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Conjugacy numbers for cyclic groups of even order

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CONJUGACY NUMBERS FOR CYCLIC

GROUPS OF EVEN ORDER

by

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ABSTRACT

Conjugacy Numbers for Cyclic Groups of Even Order

by

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Let Γ be a finite group, let X be a subset of Γ where $X^{-1} = X$ and $1 \notin X$. The *conjugacy graph* Con(Γ; X) has vertex set Γ and two vertices g, $h \in \Gamma$ are adjacent if and only if there exists $x \in X$ with $g = xhx^{-1}$. Let Ω be a group with generating set Δ. The *conjugacy number* con($\Omega;\Delta$) is defined as the minimum integer $k \geq 2$ for which there exists a nonabelian group Γ of order $k|\Omega|$ and a subset X of Γ such that Cay($\Omega;\Delta$) is isomorphic to a component of Con(Γ; X). We call this Γ a *conjugacy group* for Ω and Δ. We will calculate the conjugacy numbers for C_6, C_8 and C_{10} and identify possible conjugacy groups. Finally we will verify that certain groups of order $4n$ cannot be conjugacy groups for C_{2n}.

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CHAPTER 1

INTRODUCTION

The most widely studied graph associated with groups is the Cayley graph. When solving problems concerning groups, the use of graphs can clarify a presentation and make problems more controllable. Graphs aid in proving general results about groups and particular results about individual groups [3]. Another graph defined to aid in the understanding of group structure is the conjugacy graph. It is necessary to first discuss Cayley graphs and then we will define conjugacy graphs, which were first introduced by Bowman and Schultz in [3].

Cayley Graphs

Arthur Cayley (1821-1895) was the first to represent a group as a network of directed edges, where the vertices correspond to elements and the edges to multiplication by group generators and their inverses [11]. Let Γ be a finite group with a generating set Δ, where $\Delta^{-1} = \Delta$ and $1 \notin \Delta$. The *Cayley graph* Cay(Γ; Δ) has the vertex set Γ, where $g, h \in \Gamma$ are adjacent if and only if $h = g\delta$ for some $\delta \in \Delta$. We present the following examples to provide further understanding of a Cayley graph.

Example 1.1 Let Γ be the group S_3 and let $\Delta = \{(12), (123), (132)\}$. To find Cay($\Gamma$; Δ) we multiply each element of S_3 on the right by each element of Δ as follows:

(1)(12)=(12)	(1)(123)=(123)	(1)(132)=(132)
(12)(12)=(1)	(12)(123)=(13)	(12)(132)=(23)
(13)(12)=(132)	(13)(123)=(23)	(13)(132)=(12)
(23)(12)=(123)	(23)(123)=(12)	(23)(132)=(13)
(123)(12)=(23)	(123)(123)=(132)	(123)(132)=(1)
(132)(12)=(13)	(132)(123)=(1)	(132)(132)=(123)

The Cayley graph Cay(S_3 ; Δ) is shown in Figure 1.A, where a=(123) and b=(12). The convention is the following. The direction of an arrow labeled with a is the multiplication by a on the right, while against the arrow is multiplication by a^{-1}=(132) on the right.

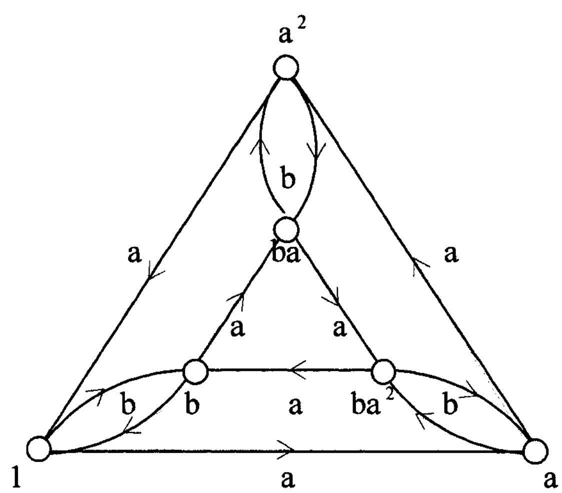


Figure 1.A The Cayley graph of S_3 where $\Delta=\{(12),(123),(132)\}$

The usefulness of Cayley graphs in solving problems involving groups can be further illustrated using Figure 1 by considering the following example as discussed in [3]. If we were asked to find the element aba^2b in S_3 where a=(123) and b=(12) we could consider the directed walk starting at 1 and quickly find that $aba^2b=a^2$. While graphs of groups do not tell anything about groups which cannot be expressed algebraically, there are still advantages to using graphs over the algebraic approach. One advantage is the speed in which conclusions can be reached using a graph with much less effort than the algebraic method. We include another example of a Cayley graph only this time the Cayley graph is simplified and we omit the directed edges.

Example 1.2 Let Γ be the group $C_4=\{1,x,x^2,x^3\}$ and let $\Delta=\{x,x^3\}$. To find Cay(Γ; Δ) we perform the following calculations similar to example 1.1:

$1\cdot x=x$	$1\cdot x^3=x^3$
$x\cdot x=x^2$	$x\cdot x^3=1$
$x^2\cdot x=x^3$	$x^2\cdot x^3=x$
$x^3\cdot x=1$	$x^3\cdot x^3=x^2$

The Cayley graph is shown in Figure 1.B.

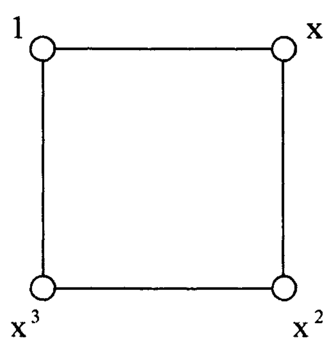


Figure 1.B The Cayley graph of C_4 where $\Delta=\{x,x^3\}$

Example 1.3 Let Γ be the group S_4 and let $\Delta=\{(12),(13),(14)\}$. Similar to example 1, to find Cay($\Gamma;\Delta$) we multiply each element in S_4 on the right by each element in Δ. The Cayley graph is shown in Figure 1.C.

