Dorsal and pectoral fin control of a biorobotic autonomous underwater vehicle

Mukund Narasimhan
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DORSAL AND PECTORAL FIN CONTROL OF A BIOROBOTIC AUTONOMOUS UNDERWATER VEHICLE

by

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Bachelor of Engineering
University of Madras, Chennai, India
2003

A thesis submitted in partial fulfillment of the requirements for the

Master of Science Degree in Electrical Engineering
Department of Electrical and Computer Engineering
Howard R. Hughes College of Engineering

Graduate College
University of Nevada, Las Vegas
August 2005
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Mukund Narasimhan

Entitled

"Dorsal and Pectoral Fin Control of a Biorobotic Autonomous Underwater Vehicle"

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Examination Committee Chair

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Dean of the Graduate College

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Examination Committee Member

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ABSTRACT

Dorsal and Pectoral Fin Control of a Biorobotic Autonomous Underwater Vehicle

by

Mukund Narasimhan

Dr. Sahjendra N. Singh, Examination Committee Chair
Professor of Electrical and Computer Engineering Department
University of Nevada, Las Vegas

This thesis involves an in-depth research on the maneuvering of bio-robotic autonomous undersea vehicles (BAUVs) using bio-mimetic swimming mechanisms. Motivation was derived from the amazing flexibility and agility the fish inherit with the help of their pectoral and dorsal fins.

In the first part of the thesis, control of BAUVs using dorsal fins is considered. The force produced by the cambering of the dorsal fins is used for control. An indirect adaptive controller is designed for depth tracking along constant trajectories even when the system parameters are not known. Next, for following time-varying trajectories, an adaptive control system for yaw plane control of BAUVs is developed. It is capable of working efficiently even when large uncertainties in the system parameters are present and system nonlinearities are dominant.
In the second part of the thesis, pectoral fin control of BAUVs is considered. The flapping of these oscillating fins provides the necessary force and moment for control. A discrete-time optimal controller for set point (constant path) control and inverse controller for tracking time varying trajectories in the yaw plane are derived. Further, an indirect adaptive control system that can accomplish depth trajectory tracking even when the model parameters are completely unknown is developed.

The performance evaluation of the controllers is done by simulation using matlab/simulink.
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CHAPTER 1

INTRODUCTION

Aquatic animals have a remarkable ability to perform swift and complex maneuvers. Currently, several physico-mechanical designs evolved in fish are inspiring robotic devices for propulsion and maneuvering purposes in AUVs. Such designs are not only expected to match the speed and flexibility of the aquatic animals, but also have the potential to reduce the wake area and achieve noiseless underwater locomotion.

To achieve this objective, it is important to understand the principles of fish locomotion. A brief introduction of the basic swimming mechanisms of fish [1] is provided in the following section.

1.1 Swimming Mechanisms of Fish

Fishes use a variety of fins (dorsal, caudal, pectoral, pelvic fins etc.) for maneuvering and propulsion. Figure 1.1 shows the various fins of a fish. The fishes can be broadly classified into two categories based on how they utilize their fins for their locomotion.

- **Median and/or Paired Fin (MPF) Locomotion**: Swimming mechanisms of aquatic animals that involve the use of their median and pectoral fins belong to this...
category. It is employed at slow speeds, offering great maneuverability and good propulsive efficiency. The pectoral fins are normally used by the fish to provide hydrostatic lift, propulsion, steering and braking. The dorsal fins are used to stabilize the fish and allow rapid changes in direction.

- **Body and/or Caudal fins (BCF) Locomotion**: Fishes that generate thrust by bending their bodies into a backward moving propulsive wave which extends to its caudal fin belong to this category. This mode of locomotion offers great thrust and acceleration.

In this thesis, the objective is to design control systems using fin-like mechanisms to provide excellent maneuvering capabilities to slow speed underwater vehicles. Thus one chooses to utilize swimming mechanisms such as the dorsal and pectoral fins which are referred to as the Median and/or Paired fins.

### 1.2 Related Research Work

Over the past 520 million years, the evolution process has produced a large number of species of fish (nearly 2500) that have a splendid ability to perform swift, complex and intricate maneuvers. Aquatic animals use their superbly streamlined bodies to exploit fluid-mechanical principles for achieving extraordinary propulsion efficiencies, acceleration and maneuverability [2]. Considerable research has been conducted to understand the principles of the swimming mechanisms of these aquatic animals [3]. Fishes are found to use a variety of fins (dorsal, caudal, pectoral, pelvic fins) for maneuvering and propulsion [1]. The dorsal fin of fish displays a diversity of
hydrodynamic functions, from a discrete thrust generating propulsor acting independently from the body, to a stabilizer generating only side forces during maneuvering [4]. Dorsal fins play an active role in generating off-axis forces during maneuvering. Furthermore, enhanced locomotor efficiency may be obtained when the caudal fin intercepts the dorsal fin wake. Biological studies are motivating researchers to design bio-robotic autonomous underwater vehicles (BAUVs) actuated by mechanisms similar to those of the fish [5]. Research has been conducted on fish morphology, locomotion and applications of biologically inspired control surfaces to rigid bodies [5-8]. Studies have also been performed on the use of flapping foil devices to produce propulsive and lifting forces [9-12].

Experiments have been conducted on the measurement and parameterization of forces and moments produced by oscillating fins [6]. Recently, an attempt to describe the hydrodynamics of fish and small underwater vehicles has been made [8,13]. Fin moments such as lead-lag, feathering and flapping are identified as the basic oscillating patterns responsible for producing large lift, side force and thrust which can be used for the control and propulsion of BAUVs [8,14-16]. Forces and moments associated with the fin movements have also been extracted from computational fluid dynamic (CFD) simulations where a number of different fin movement patterns have been considered [13,17-19]. Experimental results and CFD simulations of oscillating pectoral fins indicate that these fins produce periodic forces and moments and the oscillating parameters (the amplitude of oscillation, frequency, bias angle, phase angle, etc.) can be used as control variables for maneuvering BAUVs [8,20,21]. An
analytical representation of the unsteady hydrodynamics of oscillating foil has been obtained using Theodorsen’s theory. Readers may refer to a special issue of IEEE Journal of Oceanic Engineering on biologically inspired science and technology for autonomous underwater vehicles for excellent research review articles [4,22-24].

Considerable research has been done for controlling AUVs using traditional control surfaces [25-27]. Researchers are now focusing their attention on biologically inspired control of BAUVs using fish-like fins. Control of BAUVs using oscillating pectoral fins has been attempted [14-16]. A fuzzy control system has been developed to maneuver a robotic fish equipped with motor driven pectoral fins [16]. Here, it is noted that the application of neural networks and fuzzy controllers help in avoiding the extremely complicated analytical representation of forces and moments [16,17]. In Kato [15], an emphasis on the applicability of mechanical pectoral fins to BAUVs has been made. The design of open-loop and closed-loop discrete-time control systems of a bio-robotic AUV for the set point regulation in the dive plane using optimal control theory has been considered [28]. Recently, a discrete-time inverse controller has been designed to maneuver BAUVs along time varying trajectories in the dive plane using pectoral fins [29]. Here, parameterization of the forces and moments produced by the oscillating pectoral fins is done using computational fluid dynamics (CFD) methods and control is achieved by periodically varying the bias angle. It is noted that, for exact output trajectory control using the inverse controller, the system must be minimum phase. i.e. its zero dynamics must be stable. In [29], it is shown that one can obtain approximate trajectory control of a non-minimum phase system by
constructing a modified minimum phase system by eliminating the unstable zeros of
the original transfer function and then applying the inverse control law. A sliding
mode control system has been designed for the dive plane control of BAUVs in the
presence of surface waves by continuous cambering of dorsal fins [7]. It is noted that
this dorsal fin control system of [7] is discontinuous and uses high gain feedback for
compensating the uncertainties. It is well known that discontinuous control systems
can cause undesirable chattering phenomenon.

Although plenty of research is available in literature on the control of autonomous
underwater vehicles using conventional as well as biologically inspired techniques, very
little work has been done on the design of controllers applicable to nonlinear BAUV
models and when the exact BAUV system parameters are unknown. Thus, it is of
interest to develop adaptive controllers capable of accomplishing accurate depth and
yaw trajectory tracking even when system nonlinearities are present and the system
is not completely known.

1.3 Thesis Outline

In this thesis, the design of control systems for the dive plane and yaw plane
maneuvering of biologically inspired autonomous underwater vehicles is considered.
The first part of the thesis deals with the control of BAUVs using dorsal fins. The
force produced by the cambering of the fins is used for control. The latter part of
the thesis considers the control of the BAUVs using pectoral fins. Here, the flapping
of the pectoral fins provides the necessary force and moment for control. A brief
description of the controller designs is given below.

First, an adaptive optimal control system is designed for the dive plane maneuvering of BAUVs using dorsal fins. It is assumed here that the model parameters are completely unknown and only the depth of the vehicle is measured for feedback. Two dorsal fins are mounted in the horizontal plane on either side of the BAUV. The BAUV model considered here is non-minimum phase (i.e., the zeros of the system transfer function are unstable). The control system consists of a gradient based identifier for online parameter estimation, an observer for state estimation, and an optimal controller. The control law is derived in the second chapter. Simulation results are also presented which show that this adaptive control system accomplishes precise depth control of the BAUV using dorsal fins in spite of large uncertainties in the system parameters.

Next, an adaptive input-output feedback linearizing yaw plane control system using dorsal fins is developed. Unlike the adaptive optimal controller discussed above, the BAUV model includes non-linear hydrodynamics, and it is assumed that its hydrodynamic coefficients as well as the physical parameters are not known. For the purpose of design, a linear combination of the yaw angle tracking error and its derivatives and integral is chosen as the controlled output variable. An adaptive input-output feedback linearizing control law is derived for the trajectory control of the yaw angle. Unlike the indirect adaptive control system discussed earlier, here, the controller gains are directly tuned. The stability of the zero dynamics is also examined. Simulation results are presented for tracking exponential and sinusoidal yaw angle trajectories and
for turning maneuvers and it is shown that the adaptive control system accomplishes precise yaw angle control of the BAUV using dorsal fins in spite of the nonlinearity and large uncertainties in the system parameters.

The second part of this thesis involves the control of BAUVs using oscillating pectoral fins. An optimal controller is designed for yaw regulation of BAUV using pectoral fins. These fins are assumed to undergo a combined sway-yaw motion and the bias angle is treated as the control input which is periodically varied to accomplish the maneuver in the yaw plane. The periodic forces and moments produced by the flapping foil are parameterized using computational fluid dynamics. These forces are expanded as a Fourier series and a discrete-time model of the BAUV is developed for the purpose of control. An optimal control system for the set point control of the yaw angle is designed. An integral feedback is included in the control law for precise heading angle control. Simulation results are presented which show that the smooth yaw regulation is achieved.

It is also desirable to design control systems capable of tracking time varying trajectories. An inverse controller design is developed for this purpose. It is well known that the inverse controller can provide trajectory tracking only when the BAUV system is minimum phase. However, it turns out that the BAUV model under consideration is non-minimum phase. An approximate discrete-time system is then derived by essentially eliminating the unstable zeros from the pulse transfer function. An analytical expression of the output matrix of the approximate minimum phase system is derived. Then, an inverse control law is derived for the control of the new output vari-
able. It is shown that the controller based on the new variable accomplishes accurate trajectory following of the prescribed yaw trajectory. Simulation results show that in the closed-loop system, the yaw angle follows commanded sinusoidal trajectories and the segments of the intersample yaw trajectory remain close to the discrete-time reference trajectory. It is also found that the fins suitably located near the center of mass of the vehicle provide better maneuverability. The controller derivation is presented in chapter five.

Further, an indirect adaptive control system is designed for the dive plane control of BAUVs using pectoral fins. Here, it is assumed that the model parameters are completely unknown and only the depth of the vehicle is measured for feedback. The control algorithm used here is similar to the indirect adaptive dorsal fin controller except for the fact that it is in discrete time and the online parameter estimation scheme is done using a least-squares based identification scheme. Simulation results are also provided in chapter six which show that the BAUV accomplishes smooth depth trajectory tracking in spite of large uncertainties in the system parameters.
Figure 1.1: A diagram of a fish showing its different fins
CHAPTER 2

ADAPTIVE OPTIMAL CONTROL OF A BAUV IN THE DIVE PLANE USING DORSAL FINS

In this chapter, adaptive control of low speed bio-robotic autonomous underwater vehicles (BAUVs) in the dive plane using dorsal fins is considered. It is assumed that the model parameters are completely unknown and only the depth of the vehicle is measured for feedback. Two dorsal fins are mounted in the horizontal plane on either side of the BAUV. The normal force produced by the fins, when cambered, is used for the maneuvering. The BAUV model considered here is non-minimum phase. An indirect adaptive control system is designed for the depth control using the dorsal fins. The control system consists of a gradient based identifier for online parameter estimation, an observer for state estimation, and an optimal controller.

The organization of this chapter is as follows. Section (2.1) describes the mathematical model of the BAUV. The adaptive dorsal fin control law is obtained in Section (2.2). Section (2.3) presents the gradient estimation scheme for the estimation of the parameters. Simulation results and the caption of figures are provided in sections (2.4) and (2.5) respectively.
2.1 Dive Plane Dynamics

A schematic of the AUV model with the dorsal fins and the coordinate systems is shown in Figure 2.1. Here $O_I X_I Z_I$ is the inertial coordinate system. The vehicle is moving in the $X_I - Z_I$ plane. $X_B, Y_B, Z_B$ form the coordinate axes with the center of buoyancy as the origin, that is, $(x_B, y_B, z_B) = 0$. $x_G, y_G, z_G$ are the coordinate of the center of gravity of the vehicle.

The heave and pitch equations of motion for a neutrally buoyant vehicle are described by coupled nonlinear differential equations with respect to the moving coordinate frame $O_B X_B Z_B$. These equations describing the AUV model are [7,26]

\[
\begin{align*}
  m[w - uq - x_G \dot{q} - z_G \dot{q}] &= \frac{\rho}{2} L^4 Z_G \dot{q} + \frac{\rho}{2} L^3 [Z_G \ddot{w} + Z_u \dot{u} q] + \frac{\rho}{2} L^2 Z_{ww} \dot{w} + F_d \\
  I_y \dot{q} - m[x_G (\dot{w} - uq) - z_G w q] &= \frac{\rho}{2} L^5 M_G \dot{q} + \frac{\rho}{2} L^4 (M_G \ddot{w} + M_u \dot{u} q) + \frac{\rho}{2} L^3 M_{ww} \dot{w} + M_d \\
  -x_{GB} W \cos \theta - z_{GB} W \sin \theta \\
  \dot{z} &= w - u \theta \\
  \dot{\theta} &= q
\end{align*}
\]

Here $\theta$ is the pitch angle, $w$ is the heave velocity (along body axis $Z_B$), $x_{GB} = x_G - x_B$, $z_{GB} = z_G - z_B$, $\delta$ is the camber of the dorsal fins, and $F_d$ and $M_d$ are the net force and moment produced by the dorsal fins. Here $m$ is the mass, $I_y$ is the moment of inertia, $\rho$ is the density of sea water, and $L$ is the length of the AUV. $Z_{\dot{w}}, M_{\dot{w}}, Z_{uq}$, etc. are the hydrodynamic parameters. Camber is taken as the cross-stream deflection of the dorsal fin. Dorsal fins produce a net normal force $(z_\delta \delta)$ and a moment $(M_\delta \delta)$ proportional to the camber $\delta$ of the fins and can be continuously varied for the purpose
of control. It is also assumed that the forward velocity is held constant by a control mechanism and the lateral velocity is zero.

For the model in (2.1), \( x_{GB} \) is zero. Linearizing the system (2.1) about the equilibrium point \((w, q, \theta, z) = 0\), we obtain

\[
\begin{pmatrix}
\dot{w} \\
\dot{q} \\
\dot{\theta} \\
\dot{z}
\end{pmatrix} = A_p \begin{pmatrix}
w \\
q \\
\theta \\
z
\end{pmatrix} + B_p \delta
\] (2.2)

where \( A_p \in \mathbb{R}^{4 \times 4} \) and \( B_p \in \mathbb{R}^{4 \times 1} \) are the appropriate matrices. It is assumed that the elements of \( A_p \) and \( B_p \) are not known. Let

\[ y = z \] (2.3)

be the controlled variable. We are interested in designing an adaptive control system for the depth control and regulation of the state vector to the equilibrium point.

