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Table Games Revenue Management: A Bayesian Approach

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TABLE GAMES REVENUE MANAGEMENT:

A BAYESIAN APPROACH

By

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Abstract

With growing competition for casino floor space, the Table Games department is under increased pressure to improve revenues. Current systems in the department rely upon a supervisor's intuitive knowledge about business trends hourly to respond to business levels. By using Bayesian analysis, it is possible to develop a functional revenue management system for making optimal business decisions in the Table Game Departments.

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The mentorship of the UNLV faculty, especially Dr. Ashok Singh, Dr. Anthony Lucas, and Dr. Bo Bernhard has benefited me greatly and encouraged me to develop my research skills. I became fascinated by the ability of analytics through Dr. Lucas who has closely looked at the gaming industry, and has never shied from challenging the established paradigms. Dr. Bernhard encouraged me and others to continually look at the world as a source of inspiration and curiosity. I learned the process of statistical decision making from Dr. Singh, without whose mentorship I would not have completed this thesis. I also would like to thank Dr. Dennis Murphy for his help with the R programming language. I am indebted to all of them for their time and effort in encouraging me to become more than just a student, but a researcher, and I look forward to future collaboration with them.

Dedication

For my loving wife, Elizabeth, and son, Alexander, without your support this could never have happened. I hope to live up to all your expectations of me. Together in all things.

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Chapter One

Introduction

The table games department of a casino is, in the eyes of the industry, an essential part of the casino. It is where casinos started and has been the heart of the gaming industry for decades. Recent trends in gaming, however, have started to erode the amount of space dedicated to table games. Side bets and “carnival” games aside, table games and their management remain much the same since Nevada passed the Wide Open Gambling Act of 1931 (Kilby, Fox, & Lucas, 2005). While the hospitality industry has advanced and incorporated technology and scientific methods into their processes, table games remain unchanged. The gaming industry overall has expanded globally in new jurisdictions, and in several of these newer areas, table games is a driving profit center. This mirrors the trend in Las Vegas gaming development.

Hold percentage in the industry is a metric by which gaming volume is typically quantified. As hold percentages decline, space is made on the casino floor for games or machines that produce higher hold percentages, and thus higher profits. This is evident from the decline in the number of tables over recent years (Schwartz, 2013a). This trend is not unique to one specific type of table game, but rather the group as a whole (Schwartz, 2013b). This decline in the hold percentage, and by association profits, is a large portion of the reason for the pressure on table games. Other industries faced with similar issues have found that improved revenue management is helpful in stabilizing profits and relieving some of the pressure due to declining revenues (Buckhiester, 2011). Developing an optimal table games revenue management system that uses prior information will help maximize betting levels at the tables and potentially slow the decline of the table games hold percentage, thus potentially preventing loss of more casino “real estate” to slots.

Purpose

The purpose of this thesis is to develop and test an effective table games revenue management system which utilizes prior information to predict the future.

Definitions

The gaming industry uses unique, and sometimes ambiguous, terms for various metrics. The following terms will have the following definitions for their use in this study:

Drop: This is the mathematical calculation of gaming activity for the department and represents the net monetary value of each table. Summed for the department, it is representative of the total net cash gaming activity for the department. Casinos in Nevada use equation 1 to calculate drop (Lucas & Kilby, 2012).

$$\text{Drop}=\text{Markers Issued}-\text{Markers Redeemed}+\text{Cash}+\text{Gaming Cheques}+\text{Foreign Gaming Cheques} \tag{1}$$

Hold: This refers to the actual hold percentage, which is mathematically calculated as win/drop, and is expressed as a percentage (Lucas & Kilby, 2012).

Theoretical Framework

Revenue management (RM) is a complex process for optimizing revenue from a fixed inventory which has applications in various industries. It has its origins in the Airlines industry (Haley & Inge, 2004). By looking at the common practices of RM in other industries, it is possible to develop a, RM system that is applicable to a table games department.

Problem Statement

The aspects of table games that call for the use of a revenue management system are the variable demand of the games and the variable betting threshold of each player. Previous research has looked at other variables such as win per available seat hour, or length of each play session (Peister, 2007). While these variables are helpful, at the basic level, the hourly headcount provides a decent indicator of demand to suit our needs. As business levels increase, casino shift managers currently examine the number of players at a table and determine if opening additional games would be prudent. As games open, and fill up, the determination is then made if raising minimum bet levels is needed. This works at a reactive level, but is far from optimized. An example would be the case of low bet minimum, and the players playing at a higher minimum level without all the games being full. This potential loss of profits could be quite significant over time. For the problem at hand, we will look at the variables of estimated demand and the average bet as an indicator of the player's risk tolerance.

By predicting the next time period's expected demand (D_{player}), in term of head count, then applying the percentage of players at each average bet level (B_{min}), the number of players expected at each betting level can be predicted. After optimizing and considering overall house advantage, we can maximize profit for the next time period in a proactive manner.

Assumptions

Some assumptions are made in order to develop a solution for the problem described above. The hourly head count is assumed to be accurate, and representative of the demand of the table games to be played. Likewise, the clusters of minimum bets from the player database are assumed to represent the overall population of the gaming public that enters the casino. Since this data is taken, and being applied to, a local repeater market casino, the player's database is a representative sample of the overall population for the casino. While this may not hold true for a Strip property, few markets resemble the Las Vegas Strip in their makeup and more resemble repeater markets (Lucas & Kilby, 2012).

Scope

The scope of this study is a Las Vegas repeater market casino, located off the Strip in an affluent area, but with several competitors in close proximity and removed from the academic environment of UNLV. Data was collected over several months in 2011, and then used to build a model. The model is then calibrated using known demand up to August of 2012, adjusting the model as needed for any unexplained demand variations. Due to the proximity of high level competitors, the table games department of this property must continually adjust for the variable demand, and maximize the betting potential of all affluent players.

Justification

Two studies have looked into the development of table games revenue management systems, but neither have used the existing counts and player database to determine the plan (Chen, Tsai, & McCain, 2012; Peister, 2007). Both of these studies also utilized complex mathematical formulas that, generally, are a deterrent in a practical implementation of any system. This study looks to promote a practical system that is effective in the prediction of demand, efficient in the sense that it does not require additional labor to gather the necessary data, and simple so that that most casino managers would be able to implement it.

