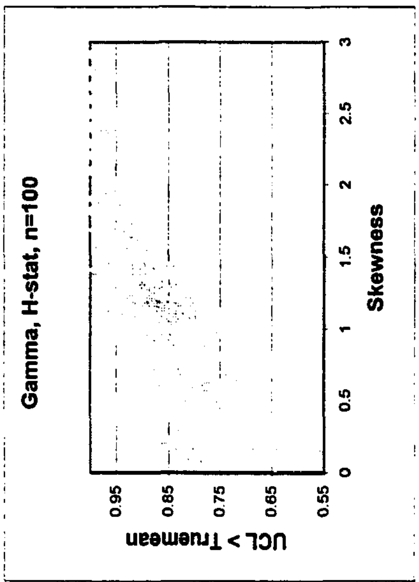
0.85
0.75
0.65
0.55
0
0.5
1
1.5
2
2.5
3
Skewness


Gamma, H-stat, n=100
UCL > Truemean
0.95
0.85
0.75
0.65
0.55
0
0.5
1
1.5
2
2.5
3
Skewness

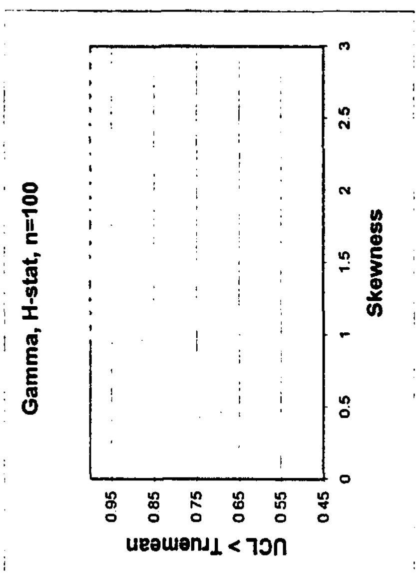
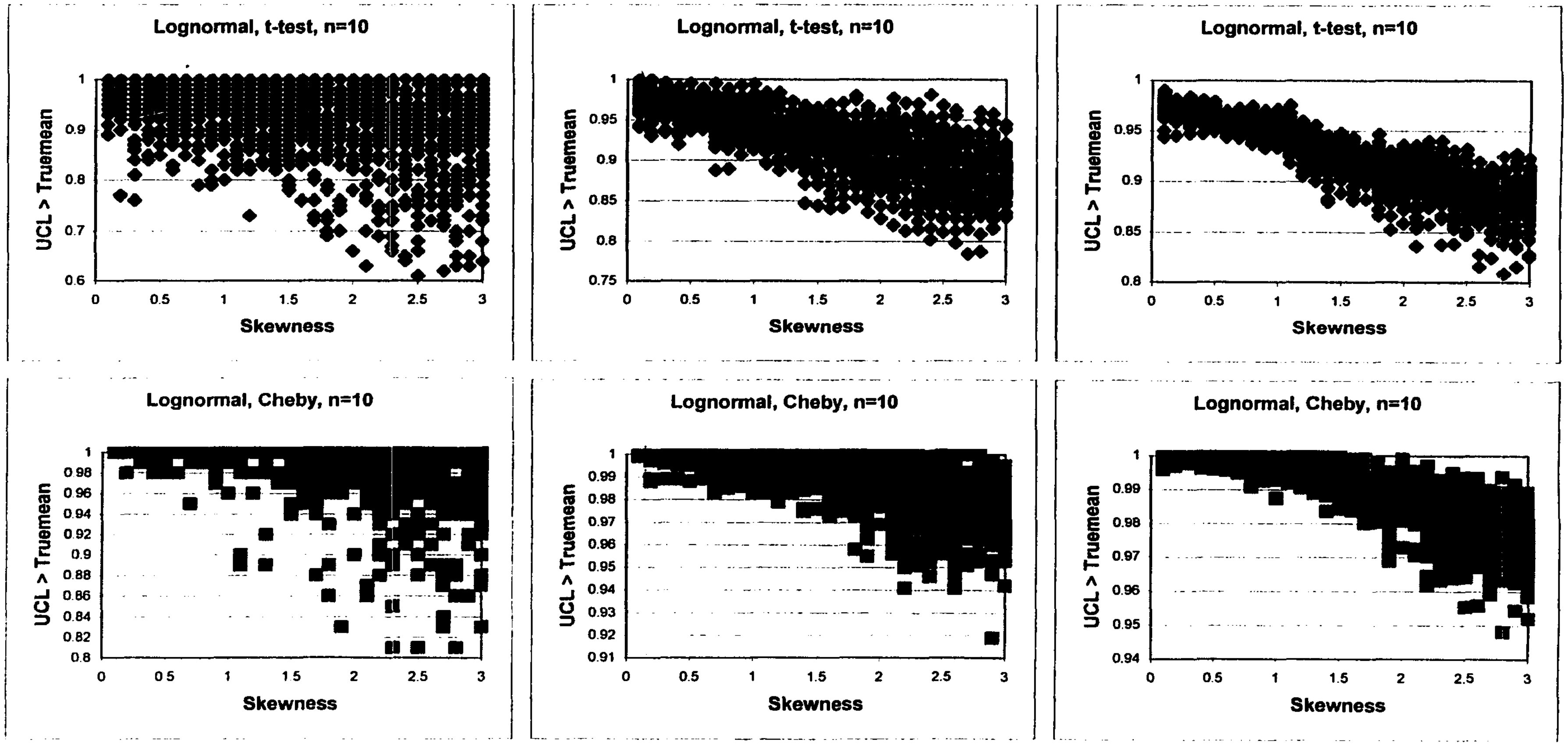
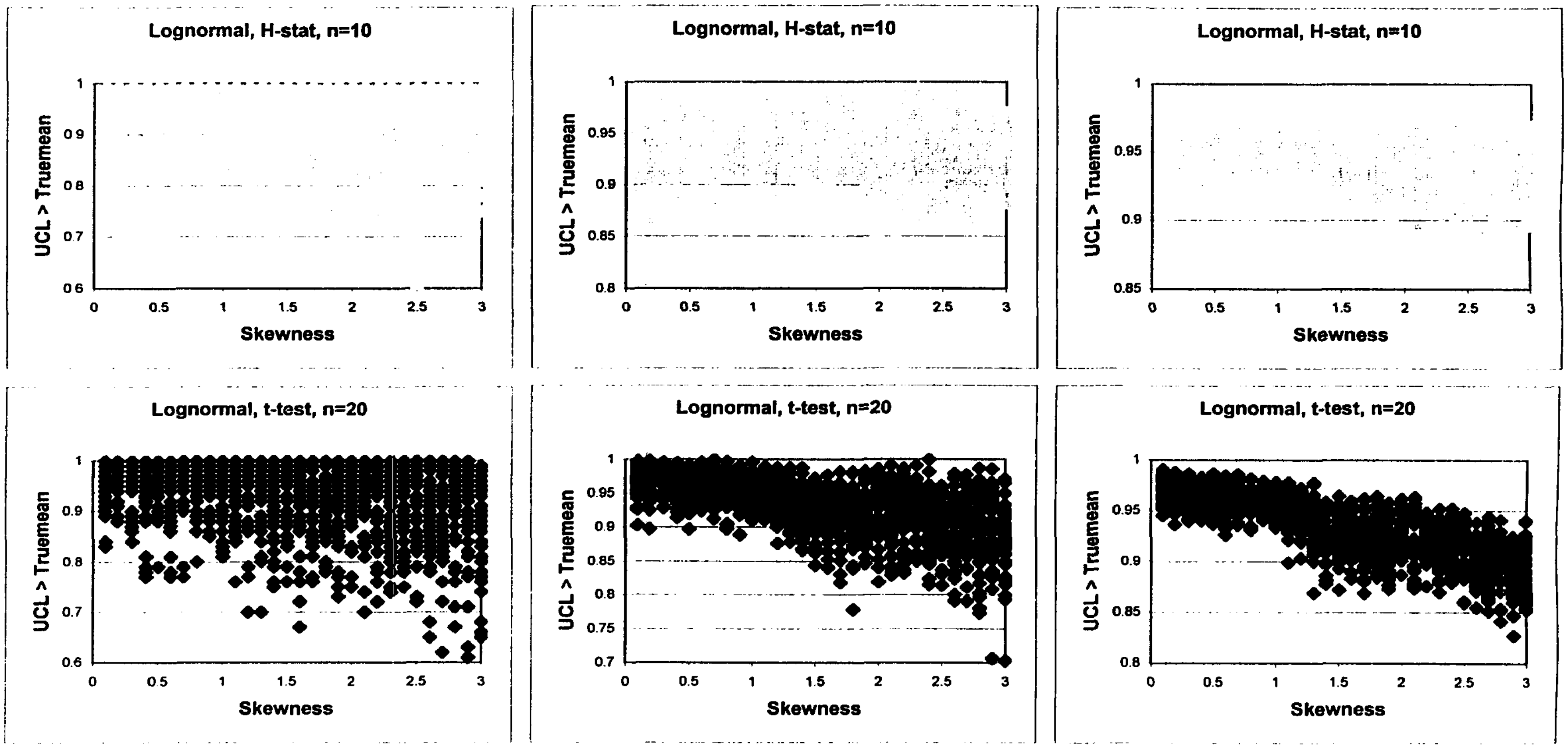

Gamma, H-stat, n=100
UCL > Truemean
0.95
0.85
0.75
0.65
0.55
0.45
0
0.5
1
1.5
2
2.5
3
Skewness

Figure 5 - Lognormal Coverage Probabilities

100 testruns 1,000 testruns 10,000 testruns

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UCL > Truemean
Skewness
Lognormal, H-stat, n=10
UCL > Truemean
Skewness
Lognormal, H-stat, n=10
UCL > Truemean
Skewness
Lognormal, t-test, n=20
UCL > Truemean
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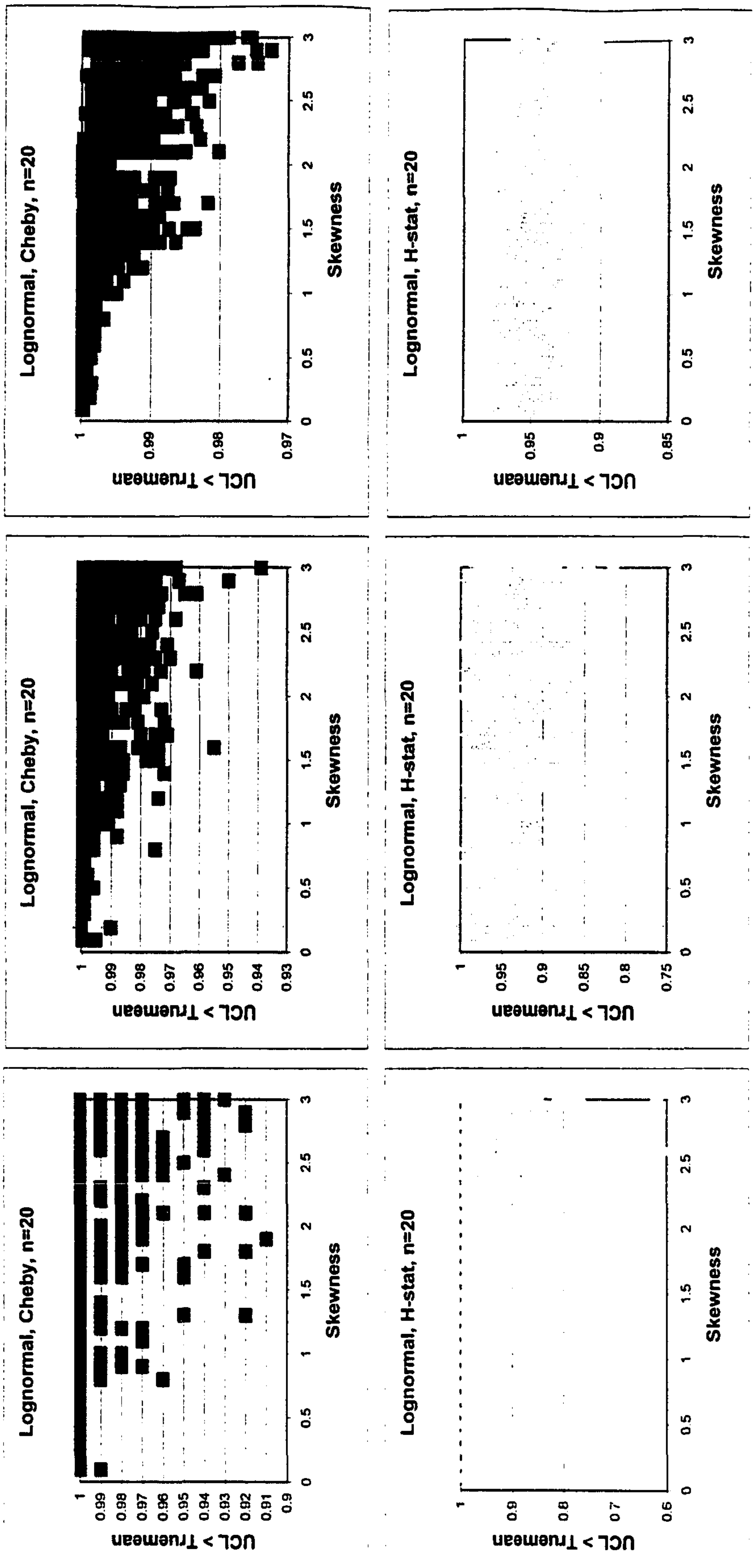
UCL > Truemean
Skewness


Lognormal, Cheby, n=20
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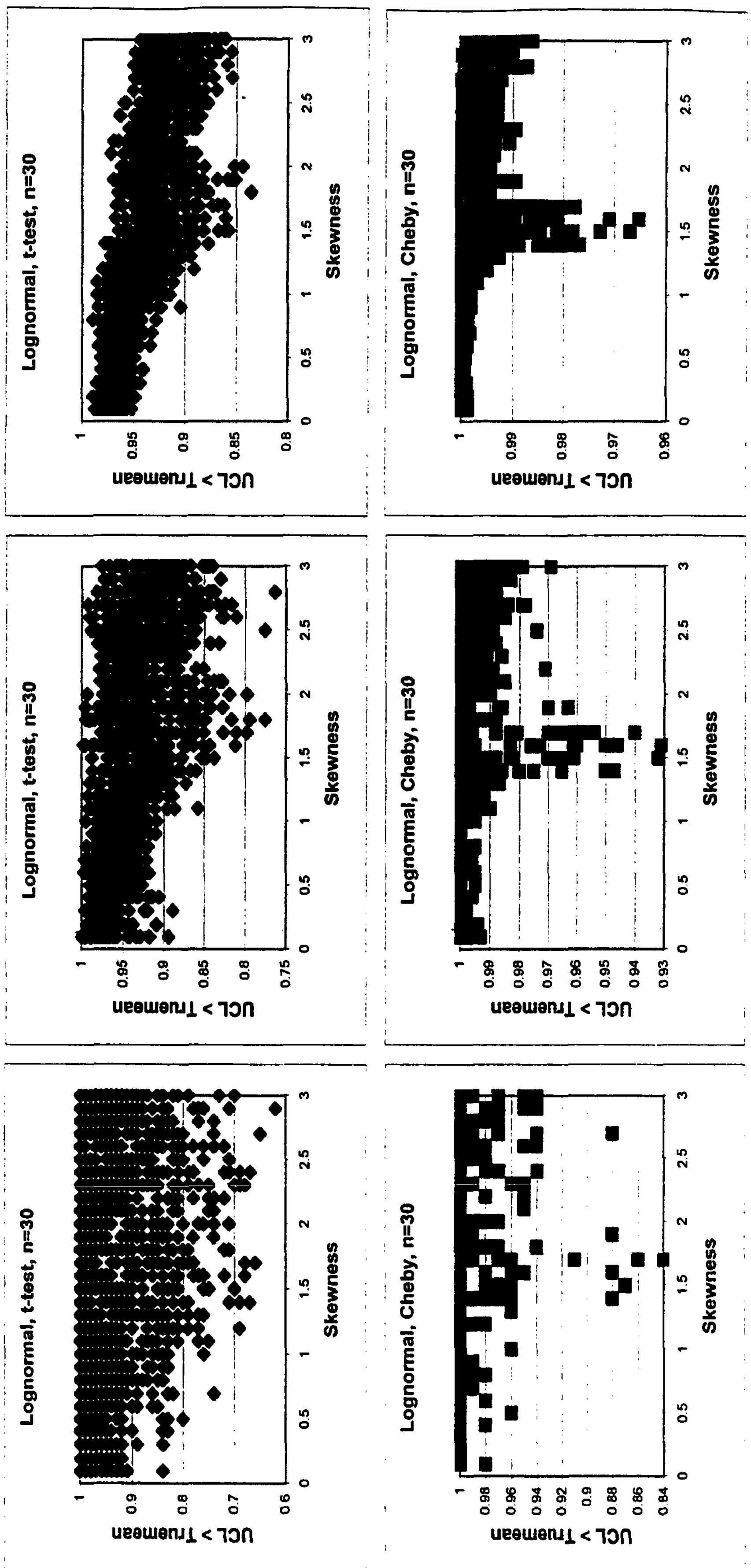
Skewness
Lognormal, Cheby, n=20
UCL > Truemean
Skewness
Lognormal, Cheby, n=20
UCL > Truemean
Skewness
Lognormal, H-stat, n=20
UCL > Truemean
Skewness
Lognormal, H-stat, n=20
UCL > Truemean
Skewness
Lognormal, H-stat, n=20
UCL > Truemean
Skewness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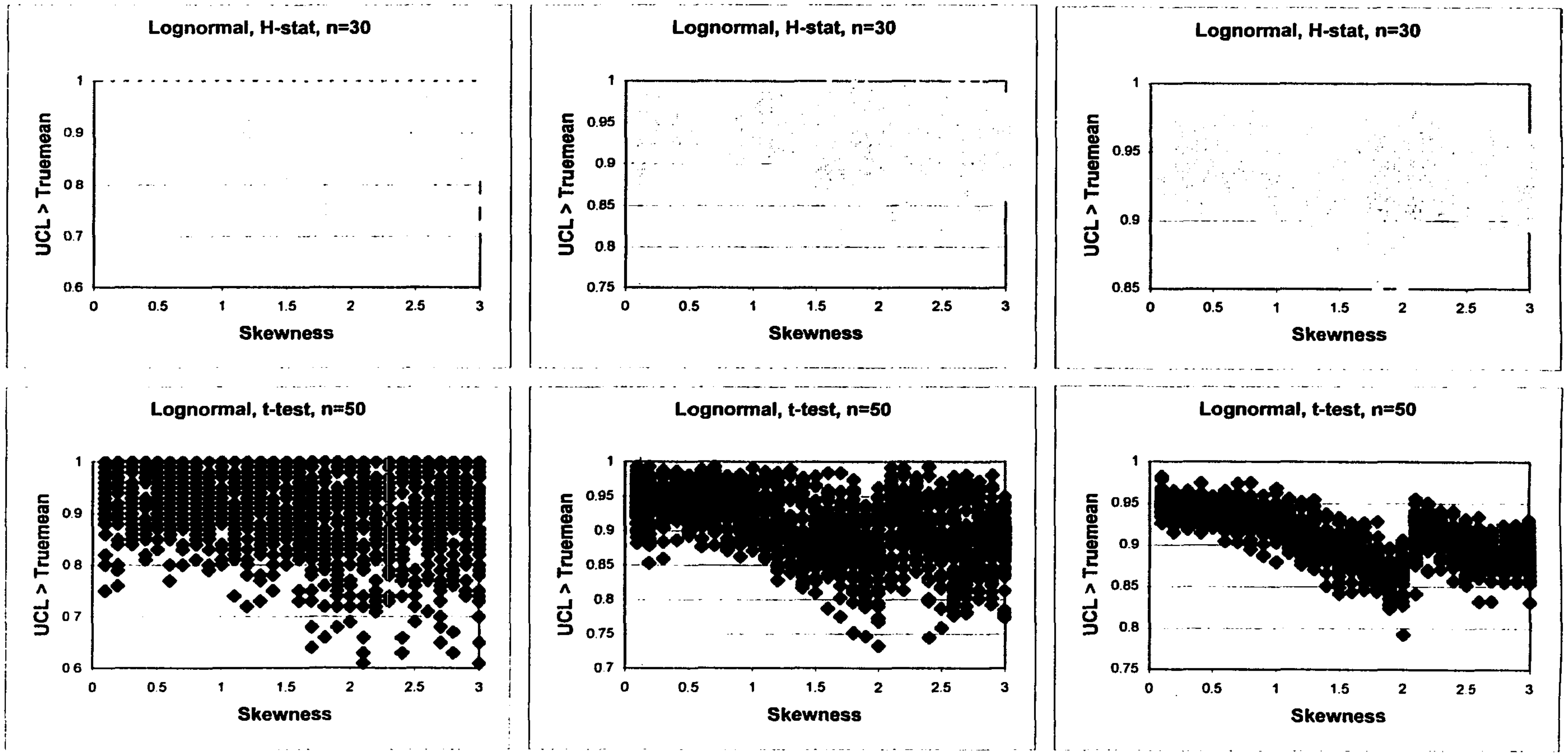
Skewness
Lognormal, t-test, n=30
UCL > Truemean
Skewness
Lognormal, t-test, n=30
UCL > Truemean
Skewness
Lognormal, Cheby, n=30
UCL > Truemean
Skewness
Lognormal, Cheby, n=30
UCL > Truemean
Skewness
Lognormal, Cheby, n=30
UCL > Truemean
Skewness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UCL > Truemean
Skewness
Lognormal, H-stat, n=30
UCL > Truemean
Skewness
Lognormal, H-stat, n=30
UCL > Truemean
Skewness
Lognormal, t-test, n=50
UCL > Truemean
Skewness
Lognormal, t-test, n=50
UCL > Truemean
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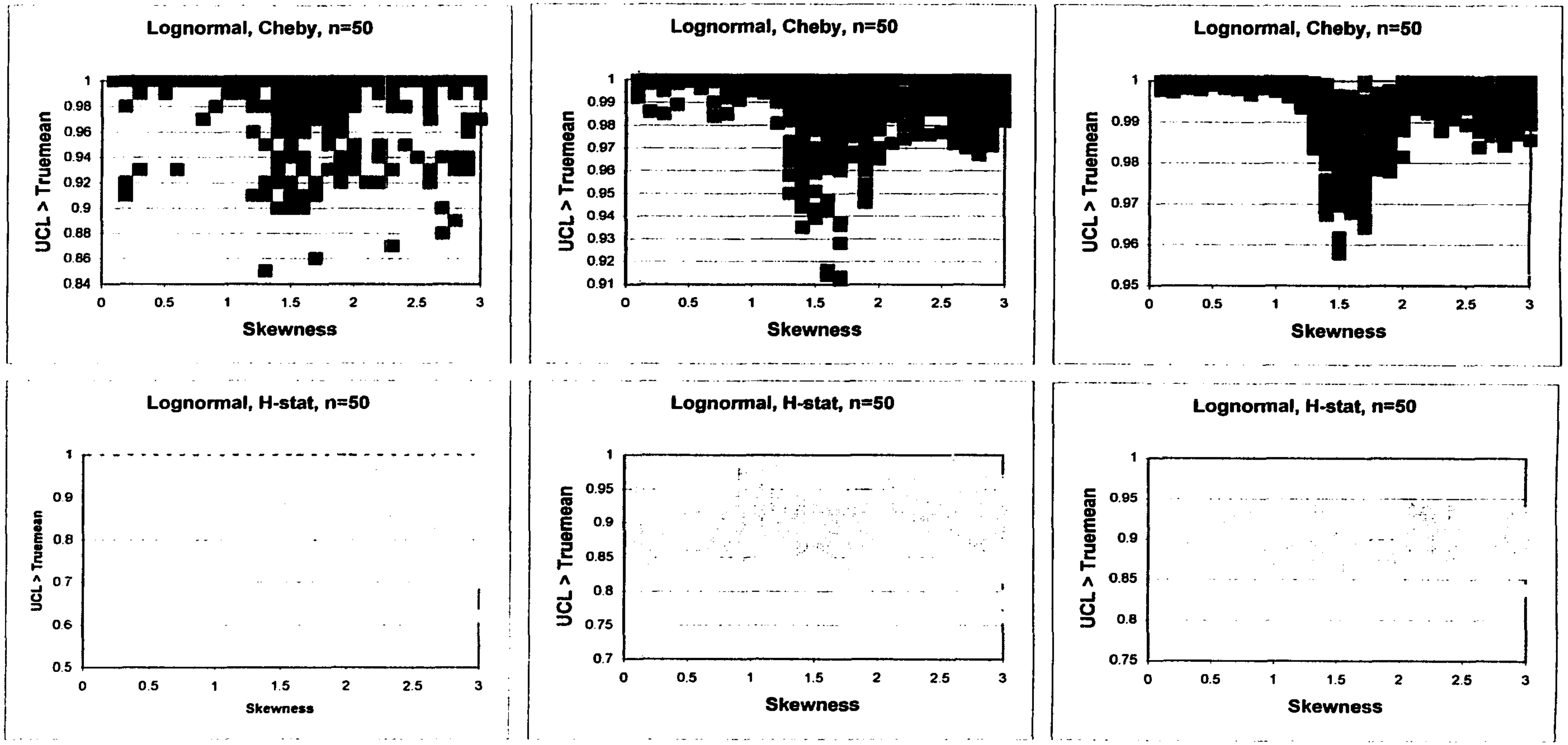
UCL > Truemean
Skewness