2.2 Adaptive Dorsal Fin Control System

In this section, the design of an adaptive control system is considered. To this end, it will be convenient to represent the system in the observer canonical form. The transfer function relating the depth and the input \( \delta \) can be shown to be of the form

\[
\frac{\hat{y}(s)}{\delta(s)} = C_p(SI - A_p)^{-1}B_p = H(s)
\] (2.4)
where \( \hat{y}(s) \) and \( \hat{\delta}(s) \) are the Laplace transforms of \( y \) and \( \delta \), respectively; and \( H(s) \) is the transfer function of the form

\[
H(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}
\]

where \( a_i \) and \( b_i \) are real numbers. The relative degree of \( H(s) \) is two. For the model under consideration one finds that the numerator has one positive root, which implies that the transfer function is non-minimum phase.

The observer canonical form realization of \( H(s) \) is given by [31]

\[
\dot{x} = \begin{bmatrix} -a_3 & 1 & 0 & 0 \\ -a_2 & 0 & 1 & 0 \\ -a_1 & 0 & 0 & 1 \\ -a_0 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ b_2 \\ b_1 \\ b_0 \end{bmatrix} \delta
\]

\[
\Delta = Ax + B\delta
\]

and the output is

\[
y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x = Cx = z
\]

We note that all the parameters \( a_i \) and \( b_i \) are unknown because \( A_p \) and \( B_p \) are not known. For the purpose of depth control, we introduced a servo compensator

\[
x_s = y - y_r = Cx - y_r
\]

where \( y_r \) is the desired depth.

Define the augmented state vector \( x_a = (x^T, x_s)^T \in \mathbb{R}^5 \). Now including \( x_s \) in the
canonical model, (2.5) gives

\[
\begin{bmatrix}
-a_3 & 1 & 0 & 0 \\
-a_2 & 0 & 1 & 0 \\
-a_1 & 0 & 0 & 1 \\
-a_0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_a \\
x_a \\
x_a \\
x_a \\
x_a
\end{bmatrix}
+
\begin{bmatrix}
0 \\
b_2 \\
b_1 \\
b_0 \\
0
\end{bmatrix}
\begin{bmatrix}
0 \\
d \\
0 \\
0 \\
-y_r
\end{bmatrix}
\]

\[\Delta = A_s x_a + b_s d + \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
-y_r
\end{bmatrix} \quad (2.7)\]

According to the servomechanism theory [30], for the regulation of the tracking error \(y - y_r\), it is necessary to obtain a control law \(d = K_c x_a\) such that the closed-loop matrix \((A_s + B_s K_c)\) is Hurwitz. Here, for the computation of \(K_c\), the linear quadratic regulation theory is used. Following Kailath [31], the optimal gain vector \(K_c\) is obtained by minimizing a performance index of the form

\[J = \frac{1}{2} \int_0^\infty (x_a^T Q_a x_a + r_a d^2) dt \quad (2.8)\]

where \(Q_a\) is a positive definite symmetric matrix (denoted as \(Q_a > 0\)) and \(r_a > 0\). The weighting matrix \(Q_a\) and the parameter \(r_a\) are chosen to obtain desirable transient responses in the closed-loop system. The feedback matrix \(K_c\) is obtained by solving the Riccati equation

\[P A_a + A_a^T P - r_a^{-1} P B_a B_a^T P + Q_a = 0 \quad (2.9)\]
The gain vector is $K_c = -r_a^{-1}B_a^TP$ and the control law is $u = K_c x$.

Let $\hat{a}_i, \hat{b}_i, \hat{A}_a$ and $\hat{B}_a$ denote the estimates of the parameters and the matrices. Since $a_i$ and $b_i$ are unknown, the matrix $P$ is obtained by using the estimated matrices $\hat{A}_a, \hat{B}_a$ in the Ricaati equation

$$P\hat{A}_a + \hat{A}_a^TP - r_a^{-1}P\hat{B}_a\hat{B}_a^TP + Q_a = 0$$

(2.10)

Then the feedback gain vector is computed as

$$\hat{K}_c = -r_a^{-1}\hat{B}_a^TP$$

and the control law is

$$u = \hat{K}_c x$$

(2.11)

For the synthesis of the control law (2.11), the complete state vector $x$ is needed which is not available. Therefore, it is essential to design an observer to obtain an estimate $\hat{x}$ of $x$. For the case of known parameters one can select an observer of the form

$$\dot{x} = A\hat{x} + B\delta + K_0(C\hat{x} - y)$$

(2.12)

For computing $K_0$, one uses the optimal control theory. For this the Ricaati equation

$$P_0A^T + AP_0 - r_0^{-1}P_0C^TP_0 + Q_0 = 0$$

(2.13)

is solved for $P_0 > 0$ where the weighting matrix $Q_0 > 0$ and $r_0 > 0$. The observer feedback matrix is then given by

$$K_0^T = -r_0^{-1}CP_0$$

(2.14)
Let \( e = x - \hat{x} \). Subtracting (2.12) from (2.5) gives the state error dynamics

\[
\dot{e} = A e + K_0 C e
\]

for which \((A + K_0 C)\) is stable. Since \( A \) and \( B \) are not known, the Riccati equation

\[
P_0 \hat{A}^T + \hat{A} P_0 - r_0^{-1} P_0 C^T C P_0 + Q_0 = 0 \tag{2.15}
\]

with the estimated parameters is solved. Then the observer feedback matrix is given by

\[
\hat{K}_0 = -r_0^{-1} C P_0 \tag{2.16}
\]

In the control law, \( \hat{x} \) is used instead of \( x \), yielding

\[
\delta = \hat{K}_e (\hat{x}^T, x_a)^T \tag{2.17}
\]

### 2.3 Adaptation Law

For the computation of the control law in the previous section, the estimates of \( \hat{a}_i \) and \( \hat{b}_i \) are required. In this section, a gradient update scheme is developed for the estimation of these parameters of the system. Let \( \theta \) be the set of all the unknown parameters of the system, i.e.,

\[
\theta = \begin{bmatrix}
\theta_b \\
\theta_a
\end{bmatrix} \in \mathbb{R}^T
\tag{2.18}
\]

where \( \theta_b = (b_2, b_1, b_0)^T \) and \( \theta_a = (a_3, a_2, a_1, a_0)^T \)

Manipulating (2.4), we get

\[
s^4 \ddot{y} = (b_2 s^2 + b_1 s + b_0) \delta - (a_3 s^2 + a_2 s + a_1 + a_0) \ddot{y} \tag{2.19}
\]
Let \( \alpha_i(s) = (s^i, s^{i-1}, ..., s^0) \). Then (2.19) can be written as

\[
s^4 \dot{y} = \theta_T^T \alpha_2(s) \dot{s} - \theta_a^T \alpha_3(s) \tag{2.20}
\]

This expression involves various derivatives of \( \delta \) and \( y \). In order to obtain an identifier which is free of derivatives of the input and output, divide (2.20) by a Hurwitz polynomial \( \Lambda(s) \) of the form (\( \lambda_i, i = 0, ..., 3 \), are real numbers)

\[
\Lambda(s) = s^4 + \lambda_3 s^3 + \lambda_2 s^2 + \lambda_1 s + \lambda_0 \tag{2.21}
\]
to obtain

\[
\frac{s^4 \dot{y}}{\Lambda(s)} = \theta_T^T \alpha_2(s) \frac{\dot{s}}{\Lambda(s)} - \theta_a^T \alpha_3(s) \frac{\dot{y}}{\Lambda(s)} \tag{2.22}
\]

Noting that \( s = \frac{d}{dt} \), (2.22) gives

\[
q(t) = \theta^T \phi(t) \tag{2.23}
\]

where

\[
q = \frac{s^4 \dot{y}}{\Lambda(s)}
\]

\[
\phi = \begin{bmatrix} \frac{\alpha_2(s) \dot{\delta}}{\Lambda(s)} \\ \frac{\alpha_3(s) \dot{y}}{\Lambda(s)} \end{bmatrix} \in \mathbb{R}^T
\]

The signals \( q \) and \( \phi \) are obtained by simply filtering the accessible input (\( \delta \)) and the output (\( y \)).

Let the estimate \( \hat{q} \) of \( q \) at time \( t \) be

\[
\hat{q}(t) = \hat{\theta}^T(t) \phi(t) \tag{2.24}
\]
and the estimation error be $\epsilon = q - \hat{q}$. Then the gradient algorithm [34] for the estimation of the parameters is obtained by minimizing the quadratic cost

$$J_0(\theta) = \frac{1}{2}(q - \hat{q})^2 = \frac{1}{2}(q - \hat{\theta}^T \phi)^2$$

(2.25)

where $\tilde{\theta} = \theta - \hat{\theta}$ is the parameter error vector and the minimizing trajectory $\hat{\theta}(t)$ is generated by

$$\dot{\theta} = -\Gamma^T \frac{\partial J_0(\theta)}{\partial \theta} = \Gamma(q - \hat{\theta}^T \phi) = \Gamma \epsilon \phi$$

(2.26)

where $\Gamma > 0$ is the scaling matrix. The estimated parameter vector $\hat{\theta}$ is used in Section 2.2 for the observer and controller design. This completes the control system design.

### 2.4 Simulation Results

In this section, the Matlab/Simulink software is used to obtain simulation results for the system (2.1) including the control law (2.16). The hydrodynamic parameters of the AUV with dorsal fins are taken from [7] and are as follows.

1. The Hydrodynamic parameters for the experimental vehicle chosen for simulation are

   $u = 3.6\text{m/s}$, $M_q = -0.16E - 3$, $M_w = -0.825E - 5$, $M_{uw} = -0.117E - 2$, $M_{wu} = 0.314E - 2$, $Z_w = -0.569E - 2$, $Z_q = -0.825E - 5$.

2. The Vehicle physical parameters are

   $x_{GB} = 0$, $z_{GB} = 0.578802$, $x_B = 0$, $z_B = 0$, $x_G = x_{GB}$, $z_G = Z_{GB}$. 

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\( L = 1.282m. \quad \rho = 1025.9Kgm^{-3}. \quad W = B = 40.7583N. \)

\[ m = 4.1548Kg. \quad I_y = 0.5732Kgm^2. \]

Substituting these values in the mathematical model, we obtain a system with two zeros and four poles. The zeros of the system are at -11.3671 and 4.5126 and the four poles are 0, -8.0079, -0.1178 and -3.7812e-007. It is seen that for a step command the depth response will diverge because the transfer function has a pole at the origin.

For the gradient update scheme, the value of the scaling matrix \( \Gamma \) is chosen as 150\( I \), where \( I \) is an identity matrix of order 7. The initial state of the system is assumed as \( x(0) = 0 \). For the computation of the optimal gain vector \( \hat{K}_c \), the value of \( Q_u \) is chosen as \( 10 \text{diag}(1000, 150, 150, 1000, 500) \) and \( r_u \) is chosen as 150. The optimal gain vector \( \hat{K}_0 \) is computed by a choice of the weighting matrix \( Q_0 = 20 \text{diag}(100, 100, 15, 1000) \) and \( r_0 = 150 \). Results are presented for commanding the BAUV to a depth of 1 (m) and 2 (m) for various uncertainties in the AUV model.

**Case 1:** Adaptive control: Target Depth = 1 (m), Estimated \( \hat{\theta}(0) \) 25% Lower

A set point of 1 (m) depth is taken as the command input. It is assumed that \( \hat{\theta}(0) = [-5.8287e - 016, -0.3348, -3.8080, 6.0943, 0.7073, 2.6742e - 007, 0]^T \), but the actual parameter vector is \( \theta = [-7.7716e - 016, -0.4464, -5.0774, 8.1257, 0.9430, 3.5657e - 007, 0]^T \), that is, \( \hat{\theta}(0) = 0.75\theta \). The simulation results are shown in Figure 2.2 (a)-(f) and Figure 2.3 (a)-(f). It can be seen that smooth depth trajectory tracking and pitch angle regulation are achieved in less than 20 sec. The maximum camber (control input) required is approximately 15mm, which can easily be provided by the dorsal fins. It is obvious that the heave velocity goes to zero when the desired depth
is attained. The estimated parameters $a_i$ and $b_i$ are also shown in the figure. One can observe that these parameters converge to certain constant values when the set point tracking is achieved. These estimated values differ from the actual values of the parameters. Of course, it is well known that the estimated parameters cannot converge to their actual values in the absence of persistent excitation, however this causes no problem in accomplishing the depth control.

Simulation results are also presented for parameter uncertainty as high as -50\% ($\hat{\theta}(0) = \frac{\theta}{2}$). As one can expect, the cambering required is high. As such results are presented by clamping the camber to a value of 20mm. It can be seen from Figure 2.4 (a)-(d) that depth tracking and pitch angle regulation are accomplished. The pitch angle and pitch rate are larger compared to the previous case. The estimated parameters also converge to larger constant values.

Simulations were also performed for an uncertainty of -80\% (i.e., $\hat{\theta}(0) = 0.2\theta$) including the control saturation of 20mm to illustrate the effectiveness of the adaptive controller for 1 (m) command. Although the depth control was accomplished, the transient response degraded considerably (Figure 2.5 (a)-(d)). It is observed that control input saturates for almost 30 sec and the pitch angle and heave responses are oscillatory in the transient period. The response time is seen to be of the order of 35 sec.

**Case 2 Adaptive Control: Target Depth = 2 (m), Estimated $\hat{\theta}(0)$ 25% Lower**

Now simulation results for a command depth of 2 (m) are presented. Smooth depth tracking is obtained as can be seen in Figure 2.6 (a)-(d). As expected, the pitch angle
and pitch rate are high compared to those of Figure 2.2 for smaller command input. Control input saturation is observed only for a very brief period (about 2 sec). There is slight increase in the response time compared to Figure 2.2.

Simulation was also done for uncertainty of +25% ($\hat{\theta}(0) = 1.25\theta$) and +50% using one and two meter command. For these uncertainties also the adaptive controller accomplished accurate depth control (These results are not shown here in order to save space). However, compared to Figures 2.2 and 2.4 for -25% and -50%, the response time was considerably longer, but the control input and pitch rate were smaller.
Figure 2.1: Schematic of the BAUV
Figure 2.2: Adaptive control: command depth = 1 (m) and $\dot{\theta}(0) = 0.75\theta$, (a) Depth ($z$), (b) Camber ($\delta$), (c) Pitch angle ($\theta$), (d) Pitch rate ($q$), (e) Heave Velocity ($w$), (f) Estimated parameter ($b_2$)

Figure 2.3: Adaptive control: command depth = 1 (m) and $\dot{\theta}(0) = 0.75\theta$
Figure 2.4: Adaptive control: command depth = 1 (m) and $\hat{\theta}(0) = 0.50\theta$, (a) Depth ($z$), (b) Camber ($\delta$), (c) Pitch angle ($\theta$), (d) Pitch rate ($q$)

Figure 2.5: Adaptive control: command depth = 1 (m) and $\hat{\theta}(0) = 0.20\theta$, (a) Depth ($z$), (b) Camber ($\delta$), (c) Pitch angle ($\theta$), (d) Pitch rate ($q$)
Figure 2.6: Adaptive control: command depth $= 2$ (m) and $\dot{\theta}(0) = 0.75\theta$, (a) Depth ($z$), (b) Camber ($\delta$), (c) Pitch angle ($\theta$), (d) Pitch rate ($q$)
CHAPTER 3

ADAPTIVE INPUT-OUTPUT FEEDBACK LINEARIZING YAW PLANE CONTROL OF A BAUV USING DORSAL FINS

This chapter presents the design of an adaptive input-output feedback linearizing dorsal fin control system for the yaw plane control of low speed bio-robotic autonomous underwater vehicles (BAUVs). The control forces are generated by cambering two dorsal fins mounted in the vertical plane on either side of the vehicle. The BAUV model includes non-linear hydrodynamics, and it is assumed that its hydrodynamic coefficients as well as the physical parameters are not known. For the purpose of design, a linear combination of the yaw angle tracking error and its derivative and integral is chosen as the controlled output variable. The stability of the zero dynamics is examined. An adaptive input-output feedback linearizing control law is derived for the trajectory control of the yaw angle. Unlike the indirect adaptive control scheme presented in the chapter two, here the controller gains are directly tuned. Also, the controller design of chapter two cannot track time varying trajectories.

