Adaptive, suboptimal and nonlinear control of an aeroelastic system

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ADAPTIVE, SUBOPTIMAL AND NONLINEAR CONTROL OF AN
AEROELASTIC SYSTEM

by

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Bachelor of Engineering
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ABSTRACT

Adaptive, Suboptimal and Nonlinear Control of an Aeroelastic System

by

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Professor of Electrical and Computer Engineering Department
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In this thesis, control systems are designed for the flutter control of a nonlinear aeroelastic system. The aeroelastic model describes the plunge and pitch motion of a wing. The model includes plunge and pitch nonlinearities, and has a single control surface for the purpose of control. First an output feedback modular adaptive control system is derived. Quasi-steady aerodynamic model is used for the output feedback modular adaptive control system. For the synthesis of the adaptive controller, it is assumed that only pitch angle and plunge displacement are measured. The control system consists of an input-to-state stabilizing controller and a passive identifier. The passive identifier provides estimates of parameters for synthesis. The second control system is based on the state dependent Riccati equation method. This design yields a suboptimal control law. Finally a nonlinear controller based on backstepping design is presented. Unsteady aerodynamic model is used for the design of the suboptimal controller using state dependent Riccati equation approach and nonlinear controller using backstepping design technique. The unsteady aerodynamic is modeled with an approximation to Theodorsens theory. For the synthesis of second and third
control systems, an observer is designed for estimating the unavailable states variables. Simulation results for each controller are presented. These results show that the designed control systems are effective in flutter suppression.
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LIST OF SYMBOLS

\( a \) = nondimensionalized distance from the midchord to the elastic axis
\( b \) = semichord of the wing
\( C(k) \) = Theodorsen's function
\( C_m \) = measurement matrix
\( c \) = nondimensional distance from the midchord to control surface hinge
\( c_\alpha, d_\alpha \) = feedback gains
\( c_\alpha \) = plunge degree of freedom structural damping coefficient
\( c_{\alpha, k, \Gamma} \) = control gains and weighting matrix
\( F_0 \) = observer gains
\( h \) = plunge displacement coordinate
\( I_\alpha \) = mass moment of inertia about the elastic axis
\( k \) = reduced frequency \((b\omega/u_\infty)\)
\( k_h \) = plunge degree of freedom structural spring constant
\( k_\alpha \) = pitch degree of freedom structural spring constant
\( L \) = estimator gains
\( L(t) \) = lift of the wing
\( M(t) \) = moment of the wing about the elastic axis
\( M, A, L \) = system matrices
\( m_e \) = mass of the plunge-pitch system
\( m_w \) = mass of the wing
\( Q, R, Q_0, R_0, Q_\alpha \) = weighting matrices
\( s_p \) = span
\( u, U \) = free stream velocity
\( V, V_0 \) = Lyapunov functions
\( \hat{x} \) = Observation error
\( v_f, y_f \) = filter input, output
\( x, y \) = state vector, output variable
\( x_\alpha \) = nondimensional distance between elastic axis and the center of mass
\( y_m \) = measured output vector
\( y_r \) = reference trajectory parameter
\( \alpha \) = pitch displacement coordinate
\( \beta \) = control surface deflection coordinate
$\beta_c$ = control input
$\gamma$ = parameter vector defining output
$\rho$ = density of air
$\omega$ = frequency of motion
$\epsilon$ = parameter used in update law
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CHAPTER 1

INTRODUCTION

Aeroelasticity deals with the science that studies the mutual interaction between aerodynamic forces and elastic forces for an aerospace vehicle. Aeroelasticity affects the stability and control performance of aerospace vehicle. Flutter is a dynamic instability resulting from the coupling of aerodynamic, elastic, and inertial forces that can result in sudden mechanical failure of an aircraft wing during flight. Active suppression of aeroelastic instabilities, such as flutter and divergence will lead to improved performance of aircraft. As new lightweight materials are incorporated into aircraft designs in efforts to save money and increase performance, active flutter suppression will become important. Flutter should be considered as important problem because it made United States third longest suspension bridge to collapse. Strong winds caused the Tacoma Narrows Bridge in Washington to collapse on Nov 7, 1940. The exact cause of its collapse was random turbulence, periodic vortex shedding and aerodynamic instability (flutter). Aerodynamic instability is the leading candidate for flutter control, hence flutter suppression should be considered as an important problem.

This thesis addresses the problem of flutter control. The model considered has two degree of freedom in plunge and pitch. The aeroelastic model includes unsteady aerodynamics, which are modeled with an approximation to Theodorsens theory. For stabilization, a single trailing edge control surface is used. Nonlinear control systems are developed for active control of flutter.
1.1 Previous Work

Nonlinear aeroelastic systems exhibit a variety of phenomena including instability, limit cycle, and even chaotic vibration [1-3]. Active control of aeroelastic instability is an important problem. Researchers have analyzed the stability properties of aeroelastic systems and designed controllers for flutter suppression [4-24]. Robust aeroservoelastic stability margins using $\mu$ method have been obtained [4]. Digital adaptive control of a linear aeroservoelastic model has been considered [5]. At the NASA Langley Research Center, a benchmark active control technology (BACT) wind-tunnel model has been designed and control algorithms for flutter suppression have been developed [6-11]. References [7] and [8] describe unsteady aerodynamic data and flutter instability for the BACT project model. The classical and minmax methods have been used to derive robust flutter control systems [9]. Robust passification techniques have been used in [10] for control. Gain scheduled controllers have been designed in [11]. Neural and adaptive control of transonic wind-tunnel model have been considered [12,13]. For an aeroelastic apparatus, tests have been performed in a wind tunnel to examine the effect of nonlinear structural stiffness and control systems have been designed using linear control theory, feedback linearizing technique, and adaptive control strategies [14-21]. A model reference variable structure adaptive control system for plunge displacement and pitch angle control has been designed using bounds on uncertain functions [18]. This approach yields a high gain feedback discontinuous control system.

A backstepping adaptive design method for flutter suppression have been adopted in [19,21]. In this approach, the aeroelastic model has been represented in an output feedback form by a suitable coordinate transformation and output feedback adaptive laws have been derived. A robust flutter control system has been presented in [20] in which a high-gain observer is used for estimating the unmeasured states and the lumped uncertain function of the model for synthesis. For the synthesis of this robust controller, precise measurement of the pitch angle and plunge displacement is
required since the high-gain observer is sensitive to measurement noise. Based on
the Euler-Lagrange theory, control of an aeroelastic model has been considered [23].
A suboptimal control law for flutter suppression using the state-dependent Riccati
equation (SDRE) method has been designed [24]. The SDRE method is applicable
to nonminimum phase systems as well, but requires the knowledge of the system
parameters.

Active output feedback control of an aeroelastic system with unsteady aerodynamics
has been designed using linear quadratic regular (LQR) approach [15]. However
for large perturbations in the state variables, nonlinearities of the model cannot be
neglected and as such it is desirable to derive control laws for the stabilization of the
nonlinear aeroelastic model with unsteady aerodynamics.

Contribution of the thesis lies in the derivation of (1) an output feedback adap-
tive control law, (2) a suboptimal nonlinear control law and (3) a nonlinear control
law designed using backstepping design technique. The aeroelastic system describes
the plunge and pitch motion of a typical wing section and has a single control surface
for flutter suppression. A modular design is used for the derivation of the adaptive
controller. It is assumed that all the system parameters are unknown to the designer,
but the sign of control effectiveness coefficient and a lower bound on its magnitude
are known. The control system consists of an input-to-state stabilizing controller and
a passive identifier. The observer-like identifier based on the gradient type adaptation
law provides the parameter update law for synthesis. This design uses a quasi-steady
aerodynamics. Then a suboptimal nonlinear control systems is designed based on the
state dependent Riccati equation (SDRE) method and, unlike the adaptive controller,
requires complete knowledge of the system parameters. In the closed-loop system,
the origin is asymptotically stable. Finally backstepping design technique is used to
derive a nonlinear controller which yield global stability. This controller also assumes
that the system parameters are not known. The suboptimal nonlinear control law
and the third controller are designed using unsteady aerodynamics and use estimated states for feedback provided by an observer. Simulation results are presented using each controller to demonstrate the flutter suppression capability.

1.2 Aeroelastic Model and Control Problem

The prototypical aeroelastic wing section is shown in Fig. 1. The governing equations of motion are provided in References [15 – 18] which are given by

\[
\begin{bmatrix}
I_\alpha & m_w x_\alpha b \\
m_w x_\alpha b & m_t
\end{bmatrix}
\begin{bmatrix}
\dot{\alpha} \\
\dot{h}
\end{bmatrix}
+
\begin{bmatrix}
c_\alpha & 0 \\
0 & c_h
\end{bmatrix}
\begin{bmatrix}
\alpha \\
h
\end{bmatrix}
+
\begin{bmatrix}
k_\alpha(\alpha) & 0 \\
0 & k_h(h)
\end{bmatrix}
\begin{bmatrix}
\alpha \\
h
\end{bmatrix}
= \begin{bmatrix}
M \\
-L
\end{bmatrix}
\tag{1}
\]

where \( \alpha \) is the pitch angle and \( h \) is the plunge displacement. In Equation (1), \( m_w \) is the mass of the wing; \( m_t \) is the total mass; \( b \) is the semichord of the wing; \( I_\alpha \) is the moment of inertia; \( x_\alpha \) is the nondimensionalized distance of the center of mass from the elastic axis; \( c_\alpha \) and \( c_h \) are the pitch and plunge damping coefficients, respectively; and \( M \) and \( L \) are the aerodynamic lift and moment.

For purposes of illustration, the function \( k_\alpha(\alpha) \) and \( k_h(h) \) are considered as polynomial nonlinearities of fourth and second degree, respectively. These are given by

\[
k_\alpha(\alpha) = k_{\alpha 0} + k_{\alpha 1} \alpha + k_{\alpha 2} \alpha^2 + k_{\alpha 3} \alpha^3 + k_{\alpha 4} \alpha^4
\]

\[
k_h(h) = k_{h 0} + k_{h 1} h^2
\tag{2}
\]

Two models of aerodynamic lift and moment are considered for design.
1.3 Quasi-Steady Aerodynamic Model:

The quasi-steady aerodynamic force and moment are of the form

\[ L = \rho U^2 b c_{l_a} s_p [\alpha + (\dot{h}/U) + \left( \frac{1}{2} - a \right) b(\dot{\alpha}/U)] + \rho U^2 b c_{m_a} s_p \beta \]

\[ M = \rho U^2 b^2 c_{m_a} s_p [\alpha + (\dot{h}/U) + \left( \frac{1}{2} - a \right) b(\dot{\alpha}/U)] + \rho U^2 b^2 c_{m_m} s_p \beta \]

where \( a \) is the nondimensionalized distance from the midchord to the elastic axis, \( s_p \) is the span, \( c_{l_a} \) and \( c_{m_a} \) are the lift and moment coefficients per angle of attack, and \( c_{l_m} \) and \( c_{m_m} \) are lift and moment coefficients per control surface deflection \( \beta \).

