Adaptive and variable structure control of a smart projectile fin by piezoelectric actuation

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ADAPTIVE AND VARIABLE STRUCTURE CONTROL OF A SMART
PROJECTILE FIN BY PIEZOELECTRIC ACTUATION

by

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Kerala University, Kerala, India
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ABSTRACT

Adaptive and Variable Structure Control of a Smart Projectile Fin by 
Piezoelectric Actuation

by

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The aim of this thesis is to develop efficient control algorithms for the control of a smart projectile fin in the presence of parameter uncertainties and aerodynamic disturbance inputs. The fin can be used to maneuver small aerial vehicles by controlling its rotation angle. The smart fin considered in this thesis consists of a hollow rigid body (or canard) within which a flexible cantilever beam with a piezoelectric active layer is mounted. The beam deforms when voltage is applied to the piezoelectric layer. The rotation angle of the fin is controlled by deforming the flexible beam which is hinged at the tip of the rigid fin. In this thesis, four kinds of fin control systems based on adaptive control and variable structure theory are done. It is assumed that the fin-beam model parameters and the aerodynamic forces are unknown to the designer. For this study, a finite dimensional state variable model of the fin-beam system obtained by a finite element method is used.
Firstly, adaptive state feedback and output feedback control laws are designed for the trajectory control of the fin angle. For the derivation of the control laws, a stable manifold which is a linear combination of the fin angle tracking error and its integral and derivative, is chosen. Secondly, an adaptive control system based on command generator tracker concept is designed using the input, output and reference command signals. The new control law is simple and unlike the first controller does not require discontinuous control signal. Next, an output feedback adaptive servoregulator, exploiting the output feedback passivity property of the system is designed for trajectory control. Finally, a model reference variable structure adaptive controller (VS-MRAC) is designed. Unlike the previous adaptive controllers, in this approach, the adaptation of parameters is not necessary, but the bounds on uncertain functions are required.

Simulation results using the designed control systems are presented. The results show that, in the closed-loop system, asymptotic trajectory tracking of the fin angle is accomplished, in spite of large uncertainties in the system and presence of aerodynamic moment.
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CHAPTER 1

INTRODUCTION

Recently, there has been growing interest in using intelligent materials in various types of structures and structural members. Intelligent materials, when embedded into the host structures provide wide range of abilities to induce strain or to sense strain. These abilities make the intelligent materials, a most adequate solution to various applications such as dynamic actuation, vibration control etc. The use of surface-mounted or bonded piezoelectric actuators for the shape control of intelligent structure has gained widespread acceptance recently. Applications can be found in many areas including the shape control of metallic or composite plates or beams [1, 2]. Applications also involve actuation of various types of aircraft structural members such as wings, fins, or rotor blade [3, 4, 5, 6]. Advantages of this approach are mainly due to the integration of the actuators into the structural members itself, thus saving the space required for servo motors, force transmission devices, or hydraulic systems [6]. This advantage becomes even more important when small aerial vehicles such as unmanned aircraft, small missiles, guided munitions, and projectiles, are examined. Piezoelectric twist actuators used for this application are based on anisotropic strain-ning of the host structure using directionally attached isotropic actuator [3] or using
piezoelectric fibers integrated into the composite structural members [4]. General formulation and solution procedures for an analytical model for a composite laminated plate with isotropic or anisotropic active layers are derived in Refs. [5, 6].

Traditionally, for the path control of missiles and projectiles, maneuvering forces and moments are generated by fin angle control using mechanical actuators which are bulky and slow. For high performance projectiles, there is a need to develop more efficient actuation mechanisms.

The emergence of small and light-weight actuators associated with smart materials has been triggered recently. Smart materials include electro-rheological fluids, shape memory alloys and piezoelectric materials. The piezoelectric materials are featured by low power consumption, high power-to-weight ratio and fast response time compared with other materials. As of now, the piezoelectric materials are normally employed as sensors as well as actuators for smart structural applications. These smart structures are primarily employed to control the static and elastodynamic responses of distributed parameter systems operating under variable service conditions. Smart airfoil - a shape changing wing is the task of current generation aerodynamicists. Recently, the development of a smart fin (fin-beam model) has been considered [7, 8].

1.1 Smart Fin

A smart fin is one which uses smart materials for the shape control of its surface. Smart materials have unique properties. This model would improve the projectile
efficiency and performance, since it is aerodynamically perfect. These fins use non-hydraulic based actuators which can be piezoelectric devices or shape memory alloys. Hydraulic actuators are heavy and on the contrary smart fins are way too light.

1.2 Types of Smart Materials

There are different types of smart materials available. Some of them are discussed here. Piezoelectric or electrostrictive material will deform when subjected to an electric charge or a variation in voltage. Electrostrictive materials produce displacements in same direction where as piezoelectric materials can deform in both directions under compression and elongation. Magnetostrictive materials undergo induced mechanical strain when subjected to a magnetic field. Memory alloys will undergo phase transformations which will produce shape changes when subjected to a thermal field.

1.3 Adaptive Control

The term adaptive control was introduced in the control literature in the late 1950s and has developed into a broad and vibrant field. Since the early 100s there has been an exponential growth in adaptive control publications, the earliest publications originated in aerospace for flight control. The need for adaptive control remains that of overcoming the difficulty of determining suitable control laws that are satisfactory over a wide range of operating points. In the 1980s, interest shifted to robust adaptive control or the adaptive control systems in the presence of perturbations. The main thrust of the research during the past decade has been in the area of robust adaptive
control where the aim is to assure the boundedness of all signals in the system even when external or internal perturbations are present. These perturbations include in time-varying parameters and structured nonlinearities as well as bounded external disturbances and the effects of the unmodeled part of the unknown plant.

1.4 Literature Review

Research on smart structure systems associated with the piezoelectric materials was initially undertaken by Bailey and Hubbard [9]. They proposed simple but effective control algorithms for transient-vibration control, constant amplitude control and constant gain control. Baz and Poh [10] worked on the vibration control of the smart structures via a modified independent modal space control by considering the effect of the bonding layer between the piezoelectric material and the host structure. Tzou [11] investigated the piezoelectric effect on the vibration control through a modal shape analysis. Choi [12] improved vibration controllability of the piezofilm actuator by attenuating undesirable chattering problem in the settling phase.

Numerous researchers have focused on vibration control of the smart structures associated with the piezoelectric materials. The design of active controllers using piezoelectric actuators for vibration, force and position control of systems have been considered by various authors [7, 9, 10, 13, 14]. It is well-known that the piezoelectric actuators can generate vibration of the flexible structures. In large structures, these vibrations have long decay times that can lead to fatigue, instability, or other problems with the operation of the structure. Flexible structures are distributed-parameter
systems having a theoretically infinite number of vibrational modes. Current design practice is to model the system with a finite number of modes and to design a control system using lumped-parameter control theory. Truncating the model may lead to performance tradeoffs when designing a control system for distributed parameter systems. There exists a wealth of distributed-parameter control theory in the literature. However, there are very few applications in the literature. The reason may be the difficulty of using distributed-parameter control theory with spatially discrete sensors and actuators.

Yim et.al., [8] proposes a smart projectile fin controller design based on a modeling error compensation approach in which the lumped uncertainties are estimated using a high-gain observer. This requires precise measurement of the fin angle for stability in the closed-loop system. A fuzzy controller has been designed in [15] for the control of this fin. Of course, for the fuzzy controller design, the designer first has to develop a number of if-then rules which often are not easy to obtain. Researchers have made considerable effort to design controllers for the control of flexible structures. Ref. [16] provides a good review of literature in which readers will find several references. For flexible structures, controller designs based on feedback linearization, passivity concepts and adaptive techniques have been attempted [14, 17, 18, 19]. But the methods based on passivity [20, 21] have advantage over other design techniques since the passivity approach does not rely on model truncation or higher-order models of the structure and is independent of the numerical values of the model data.
1.5 Thesis Contributions

This thesis treats the question of adaptively controlling the rotation angle of a smart fin developed for a subsonic projectile using piezoelectric actuation. Smart fins are deployed after the projectile reaches the apogee. They are used to guide the projectile precisely to the target. The schematic of a smart fin investigated as part of the study is shown in Figure 1.1.

The smart fin considered here has an outer hollow rigid body, within which a flexible cantilever beam with a piezoelectric active layer is mounted. The rotation angle of the fin is controlled by deforming the flexible beam which is hinged at the tip of the rigid fin.

Efficient adaptive control algorithms are developed for the control of rotation an-
gle of a projectile's fin in the presence of parameter uncertainty and disturbance inputs. Flexible structures are essentially infinite dimensional systems; however often finite dimensional models by neglecting the higher modes are used for analysis and design. The models of flexible structures are generally obtained by solving the eigenvalue problem resulting from finite element methods. However, it is well known that the resulting fidelity of model parameters degrades drastically for higher modes. Finite element approach is used to describe the dynamics of the flexible beam with active layer of piezoelectric actuator. The beam is modeled using a series of finite elements satisfying Euler-Bernoulli's theorem. The input signal is the voltage applied to piezoelectric actuator and the output variable is the rotation angle of the fin.

First, state feedback and output feedback adaptive control systems are derived for the control of rotation angle of the fin. The fin is controlled by regulating the tip deflection of the flexible beam which in turn causes the control of the rotation angle of the fin. A feedback linearizing adaptive control law is designed for the trajectory control of the fin angle of the flexible beam. For the feedback linearizing control law design, a stable manifold which is a linear combination of the fin angle tracking error and its integral and derivative, is chosen. For the purpose of design, it is assumed that the model parameters are not known. The designed adaptive control systems forces the trajectory to converge to the chosen manifold. Simulation results show that in the closed loop system, precise fin angle control is accomplished in spite of uncertainties in the system parameters, using state as well as output feedback.

Next, based on command generator tracker concept, a model reference adaptive fin
angle controller is designed. The structure of the control system is independent of the
dimension of the flexible fin-beam model. For the design of the control law, a linear
combination of the fin angle and the fin's angular rate is chosen as the controlled
output variable. In closed-loop system, the controlled output variable tracks the
reference trajectory and the fin angle asymptotically converges to the desired value
and the elastic modes converge to their equilibrium values. Simulation results show
that the fin angle is precisely controlled in spite of uncertainties in the fin-beam
parameters and the aerodynamic moment coefficients.

Next, an adaptive servoregulator is designed for the control of the fin angle and
the rejection of the disturbance inputs. Exploiting the output feedback passivity
property of the system, a direct adaptive control law is derived for the regulation
of the fin angle. Simulation results show that the adaptive servoregulator controller
accomplishes fin angle control.

Finally, a new control system based on the theory of variable structure adaptive
control using only input and output signals are derived. Here only the fin angle
measurement is required for the controller synthesis. Control using the fin angle
is very practical since measurement of flexible modes is not easy. However, it is
assumed that an upper bound on the uncertain functions is given. It is shown that
in the closed-loop system, fin angle tracks given reference trajectory. Unlike other
adaptive controllers developed in literature, controller parameter divergence cannot
occur here since integral type adaptive laws for updating controller are not used in
this design. Simulation results are presented which show good transient response and
robustness to uncertainty and unmodeled dynamics.

