Optical interferometry with detector system to measure perturbations in plastics

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OPTICAL INTERFEROMETRY WITH DETECTOR SYSTEM TO MEASURE
PERTURBATIONS IN PLASTICS

by

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ABSTRACT

Optical Interferometry with Detector System to Measure Pressure Perturbations in Plastics

by

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The Nevada Shocker is a pulsed power machine composed of a 540 kV (maximum), 4.54 kJ (maximum) Marx Bank in series with a 50 ns pulse forming Blumlein activated by a self breaking water switch. The energy released by the water dielectric Blumlein is guided by a water-filled coaxial line to a vacuum chamber containing a parallel plate diode. A cylinder of Rexolite plastic is sandwiched in between the parallel plate diode electrodes. The applied field mechanically stresses the electrodes on a molecular level generating a shock wave that changes the localized index of refraction. Further during discharge, the applied electric field also results in a change in index of refraction. Interferometers may be used to detect small, localized changes at each point in time. This thesis provides a preliminary study of interferometry probing the characteristics of Rexolite plastic under stress on a prototype of the setup modeled on the environment housing the Nevada Shocker.
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CHAPTER 1

INTRODUCTION

1.1 Thesis objective

The Nevada Shocker is a pulsed power machine composed of a 540 kV (maximum), 4.54 kJ (maximum) Marx Bank in series with a 50 ns pulse forming Blumlein activated by a self breaking water switch. The energy released by the water dielectric Blumlein is guided by a seven ohm water-filled coaxial line to a $10^{-6}$ Torr ($1.33 \times 10^{-4}$ Pa), vacuum chamber containing a parallel plate diode seven inches in diameter. The maximum possible calculated electric field and the minimum rise time are on the order of 20 MV/m and 10 ns respectively. A cylinder of Rexolite plastic one inch in diameter and one and two inches in length is sandwiched in between the parallel plate diode electrodes. The applied field mechanically stresses the electrodes on a molecular level generating a shock wave that propagates in the plastic at the speed of sound. The acoustic wave changes the localized index of refraction. Further, during discharge, the applied electric field electrically stresses the material also resulting in a change in index of refraction. Over the time duration of the evolution of the discharge prior to complete closure, these changes are assumed to be small.

Typically, interferometers may be used to detect small, localized changes at each point in time. The anticipated fractional change in the amplitude of light detected is directly coupled to the change in the index of refraction. A change in the index of
refraction alters the phase of the beam that passes through the plastic sample relative to a reference beam. By constructive-destructive interference, a fringe pattern forms. By studying the change in the fringe pattern, one may deduce information on the change in the optical path length which depends on the change in the index of refraction and the change in geometrical length. Since the plastic under test is amorphous the elasto-optic effect is assumed to be isotropic. The elasto-optic constant is a measure of the change in the index of refraction to the applied strain. In linear order, the pressure applied to the plastic is proportional to the change in the index of refraction. Consequently, one may deduce the pressure applied to the plastic under test by examining the fringe shifts. By employing a pulsed laser, the characteristics of the device under test may be studied within the interrogation time of the sampling beam typically freezing low frequency oscillations that may be encountered by both the reference and sampling beams upon passing through viewports and other environmental changes along the optical path of both beams. By synchronizing the event time with a trigger time, one may hypothesize the origin of the acoustic wave generated yielding insights to the evolution of flashover.

This thesis provides a preliminary study of interferometry probing the characteristics of Rexolite plastic under stress typically applied by the *Nevada Shocker* electrodes with the aid of a low power continuous HeNe laser. All experiments are conducted for static loads at atmospheric pressure. Hypothetically, a static study is reasonable since the acoustic wave will propagate a distance much less than the diameter of the sampling beam in the pulse duration. Because the fringe shift may be small in experiments conducted with the *Nevada Shocker*, a theoretical study to enhance the signal to be
detected above the noise level is conducted for various optical-electrical schemes within
the damage threshold of the photodetector.

1.2 Flashover on plastics

Flashover can be defined as a disruptive electrical discharge along the surface of an
insulator ultimately leading to electrical shorting or closure. Surface breakdown may
result over an isolated localized region or over a global region both within a potential
difference. It has been conjectured in literature that flashover may be a consequence of
gas streamers, triple point field enhancements, surface contamination, and prepulse
effects. Each of these effects will be discussed.

1.2.1 Gas streamer effects

Streamers, their cause and effect have been studied in detail by Raether [1] and P.
Rice Evans [2]. Both the localized jets and gaseous interface (global interface) contain
neutral molecules along with ions and electrons. When an ion-electron pair is created in
the presence of an electric field, the electron is accelerated and it creates further
ionization by colliding with the gaseous atoms that lie in the path of the electron. A chain
effect is started and an avalanche is triggered. The electrons move with high velocity
towards the anode away from the positive ions creating a space charge field which
negates the applied electric field within the avalanche. Recombination occurs in the
center of the avalanche and photons are emitted in all directions equally. Outside of the
avalanche, photon stimulated ionization and electric field accelerated electrons fuels and
enhances the formation of the avalanche. The photons that were emitted away from the
initial avalanche do not encounter any gaseous atoms in their path and will not create
further avalanches. Although the authors indicate this, the energetic photons may collide
with the chamber walls releasing adsorbed material into the vacuum region. Such photon stimulated studies have been examined by Krishnan [3] in a high vacuum environment with an 80 mJ pulsed laser. The new avalanches that form at the head and tail of the initial one will stimulate further avalanches. This chain of avalanches merges to form a streamer. The tips of the streamer travel rapidly, in time reaching the plates leading to closure in the gap.

1.2.2 Triple point effect

A triple point is a junction between a conductor, an insulator, and vacuum (or some other medium). Triple point junctions play an important role since, in certain cases, surface flashover may be initiated by the emission of electrons from the triple junction [4].

In experiments to be conducted in the Nevada Shocker, two 7.5" (190.5mm) diameter brass electrode plates securely holds a 1" (25.4mm) or 2" (50.8mm), long 1" diameter, Rexolite cylinder. A simple two-dimensional electrostatic study is conducted to illustrate the change in the electrostatic field configuration at a triple point. The Rexolite cylinder is modeled as a rectangular structure between two parallel electrodes with one dimension of infinite extent. To illustrate the effect of geometry on the triple point fields, three different electrostatic cases are studied as illustrated in Figs 1.1 to 1.3.

In Fig. 1.1, the Rexolite sample is a perfect rectangular structure. The distance of separation between the electrodes is \( d \) and the dielectric permittivity of the sample is \( \varepsilon_1 \). Since the arrangement consists of two conductor parallel plates separated by a dielectric with dielectric-vacuum boundary everywhere in a plane perpendicular to the plate surface, elementary electrostatic principles may be applied to study the fields in this
configuration. From electrostatic boundary conditions, the surface charge density, \( \rho_s \), at the dielectric-perfect conductor interface is

\[
\rho_s = D_n
\]  

(1.1)

where \( D_n \) is the normal field of the electrostatic flux density in the dielectric pointing perpendicular to and away from the conductor boundary.

Since the parallel plates are equipotential surfaces with one surface at a potential \( V_o \) relative to the second, the electric fields as obtained from Laplace's equation satisfying all boundary conditions are uniform and equal in the vacuum and plastic regions given by

\[
E_1 = E_2 = E_3 = \frac{V_o}{d}
\]  

(1.2)

where, \( E_1 \) and \( E_3 \) are the electric field intensities in the vacuum regions to the left and the right of the plastic slab respectively and \( E_2 \) is the electric field intensity in the plastic slab. Based on Eq. (1.2), the surface charge density on the conductor-vacuum and conductor dielectric interfaces are respectively

\[
\rho_{s1} = \rho_{s3} = D_{n1} = D_{n3} = \varepsilon_0 E_1 = \varepsilon_0 E_3 = \varepsilon_0 \frac{V_o}{d}
\]  

(1.3a)

\[
\rho_{s2} = D_{n2} = \varepsilon_1 E_2 = \varepsilon_1 \frac{V_o}{d}
\]  

(1.3b)

The surface charge density is directly proportional to the permittivity and therefore the surface covered by the plastic dielectric has the highest surface charge concentration.

Excluding space charge effects, the fields generated by this capacitor geometry applies a force directed parallel to the vacuum-plastic interface. Dipole effects of the material plastic may attract an electron to the plastic surface but this force is small compared to that generated by the parallel plate structure. Even though this force is small, it is
significant since there is no competing force repelling the electron normal to the surface of the plastic medium in the vacuum region between the electrodes.

A concave surface defect on the right side of the plastic sample under test is examined in Fig. 1.2. The electrostatic behavior of the system is simulated using the ESTAT module from the TRICOMP suite of programs. To view the results the VESTAT module is used. Observing the VESTAT graph in Fig. 1.4, in the regions that surround the surface defect-free Rexolite (region to the left of the slab), the electric field magnitude and direction are uniform. On the right side of the Rexolite plastic, the surface deformation tends to enhance the field magnitude in the vacuum region just below the vacuum-plastic interface in y=constant planes. The magnitude of the field in the plastic near the same interface has decreased. For boundary conditions to be satisfied, the orientation of the field lines are distorted or perturbed from their constant x-direction (in a plane parallel to the electrode surface). Effectively based on boundary conditions, the field component normal to the dielectric-vacuum interface is expelled from the dielectric and enhanced in the vacuum region. As the distance between the dielectric and electrode closes up, one can expect that the field at the triple point to be enhanced.

When the dielectric contains a convex surface defect as illustrated in Fig. 1.3, modeling codes (refer to Fig. 1.5) suggest an electric field reduction near the triple point. The polarization caused by an applied voltage leads to an enhancement of the electric field at the Rexolite-free space interface parallel to the electrode surface.
1.3 Surface contamination

Surface contaminants either absorbed or desorbed by a medium act as a source of outgassing and stimulated desorption. A contaminant can be defined as a substance whose presence is undesired and one which affects the performance of the system. Charges loosely bound to the surface of contaminant molecules may be transported to the bulk plastic medium. Consequently, upon removal from the medium, the desorbed or outgassed molecules may be ionized potentially enhancing flashover conditions. Causes and control of surface contamination are examined.

1.3.1 Causes for the occurrence and formation of surface contaminants

Surface contamination can occur due to a number of reasons and are classified as follows:

(a). Adsorbed layer type contamination resulting from undesired surface exposure to the surrounding environment [5].

(b). A chemical reaction layer formed when volatile agents such as oxygen and sulfur react with a material surface or bulk structure [5].

(c). Variable composition type of surface contaminant resulting from undesired preferential diffusion of one component type [5].

(d). Hydrocarbon surface contaminations resulting from cleaning products such as alcohol (forms a heavy layer of C-OH), oils and grease (A single fingerprint exposed to vacuum has a gas load of $10^{-5}$ torr liters/sec [6].), and inherent gases (such as desorption of CO and H$_2$). Komiya et al [7] conducted an experiment on stainless steel where there was a sharp growth observed for both carbon and hydrogen peaks only at the electron bombarded area. Coad et al [8] attributed surface migration of carbon to the fact that the
carbon peak growth was not confined to the bombardment area alone whereas the oxygen peak was confined to the bombarded area.

(e) Water vapor contamination is in a class on its own due to the abundance of water in the environment and the difficulty in removing it. To counter the water vapor contamination, the system is heated from 100 to 1000°C. DC heating techniques are used to deplete bulk water vapor oxide layers and hydrocarbons with a prolonged bake out. Pulsed heating methods lead to a rapid reduction in the surface layers or quickly melt the oxides [9].

1.3.2 Surface contamination effects and control

Surface contaminants like dielectrics, oxides and metallic particulate matter have a pronounced effect in initiating, mediating and modifying electron emission from electrically-stressed metal surfaces in a vacuum environment [10, 11]. Adsorbed gas can alter the work function for metallic emitters and result in large fluctuations in pre breakdown electron emission. Layers of surface and bulk hydrocarbon contaminants cause the formation of anode and cathode plasmas [12], which limit the system scalability and performance. Without magnetic insulation, the appearance of plasmas in high voltage gaps results in breakdown. The anode-cathode gap plasma closure leads to impedance collapse, pulse length limitations and total energy loss. The presence of surface contaminants can be controlled by these conditioning techniques:

(a). In situ heating

Heating [13-16] to 100-1000°C is one of the most widely used technique in the field of pulsed-power. Pulsed heating techniques are used to reduce the level of surface layers of contaminants and to melt oxides quickly. DC heating techniques are more appropriate
to reduce the bulk oxide layers of water vapor and hydrocarbons with a prolonged bake out.

(b). Surface coatings

Desorption from electrodes can be controlled using surface coatings [17]. Surface coatings can be used to change surface heating rates and to increase the breakdown threshold and increase surface flashover strength [18-20].

(c). Discharges

If oxygen is introduced into the system in ‘discharges’, then it removes hydrocarbons from the surfaces at a higher rate than pure physical sputtering by converting them into $H_2O, CO$ and $CO_2$ [17, 21, 22].

1.4 Mechanical analysis

Vibrational energy resulting from the acoustical wave launched by the diode electrodes on a molecular level is hypothesized to contribute to flashover by releasing ionized atoms or molecules near the plastic surface in the path of the acoustic wave. Ionized and non-ionized atoms and molecules with initial energies just below an energy threshold level (e.g., van der Waal's, work function, high field emission threshold, etc.) may be stimulated with vibrational energy sufficient enough to be emitted from the surface or bulk material medium.

The purpose of this section is to estimate the displacement of the brass surface caused by a force applied to that surface loaded with a plastic cylinder. Because the duration of the experiment is about 50 ns and the speed of sound in brass is 3499.1 m/s ($1.38 \times 10^5$ in/sec), the distance traveled by the forefront of the pulse is 175 $\mu$m ($9.252 \times 10^{-3}$ in). The speed of sound in the plastic cylinder is 2362 m/s
(9.299×10^4 in/sec) implying that the wave will move a distance of 0.1181 mm (4.65×10^{-3} in) in the cylinder. Physically, as the acoustic wave progresses into the solid brass and solid plastic materials, the surface of the brass solid continues to move in the direction of the force. It is desired to examine surface deformation in one dimension assuming the surface is rigid and does not exhibit a torsional motion and how this affects the index of refraction in the plastic medium. The solid mediums are modeled as a cascade atomic/molecular mass–spring system allowing for the support of a longitudinal acoustic wave. Because the acoustic transit time along both the brass and plastic materials is long compared to the duration of the experiment and one is interested in the change of the mediums within the first transit time, both ends of the solid mediums are assumed to fixed in space while the brass end subjected to a constant force is loaded with the plastic medium. The brass solid is the electrode of our experiment and the plastic medium is the Rexolite cylinder piece under test. Because one is looking at time scales that are short compared to an acoustic transit time in either the brass or plastic medium, complete details of both electrodes are not necessary. [Refer to Fig. 1.6]

A one inch (25.4mm) diameter Rexolite cylinder is concentrically centered on axis between two brass parallel plate disk electrodes. The diode assembly exists in a 10^{-6} Torr vacuum. The anode electrode is connected to an air cylinder mounted on a flange bellows configuration. The flange with cross sectional area A = 3167.74 mm^2 (4.91 in^2), experiences one atmosphere (ATM) of pressure [101.284 kN/m^2 (101 kPa) or, equivalently, 14.69 lb/in^2] and statically imposes this nominal pressure on the sample. For a 1" diameter sample [sample area of A_s = 506.45 mm^2 (0.785 in^2)]
\[ P_s = \frac{A}{A_s} P = 633.02 \text{kN/m}^2 (91.8125 \text{ lb/in}^2). \] Under appropriate pressure, an air cylinder attached to one of the electrodes can increase or decrease the pressure applied to the sample. Typically, the nominal pressure is used to hold the sample in place between the electrodes.

The pulse created by the Nevada Shocker dynamically stresses the electrode plate generating a shock wave in the Rexolite sample. The force acting on the Rexolite sample has to overcome the molecular inertia and binding forces of the sample geometry. The electromagnetic force acting on the surface of the brass electrode is estimated using the electromagnetic stress tensor and a quasi-static assumption. If breakdown does not occur, it is clear that as the pulse duration approaches infinity, the electromagnetic problem approaches an electrostatic condition. Consequently, quasi-electrostatics is employed. It will be shown that a small force will be generated between the electrode plates prior to breakdown.

1.4.1 Electrostatic force calculation between two parallel plates with slab dielectric using the electromagnetic stress tensor

The electromagnetic stress tensor is to be used to determine the electrostatic force between the parallel plate configuration. The dynamic changes are neglected. Further, breakdown is not assumed to exist during the initial force therefore, there is no ‘DC’ current present. The conservation of momentum theorem in integral form states

\[
\iiint \mathbf{j} \cdot d\mathbf{V} = - \iiint \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{V} - \iint \mathbf{P} \cdot d\mathbf{S} \tag{1.4}
\]

where \( \mathbf{j} \) is the momentum density vector, \( \mathbf{E} \) is the force density vector and

\[
\mathbf{P} = \frac{1}{2} \left( \mathbf{B} \cdot \mathbf{E} + \mathbf{E} \cdot \mathbf{B} \right) - \nabla \mathbf{B} - \mathbf{B} \nabla \mathbf{E} \tag{1.5}
\]
is the Maxwell stress tensor where $I$ is the identity matrix. The Maxwell stress tensor is the pressure exerted by the fields over the closed surface $S$ enclosing a volume $V$. The left hand side of the conservation relation in Eq. (1.4) describes the net force acting on the charges and currents internal to the volume enclosed by the surface. It must be understood that it is assumed that the charges are fixed to the surface of the electrodes experiencing the force. Therefore, the force imparted to the charges is in turn imparted to the material. It is therefore assumed that the charges do not have enough energy to leave the surface of the material during the duration of the pulse as might result in a high field emission scenario. Further, the surface of the material is assumed not to be deformed from its planar configuration over the duration of the experiment.

Referring to Fig. 1.6, the surface $S$ encloses the lower electrode in such a manner that the one plane of the surface lies on the surface of the plate where the electric field lines terminate. The $+z$ axis is oriented from the lower plate to the upper plate. In the conservation relation in Eq. (1.4), $d\mathbf{S}^b$ is the outward normal of the surface enclosing $V$. Therefore on the surface terminating the fields, $d\mathbf{S}^b$ points in the $+z$ direction (opposite direction of the fields in the figure). For an electrostatic approximation, $B=H=0$. Neglecting fringe effects, the electric field is uniform between the parallel plate geometry. The momentum density imparted to the plates is zero. The Maxwell stress tensor simplifies to

$$\frac{\partial}{\partial t} \int_J - \frac{1}{2} \varepsilon_j |\mathbf{E}_j|^2 \hat{z}$$

where $j=1$ or 2 for medium 1 (free space $\varepsilon_0$) and medium 2 (dielectric $\varepsilon_2 = \varepsilon_d$). Since $d\mathbf{S} = +\hat{z} d\mathbf{S}$, the net force acting on the surface charges on the plate and hence the plate is
Satisfying both boundary conditions and the concept of potential, $E_1 = E_2 = E = (V/D)$. The area $S_1$ is the area of an annular disk with inside radius of $R_1$ and outside radius $R_2$ and the area $S_2$ is that for a circular disk of radius $R_1$. Consequently replacing $\varepsilon_2$ by $\varepsilon_d = \varepsilon_0 \varepsilon_{rd}$, the force acting on the lower plate is given by

$$F_z = \frac{\pi \varepsilon_0}{2} \left( \frac{V}{D} \right) \left[ (R_2^2 - R_1^2) + \varepsilon_{rd} R_1^2 \right] = \frac{\pi \varepsilon_0}{2} \left( \frac{V}{D} \right) \left[ R_2^2 + (\varepsilon_{rd} - 1) R_1^2 \right]$$  

Table 1.1 provides values for the parameters typical for the *Nevada Shocker*. Using the parameters listed in Table 1.1, the force as computed from Eq. (1.8) yields $F_z = 32.46$ N. This force is acting on two components, the brass plate feeling the force and the plastic under test sandwiched between the plates.

### 1.4.2 Atomic mass spring model

The binding region between each layer is modeled as a spring providing a restoring force to the system acting on masses representing the mass of neighboring monolayers of the metal and/or dielectric as appropriate. Although important in the dynamics of the problem, atomic damping effects are neglected in the quasi-static study. The region is divided in such a way that the layer masses and the binding springs in each layer is the same as in any other layer. Because of geometrical symmetry, the one dimensional atomic mass-spring model of the plastic cylinder and the brass plate employed is reasonable. Based on a quasi-static approximation and neglecting damping effects, a general expression describing the change in one dimension of the sample in response to an applied force is derived.
It is argued that the net force resulting from an external electric field acting on an isolated dipole is much smaller than that acting on an isolated electron. The dipole force, as would exist in a dielectric, is dependent on the gradient of the applied field over the scale length of the dipole times the dipole moment where as the isolated ion or isolated electron force is dependent on the field amplitude and charge magnitude. Although the number of charges in a dipole could be large, the difference in external field over the length of the dipole is appreciably small. Therefore, dipole forces that would arise in the dielectric are neglected in this model.

Choosing to view the brass plate as a perfect conductor, (either based on a skin depth argument or an electrostatic argument), the external electric field draws conduction charge to the surface of the metal structure preventing the field from penetrating the surface. Since the surface charge is bound to the lattice structure of the metal by a binding force, it stands to reason that the applied field is acting on the first monolayer of the metal electrodes and hence the first mass monolayer \( m_{-1} \) in the atomic mass-spring model.

Let \( \eta_n \) represent the distance the mass \( m_n \) (mass of the \( n \)th layer) is displaced from its static equilibrium position. Figures 1.7 and 1.8 illustrate a spring-mass representation of the solid materials. The masses and springs are numbered in an increasing order in the direction of increase in \( x \), and all masses and all spring constants are identical in the respective brass or plastic regions. The \( x=0 \) position is relative to an arbitrary reference position. To deduce a general expression for the displacement of the brass surface regardless of the number of monolayers, the following three simpler problems are examined in succession: 1) Two brass monolayers and one plastic monolayer terminated
with fixed masses, 2) Three brass monolayers and one plastic monolayer terminated with fixed masses, and 3) Three brass monolayers and two plastic monolayers terminated with fixed masses.

1.4.3 Two brass monolayers and one plastic monolayer atomic mass-spring model

Hooke's law states that the restoring force due to a spring is proportional to the length that the spring is stretched and acts in the opposite direction to an applied force. Newton's second law relates the applied force to the acceleration and the spring restoring force. With this in mind, an atomic mass-spring model is developed for a three monolayer mass system characterizing two brass monolayers and one plastic monolayer with fixed load terminations as shown in Fig (1.7). For mass $m_2$ the discrete equation of motion is given by

$$m_2 \frac{d^2 \eta_2}{dt^2} = -k_b(\eta_2 - \eta_3) + k_b(\eta_1 - \eta_2)$$

(1.9)

where $\eta_2$ and $\eta_1$ are the displacements of the masses $m_2$ and $m_1$ respectively and $k_b$ is the brass spring constant. Mass $m_3$ is a rigid mass therefore $\eta_3 = 0$. As indicated in Fig. 1.7 [and Fig. 1.8], negative subscripts represent parameters in the brass medium while positive subscripts represent parameters in the plastic medium. Consequently, Eq. (1.9) reduces to

$$m_2 \frac{d^2 \eta_2}{dt^2} = -k_b(\eta_2) + k_b(\eta_1 - \eta_2)$$

(1.10)

For mass $m_1$, the equation of motion is given by

$$m_1 \frac{d^2 \eta_1}{dt^2} = -k_b(\eta_1 - \eta_2) + k_p(\eta_1 - \eta_3) + F_{EM}$$

(1.11)
This monolayer mass represents the surface of the brass electrode experiencing the external electromagnetic force, resulting from the fields between the plates. By analogy with Eq. (1.9), the equation governing the motion of the first plastic monolayer is

\[ m_1 \frac{d^2 \eta_1}{dt^2} = -k_p (\eta_1 - \eta_{-1}) + k_p (\eta_2 - \eta_1) \]  

In this scenario, the mass \( m_2 \) is a fixed mass therefore the displacement, \( \eta_2 = 0 \). Consequently, Eq. (1.12) reduces to

\[ m_1 \frac{d^2 \eta_1}{dt^2} = -k_p (\eta_1 - \eta_{-1}) - k_p (\eta_1) \]  

The external force is now considered to be a static force and the system is in a steady state. Therefore,

\[ \frac{d \eta_n}{dt} = 0 \quad \text{and} \quad \frac{d^2 \eta_n}{dt^2} = 0 \]  

Using the static steady state condition [Eq. (1.14)] and solving for \( \eta_{-2} \) in Eq. (1.10) and \( \eta_1 \) in Eq. (1.13) both in terms of \( \eta_{-1} \) and substituting into Eq. (1.11) yields the displacement at the interface given by

\[ \eta_{-1} = \frac{F_{EM}}{k_b + k_p} \left[ \frac{1}{2} \right] \]  

Note that although the number of monolayer masses in each region differs by one, the weighting of each spring constant on the displacement is the same.
1.4.4 Multiple brass and plastic monolayers atomic mass-spring model

As shown in Fig. 1.8, an additional brass monolayer is incorporated in the model as compared to that in Section 1.4.3. For mass \( m_3 \) the equation of motion is given by,

\[
m_3 \frac{d^2 \eta_2}{dt^2} = -k_b (\eta_3 - \eta_4) + k_b (\eta_2 - \eta_3)
\]

where \( \eta_3 \) and \( \eta_2 \) are the displacements of the masses \( m_3 \) and \( m_2 \) respectively. We consider the mass \( m_4 \) to be a rigid mass therefore \( \eta_4 = 0 \). Consequently, Eq (1.16) reduces to

\[
m_3 \frac{d^2 \eta_2}{dt^2} = -k_b (\eta_3 - \eta_2) + k_b (\eta_2 - \eta_3)
\]

Because \( m_3 \) is finite and \( m_2 \) is fixed, Eqs. (1.9), (1.11), and (1.13) are valid. Under the static, steady state condition [Eq. (1.14)}, \( \eta_3 \), \( \eta_2 \) and \( \eta_1 \) written in terms of \( \eta_4 \) and substituted into Eq (1.11) yields the interface displacement given by

\[
\eta_1 = \frac{F_{EM}}{\frac{k_b}{3} + \frac{k_p}{2}}
\]

Further generalizing the model, consider the addition of another plastic monolayer mass \( m_2 \), the equation of motion for mass \( m_2 \) is given by

\[
m_2 \frac{d^2 \eta_2}{dt^2} = -k_p (\eta_2 - \eta_1) + k_p (\eta_3 - \eta_2)
\]

Consequently, Eq. (1.12) is now valid along with Eqs. (1.9), (1.11), and (1.17). Under the static, steady state condition [Eq. (1.14)], \( \eta_3 \), \( \eta_2 \), \( \eta_1 \) and \( \eta_2 \) written in terms of \( \eta_4 \) and substituted into Eq. (1.11) yields the interface displacement given by
A pattern can now be deduced for the multi-monolayer geometry. By analogy assuming a many monolayer system, a generic expression for the displacement of the interface layer can be written as

$$\eta_{-1} = \frac{F_{EM}}{k_b k_p \left[ \frac{1}{3} + \frac{1}{3} \right]}$$

where \( N_{mb} \) is the number of brass mass monolayers or equivalently the number of monolayers of brass and \( N_{mpl} \) is the number of plastic mass monolayers or equivalently the number of monolayers of plastic. As expected, the larger the number of monolayers considered in the model for the same static force, the larger the displacement. To determine the number of monolayers to use in the model in the static approximation, one first determines the distance the acoustic wave has traveled inside the materials over the timeframe of the event and divides this distance by the interatomic/intermolecular distance. The expressions become invalid if this propagation distance exceeds the geometrical length of either piece.

1.4.5 Bridging the macroscopic and microscopic measurables in the spring constant

The displacement of the brass interface presented in Eq. (1.21) is expressed in terms of the microscopic (atomic/molecular) spring constant. This parameter now needs to be expressed in terms of macroscopic and microscopic values suitable for computation. Reconsider the discrete Eq. (1.9) in the continuous limit. Let all of the discrete masses and spring constants be identical given by \( m \) and \( k \) respectively. Further, let the

\[ \eta_{-1} = \frac{F_{EM}}{k_b k_p \left[ N_{mb} + \frac{1}{N_{mpl} + 1} \right]} \]
displacements, \( \eta_{-2} = \eta(x, t), \eta_{-3} = \eta(x - \Delta x, t) \) and \( \eta_{-1} = \eta(x + \Delta x, t) \). The time derivative in Eq. (1.9) is based on the position of the mass \( m_2 \) at position \( x \). Therefore, the total time derivative in Eq. (1.9) is replaced by a partial derivative. Consequently, Eq. (1.9) maybe expressed as

\[
m \frac{\partial^2 \eta(x, t)}{\partial t^2} = k \left[ \eta(x + \Delta x, t) - 2\eta(x, t) + \eta(x - \Delta x, t) \right]
\]  

(1.22)

The numerator and denominator of the right hand side of Eq. (1.22) is multiplied by \((\Delta x)^2\) and both sides of the equation are evaluated in the limit when \( \Delta x \) approaches zero in the microscopic sense. In the microscopic sense, \( m_m \) is the monolayer mass and therefore \( \Delta x \) may be no closer than an interatomic/molecular distance depending on how one defines a monolayer. For simplicity, let us define the distance as ‘a’ as illustrated in Fig. 1.9. Equation (1.22) simplifies to

\[
\frac{\partial^2 \eta(x, t)}{\partial t^2} = v_s^2 \frac{\partial^2 \eta(x, t)}{\partial x^2}
\]  

(1.23)

where

\[
v_s = \sqrt{\frac{k}{m_m}}
\]  

(1.24)

Equation (1.23) is the one dimensional wave equation for acoustic waves and \( v_s \) is the velocity of sound.

In the macroscopic view, the acoustic wave equation for longitudinal vibrations in a uniform bar may be expressed directly in terms of the modulus of elasticity (Young’s modulus) and the mass density of the bar. The macroscopic wave equation has the same form as Eq. (1.23) with the following velocity of sound [23].

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\[ v_s = \frac{Y}{\sqrt{\rho_M}} \]  \hspace{1cm} (1.25)\\

where \( Y \) is the modulus of elasticity also noted as the Young's modulus and \( \rho_M \) is the mass density of the medium. For a uniform medium of mass \( M \) and volume \( V_M \), the mass density is \( \rho_M = \frac{M}{V_M} \). Since, the velocity of sound in Eqs. (1.24) and (1.25) are the same, upon equating yields an expression for the microscopic spring constant

\[ k = \frac{Ym_m}{a^2 \rho_M} \]  \hspace{1cm} (1.26)\\

The spring constant provides a link between the macroscopic measurable for the microscopic model.

Knowing the chemical composition of the material, the mass of an atom/molecule, \( m_a \), composing the material may be determined from a chemistry handbook or periodic table. If the material is composed of a uniform mixture or interlace of molecules, the mass \( m_a \) is the effective mass representing the smallest mass measurement of this composition of molecules. Based on this representation, one may state that the medium is composed of a molecule compositions having mass \( m_a \). Consider a material of mass \( M \) and volume \( V_M \). The number of atoms/molecules inside the material is given by the ratio of the masses; \( N = \frac{M}{m_a} \). The volume of a single atom/molecule, \( V_{mol} \), may be estimated as

\[ V_{mol} = \frac{V_M}{\left(\frac{M}{m_a}\right)} \]  \hspace{1cm} (1.27)\\

Assuming a simple cubic structure to the atom/molecule, (Refer to Fig. 1.9) the atomic length of one side of the cube, 'a', or the interatomic spacing between two neighboring atoms, 'a', may be estimated as
Let $A_M$ represent the cross sectional area of the material with normal in the direction of the motion of the one dimensional acoustic wave. The number of atoms/molecules in a monolayer of material is given by

$$n_{nm} = \frac{A_M}{a^2}$$

(1.29)

The monolayer mass is then determined as

$$m_m = m_a n_{nm}$$

(1.30)

Assume a uniform medium of known mass $M$, volume $V_M$, and modulus of elasticity given by Young’s modulus, $Y$. Define the cross sectional area in which the one dimensional acoustic wave passes normal through as $A_M$. The spring constant given by Eq. (1.26) may be expressed, with the aid of Eqs. (1.27) - (1.30), as

$$k = Y \left( \frac{M}{m_a} \right)^{\frac{1}{3}} \frac{A_M}{V_M^{\frac{1}{3}}}$$

(1.31)

where $m_a$ is the mass of the atomic/molecular element composing the material.

1.4.6 Interface displacement calculation

Given the theory above, the displacement of the brass surface may be determined. This displacement changes the index of refraction at a localized region in the plastic near the electrode-plastic interface. Consequently, an effective two medium plastic similar to that shown in Figs. 1.2 and 1.3 may be envisioned. The two medium structures in the figures is a consequence of the presence of the defect when moving perpendicular to the surface of the brass electrode. As shown in the electrostatic modeling studies, changes in the index of refraction alter the field orientation and intensity at the defects or triple points. From Table 1.1 the force acting on the electrodes in the Nevada Shocker was
calculated to be 32.46 N. In literature, it is stated that the closure or full discharge occurs on a nanosecond time scale. Assuming a constant force is applied, the displacement of the electrode surface is examined at three different times namely, 0.5 ns, 1 ns and 10 ns. For these respective times, the acoustic wave in the brass mediums travels 1.75 μm, 3.5 μm and 35 μm. The brass electrode is about 1.3 cm thick. Consequently, the acoustic wave does not reach the back side of the electrode in the timeframes of interest. Similarly, the acoustic wave propagates 1.18 μm, 2.36 μm, and 23.6 μm respectively in the plastic medium. Since the plastic cylinder is about 2.54 cm in length, it is observed that the acoustic wave in the plastic medium does not reach the termination end of the plastic. The speed of sound in both mediums may be found in Table 1. These results imply that the simple theoretical model for the times of interest in the evolution of the discharge is valid. To determine the number of monolayers the acoustic wave propagates in each medium, one must determine the intermolecular distance between brass and Rexolite molecules. This is determined by first calculating the mass of each molecule. Brass is composed of various percentages of zinc (Zn) and copper (Cu). Consider the brass molecule of interest to be Cu$_x$Zn$_{1-x}$. The atomic weight of Cu and Zn are respectively 63.546 AMU and 65.39 AMU where 1 AMU=1.66 x10$^{-27}$ kg. Based on the chemical formula Cu$_x$Zn$_{1-x}$, a single brass molecule has a molecular mass of m$_{ab}$=5.34 x 10$^{-25}$ kg. Rexolite is a cross-link of di-vinyl benzene (C$_{10}$H$_{10}$) and polystyrene (C$_8$H$_8$). The atomic weight of carbon (C) and hydrogen (H) are respectively 12 AMU and 1 AMU. Based on a link of a cross-link of a single di-vinyl benzene molecule and polystyrene molecule, the molecular mass of a Rexolite molecule is m$_{ap}$=3.88x10$^{-25}$ kg. The molecular distance is computed based on Eqs. (1.27) and (1.28). Based on the model
presented, the intermolecular distance between brass molecules and between plastic molecules is respectively $a_{bb}=0.41$ nm and $a_{bp}=0.73$ nm. Consequently, in 0.5 ns, 1 ns and 10 ns, the number of monolayers that the acoustic wave in brass propagates ($N_{mb}$) is 4268 monolayers, 8537 monolayers and 85366 monolayers respectively. For the same sequence of times, the number of monolayers that the acoustic wave in plastic propagates ($N_{mp}$) is 1616 monolayers, 3233 monolayers and 32329 monolayers respectively. Using Eq. 1.31 and the appropriate values listed in Table 1.2, the spring constants can be determined as provided in Table 1.2.

With the aid of Eq. (1.21), the monolayer displacements given above, the electromagnetic force calculated in Table 1.1, and the spring constants in Table 1.2, the displacement of the electrode surface may be determined as provided in Table 1.2. It is observed that the displacement varies linearly with time. This is reasonable since the spring constant of the brass electrode is three orders of magnitude larger than that of the plastic and the numbers of monolayers that the acoustic wave propagates in each region are comparable. Consequently, Eq. (1.21) simplifies to

$$\eta_{-1} \approx \frac{N_{mb} F_{EM}}{k_b}$$

yielding the linearity observed in the computation. Upon comparing computed electrode surface displacements to the intermolecular spacing between molecules, this theoretical model tends to state that the electrode interface in the 10 ns duration moves about 0.0001 fraction of an intermolecular spacing. This displacement is probably undetectable with interferometry techniques in the optical spectrum. Even so, in the steady state limit, small forces on the order of Newtons can be detected. In this static limit, the analysis
developed is no longer valid and boundary conditions and transient limits need to be carefully considered.

1.5 Organization of thesis

Initial heuristic studies tend to indicate that the resolution of the optical/electrical interrogation system must be very sensitive if transient studies are to be examined for such small forces and short time intervals. Noise in electrical, optical and mechanical noise may inhibit one from examining signals and physics of interest. Consequently, Chapter 2 studies a number of optical/electrical systems comparing signal-to-noise ratios and photodiode input powers. The detector being employed should be able to read a signal of such low magnitude. Further, this detection should be carried in the presence of many noise sources [24] - [26]. A detection scheme is developed. To minimize noise effects due to low frequency vibrations, a pulsed laser to be used in experiments associated with the Nevada Shocker. In the pulse duration, the dynamic properties of the experiment appear static. In Chapter 3, interferometry experiments are performed using a CW He-Ne laser to study the resolution between fringe shift and changes in applied force within the limits imposed by the Nevada Shocker. The experimental setup including the polishing technique is described in detail. Chapter 4 concludes the thesis with suggestions for future work.
Fig. 1.1 A perfectly cylindrical piece of Rexolite is placed between the electrode plates.

Fig. 1.2 A piece of Rexolite with a surface deformation directed inward is placed between the electrode plates.

Fig. 1.3 A piece of Rexolite with a surface deformation directed outward is placed between the electrode plates.
Fig. 1.4 VESTAT modeling of Rexolite sample with inward surface deformation placed between electrode plates in a vacuum environment.
Fig. 1.5. VESTAT modeling of Rexolite sample with convex surface deformation placed between electrode plates in a vacuum environment.
Fig. 1.6. Parallel plate capacitor with plastic medium sandwiched in between plates. The dashed line represents the surface $S$ enclosing volume $V$ needed in the conservation of momentum equation for the force calculation exerted on the lower plate. Assume that the $+z$ axis is oriented from the lower plate to the upper plate.
Fixed mass

Brass Monolayers

$\eta_{-2}$

$m_{-2}$

$k_b$

$k_b$

Plastic Monolayers

$\eta_{-1}$

$m_{-1}$

$k_p$

$F_{EM}$

$m_1$

$k_p$

Fixed mass

Fig. 1.7 Spring mass representation of a three (two brass, one plastic) monolayer mass system
Fig. 1.8 Spring mass representation of a four (three brass, one plastic) monolayer system
Fig 1.9 Inter atomic spacing between atoms.
Table 1.1 Typical conservative parameters for the Nevada Shocker.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage; V</td>
<td>400 kV</td>
</tr>
<tr>
<td>Plate separation = plastic length; D</td>
<td>0.025 m (1&quot;)</td>
</tr>
<tr>
<td>Outside radius of brass electrode; R₂</td>
<td>0.0943 m (3.77&quot; radius; 7.54&quot; dia.)</td>
</tr>
<tr>
<td>Outside radius of plastic under test; R₁</td>
<td>0.0125 m (1/2&quot; radius; 1&quot; dia.)</td>
</tr>
<tr>
<td>Permittivity of free space; $\varepsilon_o$</td>
<td>$8.85 \times 10^{-12}$ F/m</td>
</tr>
<tr>
<td>Relative permittivity of plastic under test</td>
<td></td>
</tr>
<tr>
<td>(rexolite); $\varepsilon_{rd}$</td>
<td>2.51</td>
</tr>
<tr>
<td>EM Force exerted on brass plates</td>
<td>32.46 N</td>
</tr>
</tbody>
</table>

Table 1.2 Calculation of displacement

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity of sound through brass, $V_{sb}$</td>
<td>3499.1 m/s</td>
</tr>
<tr>
<td>Velocity of sound through brass, $V_{sp}$</td>
<td>2362 m/s</td>
</tr>
<tr>
<td>Mass of the brass plate, $M_b$</td>
<td>2.7181 kg</td>
</tr>
<tr>
<td>Mass of the plastic sample, $M_p$</td>
<td>$12.651 \times 10^{-3}$ kg</td>
</tr>
<tr>
<td>Molecular mass of brass, $m_{ab}$</td>
<td>$5.34 \times 10^{-25}$ kg</td>
</tr>
<tr>
<td>Molecular mass of plastic, $m_{ap}$</td>
<td>$3.88 \times 10^{-25}$ kg</td>
</tr>
<tr>
<td>Brass plate thickness</td>
<td>1.3 cm</td>
</tr>
<tr>
<td>Length of plastic</td>
<td>2.54 cm</td>
</tr>
<tr>
<td>Volume of brass sample, $V_{Mb}$</td>
<td>$3.5323 \times 10^{-4}$ m$^3$</td>
</tr>
<tr>
<td>Volume of plastic sample, $V_{Mp}$</td>
<td>$1.287 \times 10^{-5}$ m$^3$</td>
</tr>
<tr>
<td>Young’s Modulus for Brass, $Y_b$</td>
<td>100 GPa</td>
</tr>
<tr>
<td>Young’s Modulus for Plastic, $Y_p$</td>
<td>3.1 GPa</td>
</tr>
<tr>
<td>Cross sectional area of brass plate, $A_{Mb}$</td>
<td>$0.0272 m^2$</td>
</tr>
<tr>
<td>Cross sectional area of rexolite sample, $A_{Mp}$</td>
<td>$5.0671 \times 10^{-4}$ m$^2$</td>
</tr>
<tr>
<td>Intermolecular distance between brass molecules, $a_{ab}$</td>
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</tr>
<tr>
<td>Intermolecular distance between plastic molecules, $a_{ap}$</td>
<td>0.73 nm</td>
</tr>
<tr>
<td>Spring constant of brass, $k_b$</td>
<td>$6.62 \times 10^{18}$</td>
</tr>
<tr>
<td>Spring constant of plastic, $k_p$</td>
<td>$2.14 \times 10^{15}$</td>
</tr>
<tr>
<td>Interface displacement in 0.5 ns</td>
<td>0.0209 pm</td>
</tr>
<tr>
<td>Interface displacement in 1 ns</td>
<td>0.0418 pm</td>
</tr>
<tr>
<td>Interface displacement in 10 ns</td>
<td>0.418 pm</td>
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CHAPTER 2

A SIMPLE THEORETICAL STUDY

2.1 Introduction

The phenomenon of interference is introduced and two interferometer setups are studied. Based on the requirements of the experiment, one of the interferometric techniques is chosen. A theoretical study of that interferometer setup with detection scheme is presented in detail. Various noise sources are incorporated in the theory. The theory leads to power requirements of the laser source, detection schemes including placement and resolution, and limitations on the measurables in the plastic under test. Different detection systems are evaluated and the one best suited to the existing experimental conditions is chosen.

2.1.1 The phenomenon of interference

Consider two incoherent light beams originating from either the same source or two different sources, incident on a screen. The incoherent waves have an uncorrelated, temporally and spatially random, phase relationship. Consequently, the resultant time averaged intensity of light measured is the sum of the individual intensities of the waves. When two light beams are highly coherent, their phase relationship is fixed with time over a region in space. A high phase correlation or phase ordering exits. The resultant time averaged intensity of such coherent sources is proportional to the sums of the fields squared. Constructive and destructive interference arises thereby redistributing the
energy content of the resultant signal over the screen yielding a well defined fringe pattern. The fringe pattern is a measure of the phase relationship between the two beams. This important diagnostic is denoted as an interferometer. Interferometers require a highly coherent source hence a narrow line width about the frequency of operation.

2.1.2 Different types of interferometers

There are two classifications of interferometers: the wavefront division interferometer and the amplitude splitting interferometer. Phase splitting techniques capitalize on the spatial coherence properties of the beam. A wave interference pattern results when two or more diffracted portions of the beam interact in a constructive and/or destructive manner. Typically the multiple wavefront deformations are generated by multiple apertures in a wave barrier. Consequently, the intensity of the beam is lessened by the presence of the barrier. Further, a well defined interaction region in the plastic will be difficult to identify. The intent is to sample a small volume of the device under test and follow the evolution of the acoustic wave with time in this small volume. The amplitude splitting technique is well suited for sampling a small volume of the sample over a suitable duration in time. The amplitude splitting technique capitalizes on the temporal coherence properties of the beam. The Michelson and Mach Zender interferometers are based on amplitude splitting. Before selecting an interferometer, the two afore mentioned methods are studied in brief detail individually. Further, a short analysis on the accuracy obtained when employing these methods is provided.
2.1.3 The Michelson interferometer

The Michelson Interferometer [27], [28] as illustrated in Fig. 2.1 works on the principle of amplitude division. The interferometer setup consists of a beamsplitter with normal oriented 45° with respect to the optic axis. The beamsplitter transmits 50% of incident light and reflects the remaining 50%. Each beam propagates to separate mirrors terminating the path length. One mirror is stationary and remains fixed at all times. The other mirror is allowed to be spatially adjusted. In the initial position both the mirrors are placed at equal distances from the beamsplitter. Once the initial beam is incident on the beamsplitter and is split into two parts, both the beams travel equal distances to each of the mirrors and back to the beamsplitter, where they are recombined and directed towards a screen. An interference pattern is formed on the screen. Detectors can be placed on the screen to study the formation and movement of fringes as one mirror is moved.

2.1.4 The Mach Zender interferometer

The Mach Zender Interferometer [27, 28] setup consists of a laser head, two beam splitters and two front surface mirrors. As is seen in Fig. 2.2, the laser beam output from the head is incident on the beam splitter which transmits 50% of the incident beam and reflects the remaining 50%. Both the reflected and transmitted beams are directed towards the mirrors with normal surface vector placed at 45° relative to the incident and reflected beams. The beams reflected off the surface of the mirrors are directed to a second beamsplitter. This beam splitter combines both the beams by transmitting and reflecting one half of each of the incident beams. The resultant beams are then incident on two screens. Assuming that the geometrical path lengths are the same and that the mediums the two beams pass through are identical, the two beams will propagate the
same optical path lengths when they reach one of the two screens always yielding a complete constructive interference pattern. At the second screen, one of the two beams will have propagated through both beam splitters. The other beam was reflected from both beam splitters. Since the beam splitter has a different index of refraction compared to the surrounding medium, a difference in optical path lengths exist. In general, this results in a partial constructive or partial destructive interference pattern. Complete destructive or constructive interference are special cases on this screen.

2.1.5 Interferometer accuracy

Though the Michelson and Mach Zender interferometer methods differ from each other in the way they are setup, the overall principle behind them is the same. Each interferometer consists of splitting an incident beam by means of a beamsplitter and then combining these beams after they have traveled a certain length again by means of a beamsplitter. So in effect, a common expression can be derived, characterizing the behavior of the beam throughout the different stages in both of the interferometric techniques.

In order to better understand the process, an optical circuit diagram as shown in Fig. 2.3, which can be used to represent both the Michelson and Mach Zender interferometric techniques, is presented. As shown in the diagram, the light upon entering the system is split into two beams. For the sake of simplicity, assume that the element responsible for the division is a splitter. Beams $B_1$ and $B_2$ travel the paths $P_{11}$ and $P_{22}$ respectively through the system. They are brought together in a combiner. Considering output 1, it (denoted as wave 3) is equal to the superposition of wave 1 with wave 2. Assume that the
beam is an x-polarized wave and propagates in the z-direction. For interference to occur the beams should have the same direction of propagation in the combiner.