Defining the state vector \( q = (q_1, ..., q_4)^T = (\alpha, h, \dot{h}, \dot{\alpha})^T \in R^4 \), one obtains a state variable representation of Equation (1) in the form

\[ \dot{q} = \begin{bmatrix} 0_{2 \times 2} & I_{2 \times 2} \\ M_1 & M_2 \end{bmatrix} q + \begin{bmatrix} 0_{2 \times 2} \\ G_0 \end{bmatrix} \begin{bmatrix} k_{n_a} (\alpha) \\ k_{n_h} (h) \end{bmatrix} + \begin{bmatrix} 0_{2 \times 1} \\ b_0 \end{bmatrix} \beta \]

(4)

where \( \alpha k_{\alpha} = \alpha k_{\alpha} + k_{n_a} \), \( k_{n_h} = \alpha (k_{\alpha} + k_{x_4} \alpha^2 + k_{x_5} \alpha^3 + k_{x_6} \alpha^4) \), \( h k_{h} (h) = h_k h + k_{n_h} \), \( k_{n_{h1}} = k_{h1} h^3 \), \( k_{ij} \) are constants, \( G_0 = (g_{0ij}) \) is a \( 2 \times 2 \) constant matrix, \( b_0 = (b_{01}, b_{02})^T \) (\( T \) denotes transposition), and 0 and I denote null and identity matrices of indicated dimensions. The matrices \( M_1, M_2 \in R^{2 \times 2} \), \( G_0 \), and \( b_0 \) are easily obtained from Equation (1). It is assumed that the parameters in model Equation (4) are not known.

1.4 Unsteady Aerodynamic Model

Theodorsen derived the expressions for unsteady lift and moment, assuming harmonic motion of the airfoil, of the form [14] and are given by

\[ -L(t) = -\rho b^2 s_p (u \pi \dot{\alpha} + \pi \dot{h} - \pi b \dot{\alpha} - u T_3 \dot{h} - T_3 b \dot{\alpha}) \]

\[ -2\pi \rho u b s_p C(k) [u \alpha + \dot{h} + b (\frac{1}{2} - a) \dot{\alpha} + (1/\pi) T_{10} u \beta + b(1/2\pi) T_{11} \dot{\beta}] \]

(5)

\[ M(t) = -\rho b^2 s_p \{ \pi \left( \frac{1}{2} - a \right) u b \dot{\alpha} + \pi b^2 \left( \frac{1}{8} + a^2 \right) \dot{\alpha} + (T_4 + T_{10}) u^2 \beta + [T_1 - T_5 - (c-a) T_4 + \frac{1}{2} T_{11}] u b \dot{\beta} \]
\[-[T_7 + (c-a)T_1]b\beta + a\pi b\hat{h}) + 2\rho ut^a \alpha (\frac{1}{2} + a)C(k)\]

\[\left[ u\alpha + \dot{h} + \frac{b}{2} - a\right] + (1/\pi)T_{10}u\beta + b(1/2\pi)T_{11}\dot{\beta}\]

where $s_p$ is the span and $T_i$, ($i = 1, 4, 7, 8, 10, 11$), are described by Theodersen and depend on the elastic axis location and the control surface hinge location. The Theodersen's function $C(k)$ is a complex function of the form [14]

\[C(k) = F(k) + jG(k)\]

where $k$ is the reduced frequency ($\frac{b\omega}{u_\infty}$), and $F(k)$ and $G(k)$ are composed of Bessel functions. Jones developed an approximation to Theodersen's function for simplicity in computation which can be written as [14]

\[C(s) = 1 - \frac{0.0165s}{s + 0.0455}\frac{s}{s + 0.335s} - \frac{0.35s}{s + 0.335s} \]

\[= 0.5 + \frac{a_1 s + a_0}{s^2 + b_1 s + b_0}\]

where $s$ is the Laplace variable and

\[a_1 = 0.1080075 \frac{u}{b}, a_0 = 0.006825 \frac{u^2}{b^2}\]

\[b_1 = 0.3455 \frac{u}{b}, b_0 = 0.01365 \frac{u^2}{b^2}\]

The control surface dynamics are described by [14]

\[\ddot{\beta} + b_{c1}\dot{\beta} + b_{c0}\beta = b_{c0}\beta_c\]

where $b_{c1} = 50, b_{c0} = 2500$ and $\beta_c$ is the control input to the aeroelastic model.
1.5 System Parameters

The system parameters are given by

\[ b_s = 0.135 \text{ m} \quad m_w = 2.049 \text{ kg} \quad c_h = 27.43 \text{ Ns/m} \]
\[ c_a = 0.036 \text{ Ns} \quad \rho = 1.225 \times 0.6 \text{ kg/m}^3 \quad c_{ta} = 6.28 \]
\[ c_{\beta} = 3.358 \quad c_{ma} = (0.5 + a) c_{ta} \quad c_{m,\beta} = -0.635 \]
\[ m_t = 12.387 \text{ kg} \quad I_{\alpha} = 0.0517 + m_w a_{\alpha}^2 b_s^2 \text{ kg.m}^2 \quad x_{\alpha} = [0.0873 - (b_s + a b_s)] / b_s \]
\[ k_\alpha = 2.82(1 - 22.1 \alpha + 1315.5 \alpha^2 - 8580 \alpha^3 + 17,289.7 \alpha^4) \text{ N.m/rad} \]
\[ k_h = 2844.4 + 255.99 h^2 \text{ N/m} \]
Figure 1: Aeroelastic Model
CHAPTER 2

OUTPUT FEEDBACK MODULAR ADAPTIVE CONTROL

2.1 Introduction

The chapter presents nonlinear adaptive control systems for the flutter control of a prototypical wing section with structural nonlinearities using only output feedback. The chosen model describes the plunge and pitch motion of a wing. The model includes plunge and pitch nonlinearities, and has a single control surface for the purpose of control. Using a canonical representation of the aeroelastic system, a modular output feedback adaptive control system consisting of an input-to-state stabilizing controller and a passive identifier (an observer and adaptation law) is derived. In the closed-loop system, asymptotic stabilization of the pitch and plunge motion is accomplished. Simulation results show that the control system is effective in regulating the state vector to the origin in spite of large parameter uncertainties.

2.2 Canonical System and State Estimation

In this section, a canonical representation of the model is obtained and filters are designed for the state estimation. For a model with only pitch nonlinearity, a canonical form has been similarly used in a nonmodular design of [20]; however since the aeroelastic model (1) has both the pitch and plunge nonlinearities, there are some
differences in the structure of the filters designed here. Consider a transformation
\[ x = Tq, \]
where
\[ T = \begin{bmatrix} I_{2 \times 2} & 0_{2 \times 2} \\ -M_2 & M_1 \end{bmatrix} \tag{11} \]
Then it can be shown that
\[
\dot{x} = \begin{bmatrix} M_2 & 0_{2 \times 2} \\ M_1 & 0_{2 \times 2} \end{bmatrix} x + \begin{bmatrix} 0_{2 \times 2} \\ g \end{bmatrix} \begin{bmatrix} k_m(\alpha) \\ k_m(h) \end{bmatrix} + \begin{bmatrix} 0_{2 \times 1} \\ b \end{bmatrix} \beta \tag{12}
\]
The system Equation (12) is in an output feedback form.
Assumption 1: It is assumed that the elements of matrices \( M_1, M_2, g, b; \) and \( k_{aj}, \)
j = 0, 1, ..., 4, and \( k_{jh}, j = 0, 1, \) associated with the structural nonlinearities are not known, but the sign of each element of \( b \) is known and
\[ |b_k| \geq b_{km} \]
\((k = 1, 2), \) where the lower bound \( b_{mk} \) is given.
Define a vector of unknown parameters
\[ \theta = [b^T, M_{2(1)}, M_{2(2)}, M_{1(1)}, M_{1(2)}, g_{11}p_{1a}^T, g_{21}p_{2a}^T, g_{12}k_{h1}, g_{22}k_{h1}]^T = [b^T, a^T] \in \mathbb{R}^{20} \]
the superscript \( T \) denotes matrix transposition, \( M_{i(k)} \) denotes the \( k \)th row of \( M_i, \)
\( p_\alpha = (k_{a1}, k_{a2}, k_{a3}, k_{a4}), g = (g_{ij})(i, j = 1, 2), \) and the \( 4 \times 18 \) matrix \( F^T \) is
\[ F^T = (\beta e_3, \beta e_4, \alpha e_1, \alpha e_2, \alpha e_2, \alpha e_3, \alpha e_3, \alpha e_4, \alpha e_4, \alpha^2 e_3, \alpha^3 e_3, \alpha^4 e_3, \alpha^5 e_3, \alpha^6 e_4, \]
\[ \alpha^7 e_4, \alpha^8 e_4, \alpha^9 e_4, h^3 e_3, h^3 e_4) \]
Here \( e_k \in \mathbb{R}^4 \) denotes a column vector whose \( k \)th element is one, and the remaining elements are zero. Using the definition of matrix \( F, \) Equation (11) can be written as
\[
\dot{x} = Ax + L_0[\alpha, h]^T + F^T(\alpha, h, \beta) \theta \tag{13}
\]
where \( L_0 = (L_{1}^T, L_{2}^T)^T, \)
\[ L_1 = \begin{bmatrix} L_{41} & 0 \\ 0 & L_{22} \end{bmatrix}, A = \begin{bmatrix} -L_1 & I_{2 \times 2} \\ -L_2 & 0_{2 \times 2} \end{bmatrix} \tag{14} \]
The matrices $L_1, L_2$ are chosen so that $A$ is a stable matrix. Equation (13) is a canonical representation of system Equation (1) in which $A$ is in a special form, the regressor matrix $F$ is a function of the measured variables and the input $\beta$, and all unknown parameters of the system are included in the vector $\theta$. Based on Equation (13), certain filters are designed.

In view of Equation (13), following Ref. 26, filters are given by

$$
\dot{z} = A z + L_0[\alpha, h]^T, \dot{\Omega} = A\Omega + F'(\alpha, h, \beta) \tag{15}
$$

where $z \in \mathbb{R}^n$ and $\Omega \in \mathbb{R}^{n \times 20}$. Define a state estimate of $x$ as

$$
\hat{x} = z + \Omega^T \theta \tag{16}
$$

and let the state error be $\hat{x} = (x - \hat{x})$. In view of Equations (13-15), the error $\hat{x}$ is governed by

$$
\dot{\hat{x}} = A\hat{x} \tag{17}
$$

Because $A$ is a Hurwitz matrix, $\hat{x}(t) \to 0$ as $t \to \infty$ and, therefore, $\hat{x}(t)$ asymptotically converges to $x(t)$. Of course, $\theta$ is not known, and Equation (16) cannot be used to construct $\hat{x}(t)$; however, it is useful in the derivation of an adaptive control law. Define

$$
\Omega = [v_1, v_0, s_1, s_2, \ldots, s_{18}] = [v_1, v_0, S] \tag{18}
$$

where each column of $\Omega$ is a $4 \times 1$ vector. Because of the special structure of $F'$, it follows from Equation (15) that $v_1 = (v_{11}, \ldots, v_{14})^T$ satisfy

$$
v_1 = Av_1 + e_3 \beta, \quad v_0 = Av_0 + e_4 \beta \tag{19}
$$
2.3 Pitch Angle Control

First the derivation of the control law for the trajectory control of the pitch angle is considered.