1.6 Thesis Outline

The following is a brief summary of the outline of this thesis report. Chapter 2 introduces mathematical modeling of the projectile fin. Chapter 3 derives the adaptive state feedback and output feedback control laws. A model reference adaptive controller is designed in Chapter 4. Chapter 5 describes adaptive servoregulator. Chapter 6 describes variable structure adaptive controller. Finally, Chapter 7 includes the conclusion of this thesis and suggestions for future work.
CHAPTER 2

MATHEMATICAL MODEL

This chapter deals with the mathematical modeling of the fin-beam system. The schematic of the fin-beam model is shown in Figure 2.1. The flexible beam with a piezoelectric active layer bonded on the top surface, is hinged at one end to the fin and the other end is attached rigidly to the projectile body. The fin is free to rotate about an axis fixed to the projectile body. When the control voltage \( u(x, t) \) is applied to the actuator, the induced strain in the actuator generates the bending moment \( m \) that is expressed as [9]

\[
m = cu(x, t)
\]  \hspace{1cm} (2.1)

The constant \( c \) can be obtained by considering geometrical and material properties of the beam and piezoelectric actuator. Considering the cross sectional geometry and force equilibrium along the axial direction, the constant \( c \) can be expressed as [12]

\[
c = -d_{31} \frac{h_p + h_b}{2} E_p h_b E_b \frac{E_p h_p + E_b h_b}{b}
\]  \hspace{1cm} (2.2)

where \( d_{31} \) is the piezoelectric strain constant and \( E_p \) and \( E_b \) are Young's modulus of the piezoelectric actuator and the beam respectively. Other geometric parameters are shown in the figure.
As shown in Figure 2.1, the fin is free to rotate about the hinge joint fixed to the projectile body and one end of the beam actuator is fixed to the projectile body and the other end is connected the fin using another hinge joint fixed to the tail side of the fin. The fin is considered as rigid and its rotation angle is assumed to be small and planar.

2.1 Finite Element Approach

Finite element approach is used to describe the dynamics of the flexible beam, which is considered as composed of finite elements satisfying Euler-Bernoulli's theorem. The beam is divided into \( n \) elements with equal length of \( L_i \). The displacement \( w \) of any point on the beam element \( i \) is described in terms of nodal displacement,
where \( q_i = (w_i, \phi_i, w_{i+1}, \phi_{i+1})^T \) and \( N = (N_1, N_2, N_3, N_4) \) is the shape function vector with

\[
N_1 = \frac{1}{L_i^3} (2x_i^3 - 3x_i^2 L_i + L_i^3)
\]

\[
N_2 = \frac{1}{L_i^3} (x_i^3 L_i - 2x_i^2 L_i^2 + x_i L_i^3)
\]

\[
N_3 = \frac{1}{L_i^3} (-2x_i^3 + 3x_i^2 L_i)
\]

\[
N_4 = \frac{1}{L_i^3} (x_i^3 L_i - x_i^2 L_i^2)
\]

where \( x_i \) is the element local coordinate variable defined along the beam neutral axis.

Using Eq. (2.4), the kinetic energy of an element \( i \) becomes

\[
T_i = \frac{1}{2} \int_0^{L_i} \rho_i \dot{w}_i^2 \, dx_i = \frac{1}{2} \dot{q}_i^T M_i \dot{q}_i
\]

where \( M_i \in R^{4\times4} = \int_0^{L_i} \rho_i N^T N \, dx_i \) is a mass matrix and \( \rho_i \) is a combined density of the beam and piezoelectric actuator per unit length.

The potential energy of an element \( i \) is

\[
V_i = \frac{1}{2} \int_0^{L_i} \frac{1}{E_i I_i} (E_i I_i \frac{\partial^2 w}{\partial x_i^2} + c u(x, t))^2 \, dx_i
\]

where \( E_i I_i \) is the product of Youngs modulus of elasticity by the cross-sectional area moment of inertia for the equivalent beam of an element \( i \). If the piezoelectric actuator has a uniform geometry and that a uniform voltage is applied along its length, \( u(x, t) \)
can be assumed to be function of time only. The potential energy of an element can be further expressed as,

\[ V_i = \frac{1}{2} q_i^T K_i q_i + q_i^T \left( \int_0^{L_i} \frac{\partial^2 N}{\partial x^2_i} dx_i \right) c u(t) + \frac{1}{2} \frac{1}{E_i I_i} \epsilon^2 u^2(t) \]  

(2.7)

where the stiffness matrix \( K_i \in R^{4 \times 4} \) becomes

\[ K_i = \int_0^{L_i} E_i I_i \left( \frac{\partial^2 N}{\partial x^2_i} \right)^T \left( \frac{\partial^2 N}{\partial x^2_i} \right) dx_i \]  

(2.8)

The kinetic energy of the rigid fin is

\[ T_f = \frac{1}{2} J_f \left( \frac{\dot{w}_{n+1}}{L} \right)^2 \]  

(2.9)

where \( J_f \) is the mass moment of inertia of the fin about an axis fixed on the projectile body and \( \dot{w}_{n+1} \) is the time derivative of tip deflection \( w_{n+1} \) or \( w(L, t) \).

Using the Lagrangian dynamics, the equations of motion for the element \( i \) becomes

\[ M_i \ddot{q}_i + K_i q_i = B_i(-c u(t)), \quad i = 1, \ldots, n - 1 \]  

(2.10)

where \( B_i = (0, -1, 0, -1)^T \) which represent two concentrated moments at two nodes of the element \( i \). For the last element \( i = n \), the equation of motion including the mass of the rigid fin becomes

\[ M_n \ddot{q}_n + K_n q_n = B_n(-c u(t)) \]  

(2.11)

The equations derived for each element can be assembled after expansion and matrix
reduction from the boundary conditions of the cantilever beam as follows,

\[ M\ddot{q} + Kq = B_0u(t) \]  

(2.12)

where \( q = (w_2, \phi_2, \ldots, w_{n+1}, \phi_{n+1})^T \in \mathbb{R}^{2n}, M \in \mathbb{R}^{2n \times 2n}, K \in \mathbb{R}^{2n \times 2n} \) and \( B_0 \in \mathbb{R}^{2n \times 1} \) are appropriate matrices.

2.2 Aerodynamic Moment

The aerodynamic moment acting on the fin is a complicated function of the angle of attack of the projectile and the fin rotation angle. The data generated by the computational fluid dynamics show that the aerodynamic moment can be accurately modeled as a linear function of the fin angle with a bias term and a reasonable model can be expressed as

\[ m_a = m_{a0}(\alpha) + p_a(\alpha)\theta = m_{a0}(\alpha) + p_a(\alpha)l^{-1}e^{*T}q \]  

(2.13)

where \( \theta \) is the fin angle, \( \alpha \) is the angle of attack, \( p_a(\theta) \) is a polynomial in the angle of attack, \( p_0(\theta) = p_0 + p_1(\theta) + \ldots + p_k\theta^k \) (\( k \) is a positive integer) and \( e^{*T} \in \mathbb{R}^{2n} \) is a unit vector whose \((2n-1)^{th}\) element is one and rest are zero.

2.3 State Variable Representation

The modified fin-beam model including the aerodynamic moment takes the form

\[ M\ddot{q} + Kq = B_0u(t) + B_am_a \]  

(2.14)
where \( B_a = [0, \ldots, 0, 1, 0]^T \in \mathbb{R}^{2n} \). Substituting \( m_a \) from Eq. (2.13) in Eq. (2.14), and solving the resulting equation

\[
\dot{q} = -M^{-1}K_m q + M^{-1}B_0 u(t) + M^{-1}B_a m_a(\alpha) \quad (2.15)
\]

where \( K_m = K - p_\alpha(\alpha) l^{-1} e^* e^T \), now includes the \( \alpha \)-dependent term of the aerodynamic moment.

This reduced dynamic model using \( n \) elements can be expressed in the state variable form using the state vector \( x = (q^T, \dot{q}^T)^T \in \mathbb{R}^{4n} \) as

\[
\dot{x} = \begin{bmatrix}
0 & I_{n \times n} \\
-M^{-1}K_m & 0 \\
\end{bmatrix} x + \begin{bmatrix}
0_{n \times 1} \\
M^{-1}B_0 \\
\end{bmatrix} u(t) + \begin{bmatrix}
0_{n \times 1} \\
M^{-1}B_a \\
\end{bmatrix} m_a(\alpha) \\
\triangleq Ax + Bu + Dm_a(\alpha) \quad (2.16)
\]

where the matrices \( A, B \) and \( D \) are defined in Eq. (2.16). Since the interest is in the control of the fin angle, we associate with the system Eq. (2.13), a controlled output variable given by

\[
y = \theta \quad (2.17)
\]

where \( \theta \) is the fin angle.

Let a smooth and bounded reference fin angle trajectory \( \theta_r \) is given. We are interested in designing an adaptive control law such that in the closed-loop system the fin rotation angle asymptotically tracks \( \theta_r \) and the state variables remain bounded during maneuver.
CHAPTER 3

STATE FEEDBACK AND OUTPUT FEEDBACK CONTROL

In this chapter, state feedback and output feedback adaptive control laws are derived for the trajectory control of the rotation angle of the projectile's fin. For the purpose of design, it is assumed that the model parameters are not known. The fin is controlled by regulating the tip deflection of the flexible beam which in turn causes the control of the rotation angle of the fin. The input signal is the voltage applied to piezoelectric actuator and the output variable is chosen to be the rotation angle of the fin.

3.1 State Variable Representation

As derived in Chapter 2., the reduced dynamic model using \( n \) elements can be expressed in the state variable form using the state vector \( x = (q^T, q^T)^T \in \mathbb{R}^{4n} \) as

\[
\dot{x} = \begin{bmatrix}
0 & I_{n \times n} \\
-M^{-1}K_m & 0
\end{bmatrix} x + \begin{bmatrix}
0_{n \times 1} \\
M^{-1}B_0
\end{bmatrix} u(t) + \begin{bmatrix}
0_{n \times 1} \\
M^{-1}B_a
\end{bmatrix} m_{a0}(\alpha)
\]

\[\triangleq Ax + Bu + Dm_{a0}(\alpha) \tag{3.1}\]

where the matrices \( A, B \) and \( D \) are defined in Eq. (3.1). The controlled output variable is given by

\[y = \theta \tag{3.2}\]
where $\theta$ is the fin angle.

### 3.2 Problem Definition

Let a smooth and bounded reference fin angle trajectory $\theta_r$ be given. An adaptive control law has to be designed such that in the closed-loop system the fin rotation angle asymptotically tracks $\theta_r$ and the state variables remain bounded during maneuver. For the design of control system, it is assumed that the designer has no knowledge of the system. It is assumed here that the matrices $M^{-1}K_m$, $M^{-1}B_0$ and $M^{-1}B_a$ are unknown.

For the derivation of the control laws, a stable manifold which is a linear combination of the fin angle tracking error and its integral and derivative, is chosen. The designed adaptive control systems force the trajectory to converge to the chosen manifold. The manifold is such that the tracking error for any trajectory confined to it asymptotically tends to zero.

The fin angle is given by

$$\theta = \tan^{-1}\left(\frac{w(L, t)}{L}\right)$$

(3.3)

where $w(L, t)$ denotes the tip deflection of the flexible beam. For a small tip deflection one can approximate the fin angle as

$$\theta = \frac{w(L, t)}{L} \triangleq Cx$$

(3.4)

That is, $\theta$ is proportional to $w(L, t)$. Of course, the proposed design approach is applicable even if $\theta$ is given by the nonlinear Eq. (3.3).
Differentiating $\theta$ along the solution of Eq. (3.1) gives

$$\dot{\theta} = CAx$$

$$\ddot{\theta} = CA^2x + CABu + CADm_{a0}(\alpha)$$  \hspace{1cm} (3.5)

3.3 Adaptive State Feedback Control

In this section a state variable feedback adaptive control law is derived. For the purpose of control law derivation, a stable manifold of the form

$$S = \dot{\theta} + 2\zeta\omega_n\dot{\theta}_n + \omega_n^2\int_0^t \ddot{\theta}d\tau$$  \hspace{1cm} (3.6)

is chosen, where $\ddot{\theta} = \theta - \theta_r$ is the fin angle tracking error, $\zeta > 0$ and $\omega_n > 0$. In view of [22] one finds that if $S=0$ (that is, the trajectory is confined to the manifold $S=0$) then $\ddot{\theta}(t) \to 0$, as $t \to \infty$, and the fin angle asymptotically tracks the prescribed trajectory $\theta_r$. Therefore, it is sufficient to design a control law such that $S(t)$ tends to zero.