The electric field of beam $B_1$ can be represented by

$$E_1 = E_{01} \sin(kP_{L1} + \omega t)$$

(2.1a)

The electric field of beam $B_2$ can be represented by

$$E_2 = E_{02} \sin(kP_{L2} + \omega t)$$

(2.1b)

The electric field at the output of the combiner is given by the superposition of beams $B_1$ and $B_2$.

$$E_3 = E_1 + E_2$$

(2.2a)

From Eqs. (2.1a) and (2.1b), the resultant electric field $E_3$ can be expressed as

$$E_3 = E_{01} \sin(kP_{L1} + \omega t) + E_{02} \sin(kP_{L2} + \omega t)$$

(2.2b)

A detector is positioned at one of the outputs of the combiner to monitor the light intensity,

$$I(t) \propto E^2(t)$$

(2.3)

Based on the values of the electric field in path 1 and path 2, Eq.(2.3) can be expressed as

$$I(t) \propto \left[E_{01} \sin(kP_{L1} + \omega t) + E_{02} \sin(kP_{L2} + \omega t)\right]^2$$

(2.4a)

Expanding this expression yields,

$$I(t) \propto E_{01}^2 \sin^2(kP_{L1} + \omega t) + 2E_{01}E_{02} \sin(kP_{L1} + \omega t) \sin(kP_{L2} + \omega t) + E_{02}^2 \sin^2(kP_{L2} + \omega t)$$

(2.4b)
Using the relation
\[ 2 \sin a \sin b = \cos(a - b) - \cos(a + b) , \]  
(2.5)

Eq. (2.4b) is rewritten as
\[ I(t) \propto E_{01}^2 \sin^2 \left( kP_{L1} + \omega t \right) + E_{01}E_{02} \cos \left[ k \left( P_{L2} - P_{L1} \right) \right] - E_{01}E_{02} \cos \left[ k \left( P_{L2} + P_{L1} \right) + 2\omega t \right] + E_{02}^2 \sin^2 \left( kP_{L2} + \omega t \right) . \]  
(2.6)

As observed in Eq. (2.6), one of the two terms resulting from interference is independent of time. All remaining terms exhibit time harmonic nature. The time averaged intensity is
\[ I_{\text{ave}} \propto \frac{1}{2} E_{01}^2 + E_{01}E_{02} \cos(k\Delta P_L) + \frac{1}{2} E_{02}^2 \]  
(2.7)

where
\[ \Delta P_L = P_{L2} - P_{L1} . \]

The maximum and minimum time averaged intensities deduced from Eq. (2.7) are respectively
\[ I_{\text{ave max}} \propto \frac{1}{2} E_{01}^2 + E_{01}E_{02} + \frac{1}{2} E_{02}^2 = \frac{1}{2} (E_{01} + E_{02})^2 \]  
(2.8)

\[ I_{\text{ave min}} \propto \frac{1}{2} E_{01}^2 - E_{01}E_{02} + \frac{1}{2} E_{02}^2 = \frac{1}{2} (E_{01} - E_{02})^2 \]  
(2.9)

The light intensity at output 1 is dependent on the path difference \( P_{L2} - P_{L1} \). If both the paths have the same length, both beams 1 and 2 interfere constructively and the maximum light intensity is observed. If the path difference is \( \frac{\lambda}{2} \) then
\[ k \cdot \Delta P_L = \frac{2\pi}{\lambda} \left( \frac{\lambda}{2} \right) = \pi \]  

implying that Eq. (2.7) simplifies to Eq. (2.9) yielding a minimum intensity. If \( E_{01} \) equals \( E_{02} \), then the light intensity is zero. Both the beams interfere destructively. Consider the phase shift condition for destructive interference with a 633 nm HeNe laser beam. Ideally, pure destructive interference occurs for a path difference of \( \frac{\lambda}{2} = 316.5 \text{ nm} \) and odd integer harmonics thereof. Assuming that one can distinguish light intensity ratios down to two decimal places, spatial path differences on the order of 3 nm can be detected. Consequently, both interferometer setups reviewed offer a high degree of accuracy in measuring length.

2.1.6 Choice of interferometer setup

The Michelson interferometer shown in Fig. 2.1 requires the beam to pass through the plastic sample at least twice at two different points in time. Since the plastic sample is dynamically changing in time, the fringe pattern generated is a function of two different states of the sample over time. It is difficult to deconvolve these states to study the dynamic characteristics of the sample.

The sampling beam of the Mach Zender interferometer passes through the sample under test once. The fringe observed is uniquely attributed to a particular state of the sample. Therefore, for investigating dynamic changes in the sample, the Mach Zender interferometer is superior to the Michelson interferometer. Consequently, all remaining interferometry studies pertain to the Mach Zender type.
2.2 Optical interferometry to determine change in index of refraction

The highest resolution of optical interferometry based on the change of the index of refraction due to the compression of an optically transparent plastic under test is sought. The details leading to the compression of the plastic under test are not of importance here. The generation of the reference fringe, the shift in the fringe pattern, and the detector properties are considered in some detail. The objective is to determine the smallest change in pressure (force) applied to the sample (equivalently the smallest change in the index of refraction of the sample) that can be detected by a detector whether it is due to a fractional fringe shift or a complete fringe shift.

A Mach Zender interferometer arrangement is to be used to examine changes in the index of refraction in a Rexolite sample under dynamic stress. To examine small effects directly, two identical pieces are included in Fig. 2.4. The change in the optical path length in path B as a result of the unstressed piece under test is nullified by the second plastic piece in path A. The Mach Zender setup consists of a laser beam that is split with a beam splitter into two equal amplitude beams, one propagating along path A and the second along path B. Path A contains the reference sample. Path B contains the sample under test and a right angle wedge to initiate a phase difference (a spatial phase shift gradient) along the beam cross section in this path. The purpose of the wedge visually and analytically allows for a simple means of generating the initial fringe pattern for the unstressed case. The position of the wedge with respect to the optic axis affects the distance of separation among fringes. Suitably choosing of the distance of separation between the fringes allows for greater sensitivity in detecting fringe movement when a sudden change in optical path length occurs due to mechanical compression of the piece.
under test. In practice a wedge will not be used. A slight phase shift gradient in the beam cross section can be obtained by slightly tilting the mirror or beam splitter resulting in a non-uniform phase delay in beam cross section.

It is anticipated that a pulsed acoustic wave will be generated in the plastic under test [refer to Fig. 2.4 item 5]. This shock wave changes the intermittent spacing among atoms and molecules. It propagates through the material thereby changing the dielectric properties of the material. This in turn changes the refractive properties of the piece resulting in an overall phase shift in the wave in path B relative to the wave in path A. These two waves are combined (superimposed) at the beam splitter [refer to Fig. 2.4 item 6]. If the two beams are out of phase, a shift in fringe pattern will result. This shift provides information on the change in the index of refraction that may be related to the applied pressure. The change in the spatial distribution of the intensity of light after the last beam splitter, or equivalently, the fringe shift is to be determined assuming ideal optic conditions. An expression for the phase shift due to a change in the index of refraction and sample length is obtained. A sample compression in one dimension results in the sample expansion in the remaining dimensions as described by Poisson’s ratio. To study the phase shift, two approaches are taken. In the first approach, the observer follows a constant phase. This provides information on how far the phase shifts and in what direction. In the second approach, the observer is stationary viewing the change in the intensity. The second approach is used as a check for consistency. Finally, detector considerations (e.g., detector size and resolution) are incorporated in the model. The sensitivity of just what can be detected and how the light is to be distributed over the detector is addressed. Two sample calculations are then examined with the objective of
determining minimum change in the index of refraction. The calculations are performed based on the two different frames of reference. It is assumed throughout that the medium under stress is isotropic, the strain properties of the medium are linear, shear effects are neglected, and the light is considered to be a plane wave. The beam diameter is chosen to be small so that a localized portion of the sample can be studied. Consequently, diffraction effects are neglected and optical components are assumed ideal.

The index of refraction, \( n \), is defined as the relative factor by which electromagnetic radiation is slowed down when traveling through a material as compared to free space. In other words it is defined as the ratio of the speed of light through vacuum \( 'c' \) to the speed of the phase of the wave through the material medium or, in other words, the phase velocity \( 'v_p' \) of the wave in that medium. Consequently,

\[
n = \frac{c}{v_p} = \frac{\beta}{\beta_o}
\]

where \( c = 3 \times 10^8 \text{ms}^{-1} \) is the velocity of light in vacuum, \( \beta \) is the wavenumber of the wave in the medium and \( \beta_o \) is the wavenumber in vacuum.

For argumentative purposes, both paths A and B may be assumed to be directed in the positive \( z \) direction. Assuming an \( e^{j\omega t} \) time harmonic form of solution and that the fields are \( x \)-polarized propagating in the \( +z \) direction, the electric field in path A is expressed as

\[
E_{XA}(z) = E_{XOA}e^{-j\beta z} = E_{XOA}e^{-j\beta_o z}
\]  

(2.12a)

and the electric field in path B is expressed as,

\[
E_{XB}(z) = E_{XOB}e^{-j\beta_o z}
\]

(2.12b)

Both beam splitters are assumed to transmit 50% of the incident light and reflect the
remaining 50%. Consequently,

\[ E_{XOA} = E_{XOB} = E_{XD} \]  

(2.12c)

As observed in Fig. 2.4, the geometrical path lengths are equal when the sample under test is unstressed. We have \( l_{A3} = l_{B3} \) and \( l_{A1} + l_{S1} + l_{A2} = l_{B1} + l_{S2} + l_{B2} \). The phase delay \( \phi_a \) encountered by beam A traversing the optical path length from just before the first beam splitter [refer to Fig. 2.4 item 2] to just after the second beam splitter [refer to Fig. 2.4 item 6] (neglecting beam splitter delays) is

\[ \phi_a(x, y) = \beta_o n_o [l_{A1} + l_{A2} + l_{A3}] + \beta_o n_S l_{S1} \]  

(2.13)

As shown in Fig. 2.4, the beam in path B passes through both the right angle wedge and the sample under test. To generate straight line fringes, the back surface of the right angle wedge is located at

\[ z(x) = \frac{L}{h}(x - x_o) \]  

(2.14)

where \( L \) and \( h \) are respectively the length of the base and the height of the wedge and \( x_o \) is the position of the tip of the wedge relative to an x axis lying along the vertical side of the wedge with the \( x=0 \) origin on the z axis coinciding with the beam's axis. Refer to Fig. 2.5. Note that \( x_o \) is a negative number when the tip of the wedge is below the beam's axis. The index of refraction of the wedge is defined as \( n_p \).

The optical path length through the wedge is \( n_p z(x) \). The sample under test experiences a slight change in the index of refraction as given by \( n_{SB} = n_s + \Delta n_s \). When the sample is compressed, it expands in the z direction by length \( \Delta l_s \). The overall phase delay experienced by beam B is

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where \( I_{s2} = I_{s1} + \Delta I_s \). The first term is a phase advance due to the loss of free space resulting from the change in the lateral length (lateral relative to the direction of the pressure force applied to the sample) of the sample being compressed. The second set of terms is the phase delay due to the free space plus wedge length treated as free space in which the beam must traverse between the two beam splitters. The third term is the phase advance contribution of the wedge treated as free space. The fourth term is the phase delay of the ray passing though the wedge proper. The fifth term is the delay contribution of the wave traveling in free space over the remainder of the overall length of the wedge. The sixth term is the phase delay contribution of the sample being compressed.

The field distributed over the surface of a planar screen facing the beam with surface normal in the minus z-direction due to beam A is

\[
E_A(x, y) = E_{x0A} e^{-j\phi_0(x, y)} \hat{x}
\]

Similarly, the field distribution over the screen due to the beam B is given by

\[
E_B(x, y) = E_{x0B} e^{-j\phi(x, y)} \hat{x}
\]

The total field over the screen is,

\[
E_T(x, y) = E_A(x, y) + E_B(x, y)
\]

The time averaged intensity of the light illuminating the screen is proportional to
\[
I_{\text{ave}} \propto \frac{1}{2} \text{Re}\left[ \frac{E_A(x,y) \cdot E_A^*(x,y)}{E_B(x,y) \cdot E_B^*(x,y)} \right] \\
= \frac{1}{2} \left( |E_A(x,y)|^2 + |E_B(x,y)|^2 + |E_A(x,y) \cdot E_B^*(x,y)| + |E_A^*(x,y) \cdot E_B(x,y)| \right) \\
= \frac{1}{2} \left( |E_{XOA}|^2 + |E_{XOB}|^2 + 2 \text{Re}[E_{XOA}E_{XOB}^* e^{-j[\phi_A(x,y) - \phi_B(x,y)]}] \right) \\
= \frac{1}{2} \left( |E_{XOA}|^2 + |E_{XOB}|^2 + 2 \text{Re}[E_{XOA}E_{XOB}^* e^{-j[\phi_A(x,y) - \phi_B(x,y)]}] \right)
\]

Since the amplitudes of the fields in both path lengths are the same, then let
\[E_{XOA} = E_{XOB} = E_{XO}, \text{ yielding}
\]
\[I_{\text{ave}} \propto |E_{XO}|^2 \cos(\phi_A - \phi_B) = |E_{XO}|^2 \{1 + \cos(\phi_B - \phi_A)\} \tag{2.17b}
\]
Noting that \[I_{A_1} = I_{B_1}, I_{A_2} = I_{B_2} \text{ and } I_{A_3} = I_{B_3}, \text{ then with the aid of Eqs. (2.13) and (2.15), the phase difference between the two beams is}
\]
\[
\Delta \phi = \phi_B - \phi_A = \beta_o \left[ (n_p - n_o)z(x) + (n_s - n_o)\Delta l_s + (l_{s1} + \Delta l_s)\Delta n_s \right] \tag{2.18}
\]
where \[z(x) = \frac{L}{h} (x - x_o)\]

From Eq. (2.17b) it is observed that the intensity is a maximum when \(\cos \Delta \phi = 1\) or, consequently, \(\Delta \phi = 2\pi n\) for \(n = 0, \pm 1, 2, \ldots\). With Eq. (2.18), the change in fringe pattern is to be linked to the amount of force or pressure being applied to the sample under test.

2.2.1 Approach 1: Observer follows a constant phase on observation screen

Upon application of a stress, the fringe pattern shifts. The displacement of this shift is determined by following the change in distance of a particular constant phase. A coordinate system is placed over the surface of the screen at \(z\) equal to the position of the front surface of the screen. The \(x=0\) location lies at the intersection of the surface of the screen.
screen and the beam axis of the unstressed case. The change in phase of beam B relative to beam A is chosen to be $\phi_o$ when no pressure is applied to the piece under test. Therefore when there is no pressure applied to the piece under test, $\Delta n_S = 0$ and $\Delta l_s = 0$ (no change in lateral length where lateral is the spatial extent along the path of the unstressed beam). Choosing the beam axis as a reference, $x = 0$, the relative phase difference $\phi_o$ of the beam on axis in the unstressed case as obtained from Eq. (2.18) is

$$-\beta_o \left[ (n_p - n_o) \frac{L}{h} x_o \right] = \phi_o$$  \hspace{1cm} (2.19)

Following the change in position of the fringe from its $x=0$ position on the screen when a change in index of refraction and lateral length results yields

$$\phi_o = \beta_o \left[ (n_p - n_o) \frac{L}{h} (\bar{x} + \Delta x) \right]_{x=0} + (n_S - n_o) \Delta l_s + l_{s1} \Delta n_s + \Delta l_s \Delta n_s$$  \hspace{1cm} (2.20)

where

$$\bar{x} = x - x_o$$  \hspace{1cm} (2.21)

and $x_o$ is the location of the tip of the wedge and $x$ is the lateral position of the observer over the plane containing the detector surface. Taking the difference between Eqs. (2.20) and (2.19) and solving for $\Delta x$ yields the position of the phase $\phi_o$ shifted from its reference position. The shifted distance is given as

$$\Delta x = - \left[ \frac{(n_S - n_o) \Delta l_s + l_{s1} \Delta n_s + \Delta l_s \Delta n_s}{(n_p - n_o) \frac{L}{h}} \right]$$  \hspace{1cm} (2.22)

Assuming the plastic piece under test is isotropic, the change in the index of refraction ($\Delta n_s$) is related to the strain, S. By definition, the strain is the deformation of the stressed material changing the material's length from $l$ to a larger length $l + \Delta l$ where
\( \Delta l > 0 \). The strain is positive for medium dilatation (stretching the medium). The change in the index of refraction is given by [27, 29-31]

\[
\Delta n_s = -\frac{n_s^2}{2} \rho \frac{\Delta l}{l} \quad \text{where } \Delta l > 0
\]

and \( \rho \) is the opto-elastic constant of the medium being compressed. The minus sign is required since a compression gives rise to an increase in the index of refraction. For clarity, Eq. (2.23a) is rewritten in terms of a negative change in length \( \Delta l < 0 \) due to compression forces acting on the medium assuming that the original length of the medium is \( l_T \) as

\[
\Delta n_s = -\frac{n_s^2}{2} \rho \frac{\Delta l}{l_T} = \frac{n_s^2}{2} \rho \frac{\left| \Delta l \right|}{l_T} \quad \text{where } \Delta l < 0
\]

Although Eqs. (2.23a) and (2.23b) are equivalent, the latter relation offers a more transparent justification of signs in later expressions especially since only compression forces are considered. Refer to Fig. 2.6. Consequently, upon substituting Eq. (2.23b) into Eq. (2.22), the strain \( \left( \Delta l / l_T \right) \) produces a shift in the position of the fringe given by

\[
\Delta x = -\frac{n_s^2}{2} \rho \left( \frac{\Delta l}{l_T} \right) \left( l_s + \Delta l_s \right) - \left( n_s - n_o \right) \Delta l_s
\]

Young's modulus \( Y_M \) is the ratio of the force, \( F \), over cross sectional area \( A \) required to stretch the length of the medium from \( l \) to a larger length \( l + \Delta l \) where \( \Delta l > 0 \),

[NOTE: This is the stress.], to the strain \( \frac{\Delta l}{l} \) or equivalently [29]
Young's modulus \( Y_M \) is a positive number and has units of Newtons per square meter or (Pa) Pascals. The force, \( F \), is the outward normal force acting on the surface enclosing the volume of the medium being stretched. Equation (2.25a) may be recast in terms of the change in length, \( \Delta l_c \), resulting from compression due to an applied force, \( F_c \) as

\[
Y_M = \frac{F_c}{A} \left( \frac{\Delta l}{l} \right) = - \frac{P}{\left( \frac{\Delta l_c}{l_T} \right)} > 0
\]

where \( P = \frac{|F_c|}{A} > 0 \)

Because the applied compression force \( F_c \) is in the opposite direction of the outward normal force, its value is negative consistent with the sign of \( \Delta l_c \) such that Young's modulus is positive. The inward normal force per area acting on the medium is commonly denoted in static fluid theory as the pressure, \( P \). With this orientation (inward direction), the pressure \( P \) is a positive value. Expressing the distance displaced by the fringe to the pressure using Eqs. (2.24) and (2.25b) gives

\[
\Delta x = \frac{n_s^3 \rho}{2 Y_M} \left( -\frac{P}{Y_M} \right) (l_{S1} + \Delta l_S) - (n_s - n_o) \Delta l_S
\]

Assuming a plane wave like approximation, one should expect an infinite number of fringe patterns with spatial periodicity over the surface of an infinite in extent, two-
dimensional screen in the $z$ equal constant plane. Define the spatial distance of periodicity of the field intensity in the $x$-direction as $\lambda_x$. In other words, $\lambda_x$ is the spatial intensity wavelength in the $x$ direction. [Note: The wavelength of the beam is carefully distinguished from the spatial intensity wavelength. Since the beam propagates only in the $z$ direction, the fields do not have a wavelength component in the $x$ direction. The spatial intensity wavelength provides spatial periodicity of the distribution of power over the screen in the $x$ direction.] Normalizing Eq. (2.26a) with respect to $\lambda_x$ and neglecting very small terms $\Delta l_s \Delta n_S$, Eq. (2.26a) may be expressed as

$$\frac{\Delta x}{\lambda_x} = -\frac{\left(\frac{l_{S1}}{\lambda_x}\right)}{(n_p - n_o)\frac{L}{h}} \left[ \frac{n_s^3}{2} \rho \frac{P}{Y_M} \right] - \frac{(n_s - n_o) \Delta l_s}{(n_p - n_o)\frac{L}{h} \lambda_x}$$

(2.26b)

For simplicity, the contribution to the change in the lateral length of the sample will be neglected. Let the change in the sample length be zero, $\Delta l_s = 0$. Equation (2.26b) is approximated as

$$\frac{\Delta x}{\lambda_x} \approx -\frac{\left(\frac{l_{S1}}{\lambda_x}\right)}{(n_p - n_o)\frac{L}{h}} \left[ \frac{n_s^3}{2} \rho \frac{P}{Y_M} \right]$$

(2.26c)

It is observed that the fringe shift is linearly proportional to the pressure assuming that the index of refraction can be approximated as a constant. Assuming that the medium is elastic, Eq. (2.26c) may be used to determine an approximate value for the opto-elastic constant of the medium at least in the limit when the changes in the shift due to changes in the pressure are constant.
The spatial intensity wavelength over the surface of a screen in the x direction (the distance between fringes on a screen) may be expressed in terms of the laser wavelength. Consider the uncompressed situation with relative phase difference described by Eq. (2.19). The observer now moves a distance $\Delta x$ along the x-direction from the origin until the phase $\phi_o$ is measured. For clarity, $x_o$ in Eq. (2.21) is the position of the tip of the wedge dielectric. The distance displaced is equivalent to the intensity wavelength in the x direction, $\Delta \tilde{x} = \lambda_s$. Consequently, a $2\pi$ change in phase has resulted. In other words, the $2\pi$ phase change between two consecutive, equivalent, phase equal constant fringes on the screen is equivalent to a spatial period on the screen. The phase between two consecutive, equivalent, phase equal constant fringes is determined as

$$\beta_o \left[ (n_p - n_o) \frac{L}{h} \Delta \tilde{x} \right] = \beta_o \left[ (n_p - n_o) \frac{L}{h} \lambda_s \right] = 2\pi$$

(2.27)

The phase coefficient in free space is $\beta_o = \frac{2\pi}{\lambda}$ where $\lambda$ is the free space wavelength.

With the aid of Eq. (2.27), the spatial period on the screen can be related to the free space wavelength of the laser as

$$\lambda_s = \frac{\lambda}{(n_p - n_o) \frac{L}{h}}$$

(2.28)

Upon substituting Eq. (2.28) into the right hand side of Eq. (2.26b) and (2.26c) yields respectively,

$$\frac{\Delta x}{\lambda_s} = -\left( \frac{l_{sl}}{\lambda} \right) \left[ \frac{n_s^3}{2} \frac{P}{\rho} \frac{P}{Y_M} \right] - (n_s - n_o) \frac{\Delta I_s}{\lambda_s}$$

(2.29a)
\[
\frac{\Delta x}{\lambda_x} = \left(\frac{I_{sl}}{\Delta}\right) \left[ \frac{n_s^3}{2} \rho \frac{P}{Y_m} \right]
\]  

(2.29b)

Normalized in this manner, Eqs. (2.29a) and (2.29b) represent the fraction of a distance moved by a fringe from some reference position relative to the spatial period along the x direction on the screen based on the free space wavelength of the laser beam. These results apply only if the change in the index of refraction does not significantly refract the beam.

2.2.2 Verification study: stationary observer case- phase changes

This approach is to verify the correctness of the above relations and therefore will not be performed in detail. In this case, the observer is stationary on the screen and the change in the phase of the wave is measured. The phase difference between the two interfering beams is given by Eq. (2.18). For simplicity, the change in the lateral length of the sides of the plastic under test when a pressure is applied will be neglected. The observer is chosen to make measurements at x=0. Using Eq. (2.18), define the change in phase due only to the presence of the wedge assuming both plastic samples have the same geometry and index of refraction as

\[
\Delta \phi = -\beta_o \left[ (n_p - n_o) \frac{L}{h} x_o \right] \equiv \phi_u
\]  

(2.30a)

Now assume that there is a change in the index of refraction. The change in phase is

\[
\Delta \phi = \beta_o \left[ -(n_p - n_o) \frac{L}{h} x_o + l_{sl} \Delta n_s \right] \equiv \phi_c
\]  

(2.30b)

Neglecting small increases in the sample's width due to compression forces along it's length, the difference between the phases given by Eq. (2.30a) and (2.30b) is
\[ \varphi_c - \varphi_u = \beta_o l s_1 \Delta n_s \]  

(2.30c)

The change in the index of refraction is related to the pressure applied as given by Eqs. (2.23b) and (2.25b). Applying to Eq. (2.30c) yields

\[ \varphi_c - \varphi_u = \beta_o l s_1 \frac{n_s^3}{2} \rho \frac{P}{Y_M} \]  

(2.30d)

Effectively, Eqs. (2.29b) and (2.30d) must be equivalent. This can be seen if Eq. (2.29b) is multiplied by 2\pi yielding

\[ \frac{2\pi \Delta x}{\lambda_s} = \beta_s \Delta x = -(\varphi_c - \varphi_u) \]  

(2.31)

The minus sign in Eq. (2.31) cannot be argued without considering the change in the phase about a change in the observer's position. This may require examining the spatial change in phase for both the unstressed and stressed conditions. This detail of complexity is not of interest and hence the sign is implemented to force agreement with the technique of following the fringe shift. Consistency in expressions has been shown between the two techniques.

### 2.3 Wedge consideration

It is desired to have minimal refraction along the undisturbed beam trajectory due to the presence of the wedge. This will allow beams A and B to overlap at the final beam splitter. To this end, a constraint is developed to determine the shift of the beam relative to the desired undisturbed trajectory of the beam as a consequence of wedge geometry, optical properties, and the placement of the observation screen. The medium outside of
the wedge (medium 1) has an index of refraction $n_o$. The wedge (medium 2) has an index of refraction $n_p$. Based on Fig. 2.7, Snell’s law of refraction dictates that

$$\sin \theta_1 = \frac{n_p}{n_o} \sin \theta_2$$  \hspace{1cm} (2.32)

The geometry in Fig. 2.7 indicates that $\theta_1 = \theta_2 + \Delta \theta$. Consequently,

$$\sin \theta_1 = \sin(\theta_2 + \Delta \theta) = \sin \theta_2 \cos \Delta \theta + \sin \Delta \theta \cos \theta_2 = \frac{n_p}{n_o} \sin \theta_2$$  \hspace{1cm} (2.33)

The angle $\Delta \theta$ is required to be small, implying that the deviation of the refracted beam from its original trajectory in the absence of the wedge is small. Hence $\cos \Delta \theta \approx 1$ and $\sin \Delta \theta \approx \Delta \theta$. From the geometry, $\tan \theta_2 = L/h$. Consequently, Eq. (2.33) simplifies to

$$\Delta \theta = \frac{1}{n_o} (n_p - n_o) \frac{L}{h}$$  \hspace{1cm} (2.34)

As shown in Fig. 2.4, the distance from the backside of the wedge to the second beam splitter is $\Gamma$. Let $H$ represent the length of the refracted beam trajectory from the backside of the wedge to the beam splitter. Then, using a Taylor expansion, $H$ may be expressed as

$$H = \Gamma \left[ \cos \Delta \theta \right]^{-1} = \Gamma \left[ \sec \Delta \theta \right] = \Gamma \left[ 1 + \frac{\Delta \theta^2}{2} \right]$$  \hspace{1cm} (2.35)

The overall length of the refracted beam from the wedge to the beam splitter is now constrained such that

$$H \leq \Gamma + \frac{\lambda}{10}$$  \hspace{1cm} (2.36)
Under this constraint, the angle of deviation, $\Delta \theta$, may be determined using Eqs. (2.35) and (2.36) as

$$\Delta \theta \leq \frac{\lambda}{\sqrt{5\Gamma}} \quad (2.37)$$

Combining Eqs. (2.34) and (2.37) yields a constraint on the wedge geometry, wavelength of the beam, wedge index of refraction and position of the second beam splitter as

$$\frac{1}{n_o} (n_p - n_o) \frac{L}{h} = \Delta \theta \leq \frac{\lambda}{\sqrt{5\Gamma}} \quad (2.38)$$

Using Eqs. (2.34) and (2.28), the relationship between the laser beam wavelength and the spatial separation between consecutive fringes of equal phase can be expressed as

$$\lambda_x = \frac{\lambda}{n_o \Delta \theta} \quad (2.39a)$$

As $\Delta \theta$ approaches zero implying $n_p$ approaches $n_o$, the $x$-spatial wavelength approaches infinity as expected, a fringe pattern will not exist. Combining Eqs. (2.38) and (2.39a) yields a useful expression on the limit of the spatial wavelength of the power intensity over a screen located just after the beam splitter

$$\frac{1}{\lambda_x} \leq n_o \sqrt{\frac{1}{5\Gamma \lambda}} \quad (2.39b)$$

When setting up the experiment, it is rather easy to approximately measure or deduce the fringe pattern's spatial period over a screen just behind the beam splitter. Knowing the distance between the beam splitter and the wedge, one can determine if the angle of deviation satisfies the desired refractive constraint placed on the laser beam.
2.4 Detector considerations

The size of the detector in relation to a fringe is addressed in order for the detector to distinguish one fringe from another as the fringe pattern shifts. Assume that the detector pixel diameter is \( D \). The terminology pixel is being used since the detector assembly will be an array of detectors. In every spatial intensity wavelength along the surface of the screen, the intensity of the field reaches a maximum and a minimum. For the detector to be small enough to discriminate the difference between a minimum and a maximum and resolve various states of the signal between these extremes, an engineering approximation is used on the detector size. The detector diameter should be \textit{at least} an order of magnitude smaller than a wavelength of the intensity in order for the detector to resolve different states of the fringe. This implies that the light received by the detector is averaged over one tenth of the intensity range. Consequently,

\[
\frac{\lambda}{10} > D \tag{2.40a}
\]

or equivalently using Eqs. (2.38) and (2.39a) the spatial wavelength term can be rewritten yielding

\[
\frac{\lambda}{10(n_p - n_a) \frac{L}{h}} > D \tag{2.40b}
\]

This implies that there is a constraint on how small the spatial wavelength of the light intensity is over the screen. It is noted that the smaller spatial wavelengths provide a sharper view of the fringe pattern. With imaging software, the shift in the fringe can be detected. A lens may be used to expand the beam if necessary. A lens of this type
decreases the power density on the detector. One must also consider the detector's incident light intensity requirements (power and response time) over the detector area.

2.4.1 Optimize interferometric study with detector contribution

The surface charges on the brass electrodes that produce the electric field between the electrodes exert a force of attraction between the electrodes. This force acts to compress the insulator. The resultant strain and associated change in the refractive index, \( \Delta n_x \), are to be determined within the limits of the detector. As previously stated, an interferometer splits the light from a single source into two spatially separated beams (A and B) which are recombined at a later point in time. The value of the interferometric technique lies in its ability to resolve phase differences due to changes in both the path length and the index of refraction of the sample under test.

Consider the fringe pattern of the unstressed sample piece. In general, the scalar time averaged power flux density over the plane containing the detector surface, \( P \), is given by

\[
P = \frac{\left| \mathbf{E}_{\tau} \right|^2}{2\eta_o}
\]

(2.41)

where \( \eta_o \) is the intrinsic impedance of free space and \( \mathbf{E}_{\tau}(x, y, z = z_d) \) is the total vector electric field distribution over the detector surface at \( z = z_d \). It is assumed that the surface unit vector of the detector is anti-parallel to the flow of power. With the aid of Eqs. (2.17b) and (2.30a), the power flux density illuminating the plane containing the detector surface due to the unstressed sample is

\[
P_{ind\;u} = \frac{|E_{\infty}|^2}{2\eta_o} \left[ 1 + \cos \left( \beta_0 (n_p - n_e) \left( \frac{L}{h} \right) \right) \right]
\]

(2.42)
where \( \tilde{x} = x - x_o \) and \( E_{xo} \) is the amplitude of the electric field of either beams prior to the detector surface after being combined. Note, subscript “ind u” implies incident upon diode for unstressed case. Recall, \( x_o \) and \( x \) are respectively the location of the tip of the wedge and the lateral position of the observer over the plane containing the detector surface. It is assumed that the detector plane, the beam splitter plane, the test piece plane, and the back side of the wedge share the same \( x, y \) coordinate system translated to appropriate \( z \) equal to constant planes. The central position of the detector has not been chosen and the geometry of the detector surface has not been stated. The time averaged input power of the laser driving the system is given by \( P_{im} = (0.5|E_{im}|^2/\eta_o) \) \( A_{LB} \) where \( A_{LB} \) is the cross-sectional area of the laser beam. Assume that the laser beam is expanded to have a cross-sectional area \( A_{MB} \) prior to the first beam splitter. The beam splitters employed are 50% transmitting and 50% reflecting. Just after the second beam splitter, half of the power in beam A and half of the power in beam B is directed towards the diode surface. Consequently, the power density illuminating the surface of the detector may be expressed in terms of the laser power as

\[
\rho_{ind u} = \frac{P_{im}}{2A_{MB}} \left( 1 + \cos \left[ \beta_o \left( n_p - n_o \left( \frac{L}{h} \right) \tilde{x} \right) \right] \right)
\]  

(2.43)

The first term in parenthesis represents the contribution due to the sum of the powers. As expected, if the powers add as in the case of incoherent sources, then by power conservation assuming no loss the total power available to the detector is only half of the power of the light source. The second term in parenthesis is the contribution due to interference. If complete constructive interference results [That is, the two beams in each
part of the interferometer are coherent and in phase with each other, the total power available to the diode is equivalent to the input power of the laser.

A fractional shift of the fringe is to be detected based on the change of light intensity over the detector surface with time. Assume that the detector is centered at $x=0$. With the aid of Eq. (2.43), the power flux density due to the unstressed sample at the center of the detector is

$$\varphi_{ind,x=0} \left( \beta_0 \left( n_p - n_o \right) \frac{L}{h} \bar{x}_{x=0} \right) = \varphi_u = \frac{P_{in}}{2 A_{MB}} \left( 1 + \cos \left[ \beta_0 \left( n_p - n_o \right) \frac{L}{h} \bar{x}_{x=0} \right] \right)$$

(2.44a)

where

$$\left[ \beta_0 \left( n_p - n_o \right) \left( \frac{L}{h} \bar{x}_{x=0} \right) \right] = \varphi_u = \left[ \beta_0 \left( n_p - n_o \right) \frac{L}{h} x_o \right]$$

(2.44b)

or equivalently

$$\left( \frac{L}{h} \bar{x}_{x=0} \right) = \frac{\varphi_u}{\beta_0 \left( n_p - n_o \right)}$$

(2.44c)

The wedge geometry is dictated by both the refraction (angle of transmission) of the beam and the overall increase in the optical path length. In order for both beams A and B to give rise to a fringe pattern, the two beams must overlap each other. Therefore, Eq. (2.38) constrains the overall dimensions of the wedge. The location of the tip of the wedge is a free parameter. Therefore, satisfying the constraints given by Eqs. (2.38) and (2.44c), knowing the power density desired over the detector surface at $x=0$ for the unstressed condition, the tip of the wedge must be located at
\[ \tilde{x}_{x=0} = -x_o = \frac{\varphi_u}{\beta_o (n_p - n_o)} \left( \frac{h}{L} \right) = \frac{\varphi_u}{n_o \beta_o} \sqrt{\frac{5T}{\lambda}} \]  

(2.45)

where from Eq. (2.44a),

\[ \cos \varphi_u = \frac{2P_{ind} u A_{MB}}{P_{in}} - 1 \]  

(2.46)

for a given value of \( P_{ind} u \). Therefore, the power flux density at any point on the detector (diode) surface is

\[ P_{ind} u(x) = \frac{P_{in}}{2A_{MB}} \left( 1 + \cos \left[ \beta_o (n_p - n_o) \left( \frac{L}{h} (x - x_o) \right) \right] \right) \]  

(2.47)

The detector is assumed to be rectangular in shape with length \( L_D \) (along the \( x \) direction) and width \( W_D \). The total input diode power for the unstressed plastic under test is, upon integrating over the surface of the detector (diode) with center located at \( x=0 \),

\[ P_{ind} u = \int_{-L_D/2}^{L_D/2} \int_{-W_D/2}^{W_D/2} P_{ind} u dS \]  

(2.48a)

\[ = \frac{A_D}{2A_{MB} P_{in}} \left\{ 1 + \frac{2 \cos[\varphi_u] \sin \left[ \beta_o (n_p - n_o) \frac{L_D}{h} \right] \left[ \frac{L}{h} \left( x_o - \frac{L_D}{2} \right) \right]}{L_D \beta_o (n_p - n_o) \frac{L}{h}} \right\} \]  

(2.48b)

\[ = \frac{A_D}{2A_{MB} P_{in}} \left\{ 1 + \frac{2 \cos[\varphi_u] \sin \left[ \beta_o (n_p - n_o) \frac{L_D}{h} \right]}{L_D \beta_o (n_p - n_o) \frac{L}{h}} \right\} \]  

(2.48c)
where, $A_D = \frac{W_D L_D}{2}$ is the detector surface area. By analogy to the unstressed power density relation, using Eqs. (2.48b), (2.30a), and (2.18) and neglecting very small terms, the input power density over the plane containing the detector when the sample under test is stressed is given as

$$P_{ind,s} = \frac{P_m}{2A_{MB}} \left[ 1 + \cos \left( \beta_0 \left( n_p - n_o \left( \frac{L}{h} \right) x + \Delta n_s l_s + I_s l_s n_s \right) + \varphi_s \right) \right] (2.49)$$

where $x_o$ and $\varphi_s$ are given by Eqs(2.45) and (2.46) respectively.

For tractability in procedure and solution, the following definitions are made

$$n_p - n_o = \Delta n_{po} \quad (2.50a)$$

$$\psi_1 = -\beta_o \left( n_p - n_o \right) \frac{L}{h} x_o = \varphi_s \geq 0 \quad \text{since} \quad x_o < 0 \quad (2.50b)$$

$$\psi_2 = \beta_o \left( n_p - n_o \right) \frac{L}{h} \frac{L_D}{2} \quad (2.50c)$$

$$\psi_3 = \psi_1 + \Delta \psi \quad (2.50d)$$

$$\Delta \psi = \beta_o \left( n_s - n_o \right) \Delta l_s + \beta_o l_s \Delta n_s \quad (2.50e)$$

The total input diode power resulting from the stressed plastic under test is

$$P_{ind,s} = \int_{-L_D/2}^{L_D/2} \int_{-W_D/2}^{W_D/2} P_{ind,s} dS \quad (2.51a)$$

$$= \frac{A_D}{2A_{MB}} P_m \left[ 1 + \frac{1}{2\psi_2} \left( \sin \left( \beta_o \Delta n_{po} \frac{L}{h} \left( \frac{L_D}{2} - x_o \right) + \Delta \psi \right) \right) \right.$$

$$- \sin \left( \beta_o \Delta n_{po} \frac{L}{h} \left( -\frac{L_D}{2} - x_o \right) + \Delta \psi \right) \left] \right) \quad (2.51b)$$
\[ P_{\text{ind}, u} = \frac{A_D P_{\text{in}}}{2 A_M} \left\{ 1 + \frac{1}{\psi_2} \cos[\psi_1 + \Delta \psi] \sin[\psi_2] \right\} \]

For uniformity in notation and clarity, the power incident on the diode in the unstressed case is

\[ P_{\text{ind}, u} = \frac{A_D P_{\text{in}}}{2 A_M} \left\{ 1 + \frac{1}{\psi_2} \cos[\psi_1] \sin[\psi_2] \right\} \] (2.51c)

It is the change in the power collected by the diode that is to be detected. Therefore, the change in power between the stressed and unstressed cases is given by

\[ \Delta P_{\text{ind}} = P_{\text{ind}, s} - P_{\text{ind}, u} \]

\[ = \frac{A_D P_{\text{in}}}{2 A_M} \left\{ 1 + \frac{1}{\psi_2} \cos[\psi_1 + \Delta \psi] \sin[\psi_2] \right\} - \frac{A_D P_{\text{in}}}{2 A_M} \left\{ 1 + \frac{1}{\psi_2} \cos[\psi_1] \sin[\psi_2] \right\} \]

\[ = \frac{A_D P_{\text{in}}}{2 A_M} \frac{\sin(\psi_2)}{\psi_2} \left[ \cos[\psi_1 + \Delta \psi] - \cos[\psi_1] \right] \]

under the constraint that the wedge dimensions \((L/h)\cdot x_o\) is fixed by Eqs. (2.44c) and (2.45). Using the Taylor expansion retaining first order terms,

\[ \cos(\psi_1 + \Delta \psi) \approx \cos(\psi_1) - \Delta \psi \sin(\psi_1) - \frac{(\Delta \psi)^2}{2!} \cos(\psi_1). \] (2.53)

Equation (2.52) simplifies to

\[ \Delta P_{\text{ind}} = -\frac{A_D P_{\text{in}}}{2 A_M} \sin(\psi_2) \left[ \Delta \psi \sin(\psi_1) + \frac{(\Delta \psi)^2}{2} \cos(\psi_1) \right] \] (2.54a)

\[ \Delta P_{\text{ind}} = -\frac{A_D P_{\text{in}} (\sin(\psi_1)\sin(\psi_2))}{2 A_M} \Delta \psi - \frac{A_D P_{\text{in}} (\cos(\psi_1)\sin(\psi_2))}{4 A_M} (\Delta \psi)^2 \] (2.54b)

The measure of the elasticity of the material under test is equal to the ratio of the lateral strain to the longitudinal strain when the material is subjected to tensile stress.
This is commonly known as Poisson’s ratio. Poisson’s ratio is a positive number. Under a tensile force, the longitudinal strain is positive and the lateral strain is negative. Under a compression force, the reverse is true. Therefore, referring to Fig. 2.6 for a compression force, the change in the lateral length of the sample, \( \Delta l_s \) (where \( \Delta l_s > 0 \) for a compression force) may be related to the change in the compression (longitudinal) length of the sample, \( \Delta l_c \) (where \( \Delta l_c < 0 \) for a compression force) as

\[
\sigma_p = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}} = \frac{\Delta l_s / l_{s2}}{\Delta l_c / l_T}
\]

(2.55)

where \( l_T \) is the original length of the sample before being compressed and \( l_{s2} \) is the original length along the beam axis. This, in turn, can be related to the change in the index of refraction using Eq. (2.23b). For simplicity, the change in the lateral length of the sample will be neglected implying that \( \Delta l_s \) is forced to zero. With this approximation and Eqs. (2.54b), (2.50b), (2.50c) and (2.23b), the change in the index of refraction \([\Delta \psi \approx \beta_0 I_{s1} \Delta n_s]\) can be obtained by solving for \( \Delta \psi \) in Eq. (2.54b) yielding

\[
\Delta \psi = \left(\tan \psi_1\right) \left[ -1 \pm \left( 1 - 4 \frac{\cos \psi_1}{\sin^2 \psi_1} \frac{\Delta P_{\text{ind}}}{A_{MB} \frac{A_{p}}{P_{in}}} \frac{n_2}{\sin \psi_2} \right)^{1/2} \right]
\]

(2.56)

\[
\approx \beta_0 I_{s1} \Delta n_s = \beta_0 I_{s1} (n_{ss} - n_{su}) = -\beta_0 I_{s1} \frac{n_s^3}{2} \rho \frac{\Delta l_c}{l_T}
\]

(2.57)

where \( n_{ss} \) and \( n_{su} \) respectively are the indices of refraction of the sample (s) when stressed and unstressed and \( \Delta P_{\text{ind}} = P_{\text{ind,s}} - P_{\text{ind,u}} \). The upper sign in front of the square root in Eq. (2.56) is chosen in order for the problem to yield physically realizable results.
That is, when no change in power is experienced, \( \Delta P_{\text{ind}} \) is zero implying that there is no change in the index of refraction yielding \( \Delta \psi = 0 \).

If \( \phi_u = \phi_1 \rightarrow 0 \), then \( \tan(\phi_1) \) approaches 0 and \( \cot(\phi_1) \) approaches infinity. Equation (2.56) is rearranged into the form that the limit is transparent as indicated by

\[
\Delta \psi = \beta_o l_{s1} (n_{ss} - n_{su}) \\
= \lim_{\phi_1 \to 0} \left[ -\tan(\phi_1) + \sqrt{\tan^2(\phi_1) - \frac{\Delta P_{\text{ind}} A_M}{P_0 \cos(\phi_1) \sin[\phi_2]} \frac{4\psi_2}{A_D \cos(\phi_1) \sin[\phi_2]}} \right] \\
= \sqrt{\frac{\Delta P_{\text{ind}} A_M}{P_0 A_D \sin[\phi_2]} \frac{4\psi_2}{\cos(\phi_1) \sin[\phi_2]}} = -\beta_o l_{s1} \frac{n_s^2}{2} \rho \frac{\Delta l_c}{l_r}
\]

which is consistent with the case when the coefficient of the \( \Delta \psi \) term vanishes. When \( \psi_1 \) equals zero, the undisturbed beams in each branch of the interferometer are in phase. This implies that the undisturbed input power to the diode is at a maximum. Therefore, when the sample under test is stressed, the input power to the diode decreases from its maximum value implying that \( \Delta P_{\text{ind}} < 0 \). As a compression force is applied, \( \Delta l_c \) decreases (increases in the negative sense) implying that the expression on the right hand side of Eq. (2.58) is positive. Further, the index of refraction increases therefore \( \Delta \psi > 0 \). All equalities of Eq. (2.58) are in agreement.

In the limit that \( \psi_1 \to \pi/2 \), \( \tan(\psi_1) \) approaches infinity and \( \cot(\psi_1) \) approaches 0. Hence, Eq. (2.56) may be placed in a form such that a 0/0 limiting condition can be made. Using a binomial expansion on the square root term, Eq. (2.56) can be recast into the form
\[ \Delta \psi = \beta c J_{Si} (n_{ss} - n_{sn}) \]

\[ \approx - \lim_{\psi_1 \to \frac{\pi}{2}} \cot(\psi_1) \left[ 1 - \left( 1 - 2 \frac{\Delta P_{ind} A_{MB}}{P_m A_D} \sin^2 \psi_1 \right) \right] \]

\[ = \frac{-2 \Delta P_{ind} A_{MB}}{P_m A_D} \frac{\psi_2}{\sin \psi_2} = -\beta c J_{Si} \frac{n^2}{2} \frac{\Delta l}{l_T} \]

which is consistent with the case when the coefficient of the \((\Delta \psi)^2\) term vanishes. When \(\psi_1 = \pi/2\), the undisturbed beams in each branch of the interferometer are out of phase by 90 degrees relative to each other. They are said to be in phase quadrature. This implies that the intensity of light over the surface of the diode is approaching a mid-value from the maximum. Consequently, applying a small pressure to the sample will decrease the power of the laser delivered to the diode, decreasing \(\Delta l/c\) [increasing in the negative sense] while increasing \(\Delta \psi\). Refraction effects can be estimated using Eqs. (2.38) and (2.39a) based on the experimental setup.

### 2.5 Noise sources

Before studying signal detection with a resistor at the load for both the DC and AC cases, the noise sources that exist in these cases are examined. The noise power needs to be determined with the resolution of the measuring instrumentation kept in mind. For this simple calculation, a simple resistor will be used as a load. Both the change in voltage and ratio of this change relative to its bias condition are to be examined in the presence of noise. Further, signal-to-noise ratios (SNRs) are examined.

The two measures for noise that will be used in determining the noise power are Shot Noise (Schottky Noise) and Thermal Noise (also denoted as Johnson Noise or Nyquist Noise). Shot noise is caused by the quantized and random nature of current flow.
Current is not continuous but quantized limited by the smallest unit of electron charge. For a particular power input plus bias effect (dark current), the average current generated does not indicate what the variation in the current is or what frequencies are involved in the random variations of the current. The mean-squared shot noise current variations in a PIN diode are assumed to be uniformly distributed in frequency and have a spectral current density of

$$\langle i_{\text{sn}}^2(t) \rangle = i_{\text{sn}}^2(f) = 2eIB$$

(2.60)

where $e$ is the electron charge, $I$ is the DC current, and $B$ is the bandwidth of the signal in Hertz.

There are two sources of shot noise current, the shot noise due to dark current, $i_{\text{sd}}$, and the shot noise due to current generated by the laser signal, $i_{\text{sl}}$. In general, the modulation imposed on the laser light wave is not DC in nature. Consequently, the current due to the modulation of the laser signal may be within the bandpass of the detector and varies with time. As a result, it will be assumed that the averaging process used to obtain the shot current is over the fast time scales (high frequencies) of the random nature of current flow. The change in current resulting from the change in the laser intensity (modulation of the laser light) is assumed to be on a much slower time scale. Therefore, the DC current in the shot noise definition is now replaced by the time varying current resulting from changes in the laser intensity. Using Eq. (2.60) and omitting the averaging bar notation, these mean-squared shot noise currents are defined as

$$i_{\text{sdRMS}}^2 = 2ei_dB$$

(2.61)

$$i_{\text{slRMS}}^2(t) = 2ei_lB = 2eBP_{\text{sl}}(t)R$$

(2.62)
where, $R$ is the responsivity of the diode [A/W], $i_d$ is the dark current due to the diode bias, $i_R$ is the current generated by the PIN diode due to the laser beam incident on the diode and $P_{inc}$ is the incident power on the diode surface.

The thermal noise (Johnson noise or Nyquist noise) is caused by the vibrational kinetic energy of conduction electrons and holes at a finite temperature. If some particles are charged, vibrational kinetic energy can be coupled electrically to another device if a suitable transmission path is provided. The noise level of an electronic system is measured with respect to the theoretical thermal noise level. The thermal noise power at a certain temperature $T$ in a bandwidth $B$ is expressed by

$$P_{thn} = kTB$$

(2.63)

where $k$ is the Boltzmann constant, $T$ is the absolute temperature in degrees Kelvin, and $B$ is the bandwidth in Hz. The mean-squared value of the noise voltage and current generated by a load resistor is given by [24, 25]

$$\langle v_{thn}^2(t) \rangle = \overline{v_{thn}^2(f)} = 4kTBRL$$

and

$$\langle i_{thn}^2(t) \rangle = \overline{i_{thn}^2(f)} = \frac{4kTB}{RL}$$

(2.64)

When two signals are uncorrelated such as two noise signals or a noise signal and a real signal, the squared amplitudes of the signals add. Note that the RMS amplitudes do not add. The noise voltage and current signals are typically expressed as RMS values.

