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ANALYSIS OF THE UNSTEADY FLOW AND HEAT TRANSFER OF A DUSTY FLUID BETWEEN TWO CONCENTRIC CYLINDERS IN THE PRESENCE OF A MAGNETIC FIELD

by

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Bachelor of Engineering in Mechanical Engineering
University of Madras, India
May 2002

A thesis submitted in partial fulfillment of the requirements for the

Master of Science Degree in Mechanical Engineering
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BETWEEN TWO CONCENTRIC CYLINDERS IN THE PRESENCE OF A MAGNETIC FIELD

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MASTER OF SCIENCE IN MECHANICAL ENGINEERING

Examination Committee Chair

Dean of the Graduate College

Examination Committee Member

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Graduate College Faculty Representative
ABSTRACT

Analysis of the Unsteady Flow and Heat Transfer of a Dusty Fluid between Two Concentric Cylinders in the Presence of a Magnetic Field

by

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A pseudo-2-D numerical model for analyzing the unsteady heat transfer and flow, of an electrically conducting, viscous, incompressible fluid in a channel formed between two concentric cylinders is developed. In the model, the fluid is driven along the channel by a constant pressure gradient and an external magnetic field is applied in the direction perpendicular to the channel flow. Two cylinders are considered electrically insulated, and the surfaces are maintained at constant but different temperatures with the outer cylinder being at a higher temperature. The viscosity and electrical conductivity of the fluid are considered varying with time and temperature.

The equations governing the flow and temperature distributions are solved numerically using the finite element method. The explicit time marching procedure is used to advance the numerical solution for the dependent variables, i.e. temperature and velocity.
The thesis discusses the effect of the applied magnetic field and the variation of viscosity and electrical conductivity with temperature on the time development of the velocity and temperature distributions for the fluid.
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CHAPTER 1

INTRODUCTION

The phenomena of flow and heat transfer of a dusty fluid in a pipe or channel have been the focus of study for a number of researchers [1 - 4], because of a number of important practical applications in the fields of fluidization, combustion, use of dust gas cooling systems, centrifugal separation of matter from fluid, petroleum industry, purification of crude oil, electrostatic precipitation, polymer technology, and fluid droplet sprays.

The flow of a dusty and electrically conducting fluid through a channel in the presence of a transverse magnetic field is encountered in a variety of applications such as magnetohydrodynamic (MHD) generators, pumps, accelerators, and flow meters. In these devices, the solid particles in the form of ash or soot are suspended in the conducting fluid as a result of the corrosion and wear activities and/or the combustion processes in MHD generators and plasma MHD accelerators. The consequent effect of the presence of solid particles on the performance of such devices has led to studies of particulate suspensions in conducting fluids in the presence of externally applied magnetic fields.

1.1 Magnetohydrodynamics (MHD)

Magnetic fields influence many natural and man made flows. They are routinely used in industries to heat, pump, and stir and levitate liquid metals. There is the terrestrial
magnetic field which is maintained by fluid motion in the earth’s core, the solar magnetic field which generates sunspots and solar flares, and the galactic magnetic field which is thought to influence the formation of stars from interstellar clouds. The study of these flows is called magnetohydrodynamics (MHD) [5]. Formally, MHD is concerned with the mutual interaction of fluid flow and magnetic fields. The fluids in question must be electrically conducting and non-magnetic, like liquid metals, hot ionized gases (plasmas) and strong electrolytes. Magnetohydrodynamics can be regarded as a combination of fluid mechanics and electromagnetism, that is, the behavior of electrically conducting fluids in the presence of magnetic and electric fields [6].

1.2 A brief history of MHD

The laws of magnetism and fluid flow are hardly a twentieth-century innovation, yet MHD became a fully fledged subject only in the late 1930’s or early 1940’s [6]. The reason, probably, is that there was little incentive for nineteenth century engineers to capitalize on the possibilities offered by MHD. Thus, while there were a few isolated experiments by nineteenth-century physicists such as Faraday, the subject languished until the turn of the century. Things started to change, however, when astrophysicists realized how ubiquitous magnetic fields and plasmas are throughout the universe. This culminated in 1942 with the discovery of the Alfven wave, a phenomenon which is peculiar in MHD and important in astrophysics. (A magnetic field line can transmit transverse inertial waves, just like a plucked string). Around the same time, geophysicists began to suspect that the earth’s magnetic field was generated by dynamo action within the liquid-metal of its core, a hypothesis first put forward in 1919 by Larmor in the
context of sun's magnetic field. A period of intense research followed and continues today.

Plasma physicists, on the other hand, acquired an interest in MHD in the 1950's as the quest for controlled thermonuclear fusion gathered pace. They were particularly interested in the stability, or lack of it, of plasmas confined by magnetic fields, and great advances in stability theory were made as a result.

The development of MHD in engineering was slower and did not really get going until the 1960's. However, there was some early pioneering work by the engineer J. Hartmann, who invented the electromagnetic pump in 1918. Hartmann also undertook a systematic theoretical and experimental investigation of the flow of mercury in a homogeneous magnetic field. It is widely regarded that Hartmann was the father of liquid-metal MHD and indeed the term 'Hartmann flow' is now used to describe the duct flows in the presence of a magnetic field. Despite Hartmann's early researches, it was only in the early 1960's that MHD began to be exploited in engineering. The impetus for change came largely as a result of three technological innovations; (i) fast-breeder reactors use liquid sodium as a coolant that needs to be pumped; (ii) controlled thermonuclear fusion requires that the hot plasma be confined away from material surfaces by magnetic forces; and (iii) MHD power generation, in which ionized gas is propelled through a magnetic field, was thought to offer the prospect of improved power station efficiencies. This last innovation turned out to be quite impracticable, and its failure was rather widely publicized in the scientific community. By 1970's [7], the application of MHD for propelling rockets into space was actively researched. However as the interest in power generation declined, research into metallurgical MHD took off.
Two decades later, magnetic fields are routinely used to heat, pump, stir and levitate liquid metals in the metallurgical industries. The key point is that the Lorentz force provides a non-intrusive means of controlling the flow of metals. With constant commercial pressure to produce cheaper, better and more consistent materials, MHD provides a unique means of exercising greater control over casting and refining processes.

1.3 Applications of MHD

Enthusiasm for the subject has increased as several applications for MHD devices have become apparent. Electromagnetic flow meters and MHD pumps have been in use for some years and MHD power generation and space propulsion schemes are now being actively considered [8]. Much of the research directed towards controlled nuclear fusion can be explained in MHD terms, since though the plasma is composed of electrons and ions, it can often be regarded as a single homogeneous fluid. So over the last 20 years the subject has expanded enormously. There are varieties of applications of MHD technologies to the aerospace vehicles. One of them is the flow control around re-entry vehicles with MHD interactions. A very strong shock-wave is induced, where the air temperature can exceed 10,000K with high electrical conductivity. If the magnetic filed can be applied externally, the MHD interaction can push the shock-wave away from the vehicle and there is a possibility to reduce the thermal flux on the wall. Major efforts have been carried out around the world to develop this technology in order to improve electric conversion efficiency, increase reliability by eliminating moving parts, and reduce emissions from coal and gas plants. Another important application was in the area of using MHD as a means to propel the rockets to space. Myrabo [7] proposed a
conceptual design for a laser-riding air-breathing single-stage shuttle which would use magnetohydrodynamic forces to accelerate the engine working fluid.