The organization of the chapter is as follows. Section 3.1 describes the mathematical model of the BAUV. An adaptive dorsal fin control law for yaw plane control is obtained in Section 3.2. Simulation results and the figure captions are presented in
Sections 3.3 and 3.4, respectively.

3.1 Yaw Plane Dynamics

A schematic of the BAUV model with the dorsal fins and the coordinate systems is shown in Figure 3.1. Here $O_I X_I Y_I$ is the inertial coordinate system. The vehicle is moving in the $X_I - Y_I$ plane. $X_B, Y_B, Z_B$ form the coordinate axes with the center of buoyancy as the origin, that is, $(x_B, y_B, z_B) = 0$. $x_G, y_G, z_G$ are the coordinate of the center of gravity of the vehicle.

The yaw and sway equations of motion for a neutrally buoyant vehicle are described by coupled nonlinear differential equations with respect to the moving coordinate frame $O_B X_B Y_B$. These equations describing the AUV model are [32]

\[
\dot{\psi} = r \\
I_x \ddot{r} + m[x_G(\dot{v} + ur) + y_G vr] = \frac{\rho l^3}{2} (N_x \ddot{r} + N_y |r| r) + \frac{\rho l^4}{2} (N_x \dot{r}) \\
+ N_{ur} ur + \frac{\rho l^3}{2} (N_{uv} uv + N_{v|v|v|v|}) + N_f \\
m(\dot{v} + ur + x_G \ddot{r} - y_G r^2) = \frac{\rho l^4}{2} (Y_x \ddot{r} + Y_y |r| r) + \frac{\rho l^3}{2} (Y_x \dot{r}) \\
+ Y_{ur} ur + \frac{\rho l^2}{2} (Y_{uv} uv + Y_{v|v|v|v|}) + F_f
\]

Here $\psi$ is the yaw angle, $v$ is the lateral velocity (along body axis $Y_B$), $\delta$ is the camber of the dorsal fins, and $F_f$ and $N_f$ are the net force and moment produced by the dorsal fins. Here $m$ is the mass, $I_x$ is the moment of inertia, $\rho$ is the density of sea water, and $l$ is the length of the AUV. $Y_x, N_x, Y_{uv},$ etc., are the hydrodynamic parameters. Camber is taken as the cross-stream deflection of the dorsal fin. Dorsal fins produce
a net lateral force \((y_\delta \delta)\) and a moment \((\tau_\delta \delta)\) proportional to the camber \(\delta\) of the fins and can be continuously varied for the purpose of control. It is also assumed that the forward velocity is held constant by a control mechanism and the heave motion is zero. For the purpose of illustration, only some of the second order polynomial nonlinearities in the variables \(v\) and \(r\) are included in the model. However, it is pointed out that the design approach presented here can be easily extended to include any other polynomial nonlinearities depending on the hydrodynamic coefficients \(N_{r|v|}, Y_{r|v|}, N_{rrr},\) etc.

The global position coordinates \(X\) and \(Y\) of the vehicle are described by the kinematic equations

\[
\dot{X} = u\cos(\psi) - v\sin(\psi)
\]

\[
\dot{Y} = u\sin(\psi) + v\cos(\psi)
\]  

(3.2)

The yaw and sway equations of motion (3.1) can be compactly written as

\[
\dot{x}_p = \begin{bmatrix}
0 & 1 & 0 \\
0 & a_{22} & a_{23} \\
0 & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
\psi \\
r \\
v
\end{bmatrix}
+ \begin{bmatrix}
0 \\
b_2 \\
b_3
\end{bmatrix}
\delta + P
\begin{bmatrix}
\tau|r| \\
v|v| \\
r^2 \\
v\tau
\end{bmatrix}
\]

\[
x_p = [\psi, r, v]^T
\]

(3.3)

where \(x_p = [\psi, r, v]^T\) and \(a_{ik}, b_i\) are appropriate constants. The constant matrices \(A, b\) and \(P = [0^T_{1\times 4}, P_2^T, P_3^T]^T\) are defined in (3.3) \((T\) denotes matrix transposition), \(P_i\) is the \(i\)th row of \(P\) and \(f_n\) denotes the nonlinear vector function of \(r\) and \(v\). In the

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system representation (3.3), other nonlinear hydrodynamic functions of \( v \) and \( r \) can be included by modifying the matrix \( P \) and the vector \( f_n(r,v) \) appropriately. We note that the elements of matrices \( A, b \) and \( P \) are unknown since the physical parameters and the hydrodynamic coefficients of the vehicle are not known.

For the yaw plane maneuver, the yaw angle \( \psi \) is the key variable. Suppose that a reference trajectory generator of the form

\[
\dot{x}_r = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
a_{r0} & a_{r1} & a_{r2}
\end{bmatrix} x_r + \begin{bmatrix}
0 \\
0 \\
a_{r0}
\end{bmatrix} \psi_c^*(t) \tag{3.4}
\]

is given, where \( x_r = (x_{r1}, x_{r2}, x_{r3})^T = (\psi_r, \psi_{r}, \psi_{\dot{r}})^T, \psi_r(t) = x_{r1} \) is the reference yaw angle trajectory to be tracked and \( \psi_c^* \) is the smooth and bounded yaw angle command input. The characteristic polynomial associated with the command generator is given by

\[
\pi_r(\lambda) = \lambda^3 + a_{r2}\lambda^2 + a_{r1}\lambda + a_{r0} = (\lambda + \lambda_c)(\lambda^2 + 2\zeta_c w_c \lambda + \omega_c^2 \lambda_c) \tag{3.5}
\]

The parameters \( \lambda_c > 0, \zeta_c > 0 \) and \( w_c > 0 \) and the input \( \psi_c^*(t) \) are properly chosen to generate smooth desirable yaw angle trajectories. Note that \( \pi_r(\lambda) \) is a Hurwitz polynomial and the parameters of (3.4) are \( a_{r0} = \lambda_c w_c^2, a_{r1} = 2\zeta_c w_c \lambda_c + \omega_c^2 \lambda_c, a_{r2} = \lambda + 2\zeta_c w_c \). Let the yaw angle tracking error be

\[
\bar{\psi} = \psi - \psi_r = C(x_p - x_r) \tag{3.6}
\]

where \( C = [1, 0, 0] \) is the output matrix.

We are interested in the design of an adaptive control system for asymptotic
regulation of $\tilde{\psi}$ to zero. This way, the yaw angle can be made to follow reference yaw angle trajectories for the maneuver of the BAUV in the yaw plane.

3.2 Adaptive Dorsal Fin Control System

In this section, the design of an adaptive control system for the yaw angle control is considered. For the purpose of design, it is convenient to choose a stable manifold, which is a linear combination of the tracking error $\tilde{\psi}$ and its derivative and integral given by

$$ S = \psi + \mu_1 \dot{\psi} + \mu_0 \int_0^t \dot{\psi}(\tau) d\tau $$

where $\mu_i > 0$. Suppose that $S(t) = 0$ for $t \geq t_0$, then differentiating (3.7) gives

$$ \ddot{\psi} + \mu_1 \dot{\psi} + \mu_0 \dot{\psi} = 0 $$

(3.8)

The linear system (3.8) is a stable system by the choice of $\mu_i > 0$. This implies that $(\psi - \psi_r) \to 0$, as $t \to \infty$. Therefore, for the control of the yaw angle, it is sufficient to design a control law which accomplishes asymptotic regulation of $S(t)$ to zero.

3.2.1 Input-Output Linearizing Control Law

In this subsection, an adaptive input($\delta$)-output($S$) feedback linearizing control law is developed [33]. Differentiating $S(t)$ along the solution of (3.3) gives

$$ \dot{S} = \tilde{\dot{\psi}} + \dot{\mu}_1 \dot{\psi} + \mu_0 \ddot{\psi} = a_{22} \dot{r} + a_{23} v + P_2 f_n(r, v) + b_2 \delta - \zeta(\psi, r, t) $$

(3.9)

where $\zeta(\psi, r, t) = -\tilde{\zeta}_r + \mu_1 \dot{\psi} + \mu_0 \dot{\psi}$. Define a vector of parameters, $\theta$, and a regressor vector, $\Phi(r, v)$, as

$$ \theta = [a_{22}, a_{23}, P_2]^T \in \mathbb{R}^3 $$

$$ \Phi(r, v) = [\dot{r}, v, f_n(r, v)]^T $$

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\[ \Phi(r,v) = [r,v,r|r|, v|v|, r^2, vr]^T \in \mathbb{R}^\delta \] (3.10)

Then (3.9) can be compactly written as

\[ \dot{S} = \Phi^T \theta - \zeta + b_2 \delta \] (3.11)

If \( \theta \) and \( b_2 \) are known, one can select an input-output feedback linearizing control law of the form

\[ \delta = b_2^{-1}[\zeta - \Phi^T \theta - KS] \] (3.12)

where \( K > 0 \). Substituting (3.12) in (3.11) gives

\[ \dot{S} = -KS \] (3.13)

which implies that \( S(t) \to 0 \). This will in turn cause the regulation of \( (\psi - \psi_r) \) to zero. Note that even though the model (3.3) is nonlinear, in the closed-loop system (3.3) and (3.12), the output \( S \) satisfies a linear differential equation (3.13).

The parameter vector \( \theta \) is unknown, and, therefore, it is not possible to synthesize the control law (3.12). Now a modified control law obtained from (3.12) is considered. Let \( \hat{\theta}(t) \) be the estimate of the parameter vector \( \theta \). We choose a control law of the form

\[ \delta = \hat{\rho}_1[\zeta - \Phi^T \hat{\theta} - KS] \] (3.14)

where \( \hat{\rho}_1 \) is the estimate of \( \rho_1 \triangleq b_2^{-1} \), the inverse of the unknown parameter \( b_2 \), and \( \hat{\theta} \) is the estimate of \( \theta \). Noting that \( b_2 \rho_1 = 1 \), one has

\[ b_2 \hat{\rho}_1 = b_2(\hat{\rho}_1 - \rho_1 + \rho_1) = 1 - b_2 \rho_1 \] (3.15)
where \( \tilde{\rho}_1 = \rho_1 - \hat{\rho}_1 \) is the parameter error. For the derivation of the control law the following assumption is made.

**Assumption 1:** The sign of the parameter \( b_2 \) is known.

The transfer function of the linear model (3.3) relating \( \psi \) and \( \delta \) without the non-linearity \( f_n(r, v) \) has its numerator polynomial of degree 2. The parameter \( b_2 \) is the high frequency gain. The assumption 1 regarding the sign of the high frequency gain is also usually made for linear adaptive control designs [34]. Of course, the sign of \( b_2 \) can be determined by computing it using some nominal parameters of the model (3.3).

Substituting the control law (3.14) in (3.11) and using (3.15) gives

\[
\dot{S} = \Phi^T \theta - \zeta + b_2 \tilde{\rho}_1 [\zeta - \Phi^T \hat{\theta} - KS]
\]

\[
= \Phi^T \theta - \zeta + (1 - b_2 \tilde{\rho}_1) [\zeta - \Phi^T \hat{\theta} - KS]
\]

\[
= -KS + \Phi^T \tilde{\theta} - b_2 \tilde{\rho}_1 w_\zeta
\]

where \( \tilde{\theta} \) is the parameter error, \( \theta - \hat{\theta} \), and

\[
w_\zeta = (\zeta - \Phi^T \hat{\theta} - KS)
\]

is a known signal.

For the derivation of the adaptation law for tuning the estimate \( \hat{\theta}(t) \), the Lyapunov approach [35] is used. Consider a quadratic function given by

\[
V(S, \tilde{\theta}, \tilde{\rho}_1) = \frac{1}{2}(S^2 + \tilde{\theta}^T \Gamma \tilde{\theta} + \gamma |b_2| \tilde{\rho}_1^2)
\]
where $\Gamma$ is a positive definite symmetric weighting matrix and $\gamma > 0$. Differentiating $V$ along the solution of (3.16) gives

$$
\dot{V} = S[-KS + \Phi^T \theta - b_2 \dot{\theta}_1 w_\zeta] + \dot{\theta}^T \Gamma \ddot{\theta} + \gamma \dot{\theta}_1 \dot{\theta}_1 |b_2|
$$

$$
= -KS^2 + \dot{\theta}^T [\Phi S + \Gamma \ddot{\theta}] + \dot{\theta}_1 [-b_2 w_\zeta S + \gamma |b_2| \dot{\theta}_1]
$$

(3.19)

In order to eliminate the unknown functions in (3.19), one selects the adaptation law as

$$
\dot{\rho}_1 = -\dot{\rho}_1 = \gamma^{-1} \text{sgn}(b_2)Sw_\zeta = \gamma^{-1} \text{sgn}(b_2)S[\zeta - \Phi^T \theta - KS]
$$

$$
\dot{\theta} = -\dot{\theta} = -\Gamma^{-1} \Phi(r, v)S = -\Gamma^{-1} [r, v, r|v|, v|v|, r^2, vr]^T S
$$

(3.20)

Substituting the adaptation law (3.20) in (3.19) gives

$$
\dot{V} = -KS^2
$$

(3.21)

According to (3.21), $\dot{V}$ is negative semidefinite ($\dot{V} \leq 0$). Since $V$ is a positive definite function of $S$, $\ddot{\theta}$ and $\dot{\rho}$, it follows that $S, \ddot{\theta}, \dot{\rho} \in L_\infty[0, \infty)$, the set of bounded functions. Of course, in view of (3.7), boundedness of $S$ implies that $\dot{\psi}, r$ and $\ddot{\psi}$ are bounded. Furthermore, (3.21) gives

$$
\int_0^\infty S^2 dt = \frac{1}{K} (V(0) - V(\infty)) < \infty
$$

(3.22)

Therefore, $S \in L_2[0, \infty)$ (the set of square integrable functions). In view of (3.16), $\dot{S}$ is bounded provided that $v(t)$ is bounded. Then using the Barbalat’s Lemma [34, 35], it follows that $S(t) \to 0$ as $t \to \infty$ which in turn implies that the tracking error $\tilde{\psi}(t) = \psi(t) - \psi_r(t)$ asymptotically converges to zero.
In the above stability arguments, the boundedness of the lateral velocity during
the maneuver is required. It is well known that stability of the closed-loop system
including the feedback linearizing control law critically depends on the stability of
the zero dynamics of the system [35]. The stability of the zero dynamics is examined
in the following subsection.

3.2.2 Stability of Zero Dynamics

Zero dynamics represent the residual motion of the system, when the controlled
output $S(t)$ is identically zero. Of course, this also implies that $\tilde{\psi} = 0$ and $\dot{\tilde{\psi}} =
$ 
$r(t) - \dot{\psi}_r(t) = 0$. For simplicity, let us assume that $\psi_r$ is a constant trajectory. Then
one has $\psi = \psi_r = \text{constant}$ and $\dot{r} = \dot{\psi}_r = 0$ if $S(t) = 0$. Substituting $\psi = \psi_r$ and
$\dot{r} = 0$ in Eq. (3) gives

$$
(m_x G - \frac{\rho}{2} t^4 N_v) \dot{v} = \frac{\rho}{2} t^2 [N_{uv} uv + N_{e[v]} v |v|] + n_3 \delta
$$

$$
(m - \frac{\rho}{2} \delta Y_v) \dot{v} = \frac{\rho}{2} t^2 [Y_{uv} uv + Y_{\psi[v]} v |v|] + \delta \delta
$$

(3.23)

Eliminating the control input $\delta$ from (3.23) gives

$$
[y_\delta (m_x G - \frac{\rho}{2} t^4 N_v) - n_\delta (m - \frac{\rho}{2} \delta Y_v)] \dot{v} = \frac{\rho}{2} t^3 y_\delta [N_{uv} uv + N_{e[v]} v |v|]
$$

$$
- \frac{\rho}{2} t^2 n_\delta [Y_{uv} uv + Y_{\psi[v]} v |v|]
$$

(3.24)

Equation (3.24) can be compactly written as

$$
\dot{v} = a_v v + n_v v |v|
$$

(3.25)

where

$$
b_v = y_\delta (m_x G - \frac{\rho}{2} t^4 N_v) - n_\delta (m - \frac{\rho}{2} \delta Y_v)
$$
The zero dynamics are governed by (3.25). For stability in the closed-loop system, the system (3.25) must be stable. Linearizing (3.26) about \( v = 0 \) gives

\[
\dot{v} = a_v v \tag{3.27}
\]

In view of (3.27), the equilibrium point \( v = 0 \) of the nonlinear system (3.25) is asymptotically stable if \( a_v < 0 \), that is,

\[
b_v^{-1}[ly_s N_{uv} - n_s Y_{uv}] < 0 \tag{3.28}
\]

The system (3.25) is said to be asymptotically stable if its trajectory beginning in the neighborhood of the origin converges to \( v = 0 \) as \( t \to \infty \).