Lognormal, Cheby, n=50
UCL > Truemean
Skewness
Lognormal, Cheby, n=50
UCL > Truemean
Skewness
Lognormal, Cheby, n=50
UCL > Truemean
Skewness
Lognormal, H-stat, n=50
UCL > Truemean
Skewness
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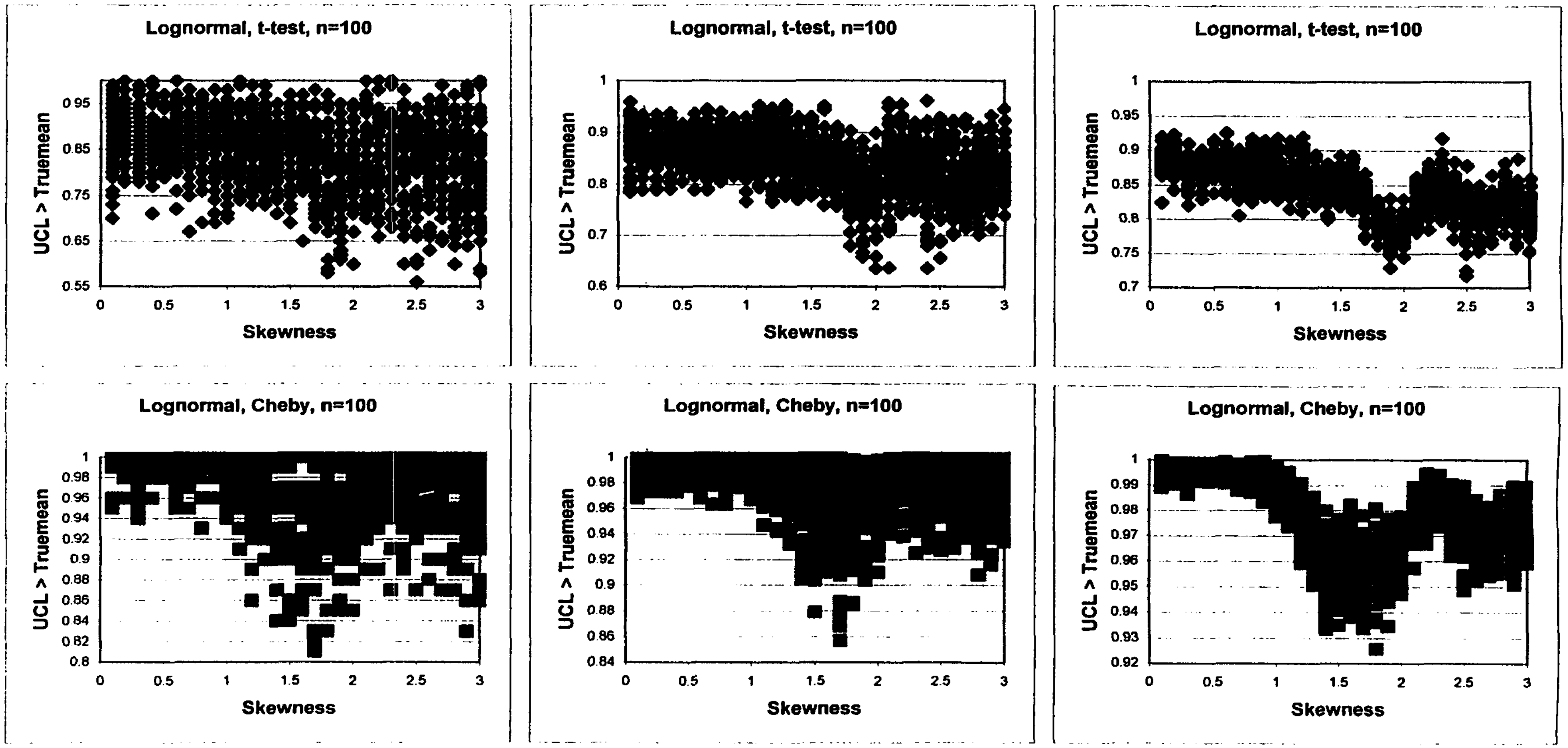
UCL > Truemean
Skewness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UCL > Truemean
Skewness
Lognormal, t-test, n=100
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Skewness
Lognormal, Cheby, n=100
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UCL > Truemean
Skewness

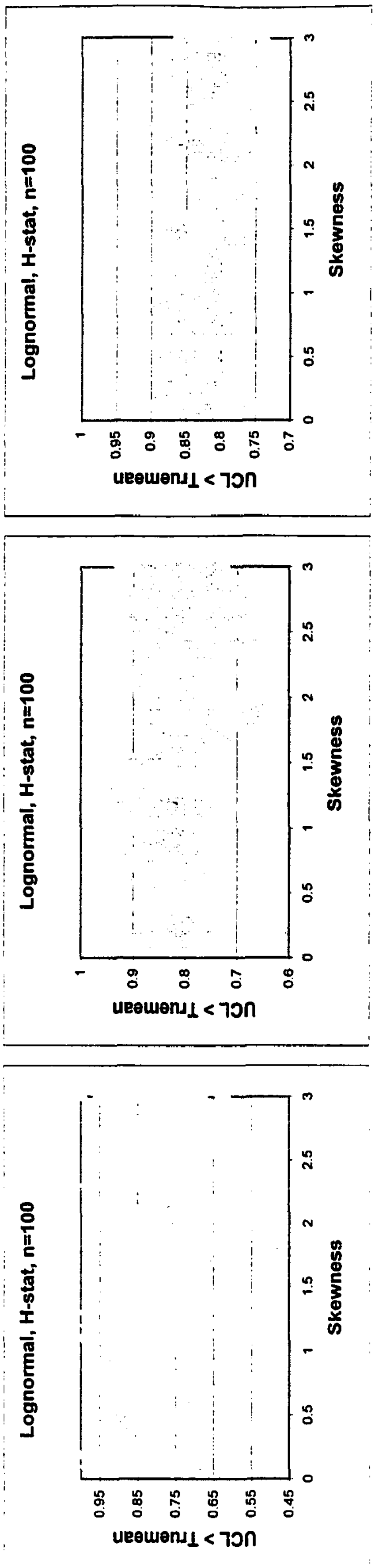
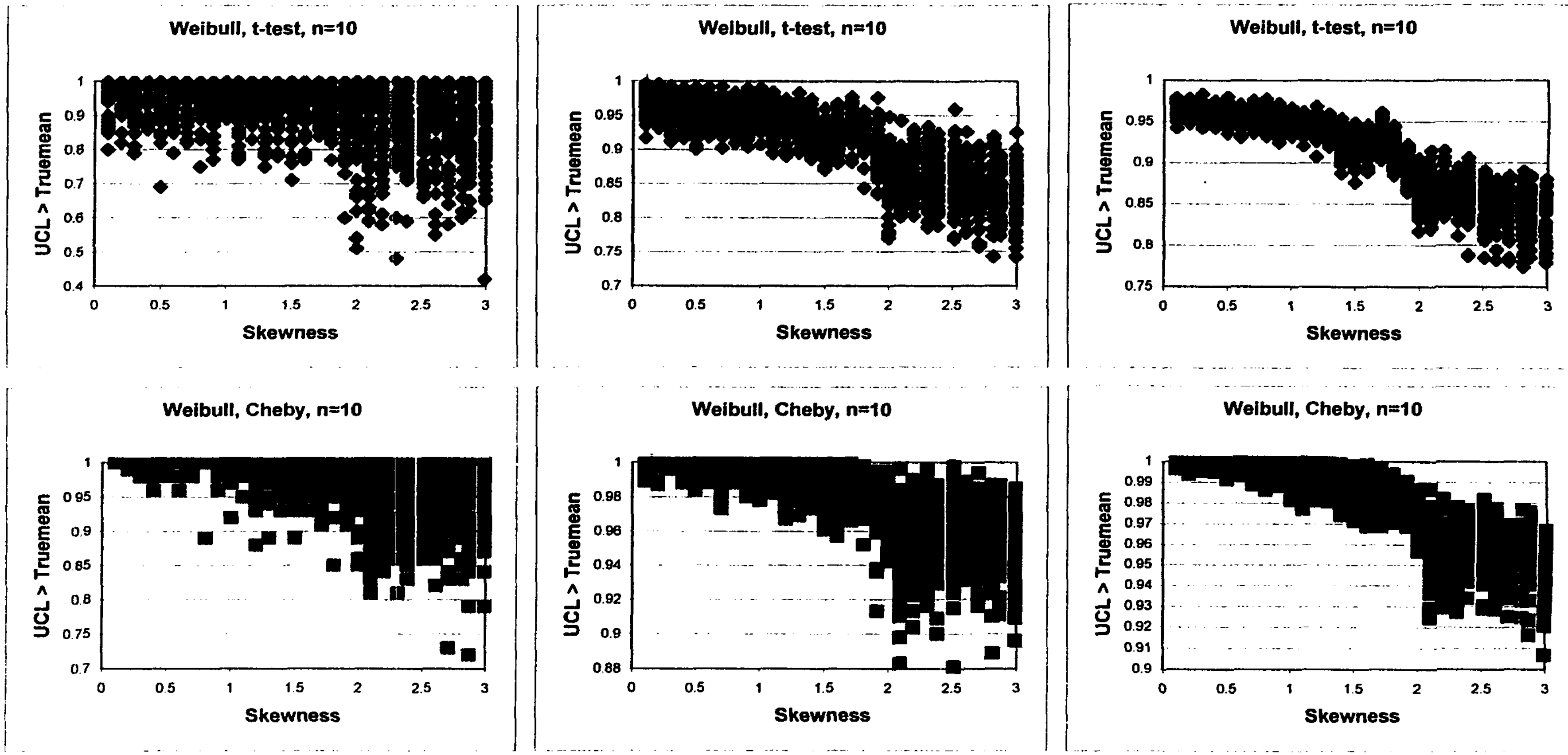
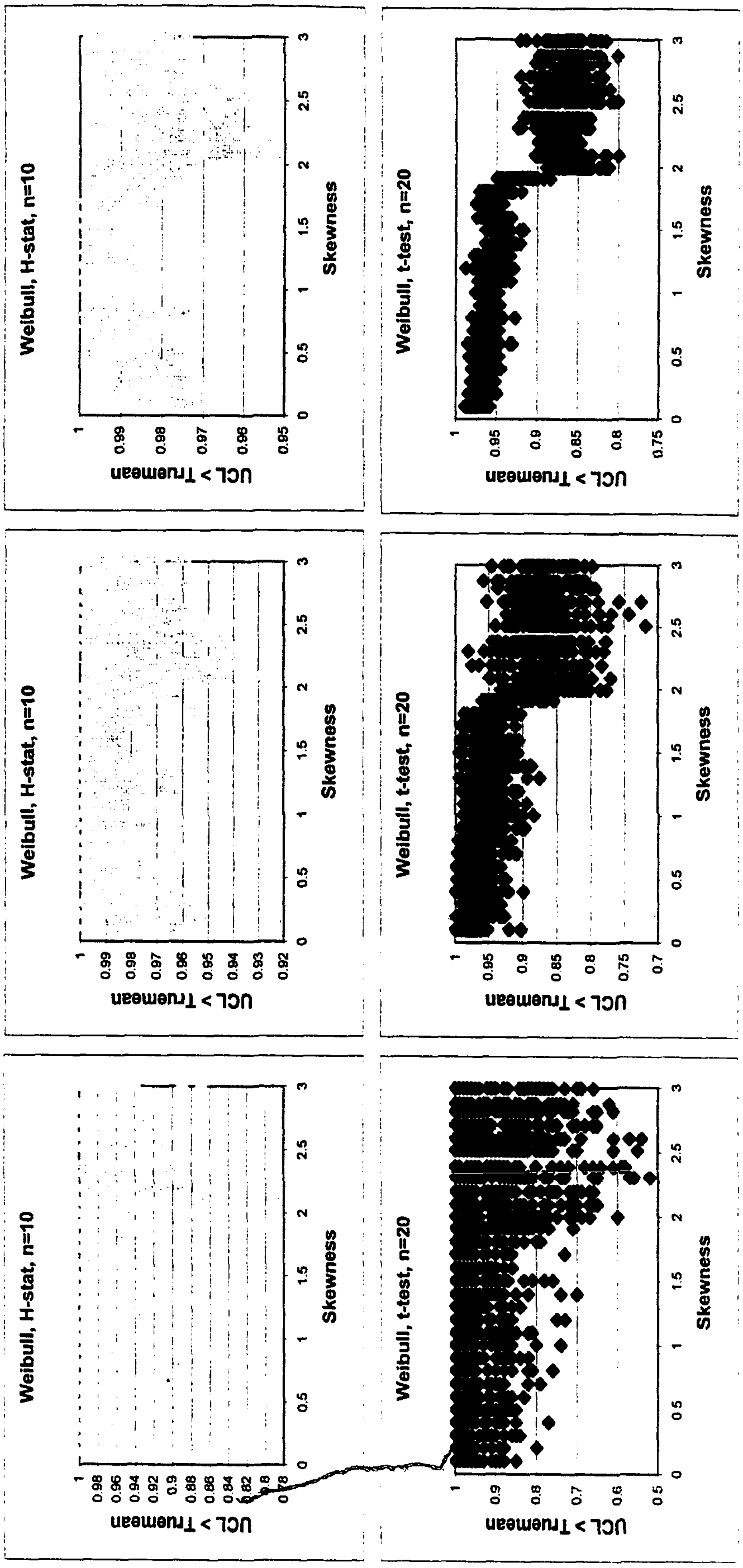
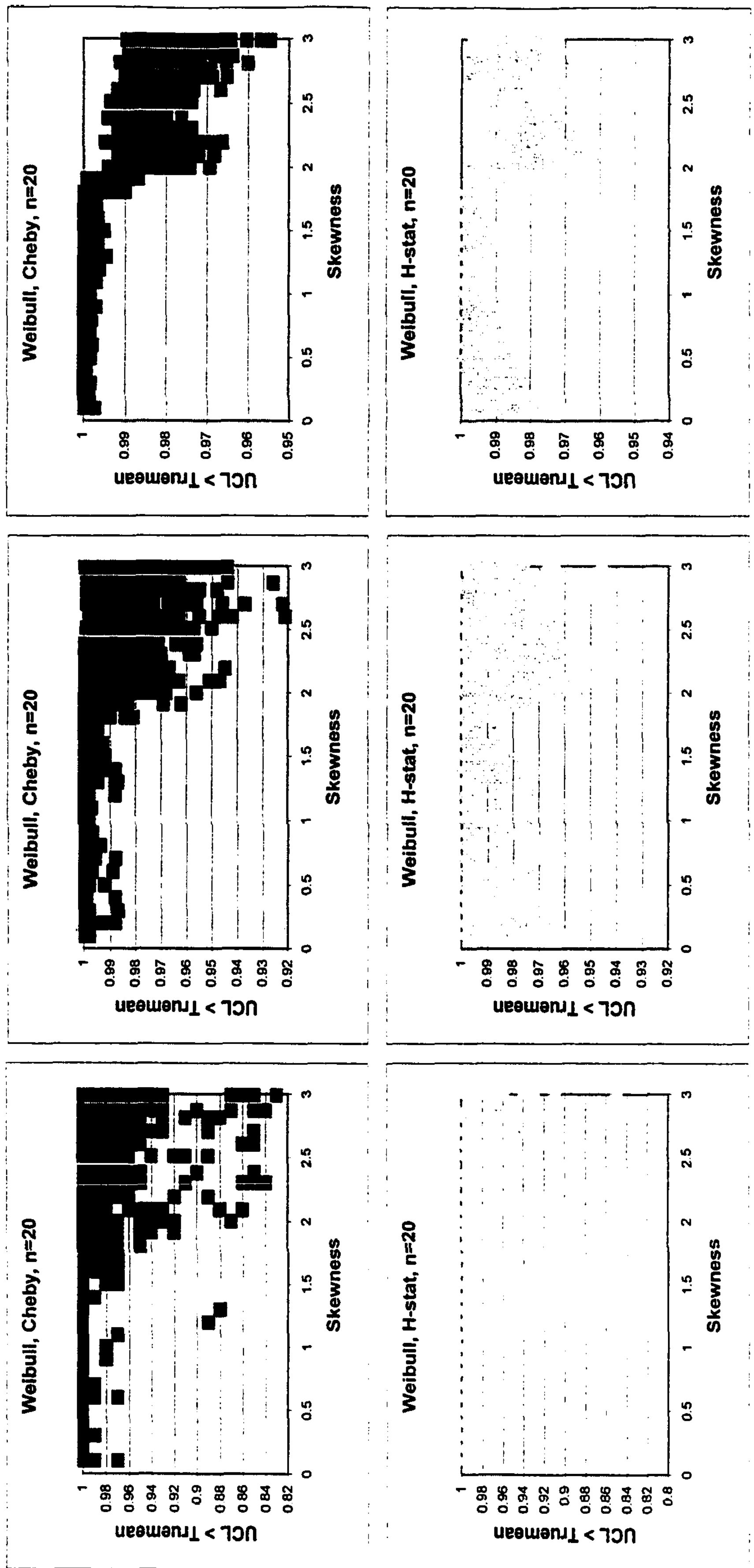

Lognormal, H-stat, n=100
UCL > Truemean
Skewness
0.95
0.85
0.75
0.65
0.55
0.45
0
0.5
1
1.5
2
2.5
3
Lognormal, H-stat, n=100
UCL > Truemean
Skewness
1
0.9
0.8
0.7
0.6
Lognormal, H-stat, n=100
UCL > Truemean
Skewness
1
0.95
0.9
0.85
0.8
0.75
0.7

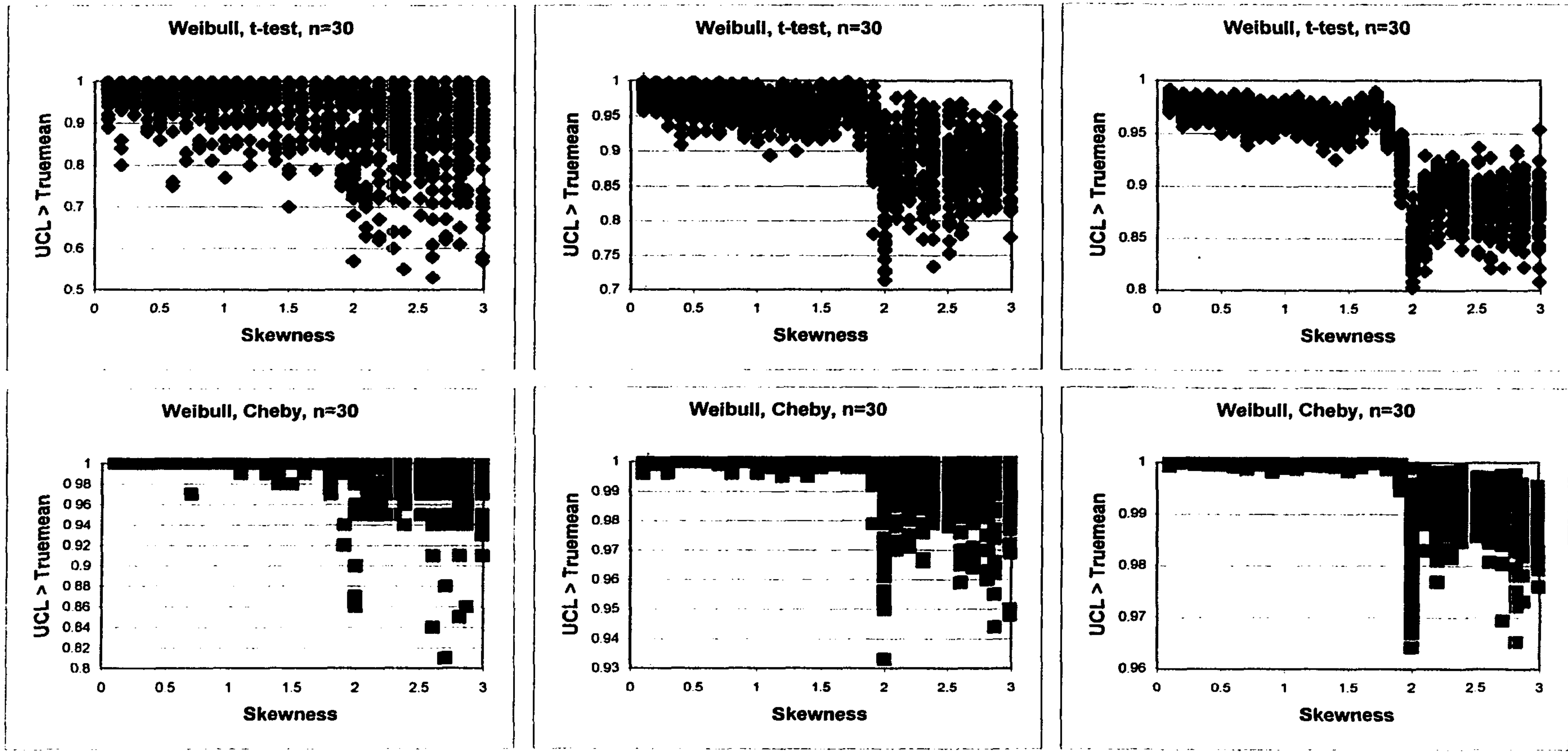
Figure 6 - Weibull Coverage Probabilities

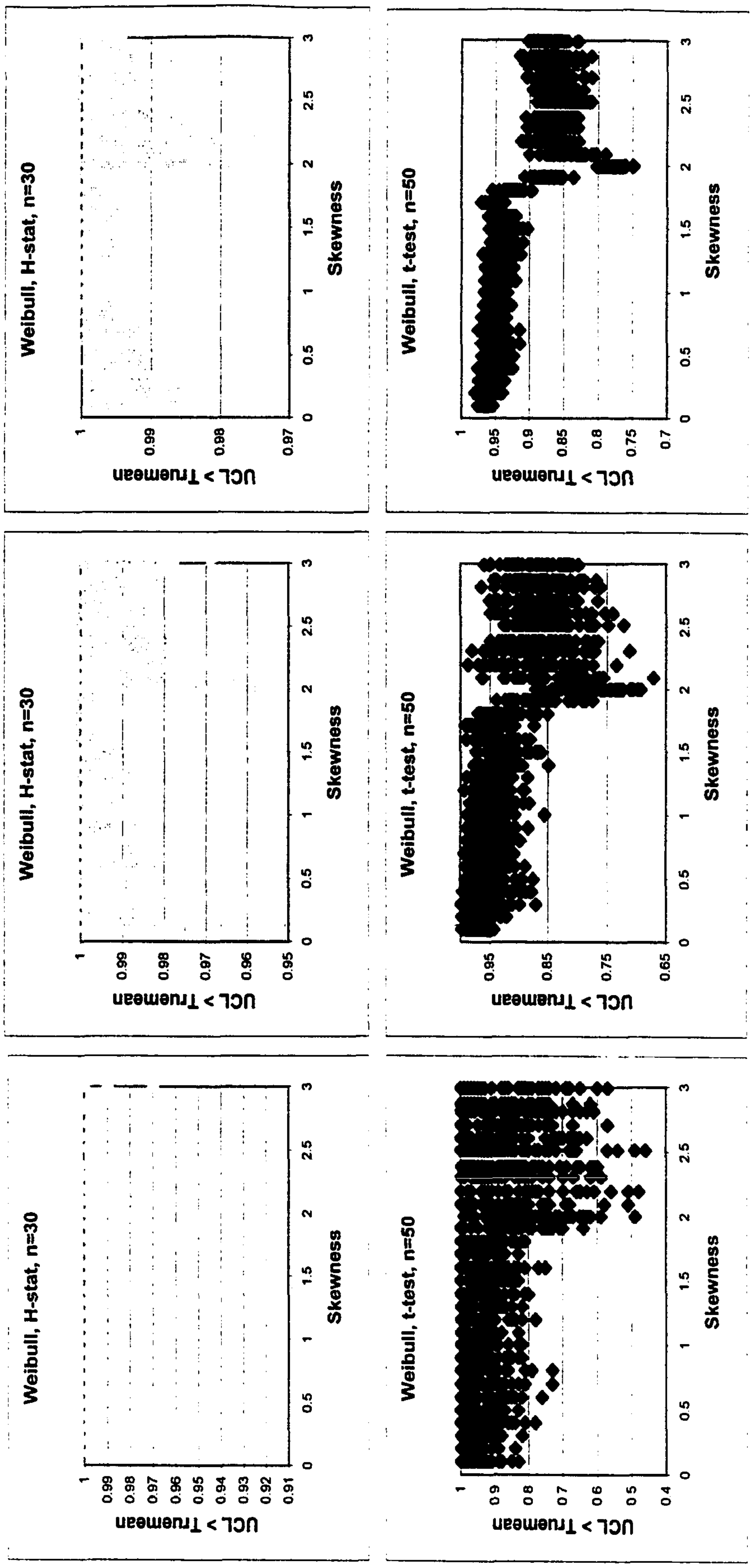
100 testruns | 1,000 testruns | 10,000 testruns

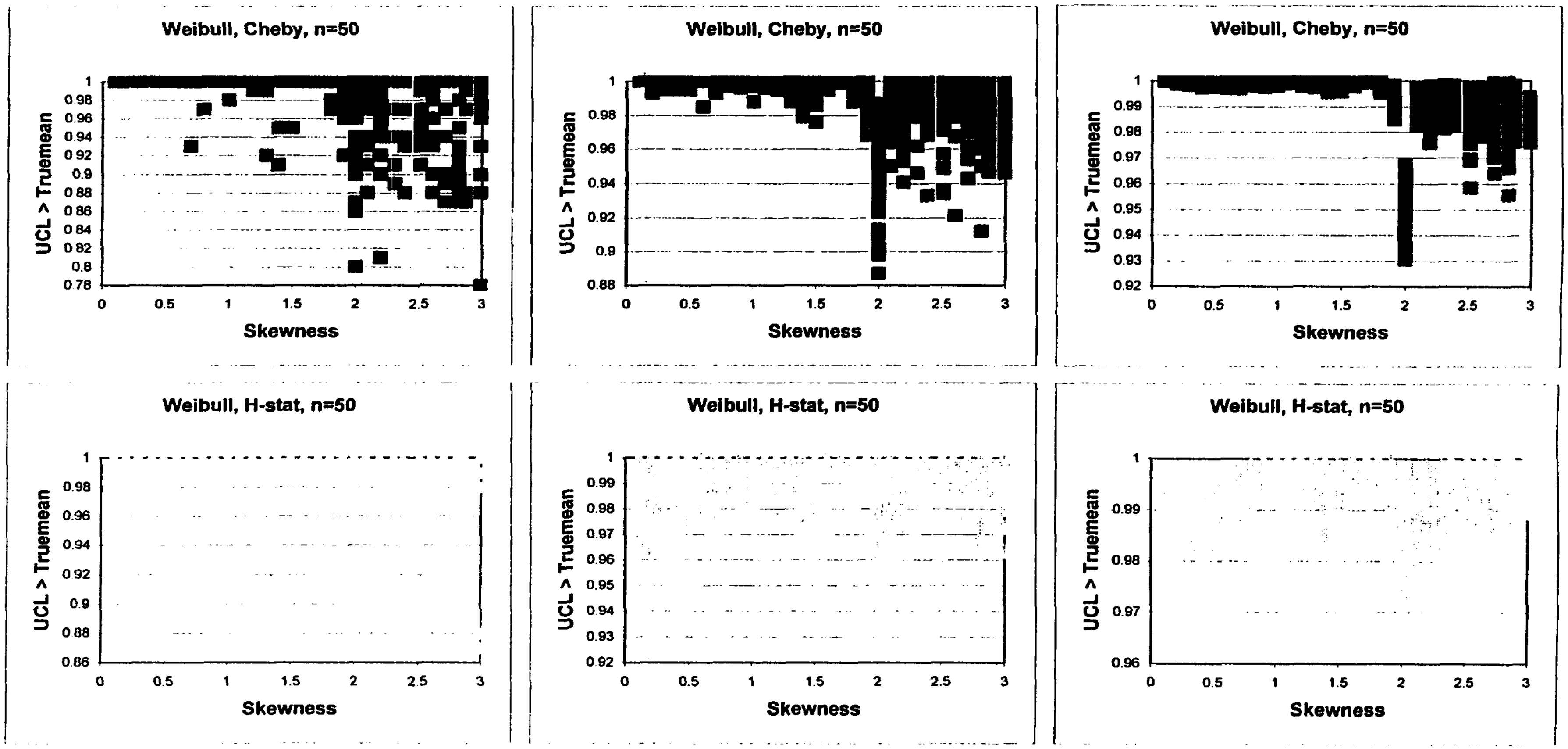


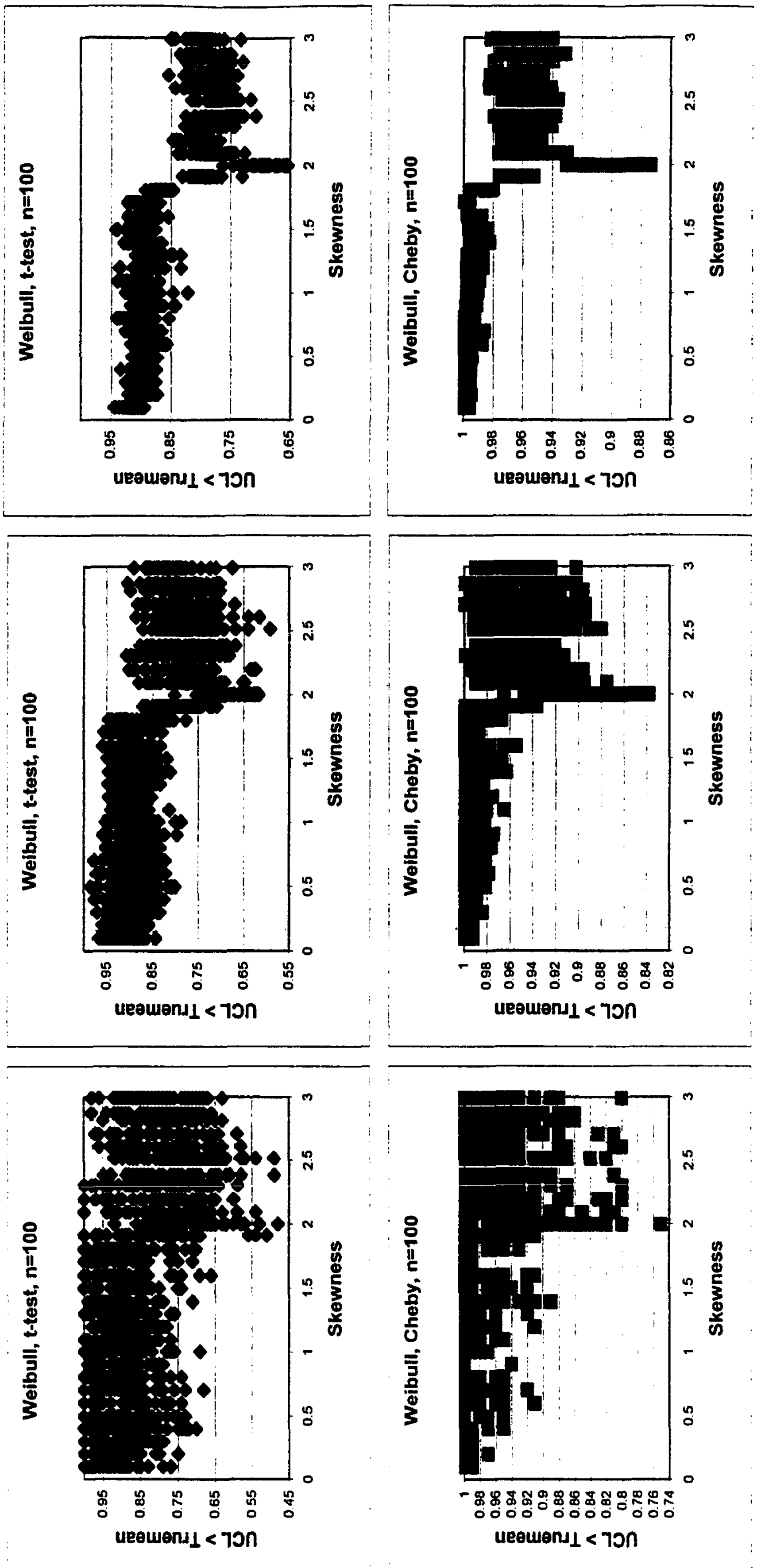

Weibull, H-stat, n=10
UCL > Truemean
Skewness
Weibull, H-stat, n=10
UCL > Truemean
Skewness
Weibull, H-stat, n=10
UCL > Truemean
Skewness
Weibull, t-test, n=20
UCL > Truemean
Skewness
Weibull, t-test, n=20
UCL > Truemean
Skewness
Weibull, t-test, n=20
UCL > Truemean
Skewness