The pulp and paper industry possesses many process areas where treatment with MHD technology can be effective. Applications include de-scaling of white, black, and green liquor lines, including equipment such as clarifiers and evaporators. Conventional shutdown and chemical cleaning for scale removal can be virtually eliminated. Further, other process systems such as boilers, coolers, heaters, and scrubbers can benefit from MHD scale removal and prevention. Closed-cycle liquid metal MHD systems using both single and two-phase flows also have been explored. Still more novel applications are in development or on the horizon. For example, recent research has shown the possibility of seawater propulsion using MHD and control of turbulent boundary layers to reduce drag [9].

Another important application of the MHD principle has been the magnetically driven imploding liner parameter space of the Atlas capacitor bank [10] developed by the Los Alamos National Laboratory. The Atlas capacitor bank was designed to magnetically drive imploding liners for use as impactors in shock and hydrodynamic experiments. This Master’s thesis research had its objective basically derived from the desire to simulate the perturbations that were developed at the magnetic field – metal liner interface during implosions. The geometry, fluid properties and the boundary conditions have all been specified with the objective of being able to apply them to the Atlas facility’s liner problem, in the future.

Several studies [11, 12, 13, 14] have been focused on the effect of the magnetic field on the flow of fluids in channels and ducts using constant physical properties. More
accurate prediction for the flow and heat transfer can be achieved by taking into account the variation of these properties with temperature [11]. Klemp et al. [12] studied the effect of temperature dependent viscosity on the entrance flow in a channel in a hydrodynamic case. Attia and Kotb [13] studied the steady MHD fully developed flow and heat transfer between two parallel plates with temperature dependent viscosity. Later Attia [14] has extended the problem to the transient state. Attia [15] also analyzed the effect of variable viscosity and the electrical conductivity on the unsteady flow and heat transfer of an electrically conducting, viscous, incompressible dusty fluid flowing between two parallel plates. The flow of a viscous incompressible fluid between two non-conducting cylinders in the presence of a radial magnetic field was investigated by Molokov and Allen [16].

This thesis research will focus on analyzing the effect of the viscosity and the electrical conductivity in the presence of a magnetic field, on the variation of temperature and velocity of a dusty fluid flowing in a channel. The study also focuses on the variation of the temperature and velocity with changes in parameters like particle dispersion for the cylindrical channel.

Chapter 2 explains the details of the problem and the various geometries associated. Chapter 3 defines the governing equations and the boundary conditions, while Chapter 4 focuses on the numerical method used for solving the governing equations, Chapter 5 discusses the results for the different geometries and finally Chapter 6 concludes the current research.
CHAPTER 2

DESCRIPTIONS OF THE PROBLEM AND GEOMETRY

Two geometries are considered for modeling the dusty fluid flow in a channel. The first geometry creates a channel between two infinitely long parallel flat plates while the second one simulates the dusty fluid flow between two infinitely long concentric cylinders. In the second geometry, two scenarios are considered, namely:

1. when the cylinders are stationary,
2. when the outer cylinder moves towards the inner cylinder.

The analysis on the system of parallel plates was done to benchmark the 2D finite element code that was developed for solving the system of two concentric cylinders. Attia [15] analyzed the same geometry by developing a finite difference code and using the same initial and boundary conditions. The results obtained using the FEM code is compared with the results from the article. In the case of the system of parallel plates (see Figure 2.1), the two plates are assumed to be electrically non-conducting. The inner plate is kept at a constant temperature of $T_1$ and the outer plate is kept at $T_2$, with $T_2 > T_1$. A constant pressure gradient is applied in the x-direction and a uniform magnetic field $B_0$ is applied in the y-direction.

Figure 2.2 shows the cylindrical geometry with the inner cylinder at a temperature $T_1$ and the outer cylinder at a temperature $T_2$, with $T_2 > T_1$. A constant pressure gradient is
applied on non-conducting cylinders in the $z$-direction with a uniform magnetic field applied in the $r$-direction.

The shape of the dust particles is assumed to be spherical, and the physical properties of the particles are taken time-independent. A uniform distribution of dust particles within the fluid is assumed; though for practical cases there might be a need to apply certain probability functions to simulate the dust particle distribution.

\[
\begin{align*}
 & y = h: & u(h, t) = u_s(h, t) = 0 & & T(h, t) = T_2, & & T_2 > T_1 \\
 & y = -h: & u(-h, t) = u_s(-h, t) = 0 & & T(-h, t) = T_1
\end{align*}
\]

Figure 2.1. Schematic diagram for the system of two parallel flat plates

Figure 2.2. Schematic diagram for the system of two concentric cylinders

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The fluid motion starts from rest, and the no-slip condition at the plates/cylinders surfaces for both the fluid and the dust particles are considered. The initial temperatures of the fluid and dust particles are assumed to be equal to those of the inner cylinder i.e., $T_1$. The fluid viscosity and the electrical conductivity are assumed to be varied with temperature.
CHAPTER 3

GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

3.1 Infinitely long parallel flat plates

The flow of the fluid in the channel between the parallel flat plates is governed by the Navier-Stokes equation [6, 15]:

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) - \sigma B_0^2 u - KN(u - u_p)$$

(1)

where $\rho$ is the density of clean fluid, $\mu$ is the viscosity of the clean fluid, $u$ is the velocity of the fluid, $u_p$ is the velocity of dust particles, $\sigma$ is the electric conductivity, $p$ is the pressure acting on the fluid, $B_0$ is the magnetic field applied in the $y$-direction, $N$ is the number of dust particles per unit volume and $K$ is Stokes constant, $K = 6\pi \mu D$, and $D$ is the average radius of the dust particles. The first three terms on the right hand side are the pressure gradient, viscosity and the Lorentz force terms, respectively. The last term represents the force term due to the relative motion between the fluid and the dust particles. It is assumed that the Reynolds number of the relative velocity is small. In such a case, the force between dust and fluid is proportional to the relative velocity [1].

The motion of the dust particles is governed by Newton’s second law [1]:

$$m_p \frac{\partial u_p}{\partial t} = K(u - u_p)$$

(2)
where $m_p$ is the average mass of the dust particles.

The initial and boundary conditions on the velocity fields are given by:

$$t = 0: \ u = u_p = 0$$

$$t > 0:$$

$$u = u_p = 0, y = -h$$ \tag{4a}

$$u = u_p = 0, y = h$$ \tag{4b}

Heat transfer takes place from the upper hot plate towards the lower cold plate by conduction through the fluids. Also there is a heat generation due to viscous dissipation as shown in Equation (1).