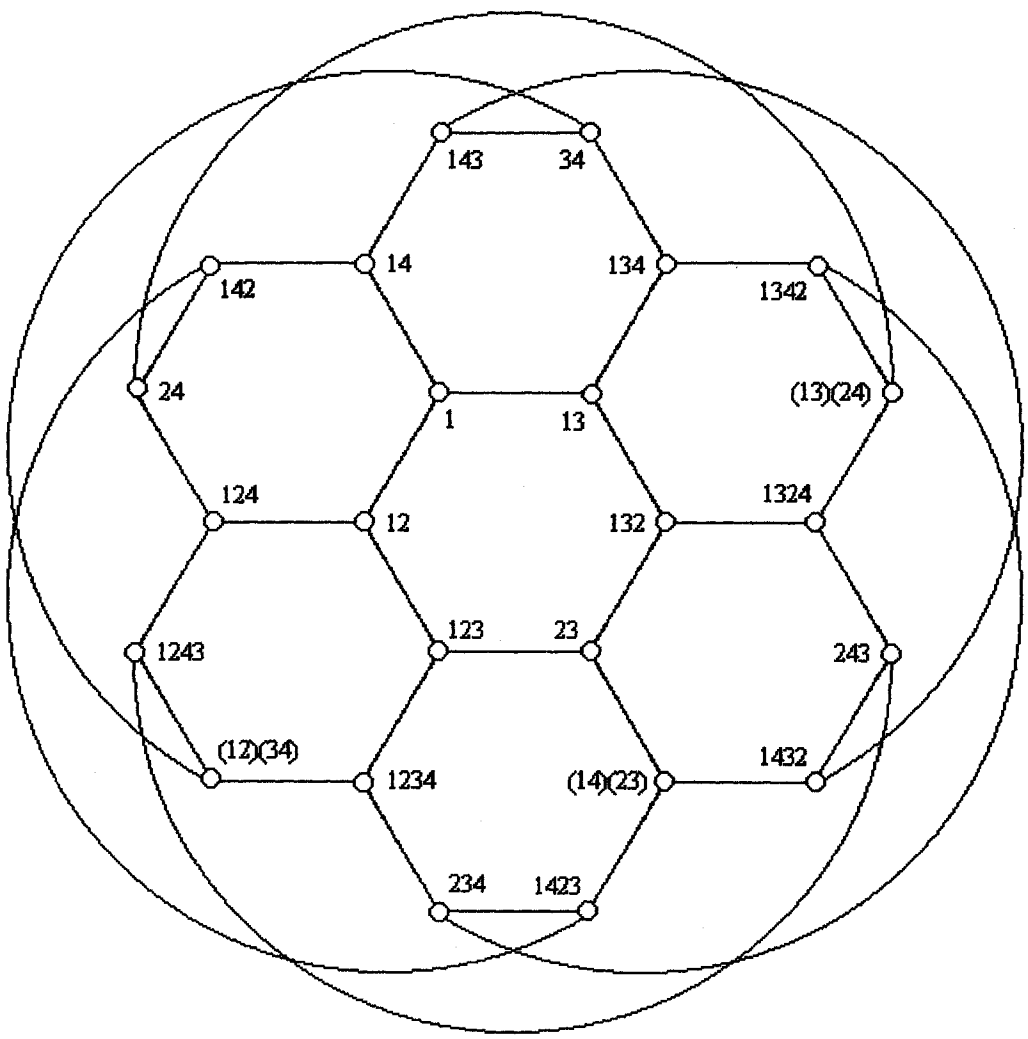


Figure 1.C The Cayley graph of S_4 where $\Delta=\{(12),(13),(14)\}$

Conjugacy Graphs

Now begins our discussion on the concept of conjugacy graphs which was introduced and studied in [3]. For a finite group Γ, let X be a subset of Γ where $X^{-1}=X$ and $1 \notin X$. The conjugacy graph Con(Γ; X) has vertex set Γ and two vertices $g, h \in \Gamma$ are adjacent if and only if there exists $x \in X$ with $g=xhx^{-1}$. If X generates Γ, then the components of Con(Γ; X) partition the vertices into the conjugacy classes of Γ. Recall that the conjugacy class for an element $g \in \Gamma$ and is defined as cl(g)=$\{xgx^{-1} : x \in \Gamma\}$ and is denoted cl(g). In general the components partition the vertices, or elements of Γ, into "conjugacy classes" where we are only allowed to do conjugation by elements of $\langle X \rangle$, the subgroup of Γ generated by X. We provide the following example to illustrate a conjugacy graph.

Example 1.4 Let Γ be the group S_3 and let $X=\{(12),(123),(132)\}$. To find Con(Γ; X) we perform the following table of calculations, where in column I each element of S_3 is conjugated by (12), in column II each element of S_3 is conjugated by (123) and in column III each element of S_3 is conjugated by (132).

Column I	Column II	Column III
(12)(1)(12)=(1)	(123)(1)(132)=(1)	(132)(1)(123)=(1)
(12) (12)(12)=(12)	(123)(12)(132)=(13)	(132)(12)(123)=(23)
(12) (13)(12)=(23)	(123)(13)(132)=(23)	(132)(13)(123)=(12)
(12) (23)(12)=(13)	(123)(23)(132)=(12)	(132)(23)(123)=(13)
(12) (123)(12)=(132)	(123)(123)(132)=(123)	(132)(123)(123)=(123)

(12) (132)(12)=(123)   (123)(132)(132)=(132)   (132)(132)(123)=(132)

Since X generates S_3, we see that the conjugacy classes of S_3 are $\{(1)\},\{(12),(13),(23)\}$ and $\{(123),(132)\}$ and these sets partition the vertices of the conjugacy graph into components, which can be seen in Figure 1.D.

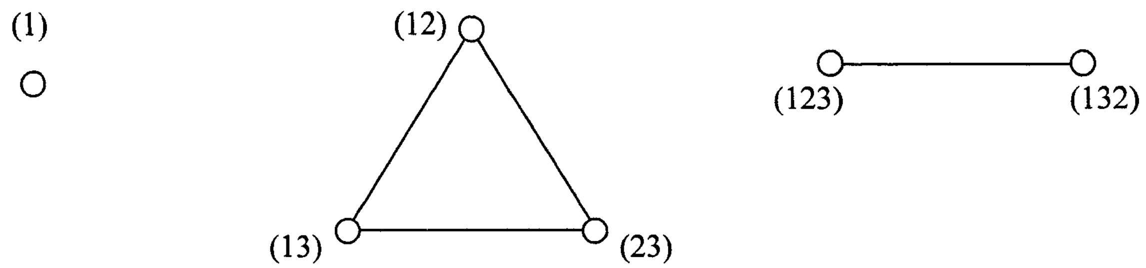


Figure 1.D The conjugacy graph of S_3 where $X=\{(12),(123),(132)\}$

Example 1.5 Let Γ be the group S_4 and let $X=\{(12),(13),(14)\}$. To find Con(S_4; X) we perform calculations similar to Example 1.4, each element of S_4 is conjugated by each element of X. Again X generates S_4. It is known that two permutations are in the same conjugacy class if and only if they have the same cycle structure [11]. Therefore the conjugacy classes of S_4 are $\{(1)\}$, $\{(12),(13),(14),(23),(24),(34)\}$, $\{(12)(34),(13)(24),(14)(23)\}$, $\{(123),(124),(132),(134),(142),(143),(234),(243)\}$ and $\{(1234),(1243),(1324),(1342),(1423),(1432)\}$. See Figure 1.E for Con(S_4; X).