The simple circuit model of a photodiode with load resistor and noise sources is provided in Fig. 2.9a. The dark current $i_d$ and the current resulting from photon-to-electrical conversion in the diode $i_i$ are correlated. Therefore their amplitudes add. The shot current and thermal noise current are uncorrelated. Therefore their powers add. Figure 2.9b displays the equivalent circuit. Again, because the noise and signal are uncorrelated,
the circuit can be simplified further as shown in Fig. 2.9c. The total current in this circuit may be determined as
\[
i_T(S&N) = \left( i_i + i_d \right)^2 + i_n^2 \text{ where } i_n^2 = i_{iRMS}^2(t) + i_{dRMS}^2 + i_{nRMS}^2
\]
(2.65)
or, equivalently,
\[
i_T(S&N)(t) = \left[ (\mathcal{R}P_{in}(t) + i_d)^2 + 2eB\mathcal{R}P_{in}(t) + 2ei_dB + \frac{4kTB}{R_L} \right]^{1/2}
\]
(2.66)
This relation will be employed in the study of signal detection.

2.6 Signal detection

The compression of the Rexolite sample is recorded by the interferometer through the movement of the fringes. To capture this information the output of the interferometer is directed towards a photodetector. To increase the detection efficiency, the photodetector can be coupled with devices like resistors or amplifiers or employ schemes like heterodyne or homodyne detection. In this section various detection schemes will be examined and their performances will be compared. There are eight signal detection schemes that are going to be investigated.

2.6.1 With resistance at load (dc case)

For the first case, the photodetector has a resistor at load. Taking the various noise sources that were examined previously into consideration, the performance of the detector in this arrangement is analyzed.

In Figs. 2.9a-c, the current generated by the photodetector and the dark current are shown as direct current sources \(i_i\) and \(i_d\) respectively. The different kinds of noise present in the circuit such as thermal noise and the shot noise current are also represented as current sources. In the first stage of circuit simplification, the linear (DC) and the noise
sources are combined separately. As explained above, the noise powers add therefore the
total noise current is the square root of the sum of the noise currents squared. The final
resultant current is the square root of the sums of the total currents squared. A
capacitance, C, is added to the circuit model of the photodiode to characterize the diode
junction capacitance. If the modulation frequency of the laser light is low enough, this
capacitance allows the transfer of energy from the photodiode to the load circuit.
Consequently, the photodiode acts like a low pass filter. The load resistance, R_L, is
assumed to be much larger than the shunt resistance, R_d, of the diode and hence is not
displayed in the noise models of Figs. 2.9a-c. Using Eq. (2.66), the voltage measured at
the load resistor due to both noise and signal is

\[ v_L(t) = R_L \left[ (R_i P_{ind}(t) + i_d) + 2eB \Re P_{ind}(t) + 2e B_i B + \frac{4kTB}{R_L} \right]^{1/2} \] 

(2.67)

Assume that the transition from P_{ind u} to P_{ind s} is modeled as a linear ramping function
starting at P_{ind u} at time t=0 to P_{ind s} at time t=T_0. For t<T_0, P_{ind} = P_{ind u}. For t>T_0,
P_{ind} = P_{ind s}. Therefore, the time varying input power on the diode may be expressed as

\[ P_{ind}(t) = \left[ \frac{P_{ind s} - P_{ind u}}{T_o} + P_{ind u} \right] \left[ U(t) - U(t-T_o) \right] + P_{ind u} U(t-T_o) \]

(2.68a)

\[ = \left[ \frac{\Delta P_{ind}}{T_o} \right] \left[ U(t) - U(t-T_o) \right] + P_{ind u} U(t-T_o) \]

\[ = P_{ind DC} + P_{ind AC}(t) \]

where
Here, \( U(t) \) is the unit step function as defined in Eq. (2.68e). Also assume that the \( R_L C \) network has a bandwidth that encompasses the bandwidth of the transient nature of the signal. Based on this assumption, the current generated by the diode is transferred directly to the resistor without loss of signal. Therefore, using Eq. (2.68a) in Eq. (2.67) yields

\[
v_{L(S,N)}(t) = R_L \left[ \left( \frac{\Delta P_{ind} AC(t)}{T_o} + P_{ind} u \right) \right] \left[ U(t) - U(t - T_o) \right]
\]

(2.68b)

\[
P_{ind DC} = P_{ind} u U(-t) + P_{ind} s U(t - T_o)
\]

(2.68c)

\[
\Delta P_{ind} = P_{ind} s - P_{ind} u
\]

(2.68d)

\[
U(t) = \begin{cases} 
1 & \text{for } t > 0 \\
0.5 & \text{for } t = 0 \\
0 & \text{for } t < 0
\end{cases}
\]

(2.68e)

The change in the load voltage across \( R_L \) relative to a change in the diode input power yields

\[
v_{L(S,N)}(t) = R_L \left[ \left( \frac{\Delta P_{ind} AC(t)}{T_o} + P_{ind} DC + i_d \right)^2 + 2eB\frac{P_{ind} AC(t) + P_{ind} DC}{R_L} \right]^{1/2} + \frac{4KB}{R_L}
\]

(2.69)

\[
dv_{L(S,N)}(t) = \frac{R_L \left[ \left( \frac{\Delta P_{ind} AC(t)}{T_o} + P_{ind} DC + i_d \right)^2 + 2eB\frac{P_{ind} AC(t) + P_{ind} DC}{R_L} \right]^{1/2}}{\left[ \left( \frac{\Delta P_{ind} AC(t)}{T_o} + P_{ind} DC + i_d \right)^2 + 2eB\frac{P_{ind} AC(t) + P_{ind} DC}{R_L} \right]^{1/2}}
\]

for all \( t \)

(2.70a)
or, equivalently,

\begin{equation}
(2.70b)
\end{equation}

\[
\begin{align*}
&= \left[ \frac{R_L [\Delta P_{ind} AC(t) + i_d] + eB\Delta P_{ind}}{2} + 2eBR_L [P_{ind} AC(t) + 2e_i d B + \frac{4kTB}{R_L}] \right]^{\frac{1}{2}} \\
& \quad \text{for } 0 < t < T_o \\
&= \left[ \frac{R_L [\Delta P_{ind} AC(t) + i_d] + eB\Delta P_{ind}}{2} + 2eBR_L [P_{ind} AC(t) + 2e_i d B + \frac{4kTB}{R_L}] \right]^{\frac{1}{2}} \\
& \quad \text{for } t < 0 \text{ and } t > T_o
\end{align*}
\]

where \( \Delta P_{ind}(t) \) is determined from Eq. (2.68a). In Eqs. (2.69) and (2.70a and b), \( P_{ind}(t) \) may be is approximated as \( P_{ind}^u \) since \( P_{ind}^u \) is approximately equal to \( P_{ind}^u \). With the aid of Eqs. (2.68a-e), Eq. (2.70) yields the change in voltage directly in terms of the change in the laser beam power incident on the diode

\begin{equation}
(2.71)
\end{equation}

\[
\begin{align*}
&= \left[ \frac{R_L [\Delta P_{ind} AC(t) + i_d] + eB\Delta P_{ind}}{2} + 2eBR_L [P_{ind} AC(t) + 2e_i d B + \frac{4kTB}{R_L}] \right]^{\frac{1}{2}} \\
& \quad \text{for } 0 < t < T_o \\
&= \left[ \frac{R_L [\Delta P_{ind} AC(t) + i_d] + eB\Delta P_{ind}}{2} + 2eBR_L [P_{ind} AC(t) + 2e_i d B + \frac{4kTB}{R_L}] \right]^{\frac{1}{2}} \\
& \quad \text{for } t < 0 \text{ and } t > T_o
\end{align*}
\]

Choosing \( dt = T_o \) and \( t = T_o \), the change in the voltage at the load resistor about the transition time interval is given by

\begin{equation}
(2.72)
\end{equation}

\[
\begin{align*}
&= \left[ \frac{R_L [\Delta P_{ind} AC(t) + i_d] + eB\Delta P_{ind}}{2} + 2eBR_L [P_{ind} AC(t) + 2e_i d B + \frac{4kTB}{R_L}] \right]^{\frac{1}{2}} \\
& \quad \text{for } 0 < t < T_o \\
&= \left[ \frac{R_L [\Delta P_{ind} AC(t) + i_d] + eB\Delta P_{ind}}{2} + 2eBR_L [P_{ind} AC(t) + 2e_i d B + \frac{4kTB}{R_L}] \right]^{\frac{1}{2}} \\
& \quad \text{for } t < 0 \text{ and } t > T_o
\end{align*}
\]
where $\Delta P_{\text{ind}}$ as given by Eqs. (2.52), (2.54b) and (2.68d) is a function of the change in the index of refraction Eq. (2.50e).

The maximum voltage or near maximum voltage can be obtained from Eq. (2.69). Again, it is approximated that the fringe shift is assumed to be small. Therefore, the maximum voltage or near maximum voltage (assuming the maximum occurs somewhere between the $0 < t < T_0$) is the larger of the two voltages at the load

$$v_L(s\&n)_{\max} = (2.73)$$

$$R_L \left[ (R_{P_{\text{ind}u}} + i_d)^2 + 2eeB R [P_{\text{ind}u}] + 2ei_d B + \frac{4kTB}{R_L} \right]^{1/2}$$

if $P_{\text{ind}u} > P_{\text{ind}s}$

$$R_L \left[ (R_{P_{\text{ind}s}} + i_d)^2 + 2eeB R [P_{\text{ind}s}] + 2ei_d B + \frac{4kTB}{R_L} \right]^{1/2}$$

if $P_{\text{ind}u} < P_{\text{ind}s}$

Another useful figure of merit is the signal-to-noise ratio (SNR). There are three SNRs that may be calculated. The first two are the SNR due to the signal power at the load resistor for the unstressed and stressed samples relative to the noise power in the electronic circuit given respectively by $SNR_u$ and $SNR_s$. These SNRs are well defined since the stressed and unstressed currents are DC currents and hence are RMS currents. The third SNR, due to the change in the signal power resulting from the unstressed to stressed transition in the index of refraction, is not well defined. The value of the time averaged signal power is dependent on where the transition occurs in the interval of time chosen. Consequently, a weighted average will result in a value between the two transition levels. Because the difference between the two levels is small, $SNR_u$ and $SNR_s$ are good measures for this value. Because the DC signal due to the laser at the transition is probably larger than the signal itself, it will out weigh the amount due to the
change in the signal power yielding no new information since the signal may not be detected on the scale of measurement. The DC component of the signal may be removed from the measurement by a differencing technique if the earlier signal is delayed by some finite amount of time. One has to optimize the amount of time delay by the amount of attenuation that will result from delay lines or electronics.

By definition,

\[
SNR = 10 \log \left( \frac{\text{time averaged signal power for ideal noiseless circuit}}{\text{noise power}} \right)
\]

\[
= 20 \log \left( \frac{V_{\text{RMS Signal}}}{V_{\text{RMS Noise}}} \right) = 20 \log \left( \frac{I_{\text{RMS Signal}}}{I_{\text{RMS Noise}}} \right)
\]

Therefore, with the aid of Eqs. (2.48c) and (2.73), the signal-to-noise ratio for the unstressed load is given by

\[
SNR_u = 10 \log \left( \frac{1}{2} \frac{R_l i_n^2}{R_l i_n^2} \right) = 20 \log \left( \frac{i_n}{i_n} \right) = 20 \log \left( \frac{R_{P_{\text{ind u}}}}{i_n} \right)
\]

\[
= 20 \log \left( \frac{RA_d P_{in}}{2AMB} \left[ 1 + \frac{1}{\nu_2} \cos[\nu_1] \sin[\nu_2] \right] \right) = 20 \log \left( \frac{eB R_{P_{\text{ind u}}} + 2e_iB + \frac{4kTB}{R_L}}{\nu_2} \right)
\]

With the aid of Eqs. (2.51b), (2.73) and (2.74), the signal to noise ratio for the stressed case is given by

72
\[ SNR_s = 10 \log \left( \frac{\frac{1}{2} R_L i_n^2}{\frac{1}{2} R_L i_n^2} \right) = 20 \log \left( \frac{i_n}{i_n} \right) = 20 \log \left( \frac{P_{\text{ind}}}{i_n} \right) \]

\[ = 20 \log \left( \frac{\mathcal{R}_D}{2A_{MB}} P_n \left\{ 1 + \frac{1}{\psi_2} \cos[\psi_1 + \Delta \psi] \sin[\psi_2] \right\} \right) \]

\[ \left\{ \frac{2eB \mathcal{R}_P}{4kTB} + \frac{2e_i B}{R_L} \right\} \]

where \( \Delta l_s \) equals zero, \( P_{\text{ind}}(t) \) in Eq. (2.73) is \( P_{\text{ind}} \) for the unstressed sample and \( P_{\text{ind}, s} \) for the stressed sample. For slight fringe shifts, \( SNR_s \) will approximately equal \( SNR_u \).

2.6.2 With resistance at the load (ac case)

The DC bias both before and after the event may be too large to observe small changes as a result of the event. The event is the stressing of the plastic. As a result, a large capacitor, \( C_0 \), is placed between the load and the scope to isolate and pass only the AC contributions of the load voltage to the scope. Refer to Fig. 2.10. The argument goes as follows: From simple circuit theory,

\[ \nu(t) = \frac{1}{C} \int i(t) dt \quad \text{and} \quad i(t) = C \frac{d\nu(t)}{dt} \]

As the capacitance goes to infinity, knowing that the time varying current and time are finite, the voltage drop across the capacitor is small to negligible. For DC, the change in the voltage is zero. Since the capacitance can never physically obtain an infinite value, the DC current is blocked by the capacitor. Separating the AC and DC components of the current passing through the capacitor, it is observed that the DC voltage component across the load resistor is equivalent to the DC component across the capacitor \( C_0 \) in Fig.
2.10. The AC voltage drop across \( C_o \) is negligible. Assuming that the measuring device has infinite input impedance to the circuit, Kirchhoff's voltage law states that the voltage measured by the scope \( V_{LAC} \) is the AC component of the load voltage across the resistor. Note that the term AC is used to imply time varying signal in this explanation.

Now consider the case when a DC blocking capacitor \( C_o \) is placed between the load and the measuring device as shown in Fig. 2.10. The AC or time varying portion of the voltage, \( V_{LAC} \), passed by the capacitor \( C_o \) is,

\[
v_{LAC(S&N)}(t) = \begin{cases} 
R_L \left[ \left\{ R_{P_{ind}}(t) - R_{P_{ind}}(u) \right\}^2 + 2eB R_{P_{ind}}(t) + 2ei_d B + \frac{4kTB}{R_L} \right]^{\frac{1}{2}} & \text{for } 0 < t < T_o \\
R_L \left[ 2eB R_{P_{ind}}(t) + 2ei_d B + \frac{4kTB}{R_L} \right]^{\frac{1}{2}} & \text{for } 0 > t \text{ and } t > T_o
\end{cases}
\]  (2.77)

Since noise is a random time varying signal, its voltage/current signature is not blocked by the blocking capacitor \( C_o \). The change in the AC voltage passed by the large capacitor due to changes in the diode input power is

\[
dv_{LAC(S&N)}(t) = \begin{cases} 
\frac{R_L}{R} \left[ R_{P_{ind}}(t) - R_{P_{ind}}(u) + eB R_{R} \right] dP_{ind}(t) & \text{for } 0 < t < T_o \\
\left[ \left\{ R_{P_{ind}}(t) - R_{P_{ind}}(u) \right\}^2 + 2eB R_{P_{ind}}(t) + 2ei_d B + \frac{4kTB}{R_L} \right]^{\frac{1}{2}} & \text{for } 0 > t \text{ and } t > T_o
\end{cases}
\]  (2.78)
Referring to Eq. (2.68b), \( P_{ind AC} - P_{ind u} = \Delta P_{ind} (t/T_o) \). Therefore, \( dP_{ind AC} (t) = \Delta P_{ind} (dt/T_o) \). Choosing \( dt = T_o \) since the rise time is linear and choosing \( t = T_o \), Eq. (2.78) yields the change in the AC voltage across the load resistor about the transition interval of the event (stressing of the plastic)

\[
\Delta V_{LAC}^{(S\&N)}(t) = \begin{cases} 
R_L \left[ \frac{\mathcal{R} \mathcal{R} \Delta P_{ind}}{\Delta P_{ind}} + eB\mathcal{R} \right]^{\frac{1}{2}} & \text{for } 0 < t < T_o \\
0 & \text{for } 0 > t \text{ and } t > T_o 
\end{cases}
\]  

(2.79)

Considering only the time frame between 0 and \( T_o \), the maximum AC load voltage occurs when

\( P_{ind AC} = P_{ind s} \) if \( P_{ind u} < P_{ind s} \)  

(2.80a)

OR

\( P_{ind AC} = P_{ind u} \) if \( P_{ind u} > P_{ind s} \)  

(2.80b)

where \( t = T_o \) in the case of Eq. (2.80a) and \( t = 0 \) in the case of Eq. (2.80b). Based on the constraints indicated in Eqs. (2.80a) and (2.80b), the maximum AC load voltage is respectively

\[
V_{LAC}^{(S\&N) MAX} = \begin{cases} 
R_L \left[ \mathcal{R} P_{ind s} \left( \mathcal{R} P_{ind s} + 2eB \right) + 2eB + \frac{4kTB}{R_L} \right]^{\frac{1}{2}} & \text{if } P_{ind u} < P_{ind s} \\
R_L \left[ 2eB \mathcal{R} + 2eB + \frac{4kTB}{R_L} \right]^{\frac{1}{2}} & \text{if } P_{ind u} > P_{ind s} 
\end{cases}
\]  

(2.81)

(2.82)
The circuit that is provided in Fig. 2.10 consists of a capacitance which blocks the DC components. Hence an AC SNR, SNR\textsubscript{AC}, can be defined based on the transition between signal levels that is well defined. To determine the SNR\textsubscript{AC} term, the definition requires the AC voltage across the load for the noiseless circuit. Consequently, over the time interval between 0 and \( T_o \), the load AC voltage for the noiseless circuit is given by

\[ v_{L,AC}\textsuperscript{(SNR)}(t) = R_L \left\{ P_{ind,AC}(t) - P_{ind,w} \right\} \]  

Substituting this relation [Eq. (2.83)] into the definition for the SNR [Eq. (2.74)] and using Eqs. (2.54b), (2.61), (2.62), (2.63), (2.64), (2.65a), and (2.68b) yields

\[
SNR\textsubscript{AC} = 10 \log \left( \frac{\text{time averaged signal power noiseless circuit}}{\text{noise power}} \right) 
\]

\[ = 10 \log \left( \frac{R_L \int_0^{T_o} R_L^2 \frac{1}{R_L} v_{L,AC}^2(t) \, dt}{T_o \int_0^{T_o} P_{ind,AC}^2(t) \, dt} \right) \]

\[ = 10 \log \left( \frac{R_L \int_0^{T_o} (\Delta P_{ind})^2 \, dt}{T_o \int_0^{T_o} i_n^2 \, dt} \right) \]

\[ \approx 10 \log \left( \frac{R_L \int_0^{T_o} (\Delta P_{ind})^2 \, dt}{2eB\Re P_{ind}(t) + 2ei_dB + \frac{4kTB}{R_L}} \right) \]

\[ \approx 10 \log \left( \frac{R_L \int_0^{T_o} (\Delta P_{ind})^2 \, dt}{2eB\Re P_{ind}(t) + 2ei_dB + \frac{4kTB}{R_L}} \right) \]
where

\[ P_{\text{ind}}(t) \approx P_{\text{ind},u} \]

\[ \psi_1 = -\beta_o \left( n_p - n_o \right) \frac{L}{h} x_o = \varphi_u \]

\[ \psi_2 = \beta_o \left( n_p - n_o \right) \frac{L L_D}{h} \]

\[ \Delta \varphi = \beta_o l s \Delta n_s \quad \text{NOTE: } \Delta l_s = 0 \text{ for simplicity.} \]

Since \( P_{\text{ind},u} \) approximately equals \( P_{\text{ind},s} \), \( P_{\text{ind}}(t) \) may be approximated as \( P_{\text{ind},u} \) over the time interval of interest. The diode has an input power limitation. Exceeding this limitation will damage the diode. The power input on the diode surface may be determined from Eqs. (2.51b) and (2.51c), where \( \psi_1 \) is given by Eq. (2.50b) with \( \Delta l_s = 0 \) (for simplicity). Since \( P_{\text{ind},u} \) approximately equals \( P_{\text{ind},s} \) for small changes in the index of refraction, Eqs. (2.51b) and (2.51c) are approximately equal. Optical input power damage thresholds, bandwidth, and the sensitivity of electronic measuring devices play a significant role in determining the laser intensity and resistive load attached to the photodiode.

Knowing the optical power damage threshold of the photodiode and the photodiode's responsivity, the maximum laser generated photodiode current may be determined. The bandwidth of the signal sought is dictated by the rise and/or fall time of the pulse. Based on the risetime of the pulse, \( t_r \), the frequency at the half power point (3 dB), \( f_{\text{HP}} \), is given by

\[ B_S = f_{\text{HP}} = \frac{0.35}{t_r} \]

(2.85a)

where \( B_S \) is the bandwidth of the pulse. The capacitance of the photodiode in shunt with the load resistance acts as a lowpass filter. Consequently, the bandwidth of the signal...
may be related to the bandwidth of this lowpass filter. For a load resistance $R_L$, the detector bandwidth (low pass filter) is

$$B_D = \frac{1}{\sqrt{R_L C_D}}$$  \hspace{1cm} (2.85b)

where $C_D$ is the detector capacitance. Equating the bandwidth of the signal, $B_S$, to the bandwidth of the detector, $B_D$, the desired load resistance may be determined. Knowing the load resistance and the detector current, the load voltage may be determined and compared to the sensitivity of the measuring instrument. Typically, real time oscilloscopes may be able to measure voltages as low as a couple of millivolts assuming that the noise signal is small in comparison. If the sensitivity parameter is out of range, one may have to compromise bandwidth (increase the load resistance) for a measurable voltage sensitivity range. The second technique is to start with the voltage sensitivity for a desired bandwidth. Equation (2.58b) dictates the required load resistance. Knowing the voltage sensitivity range desired and the load resistance, the photodiode current may be determined. With the aid of the responsivity, the diode input power may be calculated. This input power needs to be compared to the optical input power damage thresholds. If the calculated power exceeds the damage threshold, a compromise on either voltage sensitivity range or bandwidth must be made in order that the diode is not optically stressed. From these relations, the computed total mean-squared value of current, the effective time averaged power, and the input noise power may be determined.

2.6.3 With amplifier at load (dc case)

At the detector output, the signal is weak. Therefore, we introduce an amplifier stage to enhance the signal. A transimpedance amplifier is used since, for a given bandwidth, the signal to noise ratio is higher for this amplifier. That means that the transimpedance
amplifier can have a large bandwidth yet retain low noise characteristics. The AD8015 was one of the amplifiers used in the initial studies of this research effort. An illustration of this amplifier is provided in Fig. 2.11 [www.analog.com]. It is a complete, single chip solution to convert photodiode current into a differential voltage output. This low cost silicon based amplifier is ideal for systems requiring a wide dynamic range preamplifier or single ended to differential conversion. To interface the existing detector circuit, an AC resistance ($R_{ac}$) is incorporated in the circuit to increase the sensitivity performance for this application. The circuit in Fig. 2.12 models the photodetector with a transimpedance amplifier. The output of the photodetector with associated dark and shot noise currents are modeled as current sources. The equivalent noise voltage is denoted as $v_n$. It is caused due to the bipolar transistors that are used in the construction of the amplifier[www.analog.com]. The quantity $i_n$ refers to the noise current and it generates an additional noise voltage across the input signal source impedance. The impedance between the two input terminals of the amplifier is denoted as $R_i$. The signal from the output is fed back into the circuit by means of a feedback resistance $R_f$. Due to the flow of current through the feedback resistor, a noise current is generated which is denoted as $i_{nf}$. There is a shunt resistance, $R_{sh}$, that is placed parallel to the current source (photodetector) to prevent current from flowing in that path, in other words to shunt that particular electrical path.

Assuming that the signal current is higher than the dark current, the dark current ($i_d$) can be omitted. Using the superposition theorem since the circuit is linear, consider the current due to the laser ($i_l$) as the only current source driving the system. Therefore,
all remaining current sources are open circuited and voltage sources are shorted yielding
the circuit shown in Fig. 2.13. The SNR (Signal to Noise Ratio) as defined in Eq. (2.74),
requires a load resistance at the amplifier output.

Combining the shunt resistance $R_{sh}$, amplifier input resistance $R_i$ and the AC
resistance $R_{AC}$; an equivalent resistance, $R_s$ may be determined, as shown in Fig. 2.14
and given by

$$\frac{1}{R_s} = \frac{1}{R_{sh}} + \frac{1}{R_{AC}} + \frac{1}{R_i}$$

(2.86)

The feedback resistor $R_f$ connected to the inverting terminal of the transimpedance
amplifier allows for output amplification of the input signal.

A Thevenin equivalent circuit is employed on the input side of the transimpedance
amplifier as shown in Fig. 2.15. The laser current and the feedback current are equal in
magnitude but oriented in opposite directions in Fig. 2.15, therefore by Kirchhoff's
Current Law (KCL)

$$\frac{i_i R_x}{R_i} = i_i = -i_f$$

(2.87)

Consequently, the output voltage $v_o$ is

$$v_o = R_f i_f = -R_f i_i$$

(2.88)

It follows that the power due to the laser current is

$$P_L = \frac{1}{R_L} v_o^2 = \frac{R_f^2}{R_L} i_i^2$$

(2.89)

Using the method of superposition, consider that the noise current sources $i_{sl}$, $i_{sd}$
and $i_n$ are active and all other sources suppressed. Since these noise sources are parallel
to each other, the total noise current can be obtained as the root mean square sum of the individual noise currents.

\[ i_{NT} = \sqrt{i_n^2 + i_{nd}^2 + i_{a}^2} \quad (2.90) \]

By analogy to the laser current calculation, the total noise current \( i_{NT} \) is related to the feedback current as

\[ \frac{i_{NT} R_s}{R_i} = i_{NT} = -i_f \quad (2.91) \]

The output power due to the noise currents is given by

\[ P_{NT} = \frac{R_f^2}{R_L} i_{NT}^2 = \frac{R_f^2}{R_L} (i_n^2 + i_{nd}^2 + i_a^2) \quad (2.92) \]

Using the method of superposition, consider the feedback current \( i_{nf} \) as the only active current source as shown in Fig. 2.16. In an amplifier, the voltage between the inverting and noninverting terminals is a virtual short. Therefore, the voltage drop across \( R_s \) is zero implying that the feedback resistor sinks all of the current of the feedback noise source. Consequently,

\[ v_o = -R_f i_{nf} \quad (2.93) \]

The output power contribution due to the feedback current is,

\[ P_{nf} = \frac{1}{R_L} v_o^2 = \frac{1}{R_L} (R_f i_{nf})^2 \quad (2.94) \]

Using the superposition theorem, consider the noise voltage \( v_n \) as the only existing source as shown in Fig. 2.17. Again, because the input terminals of the amplifier act like a virtual short, the feedback current is directly related to the noise voltage as
Using Kirchhoff's voltage law,

\[ v_o = i_f R_f + v_n \]  \hspace{1cm} (2.96a)

\[ v_o = v_n \left( \frac{R_f}{R_s} + 1 \right) \]  \hspace{1cm} (2.96b)

The load power due to the noise voltage is,

\[ P_{nv} = \frac{R_f^2}{R_L} \left( \frac{1}{R_f} + \frac{1}{R_s} \right)^2 v_n^2 \]  \hspace{1cm} (2.97a)

where the equivalent noise voltage is given by [32]

\[ v_n = \sqrt{\frac{8kT r_m}{3}} \]  \hspace{1cm} (2.97b)

and \( r_m \) is the transresistance of the amplifier.

The total noise power \( P_n \) can be expressed as the sum of all the noise powers calculated. With the aid of Eqs. (2.92), (2.94) and (2.97), the resultant noise power at the load is

\[ P_n = P_{NT} + P_{nf} + P_{nv} = \frac{R_f^2}{R_L} (i_{sl}^2 + i_{sd}^2 + i_n^2) + \frac{R_f^2}{R_L} i_{nf}^2 + \frac{R_f^2}{R_L} \left( \frac{1}{R_f} + \frac{1}{R_s} \right)^2 v_n^2 \]  \hspace{1cm} (2.98)

\[ = \frac{R_f^2}{R_L} i_{sl}^2 + i_{sd}^2 + i_n^2 + \frac{4kT B}{R_f} + \left[ \frac{1}{R_f} + \frac{1}{R_s} \right]^2 v_n^2 \]

where the noise in the feedback loop is due solely to thermal noise of the feedback resistor. Refer to Eq. (2.64). The noise current in the circuit can be expressed as,
\[ i_{\text{noise}}^2 = \left[ \frac{i_{sl}^2 + i_{sd}^2 + i_n^2 + 4kTB}{R_f} + \left[ \frac{1}{R_f} + \frac{1}{R_s} \right]^2 \right] v_n^2 \] (2.99)

\[ = 2eB \mathbb{E} P_{\text{ind}}(t) + 2e_i B + i_n^2 + \frac{4kTB}{R_f} + \left[ \frac{1}{R_f} + \frac{1}{R_s} \right]^2 v_n^2 \]

Using the definition for the signal-to-noise ratio given by Eq. (2.74), the signal-to-noise ratio for the DC case with an amplifier before the load can be expressed as

\[ SNR_{\text{DCAMP}} = 10 \log_{10} \left[ \frac{R_f^2 i_t^2}{R_L} \right] \frac{\frac{R_f^2}{R_L} \left[ \frac{i_t^2}{i_{sl}^2 + i_{sd}^2 + i_n^2 + \frac{4kTB}{R_f} + \left[ \frac{1}{R_f} + \frac{1}{R_s} \right]^2 v_n^2} \right]}{\left[ \frac{i_t^2}{i_{sl}^2 + i_{sd}^2 + i_n^2 + \frac{4kTB}{R_f} + \left[ \frac{1}{R_f} + \frac{1}{R_s} \right]^2 v_n^2} \right]} \] (2.100)

Simplifying the Eq. (2.100) yields

\[ SNR_{\text{DCAMP}} = 10 \log_{10} \left[ \frac{i_t^2}{i_{sl}^2 + i_{sd}^2 + i_n^2 + \frac{4kTB}{R_f} + \left[ \frac{1}{R_f} + \frac{1}{R_s} \right]^2 v_n^2} \right] \] (2.101)

or equivalently

\[ SNR_{\text{DCAMP}} = 20 \log_{10} \left[ \frac{i_t}{\left( i_{sl}^2 + i_{sd}^2 + i_n^2 + \frac{4kTB}{R_f} + \left[ \frac{1}{R_f} + \frac{1}{R_s} \right]^2 v_n^2 \right)^{1/2}} \right] \] (2.102)

where \( i_{sl}, i_{sd} \) and \( i_t \) are given by Eqs. (2.62), (2.61) and the responsivity times either Eqs. (2.51b) or (2.51c) depending if the stressed or unstressed sample is being considered.

The term \( P_{\text{ind}}(t) \) in Eq.(2.62) is \( P_{\text{ind},u} \) for the unstressed sample and \( P_{\text{ind},s} \) for the stressed...
sample. For slight fringe shifts, $SNR_{DC,AMP}$ will approximately equal $SNR_{DCAMP,u}$ therefore the two notations are not distinguished in Eq. (2.102).

2.6.4 With an amplifier at load (ac case)

The DC signals are usually large making it difficult to resolve small AC effects. Consequently, one suppresses the DC signals by capacitively coupling the photodiode to the amplifier. With a large enough capacitor, the time varying signal is coupled directly to the amplifier. Because the photodiode noise is time varying, it too is coupled to the amplifier. Using Eqs. (2.74) and (2.98), the signal-to-noise ratio with an amplifier capacitively coupled to the photodiode, $SNR_{AC,AMP}$, is

$$SNR_{AC,AMP} = \text{10log} \left( \frac{\text{time averaged signal power noiseless circuit}}{\text{noise power}} \right)$$

$$= \text{10log} \left( \frac{\mathcal{R}_f \frac{R_f}{T_o} \frac{1}{T_o} \int_0^T \left( P_{ind,AC}(t) - P_{ind,DC} \right)^2 dt}{\frac{R_f}{R_L} i_{noise}^2} \right) \approx \text{10log} \left( \frac{\mathcal{R}_f \frac{R_f}{T_o} \frac{1}{T_o} \int_0^T t^2 dt}{i_{noise}^2} \right)$$

$$= \text{10log} \left( \frac{\mathcal{R}_f \frac{1}{3} \left( \mathcal{R}_{ind} \right)^2}{i_{si}^2 + i_{sd}^2 + i_{n}^2 + \frac{4kTB}{R_f} + \left[ \frac{1}{R_f} + \frac{1}{R_s} \right] v_n^2} \right)$$

$$\approx \text{10log} \left( \frac{\mathcal{R}_f \frac{1}{3} \left( \frac{A_D P_m (\sin \psi_1 \sin \psi_2)}{2A_{MB} \psi_2} - \frac{A_D P_m (\cos \psi_1 \sin \psi_2)}{4A_{MB} \psi_2} \right)^2}{2eB \mathcal{R}_{ind} + 2ei_d B + i_n^2 + \frac{4kTB}{R_f} + \left[ \frac{1}{R_f} + \frac{1}{R_s} \right] v_n^2} \right)$$

(2.103b)
2.7 Signal detection using frequency mixing techniques

Based on calculations above, it is anticipated that the signal-to-noise ratio will be small using the detection schemes above. One may increase the SNR by increasing the input power of the laser beam but one is limited to the amount of energy that may be placed in a narrow beam before breakdown of the optical elements (including the photodiode) or the plastic under test results. Some optical intensity thresholds have been investigated in a different study for Rexolite. [3] Reconsider Eq. (2.6) modified to consider the interaction of two different colors of laser light one in path 1 and one in path 2. The instantaneous intensity may be re-expressed as

\[
I(t) \propto \frac{1}{2} \left( E_{01}^2 + E_{02}^2 \right) - \frac{1}{2} \left[ E_{01}^2 \sin(2\omega_1 t + 2\Phi_1) + E_{02}^2 \sin(2\omega_2 t + 2\Phi_2) \right] + \frac{E_{01} E_{02} \left[ \cos(\Delta \omega t + \Delta \Phi) - \cos(\omega_{12} t + \Phi_{12}) \right]}{E_{01} E_{02} \left[ \cos(\Delta \omega t + \Delta \Phi) - \cos(\omega_{12} t + \Phi_{12}) \right]}
\]

where \( \Delta \omega = \omega_1 - \omega_2 \), \( \omega_{12} = \omega_1 + \omega_2 \), \( \Delta \Phi = \Phi_1 - \Phi_2 \), \( \Phi_{12} = \Phi_1 + \Phi_2 \), \( \Phi_1 = \beta_1 P_{L1} \) and \( \Phi_2 = \beta_2 P_{L2} \). As indicated in Eq. (2.15), \( \Phi \) is a function of position over the plane containing the diode surface. It is generalized that the wave in path A associated with \( P_{L1} \) in Fig. 2.4 [or path 1 in Fig. 2.3] has an angular frequency \( \omega_1 \) and the wave in path B associated with \( P_{L2} \) in Fig. 2.4 [or path 2 in Fig. 2.3] has an angular frequency \( \omega_2 \). The terms \( \beta_1 \) and \( \beta_2 \) are the appropriate wavenumbers associated with the angular source frequencies \( \omega_1 \) and \( \omega_2 \) respectively. The diode is stimulated by the intensity of light illuminating its surface. Because the diode is exposed to more than one frequency of light, a slightly different argument is employed to determine how the optical to electrical conversion of energy results. Figures 2.9a-c models the pn junction of the diode as a capacitor. For high frequencies, the capacitor acts as a short circuit. Consequently, the photodiode with load acts as a low pass filter. This suggests that the pn junction shorts the high frequency results.

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components of the instantaneous power and passes the low frequency components. Consequently, the energy content in the high frequency components is not converted to useful output electrical energy. Therefore, that portion of the total instantaneous power in Eq. (2.104) illuminating the diode surface that will contribute to the output signal of the diode, $P_I(t)$, is

$$P_I(x,t) = [P_{ave1}(x) + P_{ave2}(x)] + 2\sqrt{P_{ave1}(x)P_{ave2}(x)}\cos[\Delta \omega t + \Delta \Phi(x)]$$  \hspace{1cm} (2.105)

assuming that the difference frequency $\Delta \omega$ is below the cutoff frequency of the filter. Here, $x$ represents the placement position of the center of the diode. Due to symmetry in $y$, the power is not a function of $y$. In practice when one is detecting fringe shifts, it is desired that the light distributed over the surface of the diode, $A_D$, be nearly uniform. Equation (2.105) is written in terms of the power of each beam averaged over the period associated with its own frequency. If, the power of the beam in the path containing the sample under test is constrained to be small, then an increase in power in the reference path enhances that portion of the power signal containing the desired change in phase $\Delta \Phi$. Assume that path B [path 2] contains the plastic under test. To enhance that portion of the power contribution due to the phase information, let $P_{ave1} >> P_{ave2}$. If the frequency of the beam in both paths are the same, then

$$P_I(x,t) = P_{ave1} + 2\sqrt{P_{ave1}P_{ave2}}\cos(\Delta \Phi) = P_{ave1} \left[ 1 + 2\sqrt{\frac{P_{ave2}}{P_{ave1}}}\cos(\Delta \Phi) \right]$$  \hspace{1cm} (2.106)

Unless the $\Delta \Phi$ changes with time, it will be difficult to discern the phase information from the time averaged power of the beam in path A [path 1]. If $\Delta \Phi$ changes with time, capacitor coupling the signal to the next stage in the circuit proper can separate the two terms such that the second term is now compared to the noise floor instead of $P_{ave1}$. The
size of the coupling capacitance and load determines the cutoff frequency for the high pass filtering. The low frequency content resulting from the fringe shift is lost thereby degrading the signal. All frequencies beyond the cutoff frequency contribute to the noise implying a large noise bandwidth.

If a Bragg cell is placed in the reference path [call this in part element 7 in path A of Fig. 2.4], the frequency of the beam in this path is shifted such that $\Delta \omega$ is no longer zero but is the acoustic frequency of oscillation in the cell. Since the index of refraction of the cell changes and/or the surface of the element vibrates, the phase in path A [path 1] is in general also a function of time, $\Phi_1(t)$. Consequently, assuming $P_{ave1} \gg P_{ave2}$, Eq. (2.105) simplifies to

$$P_i(x,t) = P_{ave}(x) \left[ 1 + 2 \frac{P_{ave2}(x)}{P_{ave}(x)} \cos(\Delta \omega t + [\Phi_1(x,t) - \Phi_2(x,t)]) \right]$$

(2.107)

where $\Phi_2$ is only a function of time if a dynamic pressure is applied to the sample. If the property of the sample is in a fixed state, then $\Phi_2$ would be a constant. For purposes of Eq. 2.107, both phases $\Phi_1$ and $\Phi_2$ are treated as functions of space, $x$, and time, $t$. Pulsed power experiments occur over finite time duration. Assuming that there is enough time for beating to evolve between the two beams and the beat frequency is higher than the frequency signature of the sample under test being compressed, the constraint on the coupling capacitor [or intermediate band pass filter] may be relaxed without significantly degrading the signal. One might imagine that the acoustic wave acts as the carrier wave of the modulation resulting from dynamic compression of the plastic sample under test. A bandpass filter would significantly reduce the noise spectrum received. To isolate the change in phase that would occur in the absence of the Bragg cell, one would have to
demodulate the electrical signal at the Bragg frequency with an added synchronized time varying phase due to the Bragg cell. This type of heterodyning will be examined with the signal-to-noise ratio determined.

After a heterodyning technique has been presented, a few words will be provided to show a second more complicated optical setup that could in principle be used to perform the same function. In this setup, one may employ heterodyne or homodyne detection techniques. Note that conventional interferometry using a single frequency of light is in effect mixing a signal as in homodyne detection.

2.7.1 With heterodyne detection

In heterodyne detection the spectrum of laser modulation is shifted to an intermediate frequency (IF) range so selectivity of a photo-receiver depends on the IF filter and the bandwidth of an IF amplifier. The signal-to-noise ratio of this IF signal is robust because the reduction of the bandwidth reduces the noise contribution across the spectrum considerably. Further, in different words, the beating of the local oscillator wave with the signal wave amplifies the signal wave signature at an intermediate frequency.

Based on the system investigated by A. Rogalski and Z. Bielecki [26] as partially shown in Fig. 2.18, the beam interrogating the plastic under test and the frequency shifted reference beam are linearly combined using a beamsplitter. Passing the laser generated reference beam through an electro-optical modulator (Bragg Cell) creates the shift in frequency. Refer to Fig. 2.19. The SNR and the power in the heterodyne system shall be derived. Although the time averaging technique used above may be applied with certain qualifications made, the technique does not lend itself to clear argumentation especially if the change in frequency is on the order the optical frequency. At the expense of brevity,
this section will contain a complete time domain theory that may be extended to an interferometry setup using two different optical frequencies. Further, since the objective of heterodyning was to enhance a weak interferometer signal, the 50% transmitting 50% reflecting beam splitter constraint will be relaxed to allow for any power percentage split between the reflected and transmitted beams relative to the incident beam. This simple theory will neglect all reflection losses, consequently boundary conditions on the electric field need be satisfied in this approximation.

The time domain, linearly polarized electric field of the laser proper is defined by

\[ E_i(z,t) = E_{i0} \sin(\omega_o t - \beta_o z) \]  

(2.108)

yielding an instantaneous power density given by

\[ \phi_m(x,y,z,t) = \frac{1}{\eta_o} E_{i0}^2 \sin^2(\omega t - \beta z) = \frac{1}{2\eta_o} E_{i0}^2 \left[ 1 - \sin(2\omega_o t - 2\beta_o z) \right] \]  

(2.109)

The power entering a beam expander, \( P_m \), over the total beam cross section \( A_{LB} \) equals the power leaving the expander, \( P'_m \), over the magnified beam cross section \( A_{MB} \) assuming that one can neglect all losses. Consequently, the instantaneous power density at the beam expander is

\[ \phi'_m(x,y,t) = \frac{A_{LB}}{A_{MB}} \phi_m(x,y,t) = \frac{1}{A_{MB}} P_m(x,y,t) \]  

(2.110)

Because information of the phase of the wave is not crucial at this point, one may take \( z=0 \) providing a relation on the amplitudes of the wave as suggested in Eq. (2.110). From Eq. (2.109), the peak input power may be defined as

\[ P_{in0} = \frac{A_{LB}}{\eta_o} E_{i0}^2 \]  

(2.111)
Let $F_1$ represent the fraction of the laser power reflected to the sample under test. Consequently, the fraction of the laser power transmitted through the beam splitter available for the frequency shifted reference beam is $(1-F_1)$. Losses and phase shifts due to the presence of the beam splitter have been neglected. As shown in Fig. 2.19 (similar to Fig. 2.4), the first beam splitter divides the original beam into two beams, beams A and B. Beam A is the reference beam which propagates through a Bragg cell and an unstressed reference sample to be recombined with Beam B at the second beam splitter. Beam B passes through a wedge used to generate the desired fringe spacing and the sample plastic under test prior to being recombined with Beam A. Choosing $z=0$ at the beam splitter, the instantaneous scalar power density of Beams A and B just after the first beam splitter are respectively

$$\phi_A(x,y,z=0) = (1-F_1) \frac{P_{in}(x,y,z=0,t)}{A_{MB}}$$

$$\phi_B(x,y,z=0,t) = F_1 \frac{P_{in}(x,y,z=0,t)}{A_{MB}}$$

Relating the power densities directly to the beam fields in each path, at the frequency of the laser beam, $\omega_0$, the amplitude of the electric field in path A and B are respectively

$$E_{eA} = \sqrt{\frac{\eta_0 (1-F_1) P_{ino}}{A_{MB}}}$$

$$E_{eB} = \sqrt{\frac{\eta_0 F_1 P_{ino}}{A_{MB}}}$$

The Bragg cell is assumed to be located just after beam splitter 1 such that the distance between the two elements is zero. Assume that the length of the Bragg cell is
The Bragg cell shifts both the frequency and wavenumber (phase coefficient) as \( \omega_c + \Delta \omega \) and \( \beta_c + \Delta \beta \) where

\[
\beta_c + \Delta \beta = \frac{\omega_c + \Delta \omega}{c}
\]

and \( c \) is the speed of light. At this point, the problem is left general implying that \( \Delta \omega \) may be any value. Let \( \xi_{BC} \) represent the time averaged power conversion transmission efficiency of the Bragg cell to shift energy inputted to the cell at frequency (angular frequency) \( \omega_c \) to the shifted frequency (angular frequency) \( \omega_c + \Delta \omega \) and transmit this energy or power to the output of the cell. Consequently by definition, the amplitude of the beam in path A just after the Bragg cell may be obtained from

\[
\xi_{BC} = \frac{\phi_{i, ave}(x, y, z = l_{BC})}{\phi_{i, ave}(z, y, z = 0)}
\]

(2.114a)

where prime designates the beam's properties in the region between Bragg cell and the second beam splitter. Consequently, relating the electric fields on both sides of the Bragg cell yields the following condition on the amplitudes of the fields

\[
E_{oA}' = \sqrt{\xi_{BC}} E_{oA} = \sqrt{\frac{\eta_o (1 - F_i) \xi_{BC, P_{in}}}{A_{MB}}} E_{oA}
\]

(2.114b)

An apparent phase shift also results. Without examining the geometrical and diffraction properties of the Bragg cell with beam, the phase shift resulting from the presence of the cell is defined as

\[
\phi_{BC} = \phi_{BC}(l_{BC}, t)
\]

(2.115)

This phase shift is time varying since the acoustic wave changes the properties of the crystal medium for example, the index of refraction of the medium. As Beam A
propagates through the crystal and experiences the time varying changes occurring in the region of the crystal, a dynamic phase shift results. This change in properties of the medium extends to the physical geometry of the medium as well as its electrical properties. The field in path A just before the second beam splitter (BS2) may be expressed as

\[ E'_A(x, y, z, t)_{BS2} = E'_{oA} \sin[(\omega_o + \Delta \omega)t - (\beta_o + \Delta \beta)n_o(l_{A1} + l_{A2} + l_{A3} - l_{BC}) - (\beta_o + \Delta \beta)n_s l_{s1} - \phi_{BC}(l_{BC}, t)] \] (2.116)

Let \( F_2 \) represent the fraction of the power incident on the second beam splitter that is reflected by the beam splitter. Neglecting all loss, the fraction of the incident power transmitted by the second beam splitter is \((1-F_2)\). Since Beam A is reflected by the second beam splitter,

\[ \varphi''_A(x, y, z, t)_{BS2} = F_2 \varphi'_A(x, y, z, t)_{BS2} \] (2.117)

where double prime represents the beam properties between the second beam splitter and the detector. Similar to Eqs. (2.112a,b) and (2.113a,b), the amplitude of the electric field between the second beam splitter and the detector is given by

\[ E''_{oA} = \sqrt{F_2} E'_{oA} = \sqrt{\eta_o(1 - F_1)F_2 \xi_{BC} P_{ino} / A_{MB}} \] (2.118)

Based on Eq. (2.15), the linearly polarized field of the beam in path B probing the sample under test may be written as

\[ E_B(x, y, z, t)_{BS2} = E_{oB} \sin[\omega_o t - \phi_B(x, y, t)] \] (2.119)

where
\[
\phi_B(x, y, t) = -n_o \beta_o \Delta l_S(t) + n_o \beta_o (l_{B3} + l_{B2} + l_{B1}) - n_o \beta_o L + n_p \beta_o z(x) + n_o \beta_o [L - z(x)] + [n_s + \Delta n_s(t)] \beta_o l_{S_2}(t)
\]

where \( l_{S_2}(t) = l_{S_1} + \Delta l_{S_2}(t) \) and \( z(x) = (L/h)(x-x_0) \). Refer to Fig. 2.5. Equation (2.120) contains the time functional dependence that are implied but not explicitly written in Eq. (2.15). Following the same line of thought that led to Eqs. (2.117) and (2.118), the power density and field amplitude just after the second beam splitter is related to appropriate quantities just before the beam splitter as, respectively,

\[
\varphi''_{B}(x, y, z, t)_{BS2} = (1 - F_2) \varphi''_{B}(x, y, z, t)_{BS2}
\]

\[
E''_{oB} = \sqrt{(1 - F_2)} E_{oB} = \sqrt{\frac{\eta_o (1 - F_2) F_1 P_{ino}}{A_{MB}}}
\]

Since the phase delay caused by the beam splitter is being neglected, the phase of the wave before the beam splitter is equal to the phase just after the beam splitter. This neglect of phase contribution of the beam splitter is reasonable. Since the delay due to Beam A at frequency \( \omega_o \) at beam splitter 1 is identical to that for Beam B at beam splitter 2, the phase difference or optical path difference between the two beams as a result of the beam splitter contribution is zero.