Chapter Two

Literature Review

While there is extensive and detailed research in revenue management systems, research in the application of these systems in gaming, especially in table games, is lacking.

History of Revenue Management

The term “revenue management” can apply to a broad range of decisions made by managers. The application dates back as long as man has made business decisions. The modern theory of revenue management dates back to the Airline Deregulation Act of 1978. This allowed airlines to suddenly charge a wide variety of prices and offer new flights without government oversight. The rapid influx of new customers, the sudden growth expansion, and the technological advancement of computers allowed American Airlines to develop a system of price discrimination that allowed it to compete with lower cost airlines. This system, over time, evolved into DINAMO in 1985 which is regarded as the first large scale revenue management model (Talluri & Van Ryzin, 2004). The impact of the new yield management systems was almost immediate. American Airlines started to dominate the market, and airlines that were once profitable were soon going into bankruptcy. PeopleExpress CEO, Donald Burr, expressed what happened to cause the company’s failure and bankruptcy:

> What changed was American's ability to do widespread Yield Management in every one of our markets… we didn't get our hands around Yield Management and automation issues. . . . In my view, that's what drives airline revenues

> today more than any other factor—more than service, more than planes, more than routes (Talluri & Van Ryzin, 2004,pp.9-10).

Revenue management utilizing sophisticated mathematical models and scientific data entrenched itself into the very fabric of the airline industry. The profitability and success of any airline was attributed to the success of the airline's revenue management models. Modern revenue management systems in the airlines are extraordinarily complex and deeply integrated into the industry. This has become an impediment to its usage in other industries, as it is largely viewed as an airline industry only product, and therefore deemed unnecessary in other areas. Another area of resistance to revenue management is that airlines do not usually rank highly in customer satisfaction in regards to pricing. The perceived association between revenue management and the dissatisfaction with pricing by customers causes other areas of concern; hospitality businesses are hesitant to potentially suffer the wrath of angry consumers due to pricing disparities. Southwest Airlines, a leader in customer satisfaction in the airline industry maintains a simple rates and pricing structure, so much so that the revenue management system they utilize does not need the multitude of variables to maximize revenues due to their limited range of pricing. The revenue management model Southwest Airlines utilizes is comparably simple when evaluated against competing airlines. This model, being easier to understand and implement, holds with Southwest's management theory of greater efficiency. Though their model is simple by comparison, it is still utilized and effective (Talluri & Van Ryzin, 2004).

Revenue management does require certain conditions for the process to be effective. The first is that there must be differences in customers. If all the customers of a business or industry are uniform then there is very little to maximize. The airlines utilize price differencing between business travelers and recreational travelers. This includes price differencing for days of the week, time of the year, even time of day. Hotels also take advantage of variations among customers. While there exists variation between businesses vs. recreational travelers, even within these broad groupings there are a significant number of variations based upon demographics. In areas where you have greater variations, there is a greater potential to exploit that variation and therefore a greater potential to maximize revenues. For example, in the airline industry the amount of money a business traveler is willing to spend versus a vacation traveler is highly significant. Hotel customers share this same trait (Talluri & Van Ryzin, 2004).

The next condition would be a variation in demand. If the amount of demand is known, accurately and consistently, then there is no need for a sophisticated tool to maximize the revenue from that demand. Airline travel is extraordinarily given to seasonal, sometimes daily, fluctuations. The greater the inability to forecast demand accurately, larger the risk of management not maximizing revenues. This is when the need for a sophisticated model or tool comes into play (Talluri & Van Ryzin, 2004).

Product perishability and fixed production is the third condition needed for an effective process. Airline flights are fixed in their production, because an airline is unable to add more seats to a flight once it has reached capacity. It would not be good for business to have passengers strapped to the wings, simply because the flight was full and the airline wanted to get more people onto a flight. Likewise, once the plane takes off, those seats on that flight can never be resold. There is no inventory to be stored to sell at a later date. Once a flight has begun, the opportunity is lost forever (Talluri & Van Ryzin, 2004).

Talluri and Van Ryzin(2004) also considers quality as an indicator of price. In situations where the price of a product is a signal of its quality, a revenue management system is unlikely to be effective. I hold this as untrue, unless there is homogeneity in the market that purchases the items. The price point of a Ritz Carlton hotel is definitively a signal of quality. People largely expect to pay more for a hotel room at Ritz Carlton, than say Motel 6, due to the quality of product and service that has become synonymous with the brand. However, Ritz Carlton does have a successful revenue management system (Garrow & Ferguson, 2009).

Expansion of Revenue Management

The success of revenue management in the airline industry did lead some to look for other areas in which to apply revenue management techniques. Hotels and resorts have taken to this trend and have applied revenue management as part of their overall process (Buckhiester, 2011). By applying revenue management to room rates, casino resorts have improved profits for the casino. It can be estimated that revenues increased by anywhere from five to ten percent depending on the guest segmentation of the individual hotel (Buckhiester, 2011). The benefits of a revenue management system can be significant, while the peril of not using revenue management for an industry is just as great. A small understatement in demand could result in a one percent decrease in revenues (Peister, 2007). One percent may not make the difference between opening and closing a hotel, but it could make a significant impact on the bottom line, and "leaving money on the table" is never an ideal situation for a business. Restaurants have started to look into revenue management techniques as well (Thompson, 2010). It is becoming increasingly clear that as long as the requirements are met, and the data collected, it is possible for an industry to adopt revenue management techniques.

Status of the Gaming Industry

The history of the gaming industry is one that goes back into antiquity. For the scope of this paper, though, we will limit our purview to modern gaming, or gaming since the passage of the Wide Open Gambling Law in Nevada in 1931. It is generally perceived that the passage of this law was a turning point for Nevada, and gaming in general (Kilby, Fox & Lucas, 2005). During 1931 - 1977, gaming in the United States was largely constrained to the State of Nevada. In 1976, New Jersey voters passed a referendum to allow gambling in Atlantic City, and gambling in Atlantic City was legalized in 1977. Since then, tribal casinos have grown in popularity, and there has been a rapid expansion of commercial gambling to almost all fifty states. In 2008, states that did not have either a tribal or commercial casino numbered only seventeen, including Hawaii and Utah, which have no gaming at all (American Gaming Association, 2009). In 2012, this number declined to fifteen (American Gaming Association, 2012). Within the last decade, gaming has expanded globally. The amount of competition for gaming revenue has never been fiercer. Add to the increased competition the fact that the U.S. economy is still hurting from the economic downturn it is no surprise that the domestic gaming volumes are down (Tuttle, 2010).