The dust particles gain heat energy from the fluid by conduction through their spherical surface. Two energy equations are required to describe the temperature distributions for both the fluid and the dust particles, given by:

$$\rho c_v \frac{dT}{dt} = k \frac{\partial^2 T}{\partial y^2} + \mu \left( \frac{\partial u}{\partial y} \right)^2 + \sigma B_0^2 u^2 + \frac{\rho_p C_p}{\gamma_T} (T_p - T)$$ \tag{5}

$$\frac{\partial T_p}{\partial t} = -\frac{(T_p - T)}{\gamma_T}$$ \tag{6}

where $T$ is the temperature of the fluid, $T_p$ is the temperature of the particles, $c$ is the specific heat capacity of the fluid at constant volume, $C_p$ is the specific heat capacity of the particles, $k$ is the thermal conductivity of the fluid and $\rho_p$ is the mass of the dust particles per unit volume of the fluid. $\gamma_T = \frac{3Pr\gamma_p C_v}{2C}$ is the temperature relaxation time and $\gamma_p$ is the velocity relaxation time given as $2\rho_p D^2/9 \mu_0$. The material density of the dust particles $\rho_s$ is defined as $3\rho_s/4\pi D^3 N$ and Pr is the Prandtl number given by $\mu_0 c/k$, where $\mu_0$ is the viscosity of the fluid at $T = T_1$. The last three terms on the right hand side of
Equation (5) represent the viscous dissipation, Joule dissipation and heat conduction between the fluid and dust particles, respectively.

The initial and boundary conditions for the temperature field are given as:

\[ t = 0 : T = T_p = 0 \]  \hspace{1cm} (7a)

For \( t > 0 \), temperature at the surface of the parallel flat plates:

For lower plate: \( @ y = -h, T = T_p = T_1 \) \hspace{1cm} (7b)

For upper plate: \( @ y = h, T = T_p = T_2 \) \hspace{1cm} (7c)

The viscosity of the fluid is assumed to depend on the temperature and is defined as \( \mu = \mu_0 f_1(T) \). From [13, 14], the viscosity is assumed to vary exponentially with temperature, \( f_1(T) \) defined as \( f_1(T) = e^{-a_1(T-T_1)} \). The parameter \( a_1 \) is positive for liquids such as water, benzene or crude oil. In some gases like methane, air and helium \( a_1 \) may be negative, i.e. for gases, the coefficient of viscosity increases with temperature [17].

The electrical conductivity of the fluid is assumed to vary with temperature as \( \sigma = \sigma_0 f_2(T) \). A linear dependence for the electric conductivity with temperature is assumed to be of the form, \( \sigma = \sigma_0 [1 + b_1(T-T_1)] \), where the parameter \( b_1 \) is positive for water vapor and air and negative for liquid water and benzene [17, 18].

The problem can be simplified by rewriting the equations in the dimensionless form. The characteristic value for length is \( h \), characteristic time is \( \rho h^2/\mu_0 \) and the characteristic velocity is \( \mu_0/hp \).

The dimensionless quantities can be defined as:

\[ \tilde{t} = \frac{t \mu_0}{\rho h^2} \hspace{1cm} \tilde{p} = \frac{p \rho h^2}{\mu_0^2} \]
\[ \alpha = -\frac{d\rho}{d\bar{z}}, \quad \theta = \frac{T - T_1}{T_2 - T_1}, \quad \theta_p = \frac{T_p - T_1}{T_2 - T_1} \]

\[ f_1(\theta) = e^{-a_1(T_2-T_1)\theta} = e^{-a\theta} \]

\[ f_2(\theta) = 1 + b_1(T_2 - T_1)\theta = 1 + b\theta \]

where \( a = a_1(T_2 - T_1) \) is the viscosity parameter and \( b = b_1(T_2 - T_1) \), is the electric conductivity parameter.

Using the above characteristic variables, the dimensionless governing equations are formulated. (The hats are dropped for simplicity)

\[ \frac{\partial u}{\partial t} = \alpha + f_1(\theta)\frac{\partial^2 u}{\partial y^2} + \frac{\partial f_1(\theta)}{\partial \theta} \frac{\partial u}{\partial y} - f_2(\theta)Ha^2u - R(u - u_p) \quad (8) \]

The equation of motion of the dust particle,

\[ G \frac{\partial u_p}{\partial t} = (u - u_p) \quad (9) \]

The non-dimensional initial and the boundary conditions are given:

\[ t = 0: u = u_p = 0 \quad (10a) \]

\[ t > 0: u = u_p = 0, y = -1 \quad (10b) \]

\[ t > 0: u = u_p = 0, y = 1 \quad (10c) \]

The dimensionless forms of the two energy equations are:

\[ \frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + Ec f_1(\theta) \left( \frac{\partial u}{\partial y} \right)^2 + Ec f_2(\theta)Ha^2u^2 + \frac{2R}{3Pr} (\theta_p - \theta) \quad (11) \]

\[ \frac{\partial \theta_p}{\partial t} = -L_0(\theta_p - \theta) \quad (12) \]

The dimensionless initial and boundary condition are shown below:

\[ t = 0: \theta = \theta_p = 0 \quad (13a) \]
\( t > 0 : \theta = \theta_p = 0, y = -1 \)  

\( t > 0 : \theta = \theta_p = 1, y = 1 \)

where \( H a^2 = \sigma B^2 \mu_0 \) is the Hartmann number, \( R = KNh^2/\mu_0 \) is the particle concentration parameter, \( G = m_p \mu_0 / \rho h^2 k \) is the particle mass parameter, \( Ec = \mu_0^2 / (h^2 c \rho^2 (T_2 - T_1)) \) is the Eckert number and \( L_0 = \rho h^2 / \mu_0 \gamma_P \) is the temperature relaxation time parameter.

3.2 Infinitely long concentric cylinders

3.2.1. Fixed boundary

For the cylindrical geometry, the equations are written in the similar way as for the previous case. The coordinates are changed from the rectangular form to cylindrical form.

The flow of the fluid is governed by the Navier-Stokes equation [15, 16]:

\[
\rho \frac{\partial u}{\partial t} = -\frac{dp}{dz} + \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\partial u}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial r} (\mu \cdot u) - \sigma B_0^2 u - KN(u - u_p) \tag{14}
\]

where \( K \) is the Stokes constant, \( K = 6 \pi \mu D \). The flow of the dust particles is governed by Newton's second law of motion

\[
m_p \frac{\partial u_p}{\partial t} = K(u - u_p) \tag{15}
\]

The initial and the boundary conditions are given by:

\( t = 0 : u = u_p = 0 \)

For \( t > 0 \), the no-slip condition at the surface of the cylinders gives:

For inner cylinder: \( @ r = r_1 \)

\[ u = u_p = 0 \tag{16b} \]
For outer cylinder: \( \partial r = r_2 \)

\[ u = u_p = 0 \]  \hspace{2cm} (16c)

Two energy equations govern the temperature distributions for the dust particles and the fluid and are given by:

\[ \rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial r^2} + k \frac{1}{r} \frac{\partial T}{\partial r} + \mu \left( \frac{\partial u}{\partial r} \right)^2 + \alpha \beta u^2 + \frac{P_p C_i}{\gamma_T} (T_p - T) \]  \hspace{2cm} (17)

\[ \frac{\partial T_p}{\partial t} = \frac{(T_p - T)}{\gamma_T} \]  \hspace{2cm} (18)

The initial and boundary conditions for the temperature field are given as:

\( t = 0 : T = T_p = 0 \)  \hspace{2cm} (19a)

For \( t > 0 \), temperature at the surface of the cylinders:

For inner cylinder: \( \partial r = r_1 \)

\[ T = T_p = T_1 \]  \hspace{2cm} (19b)

For outer cylinder: \( \partial r = r_2 \)

\[ T = T_p = T_2 \]  \hspace{2cm} (19c)

The heat transfer takes place from the outer cylinder towards the inner cylinder by means of conduction through the fluid. The dimensionless quantities are defined as:

\[ \hat{t} = \frac{t \mu_0}{\rho r_2^2} \]  \hspace{2cm} \( \hat{p} = \frac{p \rho r_2^2}{\mu_0^2} \)  \hspace{2cm} \( \alpha = -\frac{d \hat{p}}{dx} \)  \hspace{2cm} \( \theta = \frac{T - T_1}{T_2 - T_1} \)  \hspace{2cm} \( \theta_p = \frac{T_p - T_1}{T_2 - T_1} \)

\[ f_1(\theta) = e^{-\alpha_1(T_2 - T_1)}  \hspace{2cm} f_2(\theta) = 1 + b_1(T_2 - T_1)\theta = 1 + b\theta \]

where \( a = a_1(T_2 - T_1) \) and \( b = b_1(T_2 - T_1) \).

