An investigation of grid sampling schemes for kriging contaminant concentrations in a riverbed

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AN INVESTIGATION OF GRID SAMPLING SCHEMES FOR
KRIGING CONTAMINANT CONCENTRATIONS
IN A RIVERBED

by

Matthew Jacob Sokol
Bachelor of Science
State University of New York, 2004

A thesis submitted in partial fulfillment
of the requirements for the

Master of Science Degree in Mathematical Sciences
Department of Mathematical Sciences
College of Sciences

Graduate College
University of Nevada, Las Vegas
December 2005
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Matthew J. Sokol

Entitled

An Investigation of Grid Sampling Schemes for Kriging Contaminant Concentrations in a Riverbed

is approved in partial fulfillment of the requirements for the degree of

Master of Science

Examination Committee Chair

Dean of the Graduate College

Graduate College Faculty Representative
ABSTRACT

An Investigation of Grid Sampling Schemes for Kriging Contaminant Concentrations in a Riverbed

by

Matthew Jacob Sokol

Dr. Ashok Singh, Examination Committee Chair
Professor of Mathematics
University of Nevada, Las Vegas

The object of this thesis is to compare grid sampling schemes for the spatial interpolation method of Kriging, specifically when Kriging is used to predict unsampled locations along a riverbed. The main concern will be finding the optimal grid spacing between samples. Kriging uses an estimated spatial covariance matrix, or variogram, to find the best linear unbiased estimate (BLUE) of contamination at an unsampled location. Since the variogram is a measure of covariance as a function of distance between sampled points, it is important to investigate the effect that sampling location distance has on the method of Kriging.
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CHAPTER 1

INTRODUCTION

Hazardous waste and its cleanup is a major problem for the United States Environmental Protection Agency. Often, however, the problem is overlooked to attend to more pressing social and economic matters. With resources being spent on other endeavors, it is important that scientists find the best way to use what is available to reasonably sample and examine a site. For this very reason, methods used for examining and sampling a contaminated site should be optimized, yielding correct results with the smallest amount of sampling, time, and money invested.

Mathematically, this is an interpolation problem (Chiles and Delfiner, 1999), where interpolation is a method of estimating the value of a function or series between two known values. By sampling a select few locations, we can use their observed values to estimate, by interpolation, the values at unsampled locations. One such method, Kriging, has become the standard mode of spatial interpolation used today. The world "Kriging" is synonymous with "optimal prediction" (Journel and Huijbregts, 1982). It uses the variogram (a measure of covariance as a function of distance between data points) to express spatial variation, and it minimizes the prediction error associated with the original observed values, as well as the values estimated by the spatial distribution.
1.1: The Variogram

Kriging is a geostatistical approach to spatial interpolation. It uses the spatial correlation structure of the data to determine the estimated value of a variable at an unsampled point. Spatial correlation is measured through the use of a variogram. This measure is defined as half of the average squared difference between two values separated by a vector $h$ (Deutsch and Journel, 1992):

$$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} (Z(x_i) - Z(y_i))^2,$$  \hspace{1cm} (1)

where $N(h)$ is the total number of pairs of points separated by vector $h$, $Z(x_i)$ is the contaminant concentration at the starting point of the vector, and $Z(y_i)$ is the concentration at the ending point of the vector. The vector can be expressed visually by the following:

![Vector $h$](image)

Figure 1: Vector $h$

Often the number of points separated by vector $h$ will be very small or none at all. Therefore a distance and direction tolerance is given (set by the sampler) so that one may capture more data points in the calculation of $N(h)$. In the figure below, a distance and
Direction tolerance is set; so that one may capture more data points for the calculation (points within the shaded area are 'captured'):

![Figure 2: Vector $h$ with 'captured' data points](image)

The distance component of $h$ is referred to as the spatial lag. This parameter strongly influences the form of the variogram. Since it is often impractical to plot a variance for each data point with respect to all the other data points, we place closely related and correlated data into these lags. Once data points are placed within a lag, the variance of all pairs of points within the lag is calculated and then averaged. We assign this average as the overall variance of the lag. The resulting average is plotted in the variogram versus the distance corresponding to the lag interval. Hence, there is one point in the variogram for each lag. There are some rules for selecting the right number of intervals (Journel and Huijbregts, 1982), but you can also arrive at an educated guess by looking at the resulting variogram. It is a compromise between the accuracy of the calculated sample-variance and a sufficient number of data points in the variogram to allow reasonable fitting of the curve. (Grauf, 2003)
For our purposes, we will use a variogram known as an omni-directional variogram. This variogram is similar to our original variogram in every way, except the angle of tolerance for determining the capture area of each lag is set to 90 degrees. This is shown in the figure below:

Figure 3: Vector $\mathbf{h}$ with a 90 degree angle of tolerance

This is the standard variogram model used in geostatistical modeling today.

Once we have created and defined the lags, we may use them to create our variogram, or in this case, our omni-directional variogram. A sample omni-directional variogram is presented below:
As we examine the variogram, we see it contains three distinct features: a sill, a range, and a nugget effect. The rate at which a variogram increases reflects the degree of dissimilarity of ever more distant data samples. A variogram can increase indefinitely if the variability has no limit at large distances (Chiles and Delfiner, 1999). If, however, the variogram does reach a certain asymptotic value, we call it the variogram’s sill. We mark the distance where the sill is found as the variogram’s range. The nugget is the $\gamma(h)$ value when $h$ is zero; this is also considered the beginning point for the modeling curve, or the variogram offset. The following figure identifies the sill, range, nugget, lag points, and modeling curve:
It is important to note that, in the cases discussed in this thesis, the variogram models will not depend on direction, which means that when the direction tolerance is set at 90 degrees, the direction of the samples and vector \( h \) have no bearing on the shape of the variogram, i.e., it is an isotropic variogram model. Since one can think of the lag spacing as 'capturing' points in a circle (because of the 90 degree angle of tolerance), it does not matter in which direction the vector originates.

Once an isotropic modeled variogram (omni-directional in this case) has been established, a modeling curve is then set to the data (as shown in Figures 4 and 5). In isotropic modeling, there are many potential models to choose from; however, for this thesis, we will focus on three major models: Spherical, Exponential, and Gaussian. They are defined as follows:

\[
\gamma(h) = \begin{cases} 
C_0 + C_1 \left( \frac{3h}{2r} - 1 - \frac{h}{2} \right)^3, & 0 \leq h \leq r \\
1, & h > 0 
\end{cases}
\]

(2)

Figure 5: Labeled omni-directional variogram using Variowin
Exponential: \( \gamma(h) = \begin{cases} C_0 + C_1 \left( 1 - e^{-\frac{h}{r}} \right), & 0 \leq h \leq r \\ 1, & h > 0 \end{cases} \) \hspace{1cm} (3)

Gaussian: \( \gamma(h) = \begin{cases} C_0 + C_1 \left( 1 - e^{-\frac{h^2}{r^2}} \right), & 0 \leq h \leq r \\ 1, & h > 0 \end{cases} \) \hspace{1cm} (4)

In each of the above models, \( h \) is the vector of importance or the lag spacing, \( C_0 \) is the model nugget value, \( C_1 \) is the model sill value, and \( r \) is the range of the variogram, as described previously.

Once a variogram model for the data has been specified, Kriging can be applied to produce predicted responses at different spatial locations. Kriging is useful in this situation because it yields interpolated estimates at locations where no data is present. For the Kriging Method presented in this thesis, we will discuss the method of Ordinary Kriging.

1.2: Ordinary Kriging

Ordinary Kriging is a widely used Kriging method. It is used often because it assumes an unknown constant mean for the contaminant concentration values. Ordinary Kriging serves to estimate a contaminant response value at a point within a region for which a variogram has been identified, using the contaminant values of neighboring locations (Wackernagel, 1995).

To derive the Ordinary Kriging equations, suppose that we know the values of contamination at the spatial locations \( x_1, x_2, \ldots, x_n \), denoted by \( Z(x_\alpha) \), where \( \alpha \) ranges
from 1 to n. To estimate the contamination at any unsampled location \( x_0 \) we use a linear combination of the sampled locations, given by:

\[
Z^*(x_0) = \sum_{a=1}^{n} \omega_a Z(x_a),
\]

(5)

where \( \omega_a \) is a constraining weight given to the contamination \( Z \) at location \( x_a \), and is derived as the weighted average variogram between all points and the data value to be estimated (Sinclair and Blackwell, 2002). The condition of unbiasedness, that is

\[
E(Z^*(x_0)) = \sum_{a} \omega_a E(Z(x_a)) = \mu,
\]

will also lead us to equation (5).