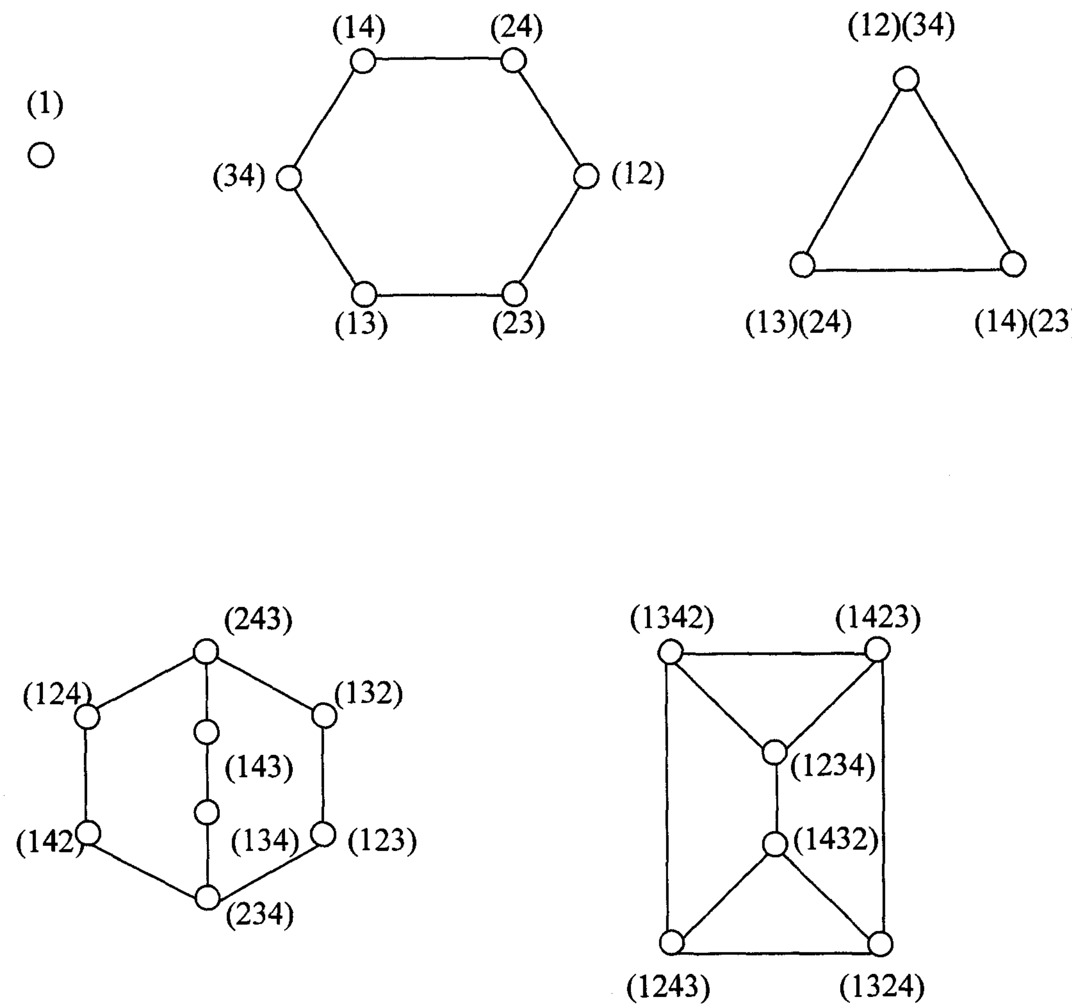


Figure 1.E The conjugacy graph of S_4 where $X = \{(12), (13), (14)\}$

CHAPTER 2

CONJUGACY NUMBERS AND CONJUGACY GROUPS

We move to the notion of a conjugacy number, which was first introduced and studied in [3]. Let Ω be a group with generating set Δ. The *conjugacy number* $\mathrm{con}(\Omega;\Delta)$ is defined as the minimum integer $k\geq 2$ for which there exists a nonabelian group Γ of order $k|\Omega|$ and a subset X of Γ such that $\mathrm{Cay}(\Omega;\Delta)$ is isomorphic to a component of $\mathrm{Con}(\Gamma; X)$. Such a group Γ is called a *conjugacy group* for Ω and Δ. For a better understanding of where this conjugacy number comes from we must provide the following information, which is discussed in [3]. Let Γ be a group and let X be a generating set such that $X^{-1} = X$ and let Ω be a subgroup of Γ. The left *Schreier coset digraph* $\overline{S}(\Gamma/\Omega;X)$ is the directed graph whose vertex set is the left cosets of Ω and there is a directed edge from $g\Omega$ to $h\Omega$ if $h\Omega=xg\Omega$ for $x\in X$. Proposition 1 from [3] states that if G is a component of the directed conjugacy graph $\overline{Con}(\Gamma;X)$ and g is a fixed vertex of G then G is isomorphic to $\overline{S}\left(\langle X\rangle/C_{\langle X\rangle}(g);X\right)$, where $C_{\langle X\rangle}(g)$ is the centralizer of g in $\langle X\rangle$. The proof [3] of Proposition 1 defines Ψ: $V(G)\rightarrow V(\overline{S}\left(\langle X\rangle/C_{\langle X\rangle}(g);X\right))$ by $\Psi(zgz^{-1})=zC_{\langle X\rangle}(g)$. To show that Ψ preserves directed edges, two adjacent vertices in G are considered. Let us call these vertices $z_1gz_1^{-1}$ and $z_2gz_2^{-1}$. Since these vertices are adjacent in G there exists $x\in X$ such that

$z_1 g z_1^{-1} = x(z_2 g z_2^{-1})x^{-1}$ which can be written as $z_1 g = x z_2 g z_2^{-1} x^{-1} z_1$ or $z_2^{-1} x^{-1} z_1 g = g z_2^{-1} x^{-1} z_1$. Hence $(z_2^{-1} x^{-1} z_1) g = g(z_2^{-1} x^{-1} z_1)$ and it follows that $z_2^{-1} x^{-1} z_1 \in C_{\langle X \rangle}(g)$. Since $z_2^{-1} x^{-1} z_1 \in C_{\langle X \rangle}(g)$ it follows that $z_1 C_{\langle X \rangle}(g) = x z_2 C_{\langle X \rangle}(g)$. Therefore $z_1 C_{\langle X \rangle}(g)$ is adjacent to $z_2 C_{\langle X \rangle}(g)$ in $\overline{S}\left(\langle X \rangle / C_{\langle X \rangle}(g); X\right)$.

Let Ω be a finite group and Δ be a generating set for Ω with $\Delta^{-1} = \Delta$. Now we consider if there exists a finite group Γ with a subset X such that Cay(Ω; Δ) is isomorphic to G where G is a component of Con(Γ; X). By Proposition 1 of [3] we know that $G \cong \overline{S}\left(\langle X \rangle / C_{\langle X \rangle}(g); X\right)$. For G to be isomorphic to Cay(Ω; Δ) then the order of G must equal the order of Ω. Since $G \cong \overline{S}\left(\langle X \rangle / C_{\langle X \rangle}(g); X\right)$ we know that $|G| = \left|\left(\langle X \rangle / C_{\langle X \rangle}(g)\right)\right|$. Therefore $|\Omega| = |\langle X \rangle| / \left|C_{\langle X \rangle}(g)\right|$. Consider $|\Gamma|$ which can be written as $|\Gamma| = \left|C_{\langle X \rangle}(g)\right| \frac{|\Gamma|}{|\langle X \rangle|} \frac{|\langle X \rangle|}{\left|C_{\langle X \rangle}(g)\right|}$ or $|\Gamma| = \left|C_{\langle X \rangle}(g)\right| \frac{|\Gamma|}{|\langle X \rangle|} |\Omega|$. Since $\left|C_{\langle X \rangle}(g)\right| \frac{|\Gamma|}{|\langle X \rangle|}$ is just an integer we can call it *k*. Hence $|\Gamma| = k|\Omega|$ for some positive integer *k*. Furthermore we can say that $k \geq 2$ since the identity of Γ is always an independent vertex of Con(Γ; X). Now we consider an example.

Example 2.1 Let Ω be C_4 with $\Delta = \{x, x^2, x^3\}$. We will verify that con(C_4; Δ)=3 and a conjugacy group for C_4 and Δ is A_4. First we show that con(C_4; Δ)≠2. For con(C_4; Δ)=2 there must exist a nonabelian group Γ such that $|\Gamma| = 2|\Omega| = 8$ where Γ has a conjugacy class of order four. There are two nonabelian groups of order eight, namely the dihedral group D_4 and the quaternion group Q. It will be shown later that D_4 has conjugacy classes of order one and two. We consider Q with presentation $\langle s,t; s^4 = 1, s^2 = t^2, sts = t \rangle$ and note that it is not difficult to show that its center is $\{1, s^2\}$ [8]. Through straightforward calculations it can be shown that the conjugacy classes of Q are $\{1\}, \{s, s^3\}, \{s^2\}, \{t, s^2t\}$ and $\{st, s^3t\}$. Therefore there does not exist a Γ of order eight with a conjugacy class of order four. Hence con(C_4; Δ) $\neq 2$.

Next we show that con(C_4; Δ)=3. By the calculations of Example 1.2 and including multiplying C_4 by x^2 on the right we get the Cay(C_4; Δ) as shown in Figure 2.A.

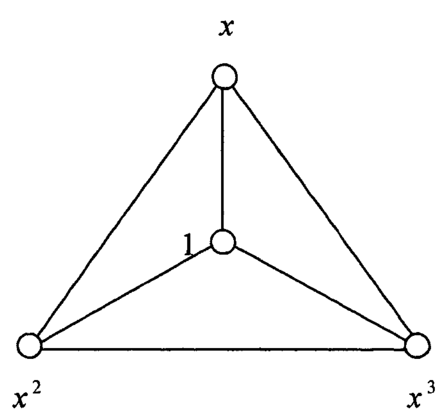


Figure 2.A The Cayley graph of C_4 where $\Delta = \{x, x^2, x^3\}$

Let Γ be A_4 with $X = \{(12)(34), (13)(24), (14)(23)\}$. We conjugate each element of A_4 by each element of X. We find that when conjugating by elements of $\langle X \rangle$ the elements of A_4 are partitioned into the following "conjugacy classes": $\{(1)\}$,

$\{(12)(34),(13)(24),(14)(23)\}$, $\{(123),(134),(142),(243)\}$ and $\{(124),(132),(143),(234)\}$.

The conjugacy class containing (123) gives the graph shown in Figure 2.B.