In view of Equations (12) and (16), the derivative of the controlled output variable $\alpha$ is given by

$$\dot{\alpha} = x_3 + M_2(1)[\alpha, h]^T = \zeta_3 + \Omega_{(3)}^T\theta + \bar{x}_3 + M_{2(1)}[\alpha, h]^T$$

$$= \zeta_3 + b_1v_{13} + b_2v_{03} + S_{(3)}a + M_{2(1)}[\alpha, h]^T + \bar{x}_3$$

$$= \zeta_3 + [0, v_{03}, (S_{(3)} + e_1^T\alpha + e_2^Th)]\theta + \bar{x}_3$$

$$= \zeta_3 + [v_{13}, v_{03}, (S_{(3)} + e_1^T\alpha + e_2^Th)]\theta + \bar{x}_3$$

(20)

where $S(t)$ is the ith row of $S$ and $e_i^T$ denotes row vector of appropriate dimension.

The design is completed in two steps following the backstepping design procedure of [26].

Step 1:

The derivative of $z_1 = \alpha$ can be written as

$$\dot{z}_1 = \zeta_3 + b_1v_{13} + \bar{\omega}^T\theta + \bar{x}_3$$

(21)

where

$$\bar{\omega}^T = [0, v_{03}, (S_{(3)} + e_1^T\alpha + e_2^Th)]$$

Let

$$z_2 = v_{13} - \alpha_1$$

(22)

be a new co-ordinate, where $\alpha_1$ is a stabilizing signal. Then Equation (21) gives

$$\dot{z}_1 = \zeta_3 + b_1(z_2 + \alpha_1) + \bar{\omega}^T\theta + \bar{x}_3$$

(23)

We select the signal $\alpha_1$ as

$$\alpha_1 = \frac{\text{sgn}(b_1)}{b_{1m}} (c_1 + l_1)z_1 + \frac{1}{b_1}\bar{\alpha}_1$$

(24)
where $b_1$ is the estimate $b_1$, $c_1 > 0$ and $l_1$ is the first damping term. Note that

$$|b_1| > b_{1m} \Rightarrow \left| \frac{b_1}{b_{1m}} \right| > 1 \Rightarrow -\frac{b_1}{b_{1m}} < -1$$

$$b_1 \frac{1}{b_1} = \frac{b_1 - \hat{b}_1 + \tilde{b}_1}{\hat{b}_1} = \frac{\hat{b}_1}{\hat{b}_1} + 1$$

where $\hat{b}_1 = b_1 - \hat{b}_1$. Using $\alpha_1$ from Equation (24) in (23), gives

$$\dot{z}_1 = \zeta_3 + b_1 z_2 - \frac{|b_1|}{b_{1m}}(c_1 + l_1)z_1 + \bar{\alpha}_1 + \bar{\alpha}_1 \bar{\omega}^T \bar{\theta} + \bar{\omega}^T \theta + \bar{x}_3$$

(25)

In view (25), we choose $\bar{\alpha}_1$ as

$$\bar{\alpha}_1 = -\zeta_3 - \bar{\omega}^T \dot{\bar{\theta}}$$

(26)

where $\dot{\bar{\theta}}$ is the estimate of $\theta$. Substituting $\bar{\alpha}_1$ in Equation (25) gives

$$\dot{z}_1 = b_1 z_2 - \frac{|b_1|}{b_{1m}}(c_1 + l_1)z_1 + \bar{\alpha}_1 \bar{\omega}^T \bar{\theta} + \bar{\omega}^T \theta + \bar{x}_3$$

(27)

Noting that $\hat{b}_1 = e_1^T \hat{\theta}$, Equation (27) gives

$$\dot{z}_1 = b_1 z_2 - \frac{|b_1|}{b_{1m}}(c_1 + l_1)z_1 + (\bar{\alpha}_1 e_1^T)\bar{\theta} + \bar{x}_3$$

(28)

Consider the first Lyapunov function

$$V_1 = z_1^2 / 2$$

(29)

Its derivative along the solution of Equation (28) is

$$\dot{V}_1 = z_1 [b_1 z_2 - \frac{|b_1|}{b_{1m}}(c_1 + l_1)z_1 + (\bar{\alpha}_1 e_1^T)\bar{\theta} + \bar{x}_3]$$

(30)

Using Young's inequality gives

$$| z_1 (\bar{\alpha}_1 e_1^T) \bar{\theta} | \leq k_1 | \bar{\omega} | + \frac{\bar{\alpha}_1}{b_1} | e_1 |^2 | z_1 |^2 + \frac{| \dot{\bar{\theta}} |^2}{4k_1}$$

$$| z_1 \bar{x}_3 | \leq d_1 | z_1 |^2 + \frac{| \bar{\omega} |^2}{4d_1} \leq d_1 | z_1 |^2 + \frac{| \bar{\omega} |^2}{4d_1}$$

(31)
where \( k_i > 0 \) and \( d_i > 0 \). Using Equation (31) in Equation (30), selecting the damping term

\[
I_1 = d_1 + k_1 | \omega_\alpha + \frac{\bar{\alpha}}{b_1} e_1^T |^2
\]

and noting that \(-|b_1|b_1^{-1} < -1\), Equation (30) gives

\[
\dot{V}_1 \leq b_1 z_1 x_2 - c_1 x_1^2 + \frac{1}{4k_1} | \dot{\theta} |^2 + \frac{1}{4d_1} | \ddot{x} |^2
\]

Step 2:
Differentiating \( z_2 \) gives

\[
\dot{z}_2 = \dot{v}_{13} - \dot{\alpha}_1
\]

Note that \( \alpha_1 \) is a function of \( y, \zeta_3, \omega, \dot{\theta} \) and \( \bar{\omega} \) is a function of \( v_{03}, S_{(3)}, \alpha, h \). Therefore, its derivative is

\[
\dot{\alpha}_1 = \frac{\partial \alpha_1}{\partial v_{03}} \dot{v}_{03} + \frac{\partial \alpha_1}{\partial \zeta_3} \dot{\zeta}_3 + \sum_{j=1}^{18} \frac{\partial \alpha_1}{\partial s_j} \ddot{s}_j + \frac{\partial \alpha_1}{\partial \dot{\theta}} \dot{\dot{\theta}} + \frac{\partial \alpha_1}{\partial \dot{x}_1} \dot{\dot{x}}_1 + \frac{\partial \alpha_1}{\partial \dot{x}_2} \dot{\dot{x}}_2
\]

where \( s_{j3} \) is the third element of vector \( s_j \) and \( a_0 \) is defined in Equation (35). The derivative of \( x_1 \) from Equation (20) is given by

\[
\dot{x}_1 = \zeta_3 + \omega^T \theta + \ddot{x}_3
\]

where

\[
\omega^T = [v_{13}, v_{03}, (S_{(3)} + e_1^T \alpha + e_2^T h)]
\]

Also from Equation (12) one has

\[
\dot{x}_2 = \dot{h} = x_4 + M_{2(3)}[\alpha, h]^T
\]

Because \( x_4 - \ddot{x}_4 = \ddot{x}_4 \), using Equation (16) in (38) gives

\[
\dot{x}_2 = \ddot{x}_4 + \dddot{x}_4 + M_{2(3)}[\alpha, h]^T
\]

\[
= \zeta_4 + \Omega_{(3)}^T \theta + M_{2(3)}[\alpha, h]^T + \dddot{x}_4
\]
Note that
\[ M_2(\alpha, h)^T = \theta_5 \alpha + \theta_6 h = (\alpha e_5^T + h e_6^T) \theta \] (40)

Using Equation (40) in (39), one has
\[ \dot{x}_2 = \zeta_4 + (\Omega_{i(4)}^T + \alpha e_5^T + h e_6^T) \theta + \ddot{x}_4 \]
\[ \simeq \zeta_4 + \omega_4^T \theta + \ddot{x}_4 \] (41)

Substituting \( \dot{x}_1 \) and \( \dot{x}_2 \) from Equations (36) and (41) in (35) gives
\[ \dot{a}_1 = a_0 + \frac{\partial a_1}{\partial x_1} (\zeta_3 + \omega^T \theta + \ddot{x}_3) + \frac{\partial a_1}{\partial x_2} (\zeta_4 + \omega^T \theta + \ddot{x}_4) + \frac{\partial a_1}{\partial \theta} \dot{\theta} \]
\[ \simeq a_1 + a_2^T \ddot{x}_a + a_3^T \theta + a_4^T \dot{\theta} \] (42)

where
\[ \ddot{x}_a = (\ddot{x}_3, \ddot{x}_4)^T \]
\[ a_1 = a_0 + \frac{\partial a_1}{\partial x_1} \zeta + \frac{\partial a_1}{\partial x_2} \zeta_4 \]
\[ a_2^T = (\frac{\partial a_1}{\partial x_1} \frac{\partial a_1}{\partial x_2}) \]
\[ a_3^T = \frac{\partial a_1}{\partial x_1} \omega^T + \frac{\partial a_1}{\partial x_2} \omega^T \]
\[ a_4^T = \frac{\partial a_1}{\partial \theta} \]

Now using Equations (19) and (42) in (34) gives
\[ \ddot{z}_2 = -l_{21} v_{11} + \beta - a_1 - a_2^T \ddot{x}_a - a_3^T \theta - a_4^T \dot{\theta} \] (43)

In view of Equation (43), the control input is chosen as
\[ \beta = -(c_2 + l_2) z_2 - \dot{b}_1 z_1 + l_{21} v_{11} + a_1 + a_2^T \dot{\theta} \] (44)

where \( c_2 > 0 \) and \( l_2 \) is the damping term yet to be determined.