Since Ordinary Kriging attempts to find the “best” linear unbiased estimated value for the unsampled location \( x_0 \), it is important that we define what is meant by the “best” estimated value. In Kriging applications, “best” is taken to be the estimate that minimizes the estimation variance given by:

\[
\sigma^2_E = E[(Z^*(x_0) - Z(x_0))^2],
\]

(6)

where \( Z^*(x_0) \) is the Kriging estimator and \( Z(x_0) \) is the true value. This expression can be represented in terms of the variogram, as follows (Sinclair and Blackwell, 2002):

\[
\sigma^2_E = -\gamma(0) - \sum_{a=1}^{n} \sum_{b=1}^{n} \omega_a \omega_b \gamma(|x_a - x_b|) + 2 \sum_{a=1}^{n} \omega_a \gamma(|x_a - x_0|).
\]

(7)

Equation (7) can be minimized with the constraint that the weights must sum to one (Sinclair and Blackwell, 2002). By minimizing the estimation variance, we obtain the system of Ordinary Kriging (OK) equations:

\[
\begin{pmatrix}
\gamma(|x_1 - x_i|) & \cdots & \gamma(|x_1 - x_n|) & 1 \\
\vdots & \ddots & \vdots & \vdots \\
\gamma(|x_n - x_i|) & \cdots & \gamma(|x_n - x_n|) & 1 \\
1 & \cdots & 1 & 0
\end{pmatrix}
\begin{pmatrix}
\omega_i \\
\vdots \\
\omega_n \\
\mu
\end{pmatrix}
= \begin{pmatrix}
\gamma(|x_n - x_0|) \\
\vdots \\
\gamma(|x_n - x_0|) \\
1
\end{pmatrix},
\]

(8)
where $\omega_1$ to $\omega_n$ are the weights assigned to this system by the variogram, $\mu$ is the Lagrange parameter (Isaaks and Srivastava, 1989), and $|x_i - x_j|$ is the distance between the points $x_i$ and $x_j$. The left hand side of the system describes the dissimilarities between the data points, while the right hand side shows the dissimilarities between each data point and the estimation point $x_0$ (Wackernagel, 1995).

The non-matrix form of the OK system can be written as:

\[
\begin{align*}
\gamma(|x_\alpha - x_0|) &= \sum_{\beta=1}^{n} \omega_\beta \gamma(|x_\alpha - x_\beta|) + \mu \\
\sum_{\beta=1}^{n} \omega_\beta &= 1
\end{align*}
\text{for } \alpha = 1, \ldots, n. \tag{9}
\]

The estimation variance of the Ordinary Kriging system can now be expressed as:

\[\sigma_{OK}^2 = -\mu - \gamma(|x_0 - x_0|) + 2 \sum_{\alpha=1}^{n} \omega_\alpha \gamma(|x_\alpha - x_0|). \tag{10}\]

It is important to note that the value produced for $Z(x_0)$ when solving the OK system is an unbiased estimate of the true $Z$-value at location $x_0$. That is to say, $E[Z^*(x_0)] = \mu$ and $Z^*(x_0) = Z(x_\alpha)$ if $x_0 = x_\alpha$. This means that Ordinary Kriging is an exact linear interpolator (Wackernagel, 1995).

The above comment can be easily proven. When $x_0$ is one of the sample points, the right hand side of equation (8) is equal to one column of the left hand side of equation (8). A weight vector $\mathbf{W}$ with a weight for that column equal to one and all other weights equal to zero is a solution for the system. As the left hand matrix is nonsingular, the unique solution (Wackernagel, 1995) is $Z^*(x_0) = Z(x_\alpha)$ when $x_0 = x_\alpha$.
Since $Z^*(x_0) = Z(x_a)$ when $x_0 = x_a$, we can say that Kriging will produce the best linear unbiased estimator (BLUE) for some un-sampled location $x_0$. This proven result will in turn be used as a form of map generation.

1.3: Map Generation with Ordinary Kriging

Kriging can be used as a spatial interpolation method to estimate values on a regular grid using irregularly spaced data (Wackernagel, 1995). Suppose again that we are given a spatial domain, or site, where contamination values $Z(x_a)$ are known at locations $x_1, x_2, \ldots, x_n$, where $\alpha$ ranges from 1 to $n$. Suppose also that the original sampled locations are randomly placed within our domain. Since our original data locations are randomly placed, one may wish to produce a map of data values that are regularly spaced along a grid to gain a better understanding of the spatial structure. In practice, the first sample location is randomly chosen, and then the uniform grid is built around it; this scheme is called 'uniform grid sampling with random start.'

This map generation can be viewed as a form of conditional simulation. Since the values assigned to each location are functions of the assigned global variogram and other sampled locations, the Kriged predicted values, or $Z$-values, can be seen as conditionally assigned on the map or conditionally simulated based on the defining function. Various texts refer to the idea of mapping as conditional simulation, or vice versa, but the terms are interchangeable and can be used according to individual preference.

Kriging is used for generating a map by following these steps in order:

- A regular grid is defined along locations within our spatial domain;
- Each node, or point, on the grid becomes the point $x_0$ in turn;
• A prediction $Z'(x_0)$ value is produced at location $x_0$ by Ordinary Kriging.

Another important aspect of map generation with Kriging is the Kriging variance. When a map is produced along a regular grid, for each location, a Kriging Standard Deviation is estimated from the OK model. This estimation error allows for evaluating the precision of the estimation in any part of the region. It should be understood that the Kriging variance is primarily a measure of the density of information around the estimation point. (Wackernagel, 1995)

1.4: Ordinary Kriging Example

We will now provide a simple example of Ordinary Kriging to illustrate the method which will be used in the rest of this paper.

Suppose we are given the following table of data values:

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Easting</th>
<th>Northing</th>
<th>$Z(x_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>61</td>
<td>139</td>
<td>477</td>
</tr>
<tr>
<td>$x_2$</td>
<td>63</td>
<td>140</td>
<td>696</td>
</tr>
<tr>
<td>$x_3$</td>
<td>64</td>
<td>129</td>
<td>227</td>
</tr>
<tr>
<td>$x_4$</td>
<td>68</td>
<td>128</td>
<td>646</td>
</tr>
<tr>
<td>$x_5$</td>
<td>71</td>
<td>140</td>
<td>606</td>
</tr>
<tr>
<td>$x_6$</td>
<td>73</td>
<td>141</td>
<td>791</td>
</tr>
<tr>
<td>$x_7$</td>
<td>75</td>
<td>128</td>
<td>783</td>
</tr>
<tr>
<td>$x_0$</td>
<td>67</td>
<td>134</td>
<td>?</td>
</tr>
</tbody>
</table>
In the above table we have not obtained a Z-value for location $x_0$. We will now illustrate the method of Kriging by finding the BLUE for the Z-value at location $x_0$.

For simplicity, we will assume an exponential variogram model with a nugget effect of zero ($C_0 = 0$), a sill value of ten ($C_1 = 10$), and a range value of ten ($r = 10$). We will now use equation (8) to solve for the corresponding weight vector $\mathbf{W}$. The system of Ordinary Kriging Equations,

$$
\begin{pmatrix}
\gamma(|x_1-x_1|) & \cdots & \gamma(|x_1-x_n|) & 1 \\
\vdots & \ddots & \vdots & \vdots \\
\gamma(|x_n-x_1|) & \cdots & \gamma(|x_n-x_n|) & 1 \\
1 & \cdots & 1 & 0
\end{pmatrix}
\begin{pmatrix}
\omega_1 \\
\vdots \\
\omega_n \\
\mu
\end{pmatrix} =
\begin{pmatrix}
\gamma(|x_n-x_0|) \\
\vdots \\
\gamma(|x_n-x_0|)
\end{pmatrix}
$$

can be solved using the numerical values provided in table 1 to produce the following:

$$
\begin{pmatrix}
0 & 4.89 & 9.56 & 9.80 & 9.74 & 9.95 & 1 \\
4.89 & 0 & 9.64 & 9.80 & 9.09 & 9.51 & 9.94 & 1 \\
9.56 & 9.64 & 0 & 7.10 & 9.80 & 9.89 & 9.64 & 1 \\
9.80 & 9.80 & 7.10 & 0 & 9.76 & 9.85 & 8.78 & 1 \\
9.74 & 9.51 & 9.89 & 9.85 & 4.89 & 0 & 9.81 & 1 \\
9.95 & 9.94 & 9.64 & 8.78 & 9.78 & 9.81 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
\omega_1 \\
\omega_2 \\
\omega_3 \\
\omega_4 \\
\omega_5 \\
\omega_6 \\
\omega_7 \\
\mu
\end{pmatrix} =
\begin{pmatrix}
9.04 \\
8.85 \\
8.26 \\
8.39 \\
8.85 \\
9.37 \\
9.50 \\
1
\end{pmatrix}
$$

The solution vector of the full rank linear system is

$$
\begin{pmatrix}
\omega_1 \\
\omega_2 \\
\omega_3 \\
\omega_4 \\
\omega_5 \\
\omega_6 \\
\omega_7 \\
\mu
\end{pmatrix} =
\begin{pmatrix}
0.11 \\
0.14 \\
0.21 \\
0.18 \\
0.16 \\
0.08 \\
0.12 \\
1.094
\end{pmatrix}
$$

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Note that all numbers were rounded to the nearest hundredth; however, \( \sum_{a=1}^{7} \omega_a = 1 \).