Based on these arguments with the detector located in the plane a distance \( l_D \) behind the second beam splitter, the field contributions of Beam A and Beam B on the detector surface may be expressed as

\[
E''_{A\ Dec}(x, y, t) = \sqrt{\frac{(1 - F_1) F_2 \eta_o P_{ino}}{A_{MB}}} \sin \left[ \omega_o + \Delta \omega \right] t - \phi_A - \vec{\phi}_{BC}(t) - \phi_{AD}
\]
\[ E_{B,\text{Dec}}(x, y, t) = \sqrt{\frac{(1 - F_2)F_1P_{\text{inc}}\eta_s}{A_{MB}}} \sin(\omega_s t - \varphi_B(x, t) - \varphi_{BD}) \]  
(2.123b)

where

\[ \tilde{\varphi}_{BC}(t) = \Delta\beta \frac{\phi_A}{\beta_o} + \Delta\beta n_s l_{s1} + \phi_{BC}(l_{BC}, t) - (\beta_o + \Delta\beta)n_o l_{BC} \]  
(2.124)

\[ \phi_{AD} = l_D n_o (\beta_o + \Delta\beta) \]  
(2.125a)

\[ \phi_{BD} = l_D n_o \beta_o \]  
(2.125b)

and \( \phi_A \) and \( \phi_B(x, t) \) are given by Eqs. (2.13) and (2.20). For simplicity, the detector is assumed to be rectangular in shape with length \( L_D \) in the \( x \) direction and \( W_D \) in the \( y \) direction. Consequently, the area of the diode is \( A_D = L_D W_D \). Defining the center location of the detector to be at \( (x_o, y_o) \), the instantaneous power captured by the diode may be expressed as

\[ P_{D}(x_o, y_o, t) = \int_{x_o - 0.5L_D}^{x_o + 0.5L_D} \int_{y_o - 0.5W_D}^{y_o + 0.5W_D} \frac{1}{\eta_s} \left[ E_{A,\text{Dec}}^\ast(x, y, t) + E_{B,\text{Dec}}^\ast(x, y, t) \right]^2 dx dy \]  
(2.126)

\[ = \frac{P_{\text{inc}}}{2} \frac{A_D}{A_{MB}} \left\{ \left[ (1 - F_1)F_2 \xi_{BC} + (1 - F_1)F_1 \right] 
\right.

\left. - \left[ (1 - F_1)F_2 \xi_{BC} \sin(2[\omega_o + \Delta \omega]t - 2[\phi_A + \tilde{\varphi}_{BC}(t) + \phi_{AD}]) \right.
\right.

\left. + \frac{(1 - F_2)F_1}{2 \nu_2} \sin(2\omega_o t - 2[\phi_B(x_o, t) + \phi_{BD}]) \sin(2\nu_2) \right] \right.

\left. + 2\sqrt{(1 - F_1)(1 - F_2)} F_1 F_2 \xi_{BC} \sin(\psi_2) \cos(\Delta \omega t - \Delta \Phi(x_o, t)) \right.

\left. - \cos(2\omega_o t - \Phi_{AB}(x_o, t)) \right\} \}

where

\[ \Delta \Phi(x, t) = -\tilde{\psi}_1(x) - \Delta \tilde{\psi}(t) + \tilde{\varphi}_{BC}(t) + \phi_{AD} - \phi_{BD} \]  
(2.127a)

\[ \Phi_{AB}(x, t) = \phi_A + \tilde{\varphi}_{BC}(t) + \phi_{AD} + \phi_B(x, t) + \phi_{BD} \]  
(2.127b)
\begin{equation}
\tilde{\psi}_1(x) = \beta_o \left( n_p - n_o \right) \frac{L}{h} x + \psi_1 \tag{2.128}
\end{equation}

and \( \psi_1, \psi_2, \) and \( \Delta \tilde{\psi}(t) \) are respectively given by Eqs. (2.50b), (2.50c), and (2.50e). The time dependence for \( \Delta \tilde{\psi}(t) \) in Eq. (2.50e) exists in both \( \Delta n_s = \Delta n_s(t) \) and \( \Delta l_s = \Delta l_s(t) \) [Refer to Eq. (2.120).] resulting from applied pressure and/or, in part, an applied large electric field. The subscript \( \alpha \) on \( x \) and \( y \) will now be dropped with the understanding that \((x,y)\) represents the placement of the center of the photodiode detector in the plane a distance \( l_D \) behind the second beam splitter relative to the axis of the beam when the device under test is undisturbed. Because the Bragg cell and phase change in the samples can be any value and the distances between any element may be varied within the constraints of the Mach Zender geometry, it is possible for the sine and cosine functions in Eq. (2.126) to be a maximum at the same time. Since the photodiode has an input power damage threshold that must be designed around, the worst case scenario for the maximum input power illuminating the diode can be determined from

\begin{equation}
P_{D \text{MAX}} = \frac{A_D}{\eta_o} \left[ E_{oa}^m + E_{ob}^m \right]^2 = \tag{2.129}
\end{equation}

\[ = P_{in} \frac{A_D}{A_{MB}} \left[ (1 - F_1)F_2 \xi_{BC} + (1 - F_2)F_1 + 2 \sqrt{(1 - F_1)(1 - F_2)}F_1F_2 \xi_{BC} \right] \]

In the case in which the beam splitters divide the power impinging on the element in half \( (F_1=F_2=0.5) \) and assuming that the efficiency of the Bragg cell is 100% \( (\xi_{BC}=1) \), the maximum possible peak power on the diode is \( (P_{in}A_D)/A_{MB} \) which is a consequence of constructive interference between Beams A and B.

As indicated in Fig. 2.9a, the pn junction acts as a capacitance along with some type of load resistance indicating that the photodiode acts as a low pass filter. Assuming that

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$\Delta \omega$ is smaller than the cutoff frequency of this filter, the contribution of the input power that is converted into useful output current of the diode is

$$P_{DI}(x,y,t) = \frac{P_{\text{ino}}}{2} \frac{A_D}{A_{MB}} [(1 - F_1)F_2 \xi_{BC} + (1 - F_2)F_1]$$

$$+ P_{\text{ino}} \frac{A_D}{A_{MB}} \sqrt{(1 - F_1)(1 - F_2)F_1F_2 \xi_{BC}} \frac{\sin(\psi_2)}{\psi_2} \cos(\Delta \omega t - \Delta \Phi(x,t))$$

The responsivity, $R$, of the photodiode is the ratio of the output current generated by the diode to the power impinging on the diode with frequency within the band pass. Therefore, the output diode current $i(x,t)$ is

$$i(x,y,t) = R P_{\text{ino}} \frac{A_D}{A_{MB}} [(1 - F_1)F_2 \xi_{BC} + (1 - F_2)F_1]$$

$$+ R P_{\text{ino}} \frac{A_D}{A_{MB}} \sqrt{(1 - F_1)(1 - F_2)F_1F_2 \xi_{BC}} \frac{\sin(\psi_2)}{\psi_2} \cos(\Delta \omega t - \Delta \Phi(x,t))$$

As illustrated in Fig. 2.18, the signal is amplified. Since one is only interested in the time varying portion of the signal, the amplifier is capacitively coupled to the detector. A typical amplifier circuit with noise sources is shown in Fig. 2.12. The coupling capacitor is not directly shown here but may be seen in Fig. 2.10. As indicated by Eq. (2.88), the AC output voltage is

$$v_{\text{out}}(x,y,t) = -R_f i(x,y,t) =$$

$$-R_f R P_{\text{ino}} \frac{A_D}{A_{MB}} \sqrt{(1 - F_1)(1 - F_2)F_1F_2 \xi_{BC}} \frac{\sin(\psi_2)}{\psi_2} \cos(\Delta \omega t - \Delta \Phi(x,t))$$

A bandpass filter with center frequency $\Delta \omega$ and 3dB cutoff frequencies that pass the frequency signature of the signal being detected. The signal is the modulation of the laser beam in path B due to dynamic pressures acting on the plastic sample in this path. The
bandpass filter isolates the signal being detected and removes much of the remaining spectrum which contains noise and other signal contributions not considered.

The bandpass filter itself generates noise but this is small compared to the amplified signal containing noise from previous stages in the optical/electrical system and the desired signal itself. Consequently, the noise contributions from this stage onward are negligible as compared to previous stages and need not be considered in the noise network.

To demodulate the signal thereby removing the Bragg cell contribution of frequency and time varying phase, a square law detector such as a diode-like device is used. To remove the Bragg cell contribution from the output voltage of the bandpass filter, a local oscillator must be synchronized with the Bragg cell and signal so to remove the frequency component $\Delta \omega$ and the time varying phase component $\tilde{\phi}_{BC}(t)$. There are a number of circuit configurations for demodulating low frequency signals. A square law detector based on voltage is chosen in the demodulation process. Therefore, the demodulated signal voltage, $v_{DMS}$, with linear and square law behavior, may be expressed as

$$v_{DMS}(x,y,t)=\alpha_1[v_{LO}(t)+v_{oMC}(x,y,t)]+\alpha_2[v_{LO}(t)+v_{oMC}(x,y,t)]^2$$  \hspace{1cm} (2.133)

where the synchronized local oscillator voltage is

$$v_{LO}(t)=V_{LO} \cos(\Delta \omega t - \tilde{\phi}_{BC}(t) + \phi_{LO})$$  \hspace{1cm} (2.134)

The quantities $\alpha_1$ and $\alpha_2$ are the linear and square law coupling coefficients. The demodulated voltage signal is
\[ v_{DMS}(x, y, t) = \alpha_1 \left[ V_{vo} \cos(\Delta \omega t - \phi_{BC}(t) + \phi_{LO}) \right] \]

\[ - R_f R_{P_{ino}} A_D \frac{A_D}{A_{MB}} \sqrt{(1 - F_1)(1 - F_2)F_1 F_2 \xi_{BC}} \sin(\psi_2) \cos(\Delta \omega t - \Delta \Phi(x, t)) \]

\[ + \alpha_2 \left[ \frac{1}{2} \left( R_f R_{P_{ino}} A_D \frac{A_D}{A_{MB}} \sin(\psi_2) \right)^2 (1 - F_1)(1 - F_2)F_1 F_2 \xi_{BC} \right] \]

\[ + \frac{1}{2} \frac{V_{vo}^2 \cos[2(\Delta \omega t - \phi_{BC}(t) + \phi_{LO})]}{\psi_2} \]

\[ + \left( R_f R_{P_{ino}} A_D \frac{A_D}{A_{MB}} \sin(\psi_2) \right)^2 (1 - F_1)(1 - F_2)F_1 F_2 \xi_{BC} \cos[2(\Delta \omega t - \Delta \Phi(x, t))] \]

\[ - \left( V_{vo} R_f R_{P_{ino}} A_D \frac{A_D}{A_{MB}} \sqrt{(1 - F_1)(1 - F_2)F_1 F_2 \xi_{BC}} \sin(\psi_2) \right) \]

\[ \times \left[ \cos(2 \Delta \omega t - [\Delta \Phi + \phi_{BC}(t) + \phi_{LO}]) \right] + \cos(-\psi_1(x) - \Delta \psi(t) + \phi_{LO} + \phi_{AD} - \phi_{BD}) \}} \]

With the aid of an ac coupling capacitor and a low pass filter with a cutoff frequency below \( \Delta \omega \) yields the desired signal form given by

\[ v_{DMSAC}(x, y, t) = -\alpha_2 V_{vo} R_f R_{P_{ino}} A_D \frac{A_D}{A_{MB}} \sqrt{(1 - F_1)(1 - F_2)F_1 F_2 \xi_{BC}} \sin(\psi_2) \]

\[ \times \cos(\psi_1(x) + \Delta \psi(t) - \phi_{LO} - \phi_{AD} + \phi_{BD}) \] (2.136)

Choosing the local oscillator phase shift to be \( \phi_{LO} = -\phi_{AD} + \phi_{BD} \) and rearranging Eq.(2.136) yields

\[ v_{DMSAC}(x, y, t) = -\alpha_2 V_{vo} R_f R_{P_{indTAC}}(x, y, t) \] (2.137a)

where

\[ P_{indTAC}(x, y, t) = P_{ino} A_D \frac{A_D}{A_{MB}} \sqrt{(1 - F_1)(1 - F_2)F_1 F_2 \xi_{BC}} \sin(\psi_2) \cos(\psi_1(x) + \Delta \psi(t)) \] (2.137b)

Equation (2.137b) has the same form as the associated time varying portion of Eq. (2.51b). Note that \( \Delta \psi \) in that relation would contain the time dependence. If \( \xi_{BC} = 1 \) and \( F_1 = F_2 = 1/2 \), Eq. (2.137b) yields the AC portion of Eq. (2.51b) identically when one
realizes that $P_{\text{in}} = 2P_{\text{in}}$ and $\tilde{\psi}_1(x)|_{x=0} = \psi_1$ implying that the center of the detector is located at the origin as assumed in those analysis.

Heterodyning with suitable electronics has recovered the desired signal. Consequently, the signal-to-noise ratio for the AC case with an amplifier given by Eqs. (2.103a,b) are the same signal-to-noise ratios for the heterodyning case with a modification in an overall constant and a narrower noise bandwidth due to the bandwidth filter with center frequency about the modulation frequency of the Bragg cell. For an equal comparison, the change in the AC power incident onto the diode given by Eq. (2.137b) is approximated by Eqs. (2.68a-e). Upon examining Eq. (2.137b) for small $\Delta \psi(t)$,

$$\cos(\tilde{\psi}_1(x) + \Delta \tilde{\psi}(t)) \approx \cos \tilde{\psi}_1(x) - \Delta \tilde{\psi}(t) \sin \tilde{\psi}_1(x) - \frac{[\Delta \tilde{\psi}(t)]^2}{2} \cos \tilde{\psi}_1(x)$$

The change in length resulting from compression due to light pressures can be neglected implying that $\Delta l_S(t)$ in $\Delta \tilde{\psi}(t)$ is zero. From Eqs. (2.23b), (2.25b) and (2.50e), a linear change in pressure yields a linear change in index of refraction which in turn justifies the linear approximation Eqs. (2.68a-e) for $P_{\text{ind AC}}$ when linear order terms in $\Delta \tilde{\psi}(t)$ dominate. Under this formalism, the change in the power incident on the diode between a stressed and unstressed condition is then approximated by Eq.(2.54b) with a second order correction term in $\Delta \psi$ retained.

For completeness, the SNR for the heterodyne case is
\[ \text{SNR}_{\text{HEV}} = 10 \log \left( \frac{\text{time averaged signal}}{\text{power noiseless circuit}} \right) \left( \frac{\text{noise power}}{} \right) \]

\[ = 10 \log \left( \frac{1}{T_0} \int_0^T V_{\text{imac}}^2(x,y,t) dt \right) \]

\[ = 10 \log \left( \frac{1}{R_f} \frac{R_f}{R_L} \int_0^T \left( P_{\text{ind}} - P_{\text{ind}} \right)^2 dt \right) \]

\[ = 10 \log \left( \frac{1}{3} \alpha^2 R_c^2 V_{\text{o2o}}^2 \frac{1}{T_0} \int_0^T \left( P_{\text{ind} - P_{\text{ind}}} \right)^2 dt \right) \]

\[ = 10 \log \left( \frac{1}{3} \alpha^2 R_c^2 V_{\text{o2o}}^2 \frac{1}{T_0} \int_0^T \left( P_{\text{ind} - P_{\text{ind}}} \right)^2 dt \right) \]

where

\[ P_{\text{in}} = P_{\text{m0}} \sqrt{(1 - F_1)(1 - F_2) F_1 F_2 \xi \omega} \quad (2.138b) \]

and \( \Delta \psi \) is given by Eq. (2.59). The denominator of Eqs. (2.103b) and (2.138) differ by the span of the bandwidth. This is the reason for the tilde \( \sim \) over \( B \) in Eq. (2.138). As indicated above, much of the noise spectrum has been filtered away due to the bandpass filter just after the amplifier with center frequency about the Bragg cell modulation frequency, \( \Delta \omega \). It is understood that the load resistor \( R_L \) in this case is the resistance in the final stage of the circuit just after the coupling capacitor and a lowpass filter with a cutoff frequency below \( \Delta \omega \).
2.7.2 Alternative frequency mixing detection systems

In homodyne detection like in heterodyne detection, a beam from the output of the interferometer beats with a beam from the local oscillator resulting in a frequency shifted wave accentuating the small interferometry signal riding on the carrier frequency which is the laser frequency. In this, the beam of the laser local oscillator is not modulated i.e., the beam is not frequency shifted. The homodyne detection system that has been studied in this case is based on the system reviewed by A. Rogalski and Z. Bielecki [26] and is illustrated in Fig. 2.21. Such a system is basically conventional interferometry. An alternative design for homodyne and heterodyne detection may be accomplished using two colors of light where in the latter case one color may be modulated at a lower frequency. A brief discussion is presented with no attempt to study the signal to noise ratio.

Figure 2.22 displays an interferometer using two colors of light, one color for the reference beam and the second color in the path interrogating the sample under test. Just after the second beam splitter which is used to linearly combine (linearly mix) the two beams, the resultant wave is transmitted through a nonlinear crystal based on a linear behavior and a square law (nonlinear) mixing behavior. Although the crystal is nonlinear, some degree of linear behavior may be present and hence it has been incorporated into the theory. The field contribution of path A and that of path B just before the nonlinear crystal may be expressed as

\[ E_A(x, y, t) = E_{aA} \cos(\omega_A t + \Phi_A) \]  
\[ E_B(x, y, t) = E_{aB} \cos(\omega_B t + \Phi_B) \]

(2.139a)

(2.139b)

Assuming that the transmitted field properties of the crystal may be treated as
\[ E_C(x,y,t) = a_1[E_A(x,y,t) + E_B(x,y,t)] + a_2[E_A(x,y,t) + E_B(x,y,t)]^2, \]  

(2.140)

the resultant field, neglecting any phase shifts due to the crystal, may be rewritten as

\[ E_C(x,y,t) = a_1[E_{oa} \cos(\omega_A t + \Phi_A) + E_{ob} \cos(\omega_B t + \Phi_B)] 
+ a_2\left[\frac{1}{2}(E_{oa}^2 + E_{ob}^2) + \frac{1}{2}(E_{oa}^2 \cos(2\omega_A t + 2\Phi_A) + E_{ob}^2 \cos(2\omega_B t + 2\Phi_B))\right] 
+ E_{oa}E_{ob}\left[\cos(\omega_A + \omega_B) t + [\Phi_A + \Phi_B]\right] \cos(\omega_B - \omega_A) t + [\Phi_B - \Phi_A]] \]  

(2.141)

The static component of the field does not propagate from the crystal. If the frequency of the beam in path B is the second harmonic \((\omega_B = 2\omega_o)\) of the beam in path A with fundamental frequency (first harmonic) \(\omega_A = \omega_o\), then, \(\omega_B - \omega_A = \omega_o\) and \(\omega_B + \omega_A = 3\omega_o\). The term of interest contains the phase difference namely \([\Phi_B - \Phi_A]\) which is associated with the first harmonic. An optical bandpass filter is placed after the nonlinear crystal to pass only the fundamental component of the beam yielding, with neglect of phase shift contributions,

\[ E_{BP}(x,y,t) = a_1E_{oa} \cos(\omega_A t + \Phi_A) + a_2E_{oa}E_{ob} \cos(\omega_A t + \Phi_B - \Phi_A)] \]  

(2.142)

It is possible that the linear and nonlinear portion of the resultant beam may be separated as a result of the birefringence leading to a physical separation of the modes as they propagate through the medium and as a result of a change in polarization. Assuming this separation can be performed, the final wave component that exists contains information on the change in the phase of the two beams in the interferometer as given by

\[ E_{BP}(x,y,t) = a_2E_{oa}E_{ob} \cos(\omega_B t + [\Phi_B - \Phi_A]) \]  

(2.143)

Now one may apply heterodyning and homodyning techniques to this wave following the block diagrams in Figs. 2.18 and 2.19 respectively. The theory for the former has already been solved in detail in the previous section. The added crystal and optical bandpass crystal will have an affect on the coherent properties of the laser. If the
polarization of the wave is affected due to the nonlinear crystal, the polarization of the reference beam will have to be adjusted for desired wave interference to occur on the surface of the photodiode. These issues along with alignment and the use of two different frequencies of light may make this a very challenging set-up.

2.8 Numerical results

Based on the theory above, a numerical study is performed to determine how complex the optical/electrical setup must be in order to detect a small change in the index of refraction based on interferometry. Both the maximum power that may be supplied to the photodiode and the signal-to-noise ratio are studied. It is anticipated that small fringe shifts will result as the pulsed electromagnetic energy causes a momentary compression force on the electrodes supporting the plastic under test. Although the SNR may be large when the sample is in a fixed end state, the information sought may not be detectable since the difference in the end states may be negligible in comparison to the magnitude of either end state. In the so called AC state (AC modulation), the DC components of the signal are blocked offering higher resolution to detect changes in the signal proper. Such resolutions may be essential when fixed end states are not accessible within the sampling time duration. Theoretical studies are performed with a 633 nm and a 532 nm laser beam.

The SNR and the load voltage as measured across a 50 Ω load are examined in the case when the detection system consists of a:

1.) diode detector with resistor load without (Eqs. 2.50b-2.50e, 2.51b, 2.51c, 2.75, 2.76), and with an AC coupling capacitor (Eqs. 2.50b-2.50e, 2.51b, 2.52, 2.84, 2.85b);
2.) diode detector cascaded with an amplifier and a 50 Ω load resistor without (Eqs. 2.50b-2.50e, 2.51c, 2.86, 2.99 with \( P_{\text{ind}}(t) = P_{\text{ind}} w, i = R P_{\text{ind}} = R P_{\text{ind}}^w, 2.102 \)) and with an AC coupling capacitor (Eqs. 2.50b-2.50e, 2.52, 2.88, 2.99, 2.103a);

and

3.) heterodyning technique (Eqs. 2.50b-2.50e, 2.52, 2.138a, 2.138b).

In all simulation studies, \( \Delta \hat{\psi}(t) \) and \( \Delta \psi(t) \) were not considered to be small. The voltages at the 50 Ω load resistor in each scheme above is deduced from the time averaged power in the numerator of the SNR relations. The shot noise in all cases has been with respect to the power incident on the diode for the undisturbed sample condition. Table 2.1 provides a number of common parameters describing the properties of the Rexolite sample, the probing laser beam, the photo-detector diode, and the wedge phase shifter. Some common constants are also listed. To correspond theory, modeling, and experiment, 0.03, 0.3, 3.0, and 32.45 N compression forces are applied to the 2.54 cm diameter Rexolite sample. With the aid of Eq. 2.25b, the respective strains experienced by the plastic for these static forces are 19.1x10^{-9}, 191x10^{-9}, 1.91x10^{-6}, and 20.6x10^{-6} respectively. Calculations from Chapter 1 have estimated a 32.45 N force to be experienced by the electrode plates as a result of the intense electric field. The dynamic changes will be substantially less. Three orders of magnitude sensitivity in the detection system are examined.

Damage thresholds of the optical components and the maximum energies/powers supplied by the laser limits the light intensity that can be employed in the optical system. Thorlab's mirrors for 532 nm light have damage thresholds on the order of 1 MW/cm^2 for CW lasers and 5 J/cm^2 for 10 ns pulses. Melles Groit beamsplitters tend to have a
damage threshold of about 1.5 J/cm$^2$ in a 1 ns pulse. Thorlab’s beamsplitter cubes have a 100W/cm$^2$ damage threshold for a 532 nm CW laser beam. The Rexolite plastic under test may withstand up to 100 mJ in a 6.5 mm diameter circular region (damage energy density threshold 300mJ/cm$^2$) from a 5 ns, 532 nm light pulse with a pulse repetition rate of 10 Hz [3]. The photo-detector has a power damage threshold of 10 mW for CW power and 20μJ for a 20 ns pulse. Although not specifically stated, it is assumed that power is to be uniformly distributed over the surface of the detector. The pulsed Nd:YAG laser with seeder can generate about 340 mJ of 532 nm light in a 3 to 5 ns pulse. The linewidth at this wavelength is 0.005 cm$^{-1}$. It was decided to constrain the laser power to be less than 200 mW focused to a spot size of 2 mm with a maximum power incident on the diode to be between 4.0 to 4.5 mW. Because a continuous laser affords one to follow the fringe shift with a video camera, a 5 mW CW HeNe laser with a 2 mm spot size is used in the experimental feasibility study associated with the experimental component of this effort. Consequently, most of this analysis will be directed to the 633 nm wavelength with power levels reaching 200 mW. These studies are then compared with some select studies at 532 nm.

Tables 2.2 – 2.4 provide the parameters for the three detection systems. The amplifier used in the heterodyne setup is equivalent to that used in the load resistor with amplifier setup. The amplifier parameters may be found in Table 2.3.

The properties of the wedge phase shifter [Refer to Eqs. (2.50b) and (2.50c)] are used to adjust the wavelength of the power intensity of the fringe over the plane of the detector [Refer to Eqs. (2.28), (2.38), and (2.39a)] and to position the appropriate phase of the fringe on the detector surface [Eq. (2.19)]. Wedge considerations exist in the terms
\( \psi_1 \) [Eq. (2.50b)] and \( \psi_2 \) [Eq. (2.50c)]. To explore the properties of the SNR and the load voltages as a function of the wedge properties, select values for \( \psi_1 \) are chosen between 0 and \( 4\pi \). Although the values are not uniformly distributed in phase, a good representation of the phases are examined. With the tip of the wedge located at \(-1\) m from the undisturbed beam axis, the properties of the wedge, \( \beta_n (n_p - n_o) \frac{L}{h} \), may be determined such that \( \psi_2 \) may be determined given the length of the active area of the photo-detector, \( L_D \). The center of the photo-detector lies on the undisturbed beam axis assuming refraction effects are not present. Because \( \psi_1 \) and \( \psi_2 \) are typically orders of magnitude different, a second set of cases are evaluated at each force. Select representative values of \( \psi_2 \) between 0 and \( 4\pi \) are chosen. Again, these values are not necessarily chosen at equal intervals in phase. Knowing \( L_D \), the properties of the wedge can be determined and \( \psi_1 \) is computed. Tables 2.5-2.8 provide the phase values examined for each of the three techniques appropriately with and without a beam modulation signature. Since experimental studies could resolve changes on the order of a Newton and because minimum detection thresholds are of interest, typical figures characterizing the properties of the optical-electrical system are presented. Figures 2.23a-i corresponds to the 0.03 N case and Figs. 2.24a-i to the 3 N case both for \( \psi_1 = 11.304 \). The Matlab code used to generate these graphs are provided in Appendix A. Programs 1 and 2 are for the 633 nm case and programs 3 and 4 are for the 532 nm case in Appendix A. It is observed in Figs. 2.23b and 2.24b that a laser input power of 200 mW may damage the diode since the power incident on the diode exceeds the 5 mW constraint. Even though the damage threshold is 10 mW, further shifts in the fringe or a non-uniform power distribution may
affect the lifetime of the diode. The 5 mW safety factor will guard against possible faults. To maintain this constraint and have optimal detection, the input of the laser for the 0.03 N case must be at 150 mW while the 3 N case must be about 125 mW. Note in the latter case, as the fringe shifts, a noticeable increase in light intensity is observed. If the optical system is biased with an input light intensity of 150 mW, the light impinging on the diode surface approaches the 5 mW level of the photo-diode. Figures 2.23c and 2.24c displays the change in the incident power on the diode between the unstressed and stressed cases. Once the input power of the laser has been established, Figs. 2.23a and 2.24a provides the SNRs for each detection constraint. If one considers just the AC modulation detection schemes alone [DC signal is blocked], it is observed that the signal-to-noise ratio increases as the detection scheme changes from the load resistor case to the amplifier with load resistor scheme to the heterodyne cases. When the DC signal is measured at the load resistor, the change in power between the stressed and unstressed cases are not distinguishable when the applied force is 0.03 N. For the 3 N case, an 18% change in the signal may be observed. From the numerators of Eqs. (2.75) [or (2.76)] and (2.100), it can be shown that the load voltage of the DC modulation detection scheme with amplifier is related to the load voltage of the detection scheme without amplifier as

\[
V_{LAMPDC} = \frac{R_f}{R_L} V_{LRDLC}
\]  

(2.144)

For a 10 kΩ feedback resistor, the voltage is amplified by a factor of 200. Although the SNR for the DC coupled signal is well above the noise level, detecting small shifts in the DC level is difficult. Blocking the DC signal offers a means to resolve small changes in the signal. Figure 2.23a tends to suggest that some of the AC coupled schemes will not
distinguish between the signal and the noise. Examining Eq. (2.138a), changes in the local oscillator voltage scales the heterodyne load voltages as

\[ v_{L,HET2} = \frac{V_{o,LO2}}{V_{o,LO1}} v_{L,HET1} \]  

(2.145)

By examining the numerators of Eqs. (2.103a) and (2.138a), the heterodyne load voltage is directly related to the AC coupled load voltage (assuming 50%-50% beamsplitters and Bragg cell transmission efficiencies of one) is

\[ v_{L,HET} = \alpha_2 V_{o,LO} v_{LAMPAC} \]  

(2.146)

This implies that the local oscillator voltage, if properly synchronized to the incoming electrical signal, has the potential of enhancing the signal to be detected in the demodulation process. Figures 2.23d-i and 2.24d-i displays how the load voltages vary with respect to either the incident power on the diode or the change in the incident power on the diode. Tables 2.5 -2.8 shows tendencies in the load voltages and the SNRs at the near optimum laser input powers within the constraints set. It is observed that the phase shifting wedge plays a significant role on the noise voltage and SNR over a wide range of forces considered. It was observed that for most points chosen, in the 3 N (as well as other cases) the SNR and load voltage were similar to other wedge phase values. An intermediate point for \( \psi_i \) was chosen to determine if this was a coincident. When \( \psi_i = 4.762 \) (blue text in Table 2.7), indeed the pattern observed was a coincidence. The phase is just right that the measurables become substantially small compared to other recorded values. Other non-recorded intermediate values in phase yield the same result. A similar observation was observed for green light as well. For all practical purposes, it appears that static forces applied from equilibrium on the order of \( 10^2 \) N (strains of

108
6x10^9) and larger are detectable based on the validity of the optical – electrical modeling studies presented.

Tables 2.9 and 2.10 display the same load voltage and SNR analyses based on the above constraints set for 532 nm laser source. The phase shifts resulting from ψ₁ and ψ₂ have been forced equivalent to that for the 633 nm light. There is a noticeable similarity in the results. Consequently, tendencies in the 633 nm study carry over to the 532 nm study.
Fig. 2.1. The Michelson interferometer.
Fig. 2.2. The Mach Zender interferometer.

Fig. 2.3. The optical circuit diagram of an interferometer setup.
Fig. 2.4. Mach Zender interferometer arrangement for detecting changes in refraction to a sample under test relative to a sham sample. 1. Laser. 2. Beam splitter. 3. Mirror. 4. Right angle wedge. 5. Sample under test. 6. Beam splitter. 7. Sample NOT under test. 8. Mirror. The beam splitter (2) generates two beams A and B. The relative geometrical path lengths are: $l_{A1} + l_{S1} + l_{A2} = l_{B1} + l_{S2} + l_{B2}$ and $l_{A3} = l_{B3}$. The length of the base of a right angle wedge is $L$. The length $\Gamma$ is dependent on the relative vertical position of the beam.
Fig. 2.5. Right angle wedge used for fringe generation.

Fig. 2.6. Illustrating the compression of the plastic, $\Delta l_c$, under test to its total unstressed vertical length $l_T$. The change in the length of the sample along the direction of the laser beam is $\Delta l_s$. Based on Poisson's ratio, $\sigma_P = \frac{\Delta l_c/l_T}{\Delta l_s/l_{s2}}$, one may estimate this length relative to the length the plastic is compressed.
Fig. 2.7. The fringe generating right angle wedge. The distance $H$ and $\Gamma$ are respectively the distance of the refracted beam and the distance between the wedge and the beam splitter. The angle between the undisturbed path of light relative the refracted light beam is $\Delta \theta = \theta_1 - \theta_2$. 

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Legend

D diameter of detector
λx spatial wavelength

Fig. 2.8. Detector size resolving a fraction of a fringe.

Fig. 2.9. Circuit diagram representing the photodetector in the RC load case (a) showing all the current sources. (b) all the dc and ac sources summed up separately to allow for a single dc source and a single ac source representation. (c) a single current source representing both dc and ac current sources.

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Fig. 2.10 Circuit diagram showing the photodetector with a resistance as load (AC case).

Fig. 2.11. Pin configuration of an AD8015 differential output transimpedance amplifier.
Fig. 2.12. Circuit diagram of the photo receiver circuit (photodetector and the transimpedance amplifier)

Fig. 2.13. Considering $i_1$ as the only active source (superposition)
Fig. 2.14. Replacing the parallel resistances by an equivalent resistance

Fig. 2.15. Thevenin equivalent circuit
Fig. 2.16. Considering the feedback current as the only active source.

Fig. 2.17. Considering the noise voltage as the only active source.
Fig. 2.18. Heterodyne detection optical receiver
Fig. 2.19. Interferometer used in heterodyne detection with Bragg cell placed in one of the arms of the interferometer.
Fig. 2.20. Possible interferometer setup used in heterodyne detection with Bragg cell placed in exterior of the Mach Zender interferometer.
Fig. 2.21. Homodyne detection optical receiver

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Fig. 2.22. Homodyne detection optical receiver with two colors of light being used in the Mach Zender interferometer.
$P_{\text{in}}$, Input Power of the laser (W) vs SNR (dB)  

Force $= 0.03 \text{N}$  
$\psi_1 = 11.304$  
$\psi_2 = 0.0022608$  
$\lambda = 633\text{nm}$

(a)

$P_{\text{ind}} (W)$ vs $P_{\text{in}} (W)$

$\psi_1 = 11.304$  
$\psi_2 = 0.0022608$

(b)
$\Delta P_{\text{ind}}$ (W) vs $P_{\text{in}}$ (W)

\[ \psi_1 = 11.304 \quad \psi_2 = 0.0022608 \]

(c)

$P_{\text{ind}}$ (W) vs $V_L$ (V) Resistor Load Unstressed DC Case

\[ \psi_1 = 11.304 \quad \psi_2 = 0.0022608 \]

(d)
$P_{\text{ind}} (W) \text{ vs } V_L (V)$ Resistor Load Stressed DC Case

$\psi_1 = 11.304 \quad \psi_2 = 0.0022608$

![Graph showing $P_{\text{ind}} (W) \text{ vs } V_L (V)$ for Resistor Load Stressed DC Case.](image)

$\Delta P_{\text{ind}} (W) \text{ vs } V_L (V)$ Resistor Load AC Case

$\psi_1 = 11.304 \quad \psi_2 = 0.0022608$

![Graph showing $\Delta P_{\text{ind}} (W) \text{ vs } V_L (V)$ for Resistor Load AC Case.](image)
P_{\text{ind}} (W) vs V_L (V) Amplifier Load DC Case

\begin{align*}
\psi_1 &= 11.304 \\
\psi_2 &= 0.0022608
\end{align*}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{dc_case_graph}
\caption{(g) Power Incident on diode (W) vs Load voltage (V) for Amplifier Load DC Case.}
\end{figure}

\Delta P_{\text{ind}} (W) vs V_L (V) Amplifier Load AC Case

\begin{align*}
\psi_1 &= 11.304 \\
\psi_2 &= 0.0022608
\end{align*}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{ac_case_graph}
\caption{(h) Change in power Incident on diode (W) vs Load voltage (V) for Amplifier Load AC Case.}
\end{figure}

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Fig. 2.23. A 0.03 N force is applied to a 2.54 cm diameter Rexolite cylinder about 2.54 cm in length.  

a) The signal-to-noise ratio for each of the detection schemes as a function of the input laser power is presented. Large SNR for the DC cases do not imply that the change in the signal signatures can be resolved.  

(b,c) The power incident on the diode and the change in the power incident on the diode can be determined based on the input laser power. These two graphs aid in determining the conditions required in order that power (power density) constraints in the optical system are maintained below damage thresholds.  

(d-i) The load voltages are obtained knowing either the incident power or the change in the incident power on the photo-diode surface.
Input Power of the laser (W) vs SNR (dB)

Force = 3N \quad \psi_1 = 11.304 \quad \psi_2 = 0.0022608 \quad \lambda = 633\text{nm}

---

(b)
P_{\text{ind}} (W) vs V_{L} (V) Amplifier Load DC Case

\begin{align*}
\psi_1 &= 11.304 \\
\psi_2 &= 0.0022608
\end{align*}

\begin{align*}
\Delta P_{\text{ind}} (W) &\text{ vs } V_{L} (V) \text{ Amplifier Load AC Case} \\
\psi_1 &= 11.304 \\
\psi_2 &= 0.0022608
\end{align*}
Fig. 2.24. A 3 N force is applied to a 2.54 cm diameter Rexolite cylinder about 2.54 cm in length. a) The signal-to-noise ratio for each of the detection schemes as a function of the input laser power is presented. Large SNR for the DC cases do not imply that the change in the signal signatures can be resolved. (b,c) The power incident on the diode and the change in the power incident on the diode can be determined based on the input laser power. These two graphs aid in determining the conditions required in order that power (power density) constraints in the optical system are maintained below damage thresholds. (d-i) The load voltages are obtained knowing either the incident power or the change in the incident power on the photo-diode surface.
Table 2.1 General parameters for AC and DC modulation computational studies relating the input power of the laser to the measured voltage at a load resistance using simple load resistor, amplifier with load resistor, and heterodyning techniques.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge of an electron, $e$</td>
<td>$1.6 \times 10^{-19}$ C</td>
</tr>
<tr>
<td>Wavelength of HeNe laser beam, $\lambda$</td>
<td>$633,nm$ and $532,nm$</td>
</tr>
<tr>
<td>Diameter of the beam, $d_b$</td>
<td>$2 \times 10^{-3}$ m</td>
</tr>
<tr>
<td>Area of the beam, $A_{LB}$</td>
<td>$3.14 \times 10^{-6}$ m$^2$</td>
</tr>
<tr>
<td>Length of the active area of the EOT 2030A detector, $L_D$</td>
<td>$400 \times 10^{-6}$ m</td>
</tr>
<tr>
<td>Active area of the EOT 2030A detector, $A_D$</td>
<td>$1.256 \times 10^{-7}$ m$^2$</td>
</tr>
<tr>
<td>EOT 2030A amplifier gain</td>
<td>26 dB</td>
</tr>
<tr>
<td>Damage threshold power of the photo-detector, $P_{th}$</td>
<td>$10,mW$</td>
</tr>
<tr>
<td>Dark current of the photo-detector, $i_d$</td>
<td>$0.1$ nA</td>
</tr>
<tr>
<td>Responsivity of the photo-detector, $\mathcal{R}$</td>
<td>$0.4$ A/W</td>
</tr>
<tr>
<td>Length of the Rexolite sample under test, $l_{S1}$</td>
<td>$25.4$ mm</td>
</tr>
<tr>
<td>Height of the Rexolite sample under test, $l_T$</td>
<td>$25.4$ mm</td>
</tr>
<tr>
<td>Refractive index of the Rexolite sample under test, $n_S$</td>
<td>$1.585$</td>
</tr>
<tr>
<td>Opto Elastic constant of the Rexolite sample under test, $\rho$</td>
<td>$0.31$</td>
</tr>
<tr>
<td>Young's modulus of Rexolite, $Y_M$</td>
<td>$3.1$ GPa</td>
</tr>
<tr>
<td>Refractive index of the glass plate wedge, $n_p$</td>
<td>$1.5$</td>
</tr>
<tr>
<td>Refractive index of free space, $n_a$</td>
<td>$1$</td>
</tr>
<tr>
<td>Location of the tip of the wedge, $x_0$</td>
<td>$-1$ m</td>
</tr>
<tr>
<td>Temperature at which the experiment is conducted (ambient temperature)</td>
<td>$293^\circ K$</td>
</tr>
<tr>
<td>Boltzmann constant, $k$</td>
<td>$1.38 \times 10^{-23}$</td>
</tr>
<tr>
<td>Speed of light, $c$</td>
<td>$3 \times 10^8$ ms$^{-1}$</td>
</tr>
<tr>
<td>Loads that are assumed to be acting on the sample, $F$</td>
<td>$0.03$, $0.3$, $3$, $32.45$ N</td>
</tr>
<tr>
<td>Maximum laser power, $P_{in}$</td>
<td>$200$ mW</td>
</tr>
<tr>
<td>Pulse duration of laser, $t_p$</td>
<td>$5 \times 10^{-9}$ s</td>
</tr>
<tr>
<td>Equivalent maximum energy of the laser, $E_{in} = P_{in}t_p$</td>
<td>$1.0 \times 10^{-9}$ J</td>
</tr>
<tr>
<td>Intrinsic impedance of free space, $\eta_0$</td>
<td>$377$ $\Omega$</td>
</tr>
</tbody>
</table>
Table 2.2 Additional input parameters and noise power computations for the simple load resistor setup without amplifier

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Load resistance, $R_L$</td>
<td>50 $\Omega$</td>
</tr>
<tr>
<td>2</td>
<td>Capacitance of the photo-detector, $C_D$</td>
<td>1.5 $pF$</td>
</tr>
<tr>
<td>3</td>
<td>Bandwidth of the photo-detector with $R_L$ load, $B = B_D$</td>
<td>13.33 GHz</td>
</tr>
<tr>
<td>4</td>
<td>Noise power due to the dark current, $i_d$</td>
<td>$2.133 \times 10^{-17} W$</td>
</tr>
<tr>
<td>5</td>
<td>Ratio of the shot noise power due to the laser current output relative to the incident power on the photo-detector</td>
<td>$1.7 \times 10^{-9}$</td>
</tr>
<tr>
<td>6</td>
<td>Noise power due to thermal noise current</td>
<td>0.215 $pW$</td>
</tr>
</tbody>
</table>

Table 2.3 Additional input parameters and noise power computations for the load resistor with amplifier setup

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Feedback resistance, $R_f$</td>
<td>10 $k\Omega$</td>
</tr>
<tr>
<td>2</td>
<td>Shunt resistance, $R_{sh}$</td>
<td>1 $k\Omega$</td>
</tr>
<tr>
<td>3</td>
<td>Alternating current resistance, $R_a$</td>
<td>7 $k\Omega$</td>
</tr>
<tr>
<td>4</td>
<td>Input resistance, $R_i$</td>
<td>5 $k\Omega$</td>
</tr>
<tr>
<td>5</td>
<td>Bandwidth of the amplifier (specs), $B = B_A$</td>
<td>1.2 GHz</td>
</tr>
<tr>
<td>6</td>
<td>Amplifier noise voltage (equivalent noise voltage for amplifier), $v_n$</td>
<td>10.38 nV</td>
</tr>
<tr>
<td>7</td>
<td>Amplifier noise current, $i_n$</td>
<td>26.5 nA</td>
</tr>
<tr>
<td>8</td>
<td>Transresistance of the amplifier, $r_m$</td>
<td>10 $k\Omega$</td>
</tr>
<tr>
<td>9</td>
<td>Noise power due to the dark current $i_d$</td>
<td>$7.68 \times 10^{-14} W$</td>
</tr>
<tr>
<td>10</td>
<td>Ratio of the shot noise power due to the laser current output relative to the incident undisturbed power $P_{ind u}$ on the photo-detector</td>
<td>$1.54 \times 10^{-10}$</td>
</tr>
<tr>
<td>11</td>
<td>Noise power due to the load current resulting from the noise voltage, $v_n$</td>
<td>$4.489 \times 10^{-16} W$</td>
</tr>
<tr>
<td>12</td>
<td>Noise power due to thermal noise current</td>
<td>3.9 $nW$</td>
</tr>
<tr>
<td>13</td>
<td>Noise power due to amplifier noise current $i_n$</td>
<td>$8.374 \times 10^{-17} W$</td>
</tr>
</tbody>
</table>
Table 2.4 Additional input parameters and noise power computations for the heterodyne setup where amplifier parameters are listed in Table 2.3

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Voltage of local oscillator, $V_{\text{LO}}$</td>
<td>2 V and 5 V</td>
</tr>
<tr>
<td>2</td>
<td>Square law coupling coefficient of the Bragg cell, $\alpha_2$</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Power conversion transmission efficiency of Bragg cell, $\xi_{BC}$</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Operating oscillation frequency of the Bragg cell, $f_B$</td>
<td>800 MHz</td>
</tr>
<tr>
<td>5</td>
<td>Rise time of the pulse, $t_r$</td>
<td>10 ns</td>
</tr>
<tr>
<td>6</td>
<td>Bandwidth of the pulse</td>
<td>35 MHz</td>
</tr>
<tr>
<td>7</td>
<td>Approximate bandwidth of input signal to diode, $\tilde{B}$</td>
<td>800 MHz</td>
</tr>
<tr>
<td>8</td>
<td>Beamsplitters, 50% reflectance-50% transmittance; $F_1$ and $F_2$</td>
<td>0.5</td>
</tr>
<tr>
<td>9</td>
<td>Noise power due to the dark current of photo-detector $i_d$</td>
<td>$2.24 \times 10^{-15}$ $W$</td>
</tr>
<tr>
<td>10</td>
<td>Ratio of the shot noise power due to the laser current output relative to the incident power on the photo-detector</td>
<td>$1.02 \times 10^{-16}$</td>
</tr>
<tr>
<td>11</td>
<td>Noise power due to the load current resulting from the noise voltage of amplifier $v_n$</td>
<td>$4.489 \times 10^{-16}$ $W$</td>
</tr>
<tr>
<td>12</td>
<td>Noise power due to thermal noise current</td>
<td>$1.33 \times 10^{-10}$ $W$</td>
</tr>
<tr>
<td>13</td>
<td>Noise power due to amplifier noise current $i_n$</td>
<td>$8.374 \times 10^{-17}$ $W$</td>
</tr>
</tbody>
</table>
Table 2.5 Evaluation of signal-to-noise ratio and load voltages for various phase delays resulting from the wedge phase shifter for a 0.03 N force (19.1x10^9 strain) acting on the sample under test when a HeNe laser (λ=633 nm) probes the sample medium.