Substituting Equation (44) in Equation (43) gives
\[ \ddot{z}_2 = -(c_2 + l_2) z_2 - \dot{b}_1 z_1 - a_2^T \dot{\theta} - a_2^T \ddot{x}_a - a_4^T \dot{\theta} \] (45)
Now consider the Lyapunov function

\[ V_2 = V_1 + z_2^2 / 2 \]  

(46)

Computing the derivation of \( V_2 \) gives

\[ \dot{V}_2 \leq b_1 z_1 z_2 - c_1 z_1^2 + \frac{1}{4 k_1} |\dot{\theta}|^2 + \frac{1}{4 d_1} |\ddot{x}|^2 + z_2 [-(c_2 + l_2) z_2 - \dot{b}_1 z_1 - a_2^T \dot{x}_\alpha - a_2^T \dot{\theta}] \]  

(47)

Noting that \( \dot{b}_1 = b_1 - \dot{b}_1 = e_1^T \dot{\theta} \) and using Young’s inequality, one has

\[ z_2 [-a_3^T + z_1 e_1^T] \dot{\theta} \leq k_2 z_2^2 - a_3 + z_1 e_1^2 + \frac{1}{4 k_3} |\dot{\theta}|^2 \]

\[ z_2 a_3^T \dot{x}_\alpha \leq k_3 z_2^2 |a_4|^2 + \frac{1}{4 k_2} |\dot{\theta}|^2 \]

\[ z_2 a_2^T \ddot{x}_\alpha \leq d_2 z_2^2 |a_2|^2 + \frac{1}{4 d_2} |\ddot{x}|^2 \]  

(48)

where \( k_i > 0 \) and \( d_i > 0 \) Using Equation (48) in (47) and selecting \( l_2 \) as

\[ l_2 = d_2 |a_2|^2 + k_2 - a_3 + z_1 e_1^2 + k_3 |a_4|^2 \]  

(49)

the derivative of \( V_2 \) becomes

\[ \dot{V}_2 \leq -c_1 z_1^2 - c_2 z_2^2 + |\ddot{x}|^2 \left[ \frac{1}{4 k_1} + \frac{1}{4 k_2} \right] + |\ddot{x}|^2 \left[ \frac{1}{4 d_1} + \frac{1}{4 d_2} \right] + |\dot{\theta}|^2 \frac{1}{4 k_3} \]  

(50)

Since \( \ddot{x} \) is bounded, one concludes from Equation (50) that \( z_1 \) and \( z_2 \) are bounded provided that \( \dot{\theta} \) and \( \ddot{\theta} \) are bounded. The desirable properties of \( \dot{\theta} \) and \( \ddot{\theta} \) are obtained using the identifier of the next section.

### 2.4 Plunge Control

In this section, the design of control system using the plunge displacement as an output is considered. Since the design can be completed following the steps of the previous section, only a brief derivation for plunge control law design is presented.

For the design of a controller, consider the plunge dynamics of interest given by

\[ \dot{y} = \dot{h} \]
\[
\dot{x}_2 = x_4 + M_2(\alpha, h)^T \tag{51}
\]

Following the steps 1 and 2 of Section 5, one obtains the control law

\[
\beta = -(c_2 + l_2)z_2 - b_2z_1 + l_2v_0 + a_1 + a_3^T \hat{\theta}
\]

where now one has \( z_1 = y = h, z_2 = v_{04} - \alpha_1, \alpha_1 = -\frac{\alpha_2}{b_{pm}}(c_1 + l_1)z_1 + \frac{\gamma_1}{b_2}, \) and

\[
\hat{\alpha}_1 = -\zeta_4 - \hat{\omega}^T \hat{\theta}
\]

Here for simplicity, symbols for certain functions identical to those used in Section 5 have been retained; however, one must note that these functions as defined here differ from those of the previous section. The identifier can also be designed following the previous section.

For the in plunge Control, the control law uses inverse of \( b_2; \) therefore the projection operator is used to modify only the update law for \( \hat{b}_2. \)

2.5 Y-Passive Observer

The design of a passive identifier is considered in this section. Define \( y = \alpha; \) then

\[
\dot{y} = \dot{\alpha} = \zeta_3 + \omega^T \theta + \tilde{x}_3 \tag{53}
\]

Consider an observer of the form [33]

\[
\dot{\hat{y}} = -(c_0 + k_0|\omega|^2)(\hat{y} - y) + \zeta_3 + \omega^T \hat{\theta}
\]

where \( c_0 = \min (c_1, c_2), k_0 = (k_1^{-1} + k_2^{-1}) \) The observation error is \( e = y - \hat{y}. \) The observation error dynamics is given by

\[
\dot{\varepsilon} = -(c_0 + k_0|\omega|^2)e + \omega^T \hat{\theta} + \tilde{x}_3
\]

For the derivation of the update law, consider a Lyapunov function,

\[
V_0 = \frac{e^2}{2} + \frac{\hat{\theta}^T \Gamma^{-1} \hat{\theta}}{2} + \frac{\tilde{x}^T \hat{P} \tilde{x}}{2c_0} \tag{56}
\]
where $\Gamma = \text{diag}(\gamma_k), \gamma_k > 0$ The derivative of $V_0$ is given by

$$V_0 = e[-\left(c_0 + k_0|\omega|^2\right)e + \omega^T \dot{\theta} + \dot{x}_3] + \dot{\theta}^T \Gamma^{-1} \dot{\theta} - \frac{|\dot{x}|^2}{2c_0} \quad (57)$$

In view of (55), the update law is chosen as

$$\dot{\theta} = -\dot{\theta} = -\Gamma \omega e \quad (58)$$

Substituting the update law in Equation (55) and using the inequality

$$|c_3| \leq \frac{c_0 e^2}{2} + \frac{1}{2c_0} |\dot{x}|^2 \quad (59)$$

gives

$$\dot{V}_0 \leq -k_0 |\omega|^2 e^2 - \frac{c_0 e^2}{2} \quad (60)$$

Nonpositivity of $\dot{V}_0$ proves that $\dot{\theta}, e \in L_\infty(0, \infty)$. Following [27], one can also show that $e, \dot{\theta} \in L_2(0, \infty)$.

This adaptation scheme may lead to division by zero because inverse of $b_1$ is needed in the control law. Therefore, it is necessary to modify the adaptation rule for the estimate $b_1$. The modification is done using the projection operator. Let $\tau_1 = \gamma_1 \omega_1 e$, where $\omega_1$ denotes the first element of $\omega$. The update law of the form

$$\dot{\theta} = \text{Proj} (\tau_1) \text{ using the projection operator is given by}$$

$$\dot{b}_1 = \tau_1 \left\{ \begin{array}{ll}
1, & \hat{b}_1 \text{sgn}(b_1) + \hat{b}_1 \text{sgn}(b_1)e^{-1}) > \hat{b}_1 \text{sgn}(b_1) \geq 0 \\
\max\{0, (\epsilon - b_1 \text{sgn}(b_1) - \hat{b}_1 \text{sgn}(b_1)e^{-1})\} & \hat{b}_1 \text{sgn}(b_1) \leq \hat{b}_1 \text{sgn}(b_1) < 0
\end{array} \right. \quad (61)$$

where $\epsilon \in (0, b_1 m)$. The update laws for the remaining parameters given in Eq. (56) are retained.

Let the initial condition be such that $\hat{b}_1(0) \text{sgn}(b_1) > b_1 m$, the lower bound. Then it can be shown following Ref. [26] (pp. 232-233), that even with the modified adaptation rule, the identifier has the following properties:

(i) $|\hat{b}_1(t)| \geq b_{m1} - \epsilon > 0, t \in [0, \infty)$

(ii) $\hat{\theta} \in L_\infty[0, \infty), \hat{\theta}_e \in L_2[0, \infty)$
This identifier has desirable properties and this permits modular design possible. Using these properties of the identifier and the ISS controller, following [26] one can conclude that all the signals in the closed-loop system are bounded and $\alpha$ tends to zero provided that the zero dynamics of the system are stable. The zero dynamics represent the residual plunge motion used in this study when the pitch angle is zero. Stability of the zero dynamics has been analyzed in [16 — 19]. For the parameters of the system [4] the zero dynamics have exponentially stable equilibrium points.

2.6 Simulation

In this section simulation results for pitch angle and plunge motion control are presented. The parameters for the system are given in the appendix. Simulation results are presented for different values of $\alpha$ and $U$. The initial conditions chosen are $x(0) = (30(deg), 0.01(m), 0, 0)^T$. The initial conditions except for $b_1$ and $b_2$ are taken to be zero. The initial states of the filters are set as $\Omega(0) = 0$ and $\zeta(0) = 0$. The design parameters are selected as $\lambda = 10$, $c_0 = 10$, $c_1 = 10$, $c_2 = 10$, $d_1 = 0.1$, $d_2 = 0.5$, $k_0 = 0.05$, $k_1 = 0.1$, $k_2 = 0.1$, $k_3 = 0.1$, $\Gamma = I_{20\times20}$, $L_{11} = L_{12} = 20$, $L_{21} = L_{22} = 100$. For simulation, the control input has been limited to 30°. Simulation results are presented for two sets of $\alpha$ and $U$. These are (S1): $\alpha = -0.8$ and $U = 20$ m/s and (S2): $\alpha = -0.6847$ and $U = 20$ m/s.

For the case (S1) the complex open-loop poles and zero of the linearized system for $\alpha$ as an output are at $(1.4965 \pm j14.1395, -3.8801 \pm j14.1635)$ and $-1.3815 \pm j19.3643$, respectively. For case (S2) for plunge displacement as an output, the poles are at $(1.4526 \pm j13.4941, -3.7054 \pm j13.8059)$ and the zeros are at $(-0.3256 \pm j7.6021)$ respectively. For each case the open-loop system is unstable, but the system has stable zeros (minimum phase system). Thus the origin of the zero dynamics is exponentially stable for each case. The open loop response of the system is shown in Figure 2. The
plot shows that the system exhibits limit cycle oscillations.

Adaptive $\alpha$ control for S1

The closed-loop system including the control law and the update law for $a = -0.8$ and $U = 20 \text{ m/s}$ is simulated. For chosen value of $a$ and $U$, one has $b_1 = -28.9919$ and $b_2 = -8.4161$. The value of the lower bound is chosen as $b_{1m} = 10$ [less than $(1/2)$th of $|b_1|$]. Selected responses are shown in fig 3. It is assumed that $b_1(0)$ is twice of the nominal value $b = (-28.9919, -8.4161)^T$. The initial conditions for the remaining elements of $\hat{\theta}$ are set to zero. Responses are shown in Fig 3. It is observed that the pitch angle is quickly controlled, and after initial oscillatory transient, the plunge displacement also decays to zero. These oscillations in the $h-$response are caused due to the complex zeros (the poles of the zero dynamics). Control saturation of only few peaks are observed.

