Nonlinear and adaptive control systems for underwater and air vehicles

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NONLINEAR AND ADAPTIVE CONTROL SYSTEMS FOR UNDERWATER
AND AIR VEHICLES

by

Pradeep Nambisan
Bachelor of Engineering
Visvesvaraya Technological University, India
2004

A thesis submitted in partial fulfillment
of the requirements for the

Master of Science Degree in Electrical Engineering
Department of Electrical and Computer Engineering
Howard R. Hughes College of Engineering

Graduate College
University of Nevada, Las Vegas
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ABSTRACT

Nonlinear and Adaptive Control Systems for Underwater and Air Vehicles

by

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This thesis considers the design of nonlinear and adaptive control systems for the control of submersibles as well as aircraft. In the first part of the thesis, control of submersibles using bow and stern hydroplanes is considered, and (i) a robust output feedback nonlinear control law using modeling error compensation, (ii) a nonlinear adaptive state feedback law using SDU decomposition; and (iii) an output feedback linear adaptive law for the dive-plane maneuvering are derived. The robust nonlinear controller with high-gain observer is designed for depth and pitch angle tracking along constant trajectories in the presence of parametric uncertainties and disturbances due to the sea waves. Next, the adaptive backstepping controller is developed to accomplish depth and pitch angle tracking. SDU decomposition of the high-frequency gain matrix is done to prevent singularity in the control law. For this design, one...
needs to know the sign of the two minors of the input matrix, but no other knowledge of the submarine parameters is required. Finally, a Model Reference Adaptive Control (MRAC) law using output feedback is derived for the linear model of the submersible.

In the second part of the thesis (i) an adaptive Variable Structure flight Control (VSC) system and (ii) an adaptive flight control system for the roll-coupled maneuvers of aircraft using the aileron, rudder and elevator inputs are derived. Again, the SDU decomposition of the high frequency gain matrix is used for the derivation of singularity free control laws. Simulations performed for the underwater and the air vehicles using Matlab and Simulink show that in the closed-loop system, desired trajectory tracking is accomplished using each of the control systems.
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CHAPTER 1

INTRODUCTION

Control of multi-input, multi-output (MIMO) underwater and air vehicles is an important research area with practical significance. Autonomous underwater and air vehicles use multiple control surfaces for performing efficient maneuvers. However, unlike single-input, single-output (SISO) systems, dynamic interactions between systems inputs and outputs in MIMO systems pose challenging design problems. Besides this, it is essential to deal with parametric uncertainties and external disturbances acting on the vehicles in order to obtain a robust autopilot. A key technical issue in the design of adaptive control laws for multi-input, multi-output (MIMO) uncertain systems is the dynamic interaction between the system inputs and outputs. Unlike the single input systems, adaptive feedback linearizing control laws for MIMO systems require online tuning of the input matrix [16, 17]. But the estimated matrix can become singular during parameter adaptation, causing unbounded control input.

1.1 Related Research Work

Researchers have made considerable effort in designing autopilots for underwater vehicles. In the past, variety of design techniques based on optimal control, Lyapunov-based...
punov stability theory, adaptive control and sliding mode control have been attempted ([4,7,9,10,11,12,14,15,16,18]). For submarine models, linear [2], optimal [5] and sliding mode controllers [3] have been developed. For the submarine model of Babaoglu (1988) equipped with bow and stern planes, a multi-model gain scheduled optimal controller has been proposed (Dumlu and Istefanopulos, 1995). But for this design the vehicle's parameters are assumed to be known. More recently, Demirci and Kerestecioglu (2003) have proposed a reconfigurable sliding mode controller for the same model, in which an observer is used to estimate the disturbance force for compensation. However, for the synthesis of the controller, the disturbance input is derived by the differentiation of the state estimation error. But this operation of differentiation is not desirable. The dive-plane controller design using single control input is simpler because unlike multiple-input systems, the question of input-output dynamic interaction does not arise. An adaptive path control system has been designed in Do et al (1997), which computes the control forces and moments, but the required control surface deflections remain to be found. The computation of surface deflections in the presence of parametric uncertainties of the input matrix is not simple. Thus it is of interest to develop multi-variable adaptive control systems for the control of underwater vehicles, which do not encounter the singularity in control law during the adaptation process.