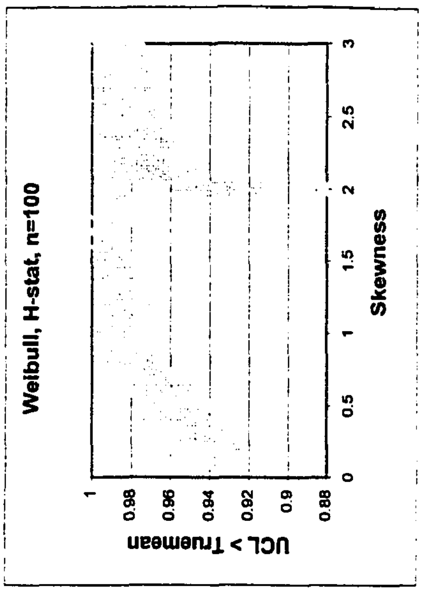

Weibull, Cheby, n=20
UCL > Truemean
Skewness
Weibull, Cheby, n=20
UCL > Truemean
Skewness
Weibull, Cheby, n=20
UCL > Truemean
Skewness
Weibull, H-stat, n=20
UCL > Truemean
Skewness
Weibull, H-stat, n=20
UCL > Truemean
Skewness
Weibull, H-stat, n=20
UCL > Truemean
Skewness

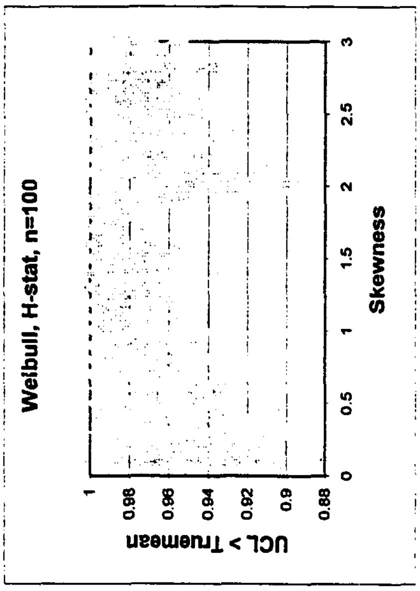

Weibull, t-test, n=30
UCL > Truemean
1
0.9
0.8
0.7
0.6
0.5
0
0.5
1
1.5
2
2.5
3
Skewness
Weibull, t-test, n=30
UCL > Truemean
1
0.95
0.9
0.85
0.8
0.75
0.7
0
0.5
1
1.5
2
2.5
3
Skewness
Weibull, t-test, n=30
UCL > Truemean
1
0.95
0.9
0.85
0.8
0
0.5
1
1.5
2
2.5
3
Skewness
Weibull, Cheby, n=30
UCL > Truemean
1
0.98
0.96
0.94
0.92
0.9
0.88
0.86
0.84
0.82
0.8
0
0.5
1
1.5
2
2.5
3
Skewness
Weibull, Cheby, n=30
UCL > Truemean
1
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0
0.5
1
1.5
2
2.5
3
Skewness
Weibull, Cheby, n=30
UCL > Truemean
1
0.99
0.98
0.97
0.96
0
0.5
1
1.5
2
2.5
3
Skewness

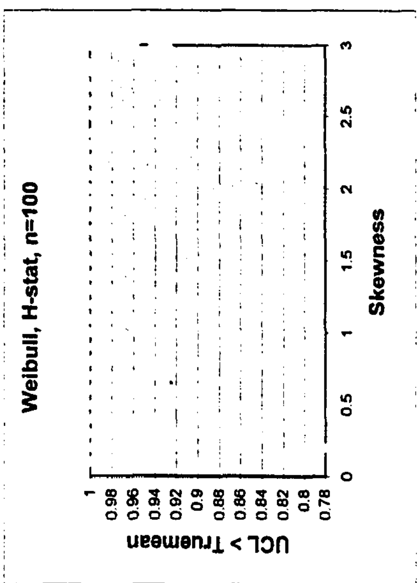

Weibull, H-stat, n=30
Weibull, H-stat, n=30
Weibull, H-stat, n=30
Weibull, t-test, n=50
Weibull, t-test, n=50
Weibull, t-test, n=50
UCL > Truemean
Skewness


Weibull, Cheby, n=50
UCL > Truemean
Skewness
Weibull, Cheby, n=50
UCL > Truemean
Skewness
Weibull, Cheby, n=50
UCL > Truemean
Skewness
Weibull, H-stat, n=50
UCL > Truemean
Skewness
Weibull, H-stat, n=50
UCL > Truemean
Skewness
Weibull, H-stat, n=50
UCL > Truemean
Skewness


Weibull, t-test, n=100
Weibull, t-test, n=100
Weibull, t-test, n=100
Weibull, Cheby, n=100
Weibull, Cheby, n=100
Weibull, Cheby, n=100
UCL > Truemean
Skewness


Weibull, H-stat, n=100
UCL > Truemean
1
0.98
0.96
0.94
0.92
0.9
0.88
0
0.5
1
1.5
2
2.5
3
Skewness

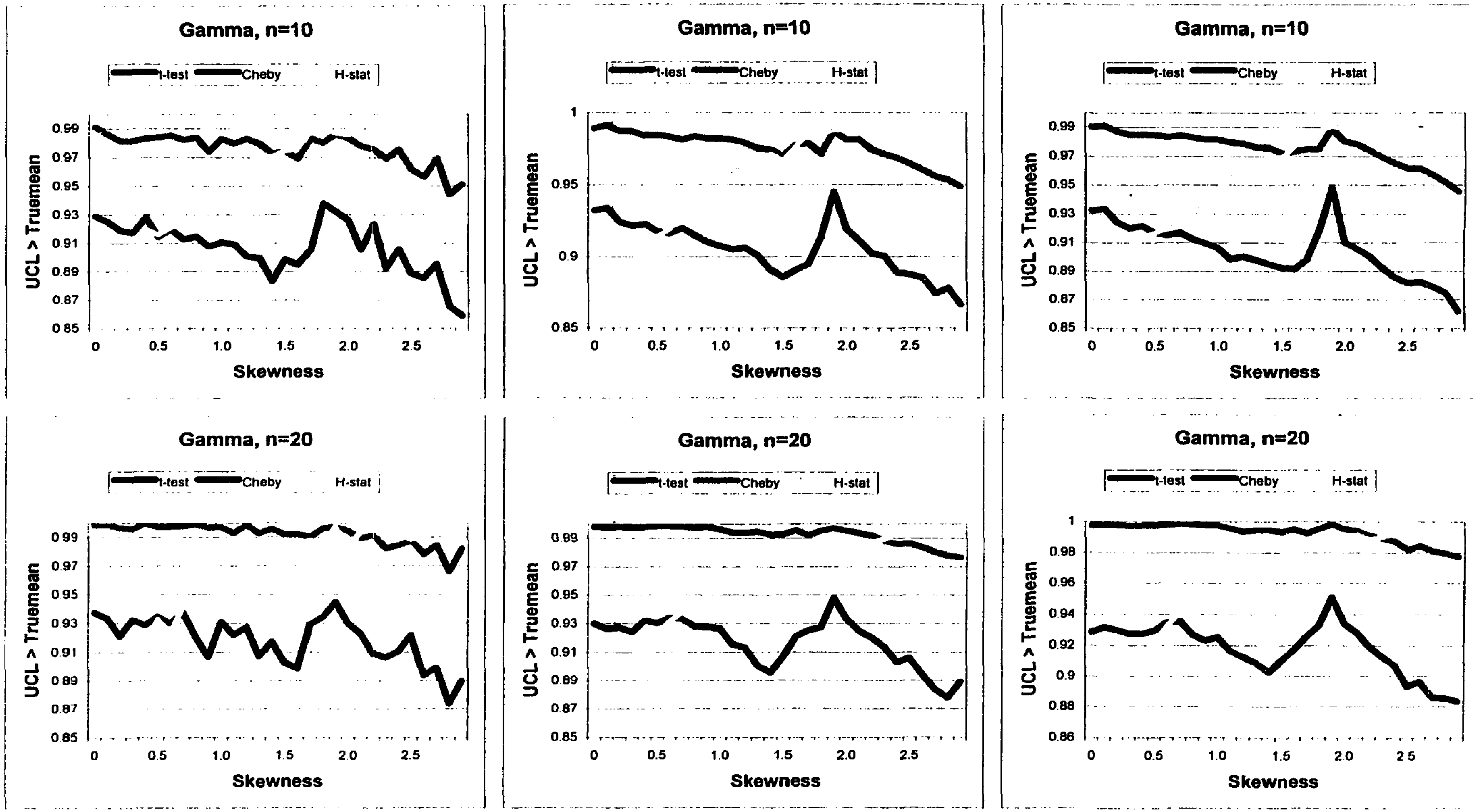

Weibull, H-stat, n=100
UCL > Truemean
1
0.98
0.96
0.94
0.92
0.9
0.88
0
0.5
1
1.5
2
2.5
3
Skewness

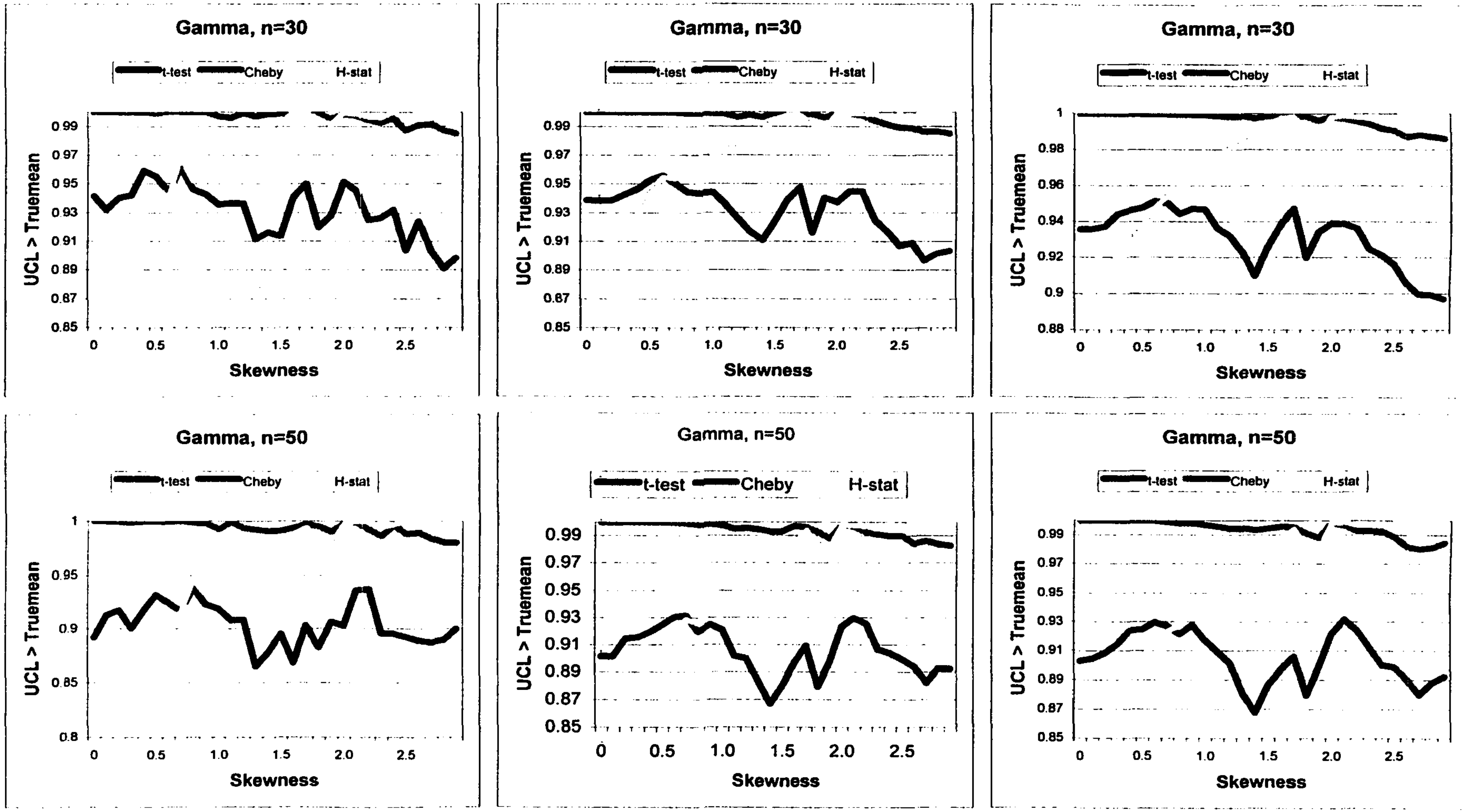

Weibull, H-stat, n=100
UCL > Truemean
1
0.98
0.96
0.94
0.92
0.9
0.88
0.86
0.84
0.82
0.8
0.78
0
0.5
1
1.5
2
2.5
3
Skewness

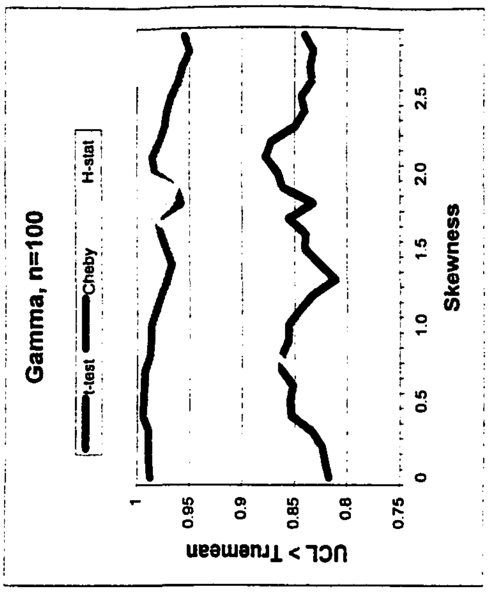
The following are the averages of the coverage at each skewness value. The results of the three test statistics are put onto a single graph for comparison.

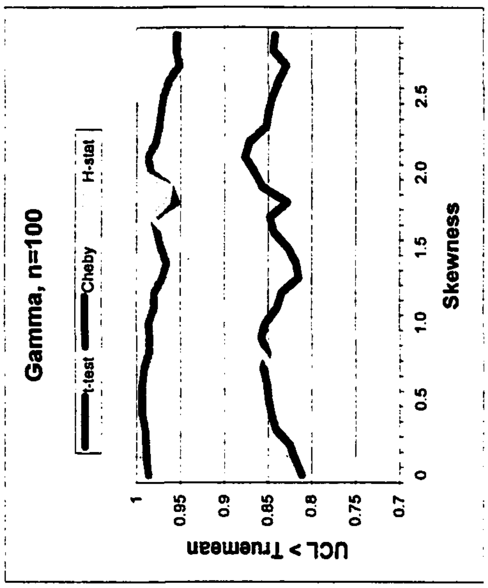
Figure 7 - Gamma Average Coverage Probabilities

100 testruns | 1,000 testruns | 10,000 testruns




Gamma, n=30
t-test Cheby H-stat
UCL > Truemean
Skewness
Gamma, n=30
t-test Cheby H-stat
UCL > Truemean
Skewness
Gamma, n=30
t-test Cheby H-stat
UCL > Truemean
Skewness
Gamma, n=50
t-test Cheby H-stat
UCL > Truemean
Skewness
Gamma, n=50
t-test Cheby H-stat
UCL > Truemean
Skewness
Gamma, n=50
t-test Cheby H-stat
UCL > Truemean
Skewness


Gamma, n=100
t-test
Cheby
H-stat
UCL > Truemean
Skewness


Gamma, n=100
t-test
Cheby
H-stat
UCL > Truemean
Skewness

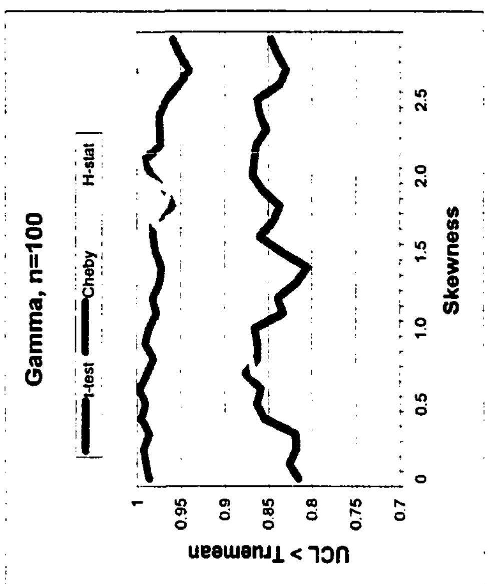
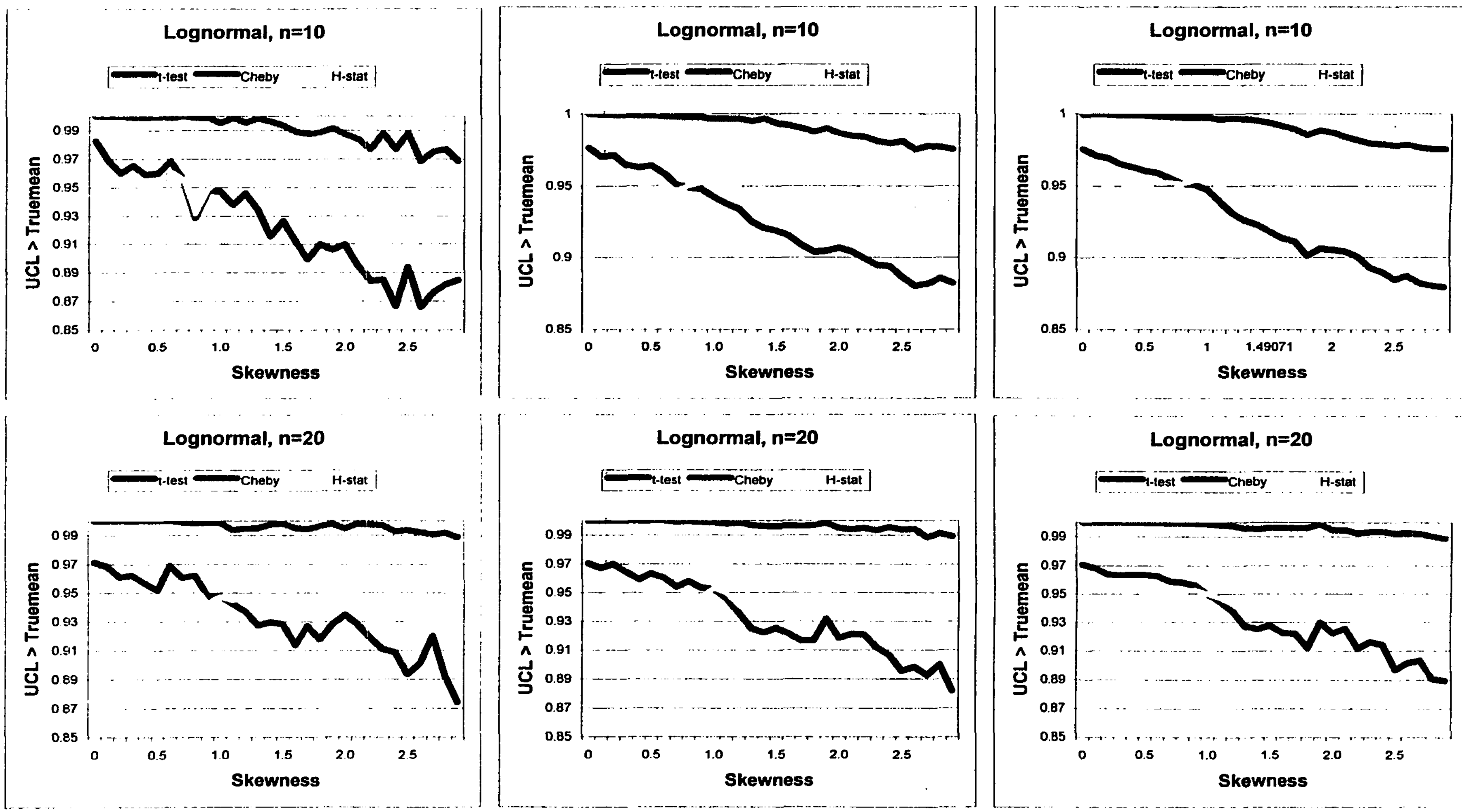
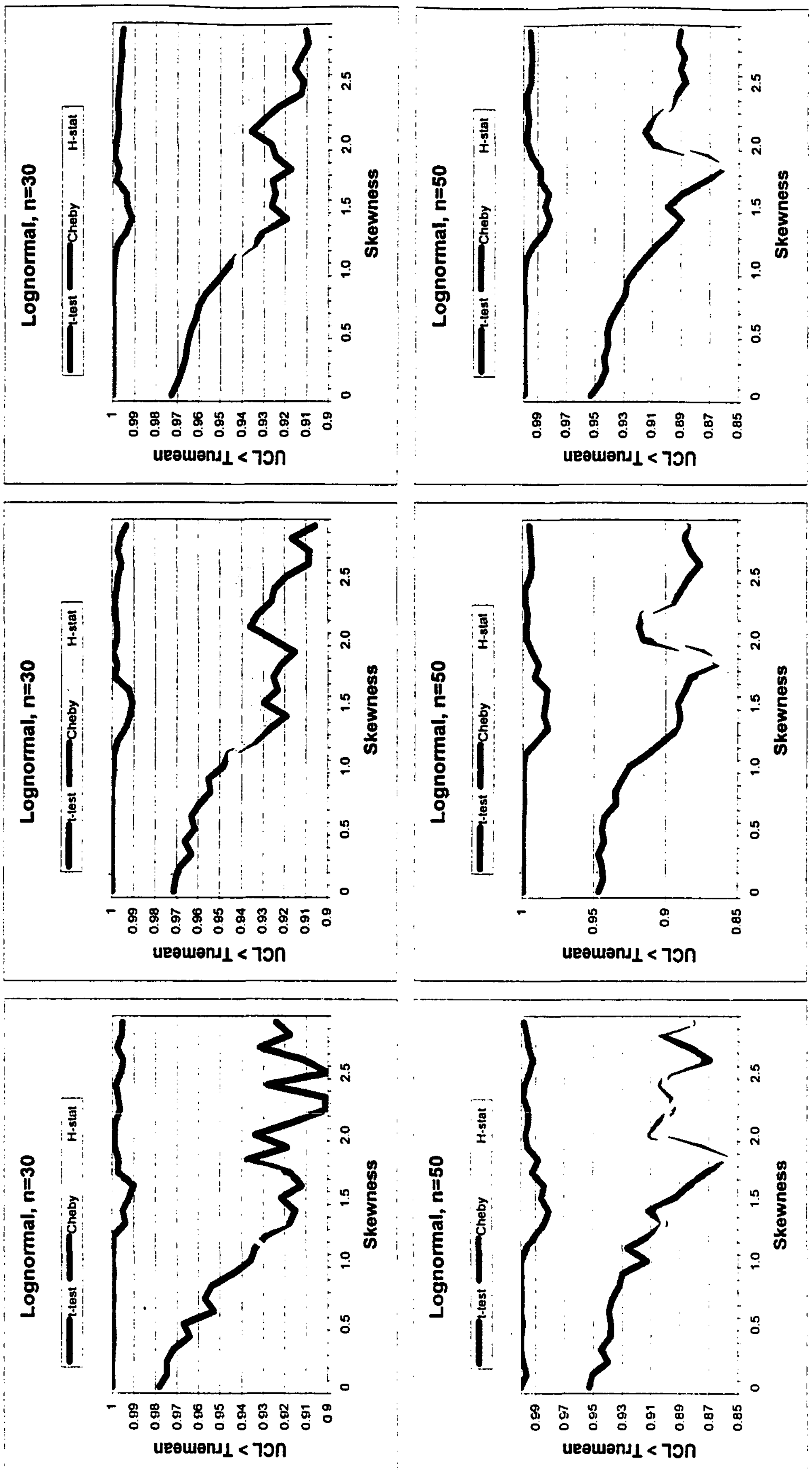
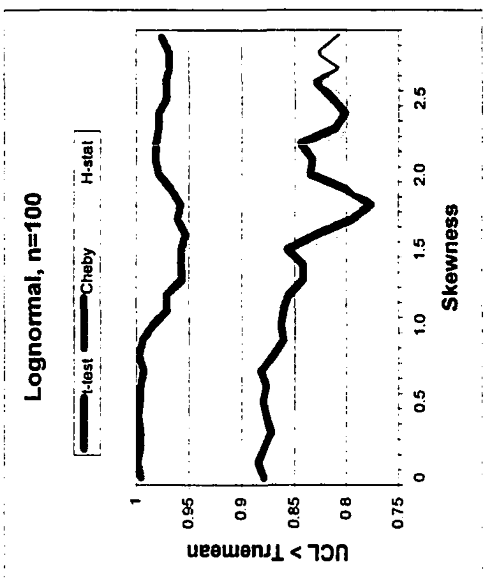