The economic recession of 2008 did not seem different from other recessions the gaming industry has weathered in the past. This recession, however, is different from previous economic downturns and the impact on gaming is significant. According to an announcement by the AGA, the gaming industry was down 5.5% in 2010 (Tuttle, 2010). In addition to this, revenue was down in eight of the 12 states with gaming (Tuttle, 2010). Gross gaming revenue in Nevada fell just short of $13 billion in 2007 (American Gaming Association, 2009) and dropped to $10.7 billion in 2012 (American Gaming Association, 2012). Most operators continue to brace for a slow recovery, while faced with increased competition due to the proliferation of gaming throughout the United States, Macau, Singapore, and other Asian jurisdictions. Moreover, the prospect of online gaming adds another level of competition. As a result, casinos continue to explore opportunities to reduce costs and increase revenues. Revenue management offers the opportunity to maximize revenue and increase profits for casinos.

Status of Table Games

Table games departments are historically the heart of the casino, as casinos began their long history with table games (Lucas & Kilby, 2012). Despite this fact, the number of table games has been in a steady decline since 2000 (see Figure 1) (Schwartz, 2013a). With a small spike in the number of table games largely due to new casino openings, the table games department of the casino is under increased pressure to produce.

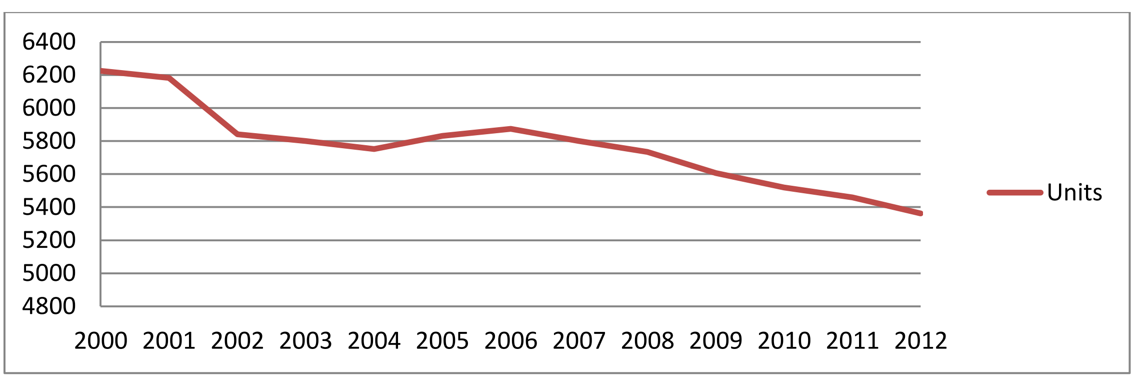

Figure 1. Decline in table games over a 12-year period.

There is a justifiable reason for the pressure on table games departments. The hold percentage of table games over the past several years is in a steady decline. This is evident from Figure 2, in which each line represents the hold percentage per game type in the state of Nevada per year (Schwartz, 2013b).

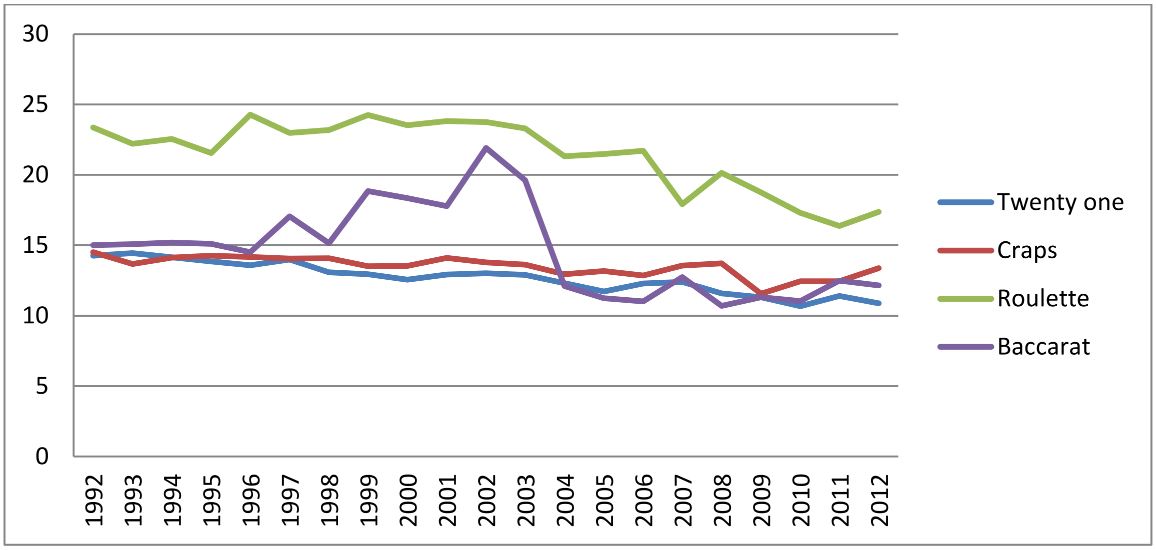

Figure 2. Annual statewide hold percentages; 1992-2012.

The negative downward trend in hold percentage adds pressure as casinos look to maximize revenue per square foot. This is what makes the need for a revenue management system in table games a critical issue.

To determine if table games fit the criteria for a revenue management system, we need to look at certain aspects of the games. The first, differencing of customers, is well established. Caesar's Entertainment revolutionized the industry with the ability to segment customers along the differencing factors (Kuyumcu, 2002). Variations in demand is evident through observation over a period. The limited demand and perishability of the product is inherent to the nature of a game. Casinos simply cannot add new tables or double book a seat to satisfy excessive demand; likewise, the perishability of the "product", e.g., one poker hand, one dice roll, or one spin of a roulette wheel is obvious. Once the event happens, there is no opportunity to recover the bet that was not placed (Peister, 2007). Since table games satisfy the necessary criteria, it is possible to apply revenue management techniques to the department.

Revenue Management in Table Games

Having established that a table games department meets the necessary criteria for a revenue management system, looking at the current systems in place, or those theoretically proposed, will help develop the system for this thesis.