$h$ Control for condition S2

The closed-loop system including the control law and the update law for $a = -0.6847$ and $U = 20m/s$ is simulated. For chosen value of $a$ and $U$, one has $b_1 = -43.8334$ and $b_2 = -10.4352$ are obtained. The value of the lower bound is chosen $b_{2m} = 5$ [less than $(1/2)$th of $|b_2|$]. Selected responses are shown in fig 4. It is assumed that $b_2(0)$ is twice of the nominal value $b = (-43.8334, -10.4352)^T$. The initial conditions for the remaining elements of $\hat{\theta}$ are set to zero. It is observed that the pitch angle is quickly controlled, and after initial oscillatory transient, the plunge displacement also decays to zero. These oscillations in the $h-$response are caused due to the complex zeros (the poles of the zero dynamics).
2.7 Summary

In this chapter, control systems for the stabilization of an aeroelastic system using quasi-steady aerodynamics based on the modular adaptive output feedback design technique were presented. Adaptive control laws for the trajectory control of $\alpha$ and $h$ were derived and filters were designed to obtain the estimate of the state vector. Each control system consists of an input-to-state stabilizing controller and an identifier. In the closed-loop system, the state vector converged to the origin and thus flutter was suppressed. The adaptive controller has several design parameters that can be adjusted to obtain desirable response characteristics. The controller designed here uses quasi-steady aerodynamics. Adaptive design for aeroelastic system with unsteady aerodynamic is quite involved and is considered as a problem for future research.
Figure 2: Open-loop response: $U = 18 \text{ m/s, } a = -0.8$
Figure 3: Closed-loop adaptive pitch angle control: $U = 18 \text{ m/s}, \alpha = -0.8$
Figure 4: Closed-loop adaptive plunge displacement control: \( U = 20 \text{ m/s}, a = -0.6847 \)
CHAPTER 3

SUBOPTIMAL CONTROL USING STATE DEPENDENT RICCATI EQUATION UNDER CONTROL CONSTRAINTS

3.1 Introduction

In this chapter nonlinear output feedback control system for the stabilization of an aeroelastic system including unsteady aerodynamics and both the plunge and pitch structural nonlinearities is presented. The plunge and pitch motion of a wing is described by the aeroelastic model. The unsteady aerodynamics are modeled using Theodorsen’s theory. A single control surface is utilized for the flutter control and it is assumed that there exists a specified hard constraint on the control input. Control of an aeroelastic system is obtained by using the State Dependent Riccati Equation method. For the synthesis of the controller, only the plunge displacement, pitch angle, and control surface deflection are measured. An observer is designed to estimate the remaining state variables of the system for feedback. A slack variable is introduced to transform the constrained control problem into an unconstrained problem and then a suboptimal control law is designed. In the closed-loop system, including the observer and nonlinear controller, the zero state is (locally) asymptotically stable, and the state vector asymptotically converges to the origin. Simulation results for various flow velocities and elastic axis are presented which show that the designed control system is effective in flutter suppression.
3.2 State Variable Representation

It will be convenient to obtain a state variable form of the complete model. The Theodorsen's function $C(s)$ can be treated as a second-order transfer function of a filter with input

$$v_f(t) = [u\alpha + \dot{h} + b(0.5 - a)\dot{\alpha} + (1/\pi)T_{10}u\beta + b(1/2\pi)T_{11}\dot{\beta}] = a_v^T x_p$$  (62)

where the vector $a_v \in \mathbb{R}^6$ is

$$a_v = [0, u, 1/\pi T_{10}u, 1, b(5 - a), b(1/2\pi)T_{11}]^T$$  (63)

and the partial state vector is $x_p = (h, \alpha, \beta, \dot{h}, \dot{\alpha}, \dot{\beta})^T \in \mathbb{R}^6$. The output of the filter is denoted as $y_f(t)$ which is related to the input $v_f(t)$ as

$$\dot{y}_f(s) = C(s)\dot{v}_f(s)$$  (64)

where $\dot{y}_f(s)$ and $\dot{v}_f(s)$ represent Laplace transforms of $y_f(t)$ and $v_f(t)$, respectively. We note that the input to the filter $C(s)$ is a linear combination of the plunge, pitch, and control surface deflection variables.

The transfer function $C(s)$ of the filter has a minimal realization of dimension 2. Although, one can derive a variety of realizations of $C(s)$, we consider a representation of the filter of the form

$$\begin{align*}
\dot{x}_f1 &= x_f2 \\
\dot{x}_f2 &= -b_0 x_f1 - b_1 x_f2 + v_f
\end{align*}$$  (65)

with its output given by

$$y_f = 0.5v_f + a_0 x_f1 + a_1 x_f2$$

$$= 0.5a_v^T x_p + a_0 x_f1 + a_1 x_f2$$  (66)

In view of (64) and (66), $L(t)$ and $M(t)$ (Eqs.(5), and (6)) can be written as

$$-L(t) = [-\rho b^2 s_p(u\pi \dot{\alpha} - u T_4 \ddot{\beta} - T_1 b \dot{\beta}) - 2\pi \rho s_p u y_f] - \pi \rho b^2 s_p (\ddot{h} - b a \ddot{\alpha})$$  (67)
Define the state vector including the filter states as

\[ x = (h, \alpha, \beta, \dot{h}, \dot{\alpha}, \dot{\beta}, x_{f1}, x_{f2})^T \in \mathbb{R}^8 \]  

(69)

Substituting \( \beta = -b_{c1} \beta - b_{c0} (\beta - \beta_c) \) from (10) and \( y_f \) from (66) in (67) and (68), one can express the terms in the square brackets as linear function of \( x \) and \( \beta_c \).

Substituting the resulting expressions of \( L \) and \( M \) in (1), collecting the terms involving \( \dot{h} \) and \( \dot{\alpha} \), solving for \( \dot{h} \) and \( \dot{\alpha} \), and using (10) gives

\[
\begin{bmatrix}
\dot{h} \\
\dot{\alpha} \\
\dot{\beta}
\end{bmatrix} = A_1 x + B_1 \beta_c + N_1 f_n(h, \alpha)
\]  

(70)

for appropriate matrix \( A_1 \in \mathbb{R}^{3 \times 8} \) and \( B_1 \in \mathbb{R}^3 \), where \( f_n \) denotes the structural nonlinearities, \( f_n(h, \alpha) = (h_n(h), k_n(a))^T \), \( N_1 = [-M_{a}^{-1}, 0_{2 \times 1}]^T \), and the matrix \( M_a \) is

\[
M_a = \begin{bmatrix}
m_x + \pi \rho b^2 s_p & m_w x_{\alpha} b - \pi \rho b^2 a s_p \\
m_w x_{\alpha} b - \pi \rho b^2 a s_p & I_a + \pi \rho b^2 s_p (1/8 + a^2)
\end{bmatrix}
\]  

(71)

The complete system including (70), (65) and (10), has a state variable representation of the form.

\[
\begin{bmatrix}
\dot{h} \\
\dot{\alpha} \\
\dot{\beta}
\end{bmatrix} = \begin{bmatrix}
O_{3 \times 3} & I_{3 \times 3} & O_{3 \times 2} \\
O_{1 \times 7} & 1 & -b_0 \\
0 & -b_1 & 0
\end{bmatrix}
\begin{bmatrix}
h \\
\alpha \\
\beta
\end{bmatrix}
+ \begin{bmatrix}
A_1 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
x \\
0 \\
0
\end{bmatrix}
+ \begin{bmatrix}
B_1 \\
0 \\
0
\end{bmatrix}
\beta_c + \begin{bmatrix}
N_1 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
f_n(h, \alpha) \\
0 \\
0
\end{bmatrix}
\]

(72)

where \( O \) and \( I \) denote null and identity matrices of indicated dimensions.

It is assumed that the control input is constrained as

\[ |\beta_c(t)| \leq \beta_{cm} \]  

(73)
where $\beta_{cm}$ is the maximum permissible magnitude of the control input $\beta_c$.

The measurement equation is

$$y_m = [h, \alpha, \beta]^T = C_m x$$  \hspace{1cm} (74)

where

$$C_m = [I_{5 \times 3} O_{3 \times 5}]$$

We are interested in designing an output feedback control law satisfying the bound (73) such that in closed-loop system, both the pitch angle and the plunge displacement asymptotically converge to zero.

### 3.3 Control Law Design

In this section, the derivation of a nonlinear controller is considered.

#### 3.3.a Observer Design

First of all, it is essential to construct an observer so that unavailable state variables $(h, \alpha, \beta, x_{f1}, x_{f2})$ can be estimated. Noting that $h, \alpha$ and $\beta$ are available, one can construct a full-order observer of the form

$$\dot{z} = A z + B \beta_c + N f_n(h, \alpha) + F_0 (y_m - \hat{y}_m)$$  \hspace{1cm} (75)

where $\dot{z}$ denotes estimated values of the state vector $x$, $F_0$ is a $8 \times 3$ matrix and $\hat{y}_m = (\dot{h}, \dot{\alpha}, \dot{\beta})^T = C_m \dot{x}$. Subtracting (75) from (72), it easily follows that the state estimation error $\ddot{x} = x - \dot{z}$ satisfies

$$\ddot{x} = (A - F_0 C_m) \ddot{x}$$  \hspace{1cm} (76)

$$\dot{x} = A_c \ddot{x}$$

For the convergence of the estimation error to zero, one computes $F_0$ so that the matrix $A_c = (A - F_0 C_m)$ is Hurwitz (i.e., its eigenvalues have negative real parts).
For this purpose one can use the pole placement or linear quadratic regulator (LQR) design approach. Here in this study, $F_0$ has been obtained using the LQR technique by minimizing the quadratic performance index.

$$J = \frac{1}{2} \int_0^\infty (z^T Q_0 z + u_0^T R_0 u_0) dt$$

(77)

for a related system $\dot{z} = A^T z + C^T_m u_0$, where $Q_0$ and $R_0$ are the positive definite symmetric weighting matrices. Then

$$F_0 = -(R_0^{-1} C_m P_0)^T$$

(78)

where $P_0$ satisfies the Riccati equation

$$A P_0 + P A^T + Q_0 - P_0 C_m^T R_0^{-1} C_m P_0 = 0$$

(79)

For this value of $F_0$, $A_c$ is a Hurwitz matrix. Therefore, the origin ($\tilde{x} = 0$) of (76) is exponentially stable and $\tilde{x}(t)$ converges to zero. Thus the observer accomplishes state estimation.

Now the design of a suboptimal control system for the regulation of state vector to the origin is considered.

### 3.3.b State Variable Feedback Control Law

This section describes a nonlinear control law based on the state-dependent Riccati equation (SDRE) method [32-35] for the flutter control assuming that the control input is constrained. The design of flutter control is applicable even if both $A$ and $B$ matrices are nonlinear functions of the state vector $x$. Of course, for the model under consideration matrix $B$ is a constant matrix.