<table>
<thead>
<tr>
<th>Phase</th>
<th>$P_{\text{ind}}$</th>
<th>$P_{\text{ind}}$</th>
<th>$P_{\text{in}}$</th>
<th>$\Delta P_{\text{ind}}$</th>
<th>SNR*</th>
<th>SNR, V$<em>{LU}$ [V], V$</em>{LS}$ [V]**</th>
<th>SNR, V$_L$ [mV]</th>
<th>Heterodyne</th>
<th>V$_{LO}$=2.0</th>
<th>Heterodyne</th>
<th>V$_{LO}$=50Ω</th>
<th>Heterodyne</th>
<th>V$_{LO}$=50Ω</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.047</td>
<td>4.5, 150, -8.0</td>
<td>55, 0.09</td>
<td>68, 18</td>
<td>-5, 0.092</td>
<td>8, 18</td>
<td>28, 36</td>
<td>38, 90</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>2.093</td>
<td>2.0, 2.0, 200, -10.8</td>
<td>58, 0.04</td>
<td>65, 8</td>
<td>0, 0.125</td>
<td>12, 25</td>
<td>32, 50</td>
<td>41, 125</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>3.14</td>
<td>6e-6, 6e-6, 200, -6.4e-4</td>
<td>-60</td>
<td>-28</td>
<td>-83</td>
<td>-72</td>
<td>-40</td>
<td>-30</td>
<td></td>
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<tr>
<td>4.187</td>
<td>2.0, 2.0, 200, 10.8</td>
<td>50, 0.04</td>
<td>65, 8</td>
<td>0, 0.125</td>
<td>12, 25</td>
<td>32, 50</td>
<td>41, 125</td>
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<tr>
<td>5.233</td>
<td>4.0, 4.0, 130, 7.0</td>
<td>52, 0.08</td>
<td>68, 160</td>
<td>-6, 0.082</td>
<td>7, 16.5</td>
<td>28, 33</td>
<td>37, 83</td>
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</tr>
<tr>
<td>7.536</td>
<td>4.0, 4.0, 150, -8.8</td>
<td>52, 0.08</td>
<td>68, 16</td>
<td>-5, 0.10</td>
<td>8, 20</td>
<td>30, 40</td>
<td>38, 100</td>
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<td>8.792</td>
<td>0.78, 0.78, 200, -7.3</td>
<td>42, 0.0148</td>
<td>59, 3.2</td>
<td>-2, 0.085</td>
<td>13, 16</td>
<td>35, 34</td>
<td>42, 85</td>
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<tr>
<td>10.048</td>
<td>0.76, 0.76, 200, 7.2</td>
<td>42, 0.0152</td>
<td>59, 3</td>
<td>-2, 0.085</td>
<td>12, 17</td>
<td>33, 34</td>
<td>42, 85</td>
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</tr>
<tr>
<td>11.304</td>
<td>4.0, 4.0, 150, 8.8</td>
<td>52, 0.08</td>
<td>68, 16</td>
<td>-2, 0.085</td>
<td>10, 21</td>
<td>30, 41</td>
<td>38, 103</td>
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</tr>
<tr>
<td>12.56</td>
<td>4.0, 4.0, 100, 3e-2</td>
<td>52, 0.08</td>
<td>68, 16</td>
<td>-52, 3.5e-4</td>
<td>-40, 0.07</td>
<td>-20, 0.14</td>
<td>-12, 0.35</td>
<td></td>
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</tr>
<tr>
<td>$\psi_2$</td>
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<td></td>
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<td>38, 68.8</td>
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<td>68, 17</td>
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<td>-28, 7.3e-3</td>
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<td>52, 0.08</td>
<td>68, 16</td>
<td>-28, 7.5e-3</td>
<td>-18, 1.5</td>
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<td>18, 7.5</td>
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<td>52, 0.075</td>
<td>68, 15</td>
<td>-21, 0.013</td>
<td>-9, 2.7</td>
<td>12, 5.7</td>
<td>20, 14.3</td>
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<td>4.0, 4.0, 200, 0.13</td>
<td>52, 0.08</td>
<td>68, 16</td>
<td>-40, 1.5e-3</td>
<td>-30, 0.3</td>
<td>-8, 0.6</td>
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<td>68, 16.5</td>
<td>-20, 0.017</td>
<td>-8, 3.3</td>
<td>15, 6.08</td>
<td>21, 15.2</td>
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<td>-29, 0.28</td>
<td>-8, 0.58</td>
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<td>68, 16.5</td>
<td>-25, 0.001</td>
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<td>10, 4.0</td>
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<tr>
<td>12.302</td>
<td>4.0, 4.0, 200, -0.225</td>
<td>52, 0.08</td>
<td>68, 16</td>
<td>-37, 0.0027</td>
<td>-22, 0.52</td>
<td>-2, 1.05</td>
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</table>

For all tables* SNR based on unstressed case, ** If a third term does not exist, then it is implied that, V$_{LU}$= V$_{LS}$.
Table 2.6 Evaluation of signal-to-noise ratio and load voltages for various phase delays resulting from the wedge phase shifter for a 0.3 N force (191x10^9 strain) acting on the sample under test when a HeNe laser (λ = 633 nm) probes the sample medium.

<table>
<thead>
<tr>
<th>Phase  ( \psi_1 ) or ( \psi_2 )</th>
<th>( P_{ind} ) ( P_{ind} ) ( P_{in} ) [mW]</th>
<th>DC Mod. ( R_L=50\Omega ) (RLDC)</th>
<th>DC Mod. ( R_L=50\Omega ) (AMPDC)</th>
<th>AC Mod. ( R_L=50\Omega ) (RLAC)</th>
<th>AC Mod. ( R_L=50\Omega ) (AMPAC)</th>
<th>Heterodyne ( V_{LO}=2.0 ) (HETAC)</th>
<th>Heterodyne ( V_{LO}=5.0 ) (HETAC)</th>
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<tbody>
<tr>
<td>( \psi_1 )</td>
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</tr>
<tr>
<td>1.047</td>
<td>4.5, 4.4150, -80</td>
<td>54, 0.09</td>
<td>67, 17.5</td>
<td>15, 0.92</td>
<td>27, 180</td>
<td>48, 360</td>
<td>56, 900</td>
</tr>
<tr>
<td>2.093</td>
<td>2.0, 1.8, 200, -10.8</td>
<td>49, 0.04, 0.036</td>
<td>64, 8</td>
<td>20, 1.24</td>
<td>34, 250</td>
<td>54, 500</td>
<td>63, 1250</td>
</tr>
<tr>
<td>3.14</td>
<td>5.5e-6, 1.75e-3, 200, 1.75</td>
<td>-10, 1.09e-7, 3.5e-5</td>
<td>-28, 2.2e-5</td>
<td>-18, 0.02</td>
<td>18, 4</td>
<td>30, 8</td>
<td>38, 20</td>
</tr>
<tr>
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<td>65, 8</td>
<td>19, 1.25</td>
<td>32, 250</td>
<td>58, 500</td>
<td>62, 1250</td>
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<td>54, 0.08</td>
<td>66, 16</td>
<td>14, 0.8</td>
<td>28, 165</td>
<td>48, 320</td>
<td>56, 800</td>
</tr>
<tr>
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<td>4.0, 3.9, 150, -88</td>
<td>54, 0.08, 0.078</td>
<td>68, 16</td>
<td>17, 1</td>
<td>29, 200</td>
<td>50, 400</td>
<td>58, 1000</td>
</tr>
<tr>
<td>8.792</td>
<td>0.78, 0.7, 200, -72</td>
<td>42, 0.0157, 0.014</td>
<td>60, 3.2</td>
<td>18, 0.83</td>
<td>34, 168</td>
<td>55, 330</td>
<td>62, 825</td>
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<td>59, 3.1</td>
<td>18, 0.86</td>
<td>34, 170</td>
<td>55, 345</td>
<td>62, 863</td>
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<tr>
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<td>68, 16</td>
<td>18, 1</td>
<td>29, 200</td>
<td>50, 400</td>
<td>58, 1000</td>
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<tr>
<td>12.56</td>
<td>4.0, 4.0, 100, -0.55</td>
<td>52, 0.08</td>
<td>68, 16</td>
<td>-28, 0.006</td>
<td>-18, 1.25</td>
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<td>60, 12375</td>
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<td>65, 9.4</td>
<td>13, 0.68</td>
<td>28, 135</td>
<td>49, 270</td>
<td>56, 675</td>
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<tr>
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<td>68, 17</td>
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<td>20, 82</td>
<td>41, 165</td>
<td>50, 413</td>
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<td>68, 16</td>
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<td>-10, 2.25</td>
<td>12, 4.5</td>
<td>19, 11.25</td>
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<td>65, 13.2</td>
<td>-18, 0.07</td>
<td>8, 14</td>
<td>28, 28</td>
<td>35, 70</td>
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<tr>
<td>4.762</td>
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<td>-8, 0.072</td>
<td>8, 15</td>
<td>28, 30</td>
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<td>-2, 0.135</td>
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<td>68, 16</td>
<td>0, 0.17</td>
<td>12, 34</td>
<td>32, 67</td>
<td>41, 168</td>
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<td>0, 0.17</td>
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<td>32, 67</td>
<td>41, 168</td>
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<td>32, 67</td>
<td>41, 168</td>
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<td>53, 0.081</td>
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<td>-20, 0.015</td>
<td>-9, 3</td>
<td>12, 6</td>
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<td>68, 16</td>
<td>-5, 0.12</td>
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<td>38, 100</td>
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<td>38, 110</td>
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<td>-16, 0.027</td>
<td>-4, 5.4</td>
<td>18, 11</td>
<td>25, 28</td>
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</table>

* SNR based on unstressed case, ** If a third term does not exist, then it is implied that, \( V_{LU} = V_{LS} \)
Table 2.7 Evaluation of signal-to-noise ratio and load voltages for various phase delays resulting from the wedge phase shifter for a 3 N force (1.91x10^6 strain) acting on the sample under test when a HeNe laser (λ=633 nm) probes the sample medium.

<table>
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<tr>
<th>Phase (\psi_1) or (\psi_2)</th>
<th>(P_{\text{ind}}), (P_{\text{ind}}), (P_{\text{in}}) [mW]</th>
<th>(\Delta P_{\text{ind}}) [(\mu)W]</th>
<th>DC Mod. (R_i=50\Omega) (RLDC)</th>
<th>DC Mod. Amplifier &amp; (R_i=50\Omega) (AMPDC)</th>
<th>AC Mod. (R_i=50\Omega) (RLAC)</th>
<th>AC Mod. Amplifier &amp; (R_i=50\Omega) (AMPAC)</th>
<th>Heterodyne (V_{LO}=2.0) (R_i=50\Omega) (HETACB)</th>
<th>Heterodyne (V_{LO}=2.0) (R_i=50\Omega) (HETAC)</th>
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<td>48, 1750</td>
<td>70, 3500</td>
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<td>-40, 2.2e-5</td>
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<td>68, 880</td>
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<td>61, 8</td>
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<td>-48, 3.4e-2</td>
<td>-27, 0.07</td>
<td>-19, 0.175</td>
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<td>45, 1200</td>
<td>66, 2400</td>
<td>75, 6100</td>
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<td>67, 16</td>
<td>38, 11</td>
<td>50, 2100</td>
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<td>52, 2000</td>
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<td>50, 400</td>
<td>58, 1000</td>
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<td>(\psi_2)</td>
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<td>47, 1150</td>
<td>68, 2300</td>
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<td>69, 4000</td>
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<td>68, 13.4</td>
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<td>40, 130</td>
<td>48, 330</td>
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<tr>
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<td>5.556</td>
<td>3.75, 3.8, 200, 128</td>
<td>54, 0.075, 0.078</td>
<td>67, 15</td>
<td>19, 1.49</td>
<td>32, 290</td>
<td>54, 600</td>
<td>61, 1490</td>
<td></td>
</tr>
<tr>
<td>6.349</td>
<td>4.0, 4.0, 200, 13</td>
<td>54, 0.08</td>
<td>68, 16</td>
<td>-1, 0.15</td>
<td>11, 30</td>
<td>32, 60</td>
<td>40, 150</td>
<td></td>
</tr>
<tr>
<td>7.342</td>
<td>4.2, 4.0, 200, -148</td>
<td>54, 0.084, 0.08</td>
<td>67, 16.8</td>
<td>20, 1.7</td>
<td>33, 340</td>
<td>54, 680</td>
<td>62, 1700</td>
<td></td>
</tr>
<tr>
<td>8.333</td>
<td>3.8, 4.0, 200, 130</td>
<td>53, 0.079, 0.082</td>
<td>67, 15.9</td>
<td>19, 1.52</td>
<td>32, 300</td>
<td>53, 600</td>
<td>61, 1500</td>
<td></td>
</tr>
<tr>
<td>9.326</td>
<td>4.0, 4.0, 200, -135</td>
<td>52, 0.08</td>
<td>68, 16.2</td>
<td>-1, 0.15</td>
<td>11, 30</td>
<td>32, 60</td>
<td>40, 150</td>
<td></td>
</tr>
<tr>
<td>10.318</td>
<td>4.2, 4.0, 200, -90</td>
<td>53, 0.084, 0.082</td>
<td>68, 16.8</td>
<td>16, 1.04</td>
<td>28, 220</td>
<td>50, 400</td>
<td>58, 1040</td>
<td></td>
</tr>
<tr>
<td>11.31</td>
<td>3.8, 4.0, 200, 98</td>
<td>53, 0.078, 0.08</td>
<td>68, 15.8</td>
<td>8, 1.15</td>
<td>30, 230</td>
<td>50, 450</td>
<td>58, 1140</td>
<td></td>
</tr>
<tr>
<td>12.302</td>
<td>4.0, 4.0, 200, -22.5</td>
<td>52, 0.08</td>
<td>68, 16</td>
<td>5.0, 0.28</td>
<td>18, 57</td>
<td>38, 120</td>
<td>48, 270</td>
<td></td>
</tr>
</tbody>
</table>
Table 2.8 Evaluation of signal-to-noise ratio and load voltages for various phase delays resulting from the wedge phase shifter for a 32.45 N force (20.6x10^-6 strain) acting on the sample under test when a HeNe laser (λ = 633 nm) probes the sample medium.

<table>
<thead>
<tr>
<th>Phase</th>
<th>P_{ind}, P_{ind}, P_{in} [mW]</th>
<th>DC Mod. R_L=50Ω (RLDC)</th>
<th>DC Mod. Amplifier &amp; R_L=50Ω (AMPDC)</th>
<th>AC Mod. Amplifier &amp; R_L=50Ω (AMPAC)</th>
<th>Heterodyne V_{LO}=2.0 R_L=50Ω (HETAC)</th>
<th>Heterodyne V_{LO}=5.0 R_L=50Ω (HETACB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ψ_1</td>
<td></td>
<td></td>
<td>SNR*, V_{LU} [V], V_{LS} [V]**</td>
<td>SNR*, V_{LU} [V]</td>
<td>SNR, V_{L} [mV]</td>
<td>SNR, V_{L} [mV]</td>
</tr>
<tr>
<td>1.047</td>
<td>4.0, 1.8, 180, -3000</td>
<td>47, 0.08, 0.037</td>
<td>70, 16</td>
<td>46, 35</td>
<td>60, 7000</td>
<td>80, 14000</td>
</tr>
<tr>
<td>2.093</td>
<td>1.2, 4.0, 120, 2600</td>
<td>57, 0.024, 0.08</td>
<td>57, 4.5</td>
<td>46, 30</td>
<td>58, 6000</td>
<td>70, 12000</td>
</tr>
<tr>
<td>3.14</td>
<td>2.75e-9, 4.0, 100, 4000</td>
<td>68, 5.5e-8, 0.055</td>
<td>-57, 1.25e-5</td>
<td>50, 47</td>
<td>62, 9000</td>
<td>82, 13000</td>
</tr>
<tr>
<td>4.187</td>
<td>1.5, 4.0, 150, 2500</td>
<td>56, 0.03, 0.08</td>
<td>58, 6</td>
<td>45, 28</td>
<td>58, 5800</td>
<td>79, 11500</td>
</tr>
<tr>
<td>5.233</td>
<td>4.0, 0.9, 130, -3300</td>
<td>41, 0.08, 0.0175</td>
<td>73, 16</td>
<td>50, 37</td>
<td>66, 7500</td>
<td>87, 16000</td>
</tr>
<tr>
<td>7.536</td>
<td>4.0, 2.6, 150, -1300</td>
<td>50, 0.08, 0.051</td>
<td>68, 16</td>
<td>40, 150</td>
<td>53, 3000</td>
<td>75, 6000</td>
</tr>
<tr>
<td>8.792</td>
<td>0.3, 3.8, 100, -3500</td>
<td>55, 0.006, 0.075</td>
<td>45, 12</td>
<td>47, 40</td>
<td>59, 8000</td>
<td>80, 16000</td>
</tr>
<tr>
<td>10.048</td>
<td>0.4, 4.0, 130, 3500</td>
<td>48, 0.008, 0.08</td>
<td>57, 1.6</td>
<td>48, 40</td>
<td>60, 7000</td>
<td>81, 17000</td>
</tr>
<tr>
<td>11.304</td>
<td>4.0, 1.5, 150, -2400</td>
<td>460, 0.08, 0.03</td>
<td>70, 16</td>
<td>46, 27</td>
<td>61, 5600</td>
<td>82, 12000</td>
</tr>
<tr>
<td>12.56</td>
<td>4.0, 0.5, 100, -4000</td>
<td>15, 0.08, &lt;0.0001</td>
<td>85, 16</td>
<td>55, 48</td>
<td>80, 9000</td>
<td>95, 18000</td>
</tr>
<tr>
<td>ψ_2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.785</td>
<td>2.3, 4.0, 170, 1600</td>
<td>55, 0.043, 0.08</td>
<td>62, 9.1</td>
<td>41, 18</td>
<td>54, 3600</td>
<td>75, 7200</td>
</tr>
<tr>
<td>1.57</td>
<td>1.5, 4.0, 130, 2500</td>
<td>56, 0.03, 0.08</td>
<td>58, 6</td>
<td>45, 28</td>
<td>58, 5800</td>
<td>79, 11500</td>
</tr>
<tr>
<td>2.381</td>
<td>4.25, 3.6, 200, -570</td>
<td>53, 0.085, 0.075</td>
<td>67, 17</td>
<td>32, 6.5</td>
<td>45, 1300</td>
<td>66, 2700</td>
</tr>
<tr>
<td>3.175</td>
<td>4.0, 4.0, 200, -61</td>
<td>52, 0.08</td>
<td>68, 16</td>
<td>12, 0.71</td>
<td>25, 141</td>
<td>46, 257</td>
</tr>
<tr>
<td>3.968</td>
<td>2.7, 4.0, 157, 1200</td>
<td>54, 0.053, 0.08</td>
<td>64, 11</td>
<td>39, 13</td>
<td>51, 2600</td>
<td>72, 5100</td>
</tr>
<tr>
<td>4.762</td>
<td>4.0, 2.8, 170, -1300</td>
<td>52, 0.08, 0.058</td>
<td>68, 13</td>
<td>40, 15</td>
<td>54, 3000</td>
<td>74, 6000</td>
</tr>
<tr>
<td>5.556</td>
<td>3.75, 4.25, 200, 550</td>
<td>55, 0.075, 0.86</td>
<td>65, 15</td>
<td>31, 6.4</td>
<td>44, 1270</td>
<td>65, 2500</td>
</tr>
<tr>
<td>6.349</td>
<td>4.0, 4.0, 200, 2.65</td>
<td>52, 0.08</td>
<td>68, 16</td>
<td>-15, 0.032</td>
<td>-2, 6.2</td>
<td>19, 12.2</td>
</tr>
<tr>
<td>7.342</td>
<td>4.2, 4.0, 200, -95</td>
<td>55, 0.084, 0.081</td>
<td>68, 16.5</td>
<td>16, 1.1</td>
<td>29, 225</td>
<td>50, 450</td>
</tr>
<tr>
<td>8.333</td>
<td>3.8, 4.0, 200, 145</td>
<td>54, 0.079, 0.08</td>
<td>65, 15.9</td>
<td>20, 1.7</td>
<td>32, 340</td>
<td>54, 675</td>
</tr>
<tr>
<td>9.326</td>
<td>4.0, 4.0, 200, -20</td>
<td>52, 0.083</td>
<td>68, 16</td>
<td>4.0.24</td>
<td>17, 46</td>
<td>38, 92</td>
</tr>
<tr>
<td>10.318</td>
<td>4.2, 3.9, 200, -18</td>
<td>52, 0.083, 0.078</td>
<td>68, 16.5</td>
<td>22, 2.1</td>
<td>35, 420</td>
<td>55, 840</td>
</tr>
<tr>
<td>11.31</td>
<td>3.8, 4.2, 200, 250</td>
<td>52, 0.076, 0.082</td>
<td>68, 15.8</td>
<td>25, 2.8</td>
<td>38, 580</td>
<td>59, 1150</td>
</tr>
<tr>
<td>12.302</td>
<td>4.1, 4.0, 200, -75</td>
<td>52, 0.082, 0.08</td>
<td>68, 16</td>
<td>14.85</td>
<td>28, 170</td>
<td>49, 340</td>
</tr>
</tbody>
</table>
Table 2.9 Evaluation of signal-to-noise ratio and load voltages for various phase delays resulting from the wedge phase shifter for a 0.03 N force (19.1x10^9 strain) acting on the sample under test when the Nd:YAG laser (λ=532 nm) probes the sample medium.

<table>
<thead>
<tr>
<th>( \psi_1 ) or ( \psi_2 )</th>
<th>( P_{\text{ind}} ), ( P_{\text{ind}} ), ( P_{\text{in}} )</th>
<th>DC Mod. ( R_L=50\Omega ) (RLDC)</th>
<th>DC Mod. Amplifier &amp; ( R_L=50\Omega ) (AMPDC)</th>
<th>AC Mod. ( R_L=50\Omega ) (RLAC)</th>
<th>AC Mod. Amplifier &amp; ( R_L=50\Omega ) (AMPAC)</th>
<th>Heterodyne ( V_L=2.0 )</th>
<th>Heterodyne ( V_L=5.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi_1 )</td>
<td>1.047</td>
<td>4.0, 4.0, 135, -9.0</td>
<td>58, 0.08</td>
<td>68, 16</td>
<td>-2, 0.11</td>
<td>10, 20</td>
<td>30, 42</td>
</tr>
<tr>
<td>( \psi_2 )</td>
<td>2.093</td>
<td>2.0, 2.0, 200, -13</td>
<td>50, 0.04</td>
<td>64, 8</td>
<td>0.0, 0.15</td>
<td>16, 30</td>
<td>38, 60</td>
</tr>
<tr>
<td>3.14</td>
<td>5.5e-6, 9e-6, 200, 3.5e-3</td>
<td>-55, 1.04e-7, 1.7e-7</td>
<td>-28, 2.2e-5</td>
<td>-70, 4.0e-5</td>
<td>-35, 8e-3</td>
<td>-25, 1.6e-3</td>
<td>-18, ***</td>
</tr>
<tr>
<td>4.187</td>
<td>2.0, 2.0, 200, 13</td>
<td>50, 0.04</td>
<td>62, 8</td>
<td>0.0, 0.15</td>
<td>16, 30</td>
<td>36, 60</td>
<td>44, ***</td>
</tr>
<tr>
<td>5.233</td>
<td>4.0, 4.0, 130, 9.0</td>
<td>56, 0.08</td>
<td>68, 16</td>
<td>-2, 0.11</td>
<td>10, 21</td>
<td>30, 42</td>
<td>40, ***</td>
</tr>
<tr>
<td>7.536</td>
<td>4.0, 4.0, 150, -8.8</td>
<td>55, 0.08</td>
<td>66, 16</td>
<td>0.0, 0.1</td>
<td>10, 20</td>
<td>22, 40</td>
<td>40, ***</td>
</tr>
<tr>
<td>8.792</td>
<td>0.78, 0.78, 200, -8.8</td>
<td>42, 0.016</td>
<td>60, 3.1</td>
<td>0.0, 0.1</td>
<td>17, 20</td>
<td>36, 40</td>
<td>46, ***</td>
</tr>
<tr>
<td>10.048</td>
<td>0.76, 0.76, 200, 8.5</td>
<td>42, 0.015</td>
<td>60, 3.1</td>
<td>0.0, 0.1</td>
<td>17, 20</td>
<td>36, 40</td>
<td>46, ***</td>
</tr>
<tr>
<td>11.304</td>
<td>4.0, 4.0, 150, 11</td>
<td>56, 0.08</td>
<td>66, 16</td>
<td>0.0, 0.12</td>
<td>10, 23</td>
<td>32, 46</td>
<td>40, ***</td>
</tr>
<tr>
<td>12.56</td>
<td>4.0, 4.0, 100, 3.2e-2</td>
<td>58, 0.08</td>
<td>70, 16</td>
<td>-50, 3.6e-4</td>
<td>-42, 0.07</td>
<td>-18, 0.15</td>
<td>-10, ***</td>
</tr>
<tr>
<td>( \psi_1 )</td>
<td>0.785</td>
<td>2.6, 2.6, 200, 12</td>
<td>53, 0.051</td>
<td>65, 10-2</td>
<td>0, 0.14</td>
<td>15, 28</td>
<td>35, 58</td>
</tr>
<tr>
<td>( \psi_2 )</td>
<td>1.57</td>
<td>2.3, 2.3, 200, -7</td>
<td>53, 0.047</td>
<td>65, 9.2</td>
<td>-8, 0.08</td>
<td>9, 16</td>
<td>30, 33</td>
</tr>
<tr>
<td>2.381</td>
<td>4.0, 4.0, 180, 4</td>
<td>52, 0.08</td>
<td>68, 16</td>
<td>-10, 0.0475</td>
<td>2.9, 4</td>
<td>22, 18.5</td>
<td>31, ***</td>
</tr>
<tr>
<td>3.175</td>
<td>4.0, 4.0, 200, 0.12</td>
<td>55, 0.08</td>
<td>68, 16</td>
<td>-42, 0.0014</td>
<td>-30, 0.275</td>
<td>-9, 0.55</td>
<td>0, ***</td>
</tr>
<tr>
<td>3.968</td>
<td>3.3, 3.3, 200, -0.75</td>
<td>52, 0.066</td>
<td>65, 13.4</td>
<td>-25, 0.0086</td>
<td>-12, 1.72</td>
<td>10, 3.5</td>
<td>18, ***</td>
</tr>
<tr>
<td>4.762</td>
<td>4.0, 4.0, 170, -0.7</td>
<td>54, 0.032</td>
<td>68, 6.4</td>
<td>-28, 80</td>
<td>-12, 1.6</td>
<td>10, 3.3</td>
<td>18, ***</td>
</tr>
<tr>
<td>5.556</td>
<td>3.75, 3.75, 200, 1.38</td>
<td>52, 0.075</td>
<td>68, 15</td>
<td>-20, 0.016</td>
<td>-8, 3.2</td>
<td>13, 6.2</td>
<td>21, ***</td>
</tr>
<tr>
<td>6.349</td>
<td>4.0, 4.0, 200, 0.16</td>
<td>52, 0.08, 0.08</td>
<td>68, 16</td>
<td>-40, 0.0018</td>
<td>-25, 0.35</td>
<td>-5.07</td>
<td>2, 1.6</td>
</tr>
<tr>
<td>7.342</td>
<td>4.1, 4.1, 200, -1.7</td>
<td>55, 0.08, 0.08</td>
<td>68, 16</td>
<td>-19, 0.02</td>
<td>-9, 4</td>
<td>15, 8</td>
<td>22, 20</td>
</tr>
<tr>
<td>8.333</td>
<td>4.0, 4.0, 200, 1.5</td>
<td>50, 0.08, 0.08</td>
<td>65, 15.8</td>
<td>-20, 0.018</td>
<td>-8, 3.5</td>
<td>14, 6.5</td>
<td>21, 17</td>
</tr>
<tr>
<td>9.326</td>
<td>4.0, 4.0, 200, -0.15</td>
<td>55, 0.08, 0.08</td>
<td>68, 16</td>
<td>-40, 0.0018</td>
<td>-38, 0.35</td>
<td>-5.07</td>
<td>1, 1.7</td>
</tr>
<tr>
<td>10.318</td>
<td>4.2, 4.2, 200, -1</td>
<td>55, 0.08, 0.08</td>
<td>68, 16</td>
<td>-22, 0.0118</td>
<td>-10, 2.25</td>
<td>10, 4.8</td>
<td>19, 11.8</td>
</tr>
<tr>
<td>11.31</td>
<td>3.8, 3.8, 200, 1.1</td>
<td>55, 0.078, 0.078</td>
<td>68, 15.5</td>
<td>-21, 0.013</td>
<td>-10, 2.6</td>
<td>11, 5</td>
<td>20, 13</td>
</tr>
<tr>
<td>12.302</td>
<td>4.0, 4.0, 200, 0.27</td>
<td>55, 0.08, 0.08</td>
<td>68, 16</td>
<td>-35, 0.0027</td>
<td>-22, 0.58</td>
<td>0, 1.2</td>
<td>8, 2.7</td>
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</tbody>
</table>
Table 2.10 Evaluation of signal-to-noise ratio and load voltages for various phase delays resulting from the wedge phase shifter for a 3N force (1.91 x 10<sup>6</sup> strain) acting on the sample under test when the Nd:YAG laser (λ = 532 nm) probes the sample medium.

<table>
<thead>
<tr>
<th>Phase (\psi_1) or (\psi_2)</th>
<th>(P_{in0}), (P_{in0}), (P_{in}) [mW]</th>
<th>DC Mod. (R_L=50\Omega) (RLDC)</th>
<th>DC Mod. Amplifier &amp; (R_L=50\Omega) (AMPDC)</th>
<th>AC Mod. (R_L=50\Omega) (RLAC)</th>
<th>AC Mod. Amplifier &amp; (R_L=50\Omega) (AMPAC)</th>
<th>Heterodyne (V_{LO}=2.0)</th>
<th>Heterodyne (V_{LO}=5.0) (R_L=50\Omega) (HETAC)</th>
<th>Heterodyne (V_{LO}=5.0) (R_L=50\Omega) (HETACB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\psi_1)</td>
<td>(\Delta P_{in}) [\mu W]</td>
<td>SNR*, (V_{LU}) [V]; (V_{LS}) [V]**</td>
<td>SNR*, (V_{LU}) [V]</td>
<td>SNR, (V_L) [mV]</td>
<td>SNR, (V_L) [mV]</td>
<td>SNR, (V_L) [mV]</td>
<td>SNR, (V_L) [mV]</td>
<td>SNR, (V_L) [mV]</td>
</tr>
<tr>
<td>1.047</td>
<td>4.5, 3.5, 150, -1050</td>
<td>52, 0.07, 0.09</td>
<td>68, 17</td>
<td>38, 12</td>
<td>51, 2500</td>
<td>71, 5400</td>
<td>80, 12000</td>
<td></td>
</tr>
<tr>
<td>2.093</td>
<td>2.0, 0.8, 200, -1100</td>
<td>42, 0.04, 0.016</td>
<td>67, 8</td>
<td>41, 13</td>
<td>58, 2500</td>
<td>78, 5000</td>
<td>85, 13000</td>
<td></td>
</tr>
<tr>
<td>3.14</td>
<td>0.26, 0.26, 200, 260</td>
<td>32, 1.1e-9, 0.005</td>
<td>-40, 2e-5</td>
<td>30.32</td>
<td>50, 600</td>
<td>70, 1250</td>
<td>76, 3000</td>
<td></td>
</tr>
<tr>
<td>4.187</td>
<td>2.0, 3.4, 200, 1400</td>
<td>54, 0.04, 0.065</td>
<td>61, 8</td>
<td>40.16</td>
<td>54, 1320</td>
<td>74, 6500</td>
<td>82, 16000</td>
<td></td>
</tr>
<tr>
<td>4.762</td>
<td>4.0, 4.0, 200, -0.018</td>
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CHAPTER 3

EXPERIMENTAL SETUP, DATA COLLECTION, AND ANALYSES

3.1 Optical modeling studies

Optical modeling may occur on a number of different levels based on approximations used. When the wavelength of light is smaller than the dimensions of the components in the system, a geometric optics or ray optics [32] approximation is commonly used. Here, the normal to the phase front of the wave, typically denoted as the ray, is exploited geometrically. Although easy to use, this technique does not consider important diffraction effects resulting from the finite nature of optical components and obstacles, and the finite nature of the beam itself. When, on the other hand, if the wave character of the light cannot be ignored such as diffraction and interference effects, then a physical optics model is employed. Theoretical modeling in the previous chapter is based on simple ray optic techniques for plane waves assuming optical components are infinite in extent. Diffraction effects are not taken into consideration. Optical modeling is performed with optical simulation programs employing both techniques. The simulation programs Rayica and Wavica developed by Optica Software executed under a Mathematica engine are used to verify theory. These programs allow for the custom modeling of the three-dimensional optical systems lending themselves to wave interference studies. A library of predefined optical components exists. Components are chosen to closely match existing optical elements used in experiment.
In order to gain confidence in the code's ability to characterize an optical system, the optical modeling codes were applied to a simple telescope arrangement in which a 532 nm Gaussian beam with a 4.255 mm (0.168") diameter spot size is to be focused to a 0.721 mm (0.028") diameter spot size to be maintained over a one meter (39.37") length. This code is present in Appendix B. One starts with a simple geometrical theory for a telescoping system consisting of two optical lenses sharing the same geometrical line of symmetry. A plane wave normally incident on a plano-convex lens transforms the plane wave into a collapsing spherical wave. The shape of the lens on the convex side is a hyperboloid (one of two sheets). As the light rays pass through the focal point of the first lens, the light rays tend to form an expanding spherical wavefront as far as the next optic is concerned. The focal distance of the first optic is \( f_1 \) from the convex surface normal along its optic axis which is the line of geometrical symmetry of the optic. The second optic is located a distance equal to its focal length, \( f_2 \), from this point of convergence such that the optic axis of each element coincide. As the wave passes into the convex side of the second plano-convex lens, the lens also with a convex shape consistent with that of a hyperboloid (one of two sheets) transforms the spherical phase front into a plane wave upon exiting the optic. Assuming a thin lens approximation where the diameter of the incident beam is \( D_1 \) and the desired diameter of the beam leaving the telescoping device is \( D_2 \), simple planar geometry of right triangles indicates that (Refer to Fig.3.1a)

\[
\tan \phi = \frac{(D_1/2)}{f_1} = \frac{(D_2/2)}{f_2}
\]

Equation (3.1) will be denoted as the telescope design principle.
The formula for the contour of the convex side of the plano-convex lens that transforms a spherical wave front to a plane wave front may be easily shown as

\[ \rho_j(\theta_j) = \frac{(n_j - 1)f_j}{n_j \cos \theta_j - 1} \]  

(3.2)

where, for the jth lens, \( \theta_j \) is the distance from the focal point to the surface of the convex side, \( \theta_j \) is the angle between \( \theta_j \) and the axis of geometrical symmetry, and \( n_j \) is the index of refraction. This relation finds application when custom building a lens for simulation.

The radius of curvature of the lens may be estimated by fitting a spherical surface over the hyperboloid using a thin lens approximation knowing the approximate diameter (in a cylindrical coordinate system) of the optic.

The overall length of the telescope was required to be less than a foot or so and to be built with commercial optics. For the converging lenses that are required either plano-convex or achromatic doublet lenses can be used. An achromatic doublet consists of a convex lens and a concave lens that are cemented together as shown in Fig. 3.2. The two lenses in an achromatic doublet are each made of different types of glasses with different values of refractive index. Achromatic doublets are chosen over plano-convex lenses for the following reasons: [33]

a. An achromatic doublet focuses the beam to a smaller spot size as compared to a plano-convex lens.

b. Rays passing through the center of the lens have a different focal length as compared to the rays passing off center of the plano-convex lens. This phenomenon is called spherical aberration. The achromatic doublet is not sensitive to the position of the beam with respect to the optic axis.
c. In the case of a white light source in the system, the focal point and the circle of least confusion are blurred by chromatic aberration, which occurs due to the refractive index varying with respect to the wavelength. By using two different glasses with different refractive indices, the achromatic doublet lens is able to cancel these aberrations.

Knowing \(D_1\) and \(D_2\) and the overall length of the telescope, combinations of conventional lenses with appropriate focal lengths were examined. Optical modeling required a number of input parameters. These parameters are listed in Table 3.1 based on the information required by the modeling package and telescope design principle. Because achromatic doublet lenses are used, the radii of curvature, the thicknesses of the cemented convex and concave lenses, focal length and aperture diameter are required.

Figure 3.3 shows the system that was designed by basically adhering to the telescope design principle based on the components with parameter descriptions listed in Table 3.1. Using the OptimizeSystem command with RayTilt option, the application tool Rayica calculates the optimal length between the lenses so as to obtain a perfectly collimated beam over the lengths of interest. Refer to Appendix B (Program 1). Rayica takes into account that the beam is a spherical wave. When using the OptimizeSystem function, the position of one component, in this case the second lens, is designated as a variable and assigned a name \(x\). On running the code, Rayica calculates the optimal value of \(x\) that would yield perfect collimation of the beam and places the objective lens at that position.

Figure 3.4 shows the optimal system with the beam perfectly collimated. Initially, the program was run using the ray tracing approach. In order to perform high level functions such as GaussianTrace, GaussianPlot, FindInterference and to determine phase
information Wavica is used. Refer to Appendix B (Program 2). These command syntaxes (e.g., GaussianTrace, GaussianPlot, FindInterference, etc.) are used to describe the functions. Taking into consideration that the beam is Gaussian, the value of the optimal distance was found to be within 0.44% of the value of the optimal distance calculated by Rayica. The optimal distance calculated by Rayica and Wavica were 428.533 and 430.415 respectively. In the previous case with ray tracing, the OptimizeSystem Function directly yielded an optimized system, whereas in this case, the optimal value for \( x \) alone is generated by calculating the position of the second lens which yield the minimum beam divergence. A separate system with the position of the second lens set to this value of \( x \) is simulated and the optimized system is generated and is displayed in Fig 3.5. In certain cases it may not be possible to discern whether the beam remains collimated over long distances from the diagram that is generated in Rayica and Wavica. This issue can be tackled by two methods. The first involves measuring the beam spot size on the screen at various distances from the output of the beamsplitter. BeamSpotSize is a rule returned by GaussianBeam that designates the Gaussian beam radius (at 1/e of its amplitude peak) in a particular transverse plane. This is done using the FindSpotSize command in Wavica. The second method involves using the FindIntensity command with the Plot2D option set to ContourPlot. The image of the beam profile as seen on a screen is given by Fig. 3.6. The laser beam is modeled with a line array of 10 rays of light as illustrated in the Fig. 3.6. Here, the beam dimension is 4.255mm (0.168”). Experimental values are used so as to allow for a comparative study. The diameter of the beam is computed in the first method by recording the SpotSize value generated by the code. The values that are obtained by this method are recorded in
Table 3.2.1. In the second case a bell graph of the SurfaceIntensity is generated as shown in Fig. 3.7. and the dimensions are obtained from the RayBoundary values that are calculated and displayed when the FindIntensity command is executed, the values are displayed in Table 3.2.2. Refer to Appendix B (Program 3). The ratio of the values obtained by the two methods was found to be 0.555 as shown in Table 3.2.2. The results obtained from the experiment were recorded in Table 3.3, which tabulates the spot size before and at various distances after the telescope. Further, the ratio of the output spot size to the input spot size is also provided. As modeled, the telescope was built (Refer to Fig. 3.8.) and tested. Burn spots on heat sensitive paper were taken before the telescope and at a number of locations after the telescope within the design region of interest. Refer to Fig. 3.9. Table 3.3 tabulates the approximate diameter of the burn marks and their ratio. The values of the focused beam diameter at various positions of the screen obtained from the theoretical, computational, and experimental methods were compared using Matlab and the plotted results are displayed in Fig 3.10. Refer to Appendix B (Program 4). Aligning the optics with the beam on the beam axis is crucial for this telescope to perform its function as designed. Experimental data was recorded when visually, the spot size of the beam on the burn paper appeared to yield the results sought. Consequently, it is not clear if optimal alignment was achieved. A 0.5 mm reticle was used to measure the laser generated burn marks. With the understanding that no effort was made to optimally align the laser beam with the telescope and burn paper, experimental results and modeling results reasonably agree. This experiment demonstrated that the modeling code is capable of predicting experimental results for practical optical systems in the laboratory.
3.1.1 Modeling the interferometer setup

With confidence in the optical modeling software established, the Mach Zender interferometer of choice is modeled using the Wavica simulation tool. Both experimental and simulation component placement and orientation are carefully coordinated for comparison purposes within the interferometer proper. The output of the interferometer in the simulation model is not magnified whereas in the experiment such magnification was deemed necessary. Since absolute measurements are not required to determine the change in the index of refraction leading to the force applied, no loss of generality results in the absence of the magnifying lens in the computational model. Figures 3.11 – 3.13 illustrate the optical system described and evaluated in this section. Refer to Appendix B (Program 5).

Since a HeNe laser is used, the light source is modeled as a Gaussian beam with a 2 mm beam diameter, a 0.001 beam divergence, and a 633 nm wavelength. Instead of using beam splitters as employed in the theoretical study (Refer to Chapter 2), a beamsplitter cube is used in experimental studies. With appropriate orientation of the cube, the difference in the optical path of the two existing beams is zero. This is not the case if a beamsplitter is used. The first beamsplitter cube, BS1, has a one inch (2.54 cm) dimension on each side and has a 50% reflectance and 50% transmittance. The angle between the normal to the face of the beamsplitter cube and the incident beam is about two degrees. Referring to Fig. 3.11, the beam is split into two separate beams, a reference beam in path A containing the optical mirror M1 and the sampling beam in path B containing the optical mirror M2 and the sample under test, W. In path A, a front surface, 25.4 mm (1") diameter, 10 mm (0.393") thick mirror M1 is rotated on a vertical
post holder axis to an angle of 45° relative to the incident beam and is held fixed. The
distances between the optical components, is equal to the distances measured in the
experimental setup and are specified in millimeters. As observed in Figs. 3.11-3.13, the
beams, one in path A and one in path B, are not identical in path length and shape due to
alignment difficulties in the experiment. The Rexolite test piece is placed in path B and is
specified by its refractive index, which in the case of Rexolite, is 1.585. In the modeling
software, the Rexolite test piece is described as a Window in the shape of a cube with a
25.4 mm (1") dimension on any one side. Also in path B, a front surface, 25.4 mm
diameter, 10 mm thick mirror M2 is mounted on an x-y translation stage with micrometer
adjustment screws. The mirror M2 is appropriately positioned with the aid of the x-y
stage and rotated about the vertical post holder in such a manner to simultaneously direct
the beam through the sample under test and force the beam to coincide with beam A.
Note that both beams A and B are maintained at the same height from the top surface of
the optics table. The second cube beamsplitter BS2 of identical geometry as BS1 is
positioned such that the beam in path A is normal to the surface and that of path B is
nearly normal to the surface. By slightly changing the position of M2, the two beams are
forced to overlap each other over a long distance (about a meter) behind the last
beamsplitter cube. Further, fine adjustments to mirror M2 allow one to change the
number of fringes per unit length as observed on an observation screen. A screen is
placed in front of the output paths of the interferometer.

For the defined optical system as shown in Figs.3.11-3.13, the FindInterference
function is employed to generate a simulation of the fringe pattern on the screen. The
FindInterference function generates the beam profiles in either path A and/or B before
they combine at the beamsplitter along with the fringe pattern that result from their
combination at the beamsplitter and at a screen placed in the output side of the
beamsplitter. There are two possible methods in which the output data can be obtained. In
the first method, for FullForm->True, GaussianBeam attempts to construct a uniform
three-dimensional grid of rays to model the spot size and beam divergence behavior.
Note, FullForm->True is an option with the GaussianBeam command syntax as it would
appear in Wavica. The wavefronts in either beam path with FullForm->True are shown in
Figs. 3.14a, b and the interference pattern generated for this system on the screen placed
at the output of the interferometer is shown in Fig. 3.15. Figures 3.14a and b show the
wavefronts. The light intensity associated with each wavefront is assigned a color
according to the VIBGYOR (violet-indigo-blue-green-yellow-orange-red) regime with
violet indicating light of the least intensity and red the highest intensity. The fringes
illustrated in Fig. 3.15 indicate that the intensity is not uniform over each fringe
displayed. Only the dark region appears to be well defined and somewhat of equal
thickness. This implies that the center of the dark region may be easier to find. Further,
the fringes are vertical. From Fig. 3.15, the distance of separation between consecutive
minima is 1.4 mm. The minimum is taken as the average horizontal position in the dark
region.

For FullForm->False, GaussianBeam creates either a single two-dimensional fan of
rays in the x-y plane or two cross-hatched fans of rays that lie in the x-y and x-z planes,
depending on whether the spots size and full divergence or complex beam parameter
contains single numbers or sets of two numbers. The interference pattern generated for
this system on the screen placed at the output of the interferometer is shown in Fig. 3.16.
For convenience, the angular orientation of mirror M2 is adjusted in the simulation until the fringe spacing in the simulation is equal to the spacing measured in the experiment. This adjustment is not necessary since relative changes are sought. To simulate the effect of various forces (pressures) acting on the sample under test in the modeling code, the change in refractive index is dictated by Eqs. (2.23b) and (2.25b). The fringe patterns produced are then digitally processed, similar to the experimental data, to a form more conducive for evaluation.

The values of the intensity of light along the screen, is exported by Mathematica as table in the .dat format. The tables are opened and saved in Kaleidagraph to a file form compatible for import to Matlab. Refer to Appendix B (Program 6). The contents in these tables are then passed to Matlab, where the graphs for the various load conditions are plotted against the values of pixel position. The exact location of the minimum relative to a predefined reference is calculated by fitting a polynomial of the second order to the minimum portion of a selected curve in the modeling data. This procedure was used since the experimental data is handled in the same fashion. By calculating the distance between the minimum of the first load case and each subsequent load, the change in fringe shifts are calculated. The measurable is in units of pixels. Knowing the physical distance between two minima in experiment and the number of pixels between minimum from modeling results, a physical distance may be associated to the pixel unit of measure. Refer to Appendix B (Program 7). When the program is run in Rayica and Movica, the pixel resolution on the screen where the interference intensity is displayed, can be defined. In this case, the pixel resolution is chosen to be 100 pixels per mm. Therefore each pixel corresponds to a distance of 0.1 μm. Once the distance between the
minima is obtained in terms of pixels, the actual distance can be computed by multiplying it with this pixel resolution.

3.1.2 Agreement between theoretical and modeling results of the interferometer system

In the theoretical analysis of the interferometer system, the wedge phase shifter characteristics described by \((n_p - n_o) \frac{L}{h}\) can assume a wide range of values with wedge tip located at \(x_o < 0\). The phase shift properties of the wedge in the theory allows an optical means to adjust the fringe pattern properties in an attempt to better simulate the behavior of the interferometer system. In the unstressed case, the phase difference of the beam on the axis can be given by Eq. (2.19) Assuming that the detector center is located at the origin, let the phase measured by the detector be \(\phi_1 = \phi_o\) where \(\phi_o\) is given by Eq. (2.19). Considering the phase difference between the two maxima is \(2\pi\), the phase of the second maximum, \(\phi_2\), can be expressed as

\[
\phi_2 = 2\pi + \phi_1 = -\beta_o \left[ (n_p - n_o) \frac{L}{h} x_o \right] + \beta_o \left[ (n_p - n_o) \frac{L}{h} l_f \right]
\]

(3.3)

where, \(l_f\) is the distance between two consecutive maxima (or minima) in the fringe pattern. Refer to Fig. 3.17a.

The quantity, \(l_f\) is measured based on experimental data for the no load case. In particular, the fringe pattern and associated shifts were filmed with a digital video camera placed behind a semi-transparent screen. A single frame extracted from the movie is analyzed. The picture (single frame) is uploaded to a computer in a Matlab environment using the imread command which imports jpg files into Matlab. Once an image is assigned to a variable, that variable is then declared as a double integer so to prevent
overflow. Matlab displays the picture superimposed with a grid containing the pixel dimensions. In the experiment, a thread attached to the screen is used to track fringe shifts. Further, its known thickness serves as a calibration measure. The thread diameter is measured to be 0.1 mm. In Matlab, the thread thickness measures one pixel. Since there are 14 pixels between two consecutive minima, the spatial period of the fringe pattern, $l_f$, is 1.4 mm.

From Eq. (3.3), the phase difference between consecutive minima can also be expressed as

$$\Delta \phi = \beta_0 \left[ (n_p - n_o) \frac{L}{h} l_f \right] = 2\pi$$

(3.4)

Rearranging yields

$$\tilde{\phi} = (n_p - n_o) \frac{L}{h} = \frac{2\pi}{\beta_0 l_f}$$

(3.5)

With the aid of Eq. (3.5) knowing the distance between consecutive minima, the characteristics of the wedge may be used to produce an equivalent fringe spacing as observed in experiment or simulation. In the theoretical method, once $\tilde{\phi}$ has been calculated assuming there is no lateral elongation of the sample, Eq. (2.22) simplifies to

$$\Delta x = \frac{\Delta n_S l_{s1}}{\tilde{\phi}}$$

(3.6)

where $l_{s1}$ is the length of the sample and $\Delta n_S$ is calculated with Eqs. (2.23b) and (2.25a) using data provided in Table 2.1 and the surface area of the ends of the irregular cylindrical Rexolite test piece of 4.88 cm². The fringe shift is a linear function of the change in the index of refraction with a slope equal to $l_{s1}/\tilde{\phi}$. For a number of arbitrary applied forces between 0 and 100 N, the Wavica modeling code displays the values of
intensity with respect to pixel position. All interference intensity patterns generated are then imported to Matlab and displayed on a single graph as shown in Fig. 3.17b. As indicated above, the minima corresponding to a particular phase is located using a curve fitting routine and plotted against its corresponding force as illustrated in Fig. 3.17c. Along with these discrete points, the theoretical curve is superimposed on the graph. The results from theory and from the modeling code agree exactly as shown in Fig. 3.17c. Refer to Appendix B (Program 8) for the calculation of theoretical values of fringe shift. The interference intensity plot generated by of the fringe pattern recorded in the experiment superposed on the fringe pattern is shown in Fig. 3.17d to illustrate the similarity in the different methods of analyses.