As before, using the above characteristic variables, the dimensionless governing equations are generated.
(25b)-(25c), the radius of the external cylinder is taken as 1, while the radius of the internal cylinder is taken to be 0.1.

3.2.2. Moving boundary

In this case, the governing equations and the boundary conditions are same as that of the fixed boundary system. Only the numerical scheme used during the solving of the equations is changed.

The governing equation in the dimensionless form for the moving boundary system is given as:

\[
\frac{\partial u}{\partial t} = \alpha + f_1(\theta) \frac{\partial^2 u}{\partial r^2} + \frac{\partial f_1(\theta)}{\partial r} \frac{\partial u}{\partial r} + f_2(\theta) \frac{1}{r} \frac{\partial u}{\partial r} - f_2(\theta) H a^2 u - R(u - u_p) \tag{26}
\]

The equation of motion of the dust particle,

\[
G \frac{\partial u_p}{\partial t} = (u - u_p) \tag{27}
\]

The non-dimensional initial and boundary conditions are given as follows:

\[
t = 0: u = u_p = 0 \tag{28a}
\]

\[
t > 0: u = u_p = 0, r_1 = 0.1 \tag{28b}
\]

\[
t > 0: u = u_p = 0, r_2 = 1 \tag{28c}
\]

The dimensionless forms of the two energy equations are given as:

\[
\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{Pr} \frac{\partial \theta}{r \partial r} + E c f_1(\theta) \left( \frac{\partial u}{\partial r} \right)^2 + E c f_2(\theta) H a^2 u^2 + \frac{2R}{3Pr} (\theta_p - \theta) \tag{29}
\]

\[
\frac{\partial \theta_p}{\partial t} = -L_\theta (\theta_p - \theta) \tag{30}
\]
The dimensionless form of the initial and boundary condition is represented as follows:

\[ t = 0 : \theta = \theta_p = 0 \]  \hspace{1cm} (31a)

\[ t > 0 : \theta = \theta_p = 0, r_i = 0.1 \]  \hspace{1cm} (31b)

\[ t > 0 : \theta = \theta_p = 1, r_2 = 1 \]  \hspace{1cm} (31c)

While solving the governing equations for the moving boundary scheme, a value for velocity is given to the outer boundary (the external cylinder). After every time step, the temperature and the velocity values of the fluid and the dust particles are re-calculated and reassigned by the numerical scheme.
CHAPTER 4

NUMERICAL METHOD

A 2-D finite element code was developed for solving the system of coupled, non-linear partial differential equations. The same code with modifications for the geometry is used for solving the governing equations for all the three cases namely – infinitely long parallel flat plates, concentric cylinders with fixed walls and concentric cylinders with moving outer wall.

The Galerkin weighted residual procedure was used to formulate the finite element method [19]. First, the mesh was generated and then the shape functions and the Jacobian matrix were derived for the system of governing equations. The detailed steps for solving the governing equations are explained in this chapter.

4.1. Element mesh

The selected element of choice is a quadrilateral consisting of four vertex nodes. In most instances, a mesh consisting of quadrilateral elements is sufficient and usually more accurate than a mesh consisting of triangular elements.

The quadrilateral mesh subdivides a region into a set of small quadrilateral domains, i.e., elements. In its simplest form, the quadrilateral becomes a rectangular element with the boundaries of the element parallel to a coordinate system. Further extension of the element, using a local natural coordinate system, results in a generalized quadrilateral of
which the rectangle is a subset. The quadrilateral mesh developed in this case more closely resembles the standard two-dimensional finite difference mesh.

4.2. Shape functions

The first step after the computational domain is divided into elements is to define the type of shape function approximation that is to be used. Consider a four noded rectangular element as shown in Figure 4.1.

![Four noded quadrilateral element](image)

This element shows in detail the shape that forms the mesh for the computational domain. The shape functions for this element are defined as:

\[
N_1 = \frac{1}{4ab} (b - x)(a - y) \\
N_2 = \frac{1}{4ab} (b + x)(a - y) \\
N_3 = \frac{1}{4ab} (b + x)(a + y) \\
N_4 = \frac{1}{4ab} (b - x)(a + y)
\]
The next step is the derivation of the derivatives of the shape functions and the Jacobian. The whole matrix with the Jacobian is represented as follows:

\[
\frac{\partial N_i}{\partial \xi} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \end{bmatrix} \frac{\partial N_i}{\partial \eta} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} J = \begin{bmatrix} \frac{\partial N_i}{\partial x} & \frac{\partial N_i}{\partial y} \end{bmatrix}
\]

where

\[
J = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}
\]

4.3. Matrix generation

The equation was discretized and arranged in a matrix of the form

\[ M \dot{T} + KT = F \]

where \( M \) is the mass matrix, \( K \) is the stiffness or conduction matrix, \( T \) is the vector of nodal unknowns with \( \dot{T} \) being their time derivatives and \( F \) is the load vector [20].

The formulation of the integrals in terms of \( \xi \) and \( \eta \) yields simple integration limits. The integral is defined as

\[
\int_a^b \int_{-\infty}^\infty F(x, y) \, dx \, dy = \int_{-\infty}^\infty \int_{-\infty}^\infty f(\xi, \eta) J |d\xi d\eta|
\]

The conduction matrix thus becomes

\[
K = \int_{\xi_1}^{\xi_2} \int_{\eta_1}^{\eta_2} K \left[ \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} \right] \, dx \, dy
\]

\[
= \int_{\xi_1}^{\xi_2} \int_{\eta_1}^{\eta_2} K \left[ \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} \right] J |d\xi d\eta|
\]
The final matrix for the system of governing equations for the case of the infinitely long parallel flat plates looks similar to that shown below:

\[
\begin{bmatrix}
\frac{\partial N_i}{\partial \xi} & \frac{\partial T_i}{\partial t} \\
\frac{\partial N_i}{\partial \tau} & \frac{\partial T_i}{\partial t} \\
\end{bmatrix}
\begin{bmatrix}
\alpha \\
R \\
\end{bmatrix}
\begin{bmatrix}
T_i \\\nT_j \\\n\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
\end{bmatrix}
\]