For global asymptotic stability, one must consider the complete nonlinear equation (3.25). Of course, the condition \( a_v < 0 \) is necessarily required for global asymptotic stability as well. To establish global result, consider a Lyapunov function

\[
W = \frac{v^2}{2} \tag{3.29}
\]

Its derivative along the solution of (3.25) gives

\[
\dot{W} = a_v v^2 + n_v v^2 |v| = v^2[a_v + n_v |v|] \tag{3.30}
\]

In view of Eq. (30), \( \dot{W} \) is negative if

\[
a_v < 0
\]
If inequalities (3.31) hold, then $\dot{W} < 0$ for all $v \neq 0$, and the solution $v(t)$ of (3.25) converges to zero for any initial condition $v(0) \in \mathbb{R}$. This implies that $v = 0$ of the zero dynamics is globally asymptotically stable.

Now let us examine the stability of the system (3.25) if $a_v < 0$ and $n_v$ is positive. For this case, $\dot{W} < 0$ only if

$$|v| < -\frac{a_v}{n_v} = \frac{|a_v|}{n_v}$$

In such a case only asymptotic stability (local) is possible.

We notice from the inequalities (3.28) and (3.31) that the stability of the zero dynamics depends on the vehicle parameters, the hydrodynamic coefficients and the location of the dorsal fins on the vehicle. In the vicinity of the origin, the convergence of the lateral velocity to the equilibrium point depends on the value of $a_v$. For a given vehicle, one can properly select the position of the dorsal fins on the vehicle to obtain stable zero dynamics and for shaping the transient characteristics of the lateral velocity. For the model under consideration, the inequality (3.28) is satisfied, and, therefore, the equilibrium point of the the zero dynamics is asymptotically stable.

### 3.3 Simulation Results

In this section, simulation results using MATLAB and SIMULINK for yaw angle control using dorsal fins are presented. Various time-varying reference trajectories are considered for tracking and the performance of the adaptive controller for each is evaluated.
The vehicle model of [7] is considered and its physical and non-dimensionalized hydrodynamic parameters are as follows.

1. The Hydrodynamic parameters for the experimental vehicle chosen for simulation are

\[ u = 3.6 m/s, \quad N_r = -0.336E - 3, \quad N_v = -0.286E - 4. \]

\[ N_{uv} = -0.613E - 2, \quad N_{ur} = -0.297E - 4, \quad Y_v = -0.619E - 2. \]

\[ Y_r = -0.286E - 4, \quad Y_{uv} = -0.137E - 1, \quad Y_{ur} = -0.0232E - 3. \]

The values of the nonlinear hydrodynamic coefficients are not available; however, in order to show robustness, simulation has been done using the following values:

\[ N_{v|\delta|} = 0.340E - 4, \quad N_{r|\delta|} = -0.303E - 4. \]

\[ Y_{v|\delta|} = -0.264E - 4, \quad Y_{r|\delta|} = 0.150E - 4. \]

The dorsal fin parameters are:

\[ y_\delta = 0.3003 (N/mm), \quad n_\delta = 0. \]

2. The Vehicle physical parameters are

\[ x_G = 0, \quad y_G = 0, \quad z_G = 0.40657E - 8. \]

\[ l = 1.282m, \quad \rho = 1025.9 Kg/m^3, \quad W = B = 40.7583N. \]

\[ m = 4.1548Kg, \quad I_z = 0.5732Kgm^2. \]

Substituting these values in the mathematical model, one finds that \( a_u < 0 \), and therefore, the zero dynamics have asymptotically stable equilibrium point. The initial condition of the vehicle is chosen as \( x_0(0) = (0, 0, 0)^T \).

The values of \( \mu_1 \) and \( \mu_0 \) of the stable mainfold \( S \) given in (3.7) are taken as 0.7 and 0.25, respectively, and the gain \( K \) in the control law (3.14) is chosen as 0.05.
Since $y_G = 0$, the variables $r^2$ and $v_r$ do not appear in the regressor vector $\Phi(r,v)$. As such $\theta$ has only four elements. For the adaptation scheme, the value of the scaling matrix $\Gamma$ is chosen as $0.01I_{4 \times 4}$, where $I_{4 \times 4}$ is an identity matrix of order 4. The initial estimates of the parameters are set to $\hat{\theta}_1(0) = \theta_1/2$, $\hat{\theta}_2(0) = \theta_2/2$, and $\hat{\theta}_3(0) = \hat{\theta}_4(0) = 0$. Here $\theta_k$ and $\hat{\theta}_k$ denote the $k$th elements of $\theta$ and $\hat{\theta}$, respectively. Thus the estimates of the first two elements of $\theta$ have 50% uncertainty, and the other two parameters associated with the nonlinearity have 100% uncertainty. The initial value of the estimate of $\rho_1$ is set to $\hat{\rho}_1 = 0.05\rho_1$ giving an uncertainty of 95%. We have taken rather a worse choice of initial estimates of the unknown parameters. However, this has been done to show the robustness of the designed controller. The value of the parameter $\gamma$ in (3.18) for the estimation of $\hat{\rho}$ is taken as 0.01.

Reference trajectories are generated using the command generator Eq. (4) with zero initial conditions. The values for the reference generator parameters $a_{r0}$, $a_{r1}$ and $a_{r2}$ are chosen to be 1, 3 and 3, respectively. Thus the poles of the command generators are at -1. The initial state $x_r$ is $x_r(0) = 0$. Simulation results are presented in the following for exponential and sinusoidal trajectory control and for turning maneuvers.

**Case 1** Adaptive control: Exponential reference trajectory tracking $\psi_c^* = 30$ (deg)

The exponential reference trajectory is generated by setting $\psi_c^*(t) = 30 \times \frac{\pi}{180}$ (rad) as the yaw command input in (3.4). The simulation results are shown in Figure 3.2 (a)-(i). It can be seen that the adaptive controller achieves accurate exponential trajectory tracking in approximately 20 sec. The control input (camber) magnitude required is less than 13 mm, which is very small and can easily be provided by
the dorsal fins. The yaw rate $r$ and the lateral velocity $v$ reach their equilibrium values after the target heading angle is reached. This is expected because the zero dynamics have asymptotically stable equilibrium point. The plots of the estimates of the adaptation parameters $\theta$ and $\hat{\rho}_1$ are also presented (Figure 3.2(e)-(i)). One can observe that these parameters converge to certain constant values after tracking is achieved. Of course, it is well known that the estimated parameters cannot converge to their actual values in the absence of persistent excitation even though yaw reference trajectory tracking is accomplished (Ioannou and Sun, 1996).

**Case 2** Adaptive Control: Sinusoidal reference trajectory tracking $\psi^*_{c} = 30 \sin(w_r t)$ (deg)

For generating the sinusoidal trajectory, the command input is chosen as $\psi^*_{c}(t) = 30 \times \frac{\pi}{180} \sin(w_r t)$ (rad), where $w_r = 0.2$ (rad/sec). The plots of the heading angle, control input, yaw rate and lateral velocity are presented in Figure 3.3(a)-(i). Smooth sinusoidal trajectory tracking is accomplished and the camber required is less than 10 mm. As one can expect, the yaw rate and the lateral velocity are also periodic. Figure 3.3(e)-(i) shows the plots of the estimates of the adaptation parameters. It is noted that, when the simulation is run for a longer time period, the estimates slowly decay to constant values. Of course, here, the results are provided for a shorter time period.

**Case 3** Adaptive control: Turning maneuvers $\psi^*_{c}(t) = 15t$ (deg)

Now we examine the turning capability of the BAUV using the designed controller. The turning circle maneuver is an important practical maneuver that BAUVs
frequently needs to perform. For constant turning rate, a smooth trajectory is generated using the command input \( \psi_c^*(t) = 15 \frac{\pi}{180} t \) (rad). It can be seen from Figure 3.4(a)-(d) that the desired constant turning rate is achieved and the lateral velocity attains constant value in the steady state. For this maneuver, the lateral velocity converges to a nonzero constant value because \( \dot{\psi}_r = \dot{\psi}_c^* \), a nonzero constant in the steady state. The nonzero equilibrium lateral velocity is caused by the appearance of a \( \dot{\psi}_c^* \)-dependent constant forcing function in the zero dynamics equation (3.25).

We observe that, after the initial transient, the BAUV traces a circular path (Figure 3.4(e)-(f)). The radius \( R_0 \) of the circle can be shown to be \( \frac{(u^2 + v^2)^{1/2}}{\psi_c} \). For the chosen turning rate, one has \( R_0 = 13.75 \) (m). It is possible to have a faster turning rate, however, that requires larger control forces. Of course, one can use dorsal fins of different dimension to generate larger control magnitude for faster turning maneuvers. The X-Y plot (Figure 3.4(e)-(f)) shows that the BAUV traces a perfect circle path. The control input magnitude is less than 20 mm. The estimated parameters converge to certain constant values after tracking is achieved and are shown in Figure 3.5(a)-(e).

Extensive simulation has been done using various other uncertainties in the parameters and the nonlinear hydrodynamic coefficients. These results show that the adaptive controller is robust and yaw angle trajectory is precisely accomplished. Moreover, there exists flexibility in the choice of design of parameters \( K, \Gamma, \mu_1, \mu_0 \) and \( a_{rk} \) for shaping the transient and steady state characteristics using reasonable camber of the dorsal fins.
Figure 3.1: Schematic of the BAUV

$X_I - Y_I$ Inertial Coordinate System

$X_B - Y_B$ Body Fixed Coordinate System

Yaw Angle

Dorsal Fin
Figure 3.2: Adaptive exponential trajectory control: $\psi^*_e = 30$ (deg), (a) Heading angle ($\psi$), (b) Camber ($\delta$), (c) Yaw rate ($r$), (d) Lateral velocity ($v$), (e) Estimated parameter ($\hat{\rho}_1$), (f)-(i) Estimated parameters ($\hat{\theta}_1$-$\hat{\theta}_4$)
Figure 3.3: Adaptive sinusoidal trajectory control: $\psi^* = 30 \times \sin(0.2t)$ (deg), (a) Heading angle ($\psi$), (b) Camber ($\delta$), (c) Yaw rate ($r$), (d) Lateral velocity ($v$), (e) Estimated parameter ($\hat{\rho}_1$), (f)-(i) Estimated parameters ($\hat{\theta}_1$-$\hat{\theta}_4$)
Figure 3.4: Adaptive turning maneuver: $\psi_*^* = 15t$ (deg), (a) Heading angle ($\psi$), (b) Camber ($\delta$), (c) Yaw rate ($r$), (d) Lateral velocity ($v$), (e) Global Positions, (f) 3-D plot of X and Y coordinates with time.

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Figure 3.5: Adaptive turning maneuver: $\psi^*_c = 15t$ (deg), (a) Estimated parameter ($\hat{\rho}_1$), (b)-(e) Estimated parameters ($\hat{\theta}_1-\hat{\theta}_4$)
CHAPTER 4

OPTIMAL YAW REGULATION OF BAUV USING PECTORAL FINS

In the previous chapters, control system design employing dorsal fins were considered. This chapter treats the question of control of a bio-robotic autonomous undersea vehicle in the yaw-plane using a bio-mimetic mechanism resembling the pectoral fins of fish. The mechanical foils are assumed to undergo a combined yaw-sway mode of oscillation with the bias angle of the foil as the key control parameter, which is periodically varied for maneuvering the BAUV. The periodic force and moment are obtained using CFD and are represented by fourier series and a discrete-time AUV model is constructed for the control system design. An optimal control law is derived for the control of the yaw angle by minimizing an appropriate quadratic performance index. The choice of performance criterion gives flexibility in shaping the transient responses.

The organization of the chapter is as follows. Section 5.1 describes the mathematical model of the BAUV. The CFD based parameterization and discrete-time representation are obtained in Section 4.2. Section 4.3 presents the optimal control law derivation. The Simulation results and the figure captions are provided in sections 4.4 and 4.5, respectively.

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4.1 Yaw Plane Dynamics

Let the vehicle be moving in the yaw plane \((X_1 - Y_1)\) plane where \(O_1X_1Y_1\) is an inertial coordinate system. \(O_BX_BY_B\) is a body fixed coordinate system, \(X_B\) is in the forward direction, and \(Y_B\) points to the right. In the moving coordinate frame \(O_BX_BY_B\) fixed at the vehicle's geometric center, the dynamics for neutrally buoyant vehicle in the yaw plane are given by [36]

\[
m(\dot{v} + Ur + X_G\dot{r} - Y_Gr^2) = Y_f \dot{r} + (Y_{fi} \dot{v} + Y_r Ur) + Y_v Uv + F_y
\]

\[
I_z \ddot{r} + m(X_G\dot{v} + X_GUr + Y_Gur) = N_f \dot{r} + (N_{fi} \dot{v} + N_r Ur) + N_v Uv + M_y
\]

\[
\dot{\psi} = r
\]

(4.1)

where \(\psi\) is the heading angle, \(r = \dot{\psi}\) is the yaw rate, \(v\) is the lateral velocity, \(x_{GB} = x_G - x_B\), \(y_{GB} = Y_G - Y_B = 0\), \(l\) = body length, \(\rho\) = density, \(m\) is the mass of the AUV, and \(I_z\) is the moment of inertia. \(Y_f\), \(N_f\), \(Y_r\), etc., are the hydrodynamic coefficients. \(F_y\) and \(M_y\) denote the net lateral force and yawing moment acting on the vehicle due to the pectoral fins. Here \(((X_B, Y_B)=0)\) and \((X_G, Y_G)\) denote the coordinates of the center of buoyancy and center of gravity \((cg)\), respectively. Although, the design approach considered in this paper can be used for speed control, here for simplicity, it is assumed that the forward velocity is held steady \((u = U)\) by a control mechanism and only lateral maneuvers are considered. In this study, only small maneuvers of the vehicle are considered. As such linearizing the equations of motion about
$v = 0, r = 0, \psi = 0$, one obtains

$$
\begin{bmatrix}
    m - Y_v & mX_G - Y_r & 0 \\
    mX_G - N_v & I_z - N_r & 0 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    \dot{v} \\
    \dot{r} \\
    \dot{\psi}
\end{bmatrix} =

\begin{bmatrix}
    Y_v U & Y_r U - mU & 0 \\
    N_v U & N_r U - mXGU & 0 \\
    0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
    v \\
    r \\
    \psi
\end{bmatrix}
$$

(4.2)

Defining the state vector $x = (v, r, \psi)^T \in \mathbb{R}^3$, solving (4.2), one obtains a state variable representation of the form

$$
\dot{x} = Ax + B_v \begin{bmatrix}
    F_y \\
    M_y \\
    0
\end{bmatrix}
$$

$$
y = [0, 0, 1]x
$$

(4.3)

for appropriate matrices $A \in \mathbb{R}^{3\times3}$ and $B_v \in \mathbb{R}^{3\times2}$, where $y$ (heading angle) is the controlled output variable.