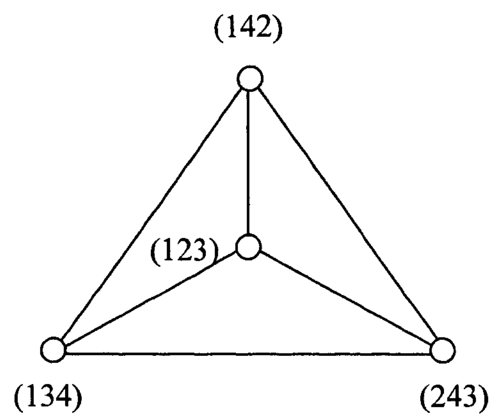


Figure 2.B The component of $\mathrm{Con}(A_4;X)$ containing cl((123))

Clearly $\mathrm{Cay}(C_4;\ \Delta)$ is isomorphic to a component of $\mathrm{Con}(A_4;\ \mathrm{X})$ therefore we can find $\mathrm{con}(C_4;\Delta)$. Using $|\Gamma|=k|\Omega|$ where $|\Gamma|=12$ and $|\Omega|=4$, we get that k=3. Hence $\mathrm{con}(C_4;\Delta)=3$ and a conjugacy group for C_4 and Δ is A_4.

CHAPTER 3

CONJUGACY NUMBERS FOR CYCLIC GROUPS OF SMALL EVEN ORDER

We will calculate con($C_n;\Delta$) for small even n. We must first discuss the following results and observations. The first is well known and covered in an elementary abstract algebra course. We include its proof for aid in understanding the concepts.

Proposition 3.1 Let $a, b \in G$ where G is a group. If $a \neq b$ then either $cl(a)=cl(b)$ or $cl(a) \cap cl(b)=\varnothing$.

Proof. Assume that $a \neq b$ and $cl(a) \cap cl(b) \neq \varnothing$. Then there exists $c \in cl(a)$ and $c \in cl(b)$. Consider $cl(a)=\{xax^{-1} : x \in G\}$. Since $c \in cl(a)$ there exists $g \in G$ such that $gag^{-1}=c$. Since $c \in cl(b)$ there exists $h \in G$ such that $hbh^{-1}=c$. Let $y \in cl(\mathrm{a})$. Then there exists $t \in G$ such that $tat^{-1}=y$. Since $a = g^{-1}hbh^{-1}g$ we get $y = tg^{-1}hbh^{-1}gt^{-1}$ = $(tg^{-1}h)b(tg^{-1}h)^{-1}$. Therefore $y \in cl(b)$ and $cl(\mathrm{a}) \subseteq cl(b)$. Let $y \in cl(b)$. Then there exists $t \in G$ such that $tbt^{-1}=y$. Since $b = h^{-1}gag^{-1}h$ we get $y = th^{-1}gag^{-1}ht^{-1}$ = $(th^{-1}g)a(th^{-1}g)^{-1}$. Therefore $y \in cl(a)$ and $cl(\mathrm{b}) \subseteq cl(a)$. Hence $cl(a)=cl(b)$. $\blacksquare$

Next we present a useful result for the dihedral groups. For an integer $n \geq 3$, define the dihedral group of order $2n$ by the presentation $D_n=\langle x, y : x^n = y^2 = 1, xyx = y\rangle$. Thus the elements of D_n are $1, x, x^2, \ldots, x^{n-1}, y, xy, x^2y, \ldots, x^{n-1}y$.

Proposition 3.2 The conjugacy classes of D_n are $\{1\},\{x^k,x^{-k}\}$ for k=1,2,…,n-1 and $\{y,xy,x^2y,...,x^{n-1}y\}$ when n is odd and the conjugacy classes of D_n are $\{1\},\{x^k,x^{-k}\}$ for k=1,2,…,n-1 , $\{y,x^2y,x^4y,...,x^{n-2}y\}$ and $\{xy,x^3y,x^5y,...,x^{n-1}y\}$ when n is even.

Proof. Consider $\left(x^iy^j\right)x^k\left(x^iy^j\right)^{-1}$ for $0\le i\le n-1$, $0\le j\le 1$ and $0\le k\le n-1$. Observe that

$$\left(x^iy^j\right)x^k\left(x^iy^j\right)^{-1}=(x^iy^j)x^k(y^{-j}x^{-i})=\begin{cases}x^ix^kx^{-i}, j=0\\ x^ix^{-k}yy^{-1}x^{-i}, j=1\end{cases}=\begin{cases}x^k, j=0\\ x^{-k}, j=1\end{cases}.$$

Therefore the conjugacy class containing x^k is $\{x^k,x^{-k}\}$ for all $0\le k\le n-1$.

Now consider $\left(x^iy^j\right)x^ky\left(x^iy^j\right)^{-1}$ for $0\le i\le n-1$, $0\le j\le 1$ and $0\le k\le n-1$. Observe that

$$\left(x^iy^j\right)x^ky\left(x^iy^j\right)^{-1}=(x^iy^j)x^ky(y^{-j}x^{-i})=\begin{cases}x^ix^kyx^{-i}, j=0\\ x^iyx^kx^{-i}, j=1\end{cases}=\begin{cases}x^ix^kx^iy, j=0\\ x^ix^{-k}yx^{-i}, j=1\end{cases}=$$

$$\begin{cases}x^{2i+k}y, j=0\\ x^{2i-k}y, j=1\end{cases}.$$

For k=0 we get $cl(y)=\{x^{2i}y|0\le i\le n-1\}$. For k=1 we get $cl(xy)=\{x^{2j+1}y|0\le j\le n-1\}$ U $\{x^{2j-1}|0\le j\le n-1\}=\{x^{2j+1}y|-1\le j\le n-1\}$.

Case 1. Let n be even. We claim that $cl(y)$ I $cl(xy)=\varnothing$. Assume, to the contrary, that $cl(y)$ I $cl(xy)\neq\varnothing$. Let $z\in cl(y)$ and $z\in cl(xy)$. For $z\in cl(y)$ we get $z=x^{2i}y$ for some i such that $0\le i\le n-1$. For $z\in cl(xy)$ we get $z=x^{2j+1}y$ for some j with $-1\le j\le n-1$. Then $x^{2i}y=x^{2j+1}y$ or $x^{2i-2j-1}=1$ or $x^{2(i-j)-1}=1$. This implies that $n|[2(i-j)-1]$. This is clearly a contradiction since n is even. Therefore $cl(y)$ I $cl(xy)=\varnothing$.

Case 2. Let n be odd. We claim that $cl(y)=cl(xy)$. We show that $xy \in cl(y)$. Since $\gcd(n,2)=1$, there exists an i, $0 \le i \le n-1$ such that $2i \equiv 1 \bmod n$. Thus $xy=x^{2i}y$ and $x^{2i}y \in cl(y)$. Since $xy=x^{2i}y \in cl(y)$ then $cl(y)=cl(xy)$.

Hence the conjugacy classes of D_n when n is even will be $\{1\},\{x^k,x^{-k}\}\ k=1,2,\ldots,n-1,\{y,x^2y,x^4y,\ldots,x^{n-2}y\}$ and $\{xy,x^3y,x^5y,\ldots,x^{n-1}y\}$ and when n is odd the conjugacy class of D_n will be $\{1\},\{x^k,x^{-k}\}$ k=1,2,…,n-1 and $\{y,xy,x^2y,x^3y,x^4y,\ldots,x^{n-2}y,x^{n-1}y\}$. W

We can say that the conjugacy classes of D_n where n is even will have orders 1, 2 and $\frac{n}{2}$ and the conjugacy classes of D_n where n is odd will have orders 1, 2 and n.

We now present a useful result for the cross product of an abelian group with a nonabelian group.

Proposition 3.3 Let G be a nonabelian group and let H be an abelian group. For $(g,h) \in G\times H$ we get $|cl(g,h)|=|cl(g)|$.