Considerable research work has also been done in developing autopilots for aircraft. Geometric nonlinear control theory has provided powerful tools for the systematic design of feedback control laws for complex nonlinear systems. For trajectory control
of nonlinear systems, the input-output and exact linearization techniques have been widely applied. The dynamical models of modern high-performance aircraft operating in a large flight envelope have significant nonlinearity. Apparently, flight controllers designed using linearized aircraft models cannot provide stability in the entire flight envelope.

The feedback linearization (dynamic input-output map inversion) technique has played an important role in the design of flight control systems for nonlinear maneuvers [24-31]. However, the application of this approach requires the complete knowledge of the system dynamics, since one must cancel the nonlinear functions appearing in the tracking error dynamics. But the assumption of complete knowledge of nonlinear aerodynamic characteristics of aircraft is unrealistic. Although, attempts have been made to analyze the robustness of inverse controllers and robust inverse flight controllers have been developed [32-34], the analysis of robustness is based on $\mu$-synthesis using linearization.

Considerable research has been done for designing nonlinear flight controllers in the presence of parametric uncertainties. Variable structure control (VSC) (sliding mode control) theory [35] has been applied for designing flight controllers for uncertain aircraft models [21], [36-38]. The variable structure (VS) control law is a discontinuous function of the state variables and for its design, the bounds on the uncertain functions are used. Unlike the VSC theory, nonlinear adaptive control methods do not require uncertainty bounds. Instead, an adaptive control system includes an adaptation mechanism for tuning the time-varying controller gains. In the past, a
variety of adaptive flight controllers have been developed [39-49]. Since the aircraft models have mismatched uncertainties, backstepping design techniques [19] have been applied to derive stable adaptive control systems [39-41], [45]. Backstepping design method is recursive in nature and the design is completed in several steps, which depends on the relative degree of the controlled output variables. For aircraft models with unknown functions, adaptive laws have been developed using neural networks for function approximation [40-48]. In this approach, unknown functions must be estimated for compensation. An adaptive-critic-based neural architecture has been presented in [49] for the design of an optimal flight controller. Recently, research has focused on adaptive design with state and control constraints [40], [45].

A key technical issue in the design of adaptive control laws for multi-input, multi-output (MIMO) uncertain systems is the dynamic interaction between the system inputs and outputs. Unlike the single input systems, adaptive feedback linearizing control laws for MIMO systems require online tuning of the input matrix [21], [39]. But the estimated matrix can become singular during parameter adaptation, causing unbounded control input. Although, one can try to use parameter projection method [50], but this method requires the knowledge of the domain of the parameter space in which the estimated input matrix is nonsingular. Of course, one can design VSC laws for uncertain nonlinear aircraft models, but it requires certain restriction on the uncertainties in the input matrix [13].

To deal with large uncertainties in the input matrix of MIMO systems, adaptive control designs based on matrix decomposition [20] have been proposed [22, 23] For
a hypersonic aircraft model, Xu et al [47] have factorized the input matrix into the product of a known regressor matrix and a diagonal matrix, and developed an adaptation scheme for tuning the parameters of the diagonal matrix for control. For adaptive design, LDU, LDS and SDU decompositions of the high-frequency gain matrix have been found to be useful. Readers may refer to Tao [22] for an adaptive backstepping design for MIMO systems based on SDU decomposition for linear systems. Of course backstepping design method is equally applicable to nonlinear systems. Recently, an adaptive law for a nonlinear aeroelastic system with two control surfaces using a matrix decomposition has been developed [51]. Adaptive laws developed based on matrix decomposition are singularity free. As such, it is of interest to design control systems for uncertain MIMO underwater and air vehicle models using matrix decomposition.

1.2 Thesis Outline

In this thesis, the design of control systems for the dive plane maneuvering of submersibles as well as the roll-coupled maneuvers of aircraft is considered.