The design is based on the Lyapunov approach [22, 23, 24]. Differentiating $S$ and using Eq. (3.5) gives

$$\dot{S} = CA^2x + CABu + CADm_{a0}(\alpha) - \dot{\theta}_r + 2\zeta\omega_n\dot{\theta} + \omega_n^2\dot{\theta}$$

$$\triangleq \nu^T x_a + b_0u + g(\theta, \dot{\theta}, t)$$  \hspace{1cm} (3.7)

where $\nu = [(CA^2)^T, CAD] \in \mathbb{R}^{4n+1}$, $b_0 = CAB \in \mathbb{R}$, $x_a = (x^T, 1) \in \mathbb{R}^{21}$ and $g(\theta, \dot{\theta}, t) = -\ddot{\theta}_r + 2\zeta\omega_n\dot{\theta} + \omega_n^2\dot{\theta}$

We note that $\nu$ is an unknown parameter vector. Let $\hat{\nu}$ and $\hat{\theta}$ be the estimates of $\nu$ and $b_0^{-1} = \rho$. Then the parameter errors are $\dot{\nu} = \nu - \hat{\nu}$ and $\dot{\rho} = \rho - \hat{\rho}$. 

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Assumption 1: The scalar parameter $b_0$ is unknown, but its sign is assumed to be known.

For the control law derivation, consider a quadratic Lyapunov function

$$V(S, \ddot{v}, \ddot{\rho}) = (S^2 + \dot{v}^T \Gamma \dot{v} + \mu |b_0| \dot{\rho}^2)/2$$  \hspace{1cm} (3.8)

where $\Gamma$ is a positive definite symmetric matrix and $\mu > 0$.

The derivative of $V$ along the solution of Eq. (3.7) is given by

$$\dot{V} = S[\dddot{v}^T x_a + b_0 u + g(\theta, \dot{\theta}, t)] + \dot{v}^T \Gamma \dot{v} + \mu |b_0| \dot{\rho} \ddot{\rho}$$  \hspace{1cm} (3.9)

In view of Eq. (3.9), we choose a control law of the form

$$u = \ddot{\rho}(-g(\theta, \dot{\theta}, t) - \ddot{v} x_a - c_s S)$$  \hspace{1cm} (3.10)

where $c_s > 0$ and $u_s = -g(\theta, \dot{\theta}, t) - \ddot{v} x_a - c_s S$.

Noting that $b_0 \ddot{\rho} = b_0 (\rho - \ddot{\rho}) = 1 - b_0 \ddot{\rho}$, one has

$$b_0 u = (1 - b_0 \ddot{\rho})(-g(\theta, \dot{\theta}, t) - \ddot{v} x_a - c_s S)$$  \hspace{1cm} (3.11)

Substituting Eq. (3.11) in Eq. (3.9) yields

$$\dot{V} = S[\dddot{v}^T x_a - c_s S - b_0 \ddot{\rho} u_s(S, x_a, \ddot{v}, t)] + \dot{v}^T \Gamma \dot{v} + \mu |b_0| \dot{\rho} \ddot{\rho}$$

$$= -c_s S^2 + \dddot{v}^T (x_a S + \Gamma \dot{v}) + \ddot{\rho} (\mu |b_0| \dot{\rho} - b_0 S u_s(S, x_a, \ddot{v}, t))$$  \hspace{1cm} (3.12)

In order to eliminate terms involving unknown parameters, we choose the adaptation law of the form

$$\dot{\nu} = -\dot{\nu} = -\Gamma^{-1} x_a S$$
\[ \dot{\rho} = -\dot{\rho} = (\text{sgn}(b_0))u_s(S, x_a, \dot{\nu}, t)\mu^{-1}S \]  

(3.13)

Substituting Eq. (3.13) in Eq. (3.12) gives

\[ \dot{V} = -c_sS^2 \]  

(3.14)

Since \( V \) is a positive definite function of \( S, \dot{\nu}, \text{and} \ \dot{\rho} \), and \( \dot{V} \) is negative semidefinite, according to Ref. [22], it follows that \( S, \dot{\nu}, \dot{\rho} \) are bounded functions (denoted as \( L_\infty[0, \infty) \)) and \( V(\infty) \) exists. Integrating Eq. (3.14) gives

\[ c_s \int_0^\infty S^2 dt = V(0) - V(\infty) < \infty \]  

(3.15)

which implies that \( S \in L_2[0, \infty) \) (square integrable function). Also in view of Eq. (3.7) \( \dot{S} \in L_\infty[0, \infty) \) if \( x \) is bounded. Then it follows from the Barbalat's lemma [24, 25], that \( S(t) \to 0 \) as \( t \to \infty \); which implies that the tracking error \( \tilde{\theta}(t) \to 0 \) as \( t \to \infty \). Thus in the closed-loop system, fin angle asymptotically follows the given reference trajectory \( \theta_r(t) \).

The above arguments for trajectory control are based on the assumption that the zero dynamics have bounded solution. Zero dynamics describe the residual motion of the fin when the fin angle is identically zero. The stability of the zero dynamics depend on the zeros of the transfer function \( \dot{\theta}(s)/\ddot{u}(s) = H(s) \), where the overbar denotes the Laplace transform and \( s \) is the Laplace variable. For the fin model, computing \( H(s) \) one finds that \( H(s) \) has purely distinct imaginary zeros. As such, when the fin angle is controlled to a constant value, the residual elastic modes will have persistent, but bounded oscillations consisting of sinusoidal signals.
For the synthesis of the control law Eq. (3.10) and the adaptation law Eq. (3.13), one needs to measure all the state variables. A sufficient number of sensors can be used to obtain the measurement of \( x \). However it is of interest to synthesize a controller using only fin angle and its derivative. We can modify the control and adaptation laws to accomplish this.

### 3.4 Adaptive Output Feedback Control

In this subsection, synthesis using only output feedback is considered. Of course one can design an adaptive feedback control laws using traditional adaptive control techniques [25, 26], however such an attempt is not made here because these adaptive controllers are quite complicated. Instead here we are interested in designing a control law which can be easily synthesized.

Let

\[
\nu^T x = \omega^T y_m + \nu_r^T x_r
\]

where \( x_r \) denotes those variables of \( x \) which are not measured, \( w \in \mathbb{R}^3 \) and \( y_m = (\theta, \dot{\theta}, 1)^T \in \mathbb{R}^3 \) is assumed to be available for feedback. We assume that \( \omega \) is an unknown parameter vector, but \( \nu_r x_r \) is treated as a disturbance input.

For the purpose of design consider a Lyapunov function

\[
W = (S^2 + \dot{\omega}^T \Gamma_1 \dot{\omega} + \mu |b_0|^2) / 2
\]

where \( \Gamma_1 > 0 \) and \( \mu > 0 \). Then its derivative along the solution of Eq. (3.7) is given by

\[
\dot{W} = S(\omega^T y_m + b_0 u + g(\theta, \dot{\theta}, t) + \nu_r^T x_r) + \dot{\omega}^T \Gamma_1 \dot{\omega} + \mu |b_0| \dot{\rho} \dot{\rho}
\]
\[ \leq S[\omega^T y_m + \omega^T y_m + b_0 u + g(\theta, \dot{\theta}, t)] + |S|k^* + \tilde{\omega}^T \Gamma_1 \dot{\omega} + \mu |b_0| \tilde{\rho} \]  
(3.18)

where \( k^* \) is constant such that \( k^* \geq |\nu^T x_r| \).

In view of Eq. (3.18), we choose a control law

\[ u = \dot{\rho}(-\omega^T y_m - g - c_1 S - k^* \text{sgn}(S)) \]

\[ \triangleq \dot{\rho} v_s \]  
(3.19)

and the adaptation rule as

\[ \dot{\omega} = -\Gamma^{-1} S y_m \]

\[ \dot{\rho} = S v_s \text{sgn}(b_0) / \mu \]  
(3.20)

Then substituting Eqs (3.19) and (3.20) in Eq. (3.18) gives

\[ \dot{W} \leq -c_1 S^2 \]

Now using a similar argument, one can show that \( \theta \rightarrow 0 \), as \( t \rightarrow \infty \), provided that \( x_a \) remains bounded during the maneuver. The results of next section indeed show that it is the case and by a proper choice of \( k^* \), fin angle control with the control law (Eq. (3.19)) is accomplished. The control law in Eq. (3.19) is discontinuous and one can use a continuous approximation of the \text{sgn} function using a \text{tanh}(s) function. However, in this case some modification in the adaptation law, Eq. (3.20) (for example \( \sigma \)-modification [25] may be required to avoid the parameter divergence.

The structure of an adaptive control system with state feedback or output feedback is shown in Figure 3.1.
3.5 Simulation Results

This section presents the simulation results for the smart fin (fin-beam model) including the state as well as output feedback systems. The mechanical properties of the simulated model are given in the appendix. Using finite element method (with \(n=5\) elements), a state-variable representation of the fin-beam model of dimension of \(20\) is obtained for simulation. The controller parameters chosen are given in the appendix. For simplicity, integral feedback is not used in the manifold Eq. (3.6) (i.e., \(\omega_n=0\)). Reference command trajectories are generated by a third order command generator given by

\[
(s + \lambda_c)(s^2 + 2\zeta_1\omega_{nc}s + \omega_{nc}^2)\theta_r(t) = w_{nc}^2\lambda_c\theta_r^* \]

where \(\theta_r^*\) is the desired fin angle. \(\zeta_1, \lambda_c\) and \(\omega_c\) are chosen to obtain desirable fin angle reference trajectories (see the appendix). Here \(\theta_r^*\) is taken for an angle of \(3^0\), which corresponds to the tip deflection of \(0.0073m\).
3.5.1 Uncontrolled Dynamics: \( u=100 \) Volts

For examining the uncontrolled behavior, the fin-beam model Eq. (3.1) without the adaptive controllers is simulated. The initial condition is set to \( x(0) = 0 \) and a constant voltage \( u = 100 \) volts is applied to the system. Selected responses are shown in Figure 3.2. We observe that each state variable has persistent but bounded oscillations. The fin angle as well as the deflection variables \( (w_2, ..., w_q) \) oscillate about nonzero average values in the steady state. Note that \( w_6 = w(L) \), the tip deflection and \( w_1 = 0 \). It is seen that amplitude of oscillations of \( w_i \) monotonically increases with \( i \) as expected. For the chosen applied voltage, the maximum fin angle is less than \( 1.75^0 \) in the steady state.

3.5.2 Adaptive State Feedback Fin Angle Control: \( \theta = 5^0, \alpha = -5^0 \)

First we present the results using the adaptive law Eq. (3.10) with state variable feedback. The initial values of the parameters chosen are \( \dot{\theta}(0)=0 \) and \( \dot{\rho}(0)=0.5776 \). The actual value of \( \rho \) is 11.5511; therefore, the estimated value of \( \rho \) is 1/20th of the actual value. We have made a worse choice of parameter estimate of \( \theta \) in order to show the robustness of the controller. The selected responses are shown in Figure 3.3. We observe that the fin angle converges to the desired value (5°) in about 2 seconds. The deflections at other points on the beam remain bounded during the maneuver and seem to converge almost to constant values. Apparently, oscillatory components of the flexible modes as well as the control input are negligible in the steady state. The control input required is about 600 volts, which is reasonable. The maximum fin angle tracking error is less than \( 2 \times 10^{-3} \) degrees, which is quite small.
3.5.3 Adaptive Output Feedback Fin Angle Control: \( \theta = 5^0, \alpha = 5^0 \)

Now the adaptive control law, Eq. (3.19) using only the measured signal \( y_m = (\theta, \dot{\theta})^T \) is synthesized and simulation results are obtained. The initial estimates of parameters are \( \hat{\omega}(0) = 0 \) and \( \hat{\rho}(0) = (1/20)\rho \). The value of \( c_1 \) is set to 20. This value has been chosen by observing the simulated responses. The remaining parameters of Figure 3.3 are retained. Simulated responses are shown in Figure 3.4. We observe that the controller using output feedback is effective in trajectory tracking and desired fin angle is smoothly obtained in less than 3 seconds. The deflections at the other points on the beam also have responses which are somewhat similar to those of Figure 3.3. The peak value of the control input remains of the same order and the maximum fin angle tracking error is \( 2 \times 10^{-3} \). It is found that one can obtain faster response using larger values of \( k^* \), however this will require larger input voltage.