Gamma, n=100
t-test
Cheby
H-stat
UCL > Truemean
Skewness

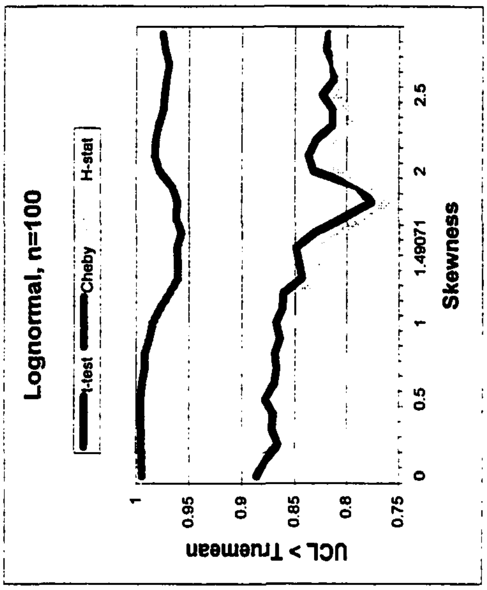
Figure 8 - Lognormal Average Coverage Probabilities

100 testruns | 1,000 testruns | 10,000 testruns




Lognormal, n=30
t-test
Cheby
H-stat
UCL > Truemean
Skewness
Lognormal, n=30
t-test
Cheby
H-stat
UCL > Truemean
Skewness
Lognormal, n=30
t-test
Cheby
H-stat
UCL > Truemean
Skewness
Lognormal, n=50
t-test
Cheby
H-stat
UCL > Truemean
Skewness
Lognormal, n=50
t-test
Cheby
H-stat
UCL > Truemean
Skewness
Lognormal, n=50
t-test
Cheby
H-stat
UCL > Truemean
Skewness


Lognormal, n=100
t-test
Cheby
H-stat
UCL > Truemean
1
0.95
0.9
0.85
0.8
0.75
0
0.5
1.0
1.5
2.0
2.5
Skewness


Lognormal, n=100
t-test
Cheby
H-stat
UCL > Truemean
1
0.95
0.9
0.85
0.8
0.75
0
0.5
1
1.49071
2
2.5
Skewness

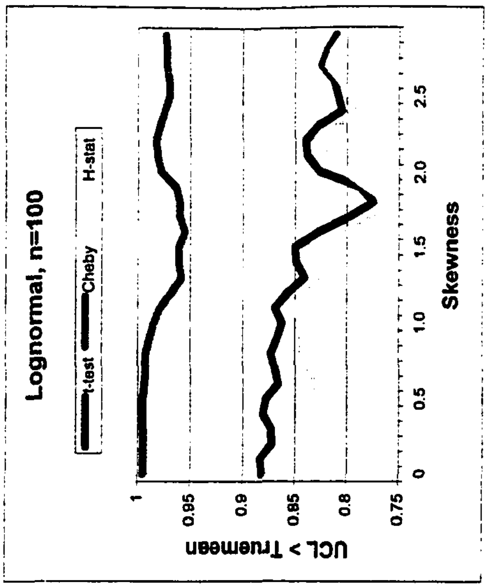
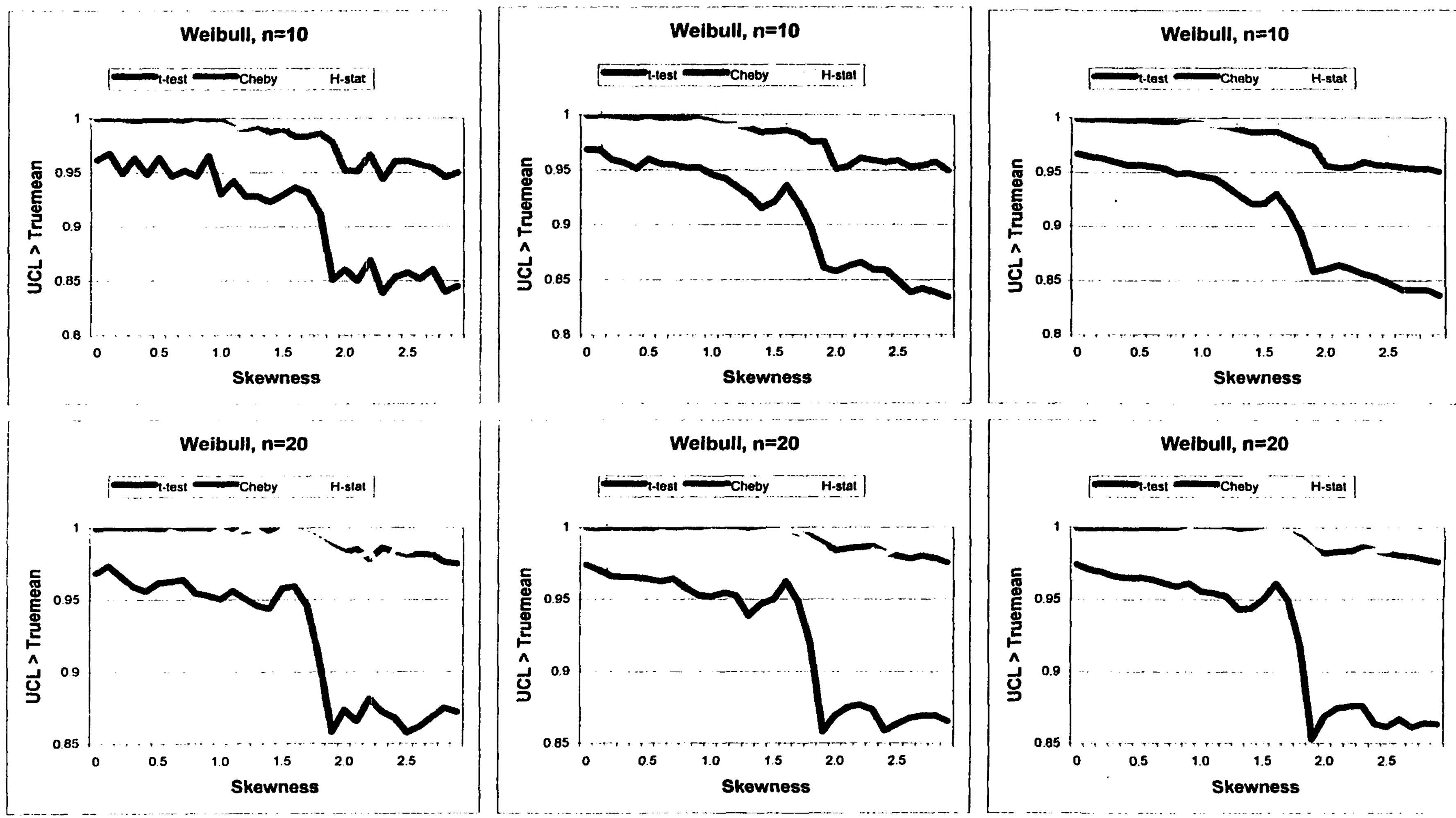
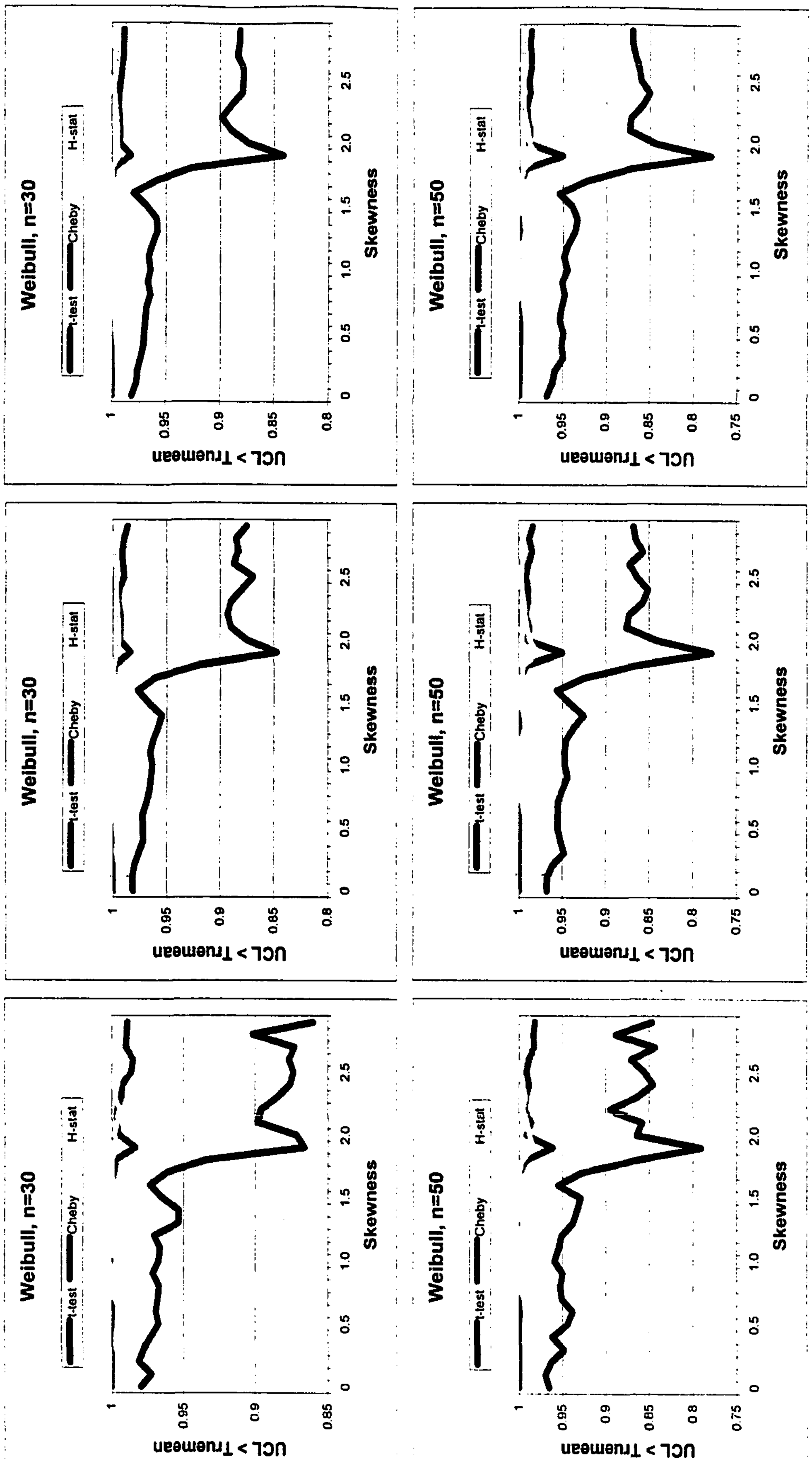
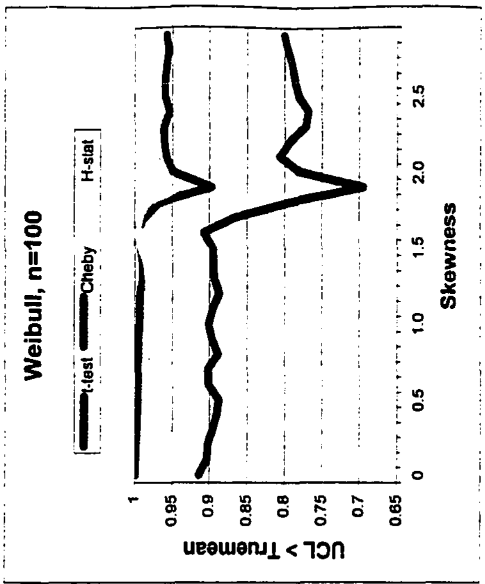

Lognormal, n=100
t-test
Cheby
H-stat
UCL > Truemean
1
0.95
0.9
0.85
0.8
0.75
0
0.5
1.0
1.5
2.0
2.5
Skewness

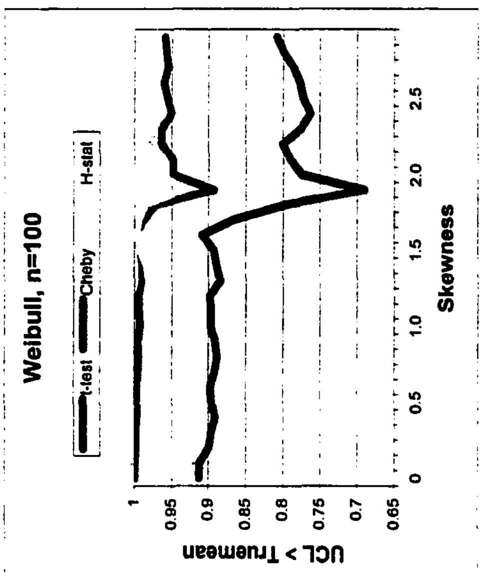
Figure 9 - Weibull Average Coverage Probabilities

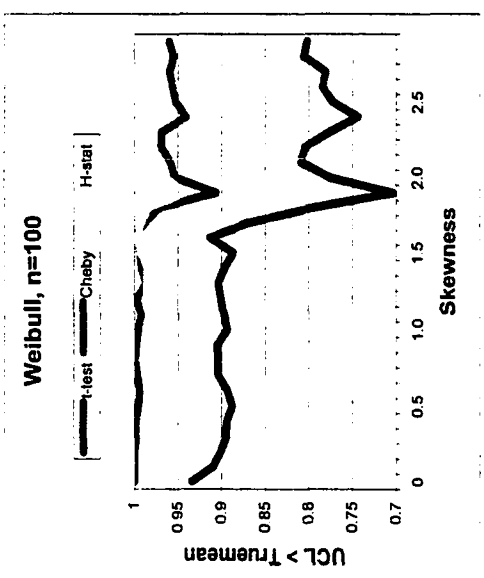
100 testruns 1,000 testruns 10,000 testruns




Weibull, n=30
Weibull, n=50
t-test
Cheby
H-stat
UCL > Truemean
Skewness


Weibull, n=100
t-test
Cheby
H-stat
UCL > Truemean
Skewness
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0
0.5
1.0
1.5
2.0
2.5


Weibull, n=100
t-test
Cheby
H-stat
UCL > Truemean
Skewness
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0
0.5
1.0
1.5
2.0
2.5


Weibull, n=100
t-test
Cheby
H-stat
UCL > Truemean
Skewness
1
0.95
0.9
0.85
0.8
0.75
0.7
0
0.5
1.0
1.5
2.0
2.5

From these graphs, we notice a few trends. First, the Chebychev test seems to give the highest reliability overall even as skewness grows. Secondly, there seems to be "jumps" in the coverage at different key points. Finally, within each skewness value, the points seemed to be evenly mixed over the varying parameter μ or β.

Looking at each of the graphs for each test, it appears that they don't look related at all as sample size increases. However, if you look carefully, you can see that as the sample size do increase, each peak and valley is exaggerated, thus indicating a higher variance of coverage depending upon sample size.

t-Statistic

Looking at the t-Statistic, although all of the average UCL coverage passes tend to decrease as skewness increases, none of them seem to look similar in their descent. In addition, it seems that each of them has a point where they "dip" suddenly. These dips will be discussed later.

Chebychev

For all three distributions, the Chebychev is highly robust and reliant, regardless of distribution and only the average only dips below 95% once; in the case of the Weibull distribution. In addition, as shown by the descriptive statistics, it does not "overshoot" the true mean by very much, indicating a very conservative estimate.

H-Statistic

The H-Statistic was only intended to be used with Lognormal distributions, however we want to see the results if someone mistakenly assumes a Lognormal distribution when in fact it is not.

For the Lognormal distribution itself, none of the values ever reach to 95%. Oddly enough, for all three distributions, as the skewness increases, the reliability goes up as well with a guaranteed acceptance for the Weibull distribution and acceptance for the Gamma when skewness approximately 1.5, regardless of the sample size. This may seem good at first, but if you look at the descriptive statistics, it will overshoot the true mean by a tremendous amount. This will give misleading conclusions about what the true mean is and may result in unnecessary cleanups.

What effect does the other parameter not dependent upon skewness have on the outcomes? Applying regression analysis to see which factors are significant in the model (skewness and the other scale parameter), we get the following summary of p-values for each distribution with skewness, the scale parameter and the interaction:

Table 7 - Gamma Distribution p-values

				Size		
		n=10	n=20	n=30	n=50	n=100
	Number of Runs					
	100					
	Beta	0.096	0.609	0.36	0.685	0.374
	Skewness	0	0	0	0.002	0.061
	Beta*Skewness	0.155	0.99	0.104	0.917	0.772
t-Stat	1,000					
	Beta	0.723	0.354	0.598	0.752	0.188
	Skewness	0	0	0	0	0
	Beta*Skewness	0.006	0.064	0.152	0.593	0.126
	10,000					
	Beta	0.453	0.49	0.842	0.804	0.849
	Skewness	0	0	0	0	0
	Beta*Skewness	0.113	0.445	0.286	0.977	0.552
		n=10	n=20	n=30	n=50	n=100
	100					
	Beta	0.31	0.855	0.271	0.875	0.77
	Skewness	0	0	0	0	0
	Beta*Skewness	0.004	0.718	0.185	0.946	0.686
Cheby	1,000					
	Beta	0.414	0.181	0.403	0.727	0.312
	Skewness	0	0	0	0	0
	Beta*Skewness	0.009	0.045	0.757	0.811	0.269
	10,000					
	Beta	0.138	0.785	0.77	0.787	0.79
	Skewness	0	0	0	0	0
	Beta*Skewness	0.008	0.689	0.838	0.842	0.5
		n=10	n=20	n=30	n=50	n=100
	100					
	Beta	0.838	0.694	0.749	0.384	0.892
	Skewness	0	0	0	0	0
	Beta*Skewness	0.883	0.153	0.685	0.433	0.495
H-Stat	1,000					
	Beta	0.05	0.111	0.086	0.496	0.693
	Skewness	0	0	0	0	0
	Beta*Skewness	0.023	0.008	0.058	0.623	0.551
	10,000					
	Beta	0.695	0.881	0.459	0.982	0.796
	Skewness	0	0	0	0	0
	Beta*Skewness	0.447	0.655	0.213	0.972	0.854

Table 8 - Lognormal Distribution p-values

				Size		
		n=10	n=20	n=30	n=50	n=100
	Number of Runs					
	100					
	Mu	0.426	0.751	0.79	0.896	0.923
	Skewness	0	0	0	0	0
	Mu*Skewness	0.695	0.273	0.179	0.412	0.352
t-Stat	**1,000**					
	Mu	0.005	0.543	0.992	0.837	0.123
	Skewness	0	0	0	0	0
	Mu*Skewness	0.002	0.156	0.451	0.38	0.202
	10,000					
	Mu	0.325	0.473	0.485	0.948	0.376
	Skewness	0	0	0	0	0
	Mu*Skewness	0.124	0.798	0.476	0.975	0.522
		n=10	**n=20**	**n=30**	**n=50**	**n=100**
	100					
	Mu	0.327	0.253	0.006	0.8	0.178
	Skewness	0	0	0	0.004	0
	Mu*Skewness	0.862	0.283	0.023	0.511	0.119
Cheby	**1,000**					
	Mu	0.327	0.253	0.006	0.8	0.178
	Skewness	0	0	0	0.004	0
	Mu*Skewness	0.862	0.283	0.023	0.511	0.119
	10,000					
	Mu	0.292	0.733	0.929	0.715	0.868
	Skewness	0	0	0	0	0
	Mu*Skewness	0.065	0.917	0.82	0.954	0.978
		n=10	**n=20**	**n=30**	**n=50**	**n=100**
	100					
	Mu	0.456	0.889	0.795	0.914	0.781
	Skewness	0.002	0.119	0.021	0	0
	Mu*Skewness	0.278	0.306	0.336	0.451	0.339
H-Stat	**1,000**					
	Mu	0.342	0.227	0.699	0.927	0.123
	Skewness	0	0	0	0	0
	Mu*Skewness	0.059	0.007	0.269	0.542	0.093
	10,000					
	Mu	0.874	0.568	0.779	0.791	0.266
	Skewness	0	0	0	0	0
	Mu*Skewness	0.352	0.962	0.574	0.83	0.471

Table 9 - Weibull Distribution p-values

				Size		
		n=10	n=20	n=30	n=50	n=100
	Number of Runs					
	100					
	Beta	0.613	0.368	0.433	0.528	0.256
	Skewness	0	0	0	0	0
	Beta*Skewness	0.072	0.07	0.425	0.073	0.712
t-Stat	**1,000**					
	Beta	0.469	0.029	0.926	0.669	0.357
	Skewness	0	0	0	0	0
	Beta*Skewness	0.876	0.002	0.59	0.471	0.933
	10,000					
	Beta	0.809	0.988	0.829	0.897	0.77
	Skewness	0	0	0	0	0
	Beta*Skewness	0.752	0.783	0.994	0.675	0.654
		n=10	n=20	n=30	n=50	n=100
	100					
	Beta	0.829	0.295	0.477	0.637	0.112
	Skewness	0	0	0	0	0
	Beta*Skewness	0.964	0.118	0.318	0.587	0.078
Cheby	**1,000**					
	Beta	0.963	0.006	0.388	0.894	0.704
	Skewness	0	0	0	0	0
	Beta*Skewness	0.497	0.003	0.357	0.645	0.696
	10,000					
	Beta	0.661	0.847	0.661	0.688	0.774
	Skewness	0	0	0	0	0
	Beta*Skewness	0.665	0.698	0.638	0.773	0.884
		n=10	n=20	n=30	n=50	n=100
	100					
	Beta	0.986	0.031	0.899	0.185	0.545
	Skewness	0.005	0	0.002	0	0
	Beta*Skewness	0.564	0	0.957	0.623	0.006
H-Stat	**1,000**					
	Beta	0.268	0	0.382	0.334	0.396
	Skewness	0	0	0.063	0	0
	Beta*Skewness	0.452	0.004	0.567	0.774	0.575
	10,000					
	Beta	0.998	0.713	0.702	0.4	0.766
	Skewness	0	0	0	0	0
	Beta*Skewness	0.638	0.923	0.269	0.554	0.607

The full ANOVA table for this summary is given in Appendix C.

As we can see in the long term under 10,000 runs, regardless of the distribution and regardless of the UCL test used, the only significant factor is skewness. Both the scale parameter and the interaction are not significant. This shows that one in general does not need to worry about the scale parameter when measuring coverage.

Discrepancies

There are a couple of discrepancies that cannot be explained. One is when the skewness is equal to 2 for both the Weibull and Gamma distributions. Mathematically, both of these distributions reduce to the same function, simplifying to the Exponential distribution. This would imply that the percentages for the different tests at that point should be very close together. However, when we analyze the data, we find out it is not always the case, especially for the t-test, where the average varies by as much as almost 17% when n=100.

Table 10 - Gamma and Weibull coverage at Skewness = 2

Skewness = 2					
100 runs					
t-test	**n=10**	**n=20**	**n=30**	**n=50**	**n=100**
Gamma	0.93	0.94	0.93	0.91	0.86
Weibull	0.85	0.86	0.87	0.79	0.70
Difference	**0.08**	**0.09**	**0.06**	**0.12**	**0.15**
Cheby					
Gamma	0.99	1.00	1.00	0.99	0.97
Weibull	0.98	0.99	0.98	0.96	0.91
Difference	**0.01**	**0.01**	**0.01**	**0.03**	**0.06**
H-Stat					
Gamma	0.99	1.00	0.99	0.98	0.96
Weibull	0.99	0.99	1.00	0.98	0.94
Difference	**0.00**	**0.01**	**0.01**	**0.01**	**0.02**
1,000 runs					
t-test	**n=10**	**n=20**	**n=30**	**n=50**	**n=100**
Gamma	0.94	0.95	0.94	0.90	0.86
Weibull	0.86	0.86	0.85	0.78	0.69
Difference	**0.08**	**0.09**	**0.09**	**0.12**	**0.17**
Cheby					
Gamma	0.99	1.00	1.00	0.99	0.96
Weibull	0.98	0.99	0.98	0.95	0.89
Difference	**0.01**	**0.01**	**0.01**	**0.04**	**0.07**
H-Stat					
Gamma	0.99	0.99	0.99	0.98	0.96
Weibull	0.99	0.99	1.00	0.98	0.93
Difference	**0.00**	**0.00**	**0.01**	**0.00**	**0.03**
10,000 runs					
t-test	**n=10**	**n=20**	**n=30**	**n=50**	**n=100**
Gamma	0.95	0.95	0.93	0.90	0.86
Weibull	0.86	0.85	0.84	0.78	0.69
Difference	**0.09**	**0.10**	**0.09**	**0.12**	**0.17**
Cheby					
Gamma	0.99	1.00	1.00	0.99	0.96
Weibull	0.97	0.98	0.98	0.95	0.90
Difference	**0.02**	**0.01**	**0.01**	**0.04**	**0.07**
H-Stat					
Gamma	0.99	0.99	0.99	0.98	0.96
Weibull	0.99	0.98	0.99	0.98	0.94
Difference	**0.00**	**0.01**	**0.00**	**0.00**	**0.03**

Why would the differences in coverage percentages increase as sample size increases? This is counterintuitive. One reason is that if we look at the standard deviations for the individual values (varying over β) we can see how large they become.

The important thing here to understand is that even if different variations on the distributions produce a close curve, there is no guarantee that the end results will be as close.

Another major enigma is that of the "dips" that occur at certain points of skewness. These dips are not because of an error in the production of random values, however the true cause is not known. This is worth further investigation.

Conclusions

We can see that as skewness increases for each of the distributions, the coverage becomes increasingly better or worse depending upon the method used.

For the different distributions, regardless of the method used, it seems on the average that once skewness grows larger than 1, the distribution becomes unstable. If at all possible, try to reduce the skewness when collecting data.

Since the t-Statistic is used for Normally distributed data, it is not recommended for data that is not certainly to be Normal, because as seen, as skewness increases, the coverage rapidly decreases (variance went only as high as 0.5 in this experiment).

Given the results of the H-Statistic, it should be avoided being used unless you are certain you are dealing with a Lognormally distributed data and with smaller sample sizes as it was seen that larger sample sizes tend to decrease the coverage. And since it's been shown that often times a Lognormal is misdiagnosed (Singh, 1997), it is discouraged from being used at all. Although the H-Statistic does give good coverage of the Gamma and Weibull distributions, it tremendously overshoots the true mean, thus will almost always suggest a cleanup which may be unnecessary.