The first part of the thesis deals with the depth and pitch angle control of submersibles. For definiteness, we consider the submarine model of Demrici and Kerestecioglu (2003), which is equipped with bow and stern hydroplanes for the depth and pitch angle control. It is assumed that the system parameters are not known.

First, the design of a robust nonlinear control law for the dive-plane maneuvering of submersibles using bow and stern hydroplanes is considered. It is assumed that the parameters of the vehicle model are unknown and that disturbance forces are acting
on the vehicle. Based on nonlinear inversion, a control law is derived for the trajectory control of the pitch angle and the depth of the vehicle. For the compensation of the unmodeled dynamics and synthesis using output feedback, a high-gain observer [29] is designed to estimate the unknown functions and unmeasurable variables. In the closed-loop system, asymptotic trajectory control of the depth and pitch angle is accomplished.

Next, an adaptive control law is derived for the control of underwater vehicles equipped with bow and stern hydroplanes for the depth and pitch angle tracking. It is assumed that the system parameters are not known, and the vehicle experiences external disturbance forces. The design is based on a back-stepping design technique [19], [22], and unlike the design of [21], uses the SDU decomposition [20] of the high frequency gain matrix to avoid singularity in the control law. This is accomplished by the factorization of the control input matrix relating the input (hydroplane deflections) and outputs (depth and pitch angle) as a product of a symmetric positive definite, a unit upper triangular, and a diagonal matrix. However, for this design, one needs to know the sign of the two minors of the input matrix, but no other knowledge of the submarine parameters is required. In the closed-loop system, it is shown that the depth and pitch angle trajectories asymptotically track the reference trajectories.

Chapter 4 presents the design of a model reference adaptive control (MRAC) law for the dive plane control of multi-input, multi-output (MIMO) submersibles using output feedback. The chosen vehicle is equipped with two hydroplanes (bow and stern) for control. It is assumed that the system parameters including the high-
frequency gain matrix are not known and only the depth and pitch angle of the vehicle are measured for synthesis. For the trajectory control of the output (depth and pitch angle), an adaptive control law is designed using Lyapunov stability theory. In the closed-loop system, the vehicle asymptotically tracks the reference trajectories. Simulation results are presented which show that in the closed-loop system, the vehicle performs the desired maneuvers in the dive plane despite the uncertainties in the model parameters.

The second part of this thesis involves the control of roll-coupled maneuvers aircraft. Chapters 5 and 6 present the design of two nonlinear control laws ((i) an integrated adaptive variable structure control law and (ii) an adaptive control law) for the roll-coupled maneuvers of aircraft model based on the decomposition of the input matrix. It is assumed that the aircraft parameters including its input matrix are unknown. For the purpose of control, the roll angle (\( \phi \)), angle of attack (\( \alpha \)), and sideslip angle (\( \beta \)) are chosen as controlled output variables; and these are controlled using aileron (\( \delta a \)), elevator (\( \delta e \)) and rudder (\( \delta r \)). For the derivation of control systems, the SDU decomposition of the 3 \( \times \) 3 input matrix (which is the high-frequency gain matrix for the linearized system) is obtained [20], [22], [23]. In this factorized form, the input matrix is expressed as the product of a positive definite symmetric matrix, a diagonal matrix and an upper triangular matrix. For the derivation, it is assumed that only the signs of the leading principal minors are known. Based on a backstepping design technique, two control laws are developed. In chapter 5, an adaptive VSC system is designed, which uses adaptation in its first step of design,
followed by a VSC law in the second step of the backstepping procedure. Unlike the VSC design without matrix decomposition, the SDU decomposition permits the derivation of the control law for arbitrary perturbations in the input matrix. Finally, in Chapter 6, a second control system is designed, which uses adaptation in each step of derivation. Unlike the adaptive VSC law, the second flight controller (adaptive law) does not require the knowledge of certain bounds on the parameter uncertainties. In the closed-loop systems, it is shown that the output tracking error asymptotically tends to zero. Simulation results are presented, which show that both the adaptive VSC law and the adaptive control law accomplish precise control of $(\phi, \alpha, \beta)$ in spite of the uncertainties in the aerodynamic and inertia parameters of the aircraft.
CHAPTER 2