Current Systems

The current method of revenue management in table games is an intuitive method employed by casino shift managers based upon years of experience. This method requires the Shift Manager to "know" the trends in business from an intuitive standpoint, and is a very reactive system. As volume or demand increases the Shift Manager opens new tables to spread players out or raise table minimums. Since table games generate revenue through a house advantage built into the game itself on each wager placed, maximizing the wagers placed in an hour is a critical component of any revenue management system for table games (Peister, 2007). Shift managers do not have the tools to accurately forecast demand; therefore, they are constantly at risk of either underestimating demand, therefore leaving "money on the table", or overestimating demand and wasting labor. The current system in a casino is highly dependent on the abilities and experience of the Shift Manager, which makes that one person, and the person's experience, very critical to the success or failure of the table games department. If that person retires, then a new Shift Manager needs either comparable experience or extensive training. Fortunately, there is literature on possible revenue management systems for table games.

Systems From Literature

There are two systems currently in the literature. Looking at these systems will allow one to determine what has already been developed for a table games revenue management system, and identifying their weaknesses provides an opportunity to develop a better system.

Survival analysis.

Peister (2007) published a revenue management system applying survival analysis. In his paper, he established the win per available seat hour (WPASH), and looked at maximizing casino win per seat hour. Utilizing processes similar to ones used by Casino Shift Managers, by manipulating the table minimums and number of open games, Peister (2007) created a distribution that sacrificed a few seats to increase the number of hands dealt at a table, while maximizing the casino win. He also identified a major data issue for any potential revenue manager; the actual demand is censored, when demand exceeds capacity there is no way to know how many players the casino "loses" due to an inability to find a seat. Peister (2007) applied a Cox survival regression to predict the survival of each seat per hour, i.e., the likelihood of a seat staying vacant throughout the entire hour. The regression model calculates this rate from equation (3):

$$h(t) = e^{(\beta_0+\beta_1 x_1+\beta_2 x_2 \dots \beta_p)} \quad (3)$$

Where *h(t)* is the hazard rate and β_0, β_1, ..., β_p are the unknown regression coefficients.

Since an analyst would have to evaluate each regression coefficient as an exponential value to determine the effect on survival, an analysis with several variables could be a lengthy and complicated process to determine the effect of each coefficient. Either way, the mathematical calculations are complicated and difficult to one without extensive mathematical and statistical training. Peister (2007) acknowledges this weakness of the model as well. One of the primary reasons for the use of the Cox Regression model is due to an unknown underlying distribution of the players (Peister, 2007).

Croston's method.

Chen, Tsai, and McCain (2012) looked to alter the landscape by measuring theoretical win rather than gross win. They sought to apply Croston's method that is able to forecast intermittent demand, since it is a simpler process to interpret. The authors also separated themselves from Peister by comparing their simulated results to actual revenue numbers from a casino. The authors used two equations (Equations 4&5) to forecast demand size Z_j and arrival times P_j (Chen et al., 2012):

$$Z_j = (1 - \alpha)Z_{j-1} + \alpha Y_j^* \tag{4}$$

$$P_j = (1 - \alpha)P_{j-1} + \alpha Q_j \tag{5}$$

They then estimate game demand at any given hour through a ratio of the two equations. Once demand is determined, a maximization equation is then applied to determine the maximum house advantage for the given demand by adjusting the spots per table, minimum bet, average wager, and table limit. This develops the table-opening plan for the shift manager, based upon the forecasted demand, for the maximization of house advantage. In their simulated data, the casino could have potentially brought in more than sixteen thousand dollars in theoretical incremental revenue for the blackjack tables on a given day. This would represent a considerable increase in available revenue (Chen et al., 2012).

Like Peister (2007), the authors' study has some weaknesses. Primarily, in their simulations, they assumed uniform distribution of betting between table minimum and table maximum (Chen et al., 2012). This is almost never the case; in fact, most Shift Managers would look upon results based on this assumption as highly suspicious. Their method also requires extensive data collection and is labor intensive. Even though this is a simpler method, this weakness still leaves room for improvement in a table games revenue management system.

Alternative Integration of Casinos Into Revenue Management

There have been other studies on incorporating revenue management in the gaming business, but most of these surround the hotel's revenue management system. This can still have a positive effect in not only the hotel's revenue numbers, but also the casino's as well (Chen et al., 2012) . Caesar's Entertainment currently takes into account player gaming history when offering room rates, therefore ensuring that the highest theoretical win gamblers are staying in the hotel. This improved their gaming win per room by approximately fifteen percent (Chen et al., 2012) . Through the data mining process, casino resorts have started to develop a better understanding of the guests to the property, and therefore have started offering more discriminating room rates based on their value as a gambler (Hendler & Hendler, 2004) . This can lead to a better development of a revenue management system for table games.

Chapter 3

Methodology

The purpose of this study is to develop an operationally efficient table games revenue management system; we must keep that goal in mind throughout. The methodology of developing a revenue management system for table games is a multi-step process. It begins with data collection and determining the best course of action for the analysis. Each step of the process adds more information to the overall system to reduce the amount of intuitive guesswork needed by the operating casino shift manager.

Research Questions

The analysis of the data begins with the analysis of demand. Hourly demand data is inherently a time series collection. Therefore, a time series analysis would be a logical plan for forecasting demand. However, after looking at the data, the realization that the miscellaneous variables to produce a reliable enough prediction model through time series analysis would be cumbersome and limited in scope. This would not suit the needs of creating an operationally efficient model, which would require the ability to update quickly and with flexibility. Additionally, time series regression requires data to be consecutive. This either requires the casino to start tracking hourly head counts, or have a large block of consecutive hourly head counts in a recent time period. Since these options may not be available at all properties, an alternative method was sought out.

A Bayesian approach allows one to utilize expert opinion and prior knowledge of a system, and is quickly and easily adaptable by using historical data and prior information to predict the demand for the next time period and each subsequent observation, updating the model and prediction using the next set of observed data

(Bolstad, 2004). This allows for a very flexible model that would adopt itself based on recent observations.

Procedure

Scope

The basis for the demand data is hourly head count data from a repeater market Las Vegas casino located in an affluent suburban neighborhood. The past several years of hourly head count data was collected; and since it was broken out by game type, it was summed to determine the complete hourly head count demand. Additionally, player betting information was pulled from the player tracking system. Gaps in the hourly demand data was used to portion the data into segments to develop the model.