By Ordinary Kriging, the predicted value \( Z'(x_0) \) is:

\[
Z'(x_0) = \sum_{a=1}^{7} \omega_a Z(x_a)
\]

\[
=(0.11)(477) + \ldots + (0.12)(783)
\]

\[
= 566.71
\]

The prediction variance at \( x_0 \) is:

\[
\sigma^2_{x_0} = E[(Z'(x_0) - Z(x_0))^2]
\]

\[
= \gamma(0) - \sum_{a=1}^{7} \sum_{\beta=1}^{7} \omega_a \omega_\beta \gamma(|x_a - x_\beta|) + 2 \sum_{a=1}^{7} \omega_a \gamma(|x_a - x_0|)
\]

For simplicity, by making the appropriate mathematical substitutions, we obtain the following: \( \sigma^2_{x_0} = 3.14 \).
CHAPTER 2

OPTIMAL GRID SPACING WHEN USING KRIGING

In this paper we will examine grid sampling schemes along a river in an attempt to find optimal sample spacing. Grid sampling is a form of sampling in which a grid of data points is formed at regularly spaced intervals. This idea is better explained visually:

![Grid Sampling vs Random Sampling](image)

Figure 6: Example of Grid sampling VS Random Sampling

Figure 6 shows two different graphs in which nine different locations were sampled. The first graph shows us spatial data points where levels of concentration were recorded at locations given by an easting and northing coordinate system. The spatial sampling in the first graph was done so that the spacing of the samples forms a grid. The second graph shows a spatial sampling where the data was sampled randomly within the easting, northing coordinate system.
In uniform grid sampling, the distance between each point is the same along the
easting axis and the northing axis. By examining different distances between each point
along a regular grid, we will attempt to find a distance in which over-sampling and under-
sampling would not yield more accurate results i.e., we would be finding the optimized
grid sampling design.

The benchmark for comparing different grid sampling designs will be Kriging
variance (KV). Since Kriging produces a KV at each data point for which the response
value is estimated, if we compute the median KV and KV maximum for a site, we will be
able to form a comparison based on estimation error.

2.1: The Procedure

To perform the comparison we will start with a sampling grid of \( n_{1,a} \) by
\( n_{1,b} \) locations, where the first subscript notation represents the grid number and the second
subscript represents the number of grid locations along that axis. We will then Kriging this
grid to produce a map of response values for some contaminant concentration at locations
on a \( m_1 \) by \( m_2 \) grid. We will set \( n_{1,a} \ll m_1 \) and \( n_{1,b} \ll m_2 \) in order to produce a much better
descriptive map of response values by Kriging the original grid of \( n_{1,a} \) by \( n_{1,b} \) locations to
a grid of \( m_1 \) by \( m_2 \) locations. Once Kriging has been performed on the \( n_{1,a} \) by \( n_{1,b} \) grid, we will record the
median and maximum KV for the \( m_1 \) by \( m_2 \) grid of response values. We then produce a
second \( n_{2,a} \) by \( n_{2,b} \) grid, in which the distance between samples along the easting and
northing axis is greater than the distance between samples on the \( n_{1,a} \) by \( n_{1,b} \) grid. We then
Krige a \( m_1 \) by \( m_2 \) grid using the \( n_{3,a} \) by \( n_{3,b} \) grid. We will record the median and maximum KV for this \( m_1 \) by \( m_2 \) grid produced by the \( n_{2,a} \) by \( n_{2,b} \) grid.

This procedure will continue until we Krige a \( n_{p,a} \) by \( n_{p,b} \) grid to produce a \( m_1 \) by \( m_2 \) grid. Again, in each case we will record the median and maximum KV for the Kriged \( m_1 \) by \( m_2 \) grid produced from a \( n_{i,a} \) by \( n_{i,b} \) grid. It is suspected that a linear relationship will be seen between the median and maximum KV of a Kriged site and the distance between sampled locations; i.e. as the distance between samples increase linearly, the KV will also increase in a linear manner. However, we will attempt to show that KV remains somewhat constant until a certain distance is reached between the sampled locations. If such a distance is found, we will also show that the KV 'jumps' or increases dramatically over a short increase in sampling distance.

To better illustrate the procedure for comparisons, a computational grid to grid Kriging algorithm is now presented:

**Step 1**

Step 1.1 → Take the originally sampled location and produce a \( n_{i,a} \) by \( n_{i,b} \) grid of response values by Kriging the original sampled location

Step 1.2 → Generate a \( m_1 \) by \( m_2 \) grid of response values by using the \( n_{i,a} \) by \( n_{i,b} \) grid

Step 1.3 → Record the median and maximum KV of the \( m_1 \) by \( m_2 \) grid of response values

**Step 2**

Step 2.1 → Take the originally sampled location and produce a \( n_{2,a} \) by \( n_{2,b} \) grid of response values by Kriging the originally sampled location
Step 2.2 → Generate a $m_1$ by $m_2$ grid of response values by using the $n_{2,a}$ by $n_{2,b}$ grid.

Step 2.3 → Record the median and maximum KV of the $m_1$ by $m_2$ grid of response values.

↓

Step N

Step N.1 → Take the originally sampled location and produce a $n_{n,a}$ by $n_{n,b}$ grid of response values by Kriging the original sampled location.

Step N.2 → Generate a $m_1$ by $m_2$ grid of response values by using the $n_a$ by $n_s$ grid.

Step N.3 → Record the median and maximum KV of the $m_1$ by $m_2$ grid of response values.

2.2: The Study Site

We apply grid sampling to riverbed Kriging on the East Fork Poplar Creek (EFPC) located in Anderson and Roane Counties in Oak Ridge, Tennessee, approximately 25 miles west of Knoxville. The EFPC site is included on the National Priorities List (NPL) provided by the Environmental Protection Agency (EPA). Although the site is listed on the NPL for releases of mercury into the water flow from a production plant, we will be looking at the arsenic content (this will be the contaminant concentration response value) found when sampling for mercury levels. Since Kriging was already applied to mercury, a comprehensive study of arsenic concentration distribution could lead to a better understanding of the overall effect of the mercury.
contamination, or how mercury may be affecting the Arsenic levels. For this paper however, the main concern is the effect of grid sampling on KV.

We will be looking at a section of the creek which is near the main contamination site for mercury. We will then break the site into two sub-regions. We will perform the computational algorithm presented above on the two sub-regions of the river, such that each constructed \( n_{i,a} \) by \( n_{i,b} \) grid computed will have the same distance between sampled locations. We record the median and maximum KV when producing each \( m_1 \) by \( m_2 \) grid.

Once we have recorded the median and maximum KV for each of the \( n_{i,a} \) by \( n_{i,b} \) grids within the two sub-regions, we can see if a dramatic increase in KV occurs, and if it does, does it occur at the same sample grid distance. If such an increase in KV does occur in each site but at different sample grid distances, we may be able to see if site structure has a bearing on that result.

2.3: The Study Site Characteristics

Before we begin the analysis of the KV for the two sub-sites, we will first present the sites graphically and then we will derive the initial variogram and variogram model from each sub-site. The site in its entirety is presented in the figure below.
In the above figure the area of the river is presented in the white shaped area, and the surrounding land area is shaped in black. The area of the river was fitted with a polygon to estimate the parameter. This is commonly done, as it saves both time and money compared to finding the exact layout for the river. We will be using the polygon layout in our Kriging algorithm, and all estimated response values will be placed into the polygon interior.

Since we want to compare two sub-regions, we will now break the site into two regions. The two sub-sites used for comparison are presented in the figures below.