NONLINEAR ROBUST OUTPUT FEEDBACK CONTROL OF SUBMERSIBLES
VIA MODELING ERROR COMPENSATION

In this chapter, the design of a nonlinear robust dive-plane control system for multivariable submersibles equipped with bow and stern hydroplanes is considered. It is assumed that the vehicle's parameters and the hydrodynamic coefficients are not known, and that disturbance forces due to the sea wave are acting on the vehicle. For the design, the depth and pitch angle are chosen as output variables. Using nonlinear input-output (pitch angle and depth) map inversion, a robust nonlinear output feedback control law for the trajectory control of the pitch angle and depth is derived. For synthesizing the robust inverse control law, the unknown functions and unmeasurable variables are estimated using a high-gain observer.

The organization of the chapter is as follows. Section 2.1 presents the mathematical model of the submarine. The feedback linearization and control law are presented in Section 2.2. Simulation results and the summary are presented in Sections 2.3 and 2.4 respectively.
2.1 Mathematical Model

A schematic of the submarine model with bow and stern hydroplanes is shown in Figure 2.1. A complete derivation of the submarine model has been developed by Richmel in an NSRDC report. We consider here the dive-plane dynamics given in Dumlu and Istefanopulos (1995). The equations of motion along the z-axis and y-axis are given by

\[
\begin{align*}
\dot{w}(t) &= \frac{Z_w'U}{Lm_3} w(t) + \frac{1}{m_3} q(Z_\theta' + m'U) \dot{\theta}(t) \\
&\quad + \frac{Z_\theta' L}{m_3} \dot{\theta}(t) + \frac{Z_{\delta B} U^2}{m_3 L} \delta B(t) + \frac{Z_{\delta S} U^2}{m_3 L} \delta S(t) \\
&\quad + \frac{2}{\rho L^2 m_3} Z_{\text{wave}}(t) \\
\dot{\theta}(t) &= \frac{M_w'}{L I_2} \dot{w}(t) + \frac{M_\theta U}{L I_2} w(t) + \frac{M_{\delta B}'}{L I_2} \dot{\theta}(t) + \frac{M_{\delta S} U^2}{L I_2} \delta B(t) \\
&\quad + \frac{M_{\delta S} U^2}{L I_2} \delta S(t) + \frac{2mg (z_G - z_B)}{\rho L^2 I_2} \dot{\theta}(t) + \frac{M_{\text{wave}}(t)}{\rho L^2 I_2} 
\end{align*}
\]  

(2.1)

(2.2)

where \(w\) is the heave velocity, \(Q\) is the rotational velocity, \(h\) is the depth error, \(\theta\) is the pitch angle, and \(\delta B\) and \(\delta S\) are the hydroplane deflections in the bow and stern planes, respectively. In this model, we assume that the forward velocity \(U = U_0\) is 8.43 ft/sec. We point out that although we have taken the linear model, the present design approach is applicable to nonlinear dive-plane dynamics. Solving for \(\dot{w}\) and \(Q = \dot{\theta}\) from (2.1) and (2.2), one obtains

\[
\begin{pmatrix}
\dot{w} \\
\dot{Q}
\end{pmatrix} = A x_p + G \delta + F d
\]

(2.3)

where \(x_p = (w, Q, \theta)^T\), an \(R^{2 \times 3}\) matrix and the matrices \(G, F \in R^{2 \times 2}\) can be easily obtained using (2.2). The matrices \(A, G, F\) are assumed to be unknown to the
designer. Since we are interested in the depth control of the submarine, we include the depth \((h)\) as a state variable satisfying the nonlinear differential equation

\[ \dot{h} = wc\cos\theta - U_0\sin\theta \]

We have retained the nonlinearity in the \(\dot{h}\) for precise depth computation.