3.2 Sample preparation

3.2.1 Surface polishing - lapping wheel technique

The Rexolite sample used for this experiment is a polished, 25.4mm long, 25.4 mm diameter cylinder. To prevent refraction that occurs when light is incident on the curved surface of the Rexolite, the sample piece is machined such that there are two flat, parallel, rectangular surfaces 11 x 25.4 mm on the cylindrical side of the sample. The machining is performed using a grinding tool that consists of a rotating base. Diamond encrusted silicon carbide paper is fixed to the base plate by means of an adhesive surface on its back. The grit on the silicon carbide paper comes in different grit sizes. For this experiment, abrasive paper with 12, 30 and 45 micron grit is used. The final stage of machining is performed by means of a felt pad surface which has a layer of French Cerium Oxide applied on it to polish the surface of the Rexolite sample. Figure 3.18a shows the grinding wheel apparatus along with the different grit plates that are mounted...
on to the grinding apparatus. Grinding usually involves two surfaces in contact with abrasive and water between them. In our case the abrasive is fixed to the base plate and a layer of water is maintained by means of a leaky bucket which has a hose with a tap at the end to control the flow of water. The bucket has to be constantly replenished with water. The slurry that is created when the abrasive erodes particles from the surface of the Rexolite is drained off the edge of the abrasive wheel through a funnel shaped structure into a slurry container. The slurry container should be emptied periodically to prevent overflow. Each effective grain of abrasive removes material in proportion to the distance through which it rolls and slides in a given time. As the radius of successive circular zones increases, the number of grain particles that pass under the sample per unit time increases. This implies that the outer parts of the tools would get worn more rapidly than the center or inner parts. On the other hand if two surfaces are rubbed together sans rotation but in a straight line motion as shown in Fig. 3.18b all the abrasive particles move at the same speed while they are between surfaces.

3.2.2 Optical density measurements

The ratio of the light intensity at a particular wavelength that is transmitted, \( I_t \), to the light at the same wavelength that is incident on a surface, \( I_i \), is called the transmittance [28],

\[
T = \frac{I_t}{I_i}
\]  

(3.7)

The optical density or absorbance is defined as the logarithm to the base 10 of the inverse of the transmittance.

\[
O.D = \log_{10}\left(\frac{1}{T}\right)
\]  

(3.8)
In order to better understand the properties of Rexolite, a test to measure the optical density of the sample was performed. The test setup is illustrated in Fig. 3.19. The rexolite sample was placed in the path of the laser beam and readings were taken before and after the beam passed through the sample. Eighteen sets of readings were taken. Based on the incident and transmitted values of power (measured by Advantest power meter) the opacity, optical density and the power diffused was calculated. The values of the incident power and the transmitted power for the 18 readings are recorded in Table 3.4 along with the values of the power dissipated. Based on the values recorded, the average transmittance was calculated to be 0.854 and the average optical density was calculated to be 1.117. The standard deviation in transmittance was found to be 0.0182 and the value of standard deviation in optical density was found to be 0.0255.

To better understand the properties of the beam passing through the Rexolite sample, pictures of the beam profile with and without the Rexolite sample in its path were taken and compared in Figs. 3.20.a-b. From the pictures it can be deduced that in the absence of the plastic sample, the beam retains a well defined intensity profile. When the Rexolite sample is placed in the path of the beam, noticeable scattering results and the beam loses the sharp definition of its features. The light has a scattering pattern typical of diffraction patterns and the size of the beam, particularly at its waist, has expanded.

3.2.3 Evaluation of the grinding technique

To test the effectiveness of the optical grinding procedure that was discussed earlier, a test was conducted to see if the surfaces were parallel to each other by measuring the distances between the surfaces at various positions along the surface using a micrometer gauge. The readings obtained are recorded in Table 3.5 below. The standard
deviation was calculated to be 0.036808 mm. The surface is mechanically flat but, on the order of the wavelength of HeNe laser light, the surface is not optically flat.

3.3 Experimental setup

Theoretical and modeling studies give way to experimental studies. Experimental studies are based on static loads. The motivation for static load studies stem from the large velocity differential between the acoustic wave and the electromagnetic wave. The electrodes take time to respond to the compression forces. The compression forces exist about an order of magnitude longer than the duration of the laser beam pulse interrogating the sample. The response is limited by the speed of sound supported by the medium. Therefore, if the duration of the applied forces is small compared to the time, the atoms in the solid redistribute in a non-uniform fashion altering the internal energy balance of the medium. Consequently, for a sampling laser beam with small spot size, the change in concentration of the medium throughout the sampling beam's cross section over the duration of the pulse is reasonably uniform as in a static load situation. The full impact of the dynamic load to its static state equilibrium is not realized. Therefore, static studies investigate the sensitivity of the static loads over a wide range of forces about the calculated 32 N force established in the Chapter 1 using a continuous wave (CW) laser.

A CW HeNe laser chosen for this experiment has a maximum rated output of 10 mW according to manufacturer specifications. It is mounted on a kinematic cylindrical laser mount, as shown in Figs. 3.21a-b. Two axes of precision angular adjustment [34] are provided with minimal irregular shift in fringe pattern observed over the duration of the experiment. With the aid of an XYZ translation stage enabled with micrometer positioning elements, the laser beam is directed to a spatial filter assembly. Scattered and
diffracted light rays are removed from the beam with the aid of a 200 µm diameter pinhole located at the focal point of an 8 mm microscope objective in the spatial filter assembly as shown in Figs. 3.22a-b. The fringe pattern is enhanced. A single convex lens placed about 25.4 mm (1") from the spatial filter collimates the beam to a spot size (beam diameter) of 6.25 mm. The collimated spatially filtered beam impinges on a mirror, MA, placed at a 45 degree angle to beam incidence such that the beam is bent at a 90° angle parallel to the optics table and enters the Mach Zender interferometer. The mirror was required due to a lack of space on the optics table. In anticipation of experiments to be conducted with the NdYAG laser, 25.4 mm diameter mirrors were chosen with high reflectivity (99.7% [34]) for wavelengths between 524 and 532 nm and the 2nd harmonic. These mirrors are designed to withstand high CW (1MW/cm²) and peak powers (5J/cm²) [34].

After the beam is deflected by mirror M4, the beam enters the interferometer through the 25.4 mm non-polarizing beamsplitter cube (refer to Figs.3.23a-b). In this experiment all the beamsplitter cubes are 400-700 nm broadband operating range. The entry and exit faces are antireflection coated while the diagonal internal surface has a broadband beam splitting coating [34]. By using a cube instead of a standard beamsplitter, the beam deflection of the through beam is almost entirely eliminated. Further, the phase delays between the split beams are also eliminated. The beamsplitter cubes are mounted on kinematic platform mounts as seen in Figs.3.24a-b. There are micrometer screws at three vertices to aid in orientation adjustments. The beam splitter is held firmly in place by means of an arm that slides down a post fixed to the platform. The laser beam is held a constant height at each point in the experimental setup.
In path A as shown in Figs. 3.9 and 3.25 there is another mirror M1 shown spaced 461.7 mm (18.18") from the center of the beamsplitter and at an angle 45° to the incident beam. The orientation of the mirror is adjusted such that the laser beam maintains a constant height above the optic table at a number of locations. Similar to the modeling studies, the beam in path A is the reference beam.

In path B, the mirror M2 is mounted on a XY translation stage as shown in Fig. 3.26 allowing for maneuverability of this beam. Mirror M2 directs the beam through the Rexolite test piece. Though initially the universal test machine was slated to be used for this test, the machine malfunctioned and another arrangement devised. The alternative setup shown in Fig. 3.27 consists of two 275 mm (10.83") threaded rods, 5.72 mm (0.225") in diameter. A rectangular metal flange with three holes is placed over these two threaded rods. Springs are placed between the base and the flange to create a back pressure when compressed. The center hole of the rectangular flange supports a 22 lbf (lbf implies pound-force) load cell fastened to it by means of washers and bolts. The load cell is connected to a controller box, as shown in Fig. 3.28 and it displays the value of the load that is acting on the piece under test. The load cell has a stainless steel platen that is fastened to it. Another platen is fastened to the base leg. The piece under test is placed between these two platens.

The center of the cube beamsplitter BS2 is placed at a distance of 440.6 mm (17.35") from mirror M2 and a distance of 253.7 mm (9.99") from mirror M1. Two concave lenses in a housing structure are placed behind the beamsplitter cube as shown in Fig. 3.29 enlarging the interference pattern generated at the output of beamsplitter BS2. A screen is placed behind the concave lens assembly at a distance of 381 mm (15") and held in place.
by a vise. A string is pasted to the screen to act as a reference. A video camera mounted on an optical rail is placed a distance 127 mm (5") from the screen. Initially a digital CCD camera was used but since it could only capture still images, the number of fringes that shifted during transition from one load to another could not be established with confidence. With video images replayed in slow motion, the number of fringes that shifted past a reference point was ascertained.

3.3.1 Problems encountered

Although the optical setup is simple on paper, difficulties were soon encountered in practice. Some of these problems with corresponding resolutions are recorded in this section.

Early on in the experimental setup, the fringe pattern was not stable. A continuous, repeatable low frequency shift in the pattern was observed. The optics table is not isolated from the building structure. Consequently, vibrations enter the setup by way of the legs of the table. Optical components responded to the vibration. The shift in the fringe pattern was drastically reduced by tightening each mount on the table.

Slight pressure at any point of the optics table resulted in a shift in the fringe pattern. Consequently, adding physical weight to the piece under test resulted in unrealistic fringe shifts. Consequently, a device was designed to change the internal forces of the system without the need of stressing the optics table with net external forces.

Due to the sensitive nature of the measurement, physically touching and operating equipment on the optical bench influenced the experimental results. To avoid this undesired effect, all interactions with select elements on the table were performed through remote control.
Due to the irregular nature of the edges of the fringe shift pattern, it was difficult to determine the spacing between fringes and their shift. Consequently, the fringe pattern was adjusted to be vertical. The light intensity of each pixel along a number of horizontal rows were examined based on light intensity. An averaging scheme was then devised to determine the center of the dark fringe and how far it shifted. This was then verified with an averaging technique envisioned with the human eye. Reference lines and slow motion video images added to the accuracy of the direction of shift as well as the number of relative shifts observed. Due to the slight irregular geometry of the sample cylinder under test, low values of forces on the order of a Newton resulted in unrealistic changes in the fringe shift. These anomalies were accounted to undetected physical shifts of the sample due to the non uniform loading of the sample.

3.3.2 Experimental procedure

Once the laser is turned on and light passes through BS1 and incident on M1 and M2, the two beams at the output of BS2 may not be parallel. In order to make the beams equidistant from each other at all distances, the mirror M2 is rotated about the mirror mount until the beams beyond BS2 are parallel to each other. The parallel nature of the beams, are ascertained by placing screens at different distances on either output path. If the beams retain the same spacing in both output paths then the beams are said to be parallel to each other. Once parallelism is obtained, the micrometer screws on the XY translator stages supporting the mirror are adjusted until the beams overlap. Once they overlap, the micrometer screws on the backside of the mirror mount M2 (refer to Fig. 3.23b) are adjusted to get better alignment and to modify fringe thickness, spacing and orientation.
In the compression assembly, two wrenches, one placed on each screw at the extreme ends of the rectangular flange, are used to afford torque and easy clamping action. By rotating one wrench by roughly the same amount as the other wrench, the platen compresses the Rexolite piece in a uniform manner implying the medium is homogeneous. Displacement suffered by the Rexolite piece is minimized due to parallel compression. If one screw alone was tightened, the Rexolite piece would be stressed to a greater degree in certain regions as opposed to others and the beam would be sampling an inhomogeneous medium. Since the experiment being discussed is an attempt to model the forces applied by the Nevada Shocker where the electrode plates are parallel to each other with uniform compression action, the setup implementing two wrenches is preferred. Initially, in the no load case, the pressure transducer attached to the upper platen is calibrated to zero. The calibration electronics and display was purchased commercially in conjunction with the eXpert 560x, a universal testing machine manufactured by ADMET. The wrenches are rotated in small increments until the bottom surface of the upper platen comes in contact with the top surface of the Rexolite piece yielding a 100 mN load. This load assures physical contact preventing the sample under test to move laterally between the faces of the platen. A video camera is switched on using a remote control so to record the dynamics of the fringe shift with minimal disturbance of the experimental setup. The lights are turned off for a higher resolution picture and the laser intensity is low to prevent saturating the film. In this case by carefully rotating the wrench so to transfer minimum disturbance through the system, the load is increased incrementally from 100 mN in steps of 200 mN till 1 N. The wrenches are usually rotated in the clockwise direction for the incremental load set and anticlockwise for the decreasing load
set. The increment is then increased in steps of 2 N until 10 N is achieved. From this point the load increment is increased in steps of 5 N until a final load of 95 N is reached. The linear regime of the load cell has a maximum load rating of 100 N. Following the incremental changes in reverse order, a second set of data is recorded. For repeatability purposes, the same experiment is performed once more for both the increasing and decreasing order in applied force. During the recording of each run, the value of each load point is verbally announced, so that the fringe pattern at discrete points on the filmed video can be associated with a load. The transitional loads between steps are also narrated along with when the start of the rotation of the wrenches, so that cause of any sudden shift of fringe can be easily attributed to load transition or sudden mechanical disturbance which may due to irregular rotation of the wrenches. This concludes the discussion on the experimental procedure.

3.4 Discussion of Theoretical, Modeling and Experimental Results

Adhering to the experimental method discussed above, there are four sets of readings that are taken. Two sets of readings are performed with an increasing load set and two sets of readings are performed with a decreasing load set. As illustrated in Fig. 3.17d, each fringe pattern that is generated at a particular load has an interference intensity curve associated with it. Using Microsoft Photo editor, the contrast of the image is modified such that the minima (dark regions) are clearly discernible. As discussed earlier in Section 3.1.2 the distance between two consecutive minima is \( l_f = 1.4 \) mm. Two white vertical reference lines forming a window for evaluating fringe shifts are drawn using Microsoft (MS) Paint at \( x = \) equal constant positions based on this distance. For reference purposes, the \( x \) axis is the horizontal axis normal to the vertical length of the
fringe. The first line is drawn at an arbitrarily determined initial pixel position. The second line is spaced 14 pixels (equal to the \( l_f = 1.4 \) mm fringe spacing) apart. All subsequent pictures have lines drawn at these pixel positions which in the case of the experiment are 175 and 189 respectively on the x-axis in the Paint window. If a minimum moves out of the window, then it indicates a minimum displacement on the order of a half fringe shift or more assuming the average position of the minimum lies at the center of the window.

This experiment was conducted at values of force ranging from 0.1 N to 100 N. It was found between approximately 0.1 to 10 N that the movement of fringes was pronounced but very irregular. This may be a consequence of the non-optical quality parallelism between sample ends and the initially non-uniform pressure over the sample surface. Though the fringe shifts were recorded for load sets less than 10 N, only those fringe shifts for loads approximately equal to 10 N or greater were analyzed to minimize the unanticipated randomness of the data. Observing the graphically enhanced picture, the position of the fringe minimum was marked with a white vertical line using MS Paint. Refer to Fig. 3.30. The graphical picture is then analyzed in Matlab and an interference intensity plot, as displayed in Fig. 3.31, is generated using a pixel color intensity routine. The pixel position of the window remains the same in all cases (pixel number 52 and 66 in Matlab corresponding respectively to pixel number 175 and 189 in MS paint). The x-coordinate pixel position of the minimum line is noted using the data cursor arrow for each load value. When plotting the data, the values of fringe shift are converted from pixels to physical quantities. Refer to Appendix B (Program 9). As discussed earlier in Section 3.2.1, each pixel is equal to 0.1 mm in distance. At load values below 10 N, the
fringe shifts observed on multiple occasions are greater than the distance between consecutive fringe minima. For force loads greater than 10 N, the relative and absolute fringe shifts are smaller than the distance between consecutive fringe minima. Since the theoretical and computational approaches model a system that is almost ideal, analyzing the three methods at 10 N or higher ensures an even ground for comparison. The value of fringe shift calculated is normalized with respect to the distance between consecutive fringe minima. The normalized shift is a relative normalized shift but the applied force (or pressure) is an absolute measurable. Because the initial point in which all relative measurements are based on in each of the four data sets is different, one cannot plot all the data points on the same graph with a common origin in their raw form. Consequently, each set of measured data is conditioned in the following manner to conform to tendencies observed in the theoretical study. The initial point in each data set is chosen as the relative origin of that data set. The measured absolute force and relative normalized shift values of each point in a particular data set is subtracted from the chosen initial point. With the initial point now as the origin, a linear line is fitted to the data set passing through the initial point. The slope is determined. With the slope known, a family of linear curves can now be drawn on an absolute plot based on absolute measured forces and absolute normalized fringe shifts. Based on physics, if the absolute force is zero, the absolute normalized fringe shift is also zero. Consequently, the line passing through the origin in the absolute plot is chosen. From this, the absolute normalized fringe shift of the initial point is calculated knowing the slope of the curve and the absolute force. Using the absolute normalized fringe shift of the initial point, the absolute normalized fringe shifts of each point in the data set is determined. This procedure is
followed for each of the four data sets. The four sets of data based on absolute values along with a fitted linear curve are then superimposed on the same plot as shown in Fig 3.32. The conditioned experimental data are displayed in symbol notation without connecting lines. The theoretical and computational curve is also superimposed in Fig. 3.32 along with the experimental results. Refer to Appendix B (Program 10). There is a large disparity in the slopes of the experimental data curves as compared to the theoretical/computational curves. It is interesting to note that 1st and 2nd load sets correspond to the first test study of increasing and decreasing loads respectively. Further, the 3rd and 4th load sets correspond to the second test study in increasing and decreasing loads respectively. Visually comparing the pair of curves obtained from the first study, reasonable agreement is observed. Similarly, visually comparing the pair of curves obtained from the second study one also observes reasonable agreement. Upon comparing curves between the two studies, the slopes vary significantly. Even so, all four loads have a slope in the same direction. This indicates that the overall behavior of the interferometer system in each of the four cases adheres to a common behavioral pattern. Since Rexolite is an elastic material, the load force versus fringe shift should be independent of how the experiment is performed assuming there are no external influences. Consequently, a single linear curve passing through the origin is fitted to all four sets of data based on absolute values as shown in Fig. 3.33. Superimposed on this plot is the curve representing the theoretical and computational results. To better understand the magnitude of variance between the theoretical/computational results and the experimental results, opto-elastic constants are determined based on the slopes of the fitted curves such that the theoretical curve may agree with the experimental results. The
opto-elastic constant, \( \rho = 0.31 \) used in the theoretical/computational study, is a material parameter obtained from literature. From Eqs. (2.23b), (2.25b), (3.5) and (3.6) and the slope of the curve in Figure 3.33 written as \( m = \frac{\Delta F}{\Delta x} \), the opto-elastic constant obtained from interferometry experiments may be expressed as

\[
\rho_{\text{exp}} = \frac{4\pi \ Y_m A_S}{\beta_o \ m l_s n_S^3}
\]

(3.9)

where \( A_S = 4.88 \times 10^{-4} \) m\(^2\) is the cross sectional area of the sample end. From Figs. 3.32 and 3.33, the slope of the linear curves fitted to the experimental data sets 1, 2, 3, 4, and all data sets combined is provided in Table 3.6. The experimental opto-elastic constant as obtained from the fitted linear curves may be found in Table 3.6 as well. The experimental values differ from literature value by factors between 2.6 and 57. Although the experimental study may have some disparity in results when repeated at these low level pressures, one concludes within the limits of validity of Eq. (2.23b) that there is an inconsistency in opto-elastic parameter values for Rexolite. More refined interferometry experiments may be required to establish a more accurate assessment of this parameter. The Matlab codes that are used in the experimental analysis are included in Appendix B.
Fig. 3.1(a) Modeling the telescope using planar geometry.

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Fig. 3.1. A (b) physical and (c) geometrical representation of the telescope design principle.

Fig. 3.2. An enlarged view of an achromatic doublet lens.
Fig. 3.3. Ray tracing of telescope setup before optimizing the distance between the lenses.

Fig. 3.4. Ray tracing of telescope setup after optimizing the distance between the lenses.

Fig. 3.5. Gaussian beam representation of telescope setup after optimizing the distance between the lenses.
Fig. 3.6. Beam profile on the screen obtained using FindSpotSize.
Fig. 3.7. Beam profile on the screen obtained using FindIntensity.
Fig. 3.8. Designed and assembled optical telescope.
Fig. 3.9. Burn paper studies before the telescope and after the telescope at various locations.
Fig. 3.10. Plot comparing the theoretical, computational and experimental methods of beam collimation.
Fig. 3.11. Top view of the Mach Zender interferometer system.
Fig. 3.12. 3D View of the Interferometer System.
Fig. 3.13. Real Time 3D view of the interferometer setup.
Fig. 3.14 Beam profiles of the wavefronts in (a) Path A in (b) Path B FullForm->True.

The units of measure on the x axis are in mm.
Fig. 3.15 Interference fringe pattern generated on the screen with FullForm->True

The units of measure on the x axis are in mm.

Fig. 3.16 Interference fringe pattern generated on the screen with FullForm->False
Fig. 3.17a. Spacing between the maxima / minima of the fringe in terms of distance plotted against intensity.
Fig. 3.17b. The interference intensity patterns generated by the computational method.
Fig. 3.17c. Graph comparing the theoretical and computational analyses.

Fig. 3.17d. Interference intensity pattern generated from experimental data.
Fig. 3.18a. The grinding machine with four plates of varying grit sizes that can be fixed on to the grinding machine, the fifth plate is attached to the grinding machine.
Fig. 3.18b Straight line stroke used in grinding.
Fig. 3.19. Setup to measure the optical density of Rexolite.
Fig. 3.20. Beam profiles (a) without Rexolite test sample placed in the beam path.

(b) with Rexolite test sample placed in the beam path.
Fig. 3.21 Kinematic mount for the laser (a). Side view and (b). Top view.
Fig. 3.22 Spatial filter (a). Side view and (b). Top View.
High energy NdYAG laser mirror

Kinematic mirror mount

Micrometer Screws for precision adjustment

Fig. 3.23 Mirror $M_A$ (a). Front View and (b). Back View.
Fig. 3.24 Beam splitter cube (a). Side view and (b). Top View
Fig. 3.25 Experimental setup including the Mach Zender Interferometer
Fig. 3.26 Mirror M2 mounted on XY translation stage.
Fig. 3.27 Test piece compression setup.
Fig. 3.28 Controller box with display indicating load measured by load cell.
Fig. 3.29 Concave lenses in lens tube placed in front of the beamsplitter BS2.
Fig. 3.30. Picture of fringe pattern indicating the window and minimum position line that have been added to aid in image processing using Matlab.
Fig. 3.31. Matlab generated intensity plot of the fringe pattern in Fig 3.30.
Fig. 3.32. Plot comparing the data obtained from the four experimental load sets, the theoretical approach and the computational approach.
Fig. 3.33. Plot comparing the average of the data obtained from the four experimental load sets, the theoretical approach and the computational approach.
Table 3.1 Parameters used in modeling the telescope using Rayica and Wavica.

<table>
<thead>
<tr>
<th>No.</th>
<th>Description of Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Focal length of the Achromatic Doublet((L_1))(mm)</td>
<td>300</td>
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<tr>
<td>2</td>
<td>Diameter of the Achromatic Doublet((L_1))(mm)</td>
<td>50.8</td>
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<td>3</td>
<td>Radius of curvature of the convex lens, (r_{11})(mm)</td>
<td>170.317</td>
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<td>Radius of curvature at the cementing layer of the achromatic doublet, (r_{12})(mm)</td>
<td>-141.605</td>
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<td>5</td>
<td>Radius of curvature of the concave lens, (r_{13})(mm)</td>
<td>-475.988</td>
</tr>
<tr>
<td>6</td>
<td>Thickness of the Convex Lens in the Achromatic Doublet((L_1))(mm)</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>Thickness of the Concave Lens in the Achromatic Doublet((L_1))(mm)</td>
<td>5</td>
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<tr>
<td>8</td>
<td>Glass type of the Convex and Concave lenses used in the Achromatic Doublet((L_1))</td>
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<td>9</td>
<td>Focal length of the Achromatic Doublet((L_2))(mm)</td>
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<td>BaFN10-SF10</td>
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<tr>
<td>17</td>
<td>Number of Rays in the Gaussian Beam</td>
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<td>18</td>
<td>Wavelength of the Gaussian Beam, (\mu m)</td>
<td>0.532</td>
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<td>19</td>
<td>The beam diameter at the input of the telescope (mm)</td>
<td>(R_{\text{in}} = 4.255, \text{mm})</td>
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<tr>
<td>20</td>
<td>Divergence of the Gaussian Beam(mrad)</td>
<td>0.01</td>
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Table 3.2.1 Computational results obtained with SpotSize values. The beam diameter at the input of the telescope $R_{ob} = 4.255$ mm

<table>
<thead>
<tr>
<th>No.</th>
<th>Distance between the screen and the second surface of Lens, $L_2$ (mm)</th>
<th>Spot Size(Beam Diameter) $d_s$ measured at the screen (mm)</th>
<th>Ratio Of $R_{ob}$ to focused Beam Diameter</th>
<th>Ratio Of change in Beam Diameter between successive positions of the screen</th>
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<td>25.4</td>
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<td>2.</td>
<td>50.8</td>
<td>0.789872</td>
<td>5.39</td>
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<td>3.</td>
<td>76.2</td>
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<td>5.51</td>
<td>0.97837</td>
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<td>4.</td>
<td>101.6</td>
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<td>0.97789</td>
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<td>5.</td>
<td>127</td>
<td>0.738616</td>
<td>5.76</td>
<td>0.977392</td>
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<td>6.</td>
<td>152.4</td>
<td>0.721531</td>
<td>5.9</td>
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<tr>
<td>7.</td>
<td>177.8</td>
<td>0.704446</td>
<td>6.04</td>
<td>0.976321</td>
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</table>

Table 3.2.2 Computational results with RayBoundary values obtained with the FindIntensity command. The beam diameter at the input of the telescope $R_{ob} = 4.255$ mm

<table>
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<tr>
<th>No.</th>
<th>Distance between the screen and the second surface of Lens, $L_2$ (mm)</th>
<th>Ray Boundary(Beam Diameter) $d_{RB}$ measured at the screen (mm)</th>
<th>Ratio Of $d_s$ to $d_{RB}$</th>
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<tbody>
<tr>
<td>1.</td>
<td>25.4</td>
<td>1.452534</td>
<td>0.555551</td>
</tr>
<tr>
<td>2.</td>
<td>50.8</td>
<td>1.421768</td>
<td>0.555556</td>
</tr>
<tr>
<td>3.</td>
<td>76.2</td>
<td>1.391004</td>
<td>0.555561</td>
</tr>
<tr>
<td>4.</td>
<td>101.6</td>
<td>1.360238</td>
<td>0.555565</td>
</tr>
<tr>
<td>5.</td>
<td>127</td>
<td>1.329474</td>
<td>0.55557</td>
</tr>
<tr>
<td>6.</td>
<td>152.4</td>
<td>1.29871</td>
<td>0.555575</td>
</tr>
<tr>
<td>7.</td>
<td>177.8</td>
<td>1.267944</td>
<td>0.555581</td>
</tr>
</tbody>
</table>
Table 3.3 Experimental results. The beam diameter at the input of the telescope $R_{ob} = 4.255 \text{mm}$

<table>
<thead>
<tr>
<th>No.</th>
<th>Distance between the screen and the second surface of Lens, $L_2$ (mm)</th>
<th>Beam Diameter measured at the screen</th>
<th>First Reading (mm)</th>
<th>Second Reading (mm)</th>
<th>Third Reading (mm)</th>
<th>Fourth Reading (mm)</th>
<th>Average Reading (mm)</th>
<th>Ratio Of $R_{ob}$ to focused Beam Diameter</th>
<th>Ratio Of change in Beam Diameter between successive positions of the screen</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>25.4</td>
<td>0.634</td>
<td>0.76</td>
<td>0.71</td>
<td>0.76</td>
<td>0.72</td>
<td>5.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>50.8</td>
<td>0.76</td>
<td>0.69</td>
<td>0.69</td>
<td>0.71</td>
<td>0.71</td>
<td>5.98</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>76.2</td>
<td>0.56</td>
<td>0.64</td>
<td>0.64</td>
<td>0.64</td>
<td>0.62</td>
<td>6.91</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>101.6</td>
<td>0.53</td>
<td>0.51</td>
<td>0.53</td>
<td>0.53</td>
<td>0.53</td>
<td>8.07</td>
<td>0.86</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>127</td>
<td>0.48</td>
<td>0.51</td>
<td>0.46</td>
<td>0.51</td>
<td>0.49</td>
<td>8.70</td>
<td>0.93</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>152.4</td>
<td>0.51</td>
<td>0.41</td>
<td>0.51</td>
<td>0.48</td>
<td>0.48</td>
<td>8.93</td>
<td>0.97</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>177.8</td>
<td>0.38</td>
<td>0.38</td>
<td>0.43</td>
<td>0.38</td>
<td>0.39</td>
<td>10.81</td>
<td>0.83</td>
<td></td>
</tr>
</tbody>
</table>
Table 3.4. In this table the values of incident power and transmitted power are measured for the 15 samples and the transmittance, optical density and power diffused are calculated.

<table>
<thead>
<tr>
<th>No.</th>
<th>Incident Power, (mW)</th>
<th>Transmitted Power, (mW)</th>
<th>Transmittance $T = \frac{I_t}{I_i}$</th>
<th>Optical Density $= \log_{10} \left( \frac{1}{T} \right)$</th>
<th>Power diff. (mW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.606</td>
<td>3.86</td>
<td>0.838</td>
<td>1.193</td>
<td>0.746</td>
</tr>
<tr>
<td>2</td>
<td>4.61</td>
<td>3.82</td>
<td>0.829</td>
<td>1.207</td>
<td>0.79</td>
</tr>
<tr>
<td>3</td>
<td>4.624</td>
<td>3.92</td>
<td>0.848</td>
<td>1.18</td>
<td>0.704</td>
</tr>
<tr>
<td>4</td>
<td>4.625</td>
<td>3.96</td>
<td>0.856</td>
<td>1.17</td>
<td>0.665</td>
</tr>
<tr>
<td>5</td>
<td>4.65</td>
<td>4</td>
<td>0.860</td>
<td>1.163</td>
<td>0.65</td>
</tr>
<tr>
<td>6</td>
<td>4.66</td>
<td>4.007</td>
<td>0.859</td>
<td>1.163</td>
<td>0.653</td>
</tr>
<tr>
<td>7</td>
<td>4.67</td>
<td>4.075</td>
<td>0.873</td>
<td>1.146</td>
<td>0.595</td>
</tr>
<tr>
<td>8</td>
<td>4.679</td>
<td>4.044</td>
<td>0.864</td>
<td>1.157</td>
<td>0.635</td>
</tr>
<tr>
<td>9</td>
<td>4.68</td>
<td>4.044</td>
<td>0.864</td>
<td>1.157</td>
<td>0.636</td>
</tr>
<tr>
<td>10</td>
<td>4.684</td>
<td>4.038</td>
<td>0.862</td>
<td>1.16</td>
<td>0.646</td>
</tr>
<tr>
<td>11</td>
<td>4.682</td>
<td>4.066</td>
<td>0.868</td>
<td>1.152</td>
<td>0.616</td>
</tr>
<tr>
<td>12</td>
<td>4.689</td>
<td>4.062</td>
<td>0.866</td>
<td>1.154</td>
<td>0.627</td>
</tr>
<tr>
<td>13</td>
<td>4.693</td>
<td>4.093</td>
<td>0.872</td>
<td>1.147</td>
<td>0.6</td>
</tr>
<tr>
<td>14</td>
<td>4.694</td>
<td>4.066</td>
<td>0.866</td>
<td>1.154</td>
<td>0.628</td>
</tr>
<tr>
<td>15</td>
<td>4.694</td>
<td>4.079</td>
<td>0.869</td>
<td>1.151</td>
<td>0.615</td>
</tr>
<tr>
<td>16</td>
<td>4.703</td>
<td>3.952</td>
<td>0.840</td>
<td>1.19</td>
<td>0.751</td>
</tr>
<tr>
<td>17</td>
<td>4.706</td>
<td>3.89</td>
<td>0.827</td>
<td>1.21</td>
<td>0.816</td>
</tr>
<tr>
<td>18</td>
<td>4.706</td>
<td>3.81</td>
<td>0.810</td>
<td>1.24</td>
<td>0.896</td>
</tr>
</tbody>
</table>

Table 3.5. The lengths measured from one surface of the Rexolite test piece to the other at different positions on the surface of the Rexolite piece.

<table>
<thead>
<tr>
<th>No.</th>
<th>Length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22.6314</td>
</tr>
<tr>
<td>2</td>
<td>22.6695</td>
</tr>
<tr>
<td>3</td>
<td>22.6949</td>
</tr>
<tr>
<td>4</td>
<td>22.733</td>
</tr>
<tr>
<td>5</td>
<td>22.6568</td>
</tr>
<tr>
<td>6</td>
<td>22.7076</td>
</tr>
</tbody>
</table>

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Table 3.6. Opto elastic parameters for different load cases calculated using the slope obtained from the graph.

<table>
<thead>
<tr>
<th>No.</th>
<th>Load case</th>
<th>x value</th>
<th>y value</th>
<th>Slope, m</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>First experimental load case</td>
<td>0.4476</td>
<td>93.1</td>
<td>207.99</td>
<td>0.0911</td>
</tr>
<tr>
<td>2.</td>
<td>Second experimental load case</td>
<td>0.5408</td>
<td>85.5</td>
<td>158.1</td>
<td>0.1198</td>
</tr>
<tr>
<td>3.</td>
<td>Third experimental load case</td>
<td>0.1176</td>
<td>92.2</td>
<td>784.01</td>
<td>0.0242</td>
</tr>
<tr>
<td>4.</td>
<td>Fourth experimental load case</td>
<td>0.0246</td>
<td>85</td>
<td>3455.28</td>
<td>0.0055</td>
</tr>
<tr>
<td>5.</td>
<td>Average experimental load case</td>
<td>1.946</td>
<td>779.2</td>
<td>400.411</td>
<td>0.0473</td>
</tr>
<tr>
<td>6.</td>
<td>Computational load case</td>
<td>0.5839</td>
<td>35</td>
<td>59.94</td>
<td>0.316</td>
</tr>
</tbody>
</table>
CHAPTER 4

CONCLUDING COMMENTS

4.0 Future Investigations

The efforts leading to this work raised a number of questions and comments that may need to be addressed prior to or when implementing this diagnostic on the *Nevada Shocker*. This chapter contains a record of these questions or comments with some brief explanation.

4.1 Physical Meaning of Signal Signature

In Chapter 1, a simple transient theory was developed to characterize the displacement of the electrode surface in the presence of the plastic under test. For the time durations examined, the displacement resulted in a motion that was about one ten thousandth of a monolayer distance. One must question if this is a measurable parameter. Within the validity of the model, it was also shown that the displacement exhibited a linear relationship with time due to the large disparity between spring constants. Even so, in the static regime, noticeable fringe shifts were recorded experimentally for forces well below the values used in the atomic/molecular mass-spring model. It is suggested that one examine the transient model with greater detail to determine when in time one may expect to be able to detect a measurable signal. Assuming this time is within the timeframe of the experiment, one can deduce the time the signal was launched. Although
a 50 ns pulsed is desired from the *Nevada Shocker*, mismatch does result in larger pulse durations.

To perform time of flight studies, a means to dependably trigger the laser with minimal jitter is necessary. Triggering has not been examined at this time.

4.2 Vibration Noise

The *Nevada Shocker* when in discharge mode sends a large amplitude signal to the Blumlein. As the Blumlein charges, the field intensifies at a water gap switch connecting the Blumlein to the transmission line leading to the experimental region housing the plastic under test. A point is reached when the water in the gap breaks down releasing the electrical energy and, as a by product, acoustical energy. Constant low frequency vibrations are also present as a result of water circulation and vacuum pump operation. These and other noise sources will mask the desired signal to be measured. Experiments have shown that small pressures to the optical bench resulting in a stress on the table results in a fringe shift. An optical scheme is discussed that may minimize some of these effects tied to the machine proper.

One approach to minimize the noise effects is to allow both the reference beam and the sampling beam in the Mach Zender to pass through the same vacuum port windows and atmospheric environment. Appropriately adjusting the optical path lengths, the undesired signal generated due to low and medium frequency vibrations can be subtracted upon combining the optical beams at the detector. In this case, the optical elements need to be isolated from the pulsed power machine.

Although this minimizes noise effects at the vacuum ports, vibrations effects will reach the sample under test due to mechanical contact of the electrodes to the machine.
proper. A statistical study of the machine's background vibrations may yield a measurable average that may be subtracted from the experimental data. Because the water switch is far from the experimental region and because the electromagnetic energy propagates at a velocity that is orders of magnitude higher than the generated acoustic wave at the switch, the experimental data may be retrieved prior to shot generated vibrations reaching the experiment. Further transient studies on this matter are required to determine the window in time that is available that data must be taken prior to the coupling of undesired shot generated acoustic energy.

Experimental studies showed that optical components must be carefully secured to an optics table especially if the table does not contain vibration absorber mechanisms. Because the coherent length of the Nd:YAG laser with seeder is less than a foot and because window vibrations are not uniform, optical components in the Mach Zender need to be closely spaced.

Tests by others (Dr. Richard Kant) have shown that if the interferometer is covered, the fringe pattern becomes more stable. It was deduced that the air circulation in the path of the two beams was responsible for random perturbations in the fringe shift.

4.3 Optical components

Although fringe patterns could be obtained with standard polishing techniques on the sample, the quality of the fringe is low. This indirectly implies that the port windows to the vacuum chamber be of a high optical grade such that the reference and sampling beams not be degraded as a result of external entities not specific to the plastic under test. It is not desired to have an optical grade polish on the plastic piece since this results in an impractical constraint in a national laboratory setting.
The Nd:YAG laser with seeder purchased for the flashover experiments requires a reduction in power level so as not to exceed the damage threshold of both the optics and the plastic in the beam paths. Though the values of power and energy are attainable, by tuning the Q-switch delay of the Nd:YAG laser power supply, the low level power output is at the cost of stability. The beam diameter varies due to the irregular flashing of the rods being pumped with flashlamp light. Since the measurements involved are of such low orders of magnitude, these effects cannot be ignored. Consequently, a Q switch delay versus stability study is required. Once a level of stability is maintained, spatial filtering of the beam will be required to minimize the irregularity in beam diameter as a result of the laser and to further reduce the energy received by optical components beyond the position of the spatial filter.

Because the signal to be measured may be well in the noise, a means to selectively enhance the signal may be required. Heterodyne techniques appear to offer some hope in selectively amplifying the light frequency of interest while minimizing the noise generated outside of a specified bandwidth. Adding another optical component complicates the experiment and may place further limitations to laser's operating state. The electronics with phase locking may be a daunting task. Even so, it may be the only alternative to detect the signal of interest. It is recommended that this work be carefully considered if more straightforward detection techniques fail as suggested in Chapter 2.

4.4 Concluding Remarks

To use this diagnostic effectively, one may have to compromise on the constraints of the problem in practice. This compromise may require a higher level of theoretical modeling and a more elaborate experimental setup than developed in Chapter 3. The
constraints on the measurable sought may have to be relaxed in order to establish a useful
detectable signal.
APPENDIX A

1. Matlab program to calculate the values of $\frac{L}{h}(n_p - n_o)$ for 633 nm.

```matlab
lambda = 633 * 10^-9; % Wavelength of a Laser
Bo = (2 * 3.14) / lambda; % Laser Wave Number (1/m)
Bo = 9.921e6;
Ld = 400e-6; % length of detector
Xl = 1
% calculating the values that lie between 2pi and 4pi
U1 = -2 * 3.14 / (Bo * Xl) % U = (n_p - n_o) * (Lh)
U2 = -4 * 3.14 / (Bo * Xl)

stepsize = (U2 - U1) / 5
ua = U1 + stepsize
ub = U1 + (2 * stepsize)
uc = U1 + (3 * stepsize)
ud = U1 + (4 * stepsize)
ue = U1 + (5 * stepsize)

% calculating the values that lie between 0 and 2pi
U3 = 2 * 3.14 / (Bo * (Ld / 2))
U4 = 4 * 3.14 / (Bo * (Ld / 2))

stepsizebb = (U6 - U5) / 6;
uab = U5 + stepsizebb
ubb = U5 + (2 * stepsizebb)
ucb = U5 + (3 * stepsizebb)
udb = U5 + (4 * stepsizebb)
ueba = U5 + (5 * stepsizebb)

stepsizeaa = (U8 - U7) / 8;
uaa = U7 + stepsizeaa
```

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uba=U7+(2*stepsizeaa)
ucd=U7+(3*stepsizeaa)
uda=U7+(4*stepsizeaa)
uea=U7+(5*stepsizeaa)
ufa=U7+(6*stepsizeaa)
uga=U7+(7*stepsizeaa)

2. Matlab program to calculate the SNR performance of above discussed detection schemes at 633nm.

clear all;close all;clc;

e=1.6*10^-19;%Charge of an electron%
lambda=633*10^-9;%Wavelength of a Laser%
Bo=(2*3.14)./lambda;%Laser Wave Number%(1/m)
Rl=50;%Load Resistance %
trt1=10e-9;%rise time of pulse signal that is generated by the marx bank discharge
% B1=0.35./trt1 %Bandwidth%
B1=1.2E9 %BANDWIDTH OF THE AMPLIFIER AD8015%
Ld=400E-6;%Length of the E0T-2030A Detector[E0T 2030A SPEC SHEET]%
Ad=3.14*(400E-6/2)^2;%ACTIVE AREA OF E0T-2030A Detector%
A=3.14*10^-6;%Area of the BEAM
lens=2.54*10^-2;%Length of the Sample%
l=2.54*10^-2;%Sample Height%
np=1.5;%Refractive index of the glass plate wedge%
no=1;%Refractive index of the air%
ns=1.585;%Refractive index of the sample%
id=0.1*10^-9;%Dark Current[E0T 2030A SPEC SHEET]%
rs=0.4;%Responsivity of the diode[from graph in the E0T 2030A SPEC SHEET]%
T=293;%Room Temperature(Kelvin)%
k=1.38*10^-23;%Boltzmann Constant%
rho=0.31;%Opto-Elastic Constant of REXOLITE%
Cd=1.5*10^-12;%capacitance of E0T 2030A PHOTODETECTOR[E0T 2030A SPEC SHEET]
Rl=50;%Resistance of the load at the scope is 50 ohms
dell=0;%change in lateral length of the sample
Pin=1e-3:0.001:20.3e-2;%power of the source laser Pin
%Pin=11.3e-2;%power of the source laser Pin
tpulse=5e-9;%pulse duration of NdYAG
%Pin=20.3e-2;
Ein=Pin*tpulse% Energy input in the system
% X1=-3.66537;
Tr=10e3;%Transresistance of the bipolar transistor stage in the amplifier[AD8015 SPEC SHEET]
X1=1;
rf=10*10^3;%Feedback resistance with the amplifier at load
\[ \text{Rac} = 7 \times 10^3 \text{ohms} \]
\[ \text{ri} = 5 \times 10^3 \text{ohms} \]
\[ \text{rsh} = 1 \times 10^3 \text{ohms} \]
\[ c = 3 \times 10^8 \text{m/s} \]

\[ U = 0.0032; \]
\[ U = 0.0037; \]
\[ U = 0.0042; \]
\[ U = 0.0047; \]
\[ U = 0.0052; \]
\[ U = 0.0057; \]
\[ U = 0.0062; \]
\[ U = 7.5960 \times 10^{-7}; \]
\[ U = 8.8620 \times 10^{-7}; \]
\[ U = 1.0128 \times 10^{-6}; \]
\[ U = 1.1394 \times 10^{-6}; \]
\[ U = 1.2660 \times 10^{-6}; \]
\[ U = 3.9563 \times 10^{-4}; \]
\[ U = 7.9125 \times 10^{-4}; \]
\[ U = 0.0012; \]
\[ U = 0.0016; \]
\[ U = 0.0020; \]
\[ U = 0.0024; \]
\[ U = 0.0028; \]
\[ U = 1.0550 \times 10^{-7}; \]
\[ U = 2.1100 \times 10^{-7}; \]
\[ U = 3.1650 \times 10^{-7}; \]
\[ U = 4.2200 \times 10^{-7}; \]
\[ U = 5.2750 \times 10^{-7}; \]

\[ \psi_1 = (-1) \times B0 \times U \times X1; \text{Eq. 2.50b} \]
\[ \psi_2 = B0 \times U \times (Ld/2); \text{Eq. 2.50c} \]
\[ Ym = 3.1 \times 10^9; \text{Youngs Modulus (Pascals)} \]

\[ \% \text{TO CALCULATE } \Delta \phi / \text{ASSUME A LOAD OF X Newtons IS ACTING ON THE PLATES} \]
\[ n = 0.03; \]
\[ \% n = 0.3; \]
\[ \% n = 32.46; \]
\[ \% n = 60; \% \text{Force acting on the plates (Newton)} \]
\[ Afr = (5.06 \times 10^{-4}) - (1.79 \times 10^{-5}) \times \text{Area over which the force is acting on the rexolite sample} \]
\[ PY1 = ((n/Afr) / Ym); \% (Pressure/Youngs Modulus) \]
\[ \Delta \text{ns} = ((n.3) \times \text{rho./2}) \times PY1; \% \text{change in refractive index} \]
\[ \% \Delta \text{ns} = 0 \text{ for unstressed case} \]
\[ \text{delpsi} = (Bo \cdot (ns-no) \cdot (dells)) + (Bo \cdot lens \cdot delnsb1); \quad \text{Eq. 2.50e, dells=0} \]

**CALCULATION OF DelPind**

\[ \text{DelPindna} = \frac{(Ad \cdot Pin \cdot (sin(psi1)) \cdot (sin(psi2)) \cdot (delpsi)) \cdot (2 \cdot A \cdot psi2)}{2 \cdot A \cdot psi2}; \quad \text{Eq. (2.54)} \]

**Change in power incident on the diode**

\[ \text{DelPindnb} = \frac{(Ad \cdot Pin \cdot (cos(psi1)) \cdot (sin(psi2)) \cdot (delpsi^2)) \cdot (4 \cdot A \cdot psi2)}{4 \cdot A \cdot psi2}; \quad \text{Eq. (2.54)} \]

**Change in power incident on the diode**

\[ \text{DelPind} = (\text{DelPindna} + \text{DelPindnb}); \quad \text{Eq. 2.54 Change in power incident on the diode} \]

\[ \text{Pindu} = (Ad \cdot (2 \cdot A)) \cdot Pin \cdot (1 + (\cos(psi1) \cdot \sin(psi2)) / (psi2)); \quad \text{Eq. 2.51c Power incident on the diode in the unstressed case} \]

\[ \text{Pinds} = (Ad \cdot (2 \cdot A)) \cdot Pin \cdot (1 + (\cos(psi1 + delpsi) \cdot \sin(psi2)) / (psi2)); \quad \text{Eq. 2.51b Power incident on the diode in the stressed case} \]

\[ i_l = \text{Pindu} \cdot rs; \quad \% \text{Current generated due to the laser incident on the diode} \]

\[ \text{DelPind} = \text{Pinds} - \text{Pindu}; \quad \% \text{DC CASE AMPLIFIER AT LOAD} \]

\[ R_{1R} = \frac{1}{(C \cdot \text{RI})} \text{Bandwidth (LPF) of the detector circuit with } R_l = \text{scope resistance and } C = \text{Diode Capacitance} \]

\[ \text{in} = (2 \cdot e \cdot B1R \cdot \text{Pindu} \cdot rs) + (2 \cdot e \cdot B1R \cdot \text{id}) + ((4 \cdot k \cdot T \cdot B1R) / R_l); \quad \% \text{noise current squared with resistance at load dc case} \]

\[ \text{isdRl} = (2 \cdot e \cdot B1R \cdot \text{id}); \quad \text{Eq. 2.61 shot noise current squared due to dark current} \]

\[ \text{islRl} = (2 \cdot e \cdot B1R \cdot \text{Pindu} \cdot rs); \quad \text{Eq. 2.62 shot noise current squared due to current generated by the diode} \]

\[ \text{ithRl} = ((4 \cdot k \cdot T \cdot B1R) / R_l); \quad \% \text{Thermal noise current squared (A}^2) \]

\[ \text{PsdRl} = \text{isdRl} \cdot R_l; \quad \% \text{Noise power due to shot noise current squared (A}^2) \text{ (dark current)} \]

\[ \text{PslRl} = \text{islRl} \cdot R_l; \quad \% \text{Noise power due to thermal noise current squared (A}^2) \]

\[ \% \text{Signal to noise ratio (SNR) FOR THE UNSTRESSED CASE} \]

\[ \text{SNRRIDCU} = 20 \cdot \text{(log10}( (rs \cdot \text{Pindu}) / (\text{sqrt}(\text{in}))) ) ; \quad \% \text{SNR with resistance at load dc unstressed case} \]

\[ \% \text{Signal to noise ratio (SNR) FOR THE STRESSED CASE} \]

\[ \text{SNRRIDCS} = 20 \cdot \text{(log10}( (rs \cdot \text{Pinds}) / (\text{sqrt}(\text{in}))) ) ; \quad \% \text{SNR with resistance at load dc stressed case} \]

\[ \% \text{AC CASE AMPLIFIER AT LOAD} \]

\[ \% \text{Pindac} = \text{Pindu} - ((e \cdot B1) / rs); \quad \% \text{a/c component of the power that is incident on the diode} \]

\[ \text{ina} = (2 \cdot e \cdot B1R \cdot \text{Pindac} \cdot rs) + (2 \cdot e \cdot B1R \cdot \text{id}) + ((4 \cdot k \cdot T \cdot B1R) / R_l); \quad \% \text{noise current squared (A}^2) \text{ with resistance at load a/c case} \]

\[ \text{SNRRIAC} = 10 \cdot \text{(log10}( (rs^2) \cdot (1/3) \cdot (\text{DelPind})^2) ) / (\text{ina})) ; \quad \% \text{Eq. 2.84 SNR with resistance at load dc case} \]

\[ \text{islRIAC} = (2 \cdot e \cdot B1R \cdot \text{Pindu} \cdot rs); \]

\[ \text{PslRIAC} = \text{islRIAC} \cdot R_l; \]

\[ \% \text{DC CASE AMPLIFIER AT LOAD} \]
inamp=26.5*10^-9;%The noise current of the amplifier(nA) [AD8015 specsheet]
rs11=(rsh.*Rac.*ri)./(rsh.*Rac)+(Rac.*ri)+(rsh.*ri);%Eq. 2.86. Source resistance
Tr=10e3;%Transresistance [AD8015 specsheet]
vn=sqrt(4*k*T*(2/3)*Tr);%Equivalent noise voltage [ART OF ELECTRONICS Pg 444]
rfv=[(rf+rs11)./(rf.*rs11)].*(vn).^2;%Load current squared(A^2) due to noise voltage
isdA=(2.*e.*B1.*id);%Eq. 2.61 the shot noise current squared(A^2) due to the dark current
islA=(2.*e.*B1.*Pinds.*rs);%Eq. 2.62 the shot current squared(A^2) noise due to the laser current
ithA=((4.*k.*T.*B1)./rf);%Thermal noise current squared(A^2)
inamps=inamp.^2;%Noise current due to amplifier noise(A^2)
Pampnoise=inamps.*(rf.^2)./Rl)%Noise power due to amplifier noise current
PsdA=isdA.*(rf.^2)./Rl)%Eq. 2.89. Noise power due to shot noise current squared(A^2)(dark current)
PslA=islA.*(rf.^2)./Rl)%Noise power due to shot noise current squared(A^2)(laser current)
PthA=ithA.*(rf.^2)./Rl)%Noise power due to shot noise current squared(A^2)(thermal current)
PlnA=rfv.*(rf.^2)./Rl)%Noise power due to shot noise current squared(A^2)(load current from noise voltage)
SNRAMPDC=20.*(log10(1/sqrt((isdA+islA+inamps+ithA+rfv))))%SNR with an amplifier(dc case)
%AC CASE AMPLIFIER AT LOAD
SNRAMPAC=10.*(log10(((rs.^2).*(1/3).*(DelPind.^2))./(isdA+islA+inamps+ithA+rfv))));%SNR with an amplifier(ac case)