Figure 7: Polygon Scatterplot of the EFPC

Figure 8: Polygon Scatterplot of EFPC sub-site 1
A variogram model will now be fitted to each sub-site as the defining function for the computation of the $n_{i,a}$ by $n_{i,b}$ grids in each sub-site. Once we have computed each of the $n_{i,a}$ by $n_{i,b}$ grids we will reestablish a variogram model for the new sample structure of that site. We will present the $n_{i,a}$ by $n_{i,b}$ grid variogram models in the next chapter.

The variogram and variogram model for EFPC sub-site 1 are presented in the figure below.
The key characteristics of this variogram model are: an exponential model with a nugget effect value of 11, a sill value of 5, and a range value of 62. In this model we notice that the variogram model $\gamma(h)$ does not seem to tend to zero as $h \to 0$. This means that the regionalized variable, Arsenic in this case, is generally not continuous and is thus very irregular (Chiles and Delfiner, 1999). In general, the nugget effect is due to measurement or positioning errors. When the nugget value is greater than the sill value, we have a case where the nugget effect has a greater bearing on the degree of prediction. Since the sill represents the limiting value for the correlation between sample distance and prediction value (Chiles and Delfiner, 1999), a large nugget effect will cause an increase in estimation error.

Although this site has a large nugget effect, when we Krig the site to a grid sample design, we will see that the resulting variogram model fit to the grid design has a much smaller nugget effect, which is attributable to the spatial grid sampling design.
Although some accuracy in prediction is lost in the initial Kriging, the main concern in this thesis is the effect of grid sampling on the prediction variance as a function of spatial distance between sample locations. Once we have established the $n_{i,a}$ by $n_{i,b}$ grids the concern shifts from our original site to the $n_{i,a}$ by $n_{i,b}$ grid sites. Since we are comparing grid sampling schemes in an attempt to find an optimal grid distance to Krige a $m_1$ by $m_2$ grid from a much larger grid, once we have established the $n_{i,a}$ by $n_{i,b}$ grids, these are considered our testing sites, not the initial site whose variogram in defined in figure 10.

Next, we present the variogram and variogram model at EFPC sub-site 2 in the figure below.

![Variogram and Model for EFPC sub-site 2](image)

Figure 11: Variogram and Model for EFPC sub-site 2

The key characteristics of this variogram model are: an exponential model with a nugget effect value of 7.5, a sill value of 5, and a range value of 900. Here we see the resulting variogram is much different from that from EFPC sub-site 1. For example, the nugget
value on this sub-site is smaller and the range for the variogram model has increased dramatically. The difference in models could have been caused by the break up of the sites. Since the spatial structures of each sub-site of the river, as well as the allocation of the grid within the site, are different, the variogram model could be a function of these two features. This would explain why the two variogram models differ greatly. Since each spatial sub-site of a larger spatial domain could have different variogram models, it points to a limitation in spatial sampling designs. This limitation will be discussed in the conclusion of this thesis.

Since each model is different, it will affect the KV produced when each $n_{x,a}$ by $n_{x,b}$ grid Kriges the $m_1$ by $m_2$ grid. Since we are only interested in finding an optimal distance for grid to grid Kriging, it is not the difference in values that is important, but rather, the distance between samples that causes a large increase in KV. Since EFPC sub-site 1 has a larger nugget effect and a shorter range value, we should see that when we Krig from grid to grid, the median and maximum KV will be larger.
CHAPTER 3

ANALYSIS OF GRID TO GRID KRIGING

In chapter 2 we established global variogram models for the two sub-regions. We will use these models, in conjunction with the grid to grid algorithm also presented in chapter 2, to analyze the KV as a function of distance between grid nodes.

Due to computational limitations, we will set the \( m_i \) by \( m_j \) grid so that the distance between each sampled locations is 20 meters, producing 2750 estimated locations in the first sub-site, and 3500 estimated locations in the second sub-site. The number of estimated points for each sub-site is different because the division of the initial region left the second sub-site larger than the first.

We will set the \( n_{i,a} \) by \( n_{i,b} \) grids in such a way so that the distance between the sampled locations along the easting and northing axis is approximately 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, and 100 meters. We will use the global variogram models to generate the \( n_{i,a} \) by \( n_{i,b} \) grids from the original sampled data sets. Once that is completed, we compute a new variogram model for the \( n_{i,a} \) by \( n_{i,b} \) data set, so that we may Krige the \( m_i \) by \( m_j \) grid.
3.1: Variogram models for EFPC sub-site 1

Presented below is a table listing the nugget value, sill value, and range value for each \( n_{i,a} \) by \( n_{i,b} \) grid variogram models.

Table 2: Variogram Models for EFPC Sub-Site 1

<table>
<thead>
<tr>
<th>Grid</th>
<th>Distance</th>
<th>Number of Samples</th>
<th>Nugget</th>
<th>Sill</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_{1,22} ) by ( n_{1,20} )</td>
<td>50</td>
<td>440</td>
<td>1.00</td>
<td>1.80</td>
<td>300</td>
</tr>
<tr>
<td>( n_{2,20} ) by ( n_{2,18} )</td>
<td>55</td>
<td>380</td>
<td>1.00</td>
<td>1.25</td>
<td>300</td>
</tr>
<tr>
<td>( n_{3,18} ) by ( n_{3,17} )</td>
<td>60</td>
<td>323</td>
<td>1.00</td>
<td>1.10</td>
<td>300</td>
</tr>
<tr>
<td>( n_{4,17} ) by ( n_{4,15} )</td>
<td>65</td>
<td>272</td>
<td>1.00</td>
<td>1.10</td>
<td>300</td>
</tr>
<tr>
<td>( n_{5,16} ) by ( n_{5,14} )</td>
<td>70</td>
<td>240</td>
<td>1.00</td>
<td>1.00</td>
<td>300</td>
</tr>
<tr>
<td>( n_{6,15} ) by ( n_{6,13} )</td>
<td>75</td>
<td>210</td>
<td>1.15</td>
<td>1.00</td>
<td>300</td>
</tr>
<tr>
<td>( n_{7,14} ) by ( n_{7,12} )</td>
<td>80</td>
<td>182</td>
<td>0.85</td>
<td>1.20</td>
<td>300</td>
</tr>
<tr>
<td>( n_{8,13} ) by ( n_{8,12} )</td>
<td>85</td>
<td>156</td>
<td>1.70</td>
<td>1.10</td>
<td>300</td>
</tr>
<tr>
<td>( n_{9,12} ) by ( n_{9,11} )</td>
<td>90</td>
<td>156</td>
<td>1.70</td>
<td>1.10</td>
<td>300</td>
</tr>
<tr>
<td>( n_{10,12} ) by ( n_{10,10} )</td>
<td>95</td>
<td>132</td>
<td>2.60</td>
<td>0.33</td>
<td>300</td>
</tr>
<tr>
<td>( n_{11,11} ) by ( n_{11,10} )</td>
<td>100</td>
<td>110</td>
<td>1.70</td>
<td>0.30</td>
<td>300</td>
</tr>
</tbody>
</table>

In general, we see that the nugget value for the models remains somewhat constant until a distance of 85 meters between grid nodes. At this distance the nugget value almost doubles. This could be a function of the distance or the number of samples within the site. At 85 meters, the number of samples observed is almost one third the number of samples observed at 50 meters. In chapter 2, we said that, large nugget effects were due partly to positioning errors. Perhaps when a distance of 85 meters between samples is observed, or as the number of samples continues to decrease, the positioning between observed
samples reaches a point when an increasing error results when Kriging, resulting in an increased nugget effect.

To illustrate the nugget effect as a function of distance between samples and the number of samples observed, the following figure is presented:

![Scatterplot of Nugget vs Distance](image)

Figure 12: Scatter plot of Nugget Value vs. Sample Distance for EFPC Sub-Site 1

As the distance continues to increase, the nugget value remains relatively constant until a distance of 85 meters is observed between samples. Past 85 meters, the increase in the nugget value induces an increase in the KV.

Having defined the variogram model parameters for each of the $n_{ix} \times n_{iy}$ grids, the next step is to perform the grid to grid Kriging algorithm shown in chapter 2. The recorded median and maximum KV is presented later in this chapter.
3.2: Variogram models EFPC sub-site 2

Presented below is a table listing the nugget value, sill value, and range value for each of the \(n_{i,a} \times n_{i,b}\) grids variogram model.