The SDRE method is suitable for the design of stabilizer even when there is a hard constraint on the input $\beta_c$. Following [34], the bounded control problem is transformed to an equivalent nonlinear regulator problem by introducing a slack variable $x_s$ which satisfies

$$\dot{x}_s = U_n$$

(80)
where $U_n$ is a new control input and $\beta_c$ takes the form of a saturation sin function given by

$$\beta_c = \text{satsin}(\beta_{cm}, x_s)$$  \hspace{1cm} (81)

where one defines

$$\text{satsin}(\beta_{cm}, x_s) = \left\{ \begin{array}{ll} \beta_{cm} & \text{for } x_c > \pi/2 \\ \beta_{cm}\sin x & \text{for } -\pi/2 \leq x \leq -\pi/2 \\ \beta_{cm} & \text{for } x_c < -\pi/2 \end{array} \right.$$  \hspace{1cm} (82)

Defining the augmented state vector as $x_a = (x^T, x_s)^T \in R^9$, the system (72) and (80) can be written as

$$\dot{x}_a = A_a(x_a) + B_a U_n$$  \hspace{1cm} (83)

where

$$A_a = \begin{bmatrix} A & \frac{B \text{satsin}(\beta_{cm}, x_s)}{x_c} \\ 0_{1 \times 8} & 0 \end{bmatrix}, B_a = \begin{bmatrix} 0_{8 \times 1} \\ 1 \end{bmatrix}$$  \hspace{1cm} (84)

Consider an optimal control (infinite-horizon regulator) problem in which for the nonlinear system, the performance index of the form

$$J_a = \frac{1}{2} \int_0^\infty (x_a^T \dot{x}_a + \epsilon U_n^2) dt$$  \hspace{1cm} (85)

$$Q_a = \begin{bmatrix} Q & 0_{8 \times 1} \\ 0_{1 \times 8} & R q_q \end{bmatrix}$$  \hspace{1cm} (86)

where

$$q_q = \left\{ \begin{array}{ll} \frac{\text{satsin}(\beta_{cm}, x_s)/x_c}{|x_s| \leq \pi/2} \\ \beta_{cm}/x_s & |x_s| \leq \pi/2 \end{array} \right.$$  \hspace{1cm} (87)

is to be minimized, where $Q(x)$ is a positive definite symmetric matrix and $R > 0$ for all $x_a \in R^9$. The weighting matrix and $Q_a(x_a)$ and the scalar function $\epsilon > 0$ are chosen properly for obtaining desirable responses in the closed-loop system. Instead of deriving an optimal control law, for simplicity, a suboptimal control law is designed using the SDRE method.

Consider a region $\Omega_a \in R^9$ of the state space surrounding the origin $x_a = 0$. For the existence of a solution using the SDRE method, the following assumption is made.
Assumption 1:
The pair $A_a(x_a), B_a$ is pointwise stabilizable at each $x_a \in \Omega$. Now for obtaining a suboptimal solution using the SDRE method, one solves the state-dependent Riccati equation given by

$$A^T_a(x_a)P(x_a) + P(x_a)A_a(x_a) - P(x_a)B_a\epsilon^{-1}B^T_aP(x_a) + Q_a(x_a) = 0 \quad (88)$$

to obtain a symmetric positive definite solution for $P(x_a)$. Then the nonlinear feedback control law is given by

$$U_a(x_a) = -\epsilon^{-1}B^T_aP(x_a)x_a \quad (89)$$

Readers may refer to [32] for the properties and capabilities of the SDRE method. It is interesting to note that the suboptimal law satisfies

$$dH(x_a, \lambda)/dU_a = 0, \quad (90)$$

where the Hamiltonian of the nonlinear optimal control problem is

$$H(x_a, \lambda) = (1/2)[x_a^TQ_a(x_a)x_a + \epsilon^{-1}U_n^2] + \lambda^T[A_a(x_a)x_a + B_aU_c] \quad (91)$$

and $\lambda \in R^n$ is the co-state or the Lagrange multiplier. Substituting the control law [89] in [83] gives the closed-loop system

$$\dot{x}_a = [A_a(x_a) - B_a\epsilon^{-1}B^T_aP(x_a)]x_a \equiv A_c(x_a)x_a \quad (92)$$

The closed-loop matrix $A_c(x_a)$ is guaranteed to be Hurwitz at every $x_a \in \Omega$ from Riccati equation theory. Since the elements of $A_a(x_a)$ are smooth functions, expanding $A_c(x_a)$ about $x_a = 0$, and using mean value theorem, one can show that the equilibrium point $x_a = 0$ of (40) is asymptotically stable. The performance of the closed-loop system depends on the matrix and the weighting matrices $Q_a(x_a)$ and $\epsilon$.  

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3.4 Simulation

In this section, the simulation results are presented. The model parameters are taken from [14] and these are collected in the appendix. Results are presented for two sets: \((S_1)(u = 19.0625\,(m/s), \, a = -0.8424)\) and \((S_2)(u = 25\,(m/s), \, a = -0.6)\) of the flow velocity and parameter \(a\). For simulation, the control input \(\beta_c\) has been limited to \(\beta_{cm} = 20\,(deg/s^2)\).

The initial condition of the model \((1)\) is \(x(0) = [0.01(m), \, 30^\circ, 0, 0, 1\,(deg/s), 0, 0, 0.1]^T\) and the estimator's initial state \(\hat{x}(0) = [0.01(m), \, 30^\circ, 0, 0, 0, 0, 0, 0]^T\). For the observer design, the weighing matrices chosen are \(Q_0 = 100I_{8\times8}\) and \(R_0 = 5I_{3\times3}\). The open-loop system has two unstable poles at \(1.4871\pm j14.8748\) for \((S_1)\) and \(1.2286\pm j13.6091\) for \((S_2)\) and the remaining poles are stable. The open-loop response for each case shows limit cycle oscillations. The plots for case \((S_1)\) are shown in Fig. 5, which show that after an initial transient the pitch angle and the plunge displacement trajectories converge to limit cycles.

Case A: Feedback Control for \(S_1\)

The complete closed-loop system including the model \((1)\), observer \((75)\), and the nonlinear feedback controller \((89)\) for case \(S_1\) is simulated. It is observed that the state vector converges to the origin. The response time is of the order of 3-4 seconds. Fig 6 shows that the filter states \(x_{f1}\) and \(x_{f2}\) also converges to zero. In the transient period, \(\beta_c\) saturates and the control surface deflection \(\beta\) remains within \((20)\, deg\).

Case B: Feedback Control for \(S_2\)

Simulation is performed for the condition \(S_2\) \((u = 25\,(m/s), \, a = -0.6)\). The initial conditions and controller parameters of case A are retained for simulation. Selected responses are shown in Fig. 7. In the transient period, plunge and pitch responses are oscillatory and the control input \(\beta_c\) saturates over a short period. It is observed that the plunge displacement, pitch angle, and the filter states converge to zero. The
maximum control surface deflection $\beta$ is less than (18) deg and the control input $\beta_c$ saturates during a segment of the transient period.

3.5 Summary

In this chapter, control of a prototypical aeroelastic wing section with structural pitch and plunge nonlinearities using a single control surface was considered. Unsteady aerodynamics were modeled with an approximation to Theodorsen's theory. For the purpose of design, a hard constraint on the control input was introduced. A minimal realization of the filter associated with the Theodorsen's function was used to obtain filtered values of signals in the lift and moment expressions. An observer was designed to obtain the estimates of unavailable states using only the plunge displacement, pitch angle, and control surface deflection measurements. A suboptimal nonlinear control law based on the state dependent Riccati equation method was derived. In the closed-loop system using output feedback, asymptotic regulation of the state vector to zero was accomplished. Simulation results were presented which showed that flutter suppression can be achieved for different flow velocities and elastic axis locations even when hard constraint on the control input is imposed. The limitation of the control system design is it achieves local stability, hence to achieve global stability new control system is designed using backstepping design technique in next chapter.
Figure 5: Open-loop response: $u = 19.0625 \text{ m/s}$, $a = -0.8424$
Figure 6: Closed-loop response: $u = 19.0625 \text{ m/s}, a=-0.8424$
Figure 7: Closed-loop response: $u = 25 \text{ m/s}$, $a = -0.6$
CHAPTER 4

OUTPUT FEEDBACK CONTROL OF AEROELASTIC SYSTEM WITH UNSTEADY AERODYNAMICS: A BACKSTEPPING DESIGN

4.1 Introduction

The chapter presents a nonlinear output feedback control system for the stabilization of an aeroelastic system with structural nonlinearities based on a backstepping design technique. The aeroelastic model describes the plunge and pitch motion of a wing. The unsteady aerodynamics are modeled with an approximation to Theodorsen’s theory. A single control surface is utilized for the flutter control. For the purpose of control law derivation, a judicious choice of output as a linear combination of the plunge displacement and pitch angle is made. Based on a backstepping design technique, a control law for the trajectory control of the chosen output variable is derived. For the synthesis of the controller, only the plunge displacement, pitch angle, and control surface deflection are measured. An observer is designed to estimate the remaining state variables of the system for feedback. In the closed-loop system, including the observer and nonlinear controller, trajectory control of the output is accomplished and the state vector asymptotically converges to the origin. Simulation results are presented which show that the designed control system is effective in flutter suppression.

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4.2 Complete System

The state variable representation has been derived in previous chapter and we take the same. The complete system including has a state variable representation of the form

\[
\begin{bmatrix}
\frac{d}{dt} x_1 \\
\frac{d}{dt} x_2 \\
\frac{d}{dt} x_3 \\
\frac{d}{dt} x_4 \\
\frac{d}{dt} x_5 \\
\end{bmatrix} =
\begin{bmatrix}
O_{3x3} & I_{3x3} & O_{3x2} \\
0 & A_1 & 0 \\
0 & 0 & -b_0 -b_1 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
x \\
B_1 \\
\beta \\
N_1 \\
f_n \\
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

where \( O \) and \( I \) denote null and identity matrices of indicated dimensions.

The measurement equation is

\[
y_m = [h, \alpha, \beta]^T \equiv C_m x
\]

where

\[
C_m = [I_{3x3} O_{3x5}]
\]

Let us consider a controlled output variable \( y \) which is a linear combination of the plunge displacement and the pitch angle of the form

\[
y = \gamma_1 h + \gamma_2 \alpha = \gamma^T(h, \alpha)^T
\]

where \( \gamma_i > 0, i = 1, 2 \), and \( \gamma = (\gamma_1, \gamma_2)^T \). The real numbers \( \gamma_1 \) and \( \gamma_2 \) are chosen such that the origin of the zero dynamics of the system is asymptotically stable [28, 29]. That is, if the output \( y \) is identically zero, then the residual motion of the system asymptotically converges to zero.

Suppose that a reference trajectory \( y_r(t) \) is given. We are interested in designing an input-output feedback linearizing control law such that the output \( y \) tracks \( y_r \) and the state vector converges to the origin. Moreover the control law is to be synthesized using only the measured signal \( y_m \).
4.3 Control Law Design

In this section, the derivation of a controller is considered.