3.6 Conclusion

In this chapter control of a smart fin (rigid fin-flexible beam model) of a projectile was considered. The flexible beam with a piezoelectric active layer was used for rotating the fin. A state variable model obtained using the finite element method was used. State-variable feedback and output feedback adaptive control laws were developed. In the closed-loop system trajectory tracking of the reference fin angle was accomplished. Simulation results obtained show that fin angle can be smoothly controlled to desirable values by using either of the controllers in spite of large parameter uncertainties.
Figure 3.2: Uncontrolled dynamics, $u = 100\text{volts}$
Figure 3.3: Adaptive State Feedback Control, $\theta = 5^\circ$, $\alpha = -5^\circ$
Figure 3.4: Adaptive Output Feedback Control, $\theta = 5^\circ$, $\alpha = -5^\circ$
CHAPTER 4

MODEL REFERENCE ADAPTIVE CONTROL

In the previous chapter, an adaptive controllers based on inverse feedback linearization technique were designed. This required synthesis using either the state feedback or discontinuous output feedback control. For state feedback, one needs to measure all the state variables. This requires sufficient number of sensors to obtain the measurement of states. In output feedback control, the controller was designed using only the fin angle and its derivative. All the other states which are not measured are treated as disturbance inputs.

4.1 Introductory Concepts

Based on the command generator tracker concept, a model reference adaptive fin angle controller is designed in this chapter. For the trajectory control of the fin angle, a judicious choice of a controlled output variable, which is a linear combination of the fin angle and fin's angular rate is chosen. The control law does not depend on the dimension of the state space of the fin-elastic beam model. This is important because controller designed based on truncated models of flexible structures may encounter instability due to control and observer spillover. In the closed-loop system
the fin angle asymptotically converges to the target fin angle generated by a command
generator and all the elastic modes converge to their equilibrium values.

The aerodynamic moment affecting the projectile fin can be expressed as (Eq. (2.13))

\[ m_a = m_{a0}(\alpha) + p_a(\alpha)\theta = m_{a0}(\alpha) + p_a(\alpha)l^{-1}e^T q \]  \hspace{1cm} (4.1)

where \( \theta \) is the fin angle, \( \alpha \) is the angle of attack, \( p_a(\theta) \) is a polynomial in the angle of
attack, \( \alpha, p_a(\theta) = p_0 + p_1(\theta) + \ldots + p_k\theta^k \) (\( k \) is a positive integer) and \( e^T \in \mathbb{R}^{2n} \) is a
unit vector whose \((2n - 1)^{th}\) element is one and rest are zero.

The modified fin-beam model including the aerodynamic moment takes the form

\[ M\ddot{q} + Kq = B_0u(t) + B_a m_a \] \hspace{1cm} (4.2)

where \( B_a = [0, \ldots, 0, 1, 0]^T \in \mathbb{R}^{2n} \). Solving Eq. (4.1) gives

\[ \ddot{q} = -M^{-1}K_m q + M^{-1}B_0u(t) + M^{-1}B_a m_{a0}(\alpha) \] \hspace{1cm} (4.3)

where \( K_m = K - p_a(\alpha)L^{-1}e^e^{*T}. \)

The eigenvalues of \( M^{-1}K_m \) are distinct positive real numbers. As such there
exists a similarity transformation matrix \( V \) formed by the eigenvectors of the matrix
\( M^{-1}K_m \) such that

\[ V^{-1}M^{-1}K_m V = \Omega^2 \] \hspace{1cm} (4.4)

where \( \Omega^2 = \text{diag}(\Omega_i^2), i = 1, \ldots, 2n; \Omega_i \neq \Omega_j, i \neq j. \)

Defining \( \eta = V^{-1}q \), one obtains from Eq. (4.3)

\[ \ddot{\eta} = -\Omega^2 \eta + V^{-1}M^{-1}B_0u(t) + V^{-1}M^{-1}B_a m_{a0}(\alpha) \]
\[ = -\Omega^2 \eta + B_1 u(t) + d_1 \] (4.5)

where \( B_1 = V^{-1}M^{-1}B_0 \in \mathbb{R}^{2n} \) and \( d_1 = V^{-1}M^{-1}B_0 m_{a0}(\alpha) \). The modal form Eq. (4.5) has no damping. However, there is nonzero structural damping for any elastic body. As such it is common to introduce a dissipation term proportional to the rate \( \dot{\eta} \). Introducing a damping term of the form \( 2D\Omega \), where \( D = \text{diag}(\zeta_i), i = 1, \ldots, 2n \), \( \zeta_i > 0 \), one obtains the system

\[ \ddot{\eta} = -2D\Omega\eta - \Omega^2 \eta + B_1 u + d_1 \] (4.6)

The fin angle in new coordinate becomes

\[ \theta = L^{-1}e^{*T}q = L^{-1}e^{*T}V\eta = C_0\eta \]

where \( C_0 = L^{-1}e^{*T}V \).

It is assumed that the system matrices \( D, \Omega, B_1 \) and \( C_0 \) are unknown. Furthermore, it is assumed that only the fin angle and the angular rate is measurable. Consider a first-order reference model of the form

\[ x_m' = A_m x_m + B_m u_m \] (4.7)

\[ y_m = C_m x_m \] (4.8)

where \( x_m \in \mathbb{R}^m \), \( y_m \in \mathbb{R} \) and \( A_m \) is a Hurwitz matrix. We are interested in designing an adaptive control system such that the fin angle asymptotically tracks the reference trajectory \( y_m \). Moreover, for synthesis only the measured angle \( \theta \) and \( \dot{\theta} \) are to be used.
4.2 Control Law Design

In this section, an adaptive controller will be designed. Defining the state vector $X = (\eta^T, \eta^T)^T \in \mathbb{R}^{4n}$, a state variable representation of Eq. (4.5) takes the form

$$
\dot{X} = \begin{bmatrix}
0_{2n \times 2n} & I_{2n \times 2n} \\
-\Omega^2 & -2D\Omega
\end{bmatrix} X + \begin{bmatrix}
0_{2n \times 1} \\
B_1
\end{bmatrix} u + \begin{bmatrix}
0_{2n \times 1} \\
d_1
\end{bmatrix}
$$

$$
\triangleq AX + Bu + d
$$

(4.9)

where $d = [0_{1 \times 2n}, d_1^T]^T$ and $B = [0_{1 \times 2n}, B_1^T]^T$. We associate with Eq. (4.9), an output variable

$$
y = CX
$$

(4.10)

where $C$ is yet to be chosen. In Eq. (4.9), $d$ is treated as a disturbance input. An ideal system is obtained when the disturbance $d$ is zero. This ideal state equation is given by

$$
\dot{X} = AX + Bu
$$

(4.11)

with its associated output given in Eq. (4.10). For the design of the controller based on the CGT method [21, 27], almost strictly positive real (ASPR) condition for the system Eq. (4.10) and Eq. (4.11) (system $\{A, B, C\}$) is required [28].

**Definition:** A system $\{A, B, C\}$ is ASPR if there exists a scalar gain $K_e$ and symmetric positive definite matrices $P, Q_a \in \mathbb{R}^{4n \times 4n}$ such that

$$
P(A - K_e BC) + (A - K_e BC)^T P = -Q_a
$$

(4.12)

$$
P B = C^T
$$

(4.13)
and if $K_e = 0$, then $\{A, B, C\}$ is said to be strictly positive real (SPR) [27].

The model with the choice of the output $y \triangleq \theta = C_0 \eta$ cannot be ASPR because the associated transfer function is of relative degree two. The ASPR condition can be satisfied if we make a judicious choice of the output variable of the form

$$y = (\dot{\theta} + \mu \theta) = C_0 \dot{\eta} + \mu C_0 \eta$$

$$= \begin{pmatrix} \mu C_0 & C_0 \end{pmatrix} X \triangleq CX$$  \hspace{1cm} (4.14)

where $\mu$ is positive real number. In the following derivation, it is assumed that $C = (\mu C_0, C_0)$; that is, the output variable is a linear combination of the fin angle and its derivative.

Mufti [21] has shown that for a choice of

$$\mu < \mu^* = \min\{2\delta \omega_i, i = 1, \ldots, 2n\}$$  \hspace{1cm} (4.15)

there exists matrix $P$ which satisfies Eq. (4.12) and Eq. (4.13) with $Q_a > 0$ for $K_e = 0$ and therefore, the system (Eq. (4.10) and Eq. (4.11)) is in fact, strictly positive real. This is also easily verified by computing the transfer function relating $y$ and $u$, which is

$$\frac{\dot{y}(s)}{\dot{u}(s)} = \sum_{i=1}^{2n} \frac{c_0 b_{1i}(s + \mu)}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} \triangleq \sum_{i=1}^{2n} H_i(s)$$  \hspace{1cm} (4.16)

where $C_0 = (c_0, \ldots, c_{0,2n})$ and $B_1 = (b_{11}, \ldots, b_{1,2n})^T$. For the model under consideration, computing the matrices $C_0$ and $B_1$, one finds that

$$c_0 b_{1i} > 0, i = 1, \ldots, 2n$$  \hspace{1cm} (4.17)
and therefore, each of the transfer functions $H_i(s)$ with $\mu < 2\delta \omega_i$ is SPR. Apparently, the parallel combination of SPR transfer functions (see Eq. (4.16)) is also SPR.

Later, even though $(A, B, C)$ is SPR, we shall introduce additional output feedback for modifying the transient characteristics. It may be noted that in fact, for any real number $K_e > 0$, the system $\{(A - K_e BC), B, C\}$ remain SPR. This is easily verified by noting that SPR system $\{A, B, C\}$ implies that there exist $P > 0$ and $Q > 0$, such that

$$A^T P + P A = -Q \tag{4.18}$$
$$PB = C^T \tag{4.19}$$

Subtracting both sides by $K_e^* P B C$ and $K_e^* C^T B^T P$ from Eq. (4.18) and noting that $PB = C^T$, one obtains

$$P(A - K_e^* BC) + (A - K_e^* BC)^T P = -Q - 2K_e^* C^T C \triangleq Q_a < 0 \tag{4.20}$$

and therefore, the same matrix $P$ solves Eq. (4.20). However, a good value of $K_e^*$ is not known. Therefore, we intend to design a controller which will adaptively seek a good output feedback gain $K_e^*$ for control.

Now the design of the controller for the asymptotic tracking of the reference fin angle trajectory $y_m(t)$ is considered. When perfect tracking occurs, i.e., $y = y_m$ for $t \geq 0$, let $X^*$, $u^*$ and $y^*$, respectively, denote the corresponding plant state, input and output trajectories of the ideal model Eq. (4.10) and Eq. (4.11). These ideal trajectories satisfy

$$\dot{X}^* = AX^* + Bu^* \tag{4.21}$$
and in view of Eq. (4.8), one has

\[ y^* = CX^* = C_m x_m = y_m \]  \hspace{1cm} (4.22)

In the command generator tracker theory [21, 27], it is assumed that the starred signals satisfy

\[ X^* = S_{11} x_m + S_{12} u_m \]

\[ u^* = S_{21} x_m + S_{22} u_m \]  \hspace{1cm} (4.23)

where \( S_{ij} \) are constant matrices. We assume here that \( u_m \) is a step function. Indeed for the existence of matrices \( S_{ij} \), one needs to satisfy only a mild (CGT) condition. For the solvability of Eq. (4.23), it is sufficient that, the transfer function \( C(sI - A)^{-1}B \) has no zeros at the origin or in common with any eigenvalue of \( A_m \) [27]. For the model under consideration, this CGT condition is satisfied if \( A_m \) is selected to have only real eigenvalues. Note that since \( H(s) \) is SPR, it has only stable zeros.