<table>
<thead>
<tr>
<th>Grid</th>
<th>Distance</th>
<th>Number of Samples</th>
<th>Nugget</th>
<th>Sill</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n_{1,28} \times n_{1,20})</td>
<td>50</td>
<td>560</td>
<td>0.4</td>
<td>5.5</td>
<td>900</td>
</tr>
<tr>
<td>(n_{2,24} \times n_{2,18})</td>
<td>55</td>
<td>475</td>
<td>0.5</td>
<td>5.5</td>
<td>1000</td>
</tr>
<tr>
<td>(n_{3,23} \times n_{3,37})</td>
<td>60</td>
<td>408</td>
<td>0.5</td>
<td>5.5</td>
<td>1000</td>
</tr>
<tr>
<td>(n_{4,22} \times n_{4,15})</td>
<td>65</td>
<td>352</td>
<td>0.5</td>
<td>5.5</td>
<td>1000</td>
</tr>
<tr>
<td>(n_{5,20} \times n_{5,14})</td>
<td>70</td>
<td>300</td>
<td>0.5</td>
<td>5.5</td>
<td>1000</td>
</tr>
<tr>
<td>(n_{6,19} \times n_{6,33})</td>
<td>75</td>
<td>266</td>
<td>0.5</td>
<td>5.5</td>
<td>1000</td>
</tr>
<tr>
<td>(n_{7,18} \times n_{7,32})</td>
<td>80</td>
<td>234</td>
<td>0.5</td>
<td>6.0</td>
<td>1000</td>
</tr>
<tr>
<td>(n_{8,16} \times n_{8,12})</td>
<td>85</td>
<td>204</td>
<td>1.0</td>
<td>5.5</td>
<td>1000</td>
</tr>
<tr>
<td>(n_{9,16} \times n_{9,11})</td>
<td>90</td>
<td>192</td>
<td>1.0</td>
<td>5.0</td>
<td>1000</td>
</tr>
<tr>
<td>(n_{10,15} \times n_{10,10})</td>
<td>95</td>
<td>165</td>
<td>1.0</td>
<td>6.0</td>
<td>1000</td>
</tr>
<tr>
<td>(n_{11,14} \times n_{11,10})</td>
<td>100</td>
<td>140</td>
<td>1.0</td>
<td>5.5</td>
<td>1000</td>
</tr>
</tbody>
</table>

We see that for this sub-site the sill and range values remain somewhat constant across the variogram models for each \(n_{i,a} \times n_{i,b}\) grid. The nugget values remain almost constant until a distance of 85 meters between each grid sample is reached. As with EFPC sub-site 1, when a distance of 85 meters between samples is observed, a spike in the nugget value occurs. To illustrate this relationship a Scatterplot of nugget value vs. sample distance is presented below.
As the distance increases from 50 meters to 80 meters, the nugget value for the variogram model remains almost constant. Once 85 meters is reached, we see a large increase in the nugget value. This will result in an increase in the KV.

Now that we have defined the variogram model parameter estimates for each of the \( n_{i,a} \) by \( n_{i,b} \) grids in this sub-site, we will perform the grid to grid Kriging algorithm shown in chapter 2 and present the recorded median and maximum KV.
3.3: Grid to Grid Kriging Analysis for EFPC sub-site 1

Having computed a \( m_1 \) by \( m_2 \) grid from each \( n_{i,d} \) by \( n_{i,d} \) grid, the median and maximum KV was recorded, and is presented in the table below:

Table 4: Grid Sampling KV Summary for EFPC Sub-Site 1

<table>
<thead>
<tr>
<th>Grid</th>
<th>Distance</th>
<th>Number of Samples</th>
<th>Median KV</th>
<th>Maximum KV</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_{1,22} ) by ( n_{1,20} )</td>
<td>50</td>
<td>440</td>
<td>1.42</td>
<td>1.51</td>
<td>6.25</td>
</tr>
<tr>
<td>( n_{2,20} ) by ( n_{2,18} )</td>
<td>55</td>
<td>380</td>
<td>1.34</td>
<td>1.43</td>
<td>7.24</td>
</tr>
<tr>
<td>( n_{3,18} ) by ( n_{3,17} )</td>
<td>60</td>
<td>323</td>
<td>1.34</td>
<td>1.41</td>
<td>8.51</td>
</tr>
<tr>
<td>( n_{4,17} ) by ( n_{4,15} )</td>
<td>65</td>
<td>272</td>
<td>1.36</td>
<td>1.43</td>
<td>10.11</td>
</tr>
<tr>
<td>( n_{5,16} ) by ( n_{5,14} )</td>
<td>70</td>
<td>240</td>
<td>1.47</td>
<td>1.54</td>
<td>11.46</td>
</tr>
<tr>
<td>( n_{6,15} ) by ( n_{6,13} )</td>
<td>75</td>
<td>210</td>
<td>1.54</td>
<td>1.61</td>
<td>13.10</td>
</tr>
<tr>
<td>( n_{7,14} ) by ( n_{7,12} )</td>
<td>80</td>
<td>182</td>
<td>1.28</td>
<td>1.34</td>
<td>15.12</td>
</tr>
<tr>
<td>( n_{8,13} ) by ( n_{8,12} )</td>
<td>85</td>
<td>156</td>
<td>2.24</td>
<td>2.31</td>
<td>17.63</td>
</tr>
<tr>
<td>( n_{9,12} ) by ( n_{9,11} )</td>
<td>90</td>
<td>156</td>
<td>2.24</td>
<td>2.31</td>
<td>17.63</td>
</tr>
<tr>
<td>( n_{10,12} ) by ( n_{10,10} )</td>
<td>95</td>
<td>132</td>
<td>2.87</td>
<td>2.89</td>
<td>20.83</td>
</tr>
<tr>
<td>( n_{11,11} ) by ( n_{11,10} )</td>
<td>100</td>
<td>110</td>
<td>2.00</td>
<td>2.02</td>
<td>25.00</td>
</tr>
</tbody>
</table>

The above table records the median KV, maximum KV, and the ratio of the number of observed samples to the number estimated. Since the first grid in the above table has 440 observed samples, and we use those to estimate 2750 samples, yielding a ratio of 6.25 estimated samples for each observed sample. We will discuss why we included this number later in this chapter.

We notice that the median and maximum KV remain somewhat constant (never increasing or decreasing by more than 20% compared to the sample before), until a
distance of 85 meters is observed between the sampled grid nodes. At distances of 85 meters and greater, the median and maximum KV increase by almost 70 percent. To illustrate this increase, consider the following two scatterplots:

Figure 14: Scatterplot of Median KV vs. Sample Distance for EFPC Sub-Site 1

Figure 15: Scatterplot of Maximum KV vs. Sample Distance for EFPC Sub-Site 1

In both scatterplots we see that when a distance of 85 meters between samples is observed, the median and maximum KV increases sharply. When this happens, we have found the threshold sampling distance in which over-sampling and under-sampling would not yield more accurate results. If we sample at a distance of 80 meters (182 total
samples), we would not get significantly more accurate results by over sampling at a
distance of 75 meters (210 total samples), nor would we have more accurate results by
under-sampling at a distance of 85 meters (156 total samples). This is the distance
realized for which we hypothesized in chapter 2.

3.4: Grid to Grid Kriging Analysis for EFPC sub-site 2

Having computed a $m_1$ by $m_2$ grid from each $n_{i,a}$ by $n_{i,b}$ grid, the median and
maximum KV was recorded, and is presented in the table below:

Table 5: Grid Sampling KV Summary for EFPC Sub-Site 2

<table>
<thead>
<tr>
<th>Grid</th>
<th>Number of Samples</th>
<th>Median KV</th>
<th>Maximum KV</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_{1,28}$ by $n_{1,20}$</td>
<td>50</td>
<td>0.73</td>
<td>0.77</td>
<td>6.25</td>
</tr>
<tr>
<td>$n_{2,24}$ by $n_{2,18}$</td>
<td>55</td>
<td>0.85</td>
<td>0.89</td>
<td>7.37</td>
</tr>
<tr>
<td>$n_{3,23}$ by $n_{3,17}$</td>
<td>60</td>
<td>0.87</td>
<td>0.91</td>
<td>8.58</td>
</tr>
<tr>
<td>$n_{3,22}$ by $n_{4,15}$</td>
<td>65</td>
<td>0.89</td>
<td>0.93</td>
<td>9.94</td>
</tr>
<tr>
<td>$n_{5,20}$ by $n_{5,14}$</td>
<td>70</td>
<td>0.92</td>
<td>0.97</td>
<td>11.67</td>
</tr>
<tr>
<td>$n_{6,19}$ by $n_{6,13}$</td>
<td>75</td>
<td>0.94</td>
<td>0.97</td>
<td>13.16</td>
</tr>
<tr>
<td>$n_{7,18}$ by $n_{7,12}$</td>
<td>80</td>
<td>0.99</td>
<td>1.05</td>
<td>14.96</td>
</tr>
<tr>
<td>$n_{8,16}$ by $n_{8,12}$</td>
<td>85</td>
<td>1.58</td>
<td>1.68</td>
<td>17.16</td>
</tr>
<tr>
<td>$n_{9,10}$ by $n_{9,11}$</td>
<td>90</td>
<td>1.55</td>
<td>1.65</td>
<td>18.23</td>
</tr>
<tr>
<td>$n_{10,15}$ by $n_{10,10}$</td>
<td>95</td>
<td>1.67</td>
<td>1.77</td>
<td>21.21</td>
</tr>
<tr>
<td>$n_{11,14}$ by $n_{11,10}$</td>
<td>100</td>
<td>1.59</td>
<td>1.69</td>
<td>25.00</td>
</tr>
</tbody>
</table>