Assumptions

In Bayesian analysis, the main assumptions reside with the player database betting information. It is assumed that the player database is representative of the population of bettors at the property as a whole. Since this is a repeater market property, the player database is extensive and the assumption of it being representative of the population is reasonable. The assumption with the demand data is that it is accurate. Since the data is gathered by observation and physical head counts, there inherently lies some error. Likewise, the head counts only occur once each hour and are not constantly tracked. In other words, if a player is betting at the time of the head count, the player is included in the count. If there is high turnover of the seat, however, this is not represented.

Forecasting Demand

Since demand is strictly positive, a two-parameter gamma distribution is fitted to the demand distributions. A two-parameter gamma distribution is composed of two independent parameters shape (α) and rate (λ); since the exact joint posterior distribution of (α, λ) is intractable, Lindley's approximation is used (Pradhan & Kundu, 2011).

The equations for the Lindley approximation (Equations. 6 & 7) use the method of maximum likelihood to estimate both α and λ based on the current observed data, in addition to the α and λ from the prior data's distribution of parameters (Pradhan & Kundu, 2011). Once a suitable segment of prior data was elicited, it was sectioned to determine the distribution of both the α and λ for each day part of the demand data. The posterior parameters for the current period are used as prior parameters for the next time period. As more observed data is gathered, the process is repeated to further refine and develop the model.

$$\widehat{\alpha_B} = \hat{\alpha} + \frac{1}{2n(\hat{\alpha}\psi'(\hat{\alpha})-1)^2}\left[-\psi''(\hat{\alpha})\hat{\alpha}^2 + \psi'(\hat{\alpha})\hat{\alpha} - 2\right] + \frac{a+c-2-d\hat{\alpha}-b\hat{\lambda}}{n(\hat{\alpha}\psi'(\widehat{\alpha})-1)} \tag{6}$$

$$\hat{\lambda} = \hat{\lambda} + \frac{\hat{\alpha}\hat{\lambda}}{2n(\hat{\alpha}\psi'(\hat{\alpha})-1)^2}\left[-\psi''(\hat{\alpha}) + 2\left(\psi'(\hat{\alpha})\right)^2 - \frac{3\psi'(\hat{\alpha})}{\hat{\alpha}}\right] + \frac{\hat{\lambda}}{n(\hat{\alpha}\psi'(\hat{\alpha})-1)}\left(\frac{c-1}{\hat{\alpha}} - d\right) + \frac{\hat{\lambda}^2\psi'(\hat{\alpha})}{n(\hat{\alpha}\psi'(\hat{\alpha})-1)}\left(\frac{a-1}{\hat{\lambda}} - b\right) \tag{7}$$

Once the Bayes estimates are calculated, a Poisson distribution updating is used to update the estimate to a new posterior distribution (See Appendix A). This is done in R with the Bolstad package. Once the final posterior is calculated given the current data, the Laplaces Demon package for R is used to calculate the highest posterior density region, and provide the 99% credible set. Demand for each game type will be determined based on historical usage of each game type.

Optimization

Once the credible set for the next time period is calculated, the appropriate table minimums and number of games to open is then configured through the optimization algorithm in Excel to maximize the revenue for the time period. The optimization algorithm will seek to maximize either table minimums, or number of games, depending on the situation. Since all metrics used in table games to determine business volume (e.g. Theoretical hold, hold, drop) can be figured as a function of both the number of games, or players at each game, and the minimum betting level, the objective function to be optimized will be a function of table minimums and number of tables.

Chapter 4

Results

The purpose of this study is to develop an operationally efficient revenue management system for table games, which is to say that we ultimately wish to reduce the amount of guesswork the casino shift manager has to do when determining the upcoming time period's number of players. To this end, the data was evaluated first to determine the forecast for demand, and then the player base was segmented via cluster analysis. The entirety of the data analysis was done in the R programming environment for statistical analysis (R Core Team, 2012)

Demand

The demand data analysis had to be done in several parts. The first part involved exploratory data analysis of the first set of data, and establishing the informative prior for the Bayesian analysis. The second step of the analysis involved updating the Bayesian Prior with the observed data for the next period. The highest posterior density for the posterior distribution is then calculated, which in turn yields the required credible set. This credible set is used to estimate the number of each betting level of player for the day part.

Data-Mining

Plotting the density of the demand, represented by the variable "ttlhc", showed that the number of players has a mixture distribution (see Figure 3).

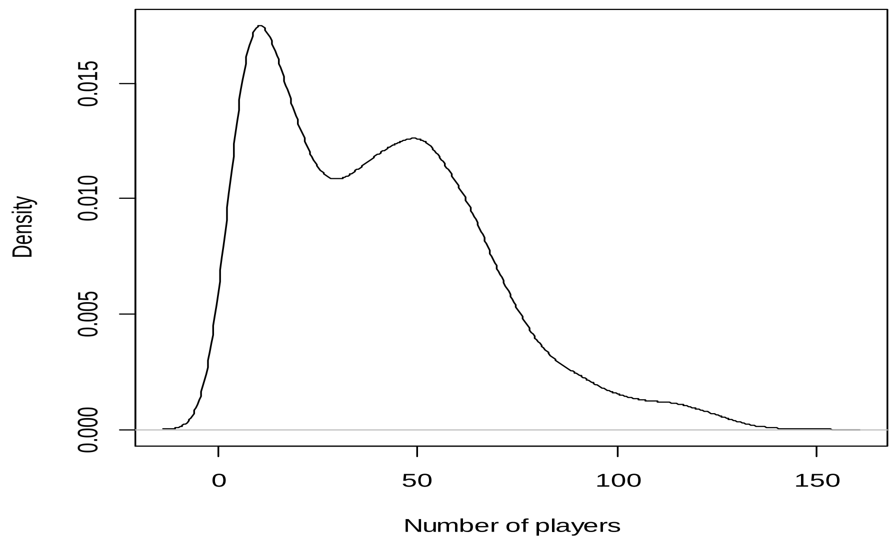

Figure 3. Mixture demand distribution for total head count of players.

The initial data analysis showed the presence of five sub-populations in the mixture distribution. The count data was separated initially into weekends (Fri-Sun), and weekdays (Mon-Thurs). This was then further split into different day parts of the demand variable. Each day part was given a unique name (see Table 1).

Table 1:

List of Variable Names and Representative Day Parts.