We are interested in developing an optimal control system for the heading angle regulation to given set points.
4.2 CFD Parameterization and Discrete-time State Variable Representation

It is assumed that the BAUV model has one pair of pectoral fins that are arranged symmetrically around the body of the AUV. Figure 4.1 shows a schematic of a typical AUV. Each fin is assumed to undergo a combined sway-yaw motion described as follows:

\[ s(t) = s_1 \sin(\omega_f t) \]

\[ \theta(t) = \beta + \theta_1 \sin(\omega_f t + \nu_1) \]  \hspace{1cm} (4.4)

where \( s \) and \( \theta \) correspond to the sway and yaw angle of the oscillating fin, respectively. The swaying is assumed to occur about the center-chord location. Furthermore, \( \omega_f, s_1, \theta_1 \) are the frequency and amplitudes of oscillations, \( \beta \) is yaw bias angle and \( \nu_1 \) is the phase difference between the yawing and swaying motions.

As a result of this flapping motion, each fin experiences a time-varying hydrodynamic force which can be resolved into a sway force component \( f_y \) and a yawing moment \( m_y \). The pectoral fin can be suitably attached to the vehicle to produce rolling, and pitching moments on the BAUV which affect its dynamics. However, since yaw-plane dynamics and maneuvering is assumed to be affected by the sway force and moment only, we limit our discussion to these components.

Since \( f_y(t) \) and \( m_y(t) \) produced by each fin are periodic functions, they can be represented by the Fourier series

\[ f_y = \sum_{n=0}^{N} (f_n^s \sin(n\omega_f t) + f_n^c \cos(n\omega_f t)) \]

\[ m_y = \sum_{n=0}^{N} (m_n^s \sin(n\omega_f t) + m_n^c \cos(n\omega_f t)) \]  \hspace{1cm} (4.5)
where it is assumed that the fins produce dominant $N$ harmonically related components and the harmonics of higher frequencies are negligible. The Fourier coefficients $f_n^a$ and $m_n^a$, $a \in \{s, c\}$, capture the characteristics of the time-varying signals $f_y(t)$ and $m_y(t)$. Parametrization of these coefficients is, therefore, needed in order to complete the equations that govern the motion of the BAUV in the yaw plane.

**CFD Based Parameterization**

A finite-difference based, Cartesian grid immersed boundary solver [37] has been used to simulate the flow past flapping foils in the current study. The key feature of this method is that simulations with complex moving bodies can be carried out on stationary non-body conformal Cartesian grids and this eliminates the need for complicated re-meshing algorithms that are usually employed with conventional Lagrangian body-conformal methods. The Eulerian form of the incompressible Navier-Stokes equations is discretized on a Cartesian mesh and boundary conditions on the immersed boundary are imposed through a “ghost-cell” procedure [38]. The method employs a second-order center-difference scheme in space and a second-order accurate fractional-step method for time advancement. The code employs the large-eddy simulation (LES) approach in order to account for the effect of the small subgrid flow scales on the large resolved scales. A Lagrangian dynamic model [39] is used to estimate the subgrid-scale eddy viscosity. The details of the numerical method and validation of the code can be found in [19].

Thin ellipsoidal foils are employed in the current study. The geometry of the foil is defined by its three major axes denoted by $a_x$, $a_y$ and $a_z$ as shown in Figure 4.2. The
surface of the foil is represented by a fine, unstructured mesh with triangular elements. Note that the foil is oriented with the \( x \)-axis along the streamwise direction and the \( z \)-axis along the spanwise direction. Furthermore, \( a_x \) is also the chord of the foil, which in these simulations is set equal to unity and \( a_y \) is the foil thickness. The ratio \( a_y/a_x \) and \( a_z/a_x \) in the current study is equal to 0.12 and 2.0, respectively. In addition to these foil geometric parameters, the following are the other key non-dimensional parameters in the current study: Reynolds number \( Re = U_{\infty}a_x/\nu \); normalized sway amplitude \( s_1/a_x \), yaw-bias angle \( \beta \), yaw amplitude \( \theta_1 \), phase advance of yawing over swaying \( \nu_1 \) and Strouhal number based on the wake thickness \( St = s_1\omega_f/U_{\infty}\pi \). In the current simulations, Reynolds number, \( s_1/a_x \), \( \psi_1 \), \( \nu_1 \) and \( St \) are fixed at value equal to 1000, 0.5, 30°, 90° and 0.6, respectively. The yaw-bias angle, \( \beta \) which is the main control parameter, is varied from 0° to 20°. A non-uniform 177 × 129 × 105 Cartesian mesh is employed in the simulations, where the grid is clustered in the region around the flapping foil and in the foil wake. The size of computational domain as well as the number of grids have been chosen so as to ensure the simulation accuracy.

In the current study, the lift and moment coefficients are defined as

\[
C_Y = \frac{f_y}{\frac{1}{2}\rho U_{\infty}^2 A_{\text{plan}}} \\
C_M = \frac{m_y}{\frac{1}{2}\rho U_{\infty}^2 A_{\text{plan}} a_x}
\]

(4.6)

where \( f_y \) and \( m_y \) are the sway force and yawing moment respectively; and \( A_{\text{plan}} \) is the projected area of the foil which is equal to \( \frac{\pi}{4} a_x a_z \) for the ellipsoidal foils. Forces and moments are calculated by directly integrating the computed pressure and shear stress on the foil surface.
The side views of wake topologies of a yawing-swaying flapping foil with different yaw bias angles, \( \beta = 0^\circ \) and \( 20^\circ \), are shown in Figure 4.3. The isosurfaces of the eigenvalue imaginary part of the velocity gradient tensor of the flow are plotted in order to clearly show the vortex topology [40]. The key feature observed in Figure 4.3(a) is the presence of two sets of interconnected vortex loops that slowly convert into vortex rings as they convect downstream in the case of \( \beta = 0^\circ \). The jets formed by these two set of rings contribute equally to the thrust production of the flapping foil and zero mean sway force is expected. As seen in Figure 4.3(b), when yaw-bias angle increases, one of those two sets of vortex rings becomes weaker and the other one grows. This asymmetry is associated with the production of a mean sway-force on the fin. Figure 4.4 shows the time-averaged streamwise velocity contours for both of these two cases. For the \( \beta = 0^\circ \) foil, two oblique jets with equal strength are observed. As yaw-bias angle increases, the lower jet becomes stronger while the upper jet essentially disappears. As a result of this, the sway force is modified significantly. Table ?? shows the changes in the mean side force coefficient for different bias angles. It can be seen that small changes in the yaw-bias angle can produce large changes in the mean side force. This clearly proves that the yaw-bias angle is an effective control parameter for precise maneuvering.

<table>
<thead>
<tr>
<th>Yaw-bias Angle</th>
<th>( C_Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0^\circ )</td>
<td>-0.057</td>
</tr>
<tr>
<td>( 10^\circ )</td>
<td>1.519</td>
</tr>
<tr>
<td>( 20^\circ )</td>
<td>2.744</td>
</tr>
</tbody>
</table>
We assume that the bias-angle (control input) $\beta$ is varied periodically and the remaining oscillation parameters are constant. It has been experimentally shown that the mean values of the side force and the yawing moment vary almost linearly with $\beta$ and the amplitudes of the fin force and moment are functions of $\beta$ [20, 21, 41].

Expanding the fin force and moment of each fin in a Taylor series about $\beta = 0$ gives

$$f_y(t, \beta) = f_y(t, 0) + \frac{\partial f_y}{\partial \beta}(t, 0)\beta + O(\beta^2)$$

$$m_y(t, \beta) = m_y(t, 0) + \frac{\partial m_y}{\partial \beta}(t, 0)\beta + O(\beta^2)$$

(4.7)

where $O(\beta^2)$ denotes higher order terms. We assume here that for a fixed $\beta \in R$, $f_y(t+T_0, \beta) = f_y(t, \beta)$ and $m_y(t+T_0, \beta) = m_y(t, \beta)$, $t > 0$ ($T_0$ denotes the fundamental period). Then the partial derivatives of $f_y$ and $m_y$ with respect to $\beta$ are also periodic functions of time. Using (4.7), one can approximately express $f_y$ and $m_y$ as

$$f_y = \sum_{n=0}^{N} f_n^a(0) \sin nwft + f_n^c(0) \cos nwft +$$

$$\sum_{n=0}^{N} \left( \frac{\partial f_n^a}{\partial \beta}(0) \sin nwft + \frac{\partial f_n^c}{\partial \beta}(0) \cos nwft \right) \beta$$

$$m_y = \sum_{n=0}^{N} m_n^a(0) \sin nwft + m_n^c(0) \cos nwft +$$

$$\sum_{n=0}^{N} \left( \frac{\partial m_n^a}{\partial \beta}(0) \sin nwft + \frac{\partial m_n^c}{\partial \beta}(0) \cos nwft \right) \beta$$

(4.8)

where $O(\beta^2)$ terms are ignored in the series expansion.

Thus, we get

$$f_y(t) = \phi^T (f_a + \beta f_b)$$
\[ m_y(t) = \phi^T(m_a + \beta m_b) \]  
\[ \phi = \begin{bmatrix} 1 & \sin \omega_{ft} & \ldots & \sin N \omega_{ft} \cos N \omega_{ft} \end{bmatrix}^T \]

where \( f_a, f_b, m_a, m_b \in \mathbb{R}^{2N+1} \) and can be obtained from (4.8).

Thus, in order to complete the equations that govern the motion of the BAUV in the yaw plane, the Fourier components of the force are needed. Figure 4.5 shows the time variation of computed fin force and the harmonic components of the fin force for yaw bias angle of \( 0^\circ \) and \( 20^\circ \), respectively. Note the even modes \( (n = 2, 4) \) for the zero bias angle have negligible contribution to the fin force. It is also seen that fin force of larger magnitude is obtained when the bias angle is increased.

Furthermore, the amplitudes of higher harmonics diminish as \( n \) increases. Table ?? and ?? show both the force and moment Fourier coefficients for the yaw-bias of \( 0^\circ \) and \( 20^\circ \) for different harmonics (see (4.8)). It is seen from the table that the Fourier coefficients of the fourth harmonic are quite small compared to the coefficients of the first harmonic. As such even four harmonic components are sufficient to capture most of the characteristics of the time-varying signals \( f_y(t) \) and \( m_y(t) \).

Table 4.2: Table showing various components of force and moment coefficient for the \( \beta_y = 0^\circ \) case.

<table>
<thead>
<tr>
<th>( n )</th>
<th>( f_n^c )</th>
<th>( f_n^s )</th>
<th>( m_n^c )</th>
<th>( m_n^s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>1</td>
<td>5.62</td>
<td>-5.16</td>
<td>0.90</td>
<td>1.11</td>
</tr>
<tr>
<td>2</td>
<td>0.08</td>
<td>-0.05</td>
<td>-0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>-1.31</td>
<td>0.8</td>
<td>-0.17</td>
<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>0.09</td>
<td>-0.02</td>
<td>-0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Discrete time State Variable representation

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Table 4.3: Table showing various components of force and moment coefficient for the \( \beta_y = 20^\circ \) case.

<table>
<thead>
<tr>
<th>n</th>
<th>( f_n^c )</th>
<th>( f_n^s )</th>
<th>( m_n^c )</th>
<th>( m_n^e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.74</td>
<td>0.00</td>
<td>-0.26</td>
<td>0.00</td>
</tr>
<tr>
<td>1</td>
<td>-6.75</td>
<td>-4.98</td>
<td>0.75</td>
<td>0.65</td>
</tr>
<tr>
<td>2</td>
<td>-0.68</td>
<td>2.54</td>
<td>0.01</td>
<td>-0.14</td>
</tr>
<tr>
<td>3</td>
<td>-1.1</td>
<td>0.63</td>
<td>-0.13</td>
<td>0.04</td>
</tr>
<tr>
<td>4</td>
<td>0.15</td>
<td>0.14</td>
<td>-0.03</td>
<td>0.04</td>
</tr>
</tbody>
</table>

The vehicle has two attached fins; therefore the net force and moment are \( F_y = 2f_y \) and \( M_y = 2d_{cgf}f_y + m_y \), where \( d_{cgf} \) is the moment arm due to the fin location (positive forward). Using (4.9), the yaw plane dynamics (4.3) can be written as

\[
\dot{x} = Ax + B\Phi(t)f_c + B\Phi(t)f_v\beta
\]  

where \( B[f_y, m_y]^T = B_0[F_y, M_y]^T \), \( f_c = (f_a^T, m_a^T)^T \in \mathbb{R}^{4N+2} \), and \( f_v = (f_b^T, m_b^T)^T \in \mathbb{R}^{4N+2} \) where

\[
\Phi(t) = \begin{bmatrix}
\phi^T(t) & 0 \\
0 & \phi^T(t)
\end{bmatrix}
\]

For the purpose of control, the bias angle is changed at a periodic interval of \( T^* \), where \( T^* \) is an integer multiple of the period \( T_0 \), i.e., \( T^* = n_0T_0 \), where \( n_0 \) is a positive integer. This way one switches the bias angle at an uniform rate of \( T^* \) seconds at the end of \( n_0 \) cycles. For the derivation of the control law, the transients introduced due to changes in the bias-angle are ignored. Since the bias is changed periodically, it will be convenient to express the continuous-time system (4.10) as a discrete-time system. The function \( \beta(t) \) now has piecewise constant values \( \beta_k \) for \( t \in [kT^*, (k+1)T^*) \), \( k = 0, 1, 2 \ldots \).
Discretizing the state equation (4.10), one obtains a discrete-time representation of the form

\[ x[(k + 1)T^*] = A_dx(kT^*) + B_d\beta_k + d \quad (4.12) \]

where \( A_d = e^{AT^*} \), \( B_0 = \int_0^{T^*} e^{A\tau}B\Phi(-s)ds \), \( B_d = B_0f_v \in R^3 \), and \( d = B_0f_c \in R^3 \).

The output variable \( (\psi) \) is

\[ y(kT^*) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x(kT^*) \equiv C_d x(kT^*) \quad (4.13) \]

4.3 Optimal Yaw Plane Control System

In this section, the design of an optimal feedback yaw-plane control law for the regulation of the yaw angle is considered. For the precise yaw control, it is desirable to include a feedback term in the control law which is proportional to the integral of the yaw tracking error. For this purpose, a new state variable \( x_s \) is introduced which satisfies

\[ x_s[(k + 1)T^*] = \psi^* - y(kT^*) + x_s(kT^*) \quad (4.14) \]

where \( \psi^* \), a constant, is the desired yaw angle and \( \psi^* - y(kT^*) \) is the tracking error.

Defining the state vector \( x_a = (x^T, x_s)^T \in R^4 \) and using equations (4.12) and (4.14), the augmented system takes the form

\[
\begin{bmatrix}
A_d & 0 \\
-C_d & 1
\end{bmatrix}
\begin{bmatrix}
x(kT^*) \\
x_s(kT^*)
\end{bmatrix} +
\begin{bmatrix}
B_d \beta_k + d \\
0 \psi^*
\end{bmatrix}
\]
\[
\dot{x}_a(kT^*) = A_a x_a(kT^*) + B_a \beta_k + d_a
\]  
(4.15)

where the constant matrices \(A_a, B_a\) and \(d_a\) are defined in (4.15).

The control of the system (4.15) can be accomplished by following the servomechanism design approach [42] in which \(d_a\) is treated as a constant disturbance input. The design is completed by computing a feedback control law of the form

\[
\beta(kT^*) = -K x_a(kT^*), k = 0, 1, 2, ...
\]  
(4.16)

where \(K\) is a constant row vector such that the closed-loop matrix

\[
A_c = (A_a - B_a K)
\]

is stable. It is well known that one can assign the eigenvalues of \(A_c\) arbitrarily if \((A_a, B_a)\) is controllable [42, 43]. For the discrete-time system, this implies that one must choose \(K\) such that the eigenvalues of \(A_c\) are strictly within the unit disk in the complex plane.

In this study, an appropriate value of \(K\) is obtained by using the linear quadratic optimal control theory [42]. For this one chooses a performance index of the form

\[
J_o = \sum_{k=0}^{\infty} x_a^T(kT^*) Q x_a(kT^*) + \beta_k^2 \mu
\]  
(4.17)

where \(Q\) is a positive definite symmetric matrix and \(\mu > 0\). The weighting matrix \(Q\) associated with \(x_a\) and the parameter \(\mu\) penalizing the level of the bias angle are chosen to provide trade-off between the convergence rate of the state variables to the equilibrium point and the bias angle magnitude.