The Chebychev Inequality is not only a reliable test that is minimally affected by skewness, but it also the only one which consistently gives a conservative estimate. Given this, if you aren't sure what distribution the data is giving you, especially if it is skewed, use the Chebychev Inequality for testing.

Further Research

It would be interesting to see how quickly or slowly it would take for each of the distributions for each of the tests to reach the full range of coverage from 0-100%. However, the skewness value would probably have to be quite high. It would also be interesting to see if there is a pattern to the "jumps" and if they are simply caused by computer error, or something that can be determined.

In addition, it would be useful to expand both the test in terms of including additional distributions (such as Normal), and include additional testing methods (Zhou, 1997).

APPENDIX A

COMPUTER CODE

The following code in MS-Visual Basic 6 was used to obtain all the raw data gathered in the thesis.

```
'output to file
Dim fso0, fso1, fso2, fso, fso3, fso4, fso5, fso6, fso7,
fso8, fso9, fso10, fso11
Dim fso12, fso13, fso14, fso15, fso16, fso17, fso18, fso20,
fso21, fso22, fso23
Dim fso24, fso25, fso26, fso27, fso28, fso29, fso30, fso31,
fso32, fso33, fso34
Dim fso35, fso36, fso37, fso38, fso39, fso40, fso41, fso42,
fso43, fso44, fso45
Dim fso46, fso47, fso48, fso49, fso50, fso51, fso52, fso53,
fso54, fso55, fso56
Dim fso57, fso58, fso59, fso60, fso61, fso62, fso63, fso64,
fso65, fso66, fso67
Dim fso68, fso69, fso70, fso71, fso72
Dim txtfile0, txtfile1, txtfile2, txtfile3, txtfile4,
txtfile5, txtfile6, txtfile7, txtfile8
Dim txtfile9, txtfile10, txtfile11, txtfile12, txtfile13,
txtfile14, txtfile15, txtfile16, txtfile17, txtfile18

'Create file outputs

Set fso10 = CreateObject("Scripting.FileSystemObject")
Set txtfile10 =
fso10.CreateTextFile("c:\stats\thesis\GammaAverageTucl100.t
xt", True)
Set fso11 = CreateObject("Scripting.FileSystemObject")
Set txtfile11 =
fso11.CreateTextFile("c:\stats\thesis\GammaAverageCucl100.t
xt", True)
Set fso12 = CreateObject("Scripting.FileSystemObject")
```

```
Set txtfile12 =
fso12.CreateTextFile("c:\stats\thesis\GammaAverageHucl100.t
xt", True)

Set fso13 = CreateObject("Scripting.FileSystemObject")
Set txtfile13 =
fso13.CreateTextFile("c:\stats\thesis\LognormalAverageTucl1
00.txt", True)
Set fso14 = CreateObject("Scripting.FileSystemObject")
Set txtfile14 =
fso14.CreateTextFile("c:\stats\thesis\LognormalAverageCucl1
00.txt", True)
Set fso15 = CreateObject("Scripting.FileSystemObject")
Set txtfile15 =
fso15.CreateTextFile("c:\stats\thesis\LognormalAverageHucl1
00.txt", True)

Set fso16 = CreateObject("Scripting.FileSystemObject")
Set txtfile16 =
fso16.CreateTextFile("c:\stats\thesis\WeibullAverageTucl100
.txt", True)
Set fso17 = CreateObject("Scripting.FileSystemObject")
Set txtfile17 =
fso17.CreateTextFile("c:\stats\thesis\WeibullAverageCucl100
.txt", True)
Set fso18 = CreateObject("Scripting.FileSystemObject")
Set txtfile18 =
fso18.CreateTextFile("c:\stats\thesis\WeibullAverageHucl100
.txt", True)

Set fso19 = CreateObject("Scripting.FileSystemObject")
Set txtfile19 =
fso19.CreateTextFile("c:\stats\thesis\GammaStdDevTucl100.tx
t", True)
Set fso20 = CreateObject("Scripting.FileSystemObject")
Set txtfile20 =
fso20.CreateTextFile("c:\stats\thesis\GammaStdDevCucl100.tx
t", True)
Set fso21 = CreateObject("Scripting.FileSystemObject")
Set txtfile21 =
fso21.CreateTextFile("c:\stats\thesis\GammaStdDevHucl100.tx
t", True)

Set fso22 = CreateObject("Scripting.FileSystemObject")
Set txtfile22 =
fso22.CreateTextFile("c:\stats\thesis\LognormalStdDevTucl10
0.txt", True)
```

```
Set fso23 = CreateObject("Scripting.FileSystemObject")
Set txtfile23 =
fso23.CreateTextFile("c:\stats\thesis\LognormalStdDevCucl10
0.txt", True)
Set fso24 = CreateObject("Scripting.FileSystemObject")
Set txtfile24 =
fso24.CreateTextFile("c:\stats\thesis\LognormalStdDevHucl10
0.txt", True)

Set fso25 = CreateObject("Scripting.FileSystemObject")
Set txtfile25 =
fso25.CreateTextFile("c:\stats\thesis\WeibullStdDevTucl100.
txt", True)
Set fso26 = CreateObject("Scripting.FileSystemObject")
Set txtfile26 =
fso26.CreateTextFile("c:\stats\thesis\WeibullStdDevCucl100.
txt", True)
Set fso27 = CreateObject("Scripting.FileSystemObject")
Set txtfile27 =
fso27.CreateTextFile("c:\stats\thesis\WeibullStdDevHucl100.
txt", True)

Set fso28 = CreateObject("Scripting.FileSystemObject")
Set txtfile28 =
fso28.CreateTextFile("c:\stats\thesis\GammaMedianTucl100.tx
t", True)
Set fso29 = CreateObject("Scripting.FileSystemObject")
Set txtfile29 =
fso29.CreateTextFile("c:\stats\thesis\GammaMedianCucl100.tx
t", True)
Set fso30 = CreateObject("Scripting.FileSystemObject")
Set txtfile30 =
fso30.CreateTextFile("c:\stats\thesis\GammaMedianHucl100.tx
t", True)

Set fso31 = CreateObject("Scripting.FileSystemObject")
Set txtfile31 =
fso31.CreateTextFile("c:\stats\thesis\LognormalMedianTucl10
0.txt", True)
Set fso32 = CreateObject("Scripting.FileSystemObject")
Set txtfile32 =
fso32.CreateTextFile("c:\stats\thesis\LognormalMedianCucl10
0.txt", True)
Set fso33 = CreateObject("Scripting.FileSystemObject")
Set txtfile33 =
fso33.CreateTextFile("c:\stats\thesis\LognormalMedianHucl10
0.txt", True)
```

```
Set fso34 = CreateObject("Scripting.FileSystemObject")
Set txtfile34 =
fso34.CreateTextFile("c:\stats\thesis\WeibullMedianTucl100.
txt", True)
Set fso35 = CreateObject("Scripting.FileSystemObject")
Set txtfile35 =
fso35.CreateTextFile("c:\stats\thesis\WeibullMedianCucl100.
txt", True)
Set fso36 = CreateObject("Scripting.FileSystemObject")
Set txtfile36 =
fso36.CreateTextFile("c:\stats\thesis\WeibullMedianHucl100.
txt", True)

Set fso37 = CreateObject("Scripting.FileSystemObject")
Set txtfile37 =
fso37.CreateTextFile("c:\stats\thesis\GammaMinTucl100.txt",
True)
Set fso38 = CreateObject("Scripting.FileSystemObject")
Set txtfile38 =
fso38.CreateTextFile("c:\stats\thesis\GammaMinCucl100.txt",
True)
Set fso39 = CreateObject("Scripting.FileSystemObject")
Set txtfile39 =
fso39.CreateTextFile("c:\stats\thesis\GammaMinHucl100.txt",
True)

Set fso40 = CreateObject("Scripting.FileSystemObject")
Set txtfile40 =
fso40.CreateTextFile("c:\stats\thesis\LognormalMinTucl100.t
xt", True)
Set fso41 = CreateObject("Scripting.FileSystemObject")
Set txtfile41 =
fso41.CreateTextFile("c:\stats\thesis\LognormalMinCucl100.t
xt", True)
Set fso42 = CreateObject("Scripting.FileSystemObject")
Set txtfile42 =
fso42.CreateTextFile("c:\stats\thesis\LognormalMinHucl100.t
xt", True)

Set fso43 = CreateObject("Scripting.FileSystemObject")
Set txtfile43 =
fso43.CreateTextFile("c:\stats\thesis\WeibullMinTucl100.txt
", True)
Set fso44 = CreateObject("Scripting.FileSystemObject")
```

```
Set txtfile44 =
fso44.CreateTextFile("c:\stats\thesis\WeibullMinCucl100.txt
", True)
Set fso45 = CreateObject("Scripting.FileSystemObject")
Set txtfile45 =
fso45.CreateTextFile("c:\stats\thesis\WeibullMinHucl100.txt
", True)

Set fso46 = CreateObject("Scripting.FileSystemObject")
Set txtfile46 =
fso46.CreateTextFile("c:\stats\thesis\GammaMaxTucl100.txt",
True)
Set fso47 = CreateObject("Scripting.FileSystemObject")
Set txtfile47 =
fso47.CreateTextFile("c:\stats\thesis\GammaMaxCucl100.txt",
True)
Set fso48 = CreateObject("Scripting.FileSystemObject")
Set txtfile48 =
fso48.CreateTextFile("c:\stats\thesis\GammaMaxHucl100.txt",
True)

Set fso49 = CreateObject("Scripting.FileSystemObject")
Set txtfile49 =
fso49.CreateTextFile("c:\stats\thesis\LognormalMaxTucl100.t
xt", True)
Set fso50 = CreateObject("Scripting.FileSystemObject")
Set txtfile50 =
fso50.CreateTextFile("c:\stats\thesis\LognormalMaxCucl100.t
xt", True)
Set fso51 = CreateObject("Scripting.FileSystemObject")
Set txtfile51 =
fso51.CreateTextFile("c:\stats\thesis\LognormalMaxHucl100.t
xt", True)

Set fso52 = CreateObject("Scripting.FileSystemObject")
Set txtfile52 =
fso52.CreateTextFile("c:\stats\thesis\WeibullMaxTucl100.txt
", True)
Set fso53 = CreateObject("Scripting.FileSystemObject")
Set txtfile53 =
fso53.CreateTextFile("c:\stats\thesis\WeibullMaxCucl100.txt
", True)
Set fso54 = CreateObject("Scripting.FileSystemObject")
Set txtfile54 =
fso54.CreateTextFile("c:\stats\thesis\WeibullMaxHucl100.txt
", True)
```

```
Set fso55 = CreateObject("Scripting.FileSystemObject")
Set txtfile55 =
fso55.CreateTextFile("c:\stats\thesis\GammaQ1Tucl100.txt",
True)
Set fso56 = CreateObject("Scripting.FileSystemObject")
Set txtfile56 =
fso56.CreateTextFile("c:\stats\thesis\GammaQ1Cucl100.txt",
True)
Set fso57 = CreateObject("Scripting.FileSystemObject")
Set txtfile57 =
fso57.CreateTextFile("c:\stats\thesis\GammaQ1Hucl100.txt",
True)

Set fso58 = CreateObject("Scripting.FileSystemObject")
Set txtfile58 =
fso58.CreateTextFile("c:\stats\thesis\LognormalQ1Tucl100.tx
t", True)
Set fso59 = CreateObject("Scripting.FileSystemObject")
Set txtfile59 =
fso59.CreateTextFile("c:\stats\thesis\LognormalQ1Cucl100.tx
t", True)
Set fso60 = CreateObject("Scripting.FileSystemObject")
Set txtfile60 =
fso60.CreateTextFile("c:\stats\thesis\LognormalQ1Hucl100.tx
t", True)

Set fso61 = CreateObject("Scripting.FileSystemObject")
Set txtfile61 =
fso61.CreateTextFile("c:\stats\thesis\WeibullQ1Tucl100.txt"
, True)
Set fso62 = CreateObject("Scripting.FileSystemObject")
Set txtfile62 =
fso62.CreateTextFile("c:\stats\thesis\WeibullQ1Cucl100.txt"
, True)
Set fso63 = CreateObject("Scripting.FileSystemObject")
Set txtfile63 =
fso63.CreateTextFile("c:\stats\thesis\WeibullQ1Hucl100.txt"
, True)

Set fso64 = CreateObject("Scripting.FileSystemObject")
Set txtfile64 =
fso64.CreateTextFile("c:\stats\thesis\GammaQ3Tucl100.txt",
True)
Set fso65 = CreateObject("Scripting.FileSystemObject")
Set txtfile65 =
fso65.CreateTextFile("c:\stats\thesis\GammaQ3Cucl100.txt",
True)
```

```
Set fso66 = CreateObject("Scripting.FileSystemObject")
Set txtfile66 =
fso66.CreateTextFile("c:\stats\thesis\GammaQ3Hucl100.txt",
True)

Set fso67 = CreateObject("Scripting.FileSystemObject")
Set txtfile67 =
fso67.CreateTextFile("c:\stats\thesis\LognormalQ3Tucl100.tx
t", True)
Set fso68 = CreateObject("Scripting.FileSystemObject")
Set txtfile68 =
fso68.CreateTextFile("c:\stats\thesis\LognormalQ3Cucl100.tx
t", True)
Set fso69 = CreateObject("Scripting.FileSystemObject")
Set txtfile69 =
fso69.CreateTextFile("c:\stats\thesis\LognormalQ3Hucl100.tx
t", True)

Set fso70 = CreateObject("Scripting.FileSystemObject")
Set txtfile70 =
fso70.CreateTextFile("c:\stats\thesis\WeibullQ3Tucl100.txt"
, True)
Set fso71 = CreateObject("Scripting.FileSystemObject")
Set txtfile71 =
fso71.CreateTextFile("c:\stats\thesis\WeibullQ3Cucl100.txt"
, True)
Set fso72 = CreateObject("Scripting.FileSystemObject")
Set txtfile72 =
fso72.CreateTextFile("c:\stats\thesis\WeibullQ3Hucl100.txt"
, True)

Dim samplesize(5) As Integer 'Sample size.  Determines
number of rows in data matrix
Dim m(50) As Single   'Stores scale parameters

'Stores skewness parameters

Dim sGamma(30) As Single
Dim sLognormal(30) As Single
Dim sWeibull(30) As Single

'Vector or random data

Dim GammaRandomVector(100) As Single
Dim LognormalRandomVector(100) As Single
Dim WeibullRandomVector(100) As Single
```

```
'Stores Chebychev, t-test, and H-Statistic UCL values for
distributions
'Size of array depends upon number of testruns (100, 1,000,
10,000)
Dim GammaCucl(100) As Single
Dim GammaTucl(100) As Single
Dim GammaHucl(100) As Single
Dim LognormalCucl(100) As Single
Dim LognormalTucl(100) As Single
Dim LognormalHucl(100) As Single
Dim WeibullCucl(100) As Single
Dim WeibullTucl(100) As Single
Dim WeibullHucl(100) As Single

Dim testruns As Integer 'number of times to run the test
Dim lowmean As Single
Dim lowvar As Single
Dim himean As Single
Dim hivar As Single
Dim n As Integer

'Number of times the UCL > truemean

Dim GammaTuclGreater As Integer
Dim GammaCuclGreater As Integer
Dim GammaHuclGreater As Integer
Dim LognormalTuclGreater As Integer
Dim LognormalCuclGreater As Integer
Dim LognormalHuclGreater As Integer
Dim WeibullTuclGreater As Integer
Dim WeibullCuclGreater As Integer
Dim WeibullHuclGreater As Integer

'Used to help generate shape and scale parameters
Dim meanstep As Single
Dim varstep As Single

'Sample mean and sample deviation
Dim GammaSampleMean As Single
Dim GammaSampleStddev As Single
Dim GammaLognormalSampleMean As Single
Dim GammaLognormalSampleVariance As Single
Dim LognormalSampleMean As Single
Dim LognormalSampleStddev As Single
Dim LognormalLognormalSampleMean As Single
Dim LognormalLognormalSampleVariance As Single
Dim WeibullSampleMean As Single
```

```
Dim WeibullSampleStddev As Single
Dim WeibullLognormalSampleMean As Single
Dim WeibullLognormalSampleVariance As Single

'Truemean for the distributions
Dim GammaTruemean As Single
Dim LognormalTruemean As Single
Dim WeibullTruemean As Single

'Value for t-statistic, depends upon sample size
Dim tvalue As Single

'Percentages for when (UCL > truemean) / number of testruns
Dim GammaTuclGreaterPercentage(7500) As Single
Dim GammaCuclGreaterPercentage(7500) As Single
Dim GammaHuclGreaterPercentage(7500) As Single
Dim LognormalTuclGreaterPercentage(7500) As Single
Dim LognormalCuclGreaterPercentage(7500) As Single
Dim LognormalHuclGreaterPercentage(7500) As Single
Dim WeibullTuclGreaterPercentage(7500) As Single
Dim WeibullCuclGreaterPercentage(7500) As Single
Dim WeibullHuclGreaterPercentage(7500) As Single

'Average values for UCL tests
Dim GammaTuclAverage As Single
Dim GammaCuclAverage As Single
Dim GammaHuclAverage As Single
Dim LognormalTuclAverage As Single
Dim LognormalCuclAverage As Single
Dim LognormalHuclAverage As Single
Dim WeibullTuclAverage As Single
Dim WeibullCuclAverage As Single
Dim WeibullHuclAverage As Single

'Counters for loops
Dim i As Integer
Dim j As Integer
Dim k As Integer
Dim t As Integer
Dim z As Integer

samplesize(1) = 10
samplesize(2) = 20
samplesize(3) = 30
samplesize(4) = 50
samplesize(5) = 100
```

```
lowmean = 0.1
lowvar = 0.1
himean = 5
hivar = 2.1

meanstep = 0.1
varstep = 0.1

'code below creates the vectors of the means and variances
to use

For lowmean = 0 To himean Step meanstep
    m(i) = lowmean
    m(i) = Round(m(i), 1)
    i = i + 1
Next

'For lowvar = 0 To hivar Step varstep
'    s(j) = lowvar
'    s(j) = Round(s(j), 1)
'    j = j + 1
'Next

'Values for Variance to get LN skewness from .1 to 3
sLognormal(1) = 0.001
sLognormal(2) = 0.004
sLognormal(3) = 0.01
sLognormal(4) = 0.018
sLognormal(5) = 0.027
sLognormal(6) = 0.038
sLognormal(7) = 0.051
sLognormal(8) = 0.066
sLognormal(9) = 0.082
sLognormal(10) = 0.099
sLognormal(11) = 0.117
sLognormal(12) = 0.136
sLognormal(13) = 0.156
sLognormal(14) = 0.176
sLognormal(15) = 0.197
sLognormal(16) = 0.218
sLognormal(17) = 0.239
sLognormal(18) = 0.261
sLognormal(19) = 0.282
sLognormal(20) = 0.304
sLognormal(21) = 0.326
sLognormal(22) = 0.347
sLognormal(23) = 0.369
```

```
sLognormal(24) = 0.39
sLognormal(25) = 0.411
sLognormal(26) = 0.432
sLognormal(27) = 0.452
sLognormal(28) = 0.47
sLognormal(29) = 0.492
sLognormal(30) = 0.512

'Values for Alpha to get Gamma skewness from .1 to 3
sGamma(1) = 400
sGamma(2) = 100
sGamma(3) = 44.4
sGamma(4) = 25
sGamma(5) = 16
sGamma(6) = 11.1
sGamma(7) = 8.2
sGamma(8) = 6.2
sGamma(9) = 4.9
sGamma(10) = 4
sGamma(11) = 3.3
sGamma(12) = 2.8
sGamma(13) = 2.4
sGamma(14) = 2
sGamma(15) = 1.8
sGamma(16) = 1.6
sGamma(17) = 1.4
sGamma(18) = 1.2
sGamma(19) = 1.1
sGamma(20) = 1
sGamma(21) = 0.91
sGamma(22) = 0.82
sGamma(23) = 0.76
sGamma(24) = 0.69
sGamma(25) = 0.64
sGamma(26) = 0.59
sGamma(27) = 0.55
sGamma(28) = 0.51
sGamma(29) = 0.47
sGamma(30) = 0.44

'Values for Alpha to get Weibull skewness from .1 to 3
sWeibull(1) = 3.22
sWeibull(2) = 2.9
sWeibull(3) = 2.64
sWeibull(4) = 2.41
sWeibull(5) = 2.22
sWeibull(6) = 2.05
```

```
sWeibull(7) = 1.9
sWeibull(8) = 1.77
sWeibull(9) = 1.66
sWeibull(10) = 1.56
sWeibull(11) = 1.48
sWeibull(12) = 1.4
sWeibull(13) = 1.33
sWeibull(14) = 1.27
sWeibull(15) = 1.21
sWeibull(16) = 1.16
sWeibull(17) = 1.11
sWeibull(18) = 1.07
sWeibull(19) = 1.03
sWeibull(20) = 1
sWeibull(21) = 0.97
sWeibull(22) = 0.94
sWeibull(23) = 0.91
sWeibull(24) = 0.89
sWeibull(25) = 0.86
sWeibull(26) = 0.84
sWeibull(27) = 0.82
sWeibull(28) = 0.8
sWeibull(29) = 0.79
sWeibull(30) = 0.77

'For i = 1 To 50
'    Debug.Print m(i)
'Next

'For j = 1 To 30
'    Debug.Print sLognormal(j)
'Next

'For j = 1 To 30
'    Debug.Print sGamma(j)
'Next

'For j = 1 To 30
'    Debug.Print sWeibull(j)
'Next

testruns = 100
k = 1

For z = 100 To 100

For n = 1 To 5
```

```
     tvalue = tstat(samplesize(n))
     For i = 1 To 50
         munot = m(i)
         For j = 1 To 30
             'LN truemean
             LognormalTruemean = Exp(m(i) + 0.5 *
sLognormal(j))
             'Gamma trumean
             GammaTruemean = m(i) * sGamma(j)
             'Weibull truemean
             WeibullTruemean = m(i) * Exp(gammln(1 + (1 /
sWeibull(j))))
             'txtfile0.writeline (truemean)
             For t = 1 To testruns
                 For v = 1 To samplesize(n)
                     'Random vector for LN
                     LognormalRandomVector(v) =
randomLognormal(m(i), sLognormal(j))
                     'Random vector for Gamma
                     GammaRandomVector(v) = m(i) *
randomGamma(sGamma(j))
                     'Random vector for Weibull
                     WeibullRandomVector(v) =
randomWeibull(sWeibull(j), m(i))
                 Next

                 GammaSampleMean =
average(GammaRandomVector, samplesize(n))
                 GammaSampleStddev =
Sqr(sampleVariance(GammaRandomVector, GammaSampleMean,
samplesize(n)))
                 GammaLognormalSampleMean =
LognormalAverage(GammaRandomVector, samplesize(n))
                 GammaLognormalSampleVariance =
LognormalSampleVariance(GammaRandomVector,
GammaLognormalSampleMean, samplesize(n))

                 LognormalSampleMean =
average(LognormalRandomVector, samplesize(n))
                 LognormalSampleStddev =
Sqr(sampleVariance(LognormalRandomVector,
LognormalSampleMean, samplesize(n)))
                 LognormalLognormalSampleMean =
LognormalAverage(LognormalRandomVector, samplesize(n))
                 LognormalLognormalSampleVariance =
LognormalSampleVariance(LognormalRandomVector,
LognormalLognormalSampleMean, samplesize(n))
```

```
                    WeibullSampleMean =
average(WeibullRandomVector, samplesize(n))
                    WeibullSampleStddev =
Sqr(sampleVariance(WeibullRandomVector, WeibullSampleMean,
samplesize(n)))
                    WeibullLognormalSampleMean =
LognormalAverage(WeibullRandomVector, samplesize(n))
                    WeibullLognormalSampleVariance =
LognormalSampleVariance(WeibullRandomVector,
WeibullLognormalSampleMean, samplesize(n))

                    GammaTucl(t) = GammaSampleMean + ((tvalue *
GammaSampleStddev) / Sqr(samplesize(n)))
                    GammaCucl(t) = GammaSampleMean + (4.47 *
GammaSampleStddev) / Sqr(samplesize(n))
                    GammaHucl(t) = (GammaLognormalSampleMean +
0.5 * GammaLognormalSampleVariance +
(Sqr(GammaLognormalSampleVariance) *
Hstat(Sqr(GammaLognormalSampleVariance), samplesize(n))) /
Sqr(samplesize(n) - 1))

                    LognormalTucl(t) = LognormalSampleMean +
((tvalue * LognormalSampleStddev) / Sqr(samplesize(n)))
                    LognormalCucl(t) = LognormalSampleMean +
(4.47 * LognormalSampleStddev) / Sqr(samplesize(n))
                    LognormalHucl(t) =
Exp(LognormalLognormalSampleMean + 0.5 *
LognormalLognormalSampleVariance +
(Sqr(LognormalLognormalSampleVariance) *
Hstat(Sqr(LognormalLognormalSampleVariance),
samplesize(n))) / Sqr(samplesize(n) - 1))

                    WeibullTucl(t) = WeibullSampleMean +
((tvalue * WeibullSampleStddev) / Sqr(samplesize(n)))
                    WeibullCucl(t) = WeibullSampleMean + (4.47
* WeibullSampleStddev) / Sqr(samplesize(n))
                    WeibullHucl(t) =
Exp(WeibullLognormalSampleMean + 0.5 *
WeibullLognormalSampleVariance +
(Sqr(WeibullLognormalSampleVariance) *
Hstat(Sqr(WeibullLognormalSampleVariance), samplesize(n)))
/ Sqr(samplesize(n) - 1))
               Next

               Call QuickSort(GammaTucl, 1, testruns)
               Call QuickSort(GammaCucl, 1, testruns)
```

```
                Call QuickSort(GammaHucl, 1, testruns)
                Call QuickSort(LognormalTucl, 1, testruns)
                Call QuickSort(LognormalCucl, 1, testruns)
                Call QuickSort(LognormalHucl, 1, testruns)
                Call QuickSort(WeibullTucl, 1, testruns)
                Call QuickSort(WeibullCucl, 1, testruns)
                Call QuickSort(WeibullHucl, 1, testruns)

                'Average of UCL values
                GammaTuclAverage = average(GammaTucl, testruns)
                GammaCuclAverage = average(GammaCucl, testruns)
                GammaHuclAverage = average(GammaHucl, testruns)
                LognormalTuclAverage = average(LognormalTucl,
testruns)
                LognormalCuclAverage = average(LognormalCucl,
testruns)
                LognormalHuclAverage = average(LognormalHucl,
testruns)
                WeibullTuclAverage = average(WeibullTucl,
testruns)
                WeibullCuclAverage = average(WeibullCucl,
testruns)
                WeibullHuclAverage = average(WeibullHucl,
testruns)

                txtfile10.writeline GammaTuclAverage
                txtfile11.writeline GammaCuclAverage
                txtfile12.writeline GammaHuclAverage
                txtfile13.writeline LognormalTuclAverage
                txtfile14.writeline LognormalCuclAverage
                txtfile15.writeline LognormalHuclAverage
                txtfile16.writeline WeibullTuclAverage
                txtfile17.writeline WeibullCuclAverage
                txtfile18.writeline WeibullHuclAverage

                'Standard Deviation of UCL values
                txtfile19.writeline
Sqr(sampleVariance(GammaTucl, GammaTuclAverage, testruns))
                txtfile20.writeline
Sqr(sampleVariance(GammaCucl, GammaCuclAverage, testruns))
                txtfile21.writeline
Sqr(sampleVariance(GammaHucl, GammaHuclAverage, testruns))
                txtfile22.writeline
Sqr(sampleVariance(LognormalTucl, LognormalTuclAverage,
testruns))
```

```
            txtfile23.writeline
Sqr(sampleVariance(LognormalCucl, LognormalCuclAverage,
testruns))
            txtfile24.writeline
Sqr(sampleVariance(LognormalHucl, LognormalHuclAverage,
testruns))
            txtfile25.writeline
Sqr(sampleVariance(WeibullTucl, WeibullTuclAverage,
testruns))
            txtfile26.writeline
Sqr(sampleVariance(WeibullCucl, WeibullCuclAverage,
testruns))
            txtfile27.writeline
Sqr(sampleVariance(WeibullHucl, WeibullHuclAverage,
testruns))

            'Median of UCL values
            txtfile28.writeline (GammaTucl(testruns / 2) +
GammaTucl((testruns / 2) + 1)) / 2
            txtfile29.writeline (GammaCucl(testruns / 2) +
GammaCucl((testruns / 2) + 1)) / 2
            txtfile30.writeline (GammaHucl(testruns / 2) +
GammaHucl((testruns / 2) + 1)) / 2
            txtfile31.writeline (LognormalTucl(testruns /
2) + LognormalTucl((testruns / 2) + 1)) / 2
            txtfile32.writeline (LognormalCucl(testruns /
2) + LognormalCucl((testruns / 2) + 1)) / 2
            txtfile33.writeline (LognormalHucl(testruns /
2) + LognormalHucl((testruns / 2) + 1)) / 2
            txtfile34.writeline (WeibullTucl(testruns / 2)
+ WeibullTucl((testruns / 2) + 1)) / 2
            txtfile35.writeline (WeibullCucl(testruns / 2)
+ WeibullCucl((testruns / 2) + 1)) / 2
            txtfile36.writeline (WeibullHucl(testruns / 2)
+ WeibullHucl((testruns / 2) + 1)) / 2

            'Min of UCL values
            txtfile37.writeline GammaTucl(1)
            txtfile38.writeline GammaCucl(1)
            txtfile39.writeline GammaHucl(1)
            txtfile40.writeline LognormalTucl(1)
            txtfile41.writeline LognormalCucl(1)
            txtfile42.writeline LognormalHucl(1)
            txtfile43.writeline WeibullTucl(1)
            txtfile44.writeline WeibullCucl(1)
            txtfile45.writeline WeibullHucl(1)
```

```
            'Max of UCL values
            txtfile46.writeline GammaTucl(testruns)
            txtfile47.writeline GammaCucl(testruns)
            txtfile48.writeline GammaHucl(testruns)
            txtfile49.writeline LognormalTucl(testruns)
            txtfile50.writeline LognormalCucl(testruns)
            txtfile51.writeline LognormalHucl(testruns)
            txtfile52.writeline WeibullTucl(testruns)
            txtfile53.writeline WeibullCucl(testruns)
            txtfile54.writeline WeibullHucl(testruns)

            'lst Quantile of UCL values
            txtfile55.writeline GammaTucl(25) + 0.25 *
(GammaTucl(26) - GammaTucl(25))
            txtfile56.writeline GammaCucl(25) + 0.25 *
(GammaCucl(26) - GammaCucl(25))
            txtfile57.writeline GammaHucl(25) + 0.25 *
(GammaHucl(26) - GammaHucl(25))
            txtfile58.writeline LognormalTucl(25) + 0.25 *
(LognormalTucl(26) - LognormalTucl(25))
            txtfile59.writeline LognormalCucl(25) + 0.25 *
(LognormalCucl(26) - LognormalCucl(25))
            txtfile60.writeline LognormalHucl(25) + 0.25 *
(LognormalHucl(26) - LognormalHucl(25))
            txtfile61.writeline WeibullTucl(25) + 0.25 *
(WeibullTucl(26) - WeibullTucl(25))
            txtfile62.writeline WeibullCucl(25) + 0.25 *
(WeibullCucl(26) - WeibullCucl(25))
            txtfile63.writeline WeibullHucl(25) + 0.25 *
(WeibullHucl(26) - WeibullHucl(25))

            '3rd Quantile of UCL values
            txtfile64.writeline GammaTucl(75) + 0.75 *
(GammaTucl(76) - GammaTucl(75))
            txtfile65.writeline GammaCucl(75) + 0.75 *
(GammaCucl(76) - GammaCucl(75))
            txtfile66.writeline GammaHucl(75) + 0.75 *
(GammaHucl(76) - GammaHucl(75))
            txtfile67.writeline LognormalTucl(75) + 0.75 *
(LognormalTucl(76) - LognormalTucl(75))
            txtfile68.writeline LognormalCucl(75) + 0.75 *
(LognormalCucl(76) - LognormalCucl(75))
            txtfile69.writeline LognormalHucl(75) + 0.75 *
(LognormalHucl(76) - LognormalHucl(75))
            txtfile70.writeline WeibullTucl(75) + 0.75 *
(WeibullTucl(76) - WeibullTucl(75))
```

```
            txtfile71.writeline WeibullCucl(75) + 0.75 *
(WeibullCucl(76) - WeibullCucl(75))
            txtfile72.writeline WeibullHucl(75) + 0.75 *
(WeibullHucl(76) - WeibullHucl(75))

            GammaTuclGreater = 0
            GammaCuclGreater = 0
            GammaHuclGreater = 0
            LognormalTuclGreater = 0
            LognormalCuclGreater = 0
            LognormalHuclGreater = 0
            WeibullTuclGreater = 0
            WeibullCuclGreater = 0
            WeibullHuclGreater = 0

            For t = 1 To testruns
                If GammaTucl(t) > GammaTruemean Then
                    GammaTuclGreater = GammaTuclGreater + 1
                End If
                If GammaCucl(t) > GammaTruemean Then
                    GammaCuclGreater = GammaCuclGreater + 1
                End If
                If GammaHucl(t) > Log(GammaTruemean) Then
                    GammaHuclGreater = GammaHuclGreater + 1
                End If

                If LognormalTucl(t) > LognormalTruemean
Then
                    LognormalTuclGreater =
LognormalTuclGreater + 1
                End If
                If LognormalCucl(t) > LognormalTruemean
Then
                    LognormalCuclGreater =
LognormalCuclGreater + 1
                End If
                If LognormalHucl(t) > LognormalTruemean
Then
                    LognormalHuclGreater =
LognormalHuclGreater + 1
                End If

                If WeibullTucl(t) > WeibullTruemean Then
                    WeibullTuclGreater = WeibullTuclGreater
+ 1
                End If
                If WeibullCucl(t) > WeibullTruemean Then