Define the state error as \( \tilde{X} = X^* - X \). Then when the disturbance input \( d \neq 0 \), one obtains from Eq. (4.9) and Eq. (4.21), the state error equation of the form

\[ \dot{\tilde{X}} = A \tilde{X} + B(u^* - u) - d \]  \hspace{1cm} (4.24)

The output error can be written as

\[ \tilde{y} = y_m - y = C_m x_m - CX = CX^* - CX = C \tilde{X} \]  \hspace{1cm} (4.25)

Using Eqs. (4.23) and (4.24) and adding and subtracting \( BK_e^* C \tilde{X} \) in Eq. (4.24), gives

\[ \dot{\tilde{X}} = (A - BK_e^* C) \tilde{X} + B [K_e \tilde{y} + S_{21} x_m + S_{22} u_m - u] - d \]  \hspace{1cm} (4.26)
where, $K^*_e > 0$. $S_{21}, S_{22}$ are treated as unknown. The feedback gain $K^*_e$ denotes an unknown gain which is considered to give a good performance.

Define $\nu = \begin{bmatrix} K^*_e & S_{21} & S_{22} \end{bmatrix}^T \in \mathbb{R}^{m+2}$, a vector of unknown parameters. Then the error equation Eq. (4.26) can be written as

$$\dot{X} = A_a \tilde{X} + B(\nu^T \Phi - u) - d \quad (4.27)$$

where $\Phi = \begin{bmatrix} \tilde{y} & x_m^T & u_m \end{bmatrix} \in \mathbb{R}^{m+2}$ is the regressor vector and $A_a = (A - BK^*_e C)$. Note that the system $\{A_a, B, C\}$ is SPR.

The control law is chosen as

$$u = \hat{\nu}^T \Phi + \nu_p^T \Phi \quad (4.28)$$

Here $\hat{\nu}$ denotes an estimate of the actual parameter vector $\nu$ and $\nu_p$ is a proportional gain vector. Substituting the control law Eq. (4.28) in Eq. (4.27) gives

$$\dot{X} = A_a \tilde{X} + B[\hat{\nu}^T \Phi - \nu_p^T \Phi] - d \quad (4.29)$$

where $\hat{\nu} = \nu - \hat{\nu}$ is the parameter estimation error.

For the derivation of the adaptation law, consider a quadratic Lyapunov function

$$W(\tilde{X}, \tilde{\nu}) = \tilde{X}^T P \tilde{X} + \tilde{\nu}^T \Gamma \tilde{\nu} \quad (4.30)$$

where $P > 0$ is the solution of Eq. (4.20) and $\Gamma$ is a positive definite symmetric matrix (denoted as $\Gamma > 0$). The function $W$ satisfies

$$\lambda_{\min}(P)||\tilde{X}||^2 + \lambda_{\min}(\Gamma)||\tilde{\nu}||^2 \leq W(\tilde{X}, \tilde{\nu}) \leq \lambda_{\max}(P)||\tilde{X}||^2 + \lambda_{\max}(\Gamma)||\tilde{\nu}||^2$$

where for any matrix $M_a > 0$, $\lambda_{\min}(M_a)$ [$\lambda_{\max}(M_a)$] denotes minimum [maximum] eigenvalue of the matrix $M_a$. 

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The derivative of $W$ along the solution of Eq. (4.29) is given by

$$
\dot{W} = \tilde{X}^T (PA_a + A_a^T P) + 2\tilde{X}^T P B [\tilde{v}^T \Phi - \nu_p^T \Phi] + 2\tilde{v}^T \Gamma \dot{\nu} - 2\tilde{X}^T P d
$$

(4.31)

Using Eq. (4.18) and Eq. (4.19) in Eq. (4.31), and choosing

$$
\nu_p = \Gamma_p \Phi \tilde{y}
$$

(4.32)

where the matrix $\Gamma_p \geq 0$ and noting that $\tilde{X}^T P B = \tilde{X}^T C^T = \dot{y}$ gives

$$
\dot{W} = -\tilde{X}^T Q_a \dot{X} + 2\tilde{v}^T (\tilde{y}^T \Phi + \Gamma \dot{\nu}) - 2\tilde{X}^T P d - 2\tilde{y}^2 \Phi^T \Gamma_p \Phi
$$

(4.33)

Eq. (4.33) involves uncertain function $\tilde{v}$. For eliminating $\tilde{v}$-dependent term in Eq. (4.33), one can select

$$
\dot{\tilde{v}} = -\dot{\nu} = \Gamma^{-1} \Phi \tilde{y}
$$

(4.34)

However, when the disturbance input $d \neq 0$, in the closed-loop system parameter divergence takes place due to the presence of $d$ in Eq. (4.33).

In order to avoid instability in the closed-loop system when $d$ is nonzero, one introduces $\sigma$-modification in Eq. (4.34) yielding

$$
\dot{\tilde{v}} = -\dot{\nu} = \Gamma^{-1} \Phi \tilde{y} - \sigma \Gamma^{-1} \dot{\nu}
$$

(4.35)

where $\sigma > 0$ is a design parameter. With $\sigma$-modification, one can show that all the signals and the tracking error are bounded in the closed-loop system if $d \neq 0$.

Substituting the parameter adaptation law Eq. (4.35) in Eq. (4.33) gives

$$
\dot{W} = -\tilde{X}^T Q_a \dot{X} - 2\tilde{y}^2 \phi^T \Gamma_p \phi + 2\sigma \tilde{v}^T \dot{\nu} - 2\tilde{X}^T P d
$$
Noting that $\ddot{v}^T \dot{v} = -\dot{v}^T (-\dot{v} + v - \nu) = -||\dot{v}||^2 + \dot{v}^T \nu$ ($||.||$ denotes the Euclidean norm), Eq. (4.33) yields,

$$\dot{W} \leq -\lambda_{\min}(Q_a)||\ddot{X}||^2 - 2\sigma||\ddot{v}||^2 + 2\sigma \dot{v}^T \nu - 2\ddot{X} P d$$  \hspace{1cm} (4.36)

Using Schwartz and Young's inequality [21], one has

$$|2\ddot{X}^T P d| \leq ||\ddot{X}||||2Pd|| \leq k_p ||\ddot{X}||^2 + \frac{||Pd||^2}{k_p}$$

$$|\dot{v}^T \nu| = ||\dot{v}||||\nu|| \leq \frac{||\dot{v}||^2}{2} + \frac{||\nu||^2}{2}$$  \hspace{1cm} (4.37)

where $k_p > 0$. Using these inequalities in Eq. (4.36) gives

$$\dot{W} \leq [\lambda_{\min}(Q_a) - k_p]||\ddot{X}||^2 - \sigma||\ddot{v}||^2 + \sigma||\nu||^2 + \frac{||Pd||^2}{k_p}$$

$$= -[\lambda_{\min}(Q_a) - k_p]||\ddot{X}||^2 - \sigma||\ddot{v}||^2 + \beta^*$$  \hspace{1cm} (4.38)

where $\beta^* = \sigma||\nu||^2 + \frac{||Pd||^2}{k_p}$ and $k_p$ is chosen such that $\lambda_{\min}(Q_a) > k_p$.

It is seen from Eq. (4.38) that for large $\ddot{X}$ and $\dot{v}$, $\dot{W}$ is negative. In fact, $\dot{W} < 0$ if $||\ddot{X}|| > \{\beta^*/[\lambda_{\min}(Q_a) - k_p]\}^{1/2} \triangleq r_{\ddot{X}}$ or $||\dot{v}|| > (\beta^*\sigma^{-1})^{1/2} \triangleq r_{\dot{v}}$.

Define

$$v^* = \lambda_{\max}(P)r_{\ddot{X}}^2 + \lambda_{\max}(\Gamma)r_{\dot{v}}^2$$  \hspace{1cm} (4.39)

and a region $S = \{(\ddot{X}, \dot{v}) \in \mathbb{R}^{n+m+2} : ||\ddot{X}|| \leq r_{\ddot{X}}$ and $||\dot{v}|| \leq r_{\dot{v}}\}$.

Then for any positive constant $v_1 \geq v^*$, the ellipsoid

$$E(v_1) = \{(\ddot{X}, \dot{v}) : V(\ddot{X}, \dot{v}) \leq v_1\}$$

contains the set $S$. Therefore, provided that $(\ddot{X}(t), \dot{v}(t)) \notin E(v_1)$, $\dot{W} < 0$ along the trajectory of the system and eventually the trajectory beginning away from $E(v_1)$
enters the set $E(v^*)$ (the set of ultimate boundedness). The subsequent motion of
the system is confined to the set $E(v^*)$. Because $\dot{W} < 0$ in $E^c(v^*)$, $(E^c(v^*))$ denotes
the complement of $E(v^*)$) along the trajectory of the system, $W(t) \leq \max(W(0), v^*)$.
This implies that for all $t \geq 0$

$$||\dot{X}(t)|| \leq \left[ \frac{W(t)}{\lambda_{\max}(P)} \right]^{1/2} \leq \frac{\max\{W(0), v^*\}}{\lambda_{\min}(P)}$$

and ultimately the tracking error satisfies

$$|\bar{y}(\infty)| \leq ||\dot{X}(\infty)|| \leq \frac{v^*}{\lambda_{\min}(P)}$$

The size of the set $E(v^*)$ depends on $P, Q_a, k_p$, and the disturbance input as well as
$\sigma$. Precise computation of the size of ultimate boundedness is rather difficult since
matrices $A, B$ are unknown and as such $P$ and $Q_a$ cannot be computed. Thus, this
analysis establishes robust stability in the closed-loop system in a qualitative sense.

It is pointed out that in the closed-loop system, instead of the asymptotic con­
vergence of the tracking error to zero, only boundedness of $\bar{y}$ as well as $\dot{\theta}$ and $\dot{X}$ is
guaranteed even if the disturbance input $d = 0$. However, it is seen in the next section
that by the proper choice of $\sigma$ and other design parameters, precise control of the fin
angle is accomplished.

4.3 Simulation Results

This section presents the simulation results for the smart fin (fin-beam model)
including the model reference adaptive control law. The mechanical properties of the
simulated model are given in the appendix. Using the finite element method (with

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n=5 elements), a state-variable representation of the fin-beam model of dimension of 20 is obtained for simulation. Of course the designed adaptive control system is independent of the order of the fin-beam model. The reference command trajectory is obtained using a first order command generator, with values $A_m = -6$, $B_m = 6$, $C_m = 1$. The values $\Gamma = 5000000(J_{2n \times 2n})$, $\Gamma_p = 0$ and $\sigma = 2 \times 10^{-9}$ are used for simulation. The damping coefficient is taken as $\zeta = 0.005$. Simulation results are shown for different fin angle commands. Based on the lowest frequency of the flexible modes ($\Omega_i = 53.343 \text{rad/sec}$), the value of $\mu = 0.5$ is selected which is less than $2\delta_i \Omega_i = 0.53$. The initial estimate of $\dot{\theta}(0)$ are taken as $[-10, 0, 0]^T$.

4.3.1 Adaptive Control: $\theta = 5^0$, $\alpha = -5^0$ and $+5^0$

Simulation results for a fin angle command of $5^0$ for angles of attack $\alpha = -5^0$ and $\alpha = +5^0$ are shown in Figure 4.1 and 4.2, respectively. It is observed that the fin angle asymptotically converges to the desired value in about 5 seconds. In the steady state, the control input $u = u^*$ ($u^*$ denotes the value in the equilibrium condition) needed to deflect the fin to an angle of $5^0$ for $\alpha = -5^0$ is 1000 volts and $\alpha = +5^0$ is 1060 volts. The deflections at other points on the beam remain bounded during the maneuver and converge to constant values, in both the cases. Note that $w_i(t) > w_i(t)$ in the figure. In spite of the $\sigma$-modification, in the adaptation law, the tracking error is almost zero in about 10 seconds. We note that there is no overshoot in the fin angle trajectory and the control input never exceeds $u^*$, the voltage required to maintain $\theta = 5^0$ in the equilibrium condition. The estimated parameters remain bounded and converge to certain constant values.
4.3.2 Adaptive Control: $\theta = -10^0$, $\alpha = -10^0$ and $+10^0$

Figure 4.3 and Figure 4.4 show the simulation results for fin angle command of $-10^0$, for the angles of attack $-10^0$ and $+10^0$, respectively. These results show that the fin angle control is achieved in about 5 seconds. We observe that for larger fin angle command the control magnitude required is larger as expected. The control magnitude required for the deflection of $-10^0$ is almost twice the voltage needed for $5^0$. It is also interesting to see that there is no overshoot for the flexible modes and they reach their equilibrium values, in both the cases. All the estimated parameters converge to constant values.