Variable name in R	Representing day part
wd1	Mon-Thurs(2AM-2PM)
wd2	Mon-Thurs(2PM-2AM)
we1	Fri-Sun(5AM-8AM)
we2	Fri-Sun(8AM-12PM)
we3	Fri-Sun(12PM-5AM)

The day part selection was done by trimming the selection until smooth density curves were provided (see Figure 4).

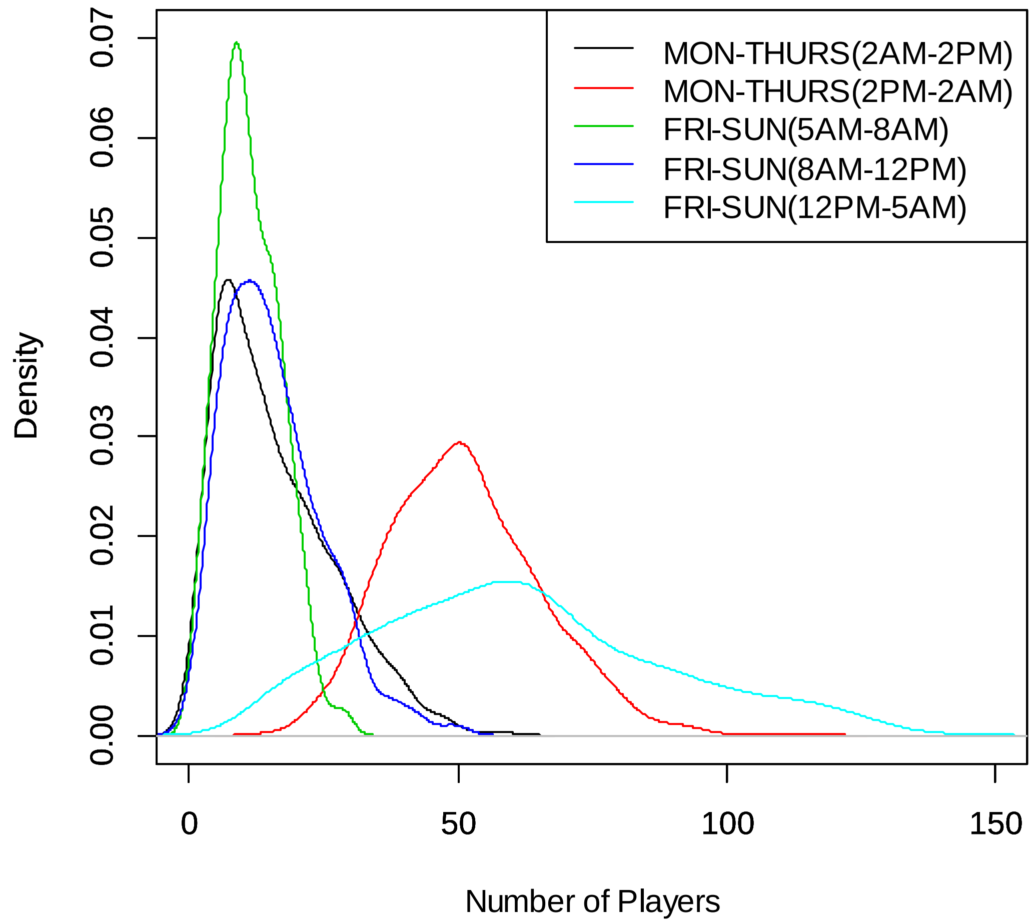

Figure 4. Density of player counts by day parts.

Establishing the Prior Distribution

Each sub-population was then sampled and the shape and rate from each was estimated and fitted with a gamma distribution. A gamma distribution was selected since the demand curve is always positive, and cannot be negative by the definition of demand. The underlying principle of the sampling method is similar to frequentist statistical measurement. The individual samples must trend toward the overall curve of the whole. By fitting each sub series of the initial demand data, we are able to estimate the overall gamma distribution of the sample, as the parts must comprise the whole. Once the alpha and lambda estimates are stored, the MLE's of each distribution can then be calculated. The MLE is calculated through the rGammaGamma package with sampling of 10,000 and a tolerance of .001. Once the MLE's were saved, we were then able to utilize equations 1 and 2 to compute the Bayes estimates of the distributions. This yielded an informative prior, which will be updated given the next set of data. Appendix B has the R code used to establish the prior.

Updating the Prior

Once the informative prior for each day part is calculated, the next step is to update the prior given the new data. The formula for updating a Bayesian distribution is simply:

$$Posterior \ \propto Prior\ X\ likelihood$$

The prior in this case is updated using the Bolstad package for R (Bolstad, 2004). Updating each prior gives the posterior distribution given the observed data, and the posterior is adjusted based on the likelihood that the observed data came from the prior distribution. This process is then repeated for the following set of observations. Each segment is updated with two sets of observed data to help refine the model. Due to the large number of new observations, the posterior distribution is dominated by the likelihood of the observed data, something that would not happen if only one or two observations were made.

Highest Posterior Density Region and Credible Set

Once the joint posterior is calculated, it is then possible to find the highest posterior density region, which will give the credible set for the distribution. The primary difference between a confidence interval and a credible set is that a confidence interval provides frequentist coverage before the data are collected, whereas the credible set is based on the observed data (Bolstad, 2004). Since this is a 99% credible interval, the data is showing that there is 99% probability that the true random parameter lies within the interval, rather than a 99% of other calculated intervals will include the constant parameter (Bolstad, 2004). The credible sets for the demand data is as follows (see Table 2).

Table 2

Credible sets for each player day parts

Day Part	Lower limit	Upper limit
• MON-THURS(2AM-2PM)	13.6743	13.9539
• MON-THURS(2PM-2AM)	44.9798	45.5202
• FRI-SUN(5AM-8AM)	9.9516	10.4916
• FRI-SUN(8AM-12PM)	13.6652	14.2490
• FRI-SUN(12PM-5AM)	54.1618	54.7625

The expected demand for each day part can then be used in conjunction with the player distribution to improve the accuracy of the Shift Manager estimates of how many games to open and how to set the table minimums.

Player Distribution

Now that the expected demand for the next time period is calculated, the Shift Manager will need to estimate the number of players coming in at certain thresholds. This is done by segmenting the player distribution by average bet. The casino player database was taken from three years' worth of player betting data with over 58,000 player betting information in it. While not all players use player's cards while playing table games, it is a reasonable assumption that such a robust sample is highly representative of the population of players for this repeater market casino. The data was analyzed by k-means clustering in the R software package. The initial evaluation of the minimum number of clusters to maximize the effective change in the sum of squares between the distances of the clusters is six (see Figure 5).