```

```
                    WeibullCuclGreater = WeibullCuclGreater
+ 1
                End If
                If WeibullHucl(t) > WeibullTruemean Then
                    WeibullHuclGreater = WeibullHuclGreater
+ 1
                End If
            Next

            GammaTuclGreaterPercentage(k) =
Round(GammaTuclGreater / testruns, 3)
            GammaCuclGreaterPercentage(k) =
Round(GammaCuclGreater / testruns, 3)
            GammaHuclGreaterPercentage(k) =
Round(GammaHuclGreater / testruns, 3)

            LognormalTuclGreaterPercentage(k) =
Round(LognormalTuclGreater / testruns, 3)
            LognormalCuclGreaterPercentage(k) =
Round(LognormalCuclGreater / testruns, 3)
            LognormalHuclGreaterPercentage(k) =
Round(LognormalHuclGreater / testruns, 3)

            WeibullTuclGreaterPercentage(k) =
Round(WeibullTuclGreater / testruns, 3)
            WeibullCuclGreaterPercentage(k) =
Round(WeibullCuclGreater / testruns, 3)
            WeibullHuclGreaterPercentage(k) =
Round(WeibullHuclGreater / testruns, 3)

            k = k + 1
        Next
    Next
Next

Set fso1 = CreateObject("Scripting.FileSystemObject")
Set txtfile1 =
fso1.CreateTextFile("c:\stats\thesis\GammaTucl" & z &
".txt", True)
Set fso2 = CreateObject("Scripting.FileSystemObject")
Set txtfile2 =
fso2.CreateTextFile("c:\stats\thesis\GammaCucl" & z &
".txt", True)
Set fso3 = CreateObject("Scripting.FileSystemObject")
Set txtfile3 =
fso3.CreateTextFile("c:\stats\thesis\GammaHucl" & z &
".txt", True)
```

```
Set fso4 = CreateObject("Scripting.FileSystemObject")
Set txtfile4 =
fso4.CreateTextFile("c:\stats\thesis\LogNormalTucl" & z &
".txt", True)
Set fso5 = CreateObject("Scripting.FileSystemObject")
Set txtfile5 =
fso5.CreateTextFile("c:\stats\thesis\LogNormalCucl" & z &
".txt", True)
Set fso6 = CreateObject("Scripting.FileSystemObject")
Set txtfile6 =
fso6.CreateTextFile("c:\stats\thesis\LogNormalHucl" & z &
".txt", True)

Set fso7 = CreateObject("Scripting.FileSystemObject")
Set txtfile7 =
fso7.CreateTextFile("c:\stats\thesis\WeibullTucl" & z &
".txt", True)
Set fso8 = CreateObject("Scripting.FileSystemObject")
Set txtfile8 =
fso8.CreateTextFile("c:\stats\thesis\WeibullCucl" & z &
".txt", True)
Set fso9 = CreateObject("Scripting.FileSystemObject")
Set txtfile9 =
fso9.CreateTextFile("c:\stats\thesis\WeibullHucl" & z &
".txt", True)

For t = 1 To 7500 'testruns * (n - 1)
    txtfile1.writeline (GammaTuclGreaterPercentage(t))
    txtfile2.writeline (GammaCuclGreaterPercentage(t))
    txtfile3.writeline (GammaHuclGreaterPercentage(t))

    txtfile4.writeline (LognormalTuclGreaterPercentage(t))
    txtfile5.writeline (LognormalCuclGreaterPercentage(t))
    txtfile6.writeline (LognormalHuclGreaterPercentage(t))

    txtfile7.writeline (WeibullTuclGreaterPercentage(t))
    txtfile8.writeline (WeibullCuclGreaterPercentage(t))
    txtfile9.writeline (WeibullHuclGreaterPercentage(t))
Next

'txtfile0.Close
txtfile1.Close
txtfile2.Close
txtfile3.Close
txtfile4.Close
txtfile5.Close
```

```
txtfile6.Close
txtfile7.Close
txtfile8.Close
txtfile9.Close
txtfile10.Close
txtfile11.Close
txtfile12.Close
txtfile13.Close
txtfile14.Close
txtfile15.Close
txtfile16.Close
txtfile17.Close
txtfile18.Close

Next

End Sub

Private Function randomLognormal(mean As Single, variance
As Single)

'Generates a random lognormal value based upon
'user given mean and variance

Dim L As Single
Dim s As Single
Dim z As Single
Dim u1 As Single
Dim u2 As Single
Dim V1 As Single
Dim V2 As Single
Dim Stdev As Single

'The Polar Method to create Random standardized normal
'Adapted from Sheldon Ross' and Donald Knuth's books
'                            by Rich Timpone

'Insure at least one loop
L = 0

Do
    'Step 1: Generate random numbers, U1 and U2
    Randomize
    u1 = Rnd
    u2 = Rnd

    'Step 2: Calculate V1, V2, and S
```

```
    V1 = 2 * u1 - 1
    V2 = 2 * u2 - 1
    s = (V1 ^ 2) + (V2 ^ 2)

    'Step 3: If S=>1 get new values for U1 and U2
    If s < 1 Then L = 1
Loop Until L = 1

'Step 4: Calculate normal- could create 2
z = (((-2 * Log(s)) / s) ^ (1 / 2)) * V1
Stdev = (variance) ^ (1 / 2)
randomLognormal = Exp((z * Stdev) + mean)

End Function

Private Function average(values() As Single, q As Integer)

Dim sum As Single

sum = 0

For i = 1 To q
    sum = sum + values(i)
Next

average = sum / q

End Function

Private Function LognormalAverage(values() As Single, q As
Integer)

Dim sum As Single
Dim copy(100) As Single

For i = 1 To q
    copy(i) = Log(values(i))
Next

For i = 1 To q
    sum = sum + copy(i)
Next

LognormalAverage = sum / q

End Function
```

```
Private Function tstat(n As Integer)

Select Case n
    Case 10
        tstat = 2.262
    Case 20
        tstat = 2.093
    Case 30
        tstat = 2.045
    Case 50
        tstat = 2.042
    Case 100
        tstat = 2.042
End Select

End Function

Private Function sampleVariance(values() As Single, xbar As
Single, n As Integer)

Dim s As Double

s = 0

For i = 1 To n
    s = s + (((values(i) - xbar) * (values(i) - xbar)) / (n
- 1))
Next

sampleVariance = s

End Function

Private Function LognormalSampleVariance(values() As
Single, ybar As Single, n As Integer)

Dim s As Single
Dim copy(100) As Single

s = 0

For i = 1 To n
    copy(i) = Log(values(i))
Next

For i = 1 To n
```

```
    s = s + (((copy(i) - ybar) * (copy(i) - ybar)) / (n -
1))
Next

LognormalSampleVariance = s

End Function

Private Function gammp(A As Single, x As Single)

Dim gamser As Single
Dim gammcf As Single
Dim gln As Single

If (x < 0 Or A <= 0) Then
    MsgBox "Invalid arguments in routine gammp"
End If
If (x < (A + 1)) Then
    Call gser(gamser, A, x, gln)
    gammp = gamser
Else
    Call gcf(gammcf, A, x, gln)
    gammp = 1 - gammcf

End If

End Function

Private Function gammq(A As Single, x As Single)

Dim gamser As Single
Dim gammcf As Single
Dim gln As Single

If (x < 0 Or A <= 0) Then
    MsgBox "Invalid arguments in routine gammq"
End If
If (x < (A + 1)) Then
    Call gser(gamser, A, x, gln)
    gammq = 1 - gamser
Else
    Call gcf(gammcf, A, x, gln)
    gammq = gammcf

End If

End Function
```

```
Private Sub gser(gamser As Single, A As Single, x As
Single, gln As Single)

Dim n As Integer
Dim sum As Single
Dim del As Single
Dim ap As Single

gln = gammln(A)

If (x <= 0) Then
    If (x < 0) Then
        MsgBox "x less than 0 in routine gser"
        gamser = 0
        Exit Sub
    End If
Else
    ap = A
    sum = 1 / A
    del = sum
    For n = 1 To 100
        ap = ap + 1
        del = del * x / ap
        sum = sum + del
        If (Abs(del) < Abs(sum) * 0.0000003) Then
            gamser = sum * Exp(-x + A * Log(x) - gln)
            Exit Sub
        End If
    Next
    MsgBox "a too large, 100 too small in routine gser"
End If

End Sub

Private Sub gcf(gammcf As Single, A As Single, x As Single,
gln As Single)

Dim an As Single
Dim b As Single
Dim c As Single
Dim d As Single
Dim del As Single
Dim h As Single

Dim i As Integer
```

```
gln = gammln(A)
b = x + 1 - A
c = 1 / 1E-30
d = 1 / b
h = d

For i = 1 To 100
    an = -i * (i - A)
    b = b + 2
    d = an * d + b
    If (Abs(d) < 1E-30) Then
        d = 1E-30
    End If
    c = b + an / c
    If (Abs(c) < 1E-30) Then
        c = 1E-30
    End If
    d = 1 / d
    del = d * c
    h = h * del
    If (Abs(del - 1) < 0.0000003) Then
        Exit For
    End If
Next

If (i > 100) Then
    MsgBox "a too large, 1000 too small in gcf"
End If

gammcf = Exp(-x + A * Log(x) - gln) * h

End Sub

Private Function gammln(xx As Single)

Dim x As Single
Dim y As Single
Dim tmp As Single
Dim ser As Double

Dim j As Integer

Dim cof(6) As Double

cof(1) = 76.1800917294715
cof(2) = -86.5053203294168
```

```
cof(3) = 24.0140982408309
cof(4) = -1.23173957245015
cof(5) = 1.20865097386618E-03
cof(6) = -5.395239384953E-06

x = xx
y = x
tmp = x + 5.5
tmp = tmp - (x + 0.5) * Log(tmp)
ser = 1.00000000019001

For j = 1 To 6
    y = y + 1
    ser = ser + cof(j) / y
Next

gammln = -tmp + Log(2.506628274631 * ser / x)

End Function

Private Function weibullSkewness(A As Single)

weibullSkewness = (2 * (Exp(gammln(1 + (1 / A)))) ^ 3 - 3 *
Exp(gammln(1 + (1 / A))) * Exp(gammln(1 + (2 / A)))) /
(Exp(gammln(1 + (2 / A))) - (Exp(gammln(1 + (1 / A)))) ^ 2)
^ 1.5 _
            + Exp(gammln(1 + (3 / A))) / (Exp(gammln(1 + (2
/ A))) - (Exp(gammln(1 + (1 / A)))) ^ 2) ^ 1.5

End Function

Private Sub Command4_Click()

Dim A As Single

Dim fsol
Set fsol = CreateObject("Scripting.FileSystemObject")
Set txtfile1 =
fsol.CreateTextFile("c:\stats\thesis\weibullskew.txt",
True)

For A = 2 To 4 Step 0.01
    txtfile1.writeline (weibullSkewness(A))
Next

txtfile1.Close
```

```
End Sub

Private Function randomGamma(A As Single)

Dim c As Single
Dim accept As Boolean
Dim u1 As Single
Dim u2 As Single
Dim gam As Single
Dim aa As Single
Dim randnum As Single

If A < 0 Then
   MsgBox "Error, a < 0"

ElseIf A = 0 Then
   gam = 0

ElseIf A >= 1 Then
   b = A - 1
   c = 3 * A - 0.75
   accept = False
   Do While accept = False
      u1 = randomNotZero
      u2 = randomNotZero
      w = u1 * (1 - u1)
      y = Sqr(c / w) * (u1 - 0.5)
      gam = b + y
      If gam >= 0 Then
         If gam = 0 Then
            gam = 0.000000001
         End If
         z = 64 * w ^ 3 * u2 ^ 2
         accept = (z <= 1 - 2 * y ^ 2 / gam)
         If accept = False Then
            If b = 0 Then
               accept = (Log(z) <= -2 * y)
            Else
               accept = (Log(z) <= 2 * (b * Log(gam / b) -
y))
            End If
         End If
      End If
   Loop

Else
   aa = 0
```

```
   b = 1 + 0.3678794 * A
   accept = False
   Do While accept = False
      p = b * randomNotZero
      If p < 1 Then
         gam = Exp(Log(p) / A)
         accept = (-Log(randomNotZero) >= gam)
      Else
         gam = -Log((b - p) / A)
         accept = (-Log(randomNotZero) >= (1 - A) *
Log(gam))
      End If
   Loop
End If

randomGamma = gam

End Function

Private Function randomExponential(mu As Single)

Dim randum As Single
Dim ans As Single

Do
    randnum = Rnd
Loop Until Not (randnum = 0)

ans = -(mu) * Log(randnum)
randomExponential = ans

End Function

Private Function RandomNormal(mean As Single, variance As
Single)

'Generates a random normal value based upon
'user given mean and variance

Dim L As Single
Dim s As Single
Dim z As Single
Dim u1 As Single
Dim u2 As Single
Dim V1 As Single
Dim V2 As Single
Dim Stdev As Single
```

```
'The Polar Method to create Random standardized normal
'Adapted from Sheldon Ross' and Donald Knuth's books
'                             by Rich Timpone

'Insure at least one loop
L = 0

Do
    'Step 1: Generate random numbers, U1 and U2
    Randomize
    u1 = Rnd
    u2 = Rnd

    'Step 2: Calculate V1, V2, and S
    V1 = 2 * u1 - 1
    V2 = 2 * u2 - 1
    s = (V1 ^ 2) + (V2 ^ 2)

    'Step 3: If S=>1 get new values for U1 and U2
    If s < 1 Then L = 1
Loop Until L = 1

'Step 4: Calculate normal- could create 2
z = (((-2 * Log(s)) / s) ^ (1 / 2)) * V1
Stdev = (variance) ^ (1 / 2)
RandomNormal = (z * Stdev) + mean

End Function

Private Sub Command5_Click()

Dim fso0, txtfile0
Dim A As Single
Dim b As Single

Set fso0 = CreateObject("Scripting.FileSystemObject")
Set txtfile0 =
fso0.CreateTextFile("c:\stats\thesis\rGamma.txt", True)

A = 1
b = 2

For i = 1 To 10000
    txtfile0.writeline (b * randomGamma(A))
Next
```

```
txtfile0.Close

End Sub

Private Sub Command6_Click()

Dim fso0, txtfile0

Set fso0 = CreateObject("Scripting.FileSystemObject")
Set txtfile0 =
fso0.CreateTextFile("c:\stats\thesis\rWeibull.txt", True)

For i = 1 To 1000
    txtfile0.writeline randomWeibull(3, 5)
Next

txtfile0.Close


End Sub

Private Function randomWeibull(A As Single, b As Single)

Dim randum As Single
Dim ans As Single

Do
    Randomize
    randnum = Rnd
Loop Until Not (randnum = 0)

ans = b * (-Log(randnum)) ^ (1 / A)
randomWeibull = ans

End Function


Private Sub Command7_Click()
Dim fso0, fso1, fso2, fso, txtfile0, txtfile1, txtfile2

Set fso0 = CreateObject("Scripting.FileSystemObject")
Set txtfile0 =
fso0.CreateTextFile("c:\stats\thesis\mean.txt", True)

Set fso1 = CreateObject("Scripting.FileSystemObject")
Set txtfile1 =
fso1.CreateTextFile("c:\stats\thesis\var.txt", True)
```

```
Dim m(50) As Single
Dim s(30) As Single

Dim lowmean As Single
Dim lowvar As Single
Dim himean As Single
Dim hivar As Single

Dim meanstep As Single
Dim varstep As Single

Dim i As Integer
Dim j As Integer
Dim k As Integer
Dim t As Integer

lowmean = 0.1
lowvar = 0.1
himean = 5
hivar = 2.1

meanstep = 0.1
varstep = 0.1

'code below creates the vectors of the means and variances
to use

s(1) = 0.001
s(2) = 0.004
s(3) = 0.01
s(4) = 0.018
s(5) = 0.027
s(6) = 0.038
s(7) = 0.051
s(8) = 0.066
s(9) = 0.082
s(10) = 0.099
s(11) = 0.117
s(12) = 0.136
s(13) = 0.156
s(14) = 0.176
s(15) = 0.197
s(16) = 0.218
s(17) = 0.239
s(18) = 0.261
s(19) = 0.282
```

```
s(20) = 0.304
s(21) = 0.326
s(22) = 0.347
s(23) = 0.369
s(24) = 0.39
s(25) = 0.411
s(26) = 0.432
s(27) = 0.452
s(28) = 0.47
s(29) = 0.492
s(30) = 0.512

For lowmean = 0 To himean Step meanstep
    m(i) = lowmean
    m(i) = Round(m(i), 1)
    i = i + 1
Next
For i = 1 To 50
    For j = 1 To 30
        txtfile0.writeline m(i)
    Next
Next

For i = 1 To 50
    For j = 1 To 30
        txtfile1.writeline s(j)
    Next
Next

txtfile0.Close
txtfile1.Close
End Sub

Private Sub Command8_Click()

Dim A As Single
Dim b As Single

Dim fso0, txtfile0

Set fso0 = CreateObject("Scripting.FileSystemObject")
Set txtfile0 =
fso0.CreateTextFile("c:\stats\thesis\test.txt", True)

A = 1

For i = 1 To 1000
```

```
    txtfile0.writeline RandomNormal(0, 1)
Next

txtfile0.Close

End Sub

Private Sub Command9_Click()

Dim A As Single

Dim fso0, txtfile0

Set fso0 = CreateObject("Scripting.FileSystemObject")
Set txtfile0 =
fso0.CreateTextFile("c:\stats\thesis\rExponential.txt",
True)

A = 1

For i = 1 To 1000
    txtfile0.writeline randomExponential(A)
Next

txtfile0.Close

End Sub

Private Function ceiling(x As Single)

If x < 0 Then
    x = Fix(x)
Else
    x = Int(x)
End If

ceiling = x
End Function

Private Function randomNotZero()

Dim randum As Single

Do
    Randomize
    randnum = Rnd
Loop Until Not (randnum = 0)
```

```
randomNotZero = randnum

End Function

Private Function Hstat(s As Single, n As Integer)

Select Case n
    Case 10
        Select Case s
            Case Is <= 0.1
                Hstat = 1.739 + 0.63 * s
            Case Is <= 0.2
                Hstat = 1.723 + 0.79 * s
            Case Is <= 0.3
                Hstat = 1.689 + 0.96 * s
            Case Is <= 0.4
                Hstat = 1.641 + 1.12 * s
            Case Is <= 0.5
                Hstat = 1.565 + 1.31 * s
            Case Is <= 0.6
                Hstat = 1.48 + 1.48 * s
            Case Is <= 0.7
                Hstat = 1.384 + 1.64 * s
            Case Is <= 0.8
                Hstat = 1.286 + 1.78 * s
            Case Is <= 0.9
                Hstat = 1.174 + 1.92 * s
            Case Is <= 1
                Hstat = 1.093 + 2.01 * s
            Case Is <= 1.25
                Hstat = 0.959 + 2.144 * s
            Case Is <= 1.5
                Hstat = 0.799 + 2.272 * s
            Case Is <= 1.75
                Hstat = 0.679 + 2.352 * s
            Case Is <= 2
                Hstat = 0.588 + 2.404 * s
            Case Is <= 2.5
                Hstat = 0.496 + 2.45 * s
            Case Is <= 3
                Hstat = 0.406 + 2.486 * s
            Case Is <= 3.5
                Hstat = 0.34 + 2.508 * s
            Case Is <= 4
                Hstat = 0.284 + 2.524 * s
            Case Is <= 4.5
```

```
            Hstat = 0.3 + 2.52 * s
        Case Is <= 5
            Hstat = 0.21 + 2.54 * s
        Case Is <= 6
            Hstat = 0.21 + 2.54 * s
        Case Is <= 7
            Hstat = 0.15 + 2.55 * s
        Case Is <= 8
            Hstat = 0.15 + 2.55 * s
        Case Is <= 9
            Hstat = 0.15 + 2.55 * s
        Case Is <= 10
            Hstat = 0.06 + 2.56 * s
        End Select
Case 20
    Select Case s
        Case Is <= 0.1
            Hstat = 1.685 + 0.37 * s
        Case Is <= 0.2
            Hstat = 1.673 + 0.49 * s
        Case Is <= 0.3
            Hstat = 1.647 + 0.62 * s
        Case Is <= 0.4
            Hstat = 1.617 + 0.72 * s
        Case Is <= 0.5
            Hstat = 1.569 + 0.84 * s
        Case Is <= 0.6
            Hstat = 1.509 + 0.96 * s
        Case Is <= 0.7
            Hstat = 1.449 + 1.06 * s
        Case Is <= 0.8
            Hstat = 1.379 + 1.16 * s
        Case Is <= 0.9
            Hstat = 1.307 + 1.25 * s
        Case Is <= 1
            Hstat = 1.244 + 1.32 * s
        Case Is <= 1.25
            Hstat = 0.128 + 1.436 * s
        Case Is <= 1.5
            Hstat = 0.983 + 1.552 * s
        Case Is <= 1.75
            Hstat = 0.863 + 1.632 * s
        Case Is <= 2
            Hstat = 0.765 + 1.688 * s
        Case Is <= 2.5
            Hstat = 0.653 + 1.744 * s
        Case Is <= 3
```

```
                Hstat = 0.543 + 1.788 * s
            Case Is <= 3.5
                Hstat = 0.459 + 1.816 * s
            Case Is <= 4
                Hstat = 0.403 + 1.832 * s
            Case Is <= 4.5
                Hstat = 0.363 + 1.842 * s
            Case Is <= 5
                Hstat = 0.309 + 1.854 * s
            Case Is <= 6
                Hstat = 0.274 + 1.861 * s
            Case Is <= 7
                Hstat = 0.22 + 1.87 * s
            Case Is <= 8
                Hstat = 0.22 + 1.87 * s
            Case Is <= 9
                Hstat = 0.22 + 1.87 * s
            Case Is <= 10
                Hstat = 0.13 + 1.88 * s
            End Select
    Case 30
        Select Case s
            Case Is <= 0.1
                Hstat = 1.67 + 0.31 * s
            Case Is <= 0.2
                Hstat = 1.66 + 0.41 * s
            Case Is <= 0.3
                Hstat = 1.64 + 0.51 * s
            Case Is <= 0.4
                Hstat = 1.604 + 0.63 * s
            Case Is <= 0.5
                Hstat = 1.568 + 0.72 * s
            Case Is <= 0.6
                Hstat = 1.518 + 0.82 * s
            Case Is <= 0.7
                Hstat = 1.458 + 0.92 * s
            Case Is <= 0.8
                Hstat = 1.402 + 1 * s
            Case Is <= 0.9
                Hstat = 1.338 + 1.08 * s
            Case Is <= 1
                Hstat = 1.293 + 1.13 * s
            Case Is <= 1.25
                Hstat = 1.167 + 1.256 * s
            Case Is <= 1.5
                Hstat = 1.037 + 1.36 * s
            Case Is <= 1.75
```

```
                Hstat = 0.917 + 1.44 * s
            Case Is <= 2
                Hstat = 0.812 + 1.5 * s
            Case Is <= 2.5
                Hstat = 0.708 + 1.552 * s
            Case Is <= 3
                Hstat = 0.588 + 1.6 * s
            Case Is <= 3.5
                Hstat = 0.51 + 1.626 * s
            Case Is <= 4
                Hstat = 0.44 + 1.646 * s
            Case Is <= 4.5
                Hstat = 0.384 + 1.66 * s
            Case Is <= 5
                Hstat = 0.348 + 1.668 * s
            Case Is <= 6
                Hstat = 0.328 + 1.672 * s
            Case Is <= 7
                Hstat = 0.22 + 1.69 * s
            Case Is <= 8
                Hstat = 0.22 + 1.69 * s
            Case Is <= 9
                Hstat = 0.22 + 1.69 * s
            Case Is <= 10
                Hstat = 0.13 + 1.7 * s
            End Select
    Case 50
        Select Case s
            Case Is <= 0.1
                Hstat = 1.659 + 0.25 * s
            Case Is <= 0.2
                Hstat = 1.65 + 0.34 * s
            Case Is <= 0.3
                Hstat = 1.632 + 0.43 * s
            Case Is <= 0.4
                Hstat = 1.605 + 0.52 * s
            Case Is <= 0.5
                Hstat = 1.561 + 0.63 * s
            Case Is <= 0.6
                Hstat = 1.526 + 0.7 * s
            Case Is <= 0.7
                Hstat = 1.472 + 0.79 * s
            Case Is <= 0.8
                Hstat = 1.416 + 0.87 * s
            Case Is <= 0.9
                Hstat = 1.36 + 0.94 * s
            Case Is <= 1
```

```
                Hstat = 1.306 + 1 * s
            Case Is <= 1.25
                Hstat = 1.21 + 1.096 * s
            Case Is <= 1.5
                Hstat = 1.075 + 1.204 * s
            Case Is <= 1.75
                Hstat = 0.967 + 1.276 * s
            Case Is <= 2
                Hstat = 0.869 + 1.332 * s
            Case Is <= 2.5
                Hstat = 0.753 + 1.39 * s
            Case Is <= 3
                Hstat = 0.633 + 1.438 * s
            Case Is <= 3.5
                Hstat = 0.543 + 1.468 * s
            Case Is <= 4
                Hstat = 0.48 + 1.486 * s
            Case Is <= 4.5
                Hstat = 0.424 + 1.5 * s
            Case Is <= 5
                Hstat = 0.379 + 1.51 * s
            Case Is <= 6
                Hstat = 0.329 + 1.52 * s
            Case Is <= 7
                Hstat = 0.263 + 1.531 * s
            Case Is <= 8
                Hstat = 0.27 + 1.53 * s
            Case Is <= 9
                Hstat = 0.19 + 1.54 * s
            Case Is <= 10
                Hstat = 0.19 + 1.54 * s
            End Select
    Case 100
        Select Case s
            Case Is <= 0.1
                Hstat = 1.651 + 0.19 * s
            Case Is <= 0.2
                Hstat = 1.643 + 0.27 * s
            Case Is <= 0.3
                Hstat = 1.625 + 0.36 * s
            Case Is <= 0.4
                Hstat = 1.601 + 0.44 * s
            Case Is <= 0.5
                Hstat = 1.565 + 0.53 * s
            Case Is <= 0.6
                Hstat = 1.525 + 0.61 * s
            Case Is <= 0.7
```

```
                Hstat = 1.477 + 0.69 * s
            Case Is <= 0.8
                Hstat = 1.435 + 0.75 * s
            Case Is <= 0.9
                Hstat = 1.379 + 0.82 * s
            Case Is <= 1
                Hstat = 1.325 + 0.88 * s
            Case Is <= 1.25
                Hstat = 1.237 + 0.968 * s
            Case Is <= 1.5
                Hstat = 1.117 + 1.064 * s
            Case Is <= 1.75
                Hstat = 1.009 + 1.136 * s
            Case Is <= 2
                Hstat = 0.911 + 1.192 * s
            Case Is <= 2.5
                Hstat = 0.795 + 1.25 * s
            Case Is <= 3
                Hstat = 0.675 + 1.298 * s
            Case Is <= 3.5
                Hstat = 0.585 + 1.328 * s
            Case Is <= 4
                Hstat = 0.508 + 1.35 * s
            Case Is <= 4.5
                Hstat = 0.452 + 1.364 * s
            Case Is <= 5
                Hstat = 0.407 + 1.374 * s
            Case Is <= 6
                Hstat = 0.357 + 1.384 * s
            Case Is <= 7
                Hstat = 0.327 + 1.389 * s
            Case Is <= 8
                Hstat = 0.25 + 1.4 * s
            Case Is <= 9
                Hstat = 0.25 + 1.4 * s
            Case Is <= 10
                Hstat = 0.16 + 1.41 * s
            End Select
End Select

End Function

Public Sub QuickSort(A() As Single, ByVal Lb As Long, ByVal
Ub As Long)
'Quicksort function needed to get a sorted list of the UCL
values.  The list needed to
```

```
'be sorted when finding descriptive statistics such as
median, min/max values, and
'quartiles .  Needs functions Qsort, InsertSort, and
Partition to function.
'Code obtained by


    Dim m As Long

    ' sort array A(lb..ub)

    Do While Lb < Ub
        ' quickly sort short lists
        If (Ub - Lb <= 12) Then
            Call InsertSort(A, Lb, Ub)
            Exit Sub
        End If

        ' partition into two segments
        m = Partition(A, Lb, Ub)

        ' sort the smallest partition to minimize stack
requirements
        If m - Lb <= Ub - m Then
            Call QuickSort(A, Lb, m - 1)
            Lb = m + 1
        Else
            Call QuickSort(A, m + 1, Ub)
            Ub = m - 1
        End If
    Loop
End Sub

Public Sub QSort(ByRef A() As Variant, ByVal Lb As Long,
ByVal Ub As Long)
    Dim lbStack(32) As Long
    Dim ubStack(32) As Long
    Dim sp As Long              ' stack pointer
    Dim lbx As Long             ' current lower-bound
    Dim ubx As Long             ' current upper-bound
    Dim m As Long
    Dim p As Long               ' index to pivot
    Dim i As Long
    Dim j As Long
    Dim t As Variant            ' temp used for exchanges

    lbStack(0) = Lb
```

```
ubStack(0) = Ub
sp = 0
Do While sp >= 0
    lbx = lbStack(sp)
    ubx = ubStack(sp)

    Do While (lbx < ubx)

        ' select pivot and exchange with 1st element
        p = lbx + (ubx - lbx) \ 2

        ' exchange lbx, p
        t = A(lbx)
        A(lbx) = A(p)
        A(p) = t

        ' partition into two segments
        i = lbx + 1
        j = ubx
        Do
            Do While i < j
                If A(lbx) <= A(i) Then Exit Do
                i = i + 1
            Loop

            Do While j >= i
                If A(j) <= A(lbx) Then Exit Do
                j = j - 1
            Loop

            If i >= j Then Exit Do

            ' exchange i, j
            t = A(i)
            A(i) = A(j)
            A(j) = t

            j = j - 1
            i = i + 1
        Loop

        ' pivot belongs in A[j]
        ' exchange lbx, j
        t = A(lbx)
        A(lbx) = A(j)
        A(j) = t
```

```
            m = j

            ' keep processing smallest segment, and stack
largest
            If m - lbx <= ubx - m Then
                If m + 1 < ubx Then
                    lbStack(sp) = m + 1
                    ubStack(sp) = ubx
                    sp = sp + 1
                End If
                ubx = m - 1
            Else
                If m - 1 > lbx Then
                    lbStack(sp) = lbx
                    ubStack(sp) = m - 1
                    sp = sp + 1
                End If
                lbx = m + 1
            End If
        Loop
        sp = sp - 1
    Loop
End Sub

Public Sub InsertSort(A() As Single, ByVal Lb As Long,
ByVal Ub As Long)
    Dim t As Variant
    Dim i As Long
    Dim j As Long

    ' sort A[Lb..Ub]
    For i = Lb + 1 To Ub
        t = A(i)

        ' shift elements down until insertion point found
        For j = i - 1 To Lb Step -1
            If A(j) <= t Then Exit For
            A(j + 1) = A(j)
        Next j

        ' insert
        A(j + 1) = t
    Next i
End Sub

Private Function Partition(A() As Single, ByVal Lb As Long,
ByVal Ub As Long) _
```

```
        As Long
    Dim t As Variant
    Dim pivot As Variant
    Dim i As Long
    Dim j As Long
    Dim p As Long

    ' partition array[lb..ub]

    ' select pivot and exchange with 1st element
    p = Lb + (Ub - Lb) \ 2
    pivot = A(p)
    A(p) = A(Lb)

    ' sort Lb+1 .. Ub based on pivot
    i = Lb
    j = Ub + 1
    Do
        Do
            j = j - 1
        Loop While j > i And A(j) > pivot
        Do
            i = i + 1
        Loop While i < j And A(i) < pivot

        If i >= j Then Exit Do

        ' swap A(i), A(j)
        t = A(i)
        A(i) = A(j)
        A(j) = t
    Loop

    ' pivot belongs in A(j)
    A(Lb) = A(j)
    A(j) = pivot
    Partition = j
End Function

'The following code below was used for testing purposes

Private Sub Command10_Click()

Dim fso0, txtfile0
Dim A As Single
Dim b As Single
```

```
Set fso0 = CreateObject("Scripting.FileSystemObject")
Set txtfile0 =
fso0.CreateTextFile("c:\stats\thesis\rLognormal.txt", True)

A = 2
b = 4

For i = 1 To 1000
    txtfile0.writeline randomLognormal(A, b)
Next

txtfile0.Close

End Sub

Private Sub Command2_Click()
' Incomplete Gamma Integral
Dim x As Single
Dim A As Single

x = 12
A = 5

MsgBox gammp(A, x)


End Sub

Private Sub Command3_Click()
Dim x(3) As Single
Dim q As Single
Dim r As Single

x(1) = 4.9
x(2) = 2.353
x(3) = 6

q = average(x, 3)
r = sampleVariance(x, q, 3)

MsgBox q
MsgBox r
End Sub
```