The optimal control law is obtained by minimizing \(J_o\) for the system

\[
x_a[(k + 1)T^*] = A_a x(kT^*) + B_a \beta_k
\]  
(4.18)
which is obtained from (4.15) by setting \( d_a = 0 \). The feedback matrix \( K \) is obtained by solving the discrete Riccati equation [42]

\[
P = Q + A_a^T P A_a - A_a^T P B_a (\mu + B_a^T P B_a) B_a^T P A_a
\]

and then setting the feedback matrix as

\[
K = -(\mu + B_a^T P B_a)^{-1} B_a^T P A_a
\]

Using the feedback law (4.16), the yaw angle can be regulated to prescribed constant values \( \psi^* \), but the BAUV cannot follow time-varying yaw angle trajectories. In the next section, an inverse control law is derived for the tracking of time-varying trajectories.

4.4 Simulation Results

In this section, simulation results using the Matlab/Simulink software is presented. The performance of the optimal controller for different values of frequencies of oscillation of the pectoral fin and for different points of attachment of the fins to the BAUV \( d_{cf} \) from the center of gravity of the AUV is examined.

The parameters of the model are taken from [36]. The AUV is assumed to move with a constant forward velocity of 0.7 (m/sec) with the help of a control mechanism. The vehicle parameters are \( l = 1.391 \) (m), mass=18.826 (kg), \( I_z = 1.77 \) (kgm²), \( X_G = -0.012 \), \( Y_G = 0 \). The hydrodynamic parameters for a forward velocity of 0.7 m/sec derived from [32] are \( Y_\dot{r} = -0.3781 \), \( Y_\dot{\psi} = -5.6198 \), \( Y_r = 1.1694 \), \( Y_\dot{\psi} = -12.0868 \), \( N_\dot{r} = -0.3781 \), \( N_\dot{\psi} = -0.8967 \), \( N_r = -1.0186 \), and \( N_\psi = -4.9587 \).
Experimental results indicate that for zero bias angle, the mean values of $f_y$ and $m_y$ are zero. Therefore, the vectors $f_a$, $f_b$, $m_a$, and $m_b$ are found to be

$$f_a = (0, -40.0893, -43.6632, -0.3885, 0.6215, 6.2154, -10.17, -0.1554, 0.6992)$$

$$f_b = (68.9975, 0.4451, -16.4704, 64.1009, -19.5864, -0.8903, -2.2257, 2.2257, 4.8966)$$

$$m_a = (0.0054, 0.6037, 0.4895, 0, -0.0054, 0, -0.0925, 0, -0.0054)$$

$$m_b = (-0.5297, -0.3739, -0.0935, -0.2493, 0.1246, 0.0312, -0.0312, 0.0935, 0)$$

It is pointed out that these parameters are obtained from the force and moment Fourier coefficients and are computed by multiplying the Fourier coefficients by $\frac{1}{2} \rho W_a U_\infty^2$ and $\frac{1}{2} \rho W_a \text{chord} U_\infty^2$ respectively, where $W_a$ is the surface area of the foil.

For simulation, the initial conditions of the vehicle are assumed to be $x(0) = 0$ and $x_s(0) = 0$. The feedback discrete control law (4.16) is simulated. The bias angle is changed to a new value every $T^* = n_0 T_0$ seconds where $T_0 = \frac{1}{f_0}$ is the fundamental period of $f_p$ and $m_p$. Choosing a small value of $n_0$ increases the transients produced due to switching. A large value of $n_0$ increases the magnitude of the intersample oscillations which is also not desirable.

The terminal state is chosen as $x^* = (0, 0, 15)^T$ with $\psi^* = 15$ (deg). Thus one desires to control the BAUV to a heading angle of 15 (deg). For optimal control design, the weighting matrix and parameter are selected as $Q = 1000I_{4 \times 4}$ and $\mu = 1.5$.

Simulation results are provided for fin frequencies of 8 Hz and 6 Hz.
**Case 1** Optimal Control: Frequency of Fin Oscillation 8 Hz

First simulation is done for the higher frequency of 8 Hz and the fin attachment point is chosen such that $d_{cgf} = 0$. The control law is updated every four cycles, i.e., $T^* = 4T_0 = 0.5$ (sec). The results are shown in Figure 4.6. The zeros of the vehicle transfer function are -1.5548 and 0.0965. It can be seen that the optimal controller achieves accurate heading angle control to the target set point in approximately 5 seconds. The control input (bias angle) magnitude required is less than 3 (deg), which is small and can easily be provided by the pectoral fins. The plots of the lateral force and moment produced by the fins are also provided in the figure. In the steady state, the lateral fin force and moment exhibit bounded periodic oscillations. The inter sample yaw angle shows oscillations of tiny amplitude, however, in the terminal phase, the sample values of yaw angle is equal to the commanded value $\psi^*$.

Simulation results for the same frequency, but for a $d_{cgf}$ value of 0.15 (m) are also presented (Figure 4.7). Now the zeros are at 1.48 and -0.75. It can be seen that set point control takes longer time as compared to the above case. It is observed that initially the vehicle heading angle swings in the wrong direction. Of course, this is because, in this case, the unstable zero has moved farther away from the region of stability (the unit disk). One can also observe that the control input (max) required has decreased compared to the previous case.

**Case 2** Optimal Control: Frequency of Fin Oscillation 6 Hz

Now, simulation is done for a lower value of fin frequency of 6 (Hz) with a $d_{cgf}$ value of 0, but the sampling period now is $T^* = 4T_0 = 2/3$ seconds. Thus compared to the
case of 8 Hz, the control is updated at a slower rate. The simulation results are shown in Figure 4.8. One can observe that the yaw angle control is accomplished, however intersample oscillations of larger magnitude compared to Figure 4.6 are present. This is an expected phenomenon because the bias angle switches after a longer period. But it is seen that the convergence time of the yaw angle is almost same. The maximum magnitude of control input required for the maneuver is also larger. The Normal force and moment were found to be less that 50 (N) and 1 (Nm), respectively.

Simulation for a $d_{cgf}$ value of 0.15 (m) was also performed (The Results are not shown here). It was observed that the magnitude of the intersample oscillations increases as the unstable zero of the system moves farther away from the unit disk in the unstable region.

Figure 4.1: Schematic of the AUV
Figure 4.2: A thin ellipsoidal foil defined in terms of a surface mesh with triangular elements.
Figure 4.3: Side view of wake structures for flow past the flapping foil with two different yaw-bias angles. (a) bias angle 0°, b) bias angle 20°
Figure 4.4: Center-plane time-averaged streamwise velocity contours for flow past the flapping foil with two different yaw-bias angles. Black lines are the streamwise velocity profiles. (a) bias angle 0°, b) bias angle 20°
Figure 4.5: Time variation of the computed force fin force and harmonic modes at two different yaw-bias angles. (a) bias angle 0°, b) bias angle 20°
Figure 4.6: Optimal Control: Frequency of Flapping = 8 Hz, $d_{cgf} = 0$ (m) for $\psi^* = 15$ (deg), (a) Heading Angle ($\psi$), (b) Bias Angle ($\beta$), (c) Yaw Rate ($r$), (d) Lateral Velocity ($v$), (e) Lateral Force ($F_y$), (f) Side Moment ($M_y$)

Figure 4.7: Optimal Control: Frequency of Flapping = 8 Hz, $d_{cgf} = 0.15$ (m) for $\psi^* = 15$ (deg), (a) Heading Angle ($\psi$), (b) Bias Angle ($\beta$), (c) Yaw Rate ($r$), (d) Lateral Velocity ($v$), (e) Lateral Force ($F_y$), (f) Side Moment ($M_y$)
Figure 4.8: Optimal Control: Frequency of Flapping = 6 Hz, $d_{cgf} = 0$ (m) for $\psi^* = 15$ (deg), (a) Heading Angle ($\psi$), (b) Bias Angle ($\beta$), (c) Yaw Rate ($\tau$), (d) Lateral Velocity ($v$), (e) Lateral Force ($F_y$), (f) Side Moment ($M_y$)
CHAPTER 5

INVERSE CONTROL OF A BAUV USING PECTORAL FINS

In the optimal control system design of chapter four, the controller is not capable of following time varying trajectories. In this chapter, the inverse control design in the yaw plane using pectoral fins of fish is considered. This controller is capable of accomplishing time varying trajectory following. The bias angle of the foil is chosen as the key control parameter and is periodically varied for maneuvering the BAUV. The periodic force and moment are obtained using CFD and are represented by fourier series and a discrete-time AUV model is constructed for the control system design. It is well known that the inverse controller works only when the system is minimum phase. However, it turns out that the pulse transfer function has unstable zeros. To overcome this obstruction, an approximate discrete-time system is constructed by eliminating the unstable zeros. An analytical expression of the output matrix of the approximate minimum phase system is derived. Then, an inverse control law is derived for the tracking of time-varying yaw angle trajectories based on the modified transfer function. It is shown that the controller designed based on the new output variable accomplishes accurate following of the yaw trajectory.
5.1 Inverse Yaw Plane Control

In this section, the design of an inverse control law for the regulation of the yaw angle is considered. The mathematical model, CFD parameterization and discrete-time state variable representation presented in chapter three is used for the derivation of the inverse control law. The state variable representation of the system is given by (refer Equation (4.3))

\[
\dot{x} = Ax + B_v \begin{bmatrix} F_y \\ M_y \end{bmatrix}
\]
\[
y = [0, 0, 1]x
\]  

(5.1)

where \( A \in \mathbb{R}^{3\times3} \) and \( B_v \in \mathbb{R}^{3\times2} \), and \( y \) (heading angle) is the controlled output variable.

Now, by the parameterization of the force and moments, one expresses them as a function of \( \beta \) and are given by (refer Equation (4.9))

\[
f_y(t) = \phi^T(f_a + \beta f_b)
\]
\[
m_y(t) = \phi^T(m_a + \beta m_b)
\]

\[
\phi = \begin{bmatrix} 1 \sin \omega_f t \ldots \sin N \omega_f t \cos N \omega_f t \end{bmatrix}^T
\]

(5.2)

where \( f_a, f_b, m_a, m_b \in \mathbb{R}^{2N+1} \)

Then, by discretization of the BAUV system model and by including the force and moment expressions, one obtains (refer Equation (4.12)),

\[
x[(k + 1)T^*] = A_dx(kT^*) + B_d\beta_k + d
\]  

(5.3)

where \( A_d = e^{AT^*} \), \( B_0 = \int_0^{T^*} e^{As} B\Phi(-s)ds \), \( B_d = B_0 f_v \in \mathbb{R}^3 \), and \( d = B_0 f_c \in \mathbb{R}^3 \).
The output variable \( (\psi) \) is

\[
y(kT^*) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x(kT^*) = C_d x(kT^*)
\] (5.4)

The transfer function relating the output \( y(kT^*) \) and the input \( \beta_k \) of (5.3) (assuming that \( d = 0 \)) is given by

\[
\frac{\hat{y}(z)}{\hat{\beta}(z)} = G(z) = C_d(zI - A_d)^{-1}B_d = k_p \frac{(z + \mu_1)(z + \mu_2)}{z^3 + a_2 z^2 + a_1 z + a_0}
\] (5.5)

where \( z \) denotes the Z-transform variable, \( \mu_i (i = 1, 2) \) are real or complex numbers and \( k_p \) and \( a_i (i = 0, 1, 2) \) are real numbers. It is assumed that the pectoral fins are attached between the cg and the nose of the vehicle. For the AUV model under consideration, the number of unstable zeros (i.e., the zeros outside the unit disk in the complex plane) depend on the distance \( d_{cgf} \) of the pectoral fins from the cg, \( \omega_f \), and the sampling time \( T^* \). It has been found that for the values of interest of the oscillation frequencies and the attachment point \( d_{cgf} \) of the fins, there exists a single unstable zero (i.e., the transfer function is nonminimum phase).

It is well known that the inverse control design can be accomplished only when the system is minimum phase (i.e., the zeros of the transfer function are stable). For this purpose, the original transfer function is simplified by ignoring its unstable zero. Let us assume that \( \mu_1 > 1 \) and \( \mu_2 < 1 \). For obtaining a minimum phase approximate system, one removes the unstable zero of \( G(z) \) but retains the zero frequency (dc) gain. Thus the approximate transfer function \( G_a(z) \) obtained from (6.1) takes the
form

\[ G_a(z) = k_p \frac{(1 + \mu_1)(z + \mu_2)}{\Delta(z)} \]  \quad (5.6)

where \( \Delta(z) = \text{det}(zI - A_d) \).

We are interested in deriving a new controlled output variable \( y_a \) such that

\[ y_a(kT^*) = C_a x(kT^*) \]  \quad (5.7)

\[ \frac{\dot{y}_a(z)}{\beta(z)} = G_a(z) = C_a (zI_n - A_d)^{-1} B_d \]  \quad (5.8)

where \( C_a \) is a new output matrix. Since the relative degree of \( G_a(z) \) is 2, one has

\[ C_a B_d = 0, \]

\[ C_a A_d B_d \neq 0 \]  \quad (5.9)

Using the Leverrier algorithm, the approximate transfer function \( G_a(z) \) can be expanded as [43]

\[ G_a(z) = \Delta^{-1}(z)[(z + a_2)C_a A_d B_d + C_a A_d^2 B_d] \]  \quad (5.10)

Comparing (5.5) and (5.9), one can easily show that

\[ C_a [B_d \quad A_d B_d \quad A_d^2 B_d + a_2 A_d B_d] = [0 \quad K_p (1 + \mu_1) \quad K_p (1 + \mu_1) \mu_2] \]  \quad (5.11)

Solving equation (5.11), one obtains the modified output matrix.

For the modified system, one has

\[ x[(k + 1)T^*] = A_d x(kT^*) + B_d \beta_k + d \]

\[ y_a(kT^*) = C_a x(kT^*) \]  \quad (5.12)
Suppose a reference trajectory $y_r(kT^*)$ is given which is to be tracked by $y_a(kT^*)$. Using Equation (5.12), one has that

$$y_a[(k + 1)T^*] = C_aA_dx(kT^*) + C_ad$$

$$y_a[(k + 2)T^*] = C_aA_d^2x(kT^*) + \sum_{i=0}^{1} C_aA_d^id + C_aA_d^{(r-1)}B_d\beta_k$$

(5.13)

In view of (5.13), for following the reference trajectory $y_r(kT^*)$, we choose the control input $\beta_k$ as

$$\beta_k = (C_aA_dB_d)^{-1}[-C_aA_d^2x(kT^*) - \sum_{i=0}^{1} C_aA_d^id + v_k]$$

(5.14)

where the signal $v_k$ is selected as

$$v_k = y_r[(k + 2)T^*] - p_1[C_aA_dx(kT^*) + C_ad - Y_r((k + 1)T^*)] + p_0(Y_a(kT^*) - Y_r(kT^*))$$

(5.15)

where $p_0$ and $p_1$ are real numbers.

Defining the tracking error $e(kT^*) = y_a(kT^*) - y_r(kT^*)$, and using the control law (5.14) and Equation (5.15) in (5.13) gives

$$e[(k + 2)T^*] + p_1e[(k + 1)T^*] + p_0e(kT^*) = 0$$

(5.16)

The tracking error equation (5.16) satisfies a second order difference equation. The characteristic polynomial associated with (5.16) is

$$(z^2 + p_1z + p_0) = 0$$

(5.17)

The parameters $p_i$ are chosen such that the roots of (5.17) are strictly within the unit disk. Then it follows that for any initial condition $x(0), e(kT^*) \to 0$ as $k \to \infty$
and the controlled output $y_a(kT^*)$ asymptotically converges to the reference sequence $y_r(kT^*)$. This completes the inverse controller design. In the next section it will be seen that the inverse controller designed based on the approximate transfer function accomplishes accurate yaw angle trajectory control.

5.2 Simulation Results

In this section, simulation results using the Matlab/Simulink software is presented. The performance of the inverse controller for different values of frequencies of oscillation of the pectoral fin and for different points of attachment of the fins to the BAUV ($d_{cgl}$) from the center of gravity of the AUV is examined.

The vehicle and hydrodynamic parameters are taken from [36] and are presented in chapter four (section 4.3). The AUV is assumed to move with a constant forward velocity of 0.7 (m/sec) with the help of a control mechanism.