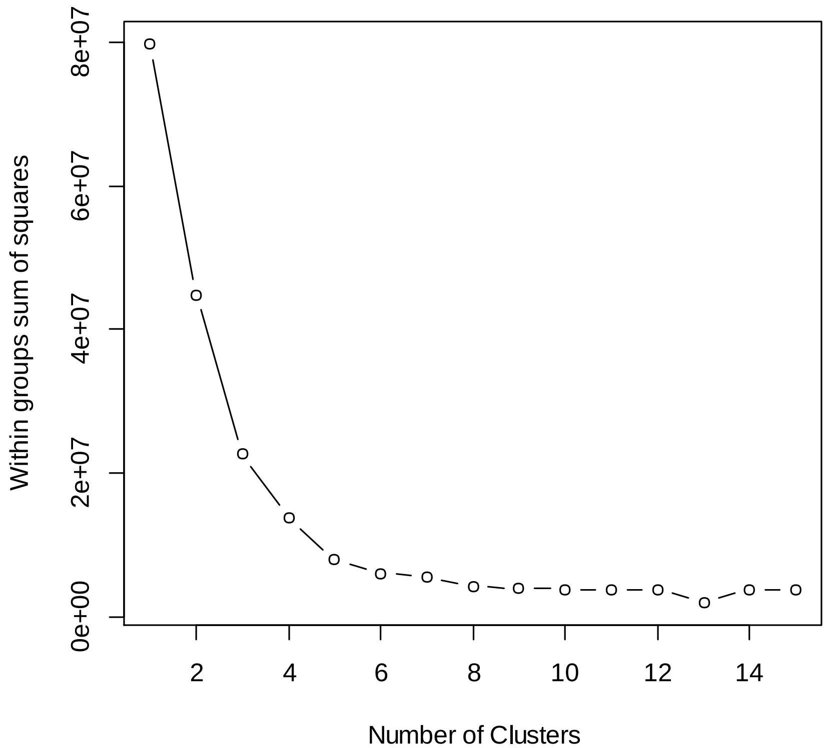

Figure 5. Cluster plot to determine the minimum number of clusters.

After assigning each player to a cluster, the mean values for each cluster is calculated. As expected, from Pareto's law, the majority of players fall into the lowest value cluster. This cluster has a mean value of $9.88 (see Table 3). Once thing to consider is that the casino typically offers $5 minimum tables, yet the average bet is almost $10. This potentially supports anecdotal theory by casino managers that a players' average bet is approximate 1.5 to two times the table minimum. Since the centers for the k-means is not preset, the center comes from the data itself, this anecdotal theory is potentially supported through this.

Table 3

Player clusters with percentages

Cluster	Average $. bet	Percentage
1	228.79	0.56
2	649.68	0.09
3	69.87	3.58
4	2,700.42	0.01
5	9.88	74.93
6	25.00	20.83

Once the player betting data is segmented, the percentage of players in each player cluster is then calculated (see Table 3). Since this is representative of the entire population of players for the casino, the assumption is that any random sample of sufficient size would have the same player distribution. Therefore, the percentages in each cluster can be applied to the expected number of players for each day part. This would give the Shift Manager the expected number of players for each betting level at each day part. The Shift Manager could then determine if tables would need to be opened to accommodate future players, and at what level the table minimums should be set.

Chapter 5

Conclusion

Introduction

The increase in competition from internal and external gaming markets has created a pressure on the table games departments in Las Vegas to maintain the historic hold percentages of a time when Las Vegas casinos was more monopolistic. Other revenue centers within the casino have implemented technology to increase performance. However, Table Games departments still utilize the intuition of Casino Shift Managers to maximize revenues, and set the number of open games and table minimums. While some Shift Managers are very experienced and there is no substitute for their level of experience, the system is still very reactionary and fragile. By utilizing a revenue management system, Shift Managers can reduce the uncertainty of their predictive estimates, and potentially increase table games revenue by optimizing the table minimums and number of open games. This study sought to answer the question: Could a table games revenue management system using Bayesian techniques be developed?

Findings

The system laid out in this paper takes into account the short-term fluctuations of demand, while not having to remake an entire theoretical model. The Bayesian updating of the distribution can occur with as little as one observation, while always estimating the next time period's demand estimate. Over time, the model will automatically adjust for unpredictable variables, such as economic changes, as this would be reflected in the demand and adjusted for without the need to rebuild the model. By allowing the distribution parameters to be random, the model is not too rigid to ignore unexpected events.

Theoretical Implications

Building on the works of Peister (2007) and Chen, Tsai, and McCain (2012), we can see that a revenue management system does help to improve table games revenue. The Bayesian model put forth in this paper will help improve this by offering an adaptive model that does not need detailed player information or rebuilding of the model if environment variables change. Likewise, since Bayesian techniques work more in tune with intuitive thought processes, it is easier for operators to grasp the concepts put forth.

Managerial Implications

Operators need to be able to reliably estimate the incoming volume to be able to adequately schedule for incoming business. By utilizing a revenue management system, operators are able to use the massive amount of data collected on a daily basis to reduce the uncertainty in the determination of the number of players for the Table Games department. In turn, this could potentially increase the revenue of the table games department over time as well.

Future Research

The fact that the minimum cluster was almost twice the amount of the current base table minimum is an area for future research. The effect of re-establishing what the bare minimum bet should be in the repeater market casino, and its effect on player satisfaction is an area that could potentially allow operators to challenge current long-standing beliefs.

The revenue management model could also be strengthened by using observational data gathered by observing players as they enter the property and noting what attracts certain players to certain games. This could be incorporated into the database so the model could then classify players to each game type.

The demand data of this research is limited, as it does not contain information in unconstrained demand. That is to say, if the demand is greater than the number of tables available in the casino, there is no way to record the amount of play that was lost due to unavailable seats. Similarly, if the table minimums were too great for a player, there is no record of players who showed interest in the game, but were priced out. The demand data is also limited in the sense of the time period observed. If a seat was observed open at the count, subsequently filled, and vacated again before the next count, the observed count would be zero. While for the purposes of this study, this limitation was negligible, this limitation could be removed through observational behavioral data gathered about players.