Proof. We show that $cl(g,h)=cl(g)\times\{h\}$. Since H is abelian observe that $(x,y)(g,h)(x^{-1},y^{-1})=(xyx^{-1},h)$ for all $y \in H$. First let $(x,y) \in cl(g,h)$. Then there exists $(a,b) \in G\times H$ such that $(a,b)(g,h)(a,b)^{-1}=(x,y)$ but $(a,b)(g,h)(a,b)^{-1}=(a,b)(g,h)(a^{-1},b^{-1})=(aga^{-1},bhb^{-1})=(aga^{-1},h)$. Since $aga^{-1} \in cl(g)$ we see that $(x,y) \in cl(g)\times\{h\}$. Now let $(x,y) \in cl(g)\times\{h\}$. Then $x \in cl(g)$ and $y=h$

and there exists an $a \in G$ such that $aga^{-1}=x$. Now $(x,y)=(aga^{-1},h)=(a,1)(g,h)(a,1)^{-1}$ and so $(x,y) \in cl(g,h)$. Then $cl(g,h)=cl(g) \times \{h\}$. Therefore $|cl(g,h)|=|cl(g)|$.W

Next we provide a useful result that was stated and proved in [3]. We include the proof here for better understanding. We remind the reader that the conjugacy number con(Ω; Δ) is defined as the minimum integer $k \geq 2$ for which there exists a nonabelian group Γ of order $k|\Omega|$ and a subset X of Γ such that Cay(Ω; Δ) is isomorphic to a component of Con(Γ; X).

Proposition 3.4 Let $n \geq 6$ be an even integer. Then con(C_n;Δ)≤ 4 for every generating set Δ of C_n.

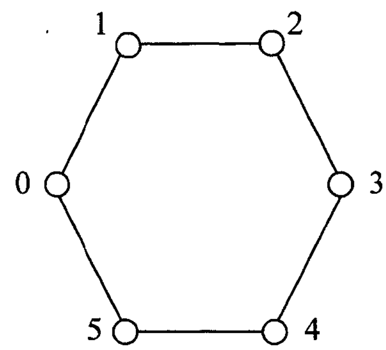
Proof. We must show that there exists a nonabelian group Γ of order $4n$ and a subset X of Γ for which Cay(Ω; Δ) is isomorphic to a component of Con(Γ; X). Let Γ be the group D_{2n} and let $X=\{x^k \mid k \in \Delta\}$. Let G be the component of Con(D_{2n}; X) containing y. We know from Proposition 3.2 that the vertices of G are $\{x^{2i}y \mid 0 \leq i \leq n-1\}$. We must show that G is isomorphic to Cay(C_n; Δ). Define a function Ψ : V(Cay(C_n;Δ))$\rightarrow V(G)$ by $\Psi(i)=x^{2i}y$. Suppose that i is adjacent to j in Cay(C_n; Δ). Therefore there exists $k \in \Delta$ such that $j \equiv (i+k) \bmod n$. Since $j \equiv (i+k) \bmod n$ we know $2j \equiv 2(i+k) \bmod 2n$ and $x^{2j}y = x^{2(i+k)}y$ in D_{2n}. Consider $x^{2(i+k)}y = x^{k+2i+k}y = x^k x^{2i} x^k y = x^k(x^{2i}y)x^{-k}$. Hence $x^{2j}y = x^k(x^{2i}y)x^{-k}$ and it follows that $x^{2i}y$ is adjacent to $x^{2j}y$ in G and Cay(C_n; Δ)$\cong G$. Note that $|\Gamma| = 4n = 4|\Omega|$. Since the conjugacy number is defined as the minimum integer k for which such a group Γ of order $k|\Omega|$ exists, we can conclude that con(C_n; Δ) ≤ 4 for an even integer $n \geq 6$. W

We now calculate con(C_n;Δ) for small even n. By Proposition 3.4 we know that con(C_n;Δ) ≤ 4 for even $n \geq 6$. In the following results we consider con(C_n;Δ) for $n = 6,8$ and 10. We begin with $n = 6$.

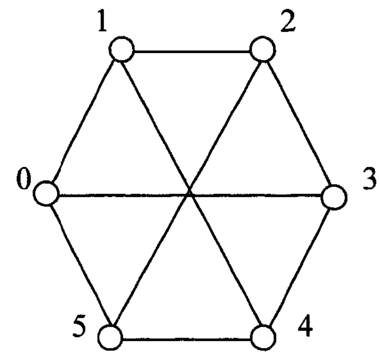
Proposition 3.5 The conjugacy number con(C_6;Δ)=3 if and only if $\Delta = \{2,3,4\}$ or $\Delta = \{1,2,4,5\}$ or $\Delta = \{1,2,3,4,5\}$.

Proof. Figure 3.A shows three possible Cayley graphs for C_6. If con(C_6;Δ)=2, then there is a nonabelian group Γ with $|\Gamma|$=12 and a subset X of Γ such that a component of the conjugacy graph Con(Γ; X) is isomorphic to the Cayley graph Cay(C_6; Δ). Note that Γ must contain a conjugacy class containing at least 6 elements. There are three nonabelian groups of order 12, namely A_4, D_6 and T, where T is the semidirect product of C_3 by C_4 [5]. For T we use the presentation $\langle s,t; s^6 = 1, s^3 = t^2, sts = t \rangle$ [10]. Through straightforward calculations, we find that A_4 has conjugacy classes of orders 1, 3 and 4. By Proposition 3.2 we know that D_6 has conjugacy classes of orders 1, 2 and 3. We can also show that T has conjugacy classes of orders 1, 2 and 3. Therefore we conclude that (C_6;Δ) $\neq 2$. We now check nonabelian groups of order 18 to verify con(C_6;Δ)=3 for a given Δ.

Cay(C_6 ; Δ) with $\Delta=\{1,5\}$



Cay(C_6 ; Δ) with $\Delta=\{1,3,5\}$



Cay(C_6 ; Δ) with $\Delta=\{2,3,4\}$

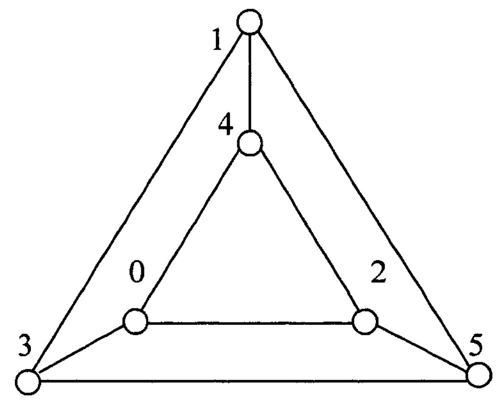


Figure 3.A Three possible Cayley graphs for C_6

The nonabelian groups of order 18 are D_9, $S_3 \times C_3$ and the semidirect product of $C_3 \times C_3$ and C_2 [5]. We check these groups for conjugacy classes of order at least 6. By

Proposition 3.2 we know that D_9 has conjugacy classes of orders 1, 2 and 9 and the conjugacy class of order 9 contains the elements $\{y, xy, x^2y, \ldots, x^8y\}$. It can be shown through tedious calculation that when the elements of cl(y) are conjugated by any x^i it produces a 9-cycle which clearly cannot give a Cayley graph on six vertices. In addition it can be shown that when the elements of cl(y) are conjugated by any $x^i y$ the result is one independent vertex and four independent edges. Finally it can be shown that if the elements of cl(y) are conjugated by any two $x^i y$ the result is a path on nine vertices and it is again impossible to obtain a Cayley graph on six vertices. Therefore D_9 is not a conjugacy group for C_6. Next we look at $S_3 \times C_3$. Since C_3 is abelian we know by Proposition 3.3 that the orders of the conjugacy classes of $S_3 \times C_3$ will be the orders of the conjugacy classes found in S_3. Since S_3 has order 6 and the identity is always in its own conjugacy class then S_3 cannot have a conjugacy class of order 6. Therefore, by Proposition 3.3, $S_3 \times C_3$ cannot have a conjugacy class of order 6.