4.3.a Observer Design

First of all, it is essential to construct an observer so that unavailable state variables $(\dot{h}, \dot{\alpha}, \dot{\beta}, x_f, x_{f2})$ can be estimated. Noting that $h, \alpha$ and $\beta$ are available, one can construct a full-order observer of the form

$$\dot{\hat{x}} = A\hat{x} + B\beta_c + Nf_n(h, \alpha) + F_0(y_m - \hat{y}_m) \tag{96}$$

where $\hat{x}$ denotes estimated values of the state vector $x$, $F_0$ is a $8 \times 3$ matrix and $\hat{y}_m = (\hat{h}, \dot{\alpha}, \dot{\beta})^T = C_m \dot{x}$. Subtracting (96) from (93), it easily follows that the state estimation error $\hat{x} = x - \hat{x}$ satisfies

$$\dot{\hat{x}} = (A - F_0 C_m) \hat{x} \tag{97}$$

$$\triangleq A_c \hat{x}$$

For the convergence of the estimation error to zero, one computes $F_0$ so that the matrix $A_c = (A - F_0 C_m)$ is Hurwitz (i.e., its eigenvalues have negative real parts). For this purpose one can use the pole placement or linear quadratic regulator (LQR) design approach [30]. Here in this study, $F_0$ has been obtained using the LQR technique by minimizing the quadratic performance index.

$$J = \frac{1}{2} \int_0^\infty (z^T Qz + u_0^T R u_0) dt \tag{98}$$

for a related system $\dot{z} = A^T z + C_m^T u_0$, where $Q$ and $R$ are the positive definite symmetric weighting matrices. Then

$$F_0 = -(R^{-1} C_m P_0)^T \tag{99}$$

where $P_0$ satisfies the Riccati equation

$$A P_0 + P A^T + Q - P_0 C_m^T R^{-1} C_m P_0 = 0 \tag{100}$$
For this value of $F_0$, $A_c$ is a Hurwitz matrix. Therefore, the origin ($\bar{x} = 0$) of (97) is exponentially stable and $\bar{x}(t)$ converges to zero. Thus the observer (26) accomplishes state estimation.

4.3.b Feedback Control: A Backstepping Design

Now the design of a feedback control system for the trajectory control of $y$ is considered. The design is completed in two steps using a backstepping design technique [25].

Step 1:

We are interested in the trajectory control of the chosen output $y(t)$. Define the tracking error $z_1$ as

$$z_1 = y - y_r = \gamma^T[h, \alpha] - y_r$$

(101)

The derivative of $z_1$ is

$$\dot{z}_1 = \dot{y} - \dot{y}_r = \gamma^T[h, \dot{\alpha}] - \dot{y}_r$$

(102)

Note that

$$\dot{h} = x_4 = \dot{x}_4 + \ddot{x}_4$$

(103)

$$\dot{\alpha} = x_5 = \dot{x}_5 + \ddot{x}_5$$

(104)

where $x_k$, $\dot{x}_k$ and $\ddot{x}_k$ denote the $k^{th}$ elements of $x$, $\dot{x}$ and $\ddot{x}$, respectively. Using (103), (104) in (102) gives

$$\dot{z}_1 = \gamma^T[\dot{x}_4, \ddot{x}_5] - \dot{y}_r + \gamma^T[\ddot{x}_4, \ddot{x}_5]$$

(105)

For the derivation of control law, $\gamma_1 \ddot{x}_4 + \gamma_2 \ddot{x}_5$ is chosen as a virtual control input. Define

$$z_2 = \gamma^T[\ddot{x}_4, \ddot{x}_5] - w_1$$

(106)

where $w_1$ is the first stabilizing signal (yet to be determined) and $z_2$ is a new coordinate. Substituting (106) in (105) gives

$$\dot{z}_1 = z_2 + w_1 - \dot{y}_r + \gamma^T[\ddot{x}_4, \ddot{x}_5]$$

(107)
Consider a Lyapunov function

$$V_1 = \frac{x_1^2}{2} + \frac{1}{d_1} \hat{x}^T P \hat{x}$$

(108)

where $d_1 > 0$ and $P$ is positive definite symmetric matrix which is the unique solution of the Lyapunov equation

$$A_c^T P + P A_c = -I_{8\times 8}$$

(109)

Because $A_c$ is a Hurwitz matrix, such a solution for $P$ exists [26]. Differentiating $V_1$ along the solution of (107) and (97) gives

$$\dot{V}_1 = z_1[z_2 + \omega_1 - \dot{y}_r + \gamma^T(\tilde{x}_4, \tilde{x}_5)^T] + \frac{1}{d_1} \hat{x}^T(A_c^T P + PA_c) \hat{x}$$

(110)

Using Schwarz' inequality [31] one has

$$|z_1 \gamma^T(\tilde{x}_4, \tilde{x}_5)^T| \leq |z_1| \times ||\gamma|| (\tilde{x}_4^2 + \tilde{x}_5^2)^{\frac{1}{2}} \leq |z_1| \times ||\gamma|| \times ||\hat{x}||$$

(111)

where $||.||$ denotes the Euclidean norm of $\hat{x}$. Using Young's inequality gives [26]

$$|z_1||\gamma||||\hat{x}|| \leq d_1 z_1^2 ||\gamma||^2 + \frac{||\hat{x}||^2}{4d_1}$$

(112)

In view of (109), (111) and (112), (110) gives

$$\dot{V}_1 \leq z_1[z_2 + \omega_1 - \dot{y}_r] + d_1 ||\gamma||^2 z_1^2 + \frac{||\hat{x}||^2}{4d_1} - \frac{1}{d_1} ||\hat{x}||^2$$

(113)

For making $\dot{V}_1$ as much negative as possible, one chooses the stabilization signal $w_1$ as

$$w_1 = -(c_1 + s_1) z_1 + \dot{y}_r$$

(114)

where $s_1 = d_1 ||\gamma||^2$ and $c_1 > 0$. Substituting (114) in (107) gives

$$\dot{z}_1 = -(c_1 + s_1) z_1 + z_2 + \gamma^T(\tilde{x}_4, \tilde{x}_5)^T$$

(115)

and using (113), one obtains

$$\dot{V}_1 \leq -c_1 z_1^2 + z_1 z_2 - \frac{3}{4d_1} ||\hat{x}||^2$$

(116)
The $z_1z_2$ term will be compensated in step 2.

Step 2:

Differentiating $z_2$ and using (105) gives

$$
\dot{z}_2 = \gamma^T[(\dot{x}_4, \dot{x}_5)^T] - \frac{\partial w_1}{\partial z_1}(\gamma^T(\dot{x}_4, \dot{x}_5)^T - \dot{y}_r + \gamma^T(\dot{x}_4, \dot{x}_5)^T) - \ddot{y}_r
$$

(117)

Substitute for $\dot{x}_4$ and $\dot{x}_5$ from (96) in (117) gives

$$
\dot{z}_2 = \gamma^T\{(A_{(4)}^T, A_{(5)}^T)^T \dot{x} + (B_{(4)}, B_{(5)})^T \beta_c + (N_{(4)}^T, N_{(5)}^T)^T f_n + (F_{(4)}^T, F_{(5)}^T)^T (y_m - C_m \dot{x})

+ (c_1 + s_1)(\dot{x}_4, \dot{x}_5)^T\} - (c_1 + s_1)\dot{y}_r - \dot{y}_r = (c_1 + s_1)\gamma^T(\dot{x}_4, \dot{x}_5)^T

\dot{z}_2 = \gamma^T(g(\dot{x}, y_m) - \dot{y}_r + \gamma^T(B_{(4)}, B_{(5)})^T \beta_c + (c_1 + s_1)\gamma^T(\dot{x}_4, \dot{x}_5)^T - (c_1 + s_1)\dot{y}_r

$$

(118)

where the function $g$ is easily obtained by equating terms in (118).

Consider the composite Lyapunov function

$$
V_2 = V_1 + \frac{t^2}{2} + \frac{1}{d_2} \dddot{x}^T P \dddot{x}
$$

(119)

where $d_2 > 0$. Then the derivative of $V_2$ is given by

$$
\dot{V}_2 \leq -c_1z_1^2 + z_1z_2 - \frac{3}{4d_1}||\dddot{x}||^2 + z_2[\gamma^T g - \dot{y}_r + \gamma^T(B_{(4)}, B_{(5)})^T \beta_c + (c_1 + s_1)

(\gamma^T(\dot{x}_4, \dot{x}_5)^T - \ddot{y}_r)] - \frac{1}{d_2}||\dddot{x}||^2
$$

(120)

For the derivation of the control law, the following assumption is made.

Assumption 1:

The parameter $\gamma_1$, $\gamma_2$ of the output are chosen such that $b^\ast = \gamma^T(B_{(4)}, B_{(5)})^T$ is nonzero.

In view of (120), we choose the control law as

$$
\beta_c = b^{\ast -1}[-(c_2 + s_2)z_2 - z_1 - \gamma^T g + \dot{y}_r + (c_1 + s_1)\dot{y}_r]
$$

(121)

where $c_2 > 0$ and $s_2 > 0$ is yet to be determined. Substituting the control law (121) in (118) gives

$$
\dot{z}_2 = -z_1 - (c_2 + s_2)z_2 + (c_1 + s_1)\gamma^T(\dot{x}_4, \dot{x}_5)^T
$$

(122)
Using the Schwarz' and Young's inequalities one has
\[
(c_1 + s_1)|z_2\gamma (\tilde{z}_4, \tilde{z}_5)^T| \leq d_2(c_1 + s_1)^2 \|\gamma\|^2 z_2^2 + \frac{||\tilde{z}||^2}{4d_2}
\]  \hspace{1cm} (123)

Using (121) and (123) in (120) gives
\[
\dot{V}_2 \leq -c_1 z_1^2 - c_2 z_2^2 - s_2 z_2^2 + d_2(c_1 + s_1)^2 \|\gamma\|^2 z_2^2 - \frac{3}{4} ||\tilde{z}||^2 (d_1^{-1} + d_2^{-1})
\]  \hspace{1cm} (124)

Choosing the damping term \( s_2 \) as
\[
s_2 = d_2(c_1 + s_1)^2 \|\gamma\|^2
\]  \hspace{1cm} (125)

(124) gives
\[
\dot{V}_2 \leq -c_1 z_1^2 - c_1 z_2^2 - \frac{3}{4} \left(\frac{1}{d_1} + \frac{1}{d_2}\right) ||\tilde{z}||^2 \leq 0
\]  \hspace{1cm} (126)

Since \( V_2 \) is a positive definite function of \( z_1, z_2 \) and \( \tilde{z} \), and \( \dot{V}_2 \leq 0 \), it follows that \( z_1, z_2 \) and \( \tilde{z} \) are bounded. One has from (126)
\[
\dot{V}_2 \leq -c^*(z_1^2 + z_2^2) \leq 0
\]  \hspace{1cm} (127)

where
\[
c^* = \min(c_i), i = 1, 2.
\]

Then integrating (127) gives
\[
c^* \int_0^\infty (z_1^2 + z_2^2) dt \leq V_2(0) - V_2(\infty) \leq V_2(0)
\]  \hspace{1cm} (128)

Thus \( z_1 \) and \( z_2 \) are square integrable. Moreover in view of (115) and (122), \( \dot{z}_1 \) and \( \dot{z}_2 \) are bounded. Therefore, using Barbalat's lemma [26], one concludes that \( z_1 \) and \( z_2 \) tend to zero as \( t \to \infty \).