APPENDIX B

RANDOM NUMBER GENERATION ALGORITHMS

The following algorithms were used to generate the random deviates:

Lognormal distribution (given paramters μ and σ^2):

1. Generate 2 unit scaled uniformly distributed variables, U1 and U2.

2. Calculate V1, V2, (U1 and U2 rescaled from -1 to 1) and S, where:

$$V1 = (2*U1)-1$$
$$V2 = (2*U2)-1$$
$$S = V1^2 + V2^2$$

3. If value calculated is not within the unit circle start again.

4. Compute standardized normally distributed random variable Z where:

$$Z = V1*(-2\ln(S)/S)^{1/2}$$

5. Return Exp(Z * σ + μ). This is known as the Polar Method.

Weibull distribution (given parameters α and β):

1. Generate random value U from a Uniform distribution on [0,1].

2. Return $\beta * [-\ln U]^{1/\alpha}$

Gamma distribution (given parameters α and β):

1. Follow one of the two algorithms depending upon value of α:

For $\alpha >= 1$, use Best's rejection algorithm XG Adapted from L. Devroye, "Non-uniform random variate generation", Springer-Verlag, New York, 1986, p. 410.

For $\alpha < 1$, use rejection algorithm GS from Ahrens, J.H. and Dieter, U. Computer methods for sampling from Gamma, Beta, Poisson and Binomial distributions. Computing, 12 (1974), 223 - 246. Adapted from Netlib Fortran routine.

2. Return β * value from step 1.

APPENDIX C

ANOVA REGRESSION RESULTS

The following is the ANOVA table generated from regression analysis.

Gamma, n=10, 100 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.90683	0.002072	437.68	0.000
Beta	-0.01171	-0.00586	0.003518	-1.67	0.096
Skewness	-0.03733	-0.01867	0.003480	-5.36	0.000
Beta*Skewness	-0.01683	-0.00842	0.005908	-1.42	0.155

Gamma, n=10, 100 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.97608	0.001000	976.31	0.000
Beta	-0.00345	-0.00172	0.001697	-1.01	0.310
Skewness	-0.02673	-0.01337	0.001679	-7.96	0.000
Beta*Skewness	-0.01646	-0.00823	0.002851	-2.89	0.004

Gamma, n=10, 100 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.960596	0.001243	772.79	0.000
Beta	-0.000865	-0.000432	0.002110	-0.20	0.838
Skewness	0.099517	0.049758	0.002088	23.83	0.000
Beta*Skewness	0.001040	0.000520	0.003545	0.15	0.883

Gamma, n=20, 100 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.91835	0.002074	442.84	0.000
Beta	0.00360	0.00180	0.003521	0.51	0.609
Skewness	-0.03505	-0.01752	0.003483	-5.03	0.000
Beta*Skewness	-0.00015	-0.00007	0.005914	-0.01	0.990

Gamma, n=20, 100 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.99202	0.000621	1597.69	0.000
Beta	-0.00038	-0.00019	0.001054	-0.18	0.855
Skewness	-0.02012	-0.01006	0.001043	-9.65	0.000
Beta*Skewness	-0.00128	-0.00064	0.001771	-0.36	0.718

Gamma, n=20, 100 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.964681	0.001282	752.28	0.000
Beta	0.001714	0.000857	0.002177	0.39	0.694
Skewness	0.101819	0.050909	0.002154	23.64	0.000
Beta*Skewness	-0.010458	-0.005229	0.003657	-1.43	0.153

Gamma, n=30, 100 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.93168	0.001928	483.26	0.000
Beta	0.00599	0.00299	0.003273	0.92	0.360
Skewness	-0.03946	-0.01973	0.003238	-6.09	0.000
Beta*Skewness	0.01787	0.00893	0.005498	1.62	0.104

Gamma, n=30, 100 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.996582	0.000375	2655.10	0.000
Beta	0.001404	0.000702	0.000637	1.10	0.271
Skewness	-0.011532	-0.005766	0.000630	-9.15	0.000
Beta*Skewness	0.002837	0.001419	0.001070	1.33	0.185

Gamma, n=30, 100 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.973438	0.001147	848.79	0.000
Beta	0.001244	0.000622	0.001947	0.32	0.749
Skewness	0.093941	0.046971	0.001926	24.38	0.000
Beta*Skewness	0.002654	0.001327	0.003270	0.41	0.685

Gamma, n=50, 100 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.90437	0.002080	434.75	0.000
Beta	-0.00286	-0.00143	0.003532	-0.41	0.685
Skewness	-0.02158	-0.01079	0.003494	-3.09	0.002
Beta*Skewness	-0.00123	-0.00062	0.005932	-0.10	0.917

Gamma, n=50, 100 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.993980	0.000529	1877.59	0.000
Beta	-0.000283	-0.000142	0.000899	-0.16	0.875
Skewness	-0.015939	-0.007969	0.000889	-8.96	0.000

Beta*Skewness	-0.000204	-0.000102	0.001510	-0.07	0.946

Gamma, n=50, 100 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.954784	0.001312	727.82	0.000
Beta	0.003883	0.001941	0.002227	0.87	0.384
Skewness	0.150499	0.075249	0.002203	34.15	0.000
Beta*Skewness	-0.005863	-0.002931	0.003741	-0.78	0.433

Gamma, n=100, 100 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.845927	0.002047	413.22	0.000
Beta	0.006184	0.003092	0.003476	0.89	0.374
Skewness	0.012870	0.006435	0.003438	1.87	0.061
Beta*Skewness	-0.003383	-0.001692	0.005838	-0.29	0.772

Gamma, n=100, 100 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.97742	0.000830	1177.52	0.000
Beta	0.00083	0.00041	0.001409	0.29	0.770
Skewness	-0.03662	-0.01831	0.001394	-13.13	0.000
Beta*Skewness	0.00191	0.00096	0.002367	0.40	0.686

Gamma, n=100, 100 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.919033	0.001479	621.39	0.000
Beta	0.000682	0.000341	0.002511	0.14	0.892
Skewness	0.259441	0.129721	0.002484	52.22	0.000
Beta*Skewness	-0.005759	-0.002880	0.004217	-0.68	0.495

Gamma, n=10, 1,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.90556	0.000798	1134.83	0.000
Beta	0.00096	0.00048	0.001355	0.35	0.723
Skewness	-0.04501	-0.02251	0.001340	-16.79	0.000
Beta*Skewness	-0.01247	-0.00623	0.002275	-2.74	0.006

Gamma, n=10, 1,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.97626	0.000367	2661.27	0.000
Beta	-0.00102	-0.00051	0.000623	-0.82	0.414
Skewness	-0.03014	-0.01507	0.000616	-24.45	0.000
Beta*Skewness	-0.00547	-0.00274	0.001046	-2.62	0.009

Gamma, n=10, 1,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.959899	0.000521	1840.69	0.000

Beta	0.003467	0.001734	0.000885	1.96	0.050
Skewness	0.102894	0.051447	0.000876	58.74	0.000
Beta*Skewness	-0.006746	-0.003373	0.001487	-2.27	0.023

Gamma, n=20, 1,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.91722	0.000905	1013.31	0.000
Beta	-0.00285	-0.00142	0.001537	-0.93	0.354
Skewness	-0.03539	-0.01770	0.001520	-11.64	0.000
Beta*Skewness	0.00958	0.00479	0.002581	1.86	0.064

Gamma, n=20, 1,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.992703	0.000216	4595.92	0.000
Beta	0.000982	0.000491	0.000367	1.34	0.181
Skewness	-0.017682	-0.008841	0.000363	-24.37	0.000
Beta*Skewness	0.002469	0.001235	0.000616	2.00	0.045

Gamma, n=20, 1,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.963406	0.000579	1663.35	0.000
Beta	-0.003135	-0.001568	0.000983	-1.59	0.111
Skewness	0.105536	0.052768	0.000973	54.24	0.000
Beta*Skewness	0.008834	0.004417	0.001652	2.67	0.008

Gamma, n=30, 1,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.93123	0.000881	1056.74	0.000
Beta	0.00158	0.00079	0.001496	0.53	0.598
Skewness	-0.03877	-0.01938	0.001480	-13.10	0.000
Beta*Skewness	-0.00720	-0.00360	0.002513	-1.43	0.152

Gamma, n=30, 1,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.996446	0.000165	6029.38	0.000
Beta	0.000469	0.000235	0.000281	0.84	0.403
Skewness	-0.012290	-0.006145	0.000278	-22.14	0.000
Beta*Skewness	-0.000292	-0.000146	0.000471	-0.31	0.757

Gamma, n=30, 1,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.972937	0.000590	1650.41	0.000
Beta	0.003444	0.001722	0.001001	1.72	0.086
Skewness	0.094363	0.047182	0.000990	47.65	0.000
Beta*Skewness	-0.006390	-0.003195	0.001681	-1.90	0.058

Gamma, n=50, 1,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.90556	0.001089	831.86	0.000
Beta	0.00117	0.00058	0.001848	0.32	0.752
Skewness	-0.02100	-0.01050	0.001828	-5.74	0.000
Beta*Skewness	-0.00332	-0.00166	0.003104	-0.54	0.593

Gamma, n=50, 1,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.994298	0.000236	4219.88	0.000
Beta	0.000279	0.000140	0.000400	0.35	0.727
Skewness	-0.015460	-0.007730	0.000396	-19.53	0.000
Beta*Skewness	0.000321	0.000161	0.000672	0.24	0.811

Gamma, n=50, 1,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.955949	0.000733	1304.83	0.000
Beta	0.001695	0.000848	0.001244	0.68	0.496
Skewness	0.145241	0.072620	0.001231	59.02	0.000
Beta*Skewness	-0.002055	-0.001028	0.002089	-0.49	0.623

Gamma, n=100, 1,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.842904	0.001163	724.97	0.000
Beta	0.005197	0.002599	0.001974	1.32	0.188
Skewness	0.015059	0.007530	0.001953	3.86	0.000
Beta*Skewness	0.010150	0.005075	0.003315	1.53	0.126

Gamma, n=100, 1,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.97713	0.000440	2219.13	0.000
Beta	0.00151	0.00076	0.000748	1.01	0.312
Skewness	-0.03403	-0.01702	0.000740	-23.01	0.000
Beta*Skewness	0.00278	0.00139	0.001256	1.11	0.269

Gamma, n=100, 1,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.916627	0.001000	916.51	0.000
Beta	-0.001340	-0.000670	0.001698	-0.39	0.693
Skewness	0.262586	0.131293	0.001680	78.16	0.000
Beta*Skewness	0.003405	0.001703	0.002852	0.60	0.551

Gamma, n=10, 10,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.90419	0.000549	1648.00	0.000
Beta	-0.00140	-0.00070	0.000931	-0.75	0.453
Skewness	-0.04697	-0.02349	0.000922	-25.49	0.000
Beta*Skewness	-0.00496	-0.00248	0.001565	-1.59	0.113

Gamma, n=10, 10,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.97581	0.000237	4125.09	0.000
Beta	-0.00119	-0.00060	0.000402	-1.48	0.138
Skewness	-0.03222	-0.01611	0.000397	-40.54	0.000
Beta*Skewness	-0.00356	-0.00178	0.000675	-2.64	0.008

Gamma, n=10, 10,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.959759	0.000384	2496.62	0.000
Beta	-0.000512	-0.000256	0.000653	-0.39	0.695
Skewness	0.104514	0.052257	0.000646	80.93	0.000
Beta*Skewness	-0.001668	-0.000834	0.001096	-0.76	0.447

Gamma, n=20, 10,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.91814	0.000557	1648.41	0.000
Beta	0.00131	0.00065	0.000946	0.69	0.490
Skewness	-0.03580	-0.01790	0.000936	-19.13	0.000
Beta*Skewness	-0.00242	-0.00121	0.001588	-0.76	0.445

Gamma, n=20, 10,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.992920	0.000128	7785.21	0.000
Beta	-0.000118	-0.000059	0.000217	-0.27	0.785
Skewness	-0.017232	-0.008616	0.000214	-40.22	0.000
Beta*Skewness	-0.000291	-0.000146	0.000364	-0.40	0.689

Gamma, n=20, 10,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.963634	0.000393	2454.52	0.000
Beta	0.000199	0.000099	0.000667	0.15	0.881
Skewness	0.105544	0.052772	0.000659	80.03	0.000
Beta*Skewness	-0.001001	-0.000500	0.001120	-0.45	0.655

Gamma, n=30, 10,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.93101	0.000496	1878.22	0.000
Beta	0.00033	0.00017	0.000842	0.20	0.842
Skewness	-0.03868	-0.01934	0.000833	-23.23	0.000
Beta*Skewness	-0.00301	-0.00151	0.001413	-1.07	0.286

Gamma, n=30, 10,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.996512	0.000091	1.1E+04	0.000
Beta	-0.000090	-0.000045	0.000154	-0.29	0.770

Skewness	-0.012204	-0.006102	0.000153	-39.94	0.000
Beta*Skewness	-0.000106	-0.000053	0.000259	-0.20	0.838

Gamma, n=30, 10,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.972603	0.000443	2194.58	0.000
Beta	0.001116	0.000558	0.000752	0.74	0.459
Skewness	0.097587	0.048793	0.000744	65.55	0.000
Beta*Skewness	-0.003149	-0.001575	0.001264	-1.25	0.213

Gamma, n=50, 10,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.90556	0.000576	1571.98	0.000
Beta	0.00048	0.00024	0.000978	0.25	0.804
Skewness	-0.02299	-0.01150	0.000968	-11.88	0.000
Beta*Skewness	-0.00009	-0.00005	0.001643	-0.03	0.977

Gamma, n=50, 10,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.994048	0.000115	8612.81	0.000
Beta	0.000106	0.000053	0.000196	0.27	0.787
Skewness	-0.016481	-0.008241	0.000194	-42.51	0.000
Beta*Skewness	-0.000131	-0.000066	0.000329	-0.20	0.842

Gamma, n=50, 10,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.956034	0.000503	1902.46	0.000
Beta	-0.000039	-0.000019	0.000853	-0.02	0.982
Skewness	0.146248	0.073124	0.000844	86.63	0.000
Beta*Skewness	-0.000100	-0.000050	0.001433	-0.03	0.972

Gamma, n=100, 10,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.844052	0.000606	1392.12	0.000
Beta	0.000393	0.000197	0.001029	0.19	0.849
Skewness	0.008945	0.004473	0.001018	4.39	0.000
Beta*Skewness	-0.002057	-0.001029	0.001729	-0.59	0.552

Gamma, n=100, 10,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.97720	0.000236	4146.07	0.000
Beta	-0.00021	-0.00011	0.000400	-0.27	0.790
Skewness	-0.03501	-0.01751	0.000396	-44.22	0.000
Beta*Skewness	-0.00091	-0.00045	0.000672	-0.68	0.500

Gamma, n=100, 10,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P

Constant		0.917700	0.000701	1308.32	0.000
Beta	0.000616	0.000308	0.001191	0.26	0.796
Skewness	0.258966	0.129483	0.001178	109.90	0.000
Beta*Skewness	-0.000737	-0.000369	0.002000	-0.18	0.854