We see that the median and maximum KV remains somewhat constant (never increasing
by more than 15% compared to the sample before) until a distance of 85 meters is
observed. At this distance, we see an increase in the median and maximum KV of almost 60 percent. To illustrate this increase, consider the following two scatterplots:

Figure 16: Scatterplot of Median KV vs. Sample Distance for EFPC Sub-Site 2

Figure 17: Scatterplot of Maximum KV vs. Sample Distance for EFPC Sub-Site 2

In both scatterplots, we see that when a distance of 85 meters between samples is observed, the median and maximum KV increases dramatically. As with EFPC sub-site 1, this is the distance at which over-sampling and under-sampling would not yield more accurate results. If we sample at a distance of 80 meters (234 total samples), we would not get more accurate results by over sampling at a distance of 75 meters (266 total
samples), nor would we have more accurate results by under-sampling at a distance of 85 meters (204 total samples). Again, this is the distance realized for which we hypothesized in chapter 2.

3.5: General Observations concerning the EFPC Sub-Sites

In chapter 2, we hypothesized that when we compute grid to grid Kriging in a riverbed, KV would remain somewhat constant until a certain distance between samples is observed, then KV would increase sharply. We saw that in both EFPC sub-sites this distance was 85 meters.

We noticed that in each sub-site, the variogram models for distances 50 to 80 meters were generally similar. However, once a distance of 85 meters was observed, the nugget value for the variogram models increased by 70 to 100 percent. This increase caused the median and maximum KV to increase sharply as well.

The variogram models for EFPC sub-site 1, with grid node distances of 80 and 85 meters are presented below to show increase in the nugget value.
Notice the increase in the nugget value for the variogram model in figure 18 compared to the model in figure 17. The same result is seen in the variogram models for the EFPC sub-site 2 at the same grid distances.
It is interesting to note that when applying grid to grid Kriging in this riverbed, a linear increase in distance does not equate to a linear increase in KV. If this fact holds for all sites, each site would have a distance in which over-sampling and under-sampling would not yield more accurate results. Since we are setting the distance between samples, this setting is a function of the size of the river and the distance at which we want to Krig a $n_{i,o}$ by $n_{i,g}$ grid to a $m_1$ by $m_2$ grid. If one wanted to generalize the results shown in this paper to sites with different grid node distances, one would have to look at the ratio of observed samples to estimated samples. Since this ratio would be the same among sites, but for different distances, we can see if a generalization exists.

3.6: Generalizing the EFPC Sub-Sites for sampling ratios

By examining the ratio at which a large increase in KV occurs, we can see if it is similar among the two sub-sites of EFPC, and form a generalization.

When we examine tables 4 and 5 of this chapter, we see that the large increase in KV occurs at a sampling ratio of 17.63 to 1 estimated to observed samples for sub-site 1, and 17.16 to 1 for sub-site 2. Here we can see that as the ratio of observed samples to estimated samples goes to 17, we get a large increase in KV. To illustrate this idea, consider the following scatterplots that show the maximum KV as a function of the estimation ratio:
In each scatterplot we see that once we increase the estimation ratio past a 15 to 1 observed to estimate ratio we get a large increase in KV. From what we have observed here, we can make a recommendation for future riverbed grid to grid Kriging. This recommendation is: For future site mapping we recommend that the observed to estimate ratio be kept under 15 to 1. However, this recommendation is subjective to this site and its limitations are talked about in the conclusion of this thesis.
CHAPTER 4

CONCLUSION

In this thesis, it was hypothesized that when we perform grid to grid Kriging in a riverbed, the Kriging Variance (KV) would remain somewhat constant until a certain critical distance between grid points was observed. Once this distance was attained, the KV would increase sharply, causing the median and maximum KV at this distance to increase greatly compared to shorter distances between sampled grid points.

We compared two sub-sites from the East Fork Poplar Creek (EFPC), and observed that at a distance of 85 meters between grid nodes a sharp increase in KV occurs. We generalized this result by looking at the ratio between observed and estimated samples. We saw that if we keep this ratio under 15 to 1, we would not yield more accurate results by over sampling with more observed samples, or by under sampling with less observed samples.

We also saw the limitations of spatial sampling. We started with a large sub-site of the EFPC and we broke the site into two sub-sites. Each sub-site came from a larger site, yet each yielded completely different variogram models, causing large differences in KV. This can illustrate the difficulties in spatial sampling, and how Kriging sub-sites can yield unexpected results, as different parts of a site may behave differently.

Future projects which could come out of this thesis include the following: (1) Seeing if the recommended ratio posed in chapter 3 holds for other sites. (2) Investigating
to see if the recommended ratio holds for other sampling schemes, not just grid to grid Kriging. (3) To investigate if the sharp increase in $K_V$ also occurs for other sampling scheme Kriging, not just grid to grid Kriging.
APPENDIX I

Presented below is the R-code used for the computations in this thesis. The programming language R can be downloaded at http://www.r-project.org, and it is a free download. The programming presented below can be run as a script in the R command window, assuming one has all the data files given in appendix 2.

```
#R-code for G101.dat
#Load R-package GeoR
library(geoR)

#Import the data file into R
G101<-read.table("g101.txt", header=TRUE)
p101<-read.table("p101.dat", header=TRUE)
geoG101<-as.geodata(G101)

varioG101<-variog(geoG101)
plot(geoG101)
plot(varioG101)

#Here we establish the variogram model for the site G101
lines.variomodel(cov.model="exp", nugget=11, cov.pars=c(5,62), max.dist=800)

#Here we establish the grid in which we want to Krige each of the larger grids to
loci<-expand.grid(seq(5600,6700,l=55),seq(11600,12600,l=50))

#Expand a grid where distance is 50 meters
loci50<-expand.grid(seq(5600,6700,l=22),seq(11600,12600,l=20))
```
loci50random<-read.table("g10150rs.txt",header=TRUE)

#Create prediction grid on the 50 meter grid
kc50<-krige.conv(geoG101,loc=loci50,krige=krige.control(cov.pars=c(5,62)))

#Import the file for the 50 meter grid
g10150<-read.table("g10150.txt",header=TRUE)

#Krige the 50 meter grid to produce a 10 meter grid
g10150<-as.geodata(g10150)
varioG10150<-variog(g10150)
plot(varioG10150)
lines.variomodel(cov.model="sph", nugget=1, cov.pars=c(1.8,300), max.dist=800)
kc50<-krige.conv(geoG10150,loc=loci,borders=p101,krige=krige.control(cov.pars=c(1.8,300), nugget=1,cov.model="sph"))

#Expanding a grid where distance is 55 meters
loci55<-expand.grid(seq(5600,6700,1=20),seq(11600,12600,1=18.20))

#Create a prediction grid on the 55 meter grid
kc55<-krige.conv(geoG101,loc=loci55,krige=krige.control(cov.pars=c(5,62)))

#Import the file for the 55 meter grid
g10155<-read.table("g10155.txt",header=TRUE)
geoG10155<-as.geodata(g10155)
varioG10155<-variog(geoG10155)
plot(varioG10155)
lines.variomodel(cov.model="sph", nugget=1, cov.pars=c(1.25,300), max.dist=800)
kc55<-krige.conv(geoG10155,loc=loci,borders=p101,krige=krige.control(cov.pars=c(1.25,300), nugget=1,cov.model="sph"))

#Expand a grid where distance is 60 meters
loci60<-expand.grid(seq(5600,6700,1=18.34),seq(11600,12600,1=16.67))

#Create a prediction grid on the 60 meter grid
kc60<-krige.conv(geoG101,loc=loci60,krige=krige.control(cov.pars=c(5,62)))
import the file for the 60 meter grid

```r
gl0160 <- read.table("gl0160.txt", header = TRUE)
geoGl0160 <- as.geodata(gl0160)
varioGl0160 <- variog(geoGl0160)
plot(varioGl0160)
lines.variomodel(cov.model = "sph", nugget = 1, cov.pars = c(1.1,300), max.dist = 800)
kc60 <-
krige.conv(geoGl0160, loc = loci, borders = pl01, krige = krige.control(cov.pars = c(1.1,300), nugget = 1, cov.model = "sph"))
```