\(\frac{\partial T}{\partial t}\) represents the first order time derivative of temperature. There are a number of ways to deal with the time marching of the dependent variables. In this case, this was solved using the so-called \(\theta\) method, which leads to the most commonly used algorithms for time integration. In the \(\theta\)-method, the time derivative is replaced by a simple difference as

\[
\frac{\partial T}{\partial t} \approx \frac{T^{n+1} - T^n}{\Delta t}
\]

where \(T^n = T(x, t_n)\) denotes the variable value at time \(t = t_n\), \(\Delta t\) is the time increment and \(t_{n+1} = t_n + \Delta t\). In general, it is assumed that \(T(x, t_n)\) is already known and is used as an initial condition to advance the solution to time level \(t_{n+1}\), where it is not yet know. Now, a relaxation parameter \(\theta\) is introduced and the solution for \(T\) is of the form

\[
T = \theta T^{n+1} + (1 - \theta) T^n, \quad t_n \leq t \leq t_{n+1}
\]

The parameter \(\theta\) is usually specified within the range of 0 to 1 and is used to control the stability of the algorithm. A value of 0 indicates that the scheme is explicit Euler scheme, 0.5 gives a second-order Crank-Nicolson-Galerkin scheme and 1 gives an implicit method. In this case, the explicit scheme was used for the time marching, i.e., \(\theta = 0\).
This scheme was followed for solving the governing equations for all the three cases. In the case of the moving outer wall problem, the mesh was regenerated after a constant time interval. The shape functions, the Jacobian and the matrix were regenerated after this particular time interval and the values for the temperature and velocity of the dust particles are calculated at the mid-point between the two walls for each and every time step.
CHAPTER 5

RESULTS AND DISCUSSION

5.1 Infinitely long parallel plates

The system of governing partial differential equations solved using the 2D finite element code has many parameters that need to be assigned values for computing the results. The following parameters are given the fixed values: \( R = 0.5 \), \( G = 0.8 \), \( \alpha = 5 \), \( Pr = 1 \), \( Ec = 0.2 \) and \( L_o = 0.7 \). These values were taken from [15] which deals with a similar case solved using the finite difference method. All the results are shown as the variation of the temperature or velocity to the time interval. The total time interval is taken as 4 (dimensionless parameter). The effects of the viscosity parameter ‘\( a \)’ and the electrical conductivity parameter ‘\( b \)’ on the temperature and the velocity of the fluid and the dust particles are studied.

Figures 5.1(a) – 5.1(d) and Figures 5.2(a) – 5.2(d) shows the effect of the viscosity parameter “\( a \)” on the time development of the velocities \( u \) and \( u_p \) and the temperatures \( T \) and \( T_p \) at the midpoint between the two plates. Figures 5.1(a) – 5.1(d) are generated for a Hartmann number \( Ha = 0 \), viscosity parameter \( a = 0 \), and electrical conductivity parameter \( b = 0 \). Figures 5.2 (a) – 5.2(d) are generated for a Hartmann number \( Ha = 0 \), viscosity parameter \( a = 0.5 \), and electrical conductivity parameter \( b = 0 \). The application
of the uniform magnetic field adds one resistive term to the momentum equation and the Joule dissipation term to the energy equation.

Figure 5.1 Variation of $T$, $T_p$, $u$, $u_p$ with time ($Ha = 0$, $a = 0$, $b = 0$)

(a) Variation of $T$ with time  
(b) Variation of $T_p$ with time 
(c) Variation of $u$ with time  
(d) Variation of $u_p$ with time
Figure 5.2 Variation of $T$, $T_p$, $u$, $u_p$ with time ($H_a = 0$, $a = 0.5$, $b = 0$)

(a) Variation of $T$ with time       (b) Variation of $T_p$ with time
(c) Variation of $u$ with time       (d) Variation of $u_p$ with time

Comparing Figures 5.1(a) – 5.1(b) and Figures 5.2(a) – 5.2(b), it is seen that the introduction of the magnetic field increases the temperature of the fluid and the dust
particles with an increase of “a” as a result of increasing the joule dissipation. As shown in Figures 5.1(c) – 5.1(d) and Figures 5.2(c) – 5.2(d), the magnetic field results in a reduction in the velocities of the fluid and the dust particles and their steady state times for all values of “a” due to its damping effects, i.e., as ‘a’ is reduced from 0.5 to 0, the values of the velocity of both the fluid as well as the dust particles decrease.

Figures 5.3(a) – 5.3(d) and Figures 5.4(a) – 5.4(d) show the effect of the viscosity parameter “b” on the time development of the temperatures $T$ and $T_p$ at the midpoint between the two plates. Figures 5.3(a) – 5.3(d) are generated for a Hartmann number $(Ha)$ of 1, viscosity parameter $(b)$ of 0, and electrical conductivity parameter $(a)$ of 0. Figures 5.4(a) – 5.4(d) are generated for a Hartmann number $Ha = 1$, viscosity parameter $b = 0.5$, and electrical conductivity parameter $a = 0$. 

(a) 

(b)
Figure 5.3 Variation of $T$, $T_p$, $u$, $u_p$ with time ($Ha = 1$, $a = 0$, $b = 0$)

(a) Variation of $T$ with time  (b) Variation of $T_p$ with time
(c) Variation of $u$ with time  (d) Variation of $u_p$ with time
Figure 5.4 Variation of $T$, $T_p$, $u$, $u_p$ with time ($Ha = 1$, $a = 0$, $b = 0.5$)

(a) Variation of $T$ with time  
(b) Variation of $T_p$ with time  
(c) Variation of $u$ with time  
(d) Variation of $u_p$ with time

Comparing Figures 5.3(a) – 5.3(b) with Figures 5.4(a) – 5.4(b), it is seen that there is a very small or negligible differences between the plots. An analysis of the results showed that the electrical conductivity parameter ‘$b$’ had little effect on the temperature and the velocity variations of both the fluid as well as the dust particles. Therefore, more emphasis was laid on the effect of the viscosity parameter ‘$a$’ on the temperature and the velocity at different Hartmann numbers. Tables 1 - 8 show the variation of the steady state values of $u$, $u_p$, $T$ and $T_p$ respectively.

Table 1. Variation of “$T$” for various values of “$a$” and “$b$” ($Ha=0$)

<table>
<thead>
<tr>
<th></th>
<th>$a = -0.5$</th>
<th>$a = 0$</th>
<th>$a = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b = -0.5$</td>
<td>0.525349</td>
<td>0.533045</td>
<td>0.542559</td>
</tr>
<tr>
<td>$b = 0$</td>
<td>0.515667</td>
<td>0.524486</td>
<td>0.531744</td>
</tr>
<tr>
<td>$b = 0.5$</td>
<td>0.506743</td>
<td>0.515126</td>
<td>0.524004</td>
</tr>
</tbody>
</table>
Table 6. Variation of “Tp” for various values of “a” and “b” (Ha = 1)

<table>
<thead>
<tr>
<th></th>
<th>a = -0.5</th>
<th>a = 0</th>
<th>a = 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>b = -0.5</td>
<td>0.476145</td>
<td>0.498667</td>
<td>0.530232</td>
</tr>
<tr>
<td>b = 0</td>
<td>0.471005</td>
<td>0.494358</td>
<td>0.527460</td>
</tr>
<tr>
<td>b = 0.5</td>
<td>0.467941</td>
<td>0.483001</td>
<td>0.523891</td>
</tr>
</tbody>
</table>