Finally we consider the semidirect product of $C_3 \times C_3$ and C_2. We use $G = \langle x, y, z : x^2 = y^3 = z^3 = 1, yz = zy, yxy = x, zxz = x \rangle$ as our presentation for the semidirect product of $C_3 \times C_3$ and C_2 [8]. Then G has the elements $1, x, y, y^2, z, z^2, xy, xy^2, xz, xz^2, yz, yz^2, y^2z, y^2z^2, xyz, xyz^2, xy^2z, xy^2z^2$. It can be shown that the conjugacy classes of G are $\{1\}$, $\{y, y^2\}$, $\{z, z^2\}$, $\{yz, y^2z^2\}$, $\{yz^2, y^2z\}$ and $\{x, xy, xy^2, xz, xz^2, xyz, xyz^2, xy^2z, xy^2z^2\}$. Consider the cl($x$) which has order 9. Let

$X=\{xz, y, y^2\}$. If cl(x) is conjugated by X we obtain the following graph, shown in Figure 3.B

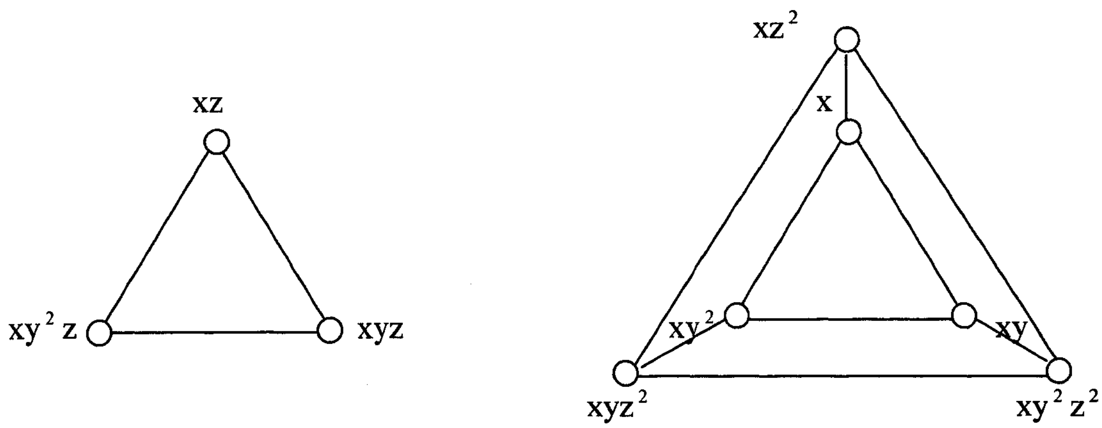


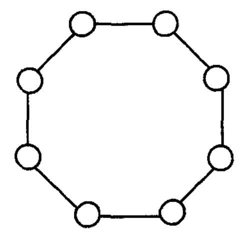
Figure 3.B Components of Con(G; X) containing cl(x)

Clearly the component of Con(G; X) containing cl(x), shown in Figure 3.B, has a component which is isomorphic to the Cayley graph Cay(C_6; Δ), where $\Delta=\{2,3,4\}$, shown in Figure 3.A. For $\Delta=\{1,2,4,5\}$ and for $\Delta=\{1,2,3,4,5\}$ let $X=\{xz, y, y^2, xyz\}$ and let $X=\{xz, y, y^2, xyz, xy^2z\}$, respectively, to obtain con(C_6; Δ)=3. Therefore the semidirect product of $C_3 \times C_3$ and C_2 is the unique conjugacy group of C_6 and we can conclude that con(C_6; Δ)=3 for $\Delta=\{2,3,4\}$, $\Delta=\{1,2,4,5\}$ and $\Delta=\{1,2,3,4,5\}$. There are two remaining generating sets, namely $\{1,5\}$ and $\{1,3,5\}$. It is not difficult to show that there is no subset X of G for which a component of Con(G; X) is isomorphic to Cay(C_6; Δ), where $\Delta=\{1,5\}$ or $\Delta=\{1,3,5\}$. W

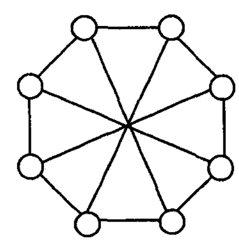
Proposition 3.6 The conjugacy number con(C_8;Δ)=3 if and only if $\Delta=\{1,2,6,7\}$ or $\Delta=\{1,3,5,7\}$ or $\Delta=\{2,3,5,6\}$ or $\Delta=\{1,2,4,6,7\}$ or $\Delta=\{1,3,4,5,7\}$ or $\Delta=\{1,2,3,5,6,7\}$ or $\Delta=\{1,2,3,4,5,6,7\}$.

Proof. Figure 3.C shows three possible Cayley graphs for C_8.

Cay(C_8 ; Δ) with Δ={1,7} or Δ={3,5}



Cay(C_8 ; Δ) with Δ={1,4,7} or Δ={3,4,5}



Cay(C_8 ; Δ) with Δ={1,3,5,7}

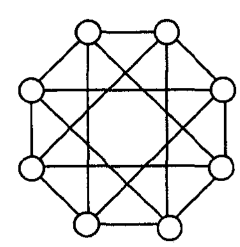


Figure 3.C Three possible Cayley graphs for C_8

If con(C_8; Δ)=2, then there exists a nonabelian group Γ of order 16 which contains a conjugacy class of order at least 8 and a subset X of Γ such that a component of the conjugacy graph Con(Γ; X) is isomorphic to the Cayley graph Cay(C_8; Δ). There are nine nonabelian groups of order 16 [5]. We begin with D_8. We know that D_8 has conjugacy classes of order 1, 2 and 4. Therefore the conjugacy graph of D_8 does not contain a component which is isomorphic to a Cayley graph of C_8.

Next we consider $D_4 \times C_2$ and $Q \times C_2$. Notice that C_2 is an abelian group therefore $D_4 \times C_2$ and $Q \times C_2$ will have conjugacy classes with orders as found in D_4 and Q, respectively. We know that D_4 has conjugacy classes of orders 1 and 2. Therefore $D_4 \times C_2$ does not contain a conjugacy class of order 8. The quaternion group, Q has order 8 and therefore it cannot contain a conjugacy class of order 8.

We now look at the quasihedral group of order 16 with presentation $\langle s, t; s^8 = t^2 = 1, st = ts^3 \rangle$ [8]. It can be shown through straightforward calculations that the conjugacy classes of this group are $\{1\}, \{s, s^3\}, \{s^2, s^6\}, \{s^4\}, \{s^5, s^7\}, \{t, s^2t, s^4t, s^6t\}$ and $\{st, s^3t, s^5t, s^7t\}$. Clearly there are no conjugacy classes of order 8.

Now we look at the modular group of order 16 with presentation $\langle s, t; s^8 = t^2 = 1, st = ts^5 \rangle$ [8]. It can be shown that the conjugacy classes of this group are $\{1\}, \{s, s^5\}, \{s^2\}, \{s^3, s^7\}, \{s^4\}, \{s^6\}, \{t, s^4t\}, \{st, s^5t\}, \{s^2t, s^6t\}$ and $\{s^3t, s^7t\}$. Therefore there is no conjugacy class of order 8.

Next we look at three of the remaining four nonabelian groups of order 16 with the presentations $\langle s,t; s^4=t^4=1, st=ts^3\rangle$, $\langle s,t; s^4=t^4=1, stst=1, ts^3=st^3\rangle$ and $\langle a,b,c; a^4=b^2=c^2=1, cbca^2b=1, bab=a, cac=a\rangle$ [8]. It can be shown that these groups only have conjugacy classes of orders 1 and 2.

Finally we look at the group with presentation $\langle s,t; s^8=1, s^4=t^2, sts=t\rangle$ [8]. The conjugacy classes of this group are $\{1\},\{t,s^2t,s^4t,s^6t\},\{st,s^3t,s^5t,s^7t\}$, $\{s,s^7\}$, $\{s^2,s^6\}$, $\{s^3,s^5\}$ and $\{s^4\}$. Hence there are no conjugacy classes of order 8. We conclude that con($C_8;\Delta$)$\neq 2$.