Defining the state vector \(x = (h, \theta, w, Q)^T \in \mathbb{R}^4\), and control input \(\delta = (\delta S, \delta B)^T\), the equations of motion and the output vector \(y = (h, \theta)^T \in \mathbb{R}^2\) are completely described by

\[
\dot{x} = f(x) + G_c\delta + F_d d \\
y = Cx
\]

(2.4)

where \(d = [Z_{\text{wave}}, M_{\text{wave}}]^T \in \mathbb{R}^2\) is the disturbance input vector, \(C = [I, O] \in \mathbb{R}^{2 \times 4}\), and \(I\) and \(O\) indicate identity and null matrices of appropriate dimensions.

The vector function \(f(x)\) is given by

\[
f(x) = \begin{bmatrix}
w\cos\theta - U_0\sin\theta \\
Q \\
A_x
\end{bmatrix}
\]

The matrices \(G_c\) and \(F_d\) are

\[
G_c = \begin{bmatrix}
O_{2 \times 2} \\
G
\end{bmatrix} \\
F_d = \begin{bmatrix}
O_{2 \times 2} \\
F
\end{bmatrix}
\]

(2.5)
Let \( y_r(t) = (h_r(t), \theta_r(t))^T \) be a given smooth reference depth and pitch angle trajectory. We are interested in deriving an adaptive control law such that the output vector \( y(t) \) asymptotically tracks the reference depth and pitch angle trajectories in spite of the parametric uncertainties and the presence of disturbance input \( d(t) \).

### 2.2 Robust Control Law for Dive-plane Maneuvers

The design of control law uses feedback linearization technique. However, for the design of the robust control law, compensation of the modeling error, in the presence of uncertainties is essential. For the derivation of the control law, the contribution of the unknown functions and external disturbances is treated as a lumped unknown vector function and a high gain observer is designed to obtain its estimate for canceling the lumped unknown function in the feedback linearizing control law.

#### 2.2.1 Feedback Linearizing Control

First, we consider the derivation of the feedback linearizing control law assuming that all the parameters and wave forces and moments in the model are known. This will be followed by the robust design. For the feedback linearization of the input (\( \delta \)) - output (\( y \)) map, the output is differentiated successively till the control input appears for the first time. Since the relative degree of each output is two, differentiating \( y \) along the solution of (2.4) yields

\[
\dot{y} = L_f C(x)
\]

\[
\ddot{y} = L_f^2 C(x) + [L_{G_c} L_f C(x)] \delta + [L_{F_3} L_f C(x)] d \quad (2.6)
\]
where the Lie derivative of $C(x)$ along the vector field $f(x)$ is

$$L_f C(x) = \frac{\partial C(x)}{\partial x} f(x)$$

$$L_f^2 C(x) = L_f L_f C(x)$$

$$L_{G_c} L_f C(x) = \left[ \frac{\partial L_f}{\partial x} C(x) \right] G_c$$

and

$$L_{F_d} L_f C(x) = \left[ \frac{\partial L_f}{\partial x} C(x) \right] F_d$$

For the vehicle model under consideration, computing the Lie derivatives gives,

$$L_f^2 C(x) = B_0 A x_p - \begin{bmatrix} (w \sin \theta + U \cos \theta) \dot{\theta} \\ 0 \end{bmatrix}$$

(2.7)

where

$$L_{G_c} L_f C(x) = B_0 G$$

$$L_{F_d} L_f C(x) = B_0 F$$

$$B_\theta = \begin{bmatrix} \cos \theta & 0 \\ 0 & 1 \end{bmatrix}$$

(2.8)

The matrix $L_{G_c} L_f C(x)$ is nonsingular as long as $|\theta| < \pi/2$. In the following derivation, it is assumed that $\theta(t)$ remains within $\pm \pi/2$.

In the presence of parametric uncertainties and disturbance forces, one can decompose the functions in the form

$$L_f^2 C(x) = a^*(y) + \Delta a(x)$$

$$L_{G_c} L_f C(x) = B_0 G^* + B_0 \Delta G$$
\begin{equation}
\Delta \triangleq B_*^\theta(\theta) + \Delta B_\theta(\theta) \tag{2.9}
\end{equation}

in which \(a^*(y), G^*\) and \(B_*^\theta(\theta)\) denote the nominal functions and matrices, and \(\Delta a(x), \Delta G\) and \(\Delta B_\theta(\theta)\) denote the uncertain portions of \(L^2_fC(x)\) and \(L^5_fC(x)\). It is noted that \(a^*(y)\) is a function of the measured variable \(y\). For the design of a feedback linearizing control, the matrix \((B_*^\theta(\theta) + \Delta B_\theta(\theta))\) must be invertible.