The system (93) is of dimension eight. Since \( y \) is of relative degree 2, there exist zero dynamics of dimension 6. It is well known that for the stability in the closed-loop system, the zero dynamics must be stable. Let us consider for simplicity that \( y_r(t) = 0 \). Of course, this is not a restriction, since we are interested in regulating \( x \) to zero. Consider a linear state transformation
\[
\Phi : x \to (y, \dot{y}, \eta^T)^T \in R^8
\]  \hspace{1cm} (129)

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That is, \((y, \dot{y}, \eta^T)^T = \Phi x\). Let \(B_1 = [b_{11}, b_{12}, b_{13}]^T\) and \(\eta \in \mathbb{R}^6\) be defined as

\[
\eta = \begin{bmatrix}
  b_{12} h - b_{11} \alpha \\
  b_{12} \dot{h} - b_{11} \dot{\alpha} \\
  b_{12} \beta - b_{13} \alpha \\
  b_{12} \dot{\beta} - b_{13} \dot{\alpha} \\
  x_{f1} \\
  x_{f2}
\end{bmatrix}
\]

The matrix \(\Phi\) can be obtained in view of (129) and the definition of \(\eta\). Computing the determinant of \(\Phi\), it is easily verified that \(\Phi\) is a nonsingular matrix if \(\gamma b_{11} + b_{12} \gamma_2 \neq 0\). In view of (93), one observes that \(\beta_c\) does not appear in \(\eta\) and it is only a function of \(x\). Thus one has

\[
\dot{\eta} = \frac{\partial \eta}{\partial x} (Ax + N f_n(h, \alpha)) = q(x) = q(\Phi^{-1}[y, \dot{y}, \eta^T]^T)
\]

where \(q(.) \in \mathbb{R}^6\) denotes a nonlinear vector function of \((y, \dot{y}, \eta)\).

The control law (121) accomplishes regulation of \(z_1 = y\) to zero. For obtaining the zero dynamics, one sets \(y\) and \(\dot{y}\) to zero. Thus the zero dynamics can be written as

\[
\dot{\eta} = q(\Phi^{-1}[0, 0, \eta^T]^T) = \Gamma(\eta)
\]

for an appropriate vector function \(\Gamma\). For stability in the closed-loop system, we assume that the equilibrium point \(\eta = 0\) of (130) is exponentially stable. The stability of system (130) can be verified by checking the eigenvalues of the linearized system

\[
\dot{\eta} = \left[\frac{\partial \Gamma(\eta)}{\partial \eta}(0)\right] \eta
\]

Alternatively, one can verify the stability of the zero dynamics by computing the transfer function of the linearized system

\[
\frac{\hat{\eta}(s)}{\hat{\beta}_c(s)} = \frac{n_p(s)}{d_p(s)}
\]

The origin of the zero dynamics is exponentially stable if \(n_p(s)\) is a Hurwitz polynomial; that is, the transfer function \([n_p(s)/d_p(s)]\) is minimum phase. It will be shown in the next section by numerical computation that indeed \(n_p(s)\) is Hurwitz for the
chosen values of $u$ and $a$. Thus as $y$ tends to zero, $\eta$ also converges to zero. Since $\Phi$ is a nonsingular matrix, one concludes that $x$ tends to the origin as well.

4.4 Simulation

In this section, the results of simulation are presented. The model parameters are taken from [14, 22] and these are collected in the appendix. Results are presented for two sets: $(S_1)(u = 19.0625(m/s), a = -0.8424)$ and $(S_2)(u = 25(m/s), a = -0.6)$ of the flow velocity and parameter $a$. For simulation, the control input $\beta_c$ has been limited. By constraining the control input, it is possible to limit the control surface deflection and deflection rate magnitudes. The inclusion of control saturation in the design process is a difficult problem for nonlinear systems, but simulation results are presented which show the effectiveness of the control system with constrained $\beta_c$ in limiting the control surface motion. The initial condition of the model (1) is $x(0) = [0.01(m), 30°, 0, 0, 1(deg/s), 0, 0, 0]^T$ and the estimator's initial state $\hat{x}(0)$ is $[0.01(m), 30°, 0, 0, 0, 0, 0, 0]^T$. For the observer design, the weighing matrices chosen are $Q = 100I_{8\times 8}$ and $R = 5I_{3\times 3}$. The controller parameters are $c_1 = 1, c_2 = 1, d_1 = 0.5$, and $d_2 = 0.5$. The selected output for control is $y = 2h + \alpha$, and therefore $\gamma = (2, 1)^T$. For condition $(S_1)$, the zeros of the transfer function $G(s) = C_0(\sigma_{I_{8\times 8}} - A)^{-1}B$ are $-1547.8, -290.8, -51.9, -1.4 \pm j5.1$ and $-6.9$ and for $(S_2)$ are $-1913.9, -397.7, -60.4, -1.8 \pm j3.7$ and $-8.5$, where $C_0 = (\gamma^T, 0_{1\times 6})$. Since $G(s)$ has stable zeros, the transfer function is minimum phase and the origin of the zero dynamics is exponentially stable. The open-loop system has two unstable poles at $1.487 \pm j14.87$ for $(S_1)$ and $1.228 \pm j13.609$ for $(S_2)$ and the remaining poles are stable. The open-loop response for each case shows limit cycle oscillations. The plots for case $(S_1)$ are shown in Fig. 8, which show that after an initial transient the pitch angle and the plunge displacement trajectories converge to limit cycles.
Case A: Feedback Control for $S_1$

The complete closed-loop system including the model (1), observer (23), and the feedback controller (48) for case $S_1$ is simulated. The control input $\beta_c$ is constrained so that its magnitude does not exceed the limiting value $\beta_{cm} = 1.8 \text{ (deg/s}^2\text{)}$ ($\beta_{cm}$ denotes the maximum value of $\beta_c$). This is a severe constraint on the control input $\beta_c$, but is effective in limiting the control surface deflection and deflection rate to small values. Responses are shown in Fig. 9. In the closed-loop system $y$ converges to zero. Moreover, the state vector $x(t)$ converges to zero as well, because the system is minimum phase. Of course, the vector $\gamma$ plays an important role in shaping the plunge and pitch responses. The response time is of the order of less than four seconds. Fig. 9 shows that the filter states $x_{f1}$ and $x_{f2}$ also converge to zero. In the transient period, $\beta_c$ saturates and the surface deflection and deflection rate remain within small values (6) deg and 40 (deg/s), respectively.

In order to examine the effect of the limiting value of $\beta_c$, simulation is done using the bound $\beta_{cm} = 2.5 \text{ (deg/s/s)}$. Responses are found to be somewhat similar to Fig. 9 and flutter is suppressed, but the surface deflection (less than 10 (deg)) and deflection rate (less than 50 (deg/s)) have increased slightly (These results are not shown here in order to save space). Of course, increase in the magnitudes of $\beta$ and $\dot{\beta}$ with $\beta_{cm}$ is expected in view of the actuator dynamics (9).

Case B: Feedback Control for $S_2$

Simulation is performed for the condition $S_2$ ($u = 25(m/s), a = -0.6$). The initial conditions and controller parameters of case A are retained for simulation. Selected responses are shown in Fig. 10. In the transient period, plunge and pitch responses are oscillatory, but converge rapidly to zero in less than four seconds. The control input $\beta_c$ saturation is observed for a period of less than two seconds, but it causes no problem in flutter suppression. It is observed that the filter states converge to zero. The maximum control surface deflection is less than (8) deg and the deflection rate
remains less than 40 (deg/s), which are reasonable.

Extensive simulation for other values of the flow speed and parameter \( a \in [-0.4, -0.9] \) has been done. It is found that in the closed-loop system, trajectories converge to zero in each case.

4.5 Summary

In this chapter, the control of a prototypical aeroelastic wing section with structural pitch and plunge nonlinearities using a single control surface was considered. Unsteady aerodynamics were modeled with an approximation to Theodorsen's theory. A minimal realization of the filter associated with the Theodorsen's function was used to obtain filtered values of signals in the lift and moment expressions. An observer was designed to obtain the estimates of unavailable states using only the plunge displacement, pitch angle, and control surface deflection measurements. A feedback linearizing control law was derived for the trajectory control of the selected controlled output variable. The output variable was judiciously chosen as a linear combination of the plunge displacement and the pitch angle which gave stable zero dynamics. The choice of the output variable played an important role in shaping the responses. Here the system is globally stable and asymptotic regulation of the state vector to zero is accomplished. Simulation results were presented which showed that flutter suppression can be achieved for different flow velocities and elastic axis locations.
Figure 8: Open-loop response: $u = 19.0625 \text{ m/s}, a = -0.8424$
Figure 9: Closed-loop response: $u = 19.0625$ m/s, $a = -0.8424$
Figure 10: Closed-loop response: \( u = 25 \, \text{m/s}, \, a = -0.6 \)
CONCLUSION

Flutter is considered to be one of the most important problems in aeroelasticity. In this thesis, three design techniques have been used to control the nonlinear aeroelastic system. The aeroelastic model has nonlinear plunge and pitch structural nonlinearity, but includes linear quasi-steady as well as unsteady aerodynamics. The aeroelastic model has two degree-of-freedom in plunge and pitch and uses a single control surface for flutter control. First an output feedback modular adaptive control was designed. This control methodology eliminates the necessity of knowledge of system parameters. Using a canonical representation of the aeroelastic system, an input-to-state stabilizing controller and a passive identifier (an observer and adaptation law) were derived. Simulation results showed that this control system is effective in spite of large parameter uncertainties.

Following the adaptive design, a suboptimal controller and a nonlinear control system based on a backstepping design were derived. For these two controllers, unsteady aerodynamic model was used. The unsteady aerodynamic was modeled with an approximation to Theodorsens theory. The suboptimal control system was derived using the state dependent Riccati equation (SDRE) approach.

For the synthesis of the controller, the plunge displacement, pitch angle and control surface deflection were measured and an observer was designed to estimate the remaining state variables for feedback. It is pointed out that where as the adaptive controller does not require any knowledge of system parameters, the other two controllers assume that the system is completely known.
Simulation results showed that the suboptimal controller provides good responses but the closed-loop system is only locally stable. The last controller yield global stability but requires high gain feedback resulting larger control inputs. Moreover, SDRE design can accommodate control constraints unlike the first and last controller.

There are several important questions remain to be answered in this area. The inclusion of nonlinear aerodynamics in design is extremely important. The problem of digital implementation of control systems is interesting. Yet another problem of interest is the prediction of onset of flutter using flight data on-line. Such a prediction will be useful in increasing the flight envelope of aircraft.
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