Table 7. Variation of “u” for various values of “a” and “b” (Ha = 1)

<table>
<thead>
<tr>
<th></th>
<th>a = -0.5</th>
<th>a = 0</th>
<th>a = 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>b = -0.5</td>
<td>0.473997</td>
<td>0.686114</td>
<td>0.776294</td>
</tr>
<tr>
<td>b = 0</td>
<td>0.471333</td>
<td>0.684204</td>
<td>0.773258</td>
</tr>
<tr>
<td>b = 0.5</td>
<td>0.469165</td>
<td>0.682067</td>
<td>0.769889</td>
</tr>
</tbody>
</table>

Table 8. Variation of “u_p” for various values of “a” and “b” (Ha = 1)

<table>
<thead>
<tr>
<th></th>
<th>a = -0.5</th>
<th>a = 0</th>
<th>a = 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>b = -0.5</td>
<td>0.649099</td>
<td>0.689253</td>
<td>0.719012</td>
</tr>
<tr>
<td>b = 0</td>
<td>0.647856</td>
<td>0.687214</td>
<td>0.715456</td>
</tr>
<tr>
<td>b = 0.5</td>
<td>0.644798</td>
<td>0.686821</td>
<td>0.710772</td>
</tr>
</tbody>
</table>

From the tables it could be noted that the increasing “a” increases the steady state velocities and temperatures for both the fluid and the dust particles for both Ha = 0 and Ha = 1. However, its effect is more pronounced for small values of “b’. Increasing the value of the parameter “b” decreases the value of the velocities u and u_p for all values of “a” and its effect is more apparent for higher values of “a”. The influence of “b” on T and T_p depends on “a”. Unless “a” has a large and negative value, increasing “b” decreases T as a result of increasing the damping magnetic forces which decreases the velocity.
Increasing the values of “Ha” increases the values of the temperatures for both the fluid and the dust particles. This could be attributed to the effect of the uniform magnetic field that is applied on the outside of the channel and the Eckert number. But the steady state velocities of both the fluid as well as the dust particles decrease with an increase in the Hartmann number.

These results were then compared to those from [15]. It deals with the very same geometry, governing equations and boundary conditions. Finite difference scheme was used for solving the governing equations in that article. The plots show similar trend in both simulations for each and every case. From Table 9 it is noted that the temperature and velocity of both the fluid and dust particles have similar values at the end of the iterations. It is also noted that the difference in the results of between both the simulations can be predicted to be less than 5%. This article was taken as a benchmarking task and the results were highly satisfactory.

Table 9. Comparison of the results from the 2 simulations (Ha = 1, a = 0.5, b = 0)

<table>
<thead>
<tr>
<th></th>
<th>FDM Code</th>
<th>FEM Code</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>0.55905</td>
<td>0.54565</td>
<td>2.39</td>
</tr>
<tr>
<td>Tp</td>
<td>0.53335</td>
<td>0.52746</td>
<td>1.11</td>
</tr>
<tr>
<td>u</td>
<td>0.79773</td>
<td>0.77326</td>
<td>3.06</td>
</tr>
<tr>
<td>up</td>
<td>0.73751</td>
<td>0.71545</td>
<td>2.98</td>
</tr>
</tbody>
</table>
5.2 System of Concentric cylinders with fixed walls

For the system of concentric cylinders with fixed walls, two sets of analysis are done, namely, the variation of viscosity parameter “a” and variation of electrical conductivity parameter “b” and their effects on the temperature and velocity of the fluid and the dust particles. For these studies, the following values are fixed for the different parameters: \( R = 0.5, \ G = 0.8, \ \alpha = 5, \ \text{Pr} = 1, \ \text{Ec} = 0.2 \) and \( L_o = 0.7 \).

Figures 5.5(a) - 5(d) show the variation of the velocities \( u \) and \( u_p \) and the temperatures \( T \) and \( T_p \) for the case of \( H_a = 1, \ a = 0.5 \) and \( b = 0 \), while Figures 5.6(a) - 5.6(d) shows the results obtained when \( H_a = 1, \ a = 0 \) and \( b = 0 \). All these results are representative of the values of the temperature and velocity of the fluid and dust particles at the midpoint of the space between the two cylinders.

![Graphs showing temperature and velocity variations](image)

(a)

(b)
Figure 5.5 Variation of $T$, $T_p$, $u$, $u_p$ with time ($Ha = 1$, $a = 0.5$, $b = 0$)

(a) Variation of $T$ with time

(b) Variation of $T_p$ with time

(c) Variation of $u$ with time

(d) Variation of $u_p$ with time
Comparing Figures 5.5(a) – 5.5(b) and Figures 5.6(a) – 5.6(b), it is noted that the temperature of the fluid and the dust particles increases with the introduction of the magnetic field, for all values of “a”. But it could be noted from comparing Figures 5.5(c) – 5.5(d) and Figures 5.6(c) – 5.6(d), there is a reduction in the velocities of the fluid and the dust particles and their steady state times for all values of “a”, i.e., as ‘a’ is reduced from 0.5 to 0, the values of the velocity of both the fluid as well as the dust particles decrease.

Figures 5.7(a) – 5.7(b) are generated for the case when Ha = 1, a = 0 and b = 0, while Figures 5.8(a) – 5.8(b) are for the case when Ha = 1, a = 0 and b = -0.5.
From Figures 5.7(a) - 5.7(b) and Figures 5.8(a) - 5.8(b), it is observed that is very small or negligible differences between the plots. The variations in the values are noted in
the 3\textsuperscript{rd} or 4\textsuperscript{th} decimal place. This implies that the electrical conductivity parameter “b” has negligible effect on the temperature and velocity variation of both the fluid and dust particles.

Effects of the variation of the Hartmann number on $u$, $u_p$, $T$ and $T_p$ are shown in the Tables 10 – 13. These values show the variation of the steady state values of $u$, $u_p$, $T$ and $T_p$ respectively at a point where the radius of the external cylinder is 0.5.

### Table 10. Variation of “$T$” for various values of “a” and Ha

<table>
<thead>
<tr>
<th>$a$</th>
<th>$a = -0.5$</th>
<th>$a = -0.25$</th>
<th>$a = 0$</th>
<th>$a = 0.25$</th>
<th>$a = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ha = 0$</td>
<td>0.758721</td>
<td>0.760099</td>
<td>0.761484</td>
<td>0.763211</td>
<td>0.765443</td>
</tr>
<tr>
<td>$Ha = 1$</td>
<td>0.758954</td>
<td>0.760386</td>
<td>0.762096</td>
<td>0.764203</td>
<td>0.766843</td>
</tr>
</tbody>
</table>

### Table 11. Variation of “$T_p$” for various values of “a” and Ha

<table>
<thead>
<tr>
<th>$a$</th>
<th>$a = -0.5$</th>
<th>$a = -0.25$</th>
<th>$a = 0$</th>
<th>$a = 0.25$</th>
<th>$a = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ha = 0$</td>
<td>0.705465</td>
<td>0.706559</td>
<td>0.707792</td>
<td>0.709320</td>
<td>0.711823</td>
</tr>
<tr>
<td>$Ha = 1$</td>
<td>0.705785</td>
<td>0.706848</td>
<td>0.708389</td>
<td>0.710277</td>
<td>0.712486</td>
</tr>
</tbody>
</table>