Additionally, the applications of this research can extend far beyond the scope of casinos, specifically table games departments. Since Bayesian techniques are generalizable, and able to update any given distribution. Demand for any industry is consistently based on an underlying gamma distribution. One potential area where these calculations can be applied to in future research are restaurants.

Restaurants could use the prediction of demand with a cluster analysis of group size and average check by group. This would require tracking of dining groups, this would allow a restaurant to maximize floor layout to increase the number of four tops or two tops based to maximize revenue and table turn around. The model proposed in this paper could be adapted to apply to bolster the current systems of revenue management in hotels as well. Comparative studies testing frequentist regression models and Bayesian predictive models would be able to highlight strengths and weaknesses of each group.

Conclusion

By using Bayesian techniques, it is possible to develop a revenue management system that would reduce uncertainty in the Shift Manager's estimations of future business. By reducing this uncertainty, the department can maximize revenues in a proactive manner, and the system can be used as a tool to assist casino managers in the better management of the table games department.

Appendix A

The demand information is assumed to follow a Poisson distribution with mean of θ, since demand will be discrete. θ will follow the Gamma distribution $g(\theta;\alpha,\lambda)$. $g^*(\theta|\underline{\boldsymbol{x}})$ then is a gamma distribution. This distribution has α_0 and λ_0 that was estimated from past data via Lindley's approximation where θ was observed to follow the gamma distribution. This was then updated via the Bolstad package in R via a Poisson likelihood. Since Gamma and Poisson are conjugate distributions, applying the Poisson likelihood to the Gamma distribution yields the Gamma posterior $g^*(\theta)$. The HPD of which provides the credible set for the estimation of demand.

Appendix B

The initial data was loaded into R as three different data sets. The initial data set provided the initial estimate of the distributions and the information for Lindley's approximation. This data, after being loaded, and separated into distinct distributions, was run through a function to segment the data, calculate the α and λ lambda of each segment via equations 8 and 9.

$$\boldsymbol{\alpha} = \frac{\bar{x}^2}{\delta^2} \tag{8}$$

$$\boldsymbol{\lambda} = \frac{\bar{x}}{\delta^2} \tag{9}$$

The R code was compiled into a function, *b.post*, and applied to each distinct distribution providing the prior gamma parameters

```
b.post<-function(x,y){

        col.names<-c("shape","rate")
        post.1<-as.data.frame(matrix(1,nrow=1,ncol=2,dimnames=list("estimate",col.names)))
        wd1break<-data.frame(1:(ceiling(nrow(weekdays_1)/10)))
        wd2break<-data.frame(1:(ceiling(nrow(weekdays_2)/10)))
        we1break<-data.frame(1:(ceiling(nrow(weekends_1)/10)))
        we2break<-data.frame(1:(ceiling(nrow(weekends_2)/10)))
        we3break<-data.frame(1:(ceiling(nrow(weekends_3)/10)))

        #define prior
        if (x==1){
              if (y==1) {z.i<-weekdays_1.t1$ttlhc;x.i<-weekdays_1}else
              if(y==2){z.i<-weekdays_1.t2$ttlhc;x.i<-
    as.data.frame(rgamma(nrow(weekdays_1),wd1$alpha.bayes,rate=wd1$lambda.bayes.rate))}


              for (i in (1:ceiling(nrow(x.i)/10))){
                   wd1break$mean[i]<-mean(x.i[((i*10)-9):(i*10),])
                   wd1break$var[i]<-var(x.i[((i*10)-9):(i*10),])}
              wd1break<-wd1break[-1]
              wd1break<-na.omit(wd1break)
              wd1break$lambda<-wd1break$mean/wd1break$var
              wd1break$alpha<-(wd1break$mean)^2/wd1break$var
              fit.wd1.lambda<-fitdistr(wd1break$lambda,"gamma")
```

```
fit.wd1.alpha<-fitdistr(wd1break$alpha,"gamma")
a<-fit.wd1.lambda$estimate[1]
c<-fit.wd1.alpha$estimate[1]
d<-1/(fit.wd1.alpha$estimate[2])
b<-1/(fit.wd1.lambda$estimate[2])
ahat<-rGammaGamma::gammaMLE(z.i,niter=10000,tol=.001)
```

This process is repeated for each distinct distribution. The function then establishes the other necessary variables and runs Lindley's approximation.

```
#establish all other variables
    n<-nrow(x.i)
    alpha.hat1<-ahat[1]
    lambda.hat1<-1/ahat[2]
    psi.prime1<-trigamma(alpha.hat1)
    psi.dblprime1<-psigamma(alpha.hat1,deriv=2)

#calculate Bayes Estimates
    alpha.hat.b<-alpha.hat1+
            (1/(2*n*(alpha.hat1*psi.prime1-1)^2))*(-
psi.dblprime1*alpha.hat1^2+psi.prime1*alpha.hat1-2)+
            (a+c-2-d*alpha.hat1-b*lambda.hat1)/(n*(alpha.hat1*psi.prime1-1))
    lambda.hat.b<-lambda.hat1+
            ((alpha.hat1*lambda.hat1)/(2*n*(alpha.hat1*psi.prime1-1)^2))*(-
psi.dblprime1+2*(psi.prime1)^2-((3*psi.prime1)/alpha.hat1))+
            (lambda.hat1/(n*(alpha.hat1*psi.prime1-1)))*((c-1)/alpha.hat1-d)+
            (((lambda.hat1)^2*psi.prime1)/(n*(alpha.hat1*psi.prime1-1)))*((a-1)/lambda.hat1-b)


#save Bayes Estimates
    if (x==1){wd1$alpha.bayes<<-alpha.hat.b
            wd1$lambda.bayes.rate<<-lambda.hat.b}else
    if (x==2){wd2$alpha.bayes<<-alpha.hat.b
            wd2$lambda.bayes.rate<<-lambda.hat.b}else
    if (x==3){we1$alpha.bayes<<-alpha.hat.b
            we1$lambda.bayes.rate<<-lambda.hat.b}else
    if (x==4){we2$alpha.bayes<<-alpha.hat.b
            we2$lambda.bayes.rate<<-lambda.hat.b}else
    if (x==5){we3$alpha.bayes<<-alpha.hat.b
            we3$lambda.bayes.rate<<-lambda.hat.b}
}
```

The prior is now ready to be updated with new data and calculate the HPD to determine the credible set.

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