For simulation, the initial conditions of the vehicle are assumed to be $x(0) = 0$ and $x_a(0) = 0$. The bias angle is changed to a new value every $T^* = n_0T_0$ seconds where $T_0 = \frac{1}{f_0}$ is the fundamental period of $f_p$ and $m_p$. Choosing a small value of $n_0$ increases the transients produced due to switching. A large value of $n_0$ increases the magnitude of the intersample oscillations which is also not desirable.

It is possible to track time-varying trajectories with the inverse controller. Simulation results for sinusoidal heading angle trajectory tracking for different fin flapping frequencies are presented. Smooth sinusoidal reference trajectories are generated by
command generators of the form

\[(E^2 + p_{c1}E + p_{c0})y_r(kT^*) = (1 + p_{c0} + p_{c1})d^* \sin(w_rkT^*)\]

where \(E\) denotes the advance operator \((Ey_r(kT^*) = y_r[(k + 1)T^*])\) and \(d^*\) is the amplitude of the sine wave and the parameters \(p_{ci}\) are chosen to be zero so that the poles of the command generator are at \(z = 0\). The reference trajectory generator is simulated using its state variable form with states \(x_r = (x_{r1}, x_{r2}, x_{r3})^T\). For the simulation, \(d^* = 15\) (deg) and \(w_r = 0.2\) (rad/s).

Simulation results for fin frequencies of 8 Hz and 6Hz are presented in the following sub-section.

**Case 1 Inverse Control: Frequency of Fin Oscillation 8 Hz**

Figure 6.1 shows the inverse controller performance for a \(d_{cgf}=0\). The sampling period is \(4T^* = \frac{1}{2}\). It can be observed that smooth heading angle trajectory control is achieved. One can observe that the modified output equal the reference trajectory at all sample instants. The maximum control input (bias angle) required is approximately 3 (deg). The lateral force and moment produced by the fins are less than 40 (N) and 0.4 (Nm), respectively. As expected, the yaw rate and the lateral velocity are sinusoidal.

Simulation results are also presented in Figure 6.2 for \(d_{cgf}=0.15\) (m). Though heading angle tracking is accomplished, the yaw trajectory is not smooth. This can be attributed to the fact that the unstable zero has larger magnitude for larger \(d_{cgf}\) and hence elimination of the unstable zero leads to a poor approximation of the original transfer function. The bias angle (control input) is little less than 10 (deg).
It is also seen that there is no significant change in the required lateral force and moment. The yaw rate and lateral velocity are much higher than for the case with smaller $d_{cgf}$.

**Case 2 Inverse Control: Frequency of Fin Oscillation 6 Hz**

Simulation for a $d_{cgf}=0$ has been performed and the results are shown in Figure 6.3. Yaw angle tracking is achieved although intersample oscillations of comparatively large magnitude are observed. The bias angle (control input) required is less than 3 deg. The lateral force and moment produced are much larger compared to a fin frequency of 8 Hz. One can also notice that the yaw rate and lateral velocity are almost same.
Figure 5.1: Inverse Control: Frequency of Flapping = 8 Hz, $d_{cg} = 0$ (m), $\psi^* = 15$ (deg), (a) Heading Angle ($\psi$) and reference trajectory ($y_r$), (b) Bias Angle ($\beta$), (c) Yaw Rate ($r$), (d) Lateral Velocity ($v$), (e) Lateral Force ($F_y$), (f) Side Moment ($M_y$)
Figure 5.2: Inverse Control: Frequency of Flapping = 8 Hz, $d_{cf} = 0.15$ (m), $\psi^* = 15$ (deg), (a) Heading Angle ($\psi$) and reference trajectory ($y_r$), (b) Bias Angle ($\beta$), (c) Yaw Rate ($r$), (d) Lateral Velocity ($v$), (e) Lateral Force ($F_y$), (f) Side Moment ($M_y$)

Figure 5.3: Inverse Control: Frequency of Flapping = 6 Hz, $d_{cf} = 0$ (m), $\psi^* = 15$ (deg), (a) Heading Angle ($\psi$) and reference trajectory ($y_r$), (b) Bias Angle ($\beta$), (c) Yaw Rate ($r$), (d) Lateral Velocity ($v$), (e) Lateral Force ($F_y$), (f) Side Moment ($M_y$)
CHAPTER 6

ADAPTIVE OPTIMAL CONTROL OF A BAUV IN THE DIVE PLANE USING PECTORAL FINS

Maneuvering of BAUVs using optimal control and inverse control techniques have been discussed in the previous chapters. However, neither of the controllers work when the system parameters are not known. In this chapter, a discrete-time indirect adaptive control approach to maneuver a low speed BAUV in the dive plane using pectoral fins is considered. It is assumed that the model parameters are unknown and only the depth of the vehicle is available for feedback. The thrust required for control is obtained by periodically varying the bias angle of the pectoral fins. The force and moment produced by the flapping of the fins are parameterized using computational fluid dynamics (CFD) and expanded as a Fourier series. A discrete-time model of the BAUV is developed for the purpose of control. The control system consists of a least-squares based identifier for online parameter estimation, an observer for state estimation, and a controller for setpoint tracking. Simulation results show that the control system accomplishes accurate depth trajectory tracking in spite of large uncertainties in the system parameters.
6.1 Dive Plane Dynamics

A schematic of the AUV model with the pectoral fins and the coordinate systems is shown in Figure 6.1. Let the vehicle be moving in the dive plane \((X_I - Z_I \text{ plane})\) where \(O_I X_I Z_I\) is an inertial coordinate system. \(O_B X_B Z_B\) is a body fixed coordinate system, where \(X_B\) is the forward direction and \(Z_B\) points down.

The heave and pitch equations of motion for a neutrally buoyant vehicle are described by coupled nonlinear differential equations with respect to the moving coordinate frame \(O_B X_B Z_B\). These equations describing the AUV model [36] are

\[
m[w - uq - x_G \dot{q} - z_G q^2] = Z_q \dot{q} + Z_\dot{w} \dot{w} + Z_{uw} w + Z_{qg} u + F_p v
\]

\[
I_y \dot{q} - m[x_G (w - uq) - z_G wq] = M_q \dot{q} + M_\dot{w} \dot{w} + M_{uw} w + M_{qG} w + M_p v
\]

\[
-x_G B W \cos \theta - z_G B W \sin \theta
\]

\[
\dot{z} = w - u \theta
\]

\[
\dot{\theta} = q
\]

(6.1)

where \(\theta\) is the pitch angle; \(w\) is the heave velocity (along body axis \(Z_B\)), \(q = \dot{\theta}\), \(x_G = x_G - x_B\), \(z_G = z_G - z_B\); and \(z\) is the depth. \(m\) and \(I_y\) denote the mass and inertia of the AUV. \(F_p\) and \(M_p\) are the net force and moment acting on the vehicle due to the pectoral fins. \(M_{\dot{w}}, M_q, M_{uw}, Z_\dot{w}, Z_q, Z_{uw}\) and \(Z_{uw}\) denote the hydrodynamic coefficients.

Although, the design approach considered in this paper can be used for speed control, here for simplicity, it is assumed that the forward velocity is held steady
(\(u = U\)) by a control mechanism and only depth maneuvers are considered. As such linearizing the equations of motion about \((w, q, \theta, z) = 0\), one obtains

\[
\begin{bmatrix}
  m - Z_w & -mX_G - Z_q & 0 \\
  -mX_G - M_w & I_y - M_q & 0 \\
  0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
w \\
q \\
z \\
\end{bmatrix}
\begin{bmatrix}
\dot{w} \\
\dot{q} \\
\dot{z} \\
\end{bmatrix}
\]

Defining the state vector \(x = (w, q, z, \theta)^T \in \mathbb{R}^4\), solving Equation (6.2), one obtains a state variable representation of the form

\[
\dot{x} = Ax + Bu
\]

\[
y = [0, 0, 1, 0] x
\]

for appropriate matrices \(A \in \mathbb{R}^{4 \times 4}\) and \(B_u \in \mathbb{R}^{4 \times 2}\), where \(y\) (heading angle) is the controlled output variable.

We are interested in designing a discrete-time adaptive control system for the depth control and regulation of the state vector to the equilibrium point.

6.2 CFD Parameterization and Discrete State Variable Representation

It is assumed that the BAUV model has one pair of pectoral fins that are arranged symmetrically around the body of the AUV. Fig. 1 shows a schematic of a typical
AUV. Each fin is assumed to undergo a combined pitch-and-heave motion described as follows:

\[ h(t) = h_1 \sin(\omega_f t) \]

\[ \psi(t) = \beta_\psi + \psi_1 \sin(\omega_f t + \nu_1) \]  

(6.4)

where \( h \) and \( \psi \) correspond to the heave and pitch angle of the oscillating fin, respectively. The pitching is assumed to occur about the center-chord location. Furthermore, \( \omega_f, h_1, \psi_1 \) are the frequency and amplitudes of oscillations, \( \beta_\psi \) is pitch bias angle and \( \nu_1 \) is the phase difference between the pitching and heaving motions.

As a result of this flapping motion, each fin experiences a time-varying hydrodynamic force which can be resolved into a thrust force component \( f_p \) and a pitching moment \( m_p \).

Since \( f_p(t) \) and \( m_p(t) \) produced by each fin are periodic functions, they can be represented by the Fourier series

\[ f_p = \sum_{n=0}^{N} (f_n^s \sin(n\omega_f t) + f_n^c \cos(n\omega_f t)) \]

\[ m_p = \sum_{n=0}^{N} (m_n^s \sin(n\omega_f t) + m_n^c \cos(n\omega_f t)) \]  

(6.5)

where it is assumed that the fins produce dominant \( N \) harmonically related components and the harmonics of higher frequencies are negligible. The Fourier coefficients \( f_n^a \) and \( m_n^a, a \in \{s, c\} \), capture the characteristics of the time-varying signals \( f_p(t) \) and \( m_p(t) \). Parametrization of these coefficients is, therefore, needed in order to complete the equations that govern the motion of the BAUV in the dive plane.

CFD Based Parameterization
A finite-difference based Cartesian grid immersed boundary solver [37] has been used to simulate the flow past the flapping foils in the current study. It is noted that the parameterization of the forces produced by the pectoral fins both in the yaw-plane and the dive-plane using CFD techniques is the same. Thus, CFD parameterization method explained in Chapter 4 (Section 4.2) is applicable to the dive plane as well. The force and moment are expressed in terms of the bias angle $\beta$ as

$$f_p(t) = \phi^T(f_a + \beta f_b)$$

$$m_p(t) = \phi^T(m_a + \beta m_b)$$

$$\phi = \begin{bmatrix} 1 & \sin \omega_f t & \ldots & \sin N \omega_f t & \cos N \omega_f t \end{bmatrix}^T$$

where $f_a, f_b, m_a, m_b \in R^{2N+1}$

**Discrete time State variable representation**

The vehicle has two attached fins; therefore the net force and moment are $F_p = 2 f_p$ and $M_p = 2 (d_{cgf} f_p + m_p)$, where $d_{cgf}$ is the moment arm due to the fin location (positive forward). Based on the CFD parameterization, the dive plane dynamics (6.3) can be written as

$$\dot{x} = Ax + B\Phi(t)f_c + B\Phi(t)f_v\beta$$

(6.7)

where $B[f_p, m_p]^T = B_v[F_p, M_p]^T$, $f_c = (f_a^T, m_a^T)^T \in R^{4N+2}$, and $f_v = (f_b^T, m_b^T)^T \in R^{4N+2}$ where

$$\Phi(t) = \begin{bmatrix} \phi^T(t) & 0 \\ 0 & \phi^T(t) \end{bmatrix}$$

(6.8)
For the purpose of control, the bias angle is changed at a periodic interval of $T^*$, where $T^*$ is an integer multiple of the period $T_0$, i.e., $T^* = n_0 T_0$, where $n_0$ is a positive integer. This way one switches the bias angle at an uniform rate of $T^*$ seconds at the end of $n_0$ cycles. For the derivation of the control law, the transients introduced due to changes in the bias-angle are ignored. Since the bias is changed periodically, it will be convenient to express the continuous-time system (6.7) as a discrete-time system. The function $\beta(t)$ now has piecewise constant values $\beta_k$ for $t \in [kT^*, (k + 1)T^*), k = 0, 1, 2......$

Discretizing the state equation (6.7), one obtains a discrete-time representation of the form

$$x([k + 1]T^*) = A_d x(kT^*) + B_d \beta_k + d_p$$

where $A_d = e^{A T^*}$, $B_0 = \int_0^{T^*} e^{A s} B \Phi(-s) ds$, $B_d = B_0 f_v \in \mathbb{R}^3$, and $d = B_0 f_c \in \mathbb{R}^3$. The output variable $(z)$ is

$$y(kT^*) = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} x(kT^*) = C_d x(kT^*)$$

6.3 Adaptive Pectoral Fin Control

In this section, the design of an adaptive control system is considered. To this end, it will be convenient to represent the system in the observer canonical form. The transfer function relating the depth and the input (bias angle $\beta_k$) for $d_p = 0$ can be shown to be of the form

$$\frac{\dot{y}(z)}{\dot{\beta}_k(z)} = C_p (Z I - A_p)^{-1} B_p = H(z)$$
where \( \hat{y}(z) \) and \( \hat{P}(z) \) are the Z-transforms of \( y \) and \( P_k \), respectively; and \( H(z) \) is the transfer function of the form

\[
H(z) = \frac{b_3 z^3 + b_2 z^2 + b_1 z + b_0}{z^4 + a_3 z^3 + a_2 z^2 + a_1 z + a_0}
\]

where \( a_i \) and \( b_i \) are real numbers. The relative degree of \( H(z) \) is one. For the model under consideration, one finds that the numerator has one unstable root and two stable roots, which implies that the transfer function is non-minimum phase.