4.4 Conclusion

In this chapter, the control of rotation angle of a smart projectile fin. The model of the fin-beam system includes the aerodynamic moment which is a function of angle of attack of the projectile. A model reference adaptive controller, based on the command generator tracker concept was designed. Only the fin angle and its derivative were required for feedback. Interestingly, the structure of the adaptive controller is simple and does not depend on the dimension of the fin-beam model. Simulation results show that the designed adaptive control system accomplishes precise fin angle control in spite of uncertainties in the fin-beam parameters and the aerodynamic moment coefficients.
Figure 4.1: Adaptive Control: $\theta = 5^0, \alpha = -5^0$
Figure 4.2: Adaptive Control: $\theta = 5^\circ, \alpha = 5^\circ$
Figure 4.3: Adaptive Control: $\theta = -10^0$, $\alpha = -10^0$
Figure 4.4: Adaptive Control: $\theta = -10^0$, $\alpha = 10^0$
CHAPTER 5

ADAPTIVE SERVOREGULATOR

In chapter 4, an adaptive controller based on the command generator tracker concept was designed. The synthesis of this controller required tuning of three parameters of the adaptive loop. This required sigma or dead-zone modification of the adaptation rule in order to avoid parameter divergence. The modification of the adaptation rule may sometimes give terminal tracking error. This chapter deals with an adaptive servoregulator designed for the control of the fin angle and the rejection of the disturbance input (aerodynamic moment). It is assumed that the system parameters are completely unknown and that only the fin angle and its derivative are measured for synthesis. A linear combination of the fin angle and fin’s angular rate is chosen as the controlled output variable. Here the controller requires tuning of a single gain and unlike in chapter 4, the controller is capable of rejecting the aerodynamic disturbance torque without any adaptive law modification. In the closed-loop system, the fin angle asymptotically converges to the target fin angle generated by a command generator.
5.1 Adaptive Servocontrol Law

In this section, an adaptive servoregulating controller will be designed. Defining the state vector $x = (\eta^T, \dot{\eta}^T)^T$, a state variable representation takes the form

$$
\dot{x} = \begin{bmatrix}
0_{2n \times 2n} & I_{2n \times 2n} \\
-\Omega^2 & -2D\Omega
\end{bmatrix} x + \begin{bmatrix}
0_{2n \times 1} \\
B_1
\end{bmatrix} u + \begin{bmatrix}
0_{2n \times 1} \\
F_1
\end{bmatrix} v
$$

$$
\triangleq A x + B u + F v \quad (5.1)
$$

We select the controlled output variable as

$$
y = (\dot{\theta} + \lambda \theta)
$$

$$
\triangleq C_0 \dot{\eta} + \lambda C_0 \eta \triangleq C x \quad (5.2)
$$

where $\lambda > 0$ is a design parameter. From Eq. (5.2), one obtains

$$
\dot{y}(s) = C(sI - A)^{-1} B \ddot{u}(s) + C(sI - A)^{-1} F \ddot{v}(s)
$$

$$
\triangleq \frac{n_p(s) \ddot{u}(s) + n_f(s) \ddot{v}(s)}{d_p(s)} \quad (5.3)
$$

where $s$ is the Laplace variable and $u$ and $v$ denote Laplace transforms of $u$ and $v$ respectively, and

$$
n_p(s) = C \text{adj}(sI - A) B
$$

$$
n_f(s) = C \text{adj}(sI - A) F
$$

$$
d_p(s) = \text{det}(sI - A)
$$

It is easily seen that

$$
d_p(s) = \sum_{i=1}^{2n} (s^2 + 2\zeta_i \Omega_i s + \Omega_i^2) \quad (5.5)
$$

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is a Hurwitz polynomial. Furthermore, computing the polynomial \( n_p(s) \) for this model, one finds that it is a Hurwitz polynomial. Therefore, the transfer function \( (n_p(s)/d_p(s)) \) is minimum phase.

The tracking error \( e_1 = y - y_m \) is

\[
e_1 = \frac{n_p(s)}{d_p(s)} \dot{u}(s) + \frac{n_f(s)}{d_p(s)} \dot{v}(s) - \dot{y}_m(s)
\]

where \( y_m \) is the constant reference trajectory is constant. For a given angle of attack, the aerodynamic moment \( v = m_{ao}(\alpha) \) acts as a constant disturbance input and it must be rejected by the controller. In order to eliminate this unknown disturbance term \( v \), let us filter each side of Eq. (5.6) with \( (\frac{s}{s+\mu}) \), where \( \mu > 0 \). For constant signals \( v \) and \( y_m \), one has \( sv = 0 \) and \( sy_m = 0 \). Therefore, the filtered equation Eq. (5.6) yields

\[
\frac{s}{s+\mu} e_1 = \frac{n_p(s)}{d_p(s)} \left( \frac{s}{s+\mu} \right) \dot{u}(s)
\]

We note that we have ignored the exponentially decaying signals in Eq. (5.7).

Defining the filtered input signal as

\[
u_f(s) = \left( \frac{s}{s+\mu} \right) \dot{u}(s)
\]

Eq. (5.7) can be expressed as

\[
e_1 = \frac{(s+\mu)n_p(s)}{sd_p(s)} \dot{u}_f(s)
\]

\[
= H(s) \dot{u}_f(s)
\]

In view of Eq. (5.9), it is sufficient to derive a control law \( u_f(t) \) such that the tracking error \( e_1(t) \) is regulated asymptotically to zero.
For the fin-beam model, \( H(s) \) is minimum phase because \( n_p(s) \) is Hurwitz and \( \mu > 0 \). Moreover, by the choice of the output \( y \), the transfer function has relative degree one. As such using a simple argument from the root-locus technique, it is easily seen that a negative feedback law of the form

\[
    u_f(t) = -K_ee_1
\]  

(5.10)

can stabilize the system Eq. (5.9), where \( K_e > 0 \). Indeed, as \( K_e \) tends to \( \infty \), the root loci of the closed-loop poles converge to finite stable zeros of \( H(s) \) and one of the pole tends to \( -\infty \) along the asymptote with angle \( \pi \). This is interesting, because it is an extremely simple control law and yet it accomplishes error regulation and disturbance (\( v \)) rejection.

Consider a minimal realization of \( H(s) \) given by

\[
    \begin{align*}
    \dot{x}_a &= A_ax_a + B_u u_f \\
    e_1 &= C_ax_a
    \end{align*}
\]  

(5.11)

where \( A_a, B_a \) and \( C_a \) are appropriate matrices. Of course, these matrices are not required for synthesis. Since \( H(s) \) is minimum phase with relative degree one, it follows that there exists a gain \( K^* > 0 \) such that [27]

\[
    P(A - K^*B_aC_a) + (A - K^*B_aC_a)^TP = -Q < 0
\]

\[
    PB_a = C_a^T
\]  

(5.12)

where \( P \) and \( Q \) are positive definite symmetric matrices. However, \( K^* \) is not known. Let \( \hat{K} \) be an estimate of \( K^* \) and consider an output feedback law

\[
    u_f = -\hat{K}e_1
\]  

(5.13)
The goal is now to adaptively tune $\tilde{K}$ to accomplish error regulation. Using Eq. (5.13) in Eq. (5.11) gives

$$\dot{x}_a = (A_a - K^*B_aC_a)x_a + (K^*B_aC_a x_a - \tilde{K}B_a e_1)$$

(5.14)

Defining the parameter error $\tilde{K} = K^* - \tilde{K}$, Eq. (5.14) gives

$$\dot{x}_a = \tilde{A}x_a + \tilde{K}B_a e_1$$

(5.15)

where $\tilde{A} = (A_a - K^*B_aC_a)$ is a Hurwitz matrix since Eq. (5.12) holds.

For the derivation of the adaptation law, consider a quadratic Lyapunov function

$$W = x_a^T P x_a + \nu \tilde{K}^2$$

(5.16)

where $\nu > 0$. The derivative of $W$ along the solution of Eq. (5.15) is given by

$$\dot{W} = x_a^T (PA_a + A_a^T P)x_a + 2x_a^T P \tilde{K}B_a e_1 + 2\gamma \tilde{K} \dot{\tilde{K}}$$

(5.17)

Using Eq. (5.12) in Eq. (5.17) and noting that $x_a^T P B_a = x_a^T C_a^T = e_1$ gives

$$\dot{W} = -x_a^T Q x_a + 2\tilde{K}(\gamma \dot{\tilde{K}} + e_1^2)$$

(5.18)

In order to eliminate $\tilde{K}$ form, the adaptation law is chosen as

$$\dot{\tilde{K}} = -\gamma \dot{\tilde{K}} = -\gamma^{-1} e_1^2$$

(5.19)

Substituting Eq. (5.19) in Eq. (5.18) gives

$$\dot{W} = -x_a^T Q x_a \leq 0$$

(5.20)

Since $W(x_a, \tilde{K})$ is positive definite and $\dot{W} \leq 0$, $x_a$ and $\tilde{K}$ are bounded. Furthermore, invoking Barbalat’s Lemma[23, 26], one can establish that $x_a$ tends to zero which in turn implies that $e_1 = C_a x_a$ converges to zero.
The control input \( u(t) \) now can be obtained using Eq. (5.8). In view of (5.8), one has

\[
\hat{u} = (s + \mu)\hat{u}_f
\]  

which yields

\[
u(t) = u_f(t) + \mu \int_0^t u_f(\tau)d\tau
\]  

Using \( u_f(t) = -\hat{K}(t)e_1(t) \) in Eq. (5.22) gives

\[
u(t) = -\hat{K}(t)e_1(t) - \mu \int_0^t \hat{K}(\tau)e_1(\tau)d\tau
\]  

For a constant \( \hat{K} \), the control input simply uses proportional and integral feedback of the tracking error.

5.2 Simulation Results

This section presents the simulation results for the smart fin (fin-beam model) including the model reference adaptive control law. The mechanical properties of the simulated model are given in the appendix. Using finite element method (with \( n=5 \) elements), a state-variable representation of the fin-beam model of dimension 20 is obtained for simulation. The aerodynamic moment Eq. (2.13) is chosen for different angles of attack of the projectile based on the CFD analysis. By a linear approximation of the data obtained by the CFD analysis, the parameters of the aerodynamic moment are found to be \( m_{a0} = -0.0022, p_a = +0.0005 \) for \( \alpha = -5^\circ \) and \( m_{a0} = -0.0028, p_a = 0.01 \) for \( \alpha = +5^\circ \). The value of \( \gamma \) and \( \mu \) is chosen to be 0.00001 and 100 respectively. The initial value of \( \hat{K} \) is taken as 500. The damping coefficient is taken as \( \zeta = 0.005 \). Simulation results are shown for different fin angle commands.
5.2.1 Adaptive Servo Control: $\theta = 5^\circ$, $\alpha = -5^\circ$ and $\alpha = 5^\circ$

Figure 6.1 and Figure 6.2 show the simulation results for fin angle of $5^\circ$, with angles of attack $-5^\circ$ and $+5^\circ$. It is observed that the fin angle asymptotically converges to the desired value in 1 second. The control input needed for the fin to deflect to an angle, $\theta = 5^\circ$ with angle of attack, $\alpha = -5^\circ$ is about 1100 volts, while that with $\alpha = 5^\circ$ is more than 1200 volts. We observe that for larger fin angle command the control input needed is much larger. We also observe that there is no overshoot for the flexible modes and they reach their equilibrium values at the steady state, in both the cases.