 Expand a grid where distance is 65 meters

```r
loci65 <- expand.grid(seq(5600, 6700, 1=16.92), seq(11600, 12600, 1=15.4))

Create a prediction grid on the 60 meter grid

```r
kc65 <- krige.conv(geoGl0160, loc = loci65, krige = krige.control(cov.pars = c(5,62)))
```

import the file for the 60 meter grid

```r
gl0165 <- read.table("gl0165.txt", header = TRUE)
geoGl0165 <- as.geodata(gl0165)
varioGl0165 <- variog(geoGl0165)
plot(varioGl0165)
lines.variomodel(cov.model = "sph", nugget = 1, cov.pars = c(1.1,300), max.dist = 800)
kc65 <-
krige.conv(geoGl0165, loc = loci, borders = pl01, krige = krige.control(cov.pars = c(1.1,300), nugget = 1, cov.model = "sph"))
```

 Expand a grid where distance is 70 meters

```r
loci70 <- expand.grid(seq(5600, 6700, 1=15.7), seq(11600, 12600, 1=14.3))

Create a prediction grid on the 70 meter grid

```r
kc70 <- krige.conv(geoGl0165, loc = loci70, krige = krige.control(cov.pars = c(5,62)))
```

import the file for the 70 meter grid

```r
gl0170 <- read.table("gl0170.txt", header = TRUE)
geoGl0170 <- as.geodata(gl0170)
varioGl0170 <- variog(geoGl0170)
plot(varioGl0170)
lines.variomodel(cov.model = "sph", nugget = 1, cov.pars = c(1.1,300), max.dist = 800)
kc70 <-
krige.conv(geoGl0170, loc = loci, borders = pl01, krige = krige.control(cov.pars = c(1.1,300), nugget = 1, cov.model = "sph"))
```
# Expand a grid where distance is 75 meters
loci75<-expand.grid(seq(5600,6700,l=14.67),seq(11600,12600,l=13.33))

# Create a prediction grid on the 75 meter grid

g10175<-read.table("g10175.txt",header=TRUE)
geoG10175<-as.geodata(g10175)
varioG10175<-variog(geoG10175)
plot(varioG10175)
lines.variomodel(cov.model="sph", nugget=1.15, cov.pars=c(1,300), max.dist=800)
kc75<-
krige.conv(geoG10175,loc=loci,borders=p101,krige=krige.control(cov.pars=c(1,300),nugget=1.15,cov.model="sph"))

# Expand a grid where distance is 80 meters
loci80<-expand.grid(seq(5600,6700,l=13.75),seq(11600,12600,l=12.5))

# Create a prediction grid on the 75 meter grid
kc80<-krige.conv(geoG101,loc=loci80,krige=krige.control(cov.pars=c(5,62)))

# Import the file for the 80 meter grid
g10180<-read.table("g10180.txt",header=TRUE)
geoG10180<-as.geodata(g10180)
varioG10180<-variog(geoG10180)
plot(varioG10180)
lines.variomodel(cov.model="sph", nugget=.85, cov.pars=c(1.2,300), max.dist=800)
kc80<-
krige.conv(geoG10180,loc=loci,borders=p101,krige=krige.control(cov.pars=c(1.2,300),nugget=.85,cov.model="sph"))

# Expand a grid where distance is 85 meters
loci85<-expand.grid(seq(5600,6700,l=12.95),seq(11600,12600,l=11.77))

# Create a prediction grid on the 85 meter grid
kc85<-krige.conv(geoG101,loc=loci85,krige=krige.control(cov.pars=c(5,62)))

# Import the file for the 85 meter grid
g10185<-read.table("g10185.txt",header=TRUE)
geoG10185<-as.geodata(g10185)
varioG10185<-variog(geoG10185)
plot(varioG10185)
lines.variomodel(cov.model="sph", nugget=1.7, cov.pars=c(1,300), max.dist=800)
kc85<-
krige.conv(geoG10185,loc=loci,borders=pl01,krige=krige.control(cov.pars=c(1,300),nugget=1.7, cov.model="sph"))

#Expand a grid where distance is 90 meters
loci90<-expand.grid(seq(5600,6700,1=12.22),seq(11600,12600,1=11.11))

#Create a prediction grid on the 90 meter grid
kc90<-krige.conv(geoG101,loc=loci90,krige=krige.control(cov.pars=c(5,62)))

#import the file for the 90 meter grid
g10190<-read.table("g10190.txt",header=TRUE)
geoG10190<-as.geodata(g10190)
varioG10190<-variog(geoG10190)
plot(varioG10190)
lines.variomodel(cov.model="sph", nugget=1.7, cov.pars=c(1,300), max.dist=800)
kc90<-
krige.conv(geoG10190,loc=loci,borders=pl01,krige=krige.control(cov.pars=c(1,300),nugget=1.7, cov.model="sph"))

#Expand a grid where distance is 95 meters
loci95<-expand.grid(seq(5600,6700,1=11.60),seq(11600,12600,1=10.53))

#Create a prediction grid on the 95 meter grid
kc95<-krige.conv(geoG101,loc=loci95,krige=krige.control(cov.pars=c(5,62)))

#import the file for the 95 meter grid
g10195<-read.table("g10195.txt",header=TRUE)
geoG10195<-as.geodata(g10195)
varioG10195<-variog(geoG10195)
plot(varioG10195)
lines.variomodel(cov.model="sph", nugget=2.6, cov.pars=c(.33,300), max.dist=800)
kc95<-
krige.conv(geoG10195,loc=loci,borders=pl01,krige=krige.control(cov.pars=c(.33,300),nugget=2.6, cov.model="sph"))
# Expand a grid where distance is 100 meters
loci100<-expand.grid(seq(5600,6700,l=11),seq(11600,12600,l=10))

# Create a prediction grid on the 100 meter grid
kc100<-krige.conv(geoGl01,loc=loci100,krige=krige.control(cov.pars=c(5,62)))

# Import the file for the 100 meter grid
gl01100<-read.table("gl01100.txt",header=TRUE)
geoGl01100<-as.geodata(gl01100)
varioGl01100<-variog(geoGl01100)
plot(varioGl01100)
lines.variomodel(cov.model="sph", nugget=1.7, cov.pars=c(.4,300), max.dist=800)
kc100<-
krige.conv(geoGl01100,loc=loci,borders=p101,krige=krige.control(cov.pars=c(.4,300),n
ugget=1.7,cov.model="sph")

# Summary of Grid Sampling KV
summary(kc50$krige.var)
supply(kc55$krige.var)
supply(kc60$krige.var)
supply(kc65$krige.var)
supply(kc70$krige.var)
supply(kc75$krige.var)
supply(kc80$krige.var)
supply(kc85$krige.var)
supply(kc90$krige.var)
supply(kc95$krige.var)
supply(kc100$krige.var)

# R-code for G102.dat

# Load R-package GeoR
library(geoR)

# Import the data file into R
G102<-read.table("g102.txt", header=TRUE)
p102<-read.table("p102.dat", header=TRUE)
geoG102<-as.geodata(G102)

varioG102<-variog(geoG102)
#plot(varioG102)

#Here we establish the variogram model for the site G102
#lines.variogram(cov.model="exp", nugget=7.5, cov.pars=c(5,900), max.dist=800)

#Our MxM grid with a distance of 20 meters between sampled locations
loci<-expand.grid(seq(6800,8200,l=70),seq(11340,12340,l=50))

#########################################################

#Expand a grid where distance is 50 meters
loci50<-expand.grid(seq(6800,8200,l=28),seq(11340,12340,l=20))

#Create prediction grid on the 50 meter grid
kc50<-krige.conv(geoG102,loc=loci50,krige=krige.control(cov.pars=c(5,900)))

#Import the file for the 50 meter grid
g10250<-read.table("g10250.txt", header=TRUE)

#Krige the 50 meter grid to produce a 20 meter grid
geoG10250<-as.geodata(g10250)
varioG10250<-variog(geoG10250)
#plot(varioG10250)
#lines.variogram(cov.model="sph", nugget=.4, cov.pars=c(5.5,900), max.dist=1000)
kc50<-krige.conv(geoG10250,loc=loci,borders=p102,krige=krige.control(cov.pars=c(5.5,900),nugget=.4,cov.model="sph"))

#########################################################

#Expand a grid where distance is 55 meters
loci55<-expand.grid(seq(6800,8200,l=24.45),seq(11340,12340,l=18.18))