### Table 12. Variation of “$u$” for various values of “a” and Ha

<table>
<thead>
<tr>
<th>$a$</th>
<th>$a = -0.5$</th>
<th>$a = -0.25$</th>
<th>$a = 0$</th>
<th>$a = 0.25$</th>
<th>$a = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ha = 0$</td>
<td>0.397458</td>
<td>0.478279</td>
<td>0.552373</td>
<td>0.633856</td>
<td>0.709845</td>
</tr>
<tr>
<td>$Ha = 1$</td>
<td>0.381235</td>
<td>0.442309</td>
<td>0.505011</td>
<td>0.572343</td>
<td>0.634789</td>
</tr>
</tbody>
</table>

### Table 13. Variation of “$u_p$” for various values of “a” and Ha

<table>
<thead>
<tr>
<th>$a$</th>
<th>$a = -0.5$</th>
<th>$a = -0.25$</th>
<th>$a = 0$</th>
<th>$a = 0.25$</th>
<th>$a = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ha = 0$</td>
<td>0.402247</td>
<td>0.474195</td>
<td>0.547311</td>
<td>0.627709</td>
<td>0.704786</td>
</tr>
<tr>
<td>$Ha = 1$</td>
<td>0.371201</td>
<td>0.438517</td>
<td>0.500501</td>
<td>0.566978</td>
<td>0.624477</td>
</tr>
</tbody>
</table>

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From the above tables it could be noted irrespective of a change in the value of the Hartmann number, increasing “a” increases the values of the steady state velocities and temperatures for both the fluid and the dust particles. It is also noted that increasing the values of “Ha” increases the values of the temperatures for both the fluid and the dust particles but the steady state velocities of both the fluid as well as the dust particles decrease with an increase in the Hartmann number.

5.3 System of Concentric cylinders with moving outer wall

In this case, the outer cylinder is moved towards the inner cylinder. The time taken for the outer cylinder to approach the inner cylinder is taken as 4 (dimensionless parameter) and the velocity of the outer cylinder is set as 0.2 In this case, the mesh is regenerated after each time step and the values for the temperature and the velocity for both the fluid and the dust particles are recalculated. The outer cylinder is stopped at a small distance before it approaches the inner cylinder. The reason for this is that the equation of state for the situation when both the cylinders touch, is unknown. As in the previous two cases, the following values are fixed for the different parameters: \( R = 0.5 \), \( G = 0.8 \), \( \alpha = 5 \), \( Pr = 1 \), \( Ec = 0.2 \) and \( L_0 = 0.7 \).
Figure 5.9 Variation of $T$, $T_p$, $u$, $u_p$ with time ($Ha = 1$, $a = 0.5$, $b = 0$)

(a) Variation of $T$ with time
(b) Variation of $T_p$ with time
(c) Variation of $u$ with time
(d) Variation of $u_p$ with time
Figure 5.10 Variation of \( T, T_p, u, u_p \) with time (Ha = 1, \( a = 0, b = 0 \))

(a) Variation of \( T \) with time  
(b) Variation of \( T_p \) with time 
(c) Variation of \( u \) with time  
(d) Variation of \( u_p \) with time 

The Figures 5.9(a) - 5.9(d) and Figures 5.10(a) - 5.10(d) shows the effect of the viscosity parameter “\( a \)” on the time development of the velocities \( u \) and \( u_p \) and the temperatures \( T \) and \( T_p \) at the mid-point between the two cylinders. Figures 5.9(a) - 5.9(d)
are generated for a Hartmann number $Ha = 1$, viscosity parameter $a = 0.5$, and electrical conductivity parameter $b = 0$. Figures 5.10(a) – 5.10(d) are generated for a Hartmann number $Ha = 1$, viscosity parameter $a = 0$, and electrical conductivity parameter $b = 0$.

From the plots it could be seen that the temperature of the fluid particles at the middle of the channel increase steeply followed by a gradual decrease towards the last stages. The increase can be attributed to the transfer of heat from the outer wall to the fluid, which was initially at a zero-temperature. The outer cylinder has a high temperature compared to the inner cylinder. As the wall keeps moving, the heat gets distributed more uniformly. The temperature profile starts decreasing when the fluid transfers the heat energy it possesses to the inner cylinder, which is maintained at a lower temperature.

As for the dust particles, the maximum temperature they possess is low compared to the fluid medium. The reason is that the dust particles transfer the heat they carry on to the fluid medium.

Comparing Figures 5.9(a) – 5.9(b) and Figures 5.10(a) – 5.10(b), it is seen that the introduction of the magnetic field increases the temperature of the fluid and the dust particles for all values of “$a$” similar to the two previous cases. From Figures 5.9(c) – 5.9(d) and Figures 5.10(c) – 5.10(d), it is noted that the decrease in the value of “$a$” decreases the values of the velocities for both the fluid and dust particles. More results are tabulated in Tables 14 - 17.

Table 14. Variation of “$T$” for various values of “$a$” and $Ha$ for moving boundary

<table>
<thead>
<tr>
<th></th>
<th>$a = -0.5$</th>
<th>$a = -0.25$</th>
<th>$a = 0$</th>
<th>$a = 0.25$</th>
<th>$a = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ha = 0$</td>
<td>0.577261</td>
<td>0.580377</td>
<td>0.584219</td>
<td>0.587008</td>
<td>0.589144</td>
</tr>
<tr>
<td>$Ha = 1$</td>
<td>0.580318</td>
<td>0.583517</td>
<td>0.585320</td>
<td>0.588978</td>
<td>0.590477</td>
</tr>
</tbody>
</table>
Table 15. Variation of “T_p” for various values of “a” and Ha for moving boundary

<table>
<thead>
<tr>
<th></th>
<th>a = -0.5</th>
<th>a = -0.25</th>
<th>a = 0</th>
<th>a = 0.25</th>
<th>a = 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ha = 0</td>
<td>0.609247</td>
<td>0.612195</td>
<td>0.615603</td>
<td>0.617909</td>
<td>0.620786</td>
</tr>
<tr>
<td>Ha = 1</td>
<td>0.611201</td>
<td>0.613857</td>
<td>0.616759</td>
<td>0.618978</td>
<td>0.621477</td>
</tr>
</tbody>
</table>

Table 16. Variation of “u” for various values of “a” and Ha for moving boundary

<table>
<thead>
<tr>
<th></th>
<th>a = -0.5</th>
<th>a = -0.25</th>
<th>a = 0</th>
<th>a = 0.25</th>
<th>a = 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ha = 0</td>
<td>0.006911</td>
<td>0.007514</td>
<td>0.008064</td>
<td>0.008619</td>
<td>0.009307</td>
</tr>
<tr>
<td>Ha = 1</td>
<td>0.006901</td>
<td>0.007499</td>
<td>0.008051</td>
<td>0.008605</td>
<td>0.009293</td>
</tr>
</tbody>
</table>

Table 17. Variation of “U_p” for various values of “a” and Ha for moving boundary

<table>
<thead>
<tr>
<th></th>
<th>a = -0.5</th>
<th>a = -0.25</th>
<th>a = 0</th>
<th>a = 0.25</th>
<th>a = 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ha = 0</td>
<td>0.022470</td>
<td>0.036195</td>
<td>0.050400</td>
<td>0.065279</td>
<td>0.073838</td>
</tr>
<tr>
<td>Ha = 1</td>
<td>0.014201</td>
<td>0.028517</td>
<td>0.042059</td>
<td>0.056978</td>
<td>0.071390</td>
</tr>
</tbody>
</table>

From the above tables it is noted that the values of the temperature and velocities of both the fluid and the dust particles increase with an increase of the viscosity parameter. Also, the temperature of the fluid and the dust particles increase with an increase in the Hartmann number, i.e., the magnetic field. While for the very same increase of Ha, the velocities of both the mediums decrease. All these values follow a similar trend as seen in the previous two cases – the parallel flat plates and the system of concentric cylinders with fixed walls.