%%%%%%% HETERODYNE CASE PARAMETERS %
Volo=2;%Voltage of the Local Oscillator 1st case=2V.
alpha=1;%Quadratic coupling efficiency.
% B4=800E6;% BANDWIDTH OF THE LOCAL OSCILLATOR
B4=0.35./trt1;%Bandwidth
rs14=(rsh.*Rac.*ri)./(rsh.*Rac)+(Rac.*ri)+(rsh.*ri);%Source resistance
vn=sqrt(4*k*T.*(2/3)*Tr);%Equivalent noise voltage
rfv=[(rf+rs14)./(rf.*rs14)].*(vn).^2;%Load current due to noise voltage
Voloa=5;%Voltage of the Local Oscillator 2nd case=5V.
isdH=(2.*e.*B4.*id);%the shot noise current squared(A^2) due to the dark current
islH=(2.*e.*B4.*Pinds.*rs);%the shot noise current squared(A^2) due to the laser current
ithH=((4.*k.*T.*B4)./rf);%Thermal noise current squared(A^2)
Pampnoise=inamps.*((rf.^2)./Rl);%Noise power due to amplifier noise current
PsdH=isdH.*(rf.^2)./Rl)%Noise power due to dark current shot noise
PslH=islH.*(rf.^2)./Rl)%Noise power due to laser current shot noise
PthH=ithH.*(rf.^2)./Rl)%Noise power due to thermal noise
PlnH=rfv.*(rf.^2)./Rl)% Noise power due to load current from noise voltage

% HETERODYNE CASE

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SNRHETAC = 10.*log10(((r.^2).*(alpha.^2).*(Volo.^2).*(l/3).*(DelPind.^2))./((isdH+isIH+inamps+ithH+rfv)))); % SNR with an amplifier (a/c case)
SNRHETACB = 10.*log10(((r.^2).*(alpha.^2).*(Voloa.^2).*(l/3).*(DelPind.^2))./((isdH+isIH+inamps+ithH+rfv)))); % SNR with an amplifier (a/c case)

plot(Pindu,SNRRIDCS,'r')
hold on
plot(Pindu,SNRIAC,'g')
hold on
plot(Pindu,SNRAMPDC,'b')
hold on
plot(Pindu,SNRAMPAC,'k')
hold on
plot(Pindu,SNRHETAC,'c')
hold on
plot(Pindu,SNRHETACB,'m')
hold on
plot(Pindu(1:20:length(Pindu)),SNRIAC(1:20:length(SNRIAC)),'k','MarkerSize',20)
hold on
plot(Pindu(1:20:length(Pindu)),SNRHETAC(1:20:length(SNRHETAC)),'k','MarkerSize',20)

hold on
title({'P_i_n_d , Power incident on the diode (W) vs SNR (dB);' ['Force = ' num2str(n) 'N ' psi1 = ' num2str(psi1) ' psi2 = ' num2str(psi2) ' lambda = ' lambda*le9 'nm']})
legend('SNRRLDC','SNRRLAC','SNRAMPDC','SNRAMPAC','SNRHETAC','SNRHETACB', 'Location','BestOutside')
xlabel('P_i_n_d , Power incident on the diode(watts)')
ylabel('SNR(dB)')
set(get(gca,'children'),'linewidth',[2]);
set(gca,'linewidth',[2],'gridlinestyle','-','fontname','timesnewroman','fontweight','bold','fontsize',[16]);
set(get(gca,'title'),'fontname','timesnewroman','fontweight','bold','fontsize',[16])
set(get(gca,'xlabel'),'fontname','timesnewroman','fontweight','bold','fontsize',[16])
set(get(gca,'ylabel'),'fontname','timesnewroman','fontweight','bold','fontsize',[16])
grid on

figure
plot(Pin,Pindu,'r')
hold on
plot(Pin,Pinds,'g')
hold on

title({'P_i_n_d (W) vs Pin (W);' ['psi1 = ' num2str(psi1) ' psi2 = ' num2str(psi2)]})

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set(get(gca,'children'), 'linewidth', [2]);
set(gca, 'linewidth', [2], 'gridlinestyle', '-', 'fontname', 'arial', 'fontweight', 'bold', 'fontsize', [16]);
set(get(gca,'title'), 'fontname', 'arial', 'fontweight', 'bold', 'fontsize', [16])
set(get(gca,'xlabel'), 'fontname', 'arial', 'fontweight', 'bold', 'fontsize', [16])
set(get(gca,'ylabel'), 'fontname', 'arial', 'fontweight', 'bold', 'fontsize', [16])
grid on

Pload3 = ((rs.^2).*(l/3).*(DelPind.^2)).*Rl;
Vload3 = sqrt(Pload3.*Rl);
figure
plot(DelPind, Vload3)
title('ΔP_i_n_d (W) vs V_L (V) Resistor Load AC Case; ['ψ1 = ' num2str(psi1) ', 'ψ2 = ' num2str(psi2)])'
xlabel('ΔP_i_n_d, Change in power incident on diode (W)')
ylabel('V_L, Load voltage (V)')
set(gca, 'children', 'linewidth', [2]);
set(gca, 'linewidth', [2], 'gridlinestyle', '-', 'fontname', 'arial', 'fontweight', 'bold', 'fontsize', [16]);
set(get(gca,'title'), 'fontname', 'arial', 'fontweight', 'bold', 'fontsize', [16])
set(get(gca,'xlabel'), 'fontname', 'arial', 'fontweight', 'bold', 'fontsize', [16])
set(get(gca,'ylabel'), 'fontname', 'arial', 'fontweight', 'bold', 'fontsize', [16])
grid on

Pload4 = (il.^2).*((rf.^2)./Rl);
Vload4 = sqrt(Pload4.*Rl);
figure
plot(Pindu, Vload4)
title('P_i_n_d (W) vs V_L (V) Amplifier Load DC Case; ['ψ1 = ' num2str(psi1) ', 'ψ2 = ' num2str(psi2)])'
xlabel('P_i_n_d, Power incident on diode (W)')
ylabel('V_L, Load voltage (V)')
set(gca, 'children', 'linewidth', [2]);
set(gca, 'linewidth', [2], 'gridlinestyle', '-', 'fontname', 'arial', 'fontweight', 'bold', 'fontsize', [16]);
set(get(gca,'title'), 'fontname', 'arial', 'fontweight', 'bold', 'fontsize', [16])
set(get(gca,'xlabel'), 'fontname', 'arial', 'fontweight', 'bold', 'fontsize', [16])
set(get(gca,'ylabel'), 'fontname', 'arial', 'fontweight', 'bold', 'fontsize', [16])
grid on

Pload5 = ((rs.^2).*(l/3).*(DelPind.^2)).*(((rf.^2)./Rl);
Vload5 = sqrt(Pload5.*Rl);
figure
plot(DelPind, Vload5)
title('ΔP_i_n_d (W) vs V_L (V) Amplifier Load AC Case; ['ψ1 = ' num2str(psi1) ', 'ψ2 = ' num2str(psi2)])'
xlabel('ΔP_i_n_d, Change in power incident on diode (W)')
ylabel('V_L, Load voltage (V)')
set(gca, 'children', 'linewidth', [2]);
3. Matlab program to calculate the values of \( \frac{L}{h} (n_p - n_o) \) for 532 nm.

```matlab
clear all;close all;clc; 
lambda=532*10^-9;%Wavelength of a Laser
Bo=(2*3.14)./lambda;%Laser Wave Number%(l/m)
Ld=400e-6;
Xl=-1;

%calculating the values that lie between 2pi and 4pi
U1=-2*3.14./(Bo*Xl);% U=(np-no).*(Lh)
U2=-4*3.14./(Bo*Xl);
U3=2*3.14./(Bo*(Ld/2));
U4=4*3.14./(Bo*(Ld/2));

stepsizedd=(U4-U3)./7;
udad=U3+stepsizedd
ubd=U3+(2*stepsizedd)
ucd=U3+(3*stepsizedd)
udd=U3+(4*stepsizedd)
ued=U3+(5*stepsizedd)
ufd=U3+(6*stepsizedd)
ugd=U3+(7*stepsizedd)
```

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stepsize=(U2-U1)/5;
ua=U1+stepsize
ub=U1+(2*stepsize)
uc=U1+(3*stepsize)
ud=U1+(4*stepsize)
ue=U1+(5*stepsize)

%calculating the values that lie between 0 and 2pi
U5=0*3.14./(Bo*Xl);
U6=-2*3.14./(Bo*Xl);
U7=0*3.14./(Bo*(Ld/2));
U8=2*3.14./(Bo*(Ld/2));

stepsizebb=(U6-U5)/6;
uab=U5+stepsizebb
ubb=U5+(2*stepsizebb)
ucb=U5+(3*stepsizebb)
udb=U5+(4*stepsizebb)
ueb=U5+(5*stepsizebb)

stepsizeaa=(U8-U7)/8;
uaa=U7+stepsizeaa
uba=U7+(2*stepsizeaa)
ucu=U7+(3*stepsizeaa)
uda=U7+(4*stepsizeaa)
uau=U7+(5*stepsizeaa)
ufa=U7+(6*stepsizeaa)
uga=U7+(7*stepsizeaa)

3. Matlab program to calculate the SNR performance of above discussed detection schemes at 532nm.

%PERFORMANCE OF THE SYSTEM WITH DIFFERENT RISE TIMES & THE RESULTING BANDWIDTHS HAS BEEN ATTEMPTED.
clear all;close all;clc;
e=1.6*10^-19;% Charge of an electron%
lambda=532*10^-9;% Wavelength of a Laser%
Bo=(2*3.14)/lambda;% Laser Wave Number%(1/m)
Rl=50;% Load Resistance%
trt1=10e-9;% Rise time of pulse signal that is generated by the marx bank discharge
%B1=0.35./trt1;% Bandwidth%
B1=1.2E9 % BANDWIDTH OF THE AMPLIFIER AD8015%
Ld=400E-6; %Length of the EOT-2030A Detector[EOT 2030A SPEC SHEET]
A=3.14*10^-6; %Area of the BEAM
Ad=3.14*(400E-6/2)^2; %ACTIVE AREA OF EOT-2030A Detector%
lens=2.54*10^-2; %Length of the Sample%
l=2.54*10^-2; %Sample Height%
np=1.5; %Refractive index of the glass plate wedge%
n=1; %Refractive index of the air%
n=1.585; %Refractive index of the sample%
id=0.1*10^-9; %Dark Current[EOT 2030A SPEC SHEET]%
rs=0.4; %Responsivity of the diode[from graph in the EOT 2030A SPEC SHEET] %
T=293; %Room Temperature(Kelvin)%
k=1.38*10^-23; %Boltzmann Constant%
rho=0.31; %Opto-Elastic Constant of REXOLITE%
Cd=1.5*10^-12; %Capacitance of EOT 2030A PHOTODETECTOR[EOT 2030A SPEC SHEET]
Rl=50; %Resistance of the load at the scope is 50 ohms
dells=0; %Change in lateral length of the sample
Pin=1e-3:0.001:20.3e-2; %Power of the source laser Pin
% Pin=11.3e-2; %Power of the source laser Pin
tpulse=5e-9; %Pulse duration of NdYAG
% %Pin=20.3e-2;
Ein=Pin*tpulse % Energy input in the system
% Xl=-3.66537;
Tr=10e3; %Transresistance of the bipolar transistor stage in the amplifier[AD8015 SPEC SHEET]
Xl=-1;
rf=10*10^3; %Feedback resistance with the amplifier at load
Rac=7*10^3; %The a/c resistance(ohms)
ri=5*10^3; %The input resistance(ohms)
rsh=1*10^3; %The shunt resistance(ohms)
c=3*10^8; %speed of light (m/s)
% U = 0.0030
% U = 0.0034
% U = 0.0038
% U = 0.0042
% U = 0.0046
% U = 0.0049
% U = 0.0053
% U = 6.3840e-007
% U = 7.4480e-007
% U = 8.5120e-007
% U = 9.5760e-007
% U = 1.0640e-006
% U = 3.3250e-004

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% U = 6.6500e-004
% U = 9.9750e-004
% U = 0.0013
% U = 0.0017
% U = 0.0020
% U = 0.0023
% U = 8.8667e-008
% U = 1.7733e-007
% U = 2.6600e-007
% U = 3.5467e-007
% U = 4.4333e-007
psil=(-l).*Bo.*U.*Xl;%Eq.2.50b
psi2=Bo.*U.*(Ld/2);%Eq.2.50c
Ym=3.1e9;%Youngs Modulus(Pascals)

% SINCE SOME VALUES DID NOT AGREE WITH THOSE AT 633 NM THESE VALUES WERE RERUN WITH PSI VALUES MADE TO MATCH
psil=11905.2133;psi2=2.381;
% psil=15873.6177;psi2=3.1747;
% psil=19842.0221;psi2=3.9684;
% psil=23810.4265;psi2=4.7621;
% psil=27778.831;psi2=5.5558;
Ym=3.1e9;%Youngs Modulus(Pascals)

%name it psi2fixed532

%TO CALCULATE del phi/ASSUME A LOAD OF X Newtons IS ACTING ON THE PLATES
% n=0.03;
% n=3;

n=60% Force acting on the plates(Newtons)
Afr=((5.06e-4)-(1.79e-5));%Area over which the force is acting on the rexolite sample
PY1=(n./Afr)./Ym;% (Pressure/Youngs Modulus)
delnsb1=((ns).^3).*(rho./2).*PY1;% change in refractive index
% delnsb1=0 for unstressed case
delpsi=(Bo.*(ns-no).*(dells))+(Bo.*lens.*delnsbl);%Eq.2.50e, dells=0
% CALCULATION OF DelPind
Pindu=(Ad./(2.*A)).*Pin.*(1+((cos(psil).*sin(psi2))./(psi2)));%Eq.2.51c Power incident on the diode in the unstressed case
Pinds=(Ad./(2.*A)).*Pin.*(1+((cos(psil1+delpsi).*sin(psi2))./(psi2)));%Eq.2.51b Power incident on the diode in the stressed case
il=Pindu.*rs;%Current generated due to the laser incident on the diode
DelPind=Pinds-Pindu;

%DC CASE RESISTANCE AT LOAD

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B1R=1./(Cd.*Rl)%Bandwidth(LPF) of the detector circuit with Rl=scope resistance and C=Diode Capacitance

\[ \text{in} = (2.*e.*B1R.*Pindu.*rs) + (2.*e.*B1R.*id) + ((4.*k.*T.*B1R)./Rl) \]% noise current squared with resistance at load dc case

\[ \text{isdRl} = (2.*e.*B1R.*id) \]% Eq. 2.61 shot noise current squared due to dark current

\[ \text{islRl} = (2.*e.*B1R.*Pindu.*rs) \]% Eq. 2.62 shot noise current squared due to current generated by the diode

\[ \text{ithRl} = ((4.*k.*T.*B1R)./Rl) \]% Thermal noise current squared (A^2)

\[ \text{PsdRl} = \text{isdRl}.*Rl \]% Noise power due to shot noise current squared (A^2) (dark current)

\[ \text{PslRl} = \text{islRl}.*Rl \]% Noise power due to shot noise current squared (A^2) (laser current)

\[ \text{PthRl} = \text{ithRl}.*Rl \]% Thermal noise current squared (A^2)

\[ \text{SNRRIDCU} = 20.\log_{10}((\text{rs}.\text{Pindu})./(\sqrt{\text{in}})) \]% Eq. 2.75. SNR with resistance at load dc unstressed case

\[ \text{SNRRIDCS} = 20.\log_{10}((\text{rs}.\text{Pinds})./(\sqrt{\text{in}})) \]% Eq. 2.76. SNR with resistance at load dc stressed case

% AC CASE RESISTANCE AT LOAD
% Pmdac = Pindu - ((e.*B1)./rs);% a/c component of the power that is incident on the diode

\[ \text{Pindac} = \text{Pinds}; \]

\[ \text{ina} = (2.*e.*B1R.*Pindac.*rs) + (2.*e.*B1R.*id) + ((4.*k.*T.*B1R)./Rl) \]% noise current squared (A^2) at load a/c case

\[ \text{SNRRIAC} = 10.\log_{10}((\text{rs}^2).*(1/3).*(\text{DelPind}^2))./(\text{ina})) \]% Eq. 2.84 SNR with resistance at load dc case

\[ \text{islRlAC} = (2.*e.*B1R.*Pindu.*rs); \]

\[ \text{PslRlAC} = \text{islRlAC}.*Rl; \]

% DC CASE AMPLIFIER AT LOAD

\[ \text{inamp} = 26.5*10^{-9}; \]% The noise current of the amplifier (nA) [AD8015 specsheet]

\[ \text{rs11} = (\text{rsh}.\text{Rac}.\text{ri})./(\text{rsh}.\text{Rac} + (\text{Rac}.\text{ri}) + (\text{rsh}.\text{ri})); \]

\[ \text{Tr} = 10.\text{e}3; \]% Transresistance [AD8015 specsheet]

\[ \text{vn} = \sqrt{4.k*T.(2/3)*Tr}; \]% Equivalent noise voltage [ART OF ELECTRONICS Pg 444]

\[ \text{rfv} = (((\text{rf} + \text{rs11})).*(\text{vn}).^2); \]% Load current squared (A^2) due to noise voltage

\[ \text{isdA} = (2.*e.*B1.*id); \]% Eq. 2.61 the shot noise current squared (A^2) due to the dark current

\[ \text{islA} = (2.*e.*B1.*Pinds.*rs); \]% Eq. 2.62 the shot current squared (A^2) noise due to the laser current

\[ \text{ithA} = ((4.*k.*T.*B1)./\text{rf}); \]% Thermal noise current squared (A^2)

\[ \text{inamps} = \text{inamp}.^2; \]% Noise current due to amplifier noise (A^2)

\[ \text{Pampnoise} = \text{inamps}.*((\text{rf}^2)./\text{Rl}); \]% Noise power due to amplifier noise current

\[ \text{Psda} = \text{isdA}.*((\text{rf}^2)./\text{Rl}); \]% Eq. 2.89 Noise power due to shot noise current squared (A^2) (dark current)

\[ \text{PsI} = \text{islA}.*((\text{rf}^2)./\text{Rl}); \]% Noise power due to shot noise current squared (A^2) (laser current)
PthA=ithA.*((rf.^2)./Rl)% Noise power due to shot noise current squared (A^2) (thermal current)
PlnA=rfv.*((rf.^2)./Rl)% Noise power due to shot noise current squared (A^2) (load current from noise voltage)
SNRAMPDC=20.*(log10(il./sqrt(isdA+islA+inamps+ithA+rfv))))% SNR with an amplifier (dc case)

% AC CASE AMPLIFIER AT LOAD
SNRAMPAC=10.*(log10(((rs.^2).*(l/3).*(DelPind.^2))./((isdA+islA+inamps+ithA+rfv))))% SNR with an amplifier (ac case)

% HETERODYNE CASE PARAMETERS%
Volo=2;% Voltage of the Local Oscillator 1st case = 2V.
alpha=1;% Quadratic coupling efficiency.
% B4=800E6;% BANDWIDTH OF THE LOCAL OSCILLATOR
B4=0.35./trt1;% Bandwidth%
rs14=(rsh.*Rac.*ri)/(((rsh.*Rac)+(Rac.*ri)+(rsh.*ri));% Source resistance
vn=sqrt(4*k*T*(2/3)*Tr);% Equivalent noise voltage
rfv=[(rf+rs14)/((rf.*rs14)].*(vn)].^2;% Load current due to noise voltage
Voloa=5;% Voltage of the Local Oscillator 2nd case = 5V.
isdH=(2.*e.*B4.*id);% the shot noise current squared (A^2) due to the dark current
islH=(2.*e.*B4.*Pinds.*rs);% the shot noise current squared (A^2) due to the laser current
ithH=((4.*k.*T.*B4)./rf);% Thermal noise current squared (A^2)

Pampnoise=inamps.*((rf.^2)./Rl)% Noise power due to amplifier current noise
PsdH=isdH.*((rf.^2)./Rl)% Noise power due to dark current shot noise
PslH=islH.*((rf.^2)./Rl)% Noise power due to laser current shot noise
PthH=ithH.*((rf.^2)./Rl)% Noise power due to thermal noise
PlnH=rfv.*((rf.^2)./Rl)% Noise power due to load current from noise voltage

% HETERODYNE CASE
SNRHETAC=10.*(log10(((rs.^2).*(alpha.^2).*(Volo.^2).*(l/3).*(DelPind.^2))./((isdH+islH+inamps+ithH+rfv))));% SNR with an amplifier (ac case)
SNRHETACB=10.*(log10(((rs.^2).*(alpha.^2).*(Voloa.^2).*(l/3).*(DelPind.^2))./((isdH+islH+inamps+ithH+rfv))));% SNR with an amplifier (ac case)

plot(Pindu,SNRRIDCS,'r')% hold on
plot(Pindu,SNRRIAC,'g')% hold on
plot(Pindu,SNRAMPDC,'b')% hold on
plot(Pindu,SNRAMPAC,'k')% hold on
plot(Pindu,SNRHETAC,'c')% hold on
plot(Pindu,SNRHETACB,'m')% hold on
plot(Pindu(1:20:length(Pindu)),SNRRIAC(1:20:length(SNRRIAC)),'k','MarkerSize',20)
hold on
plot(Pindu(1:20:length(Pindu)),SNRHETAC(1:20:length(SNRHETAC)),'k','MarkerSize',20)
hold on
title({'P_i_n_d, Power incident on the diode (W) vs SNR (dB)',['\text{Force} = ' num2str(n) 'N','\psi_1 = ' num2str(psi1),'\psi_2 = ' num2str(psi2),'\lambda = ' num2str(lambda* 1e9) 'nm']})
legend('SNRRLDC','SNRRRIAC','SNRAMPDC','SNRAMPAC','SNRHETAC','SNRHETACB','Location','BestOutside')
xlabel('P_i_n_d, Power incident on the diode (watts)')
ylabel('SNR(dB)')
set(get(gca,'children'),'linewidth',[2]);
set(gca,'linewidth',[2],'gridlinestyle','-','fontname','timesnewroman','fontweight','bold','fontsize',[16]);
set(get (gca,'title'),'fontname','timesnewroman','fontweight','bold','fontsize',[16])
set(get (gca,'xlabel'),'fontname','timesnewroman','fontweight','bold','fontsize',[16])
set(get (gca,'ylabel'),'fontname','timesnewroman','fontweight','bold','fontsize',[16])
grid on

figure
plot(Pin,Pindu,'r')
hold on
plot(Pin,Pinds,'g')
hold on
% title({'P_i_n_d, Power incident on the diode (W) vs SNR (dB)',['\text{Force} = ' num2str(n) ' N','\psi_1 = ' num2str(psi1),'\psi_2 = ' num2str(psi2),'\lambda = ' num2str(lambda* 1e9) 'nm']})
title({'P_i_n_d (W) vs Pin (W)',['\psi_1 = ' num2str(psi1),'\psi_2 = ' num2str(psi2)]})
legend('P_i_n_d_u','P_i_n_d_s','Location','BestOutside')
xlabel('P_i_n, Power of laser (W)')
ylabel('P_i_n_d, Power incident on diode (W)')
set(get(gca,'children'),'linewidth',[2]);
set(gca,'linewidth',[2],'gridlinestyle','-','fontname','arial','fontweight','bold','fontsize',[16]);
set(get (gca,'title'),'fontname','arial','fontweight','bold','fontsize',[16])
set(get (gca,'xlabel'),'fontname','arial','fontweight','bold','fontsize',[16])
set(get (gca,'ylabel'),'fontname','arial','fontweight','bold','fontsize',[16])
grid on
%
figure
plot(Pin,DelPind,'b')
hold on
title({'\text{\Delta}P_i_n_d (W) vs Pin (W)',['\psi_1 = ' num2str(psi1),'\psi_2 = ' num2str(psi2)]})

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xlabel('P_i_n, Power of laser (W)')
ylabel('{\Delta}P_i_n_d, Change in power incident on diode (W)')
set(gca,'children','linewidth',[2]);
set(gca,'linewidht',[2],'gridlinestyle','-','

Vload 1  =((Pindu.*rs).^2).*R1;
Vload 1 =sqrt(Pload 1 .*R1);
figure
plot(Pindu,Vload 1  )
title({'P_i_n_d (W) vs V_L (V) Resistor Load Unstressed DC Case ';
num2str(psi1) ' \psi_2 = ' num2str(psi2))')
xlabel('P_i_n_d , Power incident on diode (W)')
ylabel('V_L , Load voltage (V)')
set(gca,'children','linewidth',[2]);
set(gca,'linewidht',[2],'gridlinestyle','-','

Vload 2  =sqrt(Pload 2 .*R1);
figure
plot(Pinds,Vload 2  )
title({'P_i_n_d (W) vs V_L (V) Resistor Load Stressed DC Case ';
num2str(psi1) ' \psi_2 = ' num2str(psi2))')
xlabel('P_i_n_d , Power incident on diode (W)')
ylabel('V_L , Load voltage (V)')
set(gca,'children','linewidth',[2]);
set(gca,'linewidht',[2],'gridlinestyle','-','

Vload 3  =sqrt(Pload 3 .*R1);
figure
plot(DelPind,Vload 3  )
title({'{\Delta}P_i_n_d (W) vs V_L (V) Resistor Load AC Case ';
num2str(psi1) ' \psi_2 = ' num2str(psi2))')
xlabel('{\Delta}P_i_n_d , Change in power incident on diode (W)')

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ylabel('V_L , Load voltage (V)')
set(gca,'children','linewidth',[2]);
set(gca,'linewidth',[2],'gridlinestyle','-','fontname','arial','fontweight','bold','fontsize',[16]);
set(gca,'title','fontname','arial','fontweight','bold','fontsize',[16])
set(gca,'xlabel','fontname','arial','fontweight','bold','fontsize',[16])
set(gca,'ylabel','fontname','arial','fontweight','bold','fontsize',[16])
grid on

Pload4=(il.^2).*((rf.^2)./Rl);
Vload4=sqrt(Pload4.*RJ);
figure
plot(Pindu,Vload4)
title({'P_i_n_d (W) vs V_L (V) Amplifier Load DC Case';'
{\psi}_1 = ' num2str(psi1)'
{\psi}_2 = ' num2str(psi2)'
})
xlabel('P_i_n_d , Power incident on diode (W)')
ylabel('V_L , Load voltage (V)')
set(gca,'children','linewidth',[2]);
set(gca,'linewidth',[2],'gridlinestyle','-','fontname','arial','fontweight','bold','fontsize',[16]);
set(gca,'title','fontname','arial','fontweight','bold','fontsize',[16])
set(gca,'xlabel','fontname','arial','fontweight','bold','fontsize',[16])
set(gca,'ylabel','fontname','arial','fontweight','bold','fontsize',[16])
grid on

Pload5=((rs.^2).*(l/3).*(DelPind.^2)).*((rf.^2)./Rl);
Vload5=sqrt(Pload5.*R1);
figure
plot(DelPind,Vload5)
title({'{\Delta}P_i_n_d (W) vs V_L (V) Amplifier Load AC Case';'
{\psi}_1 = ' num2str(psi1)'
{\psi}_2 = ' num2str(psi2)'
})
xlabel('{\Delta}P_i_n_d, Change in power incident on diode (W)')
ylabel('V_L , Load voltage (V)')
set(gca,'children','linewidth',[2]);
set(gca,'linewidth',[2],'gridlinestyle','-','fontname','arial','fontweight','bold','fontsize',[16]);
set(gca,'title','fontname','arial','fontweight','bold','fontsize',[16])
set(gca,'xlabel','fontname','arial','fontweight','bold','fontsize',[16])
set(gca,'ylabel','fontname','arial','fontweight','bold','fontsize',[16])
grid on

Pload6=((rs.^2).*alpha.^2).*(Volo.^2).*(1/3).*(DelPind.^2)).*((rf.^2)./Rl);
Vload6=sqrt(Pload6.*RI);
figure
plot(DelPind,Vload6)
title({'{\Delta}P_i_n_d (W) vs V_L (V) Heterodyne Detection AC Case';'
{\psi}_1 = ' num2str(psi1)'
{\psi}_2 = ' num2str(psi2)'
})
xlabel('{\Delta}P_i_n_d, Change in power incident on diode (W)')
ylabel('V_L , Load voltage (V)')
set(gca,'children','linewidth',[2]);
set(gca,'linewidth',[2],'gridlinestyle','-','fontname','arial','fontweight','bold','fontsize',[16]);
set(gca,'title','fontname','arial','fontweight','bold','fontsize',[16])
set(gca,'xlabel','fontname','arial','fontweight','bold','fontsize',[16])
set(gca,'ylabel','fontname','arial','fontweight','bold','fontsize',[16])
grid on
APPENDIX B

1. Rayica program to obtain the optimal spacing between the lenses using a ray tracing approach.

\[
\text{lens1} = \text{Move}[\text{LensDoublet}[170.317,-141.605,-475.988,50.8,9.5,\text{BK7,SF5,DesignWaveLength->.5461,FocalLength->300}],60];
\text{lens2} = \text{Move}[\text{LensDoublet}[34.608,-21.460,-232.961,25.4,7.8,2.0,\text{BaFN10,SF10,DesignWaveLength->.5461,FocalLength->50.8}],\{x,410.8\},180];
\text{tharport} = \\
\{\text{Move}[\text{GaussianBeam}[2,.001,\text{NumberOfRays->11,WaveLength->.633,FullForm->True}],10],
\text{lens1},
\text{lens2},
\text{Move}[\text{Screen}[00],1500]\}
\text{TurboPlot}[\text{tharport,PlotType->TopView}];
\text{result} = \text{ConstructMeritFunction}[\text{tharport,MeritType->RayTilt}]
\text{meritfunction} = \text{RayTraceFunction/\text{result};}
\text{Plot}[\text{meritfunction}[x,0],[x,409,411]];
\text{tharporta} = \text{TurboPlot}[\text{tharport,SymbolicValues->\{}x->410\},\text{PlotType->TopView}];
\text{tharportb} = \text{OptimizeSystem}[\text{tharport,MeritType->RayTilt}]
\{\text{SymbolicValues->\{}x->445.90781509707216\},\text{NumberOfCycles->14,FinalMerit->120.99}\}
\text{result} = \text{TurboPlot}[\text{tharport,PlotType->TopView,tharportb}];
\text{FindSpotSize[result]};
\]

2. Wavica program to obtain the optimal spacing between the lenses using Wavica.

\[
\text{tharport} = \\
\{\text{Move}[\text{GaussianBeam}[4.27,.0006,\text{NumberOfRays->10,WaveLength->.532,IntrinsicMedium->Vacuum,FullForm->False}],-50],
\text{Move}[\text{LensDoublet}[170.317,-141.605,-475.988,50.8,9.5,\text{BK7,SF5,DesignWaveLength->.5461,FocalLength->300}],60],
\]
3. Simulating optical systems based on the optimal value of position of lens with screen placed at different positions and analyzing the results using both SpotSize and FindIntensity commands using Wavica.

gorta=
{
  Move[GaussianBeam[4.255,.0006,NumberOfRays->10,WaveLength->.532,IntrinsicMedium->Vacuum,FullForm->False],-50],
  Move[LensDoublet[170.317,-141.605,-232.988,50.8,9,5,BK7, SF5,DesignWaveLength->.5461,FocalLength->300],60],
  Move[LensDoublet[34.608,-21.460,-232.961.25.4.7.8.2.0,BaFN10,SF10,DesignWaveLength->.5461,FocalLength->50.8],430.415,180],
  Move[Screen[100],455.815]
}
ShowSystem[gorta,PlotType->TopView,ShowGaussian->True] FindSpotSize[gorta] FindIntensity[gorta,Plot2D->ContourPlot,FullForm->False] gprta=
{
  Move[GaussianBeam[4.255,.0006,NumberOfRays->10,WaveLength->.532,IntrinsicMedium->Vacuum,FullForm->False],-50],
  Move[LensDoublet[170.317,-141.605,-475.988,50.8,9,5,BK7, SF5,DesignWaveLength->.5461,FocalLength->300],60],
  Move[LensDoublet[34.608,-21.460,-232.961.25.4.7.8.2.0,BaFN10,SF10,DesignWaveLength->.5461,FocalLength->50.8],430.415,180],
  Move[Screen[100],455.815]
}

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ShowSystem[ghrta,PlotType->TopView,ShowGaussian->True]
FindSpotSize[ghrta]
FindIntensity[ghrta,Plot2D->ContourPlot,FullForm->False]

ghrta = 
\{ 
  Move[GaussianBeam[4.255,.0006,NumberOfRays->10,WaveLength->.532,IntrinsicMedium->Vacuum,FullForm->False],-50],
  Move[LensDoublet[170.317,-141.605,-475.988,50.8,9.5,BK7,DesignWaveLength->.5461,FocalLength->300],60],
  Move[LensDoublet[34.608,-21.460,-232.961,25.4,7.8,2.0,BaFN10,DesignWaveLength->.5461,FocalLength->50.8],430.415,180],
  Move[Screen[100],506.615]
\}

ShowSystem[gkrta,PlotType->TopView,ShowGaussian->True]
FindSpotSize[gkrta]
FindIntensity[gkrta,Plot2D->ContourPlot,FullForm->False]
gkrta = 
\{ 
  Move[GaussianBeam[4.255,.0006,NumberOfRays->10,WaveLength->.532,IntrinsicMedium->Vacuum,FullForm->False],-50],
  Move[LensDoublet[170.317,-141.605,-475.988,50.8,9.5,BK7,DesignWaveLength->.5461,FocalLength->300],60],
  Move[LensDoublet[34.608,-21.460,-232.961,25.4,7.8,2.0,BaFN10,DesignWaveLength->.5461,FocalLength->50.8],430.415,180],
  Move[Screen[100],532.015]
\}

ShowSystem[gvrta,PlotType->TopView,ShowGaussian->True]
FindSpotSize[gvrta]
FindIntensity[gvrta,Plot2D->ContourPlot,FullForm->False]
gvrta = 
\{ 
  Move[GaussianBeam[4.255,.0006,NumberOfRays->10,WaveLength->.532,IntrinsicMedium->Vacuum,FullForm->False],-50],
  Move[LensDoublet[170.317,-141.605,-475.988,50.8,9.5,BK7,DesignWaveLength->.5461,FocalLength->300],60],
  Move[LensDoublet[34.608,-21.460,-232.961,25.4,7.8,2.0,BaFN10,DesignWaveLength->.5461,FocalLength->50.8],430.415,180],
  Move[Screen[100],557.415]
\}
ShowSystem[gvrta, PlotType -> TopView, ShowGaussian -> True]
FindSpotSize[gvrta]
FindIntensity[gvrta, Plot2D -> ContourPlot, FullForm -> False]

gwrta =
{
    Move[GaussianBeam[4.255, 0.0006, NumberOfRays -> 10, Wavelength -> 0.532, IntrinsicMedium -> Vacuum, FullForm -> False], -50],
    Move[LensDoublet[170.317, -141.605, -475.988, 50.8, 9.5, BK7, SF5, DesignWavelength -> 0.5461, FocalLength -> 300], 60],
    Move[LensDoublet[34.608, -21.460, -232.961, 25.4, 7.8, 2.0, BaF10, SF10, DesignWavelength -> 0.5461, FocalLength -> 50.8], 430.415, 180],
    Move[Screen[100], 582.815]
}

ShowSystem[gwrta, PlotType -> TopView, ShowGaussian -> True]
FindSpotSize[gwrta]
FindIntensity[gwrta, Plot2D -> ContourPlot, FullForm -> False]
girt =
{
    Move[GaussianBeam[4.255, 0.0006, NumberOfRays -> 10, Wavelength -> 0.532, IntrinsicMedium -> Vacuum, FullForm -> False], -50],
    Move[LensDoublet[170.317, -141.605, -475.988, 50.8, 9.5, BK7, SF5, DesignWavelength -> 0.5461, FocalLength -> 300], 60],
    Move[LensDoublet[34.608, -21.460, -232.961, 25.4, 7.8, 2.0, BaF10, SF10, DesignWavelength -> 0.5461, FocalLength -> 50.8], 430.415, 180],
    Move[Screen[100], 608.215]
}

ShowSystem[girt, PlotType -> TopView, ShowGaussian -> True]
FindSpotSize[girt]
FindIntensity[girt, Plot2D -> ContourPlot, FullForm -> False]

4. Matlab code to generate the theoretical values for the collimation studies and to plot experimental, theoretical and computational values.

clear all; close all; clc
% theoretical analysis
clear all; close all; clc
% theoretical analysis
D = [25.4 50.8 76.2 101.6 127 152.4 177.8] % distances at which the screen is placed
F1 = 300% focal length of first lens
F2 = 50.8% focal length of first lens
dbeam = 4.255% diameter of beam at the i/p of the telescope (mm)
Notice the call to ONES to create a vector

dth=dbeam*(F2/F1)*ones(size(D)); % diameter of beam at the o/p of the
telescope

% computational analysis

dcomp=[0.806957 0.789872 0.772787 0.755701 0.738616 0.721531 0.704446];%(mm)

dexp=[0.72 0.71 0.62 0.53 0.49 0.48 0.39];%(mm)

plot(D,dth,'r.-','MarkerSize',20,'MarkerEdgeColor','k')
hold on
plot(D,dcomp,'g.-VMarkerSize',20,'MarkerEdgeColor','k')
hold on
plot(D,dexp,'b.-','MarkerSize',20,'MarkerEdgeColor','k')
hold on
grid on

5. Wavica code to simulate the experiment for different load cases.

(*WITH 0 N*)

```
{Move[GaussianBeam[2,001,WaveLength->0.633,NumberOfRays->10,FullForm->False],-50],
 Move[BeamSplitterCube[{50,50},25.4,"BS1"],{0,-11.7},88.,GraphicDesign->On],
 Move[Mirror[25.4,10,"M2"],{15.1,282},-228.01465],
 Move[Screen[150],{24.78,300},90],
 Move[Window[{25.4,25.4},25.4,GraphicDesign->Solid,ComponentMedium->(1.585)],{174.6,279}],
 Move[Mirror[25.4,10,"M1"],461.7,-45],
 Move[BeamSplitterCube[{50,50},25.4,"BS2"],{455.7,253.7},88.98,GraphicDesign->On],
 Move[Screen[150],{449,716.7},90] ;
 FindInterference[temur1,PlotDomain->{-2,2}]
 out1= FindInterference[temur1,PlotPoints->200]
 func1=InterferenceFunction/.out1
 numbers1=Table[func1[x],{x,-2,2,.01}];
 ListPlot[numbers1];
 Export["juna.dat",numbers1,"Table"]
```

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(*WITH 0.1 N LOAD*)

temur2=
{
    Move[GaussianBeam[2,.001,WaveLength->0.633,NumberOfRays->10,FullForm->False],-50],
    Move[BeamSplitterCube[{50,50},25.4,"BS1"],{0,-11.7},88.,GraphicDesign->On],
    Move[Mirror[25.4,10,"M2"],{15.1,282},-228.01465],
    Move[Screen[150],{24.78,300},90],
    Move[Window[{25.4,25.4},25.4,GraphicDesign->Solid,ComponentMedium->(1.585+0.0408*10^-6)],[174.6,279]],
    Move[Mirror[25.4,10,"M1"],461.7,-45],
    Move[BeamSplitterCube[{50,50},25.4,"BS2"],{455.7,253.7},88.98,GraphicDesign->On],
    Move[Screen[150],{449,716.7},90]
};

FindInterference[temur2,PlotDomain->{-2,2}] out2= FindInterference[temur2,PlotPoints->200]
func2=InterferenceFunction/.out2
numbers2=Table[func2[x],{x,-2,2,.01}];
ListPlot[numbers2];
Export["jung.dat",numbers2,"Table"]

(*WITH 1 N LOAD*)

temur3=
{
    Move[GaussianBeam[2,.001,WaveLength->0.633,NumberOfRays->10,FullForm->False],-50],
    Move[BeamSplitterCube[{50,50},25.4,"BS1"],{0,-11.7},88.,GraphicDesign->On],
    Move[Mirror[25.4,10,"M2"],{15.1,282},-228.01465],
    Move[Screen[150],{24.78,300},90],
    Move[Window[{25.4,25.4},25.4,GraphicDesign->Solid,ComponentMedium->(1.585+0.4079*10^-6)],[174.6,279]],
    Move[Mirror[25.4,10,"M1"],461.7,-45],
    Move[BeamSplitterCube[{50,50},25.4,"BS2"],{455.7,253.7},88.98,GraphicDesign->On],
    Move[Screen[150],{449,716.7},90]
};

FindInterference[temur3,PlotDomain->{-2,2}] out3= FindInterference[temur3,PlotPoints->200]
func3=InterferenceFunction/.out3
numbers3=Table[func3[x],{x,-2,2,.01}];
ListPlot[numbers3];
Export["j unh.dat",numbers3,"Table"]

(*WITH 32.46 N LOAD*)

temur4=
{
   Move[GaussianBeam[2.,0.001,WaveLength->0.633,NumberOfRays->10,FullForm->False],-50],
   Move[BeamSplitterCube[{50,50},25.4,"BS1"],[0,-11.7],88.,GraphicDesign->On],
   Move[Mirror[25.4,10,"M2"],[15.1,282],-228.01465],
   Move[Screen[150],[24.78,300],90],
   Move[Window[{25.4,25.4},25.4,GraphicDesign->Solid,ComponentMedium->(1.585+1.324*10^-4)],{174.6,279}],
   Move[Mirror[25.4,10,"M1"],461.7,-45],
   Move[BeamSplitterCube[{50,50},25.4,"BS2"],[455.7,253.7],88.98,GraphicDesign->On],
   Move[Screen[150],[449,716.7],90]
};

FindInterference[temur4,PlotDomain->{-2,2}]
out4= FindInterference[temur4,PlotPoints->200]
func4=InterferenceFunction/.out4
numbers4=Table[func4[x],{x,-2.,2.,0.01}];
ListPlot[numbers4];
Export["junb.dat",numbers4,"Table"]

(*WITH 60 N LOAD*)

temur5=
{
   Move[GaussianBeam[2.,0.001,WaveLength->0.633,NumberOfRays->10,FullForm->False],-50],
   Move[BeamSplitterCube[{50,50},25.4,"BS1"],[0,-11.7],88.,GraphicDesign->On],
   Move[Mirror[25.4,10,"M2"],[15.1,282],-228.01465],
   Move[Screen[150],[24.78,300],90],
   Move[Window[{25.4,25.4},25.4,GraphicDesign->Solid,ComponentMedium->(1.585+2.447*10^-4)],{174.6,279}],
   Move[Mirror[25.4,10,"M1"],461.7,-45],
   Move[BeamSplitterCube[{50,50},25.4,"BS2"],[455.7,253.7],88.98,GraphicDesign->On],
   Move[Screen[150],[449,716.7],90]
};

FindInterference[temur5,PlotDomain->{-2,2}]
out5= FindInterference[temur5,PlotPoints->200]
func5=InterferenceFunction/.out3
numbers5=Table[func5[x],{x,-2,2,.01}];
ListPlot[numbers5];
Export["junc.dat",numbers5,"Table"]

(*WITH 90N LOAD*)
temur6=

{Move[GaussianBeam[2,.001,WaveLength->0.633,NumberOfRays->10,FullForm->False],-50],
  Move[BeamSplitterCube[{50,50},25.4,\"BS1\"],{0,-11.7},88.,GraphicDesign->On],
  Move[Mirror[25.4,10,\"M2\"],{15.1,282},-228.01465],
  Move[Screen[150],{24.78,300},90],
  Move[Window[{25.4,25.4},25.4,GraphicDesign->Solid,ComponentMedium->(1.585+3.671*10^-4)],{174.6,279}],
  Move[Mirror[25.4,10,\"M1\"],461.7,-45],
  Move[BeamSplitterCube[{50,50},25.4,\"BS2\"],{455.7,253.7},88.98,GraphicDesign->On],
  Move[Screen[150],{449,716.7},90]
};

FindInterference[temur6,PlotDomain->{-2,2}]
7. Matlab code to plot the computational results and calculate the fringe shift.

function [move] = compt;

clear all; close all; clc;
[ax, bx, cx, si, ni, th, thi, aa, bb, cc, dd, ee] = newcomp;
a = 1:1:400;

% plot(ax,'rx-', 'MarkerEdgeColor', 'k');
hold on
xxaa = 23; xxab = 39; % specify limits within which the curve should be fitted
xxac = xxaa:1:xxab;
xxad = ax(xxaa:xxab)
xxadd = transpose(xxad)
xxae = polyfit(xxac, xxadd, 2);
hold on
xxaf = polyval(xxae, xxac);
plot(xxac, xxaf, 'k')
aposa = -xxae(2) / (2 * xxae(1)); % by solving the 2nd degree polynomial, the minima is obtained

plot(bx,'gx-','MarkerEdgeColor','k');
hold on
xxba = 23; xxbb = 40; % specify limits within which the curve should be fitted
xxbc = xxba:1:xxbb;
xxbd = bx(xxba:xxbb);
xxbdf = transpose(xxbd);
xxbe = polyfit(xxbc, xxbdf, 2);
hold on
xxbf = polyval(xxbe, xxbc);
plot(xxbc, xxbf, 'k')
bposa = xxbe(2) / (2 * xxbe(1)); % by solving the 2nd degree polynomial, the minima is obtained

plot(cx,'bx-','MarkerEdgeColor','k');

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hold on
xxca=25;xxcb=42; %specify limits within which the curve should be fitted
xxcc=xxca:1:xxcb;
xxcd=cx(xxca:xxcb);
xxcdd=transpose(xxcd); %this is done to change it from a 81x1 array to a 1x81 array so that the matrix dimensions agree
xxce = polyfit(xxcc,xxcdd,2);
hold on
xxcf=polyval(xxce,xxcc);
plot(xxcc,xxcf,'k')
cposa=-xxce(2)/(2*xxce(1));% 
plot(a,aa,'kx-', 'MarkerEdgeColor','b');
hold on
aaa=34;aab=52; %specify limits within which the curve should be fitted
aac=aaa:1:aab;
aad=aa(aaa:aab)
aadd=transpose(aad); %this is done to change it from a 81x1 array to a 1x81 array so that the matrix dimensions agree
aae = polyfit(aac,aadd,2);
hold on
aaf=polyval(aae,aac);
plot(aac,aaf,'b')
aaposa=-aae(2)/(2*aae(1));%by solving the 2nd degree polynomial, the minima is obtained