#Create prediction grid on the 55 meter grid
kc55<-krige.conv(geoG102,loc=loci55,krige=krige.control(cov.pars=c(5,900)))

#Import the file for the 55 meter grid
g10255<-read.table("g10255.txt", header=TRUE)

#Krige the 55 meter grid to produce a 20 meter grid
geoG10255<-as.geodata(g10255)
varioG10255<-variog(geoG10255)
#plot(varioG10255)
#lines.variomodel(cov.model="sph", nugget=.5, cov.pars=c(5.5,1000), max.dist=1000)
kc55<-
krige.conv(geoG10255,loc=loci,borders=p102,krige=krige.control(cov.pars=c(5.5,1000), nugget=.5,cov.model="sph"))

#Expand a grid where distance is 60 meters
loci60<-expand.grid(seq(6800,8200,l=23.33),seq(11340,12340,l=16.67))

#Create prediction grid on the 60 meter grid
kc60<-krige.conv(geoG102,loc=loci60,krige=krige.control(cov.pars=c(5.900)))

#Import the file for the 60 meter grid
g10260<-read.table("g10260.txt", header=TRUE)

#Krige the 60 meter grid to produce a 20 meter grid
geoG10260<-as.geodata(g10260)
varioG10260<-variog(geoG10260)
#plot(varioG10260)
#lines.variomodel(cov.model="sph", nugget=.5, cov.pars=c(5.5,1000), max.dist=1000)
kc60<-
krige.conv(geoG10260,loc=loci,borders=p102,krige=krige.control(cov.pars=c(5.5,1000), nugget=.5,cov.model="sph"))

#Expand a grid where distance is 65 meters
loci65<-expand.grid(seq(6800,8200,l=21.54),seq(11340,12340,l=15.38))

#Create prediction grid on the 60 meter grid
kc65<-krige.conv(geoG102,loc=loci65,krige=krige.control(cov.pars=c(5.900)))

#Import the file for the 65 meter grid
g10265<-read.table("g10265.txt", header=TRUE)

#Krige the 65 meter grid to produce a 20 meter grid
geoG10265<-as.geodata(g10265)
varioG10265<-variog(geoG10265)
#plot(varioG10265)
#lines.variomodel(cov.model="sph", nugget=.5, cov.pars=c(5.5,1000), max.dist=1000)
kc65<-
krige.conv(geoG10265,loc=loci,borders=p102,krige=krige.control(cov.pars=c(5.5,1000), nugget=.5,cov.model="sph"))

#lines.variomodel(cov.model="sph", nugget=.5, cov.pars=c(5.5,1000), max.dist=1000)
#Expand a grid where distance is 70 meters
loci70<-expand.grid(seq(6800,8200,l=20),seq(11340,12340,l=14.29))

#Create prediction grid on the 70 meter grid
c70<-krige.conv(geoG102,loc=loci70,krige=krige.control(cov.pars=c(5,900)))

#Import the file for the 70 meter grid
g10270<-read.table("g10270.txt", header=TRUE)

#Krige the 70 meter grid to produce a 20 meter grid
geoG10270<-as.geodata(g10270)
varioG10270<-variog(geoG10270)
plot(varioG10270)
lines.variomodel(cov.model="sph", nugget=.5, cov.pars=c(5.5,1000), max.dist=1000)
c70<-krige.conv(geoG10270,loc=loci,borders=p102,krige=krige.control(cov.pars=c(5.5,1000), nugget=.5,cov.model="sph"))

#Expand a grid where distance is 75 meters
loci75<-expand.grid(seq(6800,8200,l=18.66),seq(11340,12340,l=13.33))

#Create prediction grid on the 75 meter grid
kc75<-krige.conv(geoG102,loc=loci75,krige=krige.control(cov.pars=c(5,900)))

#Import the file for the 75 meter grid
g10275<-read.table("g10275.txt", header=TRUE)

#Krige the 75 meter grid to produce a 20 meter grid
geoG10275<-as.geodata(g10275)
varioG10275<-variog(geoG10275)
plot(varioG10275)
lines.variomodel(cov.model="sph", nugget=.5, cov.pars=c(5.5,1000), max.dist=1000)
kc75<-krige.conv(geoG10275,loc=loci,borders=p102,krige=krige.control(cov.pars=c(5.5,1000), nugget=.5,cov.model="sph"))

#Expand a grid where distance is 80 meters
loci80<-expand.grid(seq(6800,8200,l=17.50),seq(11340,12340,l=12.50))

#Create prediction grid on the 80 meter grid
kc80<-krige.conv(geoG102,loc=loci80,krige=krige.control(cov.pars=c(5,900)))

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# Import the file for the 80 meter grid

```r
gl0280 <- read.table("gl0280.txt", header=TRUE)
```

# Krige the 80 meter grid to produce a 20 meter grid

```r
geoG10280 <- as.geodata(gl0280)
varioG10280 <- variog(geoG10280)

# plot(varioG10280)
# lines.variometric(cov.model="sph", nugget=.5, cov.pars=c(6,1000), max.dist=1000)

kc80 <-
kridge.conv(geoG10280, loc=loci, borders=p102, kridge=kridge.control(cov.pars=c(6,1000), nugget=.5, cov.model="sph"))
```

# Expand a grid where distance is 85 meters

```r
loci85 <- expand.grid(seq(6800,8200,l=15.55), seq(11340,12340,l=11.11))
```

# Create prediction grid on the 85 meter grid

```r
kc85 <- kridge.conv(geoG102, loc=loci85, kridge=kridge.control(cov.pars=c(5,900)))
```

# Import the file for the 85 meter grid

```r
gl0285 <- read.table("gl0285.txt", header=TRUE)
```

# Krige the 85 meter grid to produce a 20 meter grid

```r
geoG10285 <- as.geodata(gl0285)
varioG10285 <- variog(geoG10285)

# plot(varioG10285)
# lines.variometric(cov.model="sph", nugget=1, cov.pars=c(5.5,1000), max.dist=1000)

kc85 <-
kridge.conv(geoG10285, loc=loci, borders=p102, kridge=kridge.control(cov.pars=c(5.5,1000), nugget=1, cov.model="sph"))
```

# Expand a grid where distance is 90 meters

```r
loci90 <- expand.grid(seq(6800,8200,l=15.55), seq(11340,12340,l=11.11))
```

# Create prediction grid on the 90 meter grid

```r
kc90 <- kridge.conv(geoG102, loc=loci90, kridge=kridge.control(cov.pars=c(5,900)))
```

# Import the file for the 90 meter grid

```r
gl0290 <- read.table("gl0290.txt", header=TRUE)
```

# Krige the 90 meter grid to produce a 20 meter grid

```r
geoG10290 <- as.geodata(gl0290)
```
# Expand a grid where distance is 95 meters
loci95<-expand.grid(seq(6800,8200,l=14.74),seq(11340,12340,l=10.53))

# Create prediction grid on the 95 meter grid
kc95<-krige.conv(geoG102,loc=loci95,krige=krige.control(cov.pars=c(5,900)))

# Import the file for the 95 meter grid
g10295<-read.table("g10295.txt", header=TRUE)

# Krige the 95 meter grid to produce a 20 meter grid
geoG10295<-as.geodata(g10295)
varioG10295<-variog(geoG10295)
#plot(varioG10295)
#lines.variomodel(cov.model="sph", nugget=1, cov.pars=c(5,1000), max.dist=1200)
kc95<-krige.conv(geoG10295,loc=loci,borders=p102,krige=krige.control(cov.pars=c(5,1000),nugget=1,cov.model="sph"))

# Expand a grid where distance is 100 meters
loci100<-expand.grid(seq(6800,8200,l=14),seq(11340,12340,l=10))

# Create prediction grid on the 100 meter grid
kc100<-krige.conv(geoG102,loc=loci100,krige=krige.control(cov.pars=c(5,900)))

# Import the file for the 100 meter grid
g102100<-read.table("g102100.txt", header=TRUE)

# Krige the 100 meter grid to produce a 20 meter grid
geoG102100<-as.geodata(g102100)
varioG102100<-variog(geoG102100)
#plot(varioG102100)
#lines.variomodel(cov.model="sph", nugget=1, cov.pars=c(5.5,1200), max.dist=1500)
kc100<-krige.conv(geoG102100,loc=loci,borders=p102,krige=krige.control(cov.pars=c(5.5,1200),nugget=1,cov.model="sph"))
Summary of KV

summary(kc50$krige.var)
summary(kc55$krige.var)
summary(kc60$krige.var)
summary(kc65$krige.var)
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#Summary of KV

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REFERENCES


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