Another important study has been made with regards to the particle concentration parameter R, which is dependent on the number of dust particles, and the way it affects
the maximum temperature attained by the dust particles. The particle concentration parameter is given different values and the results are analyzed. The results are tabulated in Table 18. It is found that the increase in the number of particles increases the temperature attained by the dust particles at that point of time when the outer cylinder comes to a stop. Also, a decrease in the concentration reduces the final temperature.

Table 18. Effect of “R” on different variables (Ha = 1, a = 0, b = 0)

<table>
<thead>
<tr>
<th>R</th>
<th>T</th>
<th>Tp</th>
<th>u</th>
<th>up</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.585309</td>
<td>0.615120</td>
<td>0.008044</td>
<td>0.041854</td>
</tr>
<tr>
<td>0.5</td>
<td>0.585320</td>
<td>0.616759</td>
<td>0.008051</td>
<td>0.042059</td>
</tr>
<tr>
<td>1.0</td>
<td>0.585328</td>
<td>0.618417</td>
<td>0.008058</td>
<td>0.042253</td>
</tr>
</tbody>
</table>

It is seen that there is a uniform distribution of temperature of the fluid as compared to the variation of temperature of the dust particles for the different particle concentration values. And the same trend is seen with the velocities of the fluid and the dust particles.
CHAPTER 6

CONCLUSION

In this project, the unsteady MHD flow and heat transfer of a dusty and electrically conducting fluid have been studied in the presence of an external uniform magnetic field, taking into consideration the variations of the viscosity and electric conductivity of the fluid with temperature.

Three main cases are analyzed in this, namely:

1. infinitely long parallel flat plates
2. infinitely long concentric cylinders with fixed walls
3. infinitely long concentric cylinders with moving outer wall

The proper boundary conditions were applied and the governing equations solved using the 2-D finite element code developed for this project. The results obtained for these cases are summarized below:

- The variation of the viscosity of the fluid has an apparent effect on the velocity and temperature fields for both the fluid and the dust particles.

- The variation in the electric conductivity of the fluid has no noticeable effect on the temperature distributions for the fluid and dust particles.

- The effect of the electric conductivity is less pronounced as compared to the effect of the viscosity parameter.
• In the case of the parallel flat plates, increasing “a” increases the steady state velocities and temperature for both the fluid and dust particle for all values of “b”. However, its effect is more pronounced for small values of “b”. Increasing the parameter “b” decreases the velocities \( u \) and \( u_p \) for all values of “a” and its effect is more apparent for higher values of “a”. The effect of “b” on \( T \) and \( T_p \) depends on “a”.

• The variation of the magnetic field affects the temperature and the velocity fields in different ways. While the temperature of the fluid and dust particles increase in the presence of a strong magnetic field (Hartmann number), the velocities of both the fluid and the dust particles, tend to reduce with a higher magnetic field.

• In case of the concentric cylinders, the viscosity parameter and the Hartmann number have similar effects on the temperatures and the velocities of both the fluid and the dust particles.

• For the case of moving outer wall, there is no symmetric variation of the temperatures and velocities of both the fluid and dust particle. Also, for the dust particles, the maximum temperature they possess is low compared to the fluid medium. The reason is that the dust particles transfer the heat they carry on to the fluid medium.

• The particle concentration parameter affects the maximum temperature and velocity attained by the dust particles significantly, compared to the fluid medium.
REFERENCES


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APPENDIX I

NOMENCLATURE

\[ \begin{align*}
\text{a} & \quad \text{viscosity parameter} \\
\text{b} & \quad \text{electric conductivity parameter} \\
\text{B}_0 & \quad \text{uniform magnetic field} \\
\text{c} & \quad \text{specific heat capacity of the fluid at constant volume, } \text{J kg}^{-1} \text{ K}^{-1} \\
\text{C}_s & \quad \text{specific heat capacity of the particles, } \text{J kg}^{-1} \text{ K}^{-1} \\
\text{D} & \quad \text{average radius of the dust particles, m} \\
\text{E}_c & \quad \text{Eckert number} \\
\text{G} & \quad \text{Particle mass parameter} \\
\text{Ha} & \quad \text{Hartmann number} \\
\text{k} & \quad \text{thermal conductivity of the fluid, } \text{W m}^{-1} \text{ K}^{-1} \\
\text{K} & \quad \text{Stokes constant} \\
\text{L}_0 & \quad \text{temperature relaxation time parameter} \\
\text{m}_p & \quad \text{mass of the dust particle, kg} \\
\text{N} & \quad \text{number of dust particles per unit volume} \\
\text{p} & \quad \text{pressure acting on the fluid, } \text{N m}^{-2}
\end{align*} \]
Pr Prandtl number

$r, z$ axes of the cylindrical coordinate system

$r_1$ radius of the inner cylinder, m

$r_2$ radius of the inner cylinder, m

$R$ Particle concentration parameter

$t$ time, sec

$T$ temperature of the fluid, K

$T_p$ temperature of the dust particles, K

$T_1$ temperature at the surface of the inner cylinder, K

$T_2$ temperature at the surface of the outer cylinder, K

$u$ velocity of the fluid, m sec$^{-1}$

$u_p$ velocity of the dust particles, m sec$^{-1}$

Greek Symbols

$\alpha$ constant for pressure gradient

$\gamma_p$ velocity relaxation time, sec

$\gamma_T$ temperature relaxation time, sec

$\mu$ viscosity of the fluid, cm$^2$ s$^{-1}$

$\mu_0$ viscosity of the fluid at $T = T_1$, cm$^2$ s$^{-1}$

$\theta$ Non-dimensional temperature of the fluid

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\( \theta_p \)  Non-dimensional temperature of the dust particle

\( \rho \)  density of the fluid, g cm\(^{-3}\)

\( \rho_s \)  material density of the dust particles, g cm\(^{-3}\)

\( \sigma \)  electric conductivity
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variable physical properties, Proceedings of the ASME International Mechanical

Unsteady MHD Flow and Heat Transfer of Dusty Fluid between Two Cylinders
with Variable Viscosity, Computers, Materials & Continua. (Draft submitted for
review).

Thesis title:
Analysis of the Unsteady Flow and Heat Transfer of a Dusty Fluid between Two
Concentric Cylinders in the Presence of a Magnetic Field

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