%%
%%
%%
%%
%%
%%
%%
plot(a,bb,'cx-','MarkerEdgeColor','k');
hold on
bba=47;bbb=62; %specify limits within which the curve should be fitted
bbc=bba:1:bbb;
bbd=bb(bba:bbb);
bbdd=transpose(bbd); %this is done to change it from a 81x1 array to a 1x81 array so that the matrix dimensions agree
bbe = polyfit(bbc,bbdd,2);
hold on
bbf=polyval(bbe,bbc);
plot(bbc,bbf,'k')
bbposa=-bbe(2)/(2*bbe(1));% by solving the 2nd degree polynomial, the minima is obtained

%%%
plot(a,cc,'mx-','MarkerEdgeColor','k');
hold on
cca=58;ccb=74; %specify limits within which the curve should be fitted
ccc=cca:1:ccb;
ccd=cc(cca:ccb);
ccdd=transpose(ccd); %this is done to change it from a 81x1 array to a 1x81 array so that the matrix dimensions agree
ccce = polyfit(ccc,ccdd,2)
hold on
cccf=polyval(ccce,ccc);
plot(ccc,cccf,'k')
ccposa=-ccce(2)/(2*ccce(1)); %by solving the 2nd degree polynomial, the minima is obtained

plot(a,dd,'r+-','MarkerEdgeColor','b');
hold on
dda=70;ddb=86; %specify limits within which the curve should be fitted
ddc=dda:1:ddb;
ddd=dd(dda:ddb)
dddd=transpose(ddd); %this is done to change it from a 81x1 array to a 1x81 array so that the matrix dimensions agree
dde = polyfit(ddc,dddd,2)
hold on
ddf=polyval(dde,ddc);
plot(ddc,ddf,'k')
ddposa=-dde(2)/(2*dde(1)); %by solving the 2nd degree polynomial, the minima is obtained
plot(a,ee,'g+-','MarkerEdgeColor','k'); hold on
grid minor
eea=82;eeb=97; %specify limits within which the curve should be fitted
eec=ee:1:eeb;
ceed=ee(ceed);
ceedd=transpose(ceed); %this is done to change it eom a 81x1 array to a 1x81 array so that
the maeeix dimensions agree
eee = polyfit(eec,ceedd,2);
hold on
eef=polyval(eee,eec);
plot(eec,eef,'k')
eeposa=-eee(2)/(2*eee(1)); %by solving the 2nd degree polynomial, the minima is
obtained
plot(a,th,'b+-','MarkerEdgeColor','k'); hold on
grid minor
tha=94;thb=109; %specify limits within which the curve should be fitted
thc=th:1:thb;
thd=th(thd);
thdd=transpose(thd); %this is done to change it eom a 81x1 array to a 1x81 array so that
the maeeix dimensions agree
the = polyfit(thc,thdd,2);
hold on
thf=polyval(the,thc);
plot(thc,thf,'k')
throposa=-the(2)/(2*the(1)); %by solving the 2nd degrth polynomial, the minima is
obtained
plot(a,thi,'k+-','MarkerEdgeColor','r'); hold on
grid minor
thia=106;thib=120; %specify limits within which the curve should be fitted
thic=thia:1:thib;
theid=thic(thid);
theiddd=transpose(thid); %this is done to change it thiom a 81x1 array to a 1x81 array so
that the maeeix dimensions agrthi
thie = polyfit(thic,thidd,2);
hold on
thif=polyval(thie,thic);
plot(thic,thif,'b')
thiposa=-thie(2)/(2*thie(1));%by solving the 2nd degree polynomial, the minima is obtained

plot(a,si,'c-','MarkerEdgeColor','k');
hold on
grid minor
sia=236; sib=248; % specify limits within which the curve should be fitted
sic=sia: 1: sib;
sid=sic(sia:sib);
sidd=transpose(sid); % si is done to change it from a 1x81 array to a 1x1 array so that the matrix dimensions agree
sie = polyfit(sic, sidd, 2);
hold on
sif=polyval(sie,sic);
plot(sic,sif,'k')
siposa=-sie(2)/(2*sie(1));%by solving the 2nd degree polynomial, the minima is obtained

plot(a,ni,'m+','MarkerEdgeColor','k');
hold on
grid minor
nia=309; nib=319; % specify limits within which the curve should be fitted
nic=nia: 1: nib;
nid=ni(nia:nib);
nidd=transpose(nid); % ni is done to change it from a 1x81 array to a 1x1 array so that the matrix dimensions agree
nie = polyfit(nic, nidd, 2);
hold on
nif=polyval(nie,nic);
plot(nic,nif,'k')
niposa=-nie(2)/(2*nie(1));%by solving the 2nd degree polynomial, the minima is obtained

legend('0 N','0.1 N','1 N','5 N','10 N','15 N','20 N','25 N','30 N','35 N','60 N','90 N','Location','BestOutside')
xlabel('Pixel Position')
ylabel('Intensity')
set(gca,'children','linewidth',[2]);
set(gca,'titlefontname','timesnewroman', 'fontweight','bold','fontsize',[12]);
set(get (gca,'title'),'fontname','timesnewroman', 'fontweight','bold','fontsize',[12])
set(get (gca,'xlabel'),'fontname','timesnewroman', 'fontweight','bold','fontsize',[12])
set(get (gca,'ylabel'),'fontname','timesnewroman', 'fontweight','bold','fontsize',[12])
grid on
fshftns(1)=0

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% to find the difference in distance between two lines for the .24N case and the 1.2N case
\texttt{fshftns(2)=diffa*1e-5}

% to find the difference in distance between two lines for the .24N case and the 1.2N case
\texttt{fshftns(3)=diffb*1e-5}

% to find the difference in distance between two lines for the .24N case and the 1.2N case
\texttt{fshftns(4)=diffc*1e-5}

% to find the difference in distance between two lines for the .24N case and the 1.2N case
\texttt{fshftns(5)=dbba*1e-5}

% to find the difference in distance between two lines for the .24N case and the 1.2N case
\texttt{fshftns(6)=dbbb*1e-5}

% to find the difference in distance between two lines for the .24N case and the 1.2N case
\texttt{fshftns(7)=dbbc*1e-5}

% to find the difference in distance between two lines for the .24N case and the 1.2N case
\texttt{fshftns(8)=dbbd*1e-5}

% to find the difference in distance between two lines for the .24N case and the 1.2N case
\texttt{fshftns(9)=dbbe*1e-5}

% to find the difference in distance between two lines for the .24N case and the 1.2N case
\texttt{fshftns(10)=dbbf*1e-5}

% to find the difference in distance between two lines for the .24N case and the 1.2N case
\texttt{fshftns(11)=dbbg*1e-5}

% to find the difference in distance between two lines for the .24N case and the 1.2N case
\texttt{fshftns(12)=dbbh*1e-5}

\texttt{move=[fshftns(1) fshftns(2) fshftns(3) fshftns(4) fshftns(5) fshftns(6) fshftns(7) fshftns(8) fshftns(9) fshftns(10) fshftns(11) fshftns(12)]}
8. Matlab code to calculate the theoretical set of values.

```matlab
function[delnsb,dexa,deltaxtwo,deltaxt,delF,F,fring]=lenk;

clear all;close all;clc;
fringe=1.4e-3;  % distance between minima
lambda=633e-9;  % the wavelength of HeNe laser
Bo=(2*3.14)./lambda;
Ym=3.1e9;  % Young's Modulus
l=25.4e-3;  % Length of the sample in m
n=[0 0.1 1];  % force in newtons
P=(n/((5.06e-4)-(1.79e-5)));  % pressure

delnsb=((1.585).^3).*(0.31/2).*(P./Ym)% change in refractive index

F=[0 0.1 1 5 10 15 25 30 35 60 90];  % force in newtons
P2=(F/((5.06e-4)-(1.79e-5)));  % pressure

delF=((1.585).^3).*(0.31/2).*(P2./Ym)% change in refractive index

xval=2.337e-5;
yval=4.079e-7;

m=yval./xval  % slope of the computational plot

dexam=delnsb.*(l./m)% calculating the fringe shift with the slope obtained
  % from the computational method

phi=m.* 1% calculating the quantity phi in the first method
% phi=(np-no)(L/h)=(2*3.14)./(Bo*Fringe shift)

phi2=(2*3.14)/(Bo.*fringe)% calculating phi in the second method
fring=(delF.*l)./phi2

fringa=(2*3.14)/(Bo.*phi)% calculating the distance between minima
% to see if it agrees with the fringe spacing of 1.4 mm that is
% observed in the experiment and is used in the computational method as well.

diff=fringa-fringe
```

9. Program to calculate the fringe shift from the experimental data.

```


```

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clear all; close all; clc;

%REFERENCE LINE IS AT 66

%13.1 N LOAD CONDITION
nla = imread('pr0009.jpg'); % read the image
nlta = double(nla(110:111,125:210,1)); % convert from unsigned integer 8bit to double integer so as to prevent overflow
nlsa = sum(nlta); % sums the intensity plots over the specified lines
plot(nlsa,'r*-', 'MarkerEdgeColor', 'k')
hold on
grid minor

%16.5 N LOAD CONDITION
ela = imread('pr0010.jpg'); % read the image
elta = double(ela(110:111,125:210,1)); % convert from unsigned integer 8bit to double integer so as to prevent overflow
elsa = sum(elta); % sums the intensity plots over the specified lines
plot(elsa,'g*-', 'MarkerEdgeColor', 'k')
hold on

%17.98 N LOAD CONDITION
bla = imread('pr0011.jpg'); % read the image
blta = double(bla(110:111,125:210,1)); % convert from unsigned integer 8bit to double integer so as to prevent overflow
blsa = sum(blta); % sums the intensity plots over the specified lines
plot(blsa,'b*-', 'MarkerEdgeColor', 'k')
hold on

%20.16 N LOAD CONDITION
tra = imread('pr0012.jpg'); % read the image
trta = double(tra(110:111,125:210,1)); % convert from unsigned integer 8bit to double integer so as to prevent overflow
trsa = sum(trta); % sums the intensity plots over the specified lines
plot(trsa,'k*-', 'MarkerEdgeColor', 'b')
hold on

%26.1 N
gya = imread('pr0013.jpg'); % read the image
gyta = double(gya(110:111,125:210,1)); % convert from unsigned integer 8bit to double integer so as to prevent overflow

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gy_sa=sum(gyta); % sums the inzersigy plots over the specified lines
plot(gy_sa,'y*-','MarkerEdgeColor','k')
hold on

% 29.6 N LOAD CONDITION
fra=imread('pr00I4.jpg'); % read the image
frta=double(fra(110:111,125:210,1)); % convert from unsigned integer 8bit to double
integer so as to prevent overflow
frsa=sum(frta); % sums the intensity plots over the specified lines
plot(frsa,'c*-','MarkerEdgeColor','k')
hold on

% 34.84 N CONDITION
gg_a=imread('pr00I5.jpg'); % read the image
ggta=double(gga(110:111,125:210,1)); % convert from unsigned integer 8bit to double
integer so as to prevent overflow
gg_sa=sum(ggta); % sums the intensity plots over the specified lines
plot(gg_sa,'m*-','MarkerEdgeColor','k')
hold on

% 39.8 N CONDITION
gh_a=imread('pr00I6.jpg'); % read the image
ghta=double(gha(110:111,125:210,1)); % convert from unsigned integer 8bit to double
integer so as to prevent overflow
ghsa=sum(ghta); % sums the intensity plots over the specified lines
plot(ghsa,'r.-','MarkerSize',10,'MarkerEdgeColor','k')
hold on

% 44.82 N CONDITION
ghk_a=imread('pr00I7.jpg'); % read the image
ghkta=double(ghk_a(110:111,125:210,1)); % convert from unsigned integer 8bit to double
integer so as to prevent overflow
ghksa=sum(ghkta); % sums the intensity plots over the specified lines
plot(ghksa,'g.-','MarkerSize',10,'MarkerEdgeColor','k')
hold on

nluasgposl=53;
eluasgposl=65;
bluasgposl=64;
truasgposl=64;
gyuasgposl=63;
fruasgposl=64;
gguasgposl=64;
ghuasgposl=64;
gkuasgposl=64;
dif(8)=0
dif(9)=66-nluasgpos1;
dif(10)=66-eluasgpos1;
dif(11)=66-bluasgpos1;
dif(12)=66-truasgpos1;
dif(13)=66-gyuasgpos1;
dif(14)=66-fruasgpos1;
dif(15)=66-gguasgpos1;
dif(16)=66-ghuasgpos1;
dif(17)=66-gkuasgpos1;

R(8)=0

for i=9:17
    del(i)=dif(i)-dif(i-1);R(i)=R(i-1)+del(i)
end


clear all;close all;clc;

%REFERENCE LINE IS AT 66

%50.1 N LOAD CONDITION
nla=imread('pr0018.jpg');% read the image
nlta=double(nla(110:111,125:210,1));%convert from unsigned integer 8bit to double integer so as to prevent overflow
nlsa=sum(nlta);%sums the intensity plots over the specified lines
plot(nlsa,'r')
hold on
grid minor

%56.574 N LOAD CONDITION
ela=imread('pr0019.jpg');% read the image
elta=double(ela(110:111,125:210,1));%convert from unsigned integer 8bit to double integer so as to prevent overflow
elsa=sum(elta);%sums the intensity plots over the specified lines
plot(elsa,'g')
hold on

%59.89 N LOAD CONDITION
bla=imread('pr0020.jpg');% read the image
blta = double(bla(110:111,125:210,1));  \( \text{convert from unsigned integer 8bit to double integer so as to prevent overflow} \)
blsa = sum(blta);  \( \text{sums the intensity plots over the specified lines} \)
plot(blsa,'b')  \( \text{hold on} \)

% 69.45 N LOAD CONDITION
tra = imread('pr0022.jpg');  \( \text{% read the image} \)
trta = double(tra(110:111,125:210,1));  \( \text{% convert from unsigned integer 8bit to double integer so as to prevent overflow} \)
trsa = sum(trta);  \( \text{% sums the intensity plots over the specified lines} \)
plot(trsa,'k')  \( \text{hold on} \)

% 76.1 N
gya = imread('pr0023.jpg');  \( \text{% read the image} \)
gyta = double(gya(110:111,125:210,1));  \( \text{% convert from unsigned integer 8bit to double integer so as to prevent overflow} \)
gysa = sum(gyta);  \( \text{% sums the intensity plots over the specified lines} \)
plot(gysa,'y')  \( \text{hold on} \)

% 80.54 N LOAD CONDITION
fra = imread('pr0024.jpg');  \( \text{% read the image} \)
frta = double(fra(110:111,125:210,1));  \( \text{% convert from unsigned integer 8bit to double integer so as to prevent overflow} \)
frsa = sum(frta);  \( \text{% sums the intensity plots over the specified lines} \)
plot(frsa,'c')  \( \text{hold on} \)

% 87.3 N CONDITION
gga = imread('pr0025.jpg');  \( \text{% read the image} \)
ggta = double(gga(110:111,125:210,1));  \( \text{% convert from unsigned integer 8bit to double integer so as to prevent overflow} \)
ggsa = sum(ggta);  \( \text{% sums the intensity plots over the specified lines} \)
plot(ggsa,'m')  \( \text{hold on} \)

% 93.1 N CONDITION
gha = imread('pr0026.jpg');  \( \text{% read the image} \)
ghta = double(gha(110:111,125:210,1));  \( \text{% convert from unsigned integer 8bit to double integer so as to prevent overflow} \)
ghsa = sum(ghta);  \( \text{% sums the intensity plots over the specified lines} \)
plot(ghsa,'r-*')  \( \text{hold on} \)
%97.03 N CONDITION
kka=imread('pr0027.jpg');% read the image
kkta=double(kka(110:111,125:210,1));%convert from unsigned integer 8bit to double integer so as to prevent overflow
kksa=sum(kkta);%sums the intensity plots over the specified lines
plot(kksa,'b-*')
hold on
R(17)=2;
dif(17)=2;
nluasgposl=63;
eluasgposl=62;
bluasgposl=62;
truasgposl=61;
gyuasgposl=61;
fruasgposl=61;
gguasgposl=61;
ghuasgposl=59;
kkuasgposl=61;
dif(18)=66-nluasgposl;
dif(19)=66-eluasgposl;
dif(20)=66-bluasgposl;
dif(21)=66-truasgposl;
dif(22)=66-gyuasgposl;
dif(23)=66-fruasgposl;
dif(24)=66-gguasgposl;
dif(25)=66-ghuasgposl;
dif(26)=66-kkuasgposl;
for i=18:26
    del(i)=dif(i)-dif(i-1);R(i)=R(i-1)+del(i);
end

#################################EXPERIMENT B#################################
clear all;close all;clc;

%REFERENCE LINE IS AT 66

%90.8 N LOAD CONDITION
fla = imread('pr0028.jpg');  % read the image
flta = double(fla(110:111,125:210,1));  % convert from unsigned integer 8bit to double integer so as to prevent overflow
flsa = sum(flta);  % sums the intensity plots over the specified lines
plot(flsa,'r*-','MarkerEdgeColor','k')
hold on

% 85.5 N LOAD CONDITION

nla = imread('pr0029.jpg');  % read the image
nlta = double(nla(110:111,125:210,1));  % convert from unsigned integer 8bit to double integer so as to prevent overflow
nlsa = sum(nlta);  % sums the intensity plots over the specified lines
plot(nlsa,'g*-','MarkerEdgeColor','k')
hold on
grid minor

% 78.67 N LOAD CONDITION

ela = imread('pr0030.jpg');  % read the image
elta = double(ela(110:111,125:210,1));  % convert from unsigned integer 8bit to double integer so as to prevent overflow
elsa = sum(elta);  % sums the intensity plots over the specified lines
plot(elsa,'b*-','MarkerEdgeColor','k')
hold on

% 69.18 N LOAD CONDITION

bla = imread('pr0031.jpg');  % read the image
blta = double(bla(110:111,125:210,1));  % convert from unsigned integer 8bit to double integer so as to prevent overflow
blsa = sum(blta);  % sums the intensity plots over the specified lines
plot(blta,'k*-','MarkerEdgeColor','b')
hold on

% 64.3 N LOAD CONDITION

tra = imread('pr0032.jpg');  % read the image
trta = double(tra(110:111,125:210,1));  % convert from unsigned integer 8bit to double integer so as to prevent overflow
trsa = sum(trta);  % sums the intensity plots over the specified lines
plot(trsa,'y*-','MarkerEdgeColor','k')
hold on

%59.6 N
gya = imread('pr0033.jpg'); % read the image
gyta = double(gya(110:111,125:210,1)); % convert from unsigned integer 8bit to double integer so as to prevent overflow
gysa = sum(gyta); % sums the intensity plots over the specified lines
plot(gysa, 'c*-','MarkerEdgeColor','k')
hold on

%55.07 N LOAD CONDITION
fra = imread('pr0034.jpg'); % read the image
frta = double(fra(110:111,125:210,1)); % convert from unsigned integer 8bit to double integer so as to prevent overflow
frsa = sum(frta); % sums the intensity plots over the specified lines
plot(frsa, 'm*-','MarkerEdgeColor','k')
hold on

%50.5 N CONDITION
gga = imread('pr0035.jpg'); % read the image
ggta = double(gga(110:111,125:210,1)); % convert from unsigned integer 8bit to double integer so as to prevent overflow
ggsa = sum(ggta); % sums the intensity plots over the specified lines
plot(ggsa, 'r.-','MarkerEdgeColor','k')
legend('90.8 N','85.5 N','78.67 N','69.18 N','64.3 N','59.6 N','55.07 N','50.5 N','Location','BestOutside')
hold on

gguasgpos1 = 61;
fruasgpos1 = 61;
gyuasgpos1 = 60;
truasgpos1 = 59;
bluasgpos1 = 60;
eluasgpos1 = 58;
nluasgpos1 = 61;
fluasgpos1 = 59;

dif(15) = 66-gguasgpos1;
dif(16) = 66-fruasgpos1;
dif(17) = 66-gyuasgpos1;
dif(18) = 66-truasgpos1;
dif(19) = 66-bluasgpos1;
dif(20)=66-eluasgpos1;
dif(21)=66-nluasgpos1;
dif(22)=66-fluasgpos1;

dif(14)=2
R(14)=2

for i=15:21
    del(i)=dif(i)-dif(i-1);R(i)=R(i-1)+del(i)
end

R3=[R(15) R(16) R(17) R(18) R(19) R(20) R(21)]

% difk=66-dif

clear all;close all;clc;

%REFERENCE LINE IS AT 66

%45.9 N CONDITION
gha=imread('pr0036.jpg');% read the image
ghta=double(gha(110:111,125:210,1));%convert from unsigned integer 8bit to double
integer so as to prevent overflow
ghsa=sum(ghta);%sums the intensity plots over the specified lines
plot(ghsa,'r*-','MarkerEdgeColor','k')
hold on

%43.13 N CONDITION
gka=imread('pr0038.jpg');% read the image
gkta=double(gka(110:111,125:210,1));%convert from unsigned integer 8bit to double
integer so as to prevent overflow
gksa=sum(gkta);%sums the intensity plots over the specified lines
plot(gksa,'g*-','MarkerEdgeColor','k')
hold on

%37.56 N LOAD CONDITION
nla=imread('pr0039.jpg');% read the image
nlta=double(nla(110:111,125:210,1));%convert from unsigned integer 8bit to double
integer so as to prevent overflow
nlsa=sum(nlta);%sums the intensity plots over the specified lines
plot(nlsa,'b*-','MarkerEdgeColor','k')
hold on

grid minor

%30.2 N LOAD CONDITION

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ela = imread('pr0040.jpg'); % read the image
elta = double(ela(110:111,125:210,1)); % convert from unsigned integer 8bit to double integer so as to prevent overflow
elsa = sum(elta); % sums the intensity plots over the specified lines
plot(elsa,'k*-','MarkerEdgeColor','k')
hold on

% 24.3 N LOAD CONDITION
bla = imread('pr0041.jpg'); % read the image
blta = double(bla(110:111,125:210,1)); % convert from unsigned integer 8bit to double integer so as to prevent overflow
blsa = sum(blta); % sums the intensity plots over the specified lines
plot(blsa,'y*-','MarkerEdgeColor','k')
hold on

% 17.9 N LOAD CONDITION
tra = imread('pr0042.jpg'); % read the image
trta = double(tra(110:111,125:210,1)); % convert from unsigned integer 8bit to double integer so as to prevent overflow
trsa = sum(trta); % sums the intensity plots over the specified lines
plot(trsa,'c*-','MarkerEdgeColor','k')
hold on

% 14.9 N
gya = imread('pr0043.jpg'); % read the image
gyta = double(gya(110:111,125:210,1)); % convert from unsigned integer 8bit to double integer so as to prevent overflow
gysa = sum(gyta); % sums the intensity plots over the specified lines
plot(gysa,'m*-','MarkerEdgeColor','k')
hold on

% 10.4 N LOAD CONDITION
fra = imread('pr0044.jpg'); % read the image
frta = double(fra(110:111,125:210,1)); % convert from unsigned integer 8bit to double integer so as to prevent overflow
frsa = sum(frta); % sums the intensity plots over the specified lines
plot(frsa,'r*-','MarkerEdgeColor','k')

% legend('90.8 N','85.5 N','78.67 N','69.18 N','64.3 N','59.6 N','55.07 N','50.5 N','Location','BestOutside')
hold on

fruasgpos1 = 58;
gyuasgpos1 = 65;
truasgpos1 = 65;
bluasgpos1 = 64;
eluasgpos1=64;
nluasgpos1=63;
gkuasgpos1=63;
ghuasgpos1=64;

dif(7)=fruasgpos1;
dif(8)=gyuasgpos1;
dif(9)=truasgpos1;
dif(10)=bluasgpos1;
dif(11)=eluasgpos1;
dif(12)=nluasgpos1;
dif(13)=gkuasgpos1;
dif(14)=ghuasgpos1;

dif=66-dif
R(6)=0
dif(6)= 0

for i=7:14
    del(i)=dif(i)-dif(i-1);R(i)=R(i-1)+del(i)
end

R2=[R(8) R(9) R(10) R(11) R(12) R(13) R(14)]

clear all;close all;clc;
%
% %REFERENCE LINE IS AT 66
%
%12.2 N LOAD CONDITION
nla=imread('pr0061.jpg');% read the image
nlta=double(nla(110:111,125:210,l));%convert from unsigned integer 8bit to double
integer so as to prevent overflow
nlsa=sum(nlta);%sums the intensity plots over the specified lines
plot(nlsa,'r*-','MarkerEdgeColor','k')
hold on
%
%14.8 N LOAD CONDITION
ela=imread('pr0062.jpg');% read the image
elta=double(ela(110:111,125:210,1));%convert from unsigned integer 8bit to double
integer so as to prevent overflow
elsa=sum(elta);%sums the intensity plots over the specified lines
plot(elsa,'g*-','MarkerEdgeColor','k')
hold on

%17.2 N LOAD CONDITION %
bla=imread('pr0063.jpg');% read the image
blta=double(bla(110:111,125:210,1));%convert from unsigned integer 8bit to double
integer so as to prevent overflow
blsa=sum(blta);%sums the intensity plots over the specified lines
plot(blsa,'b*-','MarkerEdgeColor','k')
hold on

%21.6 N LOAD CONDITION
tra=imread('pr0064.jpg');% read the image
trta=double(tra(110:111,125:210,1));%convert from unsigned integer 8bit to double
integer so as to prevent overflow
trsa=sum(trta);%sums the intensity plots over the specified lines
plot(trsa,'k*-','MarkerEdgeColor','r')
hold on

%25.6 N LOAD CONDITION
fra=imread('pr0065.jpg');% read the image
frta=double(fra(110:111,125:210,1));%convert from unsigned integer 8bit to double
integer so as to prevent overflow
frsa=sum(frta);%sums the intensity plots over the specified lines
plot(frsa,'y*-','MarkerEdgeColor','k')
hold on

%32.1 N CONDITION
gga=imread('pr0066.jpg');% read the image
ggta=double(gga(110:111,125:210,1));%convert from unsigned integer 8bit to double
integer so as to prevent overflow
ggsa=sum(ggta);%sums the intensity plots over the specified lines
plot(ggsa,'c*-','MarkerEdgeColor','k')
hold on

%36.2 N CONDITION
gha=imread('pr0067.jpg');% read the image
ghta=double(gha(110:111,125:210,1));%convert from unsigned integer 8bit to double
integer so as to prevent overflow
ghsa=sum(ghta);%sums the intensity plots over the specified lines
plot(ghsa,'m*-','MarkerEdgeColor','k')
hold on

%40.4 N CONDITION
gka=imread('pr0068.jpg');% read the image
gkta=double(gka(110:111,125:210,1));%convert from unsigned integer 8bit to double integer so as to prevent overflow

gksa=sum(gkta);%sums the intensity plots over the specified lines
plot(gksa,'r.-','MarkerEdgeColor','k')
hold on

%45.34 N CONDITION
pka=imread('pr0069.jpg');% read the image
pkta=double(pka(110:111,125:210,1));%convert from unsigned integer 8bit to double integer so as to prevent overflow
pksa=sum(pkta);%sums the intensity plots over the specified lines
plot(pksa,'g.-','MarkerEdgeColor','k')
hold on

%52.91 N CONDITION
tka=imread('pr0070.jpg');% read the image
tkta=double(tka(110:111,125:210,1));%convert from unsigned integer 8bit to double integer so as to prevent overflow
tksa=sum(tkta);%sums the intensity plots over the specified lines
plot(tksa,'b.-','MarkerEdgeColor','k')
hold on
grid minor

nluasgpos1=57;
elluasgpos1=56;
bluasgpos1=58;
truesgpos1=56;
fruesgpos1=57;
gguasgpos1=57;
ghuasgpos1=57;
gkuasgpos1=56;
pkuasgpos1=56;
tkuasgpos1=55

dif(1)=66-nluasgpos1;
dif(2)=66-eluasgpos1;
dif(3)=66-bluasgpos1;
dif(4)=66-truasgpos1;
dif(5)=66-fruasgpos1;
dif(6)=66-gguasgpos1;
dif(7)=66-ghuasgpos1;
dif(8)=66-gkuasgpos1;
dif(9)=66-pkuasgpos1;
dif(10)=66-tkuasgpos1;
\[ \text{dif}(10) = 0 \]
\[ \text{R}(10) = 0 \]

for \( i = 11:20 \)
\[ \text{del}(i) = \text{dif}(i) - \text{dif}(i-1); \text{R}(i) = \text{R}(i-1) + \text{del}(i) \]
end

\[ \text{R} = [\text{R}(11) \text{ R}(12) \text{ R}(13) \text{ R}(14) \text{ R}(15) \text{ R}(16) \text{ R}(17) \text{ R}(18) \text{ R}(19) \text{ R}(20)] \]

clear all; close all; clc;

%REFERENCE LINE IS AT 66

% 57.7 N LOAD CONDITION
nla = imread('pr0071.jpg');% read the image
nlta = double(nla(110:111,125:210,1));% convert from unsigned integer 8bit to double integer so as to prevent overflow
nlsa = sum(nlta);% sums the intensity plots over the specified lines
plot(nlsa,'r*-','MarkerEdgeColor','k')
hold on
grid minor

% 60.7 N LOAD CONDITION
ela = imread('pr0072.jpg');% read the image
elta = double(ela(110:111,125:210,1));% convert from unsigned integer 8bit to double integer so as to prevent overflow
elsa = sum(elta);% sums the intensity plots over the specified lines
plot(elsa,'g*-','MarkerEdgeColor','k')
hold on

% 66.4 N LOAD CONDITION
bla = imread('pr0073.jpg');% read the image
blta = double(bla(110:111,125:210,1));% convert from unsigned integer 8bit to double integer so as to prevent overflow
blsa = sum(blta);% sums the intensity plots over the specified lines
plot(blsa,'b*-','MarkerEdgeColor','k')
hold on

% 71.6 N LOAD CONDITION
tra = imread('pr0074.jpg');% read the image
trta = double(tra(110:111,125:210,1));% convert from unsigned integer 8bit to double integer so as to prevent overflow
trsa = sum(trta);% sums the intensity plots over the specified lines
plot(trsa,'k*-','MarkerEdgeColor','k')
hold on

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%76.5 N
gya=imread('pr0076.jpg');% read the image
gyta=double(gya(110:111,125:210,1));%convert from unsigned integer 8bit to double
integer so as to prevent overflow
gysa=sum(gyta);%sums the intensity plots over the specified lines
plot(gysa,'y*-','MarkerEdgeColor','k')
hold on

%81.5 N LOAD CONDITION
fra=imread('pr0077.jpg');% read the image
frta=double(fra(110:111,125:210,1));%convert from unsigned integer 8bit to double
integer so as to prevent overflow
frsa=sum(frta);%sums the intensity plots over the specified lines
plot(frsa,'c*-','MarkerEdgeColor','k')
hold on

%86.5 N CONDITION
gga=imread('pr0078.jpg');% read the image
ggta=double(gga(110:111,125:210,1));%convert from unsigned integer 8bit to double
integer so as to prevent overflow
ggsa=sum(ggta);%sums the intensity plots over the specified lines
plot(ggsa,'m*-','MarkerEdgeColor','k')
hold on

%92.2 N CONDITION
gha=imread('pr0079.jpg');% read the image
ghta=double(gha(110:111,125:210,1));%convert from unsigned integer 8bit to double
integer so as to prevent overflow
ghsa=sum(ghta);%sums the intensity plots over the specified lines
plot(ghsa,'r*-','MarkerEdgeColor','k')
hold on

%95.8 N CONDITION
gka=imread('pr0080.jpg');% read the image
gkta=double(gka(110:111,125:210,1));%convert from unsigned integer 8bit to double
integer so as to prevent overflow
gksa=sum(gkta);%sums the intensity plots over the specified lines
plot(gksa,'g*-','MarkerEdgeColor','k')
hold on

nluasgposl=54;
eluasgposl=54;
bluasgposl=54;
truasgposl=54;
gyuasgposl=54;
fruasgposl=54;

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dif(21)=66-gluasgpos1;
dif(22)=66-eluaasgpos1;
dif(23)=66-bluaasgpos1;
dif(24)=66-truaasgpos1;
dif(25)=66-gyuasgpos1;
dif(26)=66-fruaasgpos1;
dif(27)=66-gguasgpos1;
dif(28)=66-ghuaasgpos1;
dif(29)=66-gkuasgpos1;

dif(20)=11;
R(20)=11;

for i=21:29
    del(i)=dif(i-1)-dif(i);R(i)=R(i-1)+del(i)
end


#######EXPERIMENT D####################################################

clear all;close all;clc;

%REFERENCE LINE IS AT 66

%88.96 N LOAD CONDITION
ela=imread('pr0081.jpg');% read the image
elta=double(ela(110:111,125:210,1));%convert from unsigned integer 8bit to double integer so as to prevent overflow
elsa=sum(elta);%sums the intensity plots over the specified lines
plot(elsa,'g*-','MarkerEdgeColor','k')
hold on
eluasgpos1=54;

%85 N LOAD CONDITION 
bla=imread('pr0082.jpg');% read the image
blta=double(bla(110:111,125:210,1));%convert from unsigned integer 8bit to double integer so as to prevent overflow
blsa=sum(blta);%sums the intensity plots over the specified lines
plot(blta,'b*-', 'MarkerEdgeColor','k')
hold on
%
bluasgpos1=52;

%79.1 N LOAD CONDITION
tra=imread('pr0083.jpg');
trta=double(tra(110:111,125:210,1));
trsa=sum(trta);
plot(trsa,'k*-','MarkerEdgeColor','k')

hold on

%76.4 N
gya=imread('pr0084.jpg');
gyta=double(gya(110:111,125:210,1));
gysa=sum(gyta);
plot(gysa,'y*-','MarkerEdgeColor','k')

hold on

%71.3 N LOAD CONDITION
fra=imread('pr0086.jpg');
frta=double(fra(110:111,125:210,1));
frsa=sum(frta);
plot(frsa,'c*-','MarkerEdgeColor','k')

hold on

%65.6 N CONDITION
gha=imread('pr0087.jpg');
ghta=double(gha(110:111,125:210,1));
ghsa=sum(ghta);
plot(ghsa,'m*-','MarkerEdgeColor','k')

hold on

%60.7 N CONDITION

ghuaasgpos1=53;

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gka = imread('pr0088.jpg'); % read the image
gkta = double(gka(110:111,125:210,1)); % convert from unsigned integer 8bit to double integer so as to prevent overflow
gksa = sum(gkta); % sums the intensity plots over the specified lines
plot(gksa,'r.-','MarkerEdgeColor','k')
hold on

% 54.32 N CONDITION
sda = imread('pr0089.jpg'); % read the image
sdta = double(sda(110:111,125:210,1)); % convert from unsigned integer 8bit to double integer so as to prevent overflow
sdsa = sum(sdta); % sums the intensity plots over the specified lines
plot(sdsa,'g.-','MarkerEdgeColor','k')
hold on
sduasgposl = 54;

dif(16) = 66 - sduasgpos1;
dif(17) = 66 - gkuasgpos1;
dif(18) = 66 - hguasgpos1;
dif(19) = 66 - fruasgpos1;
dif(20) = 66 - gyuasgpos1;
dif(21) = 66 - truasgpos1;
dif(22) = 66 - bluasgpos1;
dif(23) = 66 - eluasgpos1;

R(15) = 12
dif(15) = 12
for i = 16:23
    del(i) = dif(i) - dif(i-1); R(i) = R(i-1) + del(i)
end
R3 = [R(16) R(17) R(18) R(19) R(20) R(21) R(22) R(23)]

clear all; close all; clc;

% REFERENCE LINE IS AT 66

% 49.8 N LOAD CONDITION
ela = imread('pr0090.jpg'); % read the image
elta = double(ela(110:111, 125:210, 1)); % convert from unsigned integer 8bit to double
integer so as to prevent overflow
elsa = sum(elta); % sums the intensity plots over the specified lines
plot(elsa, 'r*-', 'MarkerEdgeColor', 'k')
hold on
eluasgpos1 = 54;

% 43.56 N LOAD CONDITION
bla = imread('pr0091.jpg'); % read the image
blta = double(bla(110:111, 125:210, 1)); % convert from unsigned integer 8bit to double
integer so as to prevent overflow
blsa = sum(blta); % sums the intensity plots over the specified lines
plot(blsa, 'g*-', 'MarkerEdgeColor', 'k')
hold on
bluasgpos1 = 54;

% 36.37 N LOAD CONDITION
tra = imread('pr0092.jpg'); % read the image
trta = double(tra(110:111, 125:210, 1)); % convert from unsigned integer 8bit to double
integer so as to prevent overflow
trsa = sum(trta); % sums the intensity plots over the specified lines
plot(trsa, 'b*-', 'MarkerEdgeColor', 'k')
hold on
truasgpos1 = 55;

% 30.26 N
gya = imread('pr0093.jpg'); % read the image
gyta = double(gya(110:111, 125:210, 1)); % convert from unsigned integer 8bit to double
integer so as to prevent overflow
gysa = sum(gyta); % sums the intensity plots over the specified lines
plot(gysa, 'k*-', 'MarkerEdgeColor', 'k')
hold on
gyuasgpos1 = 56;

% 25.4 N LOAD CONDITION
fra = imread('pr0094.jpg'); % read the image
frta = double(fra(110:111, 125:210, 1)); % convert from unsigned integer 8bit to double
integer so as to prevent overflow
frsa = sum(frta); % sums the intensity plots over the specified lines
plot(frsa, 'y*-', 'MarkerEdgeColor', 'k')
hold on
fruasgpos1 = 56;

% 17.84 N CONDITION
gga = imread('pr0095.jpg'); % read the image

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ggta=double(gga(110:111,125:210,1)); % convert from unsigned integer 8bit to double integer so as to prevent overflow
ggsa=sum(ggta); % sums the intensity plots over the specified lines
plot(ggsa,'c*-','MarkerEdgeColor','k')
hold on

gguasgposl=54;

% 15.50 N CONDITION
gha=imread('pr0096.jpg'); % read the image
ghta=double(gha(110:111,125:210,1)); % convert from unsigned integer 8bit to double integer so as to prevent overflow
ghsa=sum(ghta); % sums the intensity plots over the specified lines
plot(ghsa,'m*-','MarkerEdgeColor','k')
hold on

gkuasgposl=55;

grid minor
title('Intensity vs Pixel Position')
xlabel('Pixel Position')
ylabel('Intensity')
set(get(gca,'title'),'fontname','timesnewroman','fontweight','bold','fontsize',[14])
set(get(gca,'xlabel'),'fontname','timesnewroman','fontweight','bold','fontsize',[14])
set(get(gca,'ylabel'),'fontname','timesnewroman','fontweight','bold','fontsize',[14])
gkuasgposl=56;

dif(8)=66-gkuasgposl;
dif(9)=66-ghuasgposl;
dif(10)=66-gguasgposl;
dif(11)=66-fruasgposl;
dif(12)=66-gyuasgposl;
dif(13)=66-truasgposl;
dif(14)=66-bluasgposl;
dif(15)=66-eluasgposl;

R(7)=0
diff(7)=0

for i=8:15
    del(i)=diff(i)-diff(i-1);R(i)=R(i-1)+del(i)
end
10. Program to plot the fringe shifts obtained in different methods

```matlab
R2=[R(9) R(10) R(11) R(12) R(13) R(14) R(15)]

delnsb,dexa,deltaxtwo,deltaxt,delF,F,fring]=lenk;
delnsbb,dexb]=vall;
[move]=compt;

dexa=dexa./1.4e-3;
dexb=dexb./1.4e-3;
move=move./1.4e-3;
fring=fring./1.4e-3
FL=[0 0.1 1 5 10 15 20 25 30 35]

% Starting point declarations from inside the array

naa = 1;
nbb = 1;
ncc = 1;
ndd = 1;

% #.............................. EXPERIMENT A #..............................

loada=[0.49 0.76 1.01 2.14 4.7 6.3 8.1 10.4 13.1 16.5 20.6 25.6 34.84 39.8 44.82 50.1 56.574 59.89 65.4 69.45 76.1 80.54 87.3 93.1 97.03];
loada=loada(1:26)
RA=[1 2 2 3 2 2 2 2 2 2 3 4 4 5 5 5 5 7 5]
RA=RA(1:16)

RA = RA.*1e-4;
NRA = (RA./1.4e-3);
NRA = NRA - NRA(naa);

nnra = NRA(naa:length(RA)),loada(naa:length(loada)),',r*','MarkerEdgeColor','k')
hold on
```

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\% Summation of x-terms and y-terms
\totalx_a = \text{sum}(\text{NRA(naa:length(RA)))}
\totaly_a = \text{sum}(\text{loada(naa:length(loada)))}

\% x_a = [\text{NRA(naa), totalx_a}];
x_a = [0, \text{totalx_a}];
\% y_a = [\text{loada(naa), totaly_a}];
y_a = [0, \text{totaly_a}];

\% \% ################################### EXPERIMENT B ###################################
\%
\% distances computed from the second load set%
loadb=[0.58 0.8 1.7 4.6 7.6 10.4 14.9 24.3 30.2 37.56 43.13 45.9 50.5 55.07 59.6 64.3 69.18 78.67 85.5 90.8];
loadb=loadb(8:20)
RB=[1 1 2 2 3 3 2 5 5 6 7 6 8 5]
RB=RB(1:13)

RB = RB.*1e-4;
NRB = (RB./1.4e-3);
NRB = NRB - NRB(nbb);

\% loadb = loadb - loadb(nbb)

\% plot(NRB(nbb:length(RB)),loadb(nbb:length(loadb)),'g*','MarkerEdgeColor','k')
nnrb = NRB(nbb:length(RB)),lloadb=loadb(nbb:length(loadb)),
hold on

\% Summation of x-terms and y-terms
\totalx_b = \text{sum}(\text{NRB(nbb:length(RB)))}
\totaly_b = \text{sum}(\text{loadb(nbb:length(loadb)))}

\% Perhaps, we could later create an IF-ELSE type of statement that we could
\% use to shorten the line we create
\x_b = [0, \text{totalx_b}];
\y_b = [0, \text{totaly_b}];
% #%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% EXPERIMENT C #%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%distances computed from the third load set%
loadc=[0.93 1.54 1.7 2.34 4.6 6 8 10.45 12.2 14.8 17.2 21.6 25.6 32.1 36.2 40.4 45.34 52.91 57.7 60.7 66.4 71.6 76.5 81.5 86.5 92.2 95.8];
loadc=loadc(10:27)
RC=[9 10 8 10 9 9 10 10 10 10 10 10 10 9 8];
RC=RC(1:18)
RC = RC.*le-4;
NRC = (RC./1.4e-3);
NRC = NRC - NRC(ncc);

% loadc = loadc - loadc(ncc)

% plot(NRC(ncc:length(RC)),loadc(ncc:length(loadc)),'b*','MarkerEdgeColor','k')
nrc = NRC(ncc:length(RC)),lloadc=loadc(ncc:length(loadc)),
hold on

% Summation of x-terms and y-terms
totalx_c = sum(NRC(ncc:length(RC)))
totaly_c = sum(loadc(ncc:length(loadc)))

% Perhaps, we could later create an IF-ELSE type of statement that we could
% use to shorten the line we create
x_c = [0, totalx_c];
y_c = [0, totaly_c];

% #%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% EXPERIMENT D #%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%distances computed from the fourth load set%
loadd=[0.64 1 2.5 5.35 7.6 10.5 13.5 15.5 17.84 25.4 30.26 36.37 43.56 49.8 54.32 60.7 65.6 71.3 76.4 79.1 85 88.96];
loadd=loadd(8:22)
RD=[12 12 10 10 11 12 12 12 13 13 13 12 14 14 12];
RD=RD(1:15)
RD = RD.*le-4;
NRD = (RD./1.4e-3);
NRD = NRD - NRD(ndd);

% shifting data to 0,0
\%
\% loadd = loadd - loadd(ndd)

\%
plot(NRD(ndd:length(RD)),loadd(ndd:length(loadd)),'k*','MarkerEdgeColor','k')
nnrd = NRD(ndd:length(RD)),lloadd=loadd(ndd:length(loadd)),
hold on

\% Summation of x-terms and y-terms
totalx_d = sum(NRD(ndd:length(RD)))
totaly_d = sum(loadd(ndd:length(loadd)))

\% Perhaps, we could later create an IF-ELSE type of statement that we could
\% use to shorten the line we create

x_d = [0, totalx_d];
y_d = [0, totaly_d];

\% Code taken from the original

SA=[lloada,lloadb,lloadc,lloadd];
DA=[nnra,nnrb,nnrc,nnrd];

figure
plot(nnra,lloada,'d','MarkerFaceColor','r')
hold on
plot(nnrb,lloadb,'o','MarkerFaceColor','g')
hold on
plot(nnrc,lloadc,'s','MarkerFaceColor','b')
hold on
plot(nnrd,lloadd,'v','MarkerFaceColor','k')
hold on

plot (x_a, y_a, 'r-')
hold on
plot (x_b, y_b, 'g-')
hold on
plot (x_c, y_c, 'b-')
hold on
plot (x_d, y_d, 'k-')
hold on

\% avglod=(loadd(1)+loadb(1)+loadc(1)+loadd(1))./4;
\% fring=fring;
\% move=move;
plot(fring,F,'m','MarkerEdgeColor','k')
hold on
grid minor
%computational
plot(move,FL,'*','MarkerEdgeColor','k','MarkerSize',5)
hold on
set(get(gca,'title'),'fontname','arial','fontweight','bold','fontsize',[14])
set(get(gca,'xlabel'),'fontname','arial','fontweight','bold','fontsize',[14])
set(get(gca,'ylabel'),'fontname','arial','fontweight','bold','fontsize',[14])
legend('1st load set','2nd load set','3rd load set','4th load set','Best fit 1st load set','Best fit 2nd load set','Best fit 3rd load set','Best fit 4th load set','Theory','Computation','Location','BestOutside')
title({'Comparision of Experimental,Theory';'and Computational Analyses'})
xlabel('Fringe Shift (Number of Fringes)')
ylabel('Force (Newtons)')
set(get(gca,'children'),'linewidth',[1]);
set(gca,'linewidth',[1],'gridlinestyle','-');
set(gca,'title','fontname','timesnewroman','fontweight','bold','fontsize',[14])
set(gca,'xlabel','fontname','timesnewroman','fontweight','bold','fontsize',[14])
set(gca,'ylabel','fontname','timesnewroman','fontweight','bold','fontsize',[14])
grid on
axis([0 1.5 0 100])

finalfringe=totalx_a+totalx_b+totalx_c+totalx_d;
finalload=totaly_a+totaly_b+totaly_c+totaly_d;

ff=[0,finalfringe]
fl=[0,finalload]

finalfringe=totalx_a+totalx_b+totalx_c+totalx_d;
finalload=totaly_a+totaly_b+totaly_c+totaly_d;
avgff=finalfringe/4;
avgfl=finalload/4;
aff=[0,avgff]
afl=[0,avgfl]

figure
plot(nnra,lloada,'d','MarkerFaceColor','r')
hold on
plot(nnrb,lloadb,'o','MarkerFaceColor','g')
hold on
plot(nnrc,lloadc,'s','MarkerFaceColor','b')
hold on
plot(nnrd,lloadd,'v','MarkerFaceColor','k')
hold on
plot (aff,afl, 'k--')
hold on
plot(fring,F,'rVMarkerEdgeColor',k')
hold on
grid minor
%computational
plot(move,FL,'*VMarkerEdgeColor',k','MarkerSize',5)
hold on
title({'Comparision of Experimental,Theory';{'and Computational Analyses'}})
xlabel('No. Of Fringes Shifted')
ylabel('Load(N)')
set(get(gca,'title'),'fontname','ariar','fontweight','bold','fontsize',[14])
set(get(gca,'xlabel'),'fontname','arial','fontweight','bold','fontsize',[14])
set(get(gca,'ylabel'),'fontname','arial','fontweight','bold','fontsize',[14])
% axis([0 0.75 10 100])
legend('1st load set ','2nd load set ','3rd load set ','4th load set ','Experimental data average','Theory','Computation','Location','BestOutside')
xlabel('Fringe Shift (Number of Fringes)')
ylabel('Force (Newtons)')
set(get(gca,'children'),'linewidth',[1]);
set(gca,'linewidth',1,'gridlinestyle',-'
','fontname','timesnewroman','fontweight','bold','fontsize',[14]);
set(get(gca,'title'),'fontname','timesnewroman','fontweight','bold','fontsize',[14])
set(get(gca,'xlabel'),'fontname','timesnewroman','fontweight','bold','fontsize',[14])
set(get(gca,'ylabel'),'fontname','timesnewroman','fontweight','bold','fontsize',[14])
grid on
axis([-0.2 1 0 100])
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