The observer canonical form realization of \( H(z) \) is given by [31]

\[
\begin{bmatrix}
-a_3 & 1 & 0 & 0 \\
-a_2 & 0 & 1 & 0 \\
-a_1 & 0 & 0 & 1 \\
-a_0 & 0 & 0 & 0
\end{bmatrix} x(k+1) = \begin{bmatrix} b_3 \\ b_2 \\ b_1 \\ b_0 \end{bmatrix} x(k) + \begin{bmatrix} d_3 \\ d_2 \\ d_1 \\ d_0 \end{bmatrix} \beta_k + \Delta
\]

\[
x(k+1) = Ax + B\beta_k + d_d
\]

and the output is

\[
y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x(k) = Cx(k) = z
\]

We note that all the parameters \( a_i \) and \( b_i \) are unknown because \( A_d \) and \( B_d \) are not known. For the purpose of depth control, we introduce a servo compensator

\[
x_s(k+1) = x_s(k) + y(k) - y_r(k) = x_s(k) + Cx(k) - y_r(k)
\]

where \( y_r \) is the desired depth.
Define the augmented state vector \( x_a = (x^T, x_s)^T \in \mathbb{R}^5 \). Now including \( x_s \) in the canonical model, (6.12) gives

\[
x_a(k+1) = \begin{bmatrix}
-a_3 & 1 & 0 & 0 & 0 \\
-a_2 & 0 & 1 & 0 & 0 \\
-a_1 & 0 & 0 & 1 & 0 \\
-a_0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 \\
\end{bmatrix} x_a(k) + \begin{bmatrix}
b_3 \\
b_2 \\
b_1 \\
b_0 \\
0 \\
\end{bmatrix} \beta_k + \begin{bmatrix}
d_3 \\
d_2 \\
d_1 \\
d_0 \\
-y_r \\
\end{bmatrix}
\]

\[
\Delta \equiv A_s x_a(k) + b_s \beta_k + \begin{bmatrix}
d_d \\
-y_r \\
\end{bmatrix}
\tag{6.14}
\]

According to the servomechanism theory [30], for the regulation of the tracking error \( (y - y_r) \), it is necessary to obtain a discrete control law \( \beta_k = K_c x_a(k) \) such that the closed-loop matrix \( (A_s + b_s K_c) \) is Hurwitz. Here, for the computation of \( K_c \), the linear quadratic regulator theory is used. Following [31], the optimal gain vector \( K_c \) is obtained by minimizing a performance index of the form

\[
J_o = \sum_{k=0}^{\infty} x_a^T(kT^*) Q x_a(kT^*) + \beta_k^2 \mu
\tag{6.15}
\]

where \( Q \) is a positive definite symmetric matrix and \( \mu > 0 \). The weighting matrix \( Q \) associated with \( x_a \) and the parameter \( \mu \) penalizing the level of the bias angle are chosen to provide trade-off between the convergence rate of the state variables to the equilibrium point and the bias angle magnitude. The feedback matrix \( K_c \) is obtained by solving the discrete time Ricaati equation [42]

\[
P = Q + A_s^T P A_s - A_s^T P B_a (\mu + B_a^T P B_a) B_a^T P A_s
\tag{6.16}
\]
and then setting the feedback matrix as

\[ K_c = -(\mu + B_a^T P B_a)^{-1} B_a^T P A_a \]  \hspace{1cm} (6.17)

Let \( \hat{a}_i, \hat{b}_i, \hat{A}_a \) and \( \hat{B}_a \) denote the estimates of the parameters and the matrices. Since \( a_i \) and \( b_i \) are unknown, the matrix \( P \) is obtained by using the estimated matrices \( \hat{A}_a, \hat{B}_a \) in the Riccati equation

\[ Q + \hat{A}_a^T \hat{P} \hat{A}_a - \hat{A}_a^T \hat{P} \hat{B}_a (\mu + \hat{B}_a^T \hat{P} \hat{B}_a) \hat{B}_a^T \hat{P} \hat{A}_a \]  \hspace{1cm} (6.18)

Then the feedback gain vector is computed as

\[ \hat{K}_c = -(\mu + \hat{B}_a^T P \hat{B}_a)^{-1} \hat{B}_a^T P \hat{A}_a \]  \hspace{1cm} (6.19)

and the control law is

\[ u = \hat{K}_c x_a(k) \]  \hspace{1cm} (6.20)

For the synthesis of the control law (6.20), the state vector \( x \) is needed which is not available. Therefore, it is essential to design an observer to obtain an estimate \( \hat{x} \) of \( x \). For the case of known parameters one can select an observer of the form (assuming \( d_d = 0 \))

\[ \hat{x}(k + 1) = A \hat{x}(k) + B \beta_k + K_0 (C \hat{x}(k) - y) \]  \hspace{1cm} (6.21)

Let \( \hat{x} = x - \hat{x} \). Substituting (6.21) from (6.14) gives the state error dynamics

\[ \hat{x}(k + 1) = A_d \hat{x}(k) + K_0 C_d \hat{x}(k) \]  \hspace{1cm} (6.22)

The matrix \( K_0 \) is selected such that \( (A + K_0 C) \) is Schur (i.e. the eigen values of \( (A + K_0 C) \) are withing the unit disk). For computing \( K_0 \) in (6.21), one uses the pole
placement technique. For this, a set of four stable poles (all poles within the unit disk) is selected. Since $A$ and $B$ are not known, $\hat{K}_0$ is computed such that the eigenvalues of $\hat{A} + \hat{K}_0C$ are placed on the selected poles. In the control law, $\hat{x}$ is used instead of $x$, yielding

$$\beta = \hat{K}_c(\hat{x}^T, x_a)$$

(6.23)

Then, according to servomechanism theory, output regulation is accomplished.

### 6.4 Adaptation Law

For the computation of the control law in the previous section, the estimates of $\hat{a}_i$ and $\hat{b}_i$ and the disturbance $d_d$ are required. In this section, a least squares based update scheme is developed for the estimation of these parameters of the system.

Manipulating the observer canonical form of (6.11), we get

$$y(k) = [1, 0, 0, 0]x(k) = z$$

$$y(k + 1) = -a_3 y(k) + x_2(k) + b_3 u(k) + d_3$$

$$y(k + 2) = -a_3 y(k + 1) - a_2 y(k) + x_3(k) + b_3 u(k + 1) + d_3 + b_2 u(k) + d_2$$

$$y(k + 3) = -a_3 y(k + 2) - a_2 y(k + 1) - a_1 y(k) + x_4(k) + b_3 u(k + 2) + d_3$$

$$+ b_2 u(k + 1) + d_2 + b_1 u(k) + d_1$$

$$y(k + 4) = -a_3 y(k + 3) - a_2 y(k + 2) - a_1 y(k + 1) - a_0 y(k) + b_3 u(k + 3)$$

$$+ d_3 + b_2 u(k + 2) + d_2 + b_1 u(k + 1) + d_1 + b_0 u(k) + d_0$$

(6.24)
Thus, rearranging (6.24), one obtains

\[
y(k) = [-a_3, -a_2, -a_1, -a_0, -b_3, -b_2, -b_1, -b_0, d_{\text{sum}}] \begin{bmatrix} y(k-1) \\ y(k-2) \\ y(k-3) \\ y(k-4) \\ u(k-1) \\ u(k-2) \\ u(k-3) \\ u(k-4) \\ 1 \end{bmatrix} = \theta^T \phi(k)
\]

(6.25)

It is important to note that an estimate of the disturbance sum \(d_{\text{sum}} = d_0 + d_1 + d_2 + d_3\) is necessary in (6.25) to cancel its effects.

Now, the least squares algorithm for the estimation of the parameters is obtained by minimizing the quadratic cost given by

\[
J_N(\theta) = \frac{1}{2} \sum_{k=1}^{N} (y(k) - \phi(k-1)^T \theta)^2 + \frac{1}{2} (\theta - \hat{\theta}(0))^T P_0^{-1}(\theta - \hat{\theta}(0)) \tag{6.26}
\]

where \(P_0\) is a positive definite matrix. Basically, the cost function represents the sum of squares of the errors \(y(k) - \phi(k-1)^T \theta\). The minimizing trajectory \(\hat{\theta}(k)\) is generated by

\[
\dot{\hat{\theta}}(k) = \hat{\theta}(k-1) + \frac{P(k-2) \phi(k-1)}{1 + \phi(k-1)^T P(k-2) \phi(k-1)} [y(k) - \phi(k-1)^T \hat{\theta}(k-1)] \tag{6.27}
\]
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where

\[ P(k - 1) = P(k - 2) - \frac{P(k - 2)\phi(k - 1)\phi(k - 1)^T P(k - 2)}{1 + \phi(k - 1)^T P(k - 2) \phi(k - 1)} \]

Here, \( \theta(0) \) is given and \( P(-1) = P_0 \). The estimated parameter vector \( \hat{\theta} \) is used in Section 6.4 for the observer and controller design.

### 6.5 Simulation results

In this section, simulation results using the Matlab/Simulink software is presented. The performance of the adaptive for different uncertainty conditions is examined.

The parameters of the model are taken from [36]. The AUV is assumed to move with a constant forward velocity of 0.7 (m/sec) with the help of a control mechanism. The vehicle parameters are \( l = 1.391 \) (m), mass=18.826 (kg), \( I_y = 1.77 \) (kgm\(^2\)), \( X_G = -0.012 \), \( Z_G = 0 \), \( d_{cgf} = 0 \). The hydrodynamic parameters for a forward velocity of 0.7 m/sec derived from [32] are \( Z_q = 1.83 \), \( Z_w = -27.2 \), \( Z_q = -56.6 \), \( Z_w = -58.5 \), \( M_q = -4.34 \), \( M_w = 1.83 \), \( M_q = -4.93 \), and \( M_w = 24 \).

The discrete-time zeros of the BAUV model for a \( d_{cgf} = 0 \) and frequency of oscillation = 8 Hz are 4.3450, -0.5868 and 0.1858. Thus the system has one unstable zero and two stable zeros. The poles of the discrete transfer function are 1.0000, 0.9859, 0.0906 + 0.4326i and 0.0906 - 0.4326i.

Experimental results indicate that for zero bias angle, the mean values of \( f_p \) and \( m_p \) are zero. Therefore, the vectors \( f_a \), \( f_b \), \( m_a \), and \( m_b \) are found to be

\[ f_a = (0, -40.0893, -43.6632, -0.3885, 0.6215, 6.2154, -10.17, -0.1554, 0.6992) \]

\[ f_b = (68.9975, 0.4451, -16.4704, 64.1009, -19.5864, -0.8903, -2.2257, 2.2257, 4.8966) \]
\[ m_a = (0.0054, 0.6037, 0.4895, 0, -0.0054, 0, -0.0925, 0, -0.0054) \]

\[ m_b = (-0.5297, -0.3739, -0.0935, -0.2493, 0.1246, 0.0312, -0.0312, 0.0935, 0) \]

It is pointed out that these parameters are obtained from the force and moment Fourier coefficients and are computed by multiplying the Fourier coefficients by \( \frac{1}{2} \rho W_a U_\infty^2 \) and \( \frac{1}{2} \rho W_a chord U_\infty^2 \) respectively, where \( W_a \) is the surface area of the foil. For simulation, the initial conditions of the vehicle are assumed to be \( x(0) = 0 \) and \( x_a(0) = 0 \).

In this section, the discrete control law (6.23) is simulated. The bias angle is changed to a new value every \( T^* = n_0 T_0 \) seconds where \( T_0 = \frac{1}{f_0} \) is the fundamental period of \( f_p \) and \( m_p \). Choosing a small value of \( n_0 \) increases the transients produced due to switching. A large value of \( n_0 \) increases the magnitude of the intersample oscillations which is also not desirable. For simulation purposes, a frequency of 8 Hz is chosen.

For the controller design, the weighting matrix and parameter are selected as \( Q = 1000I_{5x5} \) and \( \mu = 1.5 \) for the computation of \( K_c \). The observer poles are selected as \( 0.0418+0.2j, 0.0418-0.2j, 0.5, \) and \( 0.64 \). All these values are chosen so as to obtain good transient response and less control input (bias angle).

Results are presented for commanding the BAUV to a depth of 2 (m) and 3 (m) for various uncertainties in the AUV model.

**Case 1** Target depth = 2 (m), Estimated \( \dot{\theta}(0) \) 25% Lower

A set point of 2 (m) depth is taken as the command input. It is assumed that \( \dot{\theta}(0) = 0.75 * \theta = 0.75 * [1.6254, -1.1559, 0.4250, -0.1445, -1.6228, 6.4004, 3.0044, -0.7686, -0.0534]^T \). The simulation results are shown in Figure 6.2 (a)-(f) and Figure
6.3. It can be seen that smooth depth trajectory tracking and pitch angle regulation are achieved in approximately 60 sec. The maximum bias angle (control input) required is approximately 1 (deg), which can be easily provided by the pectoral fins. The estimated parameters $a_i$ and $b_i$ are also shown (Figure 6.3). One can observe that these parameters converge to certain constant values when the set point tracking is achieved. These estimated values differ from the actual values of the parameters. Of course, it is well known that the estimated parameters cannot converge to their actual values in the absense of persistent excitation, however this causes no problem in accomplishing the depth control.

Simulation results are also presented for parameter uncertainty as high as -50% ($\hat{\theta}(0) = \frac{\theta}{2}$). As one can expect, the bias angle required is high. As such results are presented by clamping the bias angle to a value of 10 (deg). It can be seen from Figure 6.4 that depth tracking and pitch angle regulation are accomplished.

Simulations were also performed for an uncertainty of -60% (i.e. $\hat{\theta}(0) = 0.4\theta$) including the control saturation of 10 (deg) to illustrate the effectiveness of the adaptive controller for 2 (m) depth command. Although the depth control was accomplished, the transient response degraded considerably (Figure 6.5).

**Case 2** Target depth = 3 (m), Estimated $\hat{\theta}(0)$ 25% Lower

Now simulation results for a command depth of 3 (m) are presented. Smooth depth tracking is obtained as can be seen in Figure 6.6. As expected, the pitch angle and pitch rate are high compared to those of Figure 2 for smaller command input. Simulation was also done for uncertainty of +25% ($\hat{\theta}(0) = 1.25\theta$) and +50% using
two and three meter command. for these uncertainties also the adaptive controller accomplishes accurate depth control (results not shown).

Figure 6.1: Schematic of the BAUV
Figure 6.2: Adaptive control: command depth = 2 (m) and $\hat{\theta}(0) = 0.75\theta$, (a) Depth ($z$), (b) Bias angle ($\beta$), (c) Pitch angle ($\theta$), (d) Pitch rate ($q$), (e) Heave velocity ($w$), (f) Normal force ($F_p$)

Figure 6.3: Adaptive control: command depth = 2 (m) and $\hat{\theta}(0) = 0.75\theta$, (a) Moment, ($M_p$), (b)-(f) Estimated parameters ($\theta(1) - \theta(9)$)
Figure 6.4: Adaptive control: command depth = 2 (m) and $\dot{\theta}(0) = 0.50\theta$, (a) Depth ($z$), (b) Bias angle ($\beta$), (c) Pitch angle ($\theta$), (d) Pitch rate ($q$)

Figure 6.5: Adaptive control: command depth = 2 (m) and $\dot{\theta}(0) = 0.40\theta$, (a) Depth ($z$), (b) Bias angle ($\beta$), (c) Pitch angle ($\theta$), (d) Pitch rate ($q$)
Figure 6.6: Adaptive control: command depth = 3 (m) and $\hat{\theta}(0) = 0.75\theta$, (a) Depth ($z$), (b) Bias angle ($\beta$), (c) Pitch angle ($\theta$), (d) Pitch rate ($q$)
CHAPTER 7

CONCLUSION

The design of control systems for the dive plane and yaw plane maneuvering of biologically inspired autonomous underwater vehicles was considered. The first part of the thesis dealt with the control of BAUVs using dorsal fins. The force produced by the cambering of the fins was used for control. The latter part of the thesis considered the control of the BAUVs using pectoral fins. Here, the flapping of the pectoral fins was used to provide the necessary force and moment for control.

Firstly, the design of control systems for the dive plane maneuver of BAUVs using dorsal fins was considered. An adaptive control law was derived for precise depth control. The control system consists of a gradient based identifier for parameter estimation, an observer for state estimation, and an optimal feedback controller. Simulation results were provided which showed that the depth control and pitch angle regulation can be accomplished quite effectively using dorsal fins in spite of large uncertainties in the system parameters.

Next, an adaptive input-output feedback linearizing control system for the control of BAUVs in the yaw plane using dorsal fins was developed. The vehicle model included nonlinear hydrodynamic forces and it was assumed that all the system pa-
rameters were unknown. For the derivation of the controller, a stable manifold was formed by a linear combination of the tracking error, its derivative and integral. A direct adaptive control system was designed in which the controller gains are directly tuned. In the closed-loop system, yaw angle trajectory control was accomplished. The stability of the zero dynamics was also examined. Simulation results were provided which showed that the yaw angle tracking of exponential and sinusoidal trajectories and turning maneuvers can be accomplished quite effectively using dorsal fins in spite of the nonlinearity and large uncertainties in the system parameters.

In chapters three and four, optimal as well as inverse yaw plane control of a biorobotic AUV using pectoral-like fins was considered. For maneuvering the BAUV, the bias angle was treated as control input. CFD and Fourier Series Expansion were used to parameterize the effect of this control input on the hydrodynamical force and moment produced by the flapping foil. For the purpose of design, a discrete-time model was obtained and a minimum phase representation was derived for controller design. Then an optimum control law for the regulation of the yaw angle to set points and an inverse control law for the trajectory control of the modified output were derived. The bias angle was updated at regular intervals (multiple of the fundamental period). In the closed-loop system, the modified output and the actual yaw trajectory were found to be sufficiently close to the desirable heading angle commands. From these results, one concludes that accurate yaw angle control along time-varying paths can be accomplished using oscillating fins. Furthermore, improved performance of the control system can be obtained when the frequency of oscillation of the fins increases.
Furthermore, in chapter five, an adaptive optimal control system using pectoral fins for the dive plane maneuver of BAUVs was considered. A discrete-time adaptive control law was derived for precise depth control. The control system consists of a least squares based identifier for parameter estimation, an observer for state estimation, and stable feedback controller. Simulation results were provided which showed that the depth control and pitch angle regulation can be accomplished quite effectively using dorsal fins in spite of large uncertainties in the system parameters.
REFERENCES


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29. S. N. Singh, A. Simha and R. Mittal, Biorobotic AUV Maneuvering by Pectoral


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