5.3 Conclusion

An adaptive servoregulator was designed for the control of fin angle. Simulation results show that the designed adaptive control system accomplishes precise fin angle control in spite of uncertainties in the fin-beam parameters and the aerodynamic moment coefficients.
Figure 5.1: Adaptive Servo Control: $\theta = 5^\circ, \alpha = -5^\circ$
Figure 5.2: Adaptive Servo Control: $\theta = 5^0, \alpha = 5^0$
CHAPTER 6

VARIABLE STRUCTURE ADAPTIVE CONTROL

The adaptive control systems designed in the previous chapters require integral type adaptation schemes. These adaptation schemes are not robust unless some modifications in the update laws are done. Certainly it is useful to explore other adaptive designs in which dynamic adaptation of parameters is avoided.

In this chapter, based on the variable structure model reference adaptive control (VS-MRAC) theory, a new control system for the control of a projectile fin is designed in this chapter. For the derivation of the control law, it is assumed that the parameters in the model are unknown, and only the fin angle is measured for feedback. Control using only the fin angle measurement is very practical since measurement of flexible modes is not easy. However, it is assumed that an upper bound on the uncertain functions is given. Significant advantages of VS-MRAC designs are: nice transient behavior, disturbance rejection capability, and insensitivity to plant nonlinearities or parameter variations.
6.1 Fin Model and Control Problem

Defining the state vector $\mathbf{x}_f = (\eta^T, \dot{\eta}^T)^T \in \mathbb{R}^n$, a state variable representation takes the form

$$\mathbf{x}_f = \begin{bmatrix} 0_{2n \times 2n} & I_{2n \times 2n} \\ -\Omega^2 & -2D\Omega \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0_{2n \times 1} \\ B_1 \end{bmatrix} u + \begin{bmatrix} 0_{2n \times 1} \\ F_1 \end{bmatrix} v$$

$$\dot{\mathbf{x}}_f = A_f \mathbf{x}_f + b_f u + F v \quad (6.1)$$

We select the controlled output variable as

$$y = \theta = h_f \mathbf{x}_f \quad (6.2)$$

Let $y_m$ be a smooth reference trajectory generated by a reference model. We are interested in deriving a VS-MRAC control law $u(t)$ such that the fin angle tracking error

$$e_0 = y - y_m$$

asymptotically tends to zero and the elastic modes remain bounded during maneuver. Furthermore, for a constant set point control of fin angle, it is desired that the flexible modes converge to their equilibrium values. By suitable choice of the reference trajectory $y_m$, desirable fin angle control is accomplished.

Consider the input-output representation of the system Eq. (6.1) given by

$$y(s) = h_f(sI - A_f)^{-1} b_f u(s) + h_f(sI - A_f)^{-1} F v(s) \quad (6.3)$$

$$\triangleq k_p n_p(s) u(s) + n_f(s) v(s) \quad (6.4)$$

$$d_p(s)$$

where $s$ denotes the differential operator or the Laplace variable.
From Eq. (6.4) one has
\[ y = W(s)[u + W_v v] \]
\[ = W(s)[u + g(v)] \] (6.5)
where \( W(s) = k_p n_p(s) d_p^{-1}(s) \) and \( W_v = n_v(s)(k_p n_p)^{-1}(s) \). Since the polynomial \( n_p(s) \) is Hurwitz, it follows that \( W_v(s) \) is a stable transfer matrix. Thus the function \( g \) is bounded.

For the projectile fin model, the transfer function \( W(s) \) has the following properties.

(P1) The relative degree (\( n^* \)) of \( W(s) \) is 2;

(P2) \( W(s) \) is minimum phase.

The property (P1) follows easily since the second derivative of \( y \) explicitly depends on the control input \( u \).

Consider a reference model of relative degree 2 with input \( r \) and output \( y_m \) given by
\[ y_m = W_m(s)r \] (6.6)
\[ W_m(s) = \frac{k_m}{s^2 + \alpha_{m1}s + \alpha_{m2}} = \frac{k_m}{d_m(s)} \]
where the poles of \( W_m \) are assumed to be stable. Now a control law will be derived for tracking the reference trajectory \( y_m \).

6.2 VS-MRAC Law Design

For the design of a variable structure adaptive controller, consider the input-output representation of the system given in Eq. (6.5). Then a controllable and
observable representation of Eq. (6.5) is given by

\[ \dot{x} = Ax + b(u + g(v)) \]  

\[ y = h^T x \]  

(6.7)

It is pointed out that the knowledge of matrices $A$, $b$, $h$ and $g$ are not required for the design of the control system.

For the synthesis of the controller, now the following filters are introduced.

\[ \dot{\omega}_1 = F\omega_1 + \nu u \]

\[ \dot{\omega}_2 = F\omega_2 + \nu y \]  

(6.8)

where $\omega_1, \omega_2 \in \mathbb{R}^{n-1}$,

\[ F = \begin{bmatrix} -\lambda_{n-2} & -\lambda_{n-1} & \ldots & -\lambda_0 \\ \vdots & I & \vdots & \vdots \\ 1 & 0 & \ldots & 0 \end{bmatrix} \]

\[ \nu = \begin{bmatrix} 1 \\ 0^T \end{bmatrix} \]

where $I$ and $0$ denote identity and null matrices of appropriate dimensions and $\lambda_i$ are coefficients of the polynomial

\[ \Lambda(s) = s^{n-1} + \lambda_{n-2}s^{n-2} + \ldots + \lambda_1 s + \lambda_0 = \text{det}(sI - F) \]  

(6.9)

Define $\omega = [\omega_1^T, y, \omega_2^T, r]^T \in \mathbb{R}^{2n}$. The control law is to be synthesized using only the regressor vector $\omega$.

Since $W(s)$ and $W_m(s)$ have same relative degree 2, for $g(v) = 0$, there exists a unique constant vector $\theta^* = [\theta_{\omega_1}^*, \theta_y^*, \theta_{\omega_2}^*, \theta_r^*]^T \in \mathbb{R}^{2n}$, such that the transfer function
of the closed-loop system with the control input

\[ u^* = \theta^* T \omega = (\theta_{\omega_1}^*)^T \omega_1 + \theta_{\omega_2}^* y + (\theta_{\omega_2}^*)^T + \theta_{\gamma}^* r \]  

(6.10)

matches the transfer function \( W_m(s) \) exactly, i.e.,

\[ y = W(s)u = W(s)\theta^* T \omega = W_m(s)r \]  

(6.11)

For model matching, the parameter vector \( \theta^* \) must satisfy [25,26]

\[ \theta_{\omega}^* = \frac{k_m}{k_p} \]  

(6.12)

\[ \theta_{\omega}^* \alpha(s) d_p(s) + k_p (\theta_{\omega_2}^* \alpha(s) + \theta_{\gamma}^* \Lambda(s)) n_p(s) = \Lambda(s) d_p(s) - n_p(s) \Lambda(s) d_m(s) \]  

(6.13)

where \( \alpha(s) = [s^{N-2}, ... , s, 1] \) and \( \theta_{\omega}^* \) denotes the \( k^{th} \) element of \( \theta^* \).

Solving Eq. (6.13) gives the parameter vector \( \theta^* \).

Define \( \kappa^* = (\theta_{\gamma}^*)^{-1} = k_p / k_m \), and \( \bar{u} = u - u^* \). For the fin-beam model, \( k_p > 0 \) and the chosen parameter \( k_m \) is positive, and therefore both \( \kappa^* \) and \( \theta^* \) are positive.

Defining the vector \( X^T = (x^T, \omega_1^T, \omega_2^T)^T \in \mathbb{R}^{3n-2} \), the composite system Eqs. (6.7) and (6.8) can be written as

\[ \dot{X} = A_a X + b_a u + b_y g \]  

(6.14)

\[ y = h_c^T X \]

where

\[
A_a = \begin{bmatrix} A & 0 & 0 \\ 0 & F & 0 \end{bmatrix}, \quad b_a = \begin{bmatrix} b \\ \nu \end{bmatrix}, \quad b_y = \begin{bmatrix} b \\ 0 \end{bmatrix}, \quad h_c^T = \begin{bmatrix} h^T \\ 0 \end{bmatrix}.
\]
Define
\[
\begin{bmatrix}
\omega_1 \\
y \\
\omega_2
\end{bmatrix}
= \begin{bmatrix}
0 & I & 0 \\
h^T & 0 & 0 \\
0 & 0 & I
\end{bmatrix} \triangleq PX \quad (6.15)
\]
Now adding and subtracting $b_au^*$ in Eq. (6.14) and using Eqs. (6.10) and (6.15), gives
\[
\dot{X} = A_cX + b_\alpha * \tilde{u} + b_c r + b_0 g \quad (6.16)
\]
\[
y = h^T_c X
\]
where $A_c = A_a + b_u[\theta^{*T}_{\omega_1}, \theta^{*T}_{\omega_2}, \theta^{*T}_{\omega_3}]P$ and $b_c = \theta^{*T}_r b_u$. For $u = u^*$ (i.e. $\tilde{u} = 0$) and $g = 0$, one has $W_m = h^T_c (sI - A_c)^{-1} b_c$. Therefore, the output of Eq. (6.16), ignoring the exponentially decaying signals due to initial conditions, which is not essential for derivation, can be written as
\[
y = W_m(s)r + \kappa^* W_m(s) \tilde{u} + g_c(v) \quad (6.17)
\]
where $g_c = \tilde{W}_m(s) g$, $\tilde{W}_m(s) = h^T_c (sI - A_c)^{-1} b_g$. Here $\tilde{W}_m(s)$ is a stable transfer function, and therefore, $g_c$ is bounded for any bounded function $g$.

Since $W_m = h^T_c (sI - A)^{-1} b_c$, a non-minimal realization of the reference model (Eq. (6.6)) is
\[
\dot{X}_m = A_c X_m + b_c r \quad (6.18)
\]
\[
y_m = h^T_c X_m
\]
where $X_m \in \mathbb{R}^{3n-2}$. 

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Let the state vector error be \( e = X - X_m \). Subtracting Eq. (6.18) from Eq. (6.16) gives the error equation and the tracking error \((y - y_m)\) given by

\[
\dot{e} = A_c e + b_c \kappa^* \ddot{u} + b_y g
\]

\( e_0 = h_c^T e \)

Using Eq. (6.19), the output tracking error can be written as \( e_0 = \kappa^* W_m(s) \ddot{u} + g_c(v) \).

For the synthesis of the controller, it is essential to introduce a chain of auxiliary errors \( (e'_i) \). Since the relative degree \( n^* \) of the reference model is two, \( W_m(s) \) is not SPR (strictly positive real). To circumvent this difficulty, a polynomial \( L(s) \) given by

\[
L(s) = \frac{s + \delta}{\delta}, \delta > 0
\]

is chosen so that the transfer function \( W_m(s)L(s) \) is SPR. Now, we introduce the following set of filtered signals:

\[
\chi_0 = L^{-1} \chi_1
\]

\[
\xi_0 = L^{-1} \xi_1
\]

where \( \chi_1 = u, \xi_1 = \omega \) and \( \xi_i = (\xi_{i1}, ..., \xi_{in})^T \in \mathbb{R}^{2n}, (i = 0, 1) \). These signals are used to generate a chain of auxiliary error signals \( e'_i (i = 0, 1) \). Based on the results of Refs. [30,32], the complete algorithm for the fin angle control is summarized in Table 1.
Table 1: VS-MRAC Algorithm

<table>
<thead>
<tr>
<th>Auxiliary Errors</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_a = \kappa_{nom}W_mL[u_0 - L^{-1}u_1]$</td>
<td></td>
</tr>
<tr>
<td>$e_0 = y - y_m; e_0' = e_0 - y_a$</td>
<td></td>
</tr>
<tr>
<td>$e_1' = (u_0)_{eq} - L^{-1}(u_1)$</td>
<td></td>
</tr>
</tbody>
</table>

| Modulation Functions | $f_0 \geq \bar{\kappa} | x_0 - \theta^T_{nom}\xi_0 | + \sum_{j=1}^{2n} \bar{\theta}_{ij} | \xi_{ij} | + \bar{\gamma}_0 + \epsilon_0$ | |
|----------------------|-------------------------------------------------|---|
| $f_1 \geq \sum_{j=1}^{2n} \bar{\theta}_{ij} | \xi_{ij} | + \bar{\gamma}_1 + \epsilon_1$ | |