Next we show that con($C_8;\Delta$)=3 for a given Δ. To verify that con($C_8;\Delta$)=3, we must show that there exists a nonabelian group of order 24 with a conjugacy class of order at least 8 with a component of the conjugacy graph Con(Γ; X) isomorphic to the Cayley graph Cay(C_8; Δ). There are 12 nonabelian groups of order 24, we show that S_4 works. The conjugacy classes of S_4 are $\{(1)\},\{(12),(13),(14),(23),(24),(34)\}$, $\{(12)(34),(13)(24),(14)(23)\}$, $\{(123),(124),(132),(134),(142),(143),(234),(243)\}$ and$\{$(1234),(1243),(1324),(1342),(1423),(1432) $\}$. We have a conjugacy class of order 8. We must verify that there does exist an X which will give a component of Con(S_4; X) that is isomorphic to Cay(C_8; Δ). Let X=$\{(12)(34),(14)(23),(1234),(1432)\}$. The component of the conjugacy graph, Con(S_4; X), containing the element (123) is shown in Figure 3.D.

Clearly Con(S_4 ; X) has a component which is isomorphic to the Cayley graph of C_8 where $\Delta=\{1,3,5,7\}$ which is shown in Figure 3.C. Therefore con($C_8;\Delta$)=3 for $\Delta=\{1,3,5,7\}$. For $\Delta=\{1,2,6,7\}$ and $\Delta=\{2,3,5,6\}$ let $X=\{(24),(12)(34),(1234),$ $(1432)\}$. For $\Delta=\{1,2,4,6,7\}$ and $\Delta=\{2,3,4,5,6\}$ let $X=\{(24),(12)(34),(13)(24),(1234),$ $(1432)\}$. For $\Delta=\{1,3,4,5,7\}$ let $X=\{(24),(13)(24),(1234),(1432)\}$. For $\Delta=$ $\{1,2,3,5,6,7\}$ let $X\{(13),(24),(12)(34),(14)(23),(1234),(1432)\}$. For $\Delta=$ $\{1,2,3,4,5,6,$ $7\}$ let $X=\{(13),(24),(12)(34),(13)(24),(14)(23),(1234),(1432)\}$. Therefore we conclude that con(C_8 ; Δ)=3 for the given Δ and a conjugacy group for C_8 and for the given Δ is S_4. There are four remaining generating sets, namely $\Delta=\{1,7\}$, $\Delta=\{3,5\}$, $\Delta=\{1,4,7\}$ and $\Delta=\{3,4,5\}$. For these generating sets it is not difficult to show that there does not exist a subset X of S_4 for which a component of Con(S_4 ; X) is isomorphic to Cay(C_8 ; Δ). For thoroughness we check the remaining 11 nonabelian groups of order 24 and show that none of these satisfy the required conditions [5].

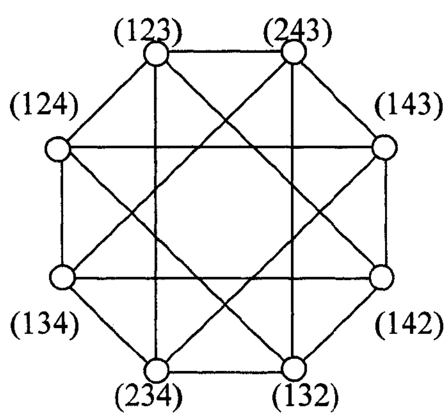


Figure 3.D The component of Con(S_4 ; X) containing cl((123))

For the groups $S_3 \times C_4$ and $S_3 \times C_2 \times C_2$, we observe that since C_4 and $C_2 \times C_2$ are abelian we know by Proposition 3.3 that both groups will have the conjugacy classes as found in S_3. Since S_3 has order 6 a conjugacy class of order 8 is impossible.

Now we look at $A_4 \times C_2$. Since C_2 is abelian we consider the conjugacy classes of A_4. We know A_4 has conjugacy classes of orders 1, 3 and 4. Therefore $A_4 \times C_2$ cannot have a conjugacy class of order 8. Similarly we know that $T \times C_2$ will have conjugacy classes of orders 1, 2 and 3.

Next we consider $D_4 \times C_3$ and $Q \times C_3$. Since C_3 is abelian we look at D_4 and Q which both have order 8 and therefore cannot contain a conjugacy class of order 8. The remaining five groups are found by taking the direct product of any group of order 8 and a group of order 3 [3]. Since there are no nonabelian groups of order 3 and any group of order 8 will not have a conjugacy class of order 8 we conclude that these groups will not give a conjugacy number of three. Therefore we conclude that con($C_8;\Delta$)=3 with the given Δ and the unique Γ which gives con($C_8;\Delta$)=3 is the permutation group S_4. W

Proposition 3.7 The conjugacy number con($C_{10};\Delta$)=4 for every generating set Δ.

Proof. To see that con($C_{10};\Delta$)$\neq 2$, we note that if con($C_{10};\Delta$)=2 then there exists a nonabelian group of order 20 with a conjugacy class of order 10 those graph is isomorphic to the Cayley graph of C_{10}. There are three nonabelian groups to check [5]. We know that D_{10} has conjugacy classes of orders 1, 2 and 5. It can be shown that the semidirect product of C_5 and C_4 has conjugacy classes of orders 1, 2 and 5. Finally we consider the Frobenius of order 20 with presentation $\langle s,t : s^4 = t^5 = 1, ts = st^2 \rangle$ [8]. It can also be shown that the Frobenius group of order 20 has conjugacy classes of orders 1, 4 and 5. Therefore con(C_{10}; Δ) $\neq$ 2.

Now we show that con(C_{10}; Δ)$\neq$3. If con(C_{10}; Δ)=3, then there exists a nonabelian group of order 30 with a conjugacy class of order 10. There are three nonabelian groups of order 30 [5]. We know that D_{15} has conjugacy classes of orders 1, 2 and 15. Similar to the conjugacy class of order 9 of D_9, we know that the conjugacy class of order 15 of D_{15} contains the elements $\{y, xy, x^2y, ..., x^{14}y\}$. It can be shown through tedious calculation that when the elements of cl(y) are conjugated by any x^i it produces a 15-cycle which clearly cannot give a Cayley graph on ten vertices. In addition it can be shown that when the elements of cl(y) are conjugated by any $x^i y$ the result is one independent vertex and seven independent edges. Finally it can be shown that if the elements of cl(y) are conjugated by any two $x^i y$ the result is a path on 15 vertices and is again impossible to obtain a Cayley graph on ten vertices. Therefore D_{15} is not a conjugacy group for C_{10}. Then we check $D_5 \times C_3$ and $D_3 \times C_5$. The group $D_5 \times C_3$ will have conjugacy classes as found in D_5 and $D_3 \times C_5$ will have conjugacy classes as found in D_3, neither of which contains a conjugacy class of order 10. Therefore con(C_{10}; Δ) $\neq$ 3, and by Proposition 3.4 we can conclude that con(C_{10}; Δ)=4. W

CHAPTER 4

GROUPS THAT CANNOT BE CONJUGACY GROUPS FOR CYCLIC GROUPS OF EVEN ORDER

Now we eliminate certain groups of order $4n$ as possible conjugacy groups for C_{2n}. The following results will show that the dihedral group D_{2n}, any group of order $4n$ which is a cross product of an abelian group with a nonabelian group, the symmetric group of order $4n$ as well as the alternating groups of order $4n$ are not conjugacy groups for C_{2n}. We begin with the dihedral groups, D_{2n}.

Proposition 4.1 Let Γ be the dihedral group, D_{2n}, $n \geq 3$ with generating set X and let Ω be the group C_{2n} with generating set Δ. Then Con(D_{2n}; X) does not contain a component which is isomorphic to Cay(C_{2n}; Δ).

Proof. Assume, to the contrary, that Con(D_{2n}; X) does contain a component which is isomorphic to Cay(C_{2n}; Δ). Then con(C_{2n}; Δ)=2 and D_{2n} contains a conjugacy class of order $2n$. This is a contradiction since D_{2n} has conjugacy classes of order 1,2 and n. Hence Con(D_{2n}; X) does not contain a component which is isomorphic to Cay(C_{2n}; Δ). W

Furthermore we can conclude that D_{2n} is not a conjugacy group for C_{2n} which will give a conjugacy number of two. Now we consider groups that are the cross product of an abelian and nonabelian group. We present the following result.