Lognormal, n=10, 100 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.92262	0.001825	505.65	0.000
Mu	0.00493	0.00247	0.003098	0.80	0.426
Skewness	-0.10944	-0.05472	0.003060	-17.88	0.000
Mu*Skewness	-0.00407	-0.00204	0.005196	-0.39	0.695

Lognormal, n=10, 100 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.99063	0.000624	1587.97	0.000
Mu	0.00208	0.00104	0.001059	0.98	0.327
Skewness	-0.02935	-0.01467	0.001046	-14.02	0.000
Mu*Skewness	0.00062	0.00031	0.001776	0.17	0.862

Lognormal, n=10, 100 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.944035	0.001528	617.77	0.000
Mu	0.003865	0.001933	0.002594	0.74	0.456
Skewness	-0.015818	-0.007909	0.002563	-3.09	0.002
Mu*Skewness	-0.009439	-0.004719	0.004351	-1.08	0.278

Lognormal, n=20, 100 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.93317	0.001770	527.28	0.000
Mu	0.00191	0.00095	0.003005	0.32	0.751
Skewness	-0.07830	-0.03915	0.002968	-13.19	0.000
Mu*Skewness	0.01105	0.00553	0.005039	1.10	0.273

Lognormal, n=20, 100 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.996442	0.000381	2618.64	0.000
Mu	0.001479	0.000739	0.000646	1.14	0.253
Skewness	-0.008903	-0.004451	0.000638	-6.97	0.000
Mu*Skewness	0.002328	0.001164	0.001084	1.07	0.283

Lognormal, n=20, 100 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.942428	0.001616	583.09	0.000
Mu	-0.000767	-0.000384	0.002744	-0.14	0.889
Skewness	-0.008458	-0.004229	0.002711	-1.56	0.119
Mu*Skewness	0.009435	0.004718	0.004602	1.03	0.306

Lognormal, n=30, 100 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.93481	0.001842	507.55	0.000
Mu	0.00166	0.00083	0.003127	0.27	0.790
Skewness	-0.06571	-0.03285	0.003089	-10.64	0.000
Mu*Skewness	0.01412	0.00706	0.005245	1.35	0.179

Lognormal, n=30, 100 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.997631	0.000305	3271.65	0.000
Mu	0.002865	0.001432	0.000518	2.77	0.006
Skewness	-0.004159	-0.002080	0.000511	-4.07	0.000
Mu*Skewness	0.003943	0.001972	0.000868	2.27	0.023

Lognormal, n=30, 100 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.936793	0.001750	535.39	0.000
Mu	-0.001544	-0.000772	0.002971	-0.26	0.795
Skewness	-0.013539	-0.006770	0.002935	-2.31	0.021
Mu*Skewness	0.009594	0.004797	0.004982	0.96	0.336

Lognormal, n=50, 100 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.90960	0.001950	466.43	0.000
Mu	0.00086	0.00043	0.003311	0.13	0.896
Skewness	-0.07119	-0.03559	0.003271	-10.88	0.000
Mu*Skewness	-0.00911	-0.00456	0.005553	-0.82	0.412

Lognormal, n=50, 100 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.994865	0.000493	2019.29	0.000
Mu	-0.000424	-0.000212	0.000836	-0.25	0.800
Skewness	-0.004742	-0.002371	0.000826	-2.87	0.004
Mu*Skewness	-0.001846	-0.000923	0.001403	-0.66	0.511

Lognormal, n=50, 100 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.89889	0.001948	461.34	0.000
Mu	0.00072	0.00036	0.003308	0.11	0.914
Skewness	-0.02600	-0.01300	0.003268	-3.98	0.000
Mu*Skewness	-0.00836	-0.00418	0.005548	-0.75	0.451

Lognormal, n=100, 100 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.84274	0.001916	439.77	0.000
Mu	-0.00063	-0.00032	0.003253	-0.10	0.923
Skewness	-0.08637	-0.04318	0.003214	-13.44	0.000

Mu*Skewness	-0.01016	-0.00508	0.005457	-0.93	0.352

Lognormal, n=100, 100 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.97852	0.000839	1165.78	0.000
Mu	-0.00384	-0.00192	0.001425	-1.35	0.178
Skewness	-0.02965	-0.01483	0.001408	-10.53	0.000
Mu*Skewness	-0.00745	-0.00372	0.002390	-1.56	0.119

Lognormal, n=100, 100 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.81759	0.001995	409.74	0.000
Mu	0.00189	0.00094	0.003388	0.28	0.781
Skewness	-0.04624	-0.02312	0.003347	-6.91	0.000
Mu*Skewness	-0.01087	-0.00544	0.005682	-0.96	0.339

Lognormal, n=10, 1,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.92479	0.000694	1332.40	0.000
Mu	-0.00669	-0.00335	0.001178	-2.84	0.005
Skewness	-0.10198	-0.05099	0.001164	-43.80	0.000
Mu*Skewness	-0.01231	-0.00615	0.001976	-3.11	0.002

Lognormal, n=10, 1,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.99063	0.000624	1587.97	0.000
Mu	0.00208	0.00104	0.001059	0.98	0.327
Skewness	-0.02935	-0.01467	0.001046	-14.02	0.000
Mu*Skewness	0.00062	0.00031	0.001776	0.17	0.862

Lognormal, n=10, 1,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.944334	0.000565	1670.43	0.000
Mu	-0.001826	-0.000913	0.000960	-0.95	0.342
Skewness	-0.014079	-0.007039	0.000948	-7.42	0.000
Mu*Skewness	-0.006083	-0.003042	0.001610	-1.89	0.059

Lognormal, n=20, 1,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.93209	0.000846	1101.56	0.000
Mu	0.00175	0.00087	0.001437	0.61	0.543
Skewness	-0.08363	-0.04182	0.001419	-29.46	0.000
Mu*Skewness	0.00684	0.00342	0.002409	1.42	0.156

Lognormal, n=20, 1,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.996442	0.000381	2618.64	0.000

Mu	0.001479	0.000739	0.000646	1.14	0.253
Skewness	-0.008903	-0.004451	0.000638	-6.97	0.000
Mu*Skewness	0.002328	0.001164	0.001084	1.07	0.283

Lognormal, n=20, 1,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.941024	0.000711	1322.66	0.000
Mu	0.002918	0.001459	0.001208	1.21	0.227
Skewness	-0.016364	-0.008182	0.001193	-6.86	0.000
Mu*Skewness	0.011026	0.005513	0.002026	2.72	0.007

Lognormal, n=30, 1,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.93759	0.000869	1079.30	0.000
Mu	0.00003	0.00001	0.001475	0.01	0.992
Skewness	-0.06400	-0.03200	0.001457	-21.96	0.000
Mu*Skewness	0.00373	0.00186	0.002474	0.75	0.451

Lognormal, n=30, 1,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.997631	0.000305	3271.65	0.000
Mu	0.002865	0.001432	0.000518	2.77	0.006
Skewness	-0.004159	-0.002080	0.000511	-4.07	0.000
Mu*Skewness	0.003943	0.001972	0.000868	2.27	0.023

Lognormal, n=30, 1,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.939410	0.000786	1194.43	0.000
Mu	0.001034	0.000517	0.001335	0.39	0.699
Skewness	-0.013063	-0.006531	0.001319	-4.95	0.000
Mu*Skewness	0.004950	0.002475	0.002240	1.11	0.269

Lognormal, n=50, 1,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.90924	0.001076	844.95	0.000
Mu	0.00075	0.00038	0.001827	0.21	0.837
Skewness	-0.07245	-0.03623	0.001805	-20.07	0.000
Mu*Skewness	0.00538	0.00269	0.003064	0.88	0.380

Lognormal, n=50, 1,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.994865	0.000493	2019.29	0.000
Mu	-0.000424	-0.000212	0.000836	-0.25	0.800
Skewness	-0.004742	-0.002371	0.000826	-2.87	0.004
Mu*Skewness	-0.001846	-0.000923	0.001403	-0.66	0.511

Lognormal, n=50, 1,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.89872	0.001032	870.95	0.000
Mu	0.00032	0.00016	0.001752	0.09	0.927
Skewness	-0.02536	-0.01268	0.001731	-7.33	0.000
Mu*Skewness	0.00358	0.00179	0.002938	0.61	0.542

Lognormal, n=100, 1,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.84212	0.001250	673.88	0.000
Mu	-0.00654	-0.00327	0.002122	-1.54	0.123
Skewness	-0.07953	-0.03976	0.002096	-18.97	0.000
Mu*Skewness	-0.00909	-0.00455	0.003558	-1.28	0.202

Lognormal, n=100, 1,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.97852	0.000839	1165.78	0.000
Mu	-0.00384	-0.00192	0.001425	-1.35	0.178
Skewness	-0.02965	-0.01483	0.001408	-10.53	0.000
Mu*Skewness	-0.00745	-0.00372	0.002390	-1.56	0.119

Lognormal, n=100, 1,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.81806	0.001255	652.00	0.000
Mu	-0.00658	-0.00329	0.002130	-1.54	0.123
Skewness	-0.03883	-0.01941	0.002104	-9.23	0.000
Mu*Skewness	-0.01203	-0.00601	0.003573	-1.68	0.093

Lognormal, n=10, 10,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.92492	0.000403	2295.72	0.000
Mu	-0.00135	-0.00067	0.000684	-0.98	0.325
Skewness	-0.10371	-0.05186	0.000676	-76.74	0.000
Mu*Skewness	-0.00353	-0.00177	0.001147	-1.54	0.124

Lognormal, n=10, 10,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.99041	0.000146	6782.96	0.000
Mu	-0.00052	-0.00026	0.000248	-1.05	0.292
Skewness	-0.02763	-0.01381	0.000245	-56.41	0.000
Mu*Skewness	-0.00154	-0.00077	0.000416	-1.85	0.065

Lognormal, n=10, 10,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.944864	0.000317	2978.78	0.000
Mu	-0.000170	-0.000085	0.000539	-0.16	0.874
Skewness	-0.014961	-0.007481	0.000532	-14.06	0.000
Mu*Skewness	-0.001683	-0.000841	0.000903	-0.93	0.352

Lognormal, n=20, 10,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.93352	0.000468	1993.81	0.000
Mu	0.00114	0.00057	0.000795	0.72	0.473
Skewness	-0.08070	-0.04035	0.000785	-51.38	0.000
Mu*Skewness	0.00068	0.00034	0.001333	0.26	0.798

Lognormal, n=20, 10,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.996366	0.000081	1.2E+04	0.000
Mu	0.000094	0.000047	0.000138	0.34	0.733
Skewness	-0.010068	-0.005034	0.000136	-36.94	0.000
Mu*Skewness	-0.000048	-0.000024	0.000231	-0.10	0.917

Lognormal, n=20, 10,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.942033	0.000413	2278.45	0.000
Mu	0.000803	0.000401	0.000702	0.57	0.568
Skewness	-0.014416	-0.007208	0.000693	-10.39	0.000
Mu*Skewness	0.000111	0.000055	0.001177	0.05	0.962

Lognormal, n=30, 10,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.93732	0.000521	1799.45	0.000
Mu	0.00123	0.00062	0.000884	0.70	0.485
Skewness	-0.06603	-0.03302	0.000874	-37.79	0.000
Mu*Skewness	0.00211	0.00106	0.001483	0.71	0.476

Lognormal, n=30, 10,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.997602	0.000091	1.1E+04	0.000
Mu	0.000028	0.000014	0.000155	0.09	0.929
Skewness	-0.004307	-0.002154	0.000153	-14.10	0.000
Mu*Skewness	-0.000118	-0.000059	0.000259	-0.23	0.820

Lognormal, n=30, 10,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.939464	0.000473	1987.66	0.000
Mu	0.000451	0.000226	0.000802	0.28	0.779
Skewness	-0.015604	-0.007802	0.000793	-9.84	0.000
Mu*Skewness	0.001514	0.000757	0.001346	0.56	0.574

Lognormal, n=50, 10,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.90979	0.000565	1610.90	0.000
Mu	-0.00013	-0.00006	0.000959	-0.07	0.948

Skewness	-0.06750	-0.03375	0.000947	-35.63	0.000
Mu*Skewness	0.00010	0.00005	0.001608	0.03	0.975

Lognormal, n=50, 10,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.994930	0.000166	5988.17	0.000
Mu	0.000206	0.000103	0.000282	0.37	0.715
Skewness	-0.006046	-0.003023	0.000279	-10.85	0.000
Mu*Skewness	-0.000054	-0.000027	0.000473	-0.06	0.954

Lognormal, n=50, 10,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.89900	0.000529	1700.86	0.000
Mu	-0.00048	-0.00024	0.000897	-0.26	0.791
Skewness	-0.02284	-0.01142	0.000887	-12.88	0.000
Mu*Skewness	-0.00065	-0.00032	0.001505	-0.21	0.830

Lognormal, n=100, 10,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.84270	0.000670	1257.07	0.000
Mu	-0.00201	-0.00101	0.001138	-0.88	0.376
Skewness	-0.08401	-0.04201	0.001124	-37.36	0.000
Mu*Skewness	-0.00245	-0.00122	0.001909	-0.64	0.522

Lognormal, n=100, 10,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.97834	0.000318	3072.20	0.000
Mu	-0.00018	-0.00009	0.000541	-0.17	0.868
Skewness	-0.02837	-0.01418	0.000534	-26.56	0.000
Mu*Skewness	0.00005	0.00003	0.000907	0.03	0.978

Lognormal, n=100, 10,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.81912	0.000722	1134.90	0.000
Mu	-0.00273	-0.00136	0.001225	-1.11	0.266
Skewness	-0.04397	-0.02199	0.001211	-18.16	0.000
Mu*Skewness	-0.00296	-0.00148	0.002055	-0.72	0.471

Weibull, n=10, 100 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.91045	0.002047	444.73	0.000
Beta	0.00352	0.00176	0.003476	0.51	0.613
Skewness	-0.14094	-0.07047	0.003420	-20.60	0.000
Beta*Skewness	0.02094	0.01047	0.005807	1.80	0.072

Weibull, n=10, 100 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P

Constant		0.98045	0.000855	1146.71	0.000
Beta	0.00063	0.00031	0.001452	0.22	0.829
Skewness	-0.05966	-0.02983	0.001428	-20.88	0.000
Beta*Skewness	0.00022	0.00011	0.002425	0.05	0.964

Weibull, n=10, 100 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.987457	0.000680	1452.64	0.000
Beta	0.000041	0.000020	0.001154	0.02	0.986
Skewness	-0.006461	-0.003230	0.001136	-2.84	0.005
Beta*Skewness	0.002226	0.001113	0.001928	0.58	0.564

Weibull, n=20, 100 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.92335	0.002034	454.01	0.000
Beta	0.00622	0.00311	0.003453	0.90	0.368
Skewness	-0.12806	-0.06403	0.003398	-18.84	0.000
Beta*Skewness	0.02093	0.01047	0.005769	1.81	0.070

Weibull, n=20, 100 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.99248	0.000572	1734.31	0.000
Beta	0.00204	0.00102	0.000972	1.05	0.295
Skewness	-0.02575	-0.01287	0.000956	-13.47	0.000
Beta*Skewness	0.00508	0.00254	0.001623	1.56	0.118

Weibull, n=20, 100 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.989843	0.000661	1497.79	0.000
Beta	0.004846	0.002423	0.001122	2.16	0.031
Skewness	-0.009884	-0.004942	0.001104	-4.48	0.000
Beta*Skewness	0.015418	0.007709	0.001875	4.11	0.000

Weibull, n=30, 100 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.93515	0.001893	493.94	0.000
Beta	-0.00504	-0.00252	0.003214	-0.78	0.433
Skewness	-0.12750	-0.06375	0.003163	-20.15	0.000
Beta*Skewness	-0.00857	-0.00429	0.005370	-0.80	0.425

Weibull, n=30, 100 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.996328	0.000402	2475.69	0.000
Beta	-0.000971	-0.000486	0.000683	-0.71	0.477
Skewness	-0.013349	-0.006675	0.000672	-9.93	0.000
Beta*Skewness	-0.002279	-0.001139	0.001142	-1.00	0.318

Weibull, n=30, 100 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.997188	0.000306	3260.22	0.000
Beta	-0.000131	-0.000066	0.000519	-0.13	0.899
Skewness	0.003173	0.001587	0.000511	3.10	0.002
Beta*Skewness	0.000093	0.000047	0.000868	0.05	0.957

Weibull, n=50, 100 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.91291	0.002152	424.16	0.000
Beta	-0.00461	-0.00230	0.003654	-0.63	0.528
Skewness	-0.13541	-0.06770	0.003596	-18.83	0.000
Beta*Skewness	-0.02192	-0.01096	0.006105	-1.79	0.073

Weibull, n=50, 100 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.994131	0.000563	1765.05	0.000
Beta	-0.000903	-0.000452	0.000956	-0.47	0.637
Skewness	-0.018941	-0.009470	0.000941	-10.06	0.000
Beta*Skewness	-0.001738	-0.000869	0.001598	-0.54	0.587

Weibull, n=50, 100 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.994044	0.000474	2097.20	0.000
Beta	0.002133	0.001066	0.000805	1.33	0.185
Skewness	0.005823	0.002912	0.000792	3.68	0.000
Beta*Skewness	-0.001321	-0.000661	0.001344	-0.49	0.623

Weibull, n=100, 100 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.85173	0.002162	393.95	0.000
Beta	-0.00834	-0.00417	0.003671	-1.14	0.256
Skewness	-0.16829	-0.08414	0.003612	-23.29	0.000
Beta*Skewness	0.00453	0.00226	0.006133	0.37	0.712

Weibull, n=100, 100 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.97929	0.000873	1121.83	0.000
Beta	0.00472	0.00236	0.001482	1.59	0.112
Skewness	-0.05836	-0.02918	0.001458	-20.01	0.000
Beta*Skewness	0.00872	0.00436	0.002476	1.76	0.078

Weibull, n=100, 100 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.977216	0.000835	1169.88	0.000
Beta	-0.001717	-0.000859	0.001418	-0.61	0.545
Skewness	0.023354	0.011677	0.001396	8.37	0.000

Beta*Skewness	0.013064	0.006532	0.002369	2.76	0.006

Weibull, n=10, 1,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.90975	0.000772	1178.45	0.000
Beta	-0.00190	-0.00095	0.001311	-0.72	0.469
Skewness	-0.15027	-0.07514	0.001290	-58.26	0.000
Beta*Skewness	-0.00069	-0.00034	0.002190	-0.16	0.876

Weibull, n=10, 1,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.97959	0.000367	2671.96	0.000
Beta	0.00006	0.00003	0.000622	0.05	0.963
Skewness	-0.05779	-0.02889	0.000613	-47.17	0.000
Beta*Skewness	0.00141	0.00071	0.001040	0.68	0.497

Weibull, n=10, 1,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.987704	0.000288	3427.60	0.000
Beta	0.001083	0.000542	0.000489	1.11	0.268
Skewness	-0.005916	-0.002958	0.000481	-6.14	0.000
Beta*Skewness	0.001229	0.000615	0.000817	0.75	0.452

Weibull, n=20, 1,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.92405	0.000877	1053.26	0.000
Beta	0.00653	0.00327	0.001489	2.19	0.029
Skewness	-0.13246	-0.06623	0.001466	-45.19	0.000
Beta*Skewness	0.01573	0.00786	0.002488	3.16	0.002

Weibull, n=20, 1,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.99308	0.000223	4458.83	0.000
Beta	0.00207	0.00104	0.000378	2.74	0.006
Skewness	-0.02496	-0.01248	0.000372	-33.54	0.000
Beta*Skewness	0.00376	0.00188	0.000632	2.98	0.003

Weibull, n=20, 1,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.990655	0.000277	3580.02	0.000
Beta	0.003458	0.001729	0.000470	3.68	0.000
Skewness	-0.010413	-0.005206	0.000462	-11.26	0.000
Beta*Skewness	0.004478	0.002239	0.000785	2.85	0.004

Weibull, n=30, 1,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.93473	0.000965	969.09	0.000

Beta	-0.00030	-0.00015	0.001638	-0.09	0.926
Skewness	-0.12763	-0.06382	0.001611	-39.60	0.000
Beta*Skewness	0.00295	0.00148	0.002736	0.54	0.590

Weibull, n=30, 1,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.996518	0.000173	5763.40	0.000
Beta	0.000507	0.000254	0.000294	0.86	0.388
Skewness	-0.012611	-0.006305	0.000289	-21.83	0.000
Beta*Skewness	0.000903	0.000452	0.000490	0.92	0.357

Weibull, n=30, 1,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.997183	0.000132	7563.67	0.000
Beta	0.000392	0.000196	0.000224	0.87	0.382
Skewness	0.000820	0.000410	0.000220	1.86	0.063
Beta*Skewness	0.000428	0.000214	0.000374	0.57	0.567

Weibull, n=50, 1,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.91201	0.001173	777.57	0.000
Beta	0.00170	0.00085	0.001991	0.43	0.669
Skewness	-0.13490	-0.06745	0.001960	-34.42	0.000
Beta*Skewness	0.00479	0.00240	0.003327	0.72	0.471

Weibull, n=50, 1,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.994035	0.000319	3115.77	0.000
Beta	-0.000145	-0.000072	0.000542	-0.13	0.894
Skewness	-0.018176	-0.009088	0.000533	-17.05	0.000
Beta*Skewness	0.000835	0.000418	0.000905	0.46	0.645

Weibull, n=50, 1,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.993477	0.000252	3947.17	0.000
Beta	0.000826	0.000413	0.000427	0.97	0.334
Skewness	0.004137	0.002068	0.000421	4.92	0.000
Beta*Skewness	0.000410	0.000205	0.000714	0.29	0.774

Weibull, n=100, 1,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.84833	0.001418	598.31	0.000
Beta	0.00444	0.00222	0.002407	0.92	0.357
Skewness	-0.16341	-0.08170	0.002369	-34.49	0.000
Beta*Skewness	-0.00067	-0.00034	0.004022	-0.08	0.933

Weibull, n=100, 1,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.97775	0.000583	1676.95	0.000
Beta	0.00075	0.00038	0.000990	0.38	0.704
Skewness	-0.05889	-0.02945	0.000974	-30.23	0.000
Beta*Skewness	0.00129	0.00065	0.001654	0.39	0.696

Weibull, n=100, 1,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.975384	0.000542	1798.14	0.000
Beta	0.001564	0.000782	0.000921	0.85	0.396
Skewness	0.022859	0.011429	0.000906	12.61	0.000
Beta*Skewness	-0.001727	-0.000863	0.001539	-0.56	0.575

Weibull, n=10, 10,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.90933	0.000536	1697.08	0.000
Beta	0.00044	0.00022	0.000910	0.24	0.809
Skewness	-0.15118	-0.07559	0.000895	-84.44	0.000
Beta*Skewness	-0.00096	-0.00048	0.001520	-0.32	0.752

Weibull, n=10, 10,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.97964	0.000255	3838.72	0.000
Beta	0.00038	0.00019	0.000433	0.44	0.661
Skewness	-0.05818	-0.02909	0.000426	-68.23	0.000
Beta*Skewness	0.00063	0.00031	0.000724	0.43	0.665

Weibull, n=10, 10,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.987603	0.000203	4853.12	0.000
Beta	0.000001	0.000001	0.000345	0.00	0.998
Skewness	-0.006146	-0.003073	0.000340	-9.04	0.000
Beta*Skewness	0.000543	0.000272	0.000577	0.47	0.638

Weibull, n=20, 10,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.92405	0.000630	1465.60	0.000
Beta	0.00003	0.00002	0.001070	0.02	0.988
Skewness	-0.13656	-0.06828	0.001053	-64.82	0.000
Beta*Skewness	-0.00098	-0.00049	0.001788	-0.28	0.783

Weibull, n=20, 10,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.99287	0.000136	7300.08	0.000
Beta	0.00009	0.00004	0.000231	0.19	0.847
Skewness	-0.02520	-0.01260	0.000227	-55.45	0.000
Beta*Skewness	0.00030	0.00015	0.000386	0.39	0.698

Weibull, n=20, 10,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.990693	0.000186	5325.25	0.000
Beta	0.000233	0.000116	0.000316	0.37	0.713
Skewness	-0.011170	-0.005585	0.000311	-17.97	0.000
Beta*Skewness	0.000103	0.000051	0.000528	0.10	0.923

Weibull, n=30, 10,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.93470	0.000691	1351.80	0.000
Beta	0.00051	0.00025	0.001174	0.22	0.829
Skewness	-0.12488	-0.06244	0.001155	-54.05	0.000
Beta*Skewness	0.00003	0.00001	0.001961	0.01	0.994

Weibull, n=30, 10,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.996384	0.000108	9221.68	0.000
Beta	-0.000161	-0.000080	0.000183	-0.44	0.661
Skewness	-0.012861	-0.006431	0.000181	-35.62	0.000
Beta*Skewness	-0.000289	-0.000144	0.000306	-0.47	0.638

Weibull, n=30, 10,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.997009	0.000084	1.2E+04	0.000
Beta	0.000109	0.000054	0.000142	0.38	0.702
Skewness	0.001679	0.000840	0.000140	6.00	0.000
Beta*Skewness	-0.000525	-0.000263	0.000238	-1.10	0.269

Weibull, n=50, 10,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.91184	0.000807	1129.64	0.000
Beta	-0.00036	-0.00018	0.001370	-0.13	0.897
Skewness	-0.13240	-0.06620	0.001349	-49.09	0.000
Beta*Skewness	-0.00192	-0.00096	0.002290	-0.42	0.675

Weibull, n=50, 10,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.993907	0.000228	4358.58	0.000
Beta	-0.000311	-0.000155	0.000387	-0.40	0.688
Skewness	-0.018465	-0.009233	0.000381	-24.23	0.000
Beta*Skewness	-0.000374	-0.000187	0.000647	-0.29	0.773

Weibull, n=50, 10,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.993417	0.000146	6787.61	0.000
Beta	-0.000419	-0.000209	0.000248	-0.84	0.400

Skewness	0.005233	0.002617	0.000245	10.70	0.000
Beta*Skewness	-0.000492	-0.000246	0.000415	-0.59	0.554

Weibull, n=100, 10,000 testruns, t-test

Term	Effect	Coef	SE Coef	T	P
Constant		0.84968	0.001020	833.35	0.000
Beta	-0.00101	-0.00051	0.001731	-0.29	0.770
Skewness	-0.16088	-0.08044	0.001703	-47.22	0.000
Beta*Skewness	0.00259	0.00130	0.002892	0.45	0.654

Weibull, n=100, 10,000 testruns, Chebychev

Term	Effect	Coef	SE Coef	T	P
Constant		0.97813	0.000433	2259.13	0.000
Beta	0.00042	0.00021	0.000735	0.29	0.774
Skewness	-0.05754	-0.02877	0.000723	-39.77	0.000
Beta*Skewness	0.00036	0.00018	0.001228	0.15	0.884

Weibull, n=100, 10,000 testruns, H-stat

Term	Effect	Coef	SE Coef	T	P
Constant		0.975332	0.000372	2623.59	0.000
Beta	-0.000375	-0.000188	0.000631	-0.30	0.766
Skewness	0.024339	0.012170	0.000621	19.59	0.000
Beta*Skewness	0.001086	0.000543	0.001054	0.51	0.607

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