<table>
<thead>
<tr>
<th>Control Laws</th>
<th>$u_i = f_i sgn(e_i'), i = 0, 1$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$u = -u_1 + u_{nom}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u_{nom} = \theta^T_{nom}\omega$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the above table, $\theta_{nom}$ and $\kappa_{nom} \neq 0$ are chosen nominal values of the parameters $\theta^*$ and $\kappa^*$, respectively, $\epsilon_0 > 0, \epsilon_1 > 0$, and the upper bounds $\bar{\theta}_{ij}$ ($i = 0, 1$ and $j = 1, ..., 2n$), $\bar{\kappa}$, and $\bar{\gamma}_i$ ($i = 0, 1$) are defined as

$$\bar{\theta}_{ij} > \rho | \theta^*_{j} - \theta_{j,nom} |, \bar{\kappa} > | \rho - 1 |$$

$$\bar{\gamma}_0 > \sup_{t \geq 0}(k_{nom}W_mL)^{-1}g_c = \sup_{t \geq 0}(k_{nom}W_mL)^{-1}\bar{W}_mg$$ (6.20)  

$$\bar{\gamma}_1 > \sup_{t \geq 0}(\theta^*_{r}W_m^{-1})g_c = \sup_{t \geq 0}(\theta^*_{r}W_m^{-1})\bar{W}_mg$$
where $\rho = \kappa^*/\kappa_{nom}$. Note that $(k_{nom}W_mL)^{-1}W_m$ is a strictly proper stable transfer function, $\theta^*W_m^{-1}W_m$ is a proper stable transfer function, and $g$ is a bounded signal. Note that $y_a$ can be interpreted as a predicted output error and hence $e_0'$ is a prediction error [30]. The complete closed-loop system is shown in Fig. 6.1.

The equivalent control $(u_0)_{eq}$ can be obtained by setting $e_0'(t) \equiv 0$ in the dynamical system governing the error $e_0'$ and can be approximately obtained from $u_0$ by means of a low-pass filter (averaging filter) with high enough cut-off frequency denominated averaging filter [31]. The inclusion of such averaging filters in the stability analysis has been considered in Ref. [30].
Now consider the **VS-MRAC** law of Table 1. Then for any trajectory the closed-loop system has the following properties:

a) The errors $e_i^i$ ($i = 0, 1$) all converge to zero in finite time;

b) The fin angle tracking error $e_0^i$ converges exponentially to zero.

From Table 1, we find that the control law is a discontinuous function of $e_i^i$. It is well known that this discontinuity in the control law causes undesirable control chattering. For avoiding control chattering, one uses a smooth continuous approximation of the switching functions. For this, one replaces $u = sgn(e_i^i)$ by $u = sat(e_i^i)$, where $sat(\eta)$ is defined as $sat(\eta) = sign(\eta)$ if $|\eta| \Delta$, and $sat(\eta) = (\eta/\delta)$ if $|\eta| \Delta$. Here $\Delta$ is the bounded layer thickness.

From Table 1, we observe that for the synthesis of the control law, it is essential to compute the modulation functions $f_i$. Of course, these functions $f_i$ can be computed on-line using signals $\xi_i$ and $\chi_0$, and the bounds $\tilde{g}_i$ on uncertain functions. However, implementation of this control law using on-line computation of the bounds $f_i$ is extremely complicated. A simple control law can be synthesized by using constant overestimated values of the modulation functions. The simplified control system has been termed as the **Relay VS-MRAC** [30].

### 6.3 Simulation Results

In this section, simulation results for the closed-loop system with the control law derived in Table 1 are presented. The closed-loop system is shown in Figure 6.1. The mechanical properties of the simulated model are given in the appendix. Using
the finite element method (with \( n=5 \) elements), a state-variable representation of the fin-beam model of dimension of 20 is obtained for simulation.

The reference model is chosen as

\[
W_m = \frac{\lambda_m^2}{(s + \lambda_m)^2}
\]  

(6.21)

where \( \lambda_m \) is 0.1. We choose \( L = (s + \lambda_m)/\lambda_m \) so that \( W_mL \) is SPR. For the computation of \( u_{nom} = \theta_{nom}r \), the nominal value of \( \theta \) was arbitrarily chosen as \( \theta_{nom} = (0, ..., 0, 1)^T \in \mathbb{R}^n \), giving \( u_{nom} = r \). This is rather an unfavorable choice of estimate of \( \theta^* \). In the saturation function, the boundary layer thickness set to \( \Delta = 0.5 \). A simplified relay type controller was synthesized by using constant modulation functions as \( f_0 = 1000000 \) and \( f_1 = 1000000 \).

6.3.1 Adaptive Control: \( \theta = 5^\circ, \alpha = -5^\circ, 5^\circ \)

Simulation results for a fin angle command of 5° for angle of attack \( \alpha = -5^\circ \) are shown in Figure 6.2. Figure 6.3 shows the simulation results for fin angle command of 5° with angle of attack, \( \alpha = 5^\circ \). It is observed that the fin angle asymptotically converges to the desired value in less than 1 second in both the cases. In the steady state, the control input needed to deflect the fin to an angle of 5° for \( \alpha = -5^\circ \) is around 1000 volts. The deflections at other points on the beam remain bounded during the maneuver and converge to constant values. The tracking error is of the order of \( 10^{-3} \). We note that there is no overshoot in the fin angle trajectory and the control input never exceeds \( u^* \), the voltage required to maintain \( \theta = 5^\circ \) in the equilibrium condition.

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6.4 Conclusions

Based on the variable structure model reference adaptive control theory, a new control law for the control of fin angle of a projectile fin was presented. In the fin-beam model, disturbance input was assumed to be present. A variable structure model reference adaptive control system was synthesized using only measurement on the fin angle and measurement of other flexible modes was not needed for control. Interesting, unlike usual adaptive controller, the derived VS-MRAC system does not have integral type adaptation law for updating the parameters of controller. This structure of adaptive controller has significant advantage over other adaptive schemes, since in this case controller parameter divergence cannot occur. In the closed-loop system, fin angle tracked the reference trajectory, and stabilization of flexible modes was accomplished. Extensive simulation results were presented which showed good transient characteristics of the designed controller in spite of the presence of uncertainties in the system parameters and aerodynamic disturbance input.
Figure 6.2: Variable Structure Control: $\theta = 5^0$, $\alpha = -5^0$
Figure 6.3: Variable Structure Control: \( \theta = 5^0, \alpha = 5^0 \)
CHAPTER 7

CONCLUSION

This thesis considered the control of rotation angle of a smart projectile fin. The flexible beam with a piezoelectric active layer was used for rotating the fin. In the second chapter, the mathematical model of the fin-beam system was described and a state-variable model was obtained using the finite element method.

In the third chapter, a state feedback adaptive control law was designed for the trajectory control of the fin angle. This was followed by the design of an adaptive control system using only output feedback. The effect of aerodynamic moment was not considered here. In the closed-loop system, asymptotic trajectory tracking of the fin angle was accomplished. Simulation results showed that trajectory control of the fin angle was accomplished in spite of large uncertainties using the state as well as output feedback and the flexible modes remain bounded during maneuvers.

The fourth chapter considered a model reference adaptive control for the projectile fin. This model included the aerodynamic moment which is a function of angle of attack of the projectile. Based on the command generator tracker concept, a model reference adaptive fin angle controller was designed. The structure of the control system designed was independent of the dimension of the flexible fin-beam model. In
the closed-loop system, the controlled output variable tracked the reference trajectory
and the fin angle asymptotically converged to the desired value. The elastic modes
converged to their equilibrium values. Simulation results presented showed that, the
fin angle was precisely controlled in spite of uncertainties in the fin-beam parameters
and the aerodynamic moment coefficients.

In the fifth chapter, an adaptive servoregulator was designed for the control of the
fin angle and the rejection of the disturbance input (aerodynamic moment). It was
assumed that the system parameters are completely unknown and that only fin angle
and its derivative are measured for synthesis of the control law. A linear combination
of the fin angle and the fin's angular rate was chosen as the controlled output variable.
Computer simulations performed showed that in the closed-loop system, the fin angle
was precisely controlled in spite of uncertainties in the fin-beam parameters and the
aerodynamic moment coefficients.

Finally in chapter 6, a control system was designed for the control of the flexible
beam based on the theory of variable structure model reference adaptive control (VS-
MRAC) using only input and output signals. It was assumed that an upper bound on
the uncertain functions is given. This adaptive controller has significant advantage
over other adaptive schemes, since in this case controller parameter divergence cannot
occur. Extensive simulation results presented showed good transient characteristics
of the designed controller in spite of the presence of unmodeled dynamics, uncertainty
in system parameters, and disturbance input.

The real time control of the fin using adaptive servoregulator (described in chapter
5) has been done in the laboratory. The reference fin angle was set to $3^\circ$ during this real time simulation, as voltage was constrained to a maximum value of $+1500$ volts giving maximum angle of $4.5^\circ$. The basic filter of the form $\frac{1}{(\tau s + 1)}$ was used to reduce the noise of the signal coming from the encoder. Test results showed that fin angle was controlled very close to the target fin angle of $3^\circ$. It is seen that the control of fin using adaptive servoregulation is very practical than any other methods developed in this thesis. Higher fin angles can be achieved by choosing different configurations of the actuator (like biomorph configuration). The effect of aerodynamic moment is not yet included into real time simulation, but wind tunnel test is of interest in future.
APPENDIX

SYSTEM PARAMETERS

The mechanical parameters for simulation in chapter 3 are taken as
\[ \rho_b = 2,700 \text{ (kg/m}^3\text{)} \quad \rho_p = 7,500 \text{ (kg/m}^3\text{)} \]
\[ E_b = 970 \text{ GPa} \quad E_p = 63 \text{ GPa} \]
\[ L = 140 \text{ mm} \quad b = 25 \text{ mm} \]
\[ h_b = 0.5 \text{ mm} \quad h_p = 0.127 \text{ mm} \]
\[ d_{31} = 1.8 E - 10 \text{ (m/volt)} \]

By a linear approximation of the data obtained by the CFD analysis, the parameters of the aerodynamic moment are found to be
\[ m_{a0} = -0.0022, \quad p_a = +0.0005 \text{ for } \alpha = -5^\circ \]
\[ m_{a0} = -0.0028, \quad p_a = +0.01 \text{ for } \alpha = -5^\circ \]

The controller parameters in chapter 3 are chosen as
\[ \zeta = 0.707 \quad \omega_n = 50 \]
\[ \mu = 5 \quad c_s = 20 \]
\[ \Gamma = I_{20 \times 20} \quad \hat{\rho}(0) = 0.309 \]
\[ \hat{\theta}_0 = 0_{20 \times 1} \quad \omega_{nc} = 2 \]
\[ y_r(0) = 0 \quad \lambda = 4. \]
The mechanical parameters for simulation in chapters 4, 5 and 6 are taken as

\[ \rho_b = 8,300(Kg/m^2) \quad \rho_p = 7,500(Kg/m^2) \]
\[ E_b = 95GPa \quad E_p = 15.89GPa \]
\[ L = 109.4mm \quad b = 25mm \]
\[ h_b = 0.4mm \quad h_p = 0.3mm \]
\[ d_{33} = 170E - 12(m/volt) \]

The controller parameters used for simulation in chapter 4 are

\[ A_m = -6 \quad B_m = 6 \quad C_m = 1 \]
\[ \Gamma = 5,000,000(I_{2n \times 2n}) \]
\[ \Gamma_p = 0 \quad \sigma = 2 \times 10^{-9} \]
\[ \zeta_i = 0.005 \quad \mu = 0.5 \]
\[ \hat{u}(0) = [10, 0, 0]^T \]

The controller parameters chosen for simulation in chapter 5 are

\[ \gamma = 0.00001 \quad \mu = 100 \]
\[ \dot{K} = 500 \quad \delta = 0.005 \]
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