Proposition 4.2 Let $n \geq 3$ be an integer. Let Γ be a group of order $4n$ such that Γ is isomorphic to $G \times H$, where G is a nonabelian group and H is a nontrivial abelian group. Then Γ is not a conjugacy group for C_{2n},.

Proof. Assume, to the contrary, that Γ is a conjugacy group for C_{2n}. Then the conjugacy number of C_{2n} is two and Γ contains a conjugacy class of order at least $2n$. By Proposition 3.3 we know that Γ will have the conjugacy classes of orders as those found in G. Since $|G| = \frac{4n}{|H|}$, the largest $|G|$ can be occurs when $|H|$=2, and here we get $|G| = \frac{4n}{|H|}$=$2n$. This is a contradiction since if $|G| = 2n$, it is impossible for G and hence for Γ, to contain a conjugacy class of order at least $2n$. Thus Γ is not a conjugacy group for C_{2n}. W

Next we will discuss the symmetric groups of order $4n$ but we must first provide the following result from [11]. Let π be a permutation of S_m. A cycle containing k elements is called a *k-cycle*. The *cycle type* of π is an expression of the form $\left(1^{n_1}, 2^{n_2}, \ldots, m^{n_m}\right)$ where n_k is the number of cycles of length k in π. We know that for any group G, where $g \in G$ and $Z_g = \{h \in G : hgh^{-1} = g\}$, that $|cl(g)| = \dfrac{|G|}{|Z_g|}$. Now let $G = S_m$. Proposition 1.1.1 of [6] says that if $m = n_1 + 2n_2 + ... + mn_m$ and $\pi \in S_m$, then $|Z_\pi|$ depends only on $n_1, n_2, ..., n_m$ and $|Z_\pi| = 1^{n_1} n_1! 2^{n_2} n_2! ... m^{n_m} n_m!$. The proof is straightforward. It states that any $h \in Z_\pi$ can either permute the cycles of length i among themselves in $n_i!$ ways or perform a cyclic rotation on each of the individual cycles in i^{n_i} ways. Now the equation $|cl(\pi)| = \dfrac{|G|}{|Z_\pi|}$, can be specialized for the symmetric group S_m to $|cl(\pi)| = \dfrac{m!}{1^{n_1} n_1! 2^{n_2} n_2! ... m^{n_m} n_m!}$ [7], [9]. The following proposition shows that a conjugacy number of two cannot be obtained by considering the symmetric group of order $4n$ as a possible conjugacy group for C_{2n}.

Proposition 4.3 The symmetric group, S_m m>3, of order $4n$ is not a conjugacy group for C_{2n}.

Proof. Assume, to the contrary, that S_m is a conjugacy group for C_{2n}. Then S_m contains a conjugacy class of order at least $2n$. We know that $|S_m| = 4n$ and $|C_{2n}| = 2n$. Since $m! = 4n$ we get that $\frac{1}{2}m! = 2n$ or $|C_{2n}| = \frac{1}{2}m!$. It can be shown that the (m-1)-cycles form the conjugacy class of largest order. We know that $|cl(\pi)| = \dfrac{m!}{1^{n_1} n_1! 2^{n_2} n_2! ... m^{n_m} n_m!}$. For the ($m$-1)-cycles, we get

$|cl(\pi)| = \dfrac{m!}{1^1 1! 2^0 0! \ldots (m-2)^0 0! (m-1)^1 1! m^0 0!} = \dfrac{m!}{m-1} = (m-2)!m$. Therefore the order of the conjugacy class containing the (m-1)-cycles is (m-2)!m. Now, we know m>3 so

m-1>2

$\frac{1}{2}(m-1) > 1$

$\frac{1}{2}(m-1)(m-2)!m > (m-2)!m$

$\frac{1}{2}m! > (m-2)!m$

This is a contradiction. The largest conjugacy class is too small to give the Cayley graph for C_{2n}. Since $\frac{1}{2}m! > (m-2)!m$, S_m cannot contain a conjugacy class of order at least $\frac{1}{2}m!$. W

Finally we consider the alternating group A_m, of order 4n. It will be shown that a conjugacy number of two cannot be obtained by considering the alternating group of order 4n as a possible conjugacy group for C_{2n}. For $m \leq 3$, A_m cannot have order 4n and we can easily verify that A_4 and A_5 do not work. We consider m>5.

Proposition 4.4 Let m>5 be an integer. The alternating group A_m of order 4n is not a conjugacy group for C_{2n}.

Proof. We know that $|A_m| = 4n = \frac{1}{2}m!$ and $|C_{2n}| = 2n = \frac{1}{4}m!$. For A_m to be a conjugacy group for C_{2n}, A_m must contain a conjugacy class of order at least $2n$. We know that the orders of the conjugacy classes of A_m will be less than or equal to the orders of the conjugacy classes as found in S_m. Since the conjugacy classe of largest order in S_m is the conjugacy class containing the (m-1)-cycles, we must show that the order of the (m-1)-cycles is less than the order of C_{2n}. We know that the order of the conjugacy class containing the (m-1)-cycles is (m-2)!m in S_m. We know m>5

so m-1>4

$(m-2)!(m-1)m>(m-2)!4m$

$\frac{1}{4}m!>(m-2)!m.$

Hence it is impossible for A_m to contain a conjugacy class of order greater than or equal to the order of C_{2n} and A_m with order $4n$ is not a conjugacy group for C_{2n}. W

CHAPTER 5

CONCLUDING REMARKS AND OPEN QUESTIONS

The initial research on conjugacy graphs and conjugacy numbers was done in [2]. The primary motivation of [2] was the problem of determining which Cayley graphs can be realized as conjugacy graphs. Our research here began by studying cyclic groups of small even order to find any emerging patterns with what we defined as possible conjugacy groups. As we tested all appropriate possible conjugacy groups for these cyclic groups we noticed that we never found a group which gave a conjugacy number of two. This led us to believe that Proposition 3.4 which was initially proved in [2] could eventually be modified to eliminate two as a possible conjugacy number, leaving three and four as the only possible conjugacy number for cyclic groups of even order. This is left as an open question since there are still classifications of groups to be eliminated. Here we have eliminated the dihedral group D_{2n}, any group which is a cross product of an abelian group with a nonabelian group, the symmetric group as well as the alternating groups, all of order $4n$, as possible conjugacy groups for C_{2n}.

Continued research could discover the pattern for which conjugacy groups gives a conjugacy number of three versus a conjugacy number of four for cyclic groups of even order. Here we only considered conjugacy graphs and conjugacy numbers for cyclic groups of even order. There is still plenty of information awaiting discovery.

BIBLIOGRAPHY

[1] B. Bellobas, *Graph Theory An Introductory Course*, Springer-Verlag, New York, 1979

[2] H. Bowman and M. Schultz, Conjugacy Graphs. *Journal of Combinatorial Mathematics and Combinatorial Computing,* **42** (2002) 167-175

[3] W. Burnside, *Theory of Groups of Finite Order*, Second ed., Dover Publications, Inc, New York, 1955.

[4] J. Dixon and B. Mortimer, *Permutation Groups*, Springer-Verlag, New York, 1996

[5] D. Gorenstein, *Finite Groups,* Harper and Row, New York, 1968

[6] I. Grossman and W. Mangus, *Groups and Their Graphs*, Mathematical Association of America, Washington, 1992

[7] L. C. Hsu and P. J.-S. Shiue, Cycle Indicators and Special Functions, *Annals of Combinatorics,* **5** (2001) 179-196

[8] J. Pedersen, and W. E. Clark, (1994). *A Catalogue of Algebraic Systems*. Retrieved May 5, 2004 from University of South Florida website

[9] J. Riordan, *An Introduction to Combinatorial Analysis*, Wiley, 1958

[10] J. Rotman, *An Introduction to the Theory of Groups*, Fourth ed., New York, 1995

[11] B.Sagan, *The Symmetric Group*, Second ed., Springer-Verlag, New York, 2001

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