**Assumption 1:** It is assumed that the uncertain portion of \(\Delta G\) satisfies

\begin{equation}
\|B_\theta \Delta G\| < \|B_\theta G^*\| \tag{2.10}
\end{equation}

where for any matrix \(N\), \(\|N\| = \left(\lambda_{\text{max}}(N^T N)\right)^{1/2}\) (\(\lambda_{\text{max}}\) denotes the largest eigenvalue). Using (2.9) in (2.6) gives

\[
\dot{y} = a^*(y) + \Delta a(x) + B_\theta(G^* + \Delta G)\delta + B_\theta Fd
\]

\[
= a^*(y) + B_\theta G^*\delta + [\Delta a(x) + B_\theta(\Delta G\delta + Fd)] \tag{2.11}
\]

Define an uncertain vector function (appearing in (2.11))

\[
\eta(x, \delta, t) = \Delta a(x) + B_\theta(\Delta G\delta + Fd) \tag{2.12}
\]

The argument \(t\) denotes the dependence of \(\eta\) on \(d(t)\). Now using (2.11) and (2.12) the tracking error can be written as

\[
\tilde{y} = a^*(y) + B_\theta G^*\delta + \eta(x, \delta, t) - \gamma \tag{2.13}
\]

where \(\tilde{y} = y - y_r\).

In view of (2.13), assuming that all parameters and the disturbance input \(d(t)\) are known, one can select a feedback linearizing control law of the form

\[
\delta = (B_\theta G^*)^{-1}[-a^*(y) - \eta(x, \delta, t) + \gamma + \delta_0] \tag{2.14}
\]
The additional control signal \( \delta_a \) in (2.14) is selected as

\[
\delta_a = -K_2 \dot{y} - K_1 y - K_0 x_s
\]  
(2.15)

with

\[
x_s = \dot{y}
\]  
(2.16)

where \( K_i > 0 \) are the design parameters. The control law includes integral error feedback as well as proportional and derivative feedback.

Using the control law (2.14) in (2.13) gives

\[
\ddot{y} + A_2 \dot{y} + A_0 \int_0^t \dot{y}(\tau) d\tau = 0
\]  
(2.17)

The feedback gains \( K_i \) are properly chosen so that the polynomial

\[
\Delta_A(\lambda) = (\lambda^3 + K_2 \lambda^2 + K_1 \lambda + K_0)
\]

is Hurwitz. For such a choice of the feedback gains, the tracking error \( \bar{y}(t) \to 0 \); and therefore, \( h \to h_r \) and \( \theta \to \theta_r \) as \( t \to \infty \). Of course, the reference trajectory \( \theta_r(t) \) is chosen to be zero. Although, in the closed loop system, the trajectory tracking is accomplished, the control law (2.14) - (2.16) cannot be implemented, because the system parameters and the disturbance input \( d(t) \) are known. Furthermore, only \( y = (h, \theta)^T \) is measured, and its derivatives, which are functions of \( w \) and \( \dot{\theta} \), are not available for feedback. For the synthesis of the control law in the presence of uncertainties, one must obtain an estimate of \( \eta \) as well as \( \dot{y} \) since \( \bar{y} \) is not measured.

2.2.2 High-gain Observer and Robust Control Law

In this subsection, the control law (2.14) is modified to obtain a robust controller.
First a high-gain observer is designed to obtain an estimate of $\eta$ and $\dot{y}$. For the design of an estimator, $\eta$ is treated as a state vector. Differentiating $\eta$, it follows that for an appropriate vector function $f_\eta$, one has

$$\dot{\eta} = \frac{\partial \eta}{\partial x} (f(x) + G_c \delta + F_d d) + \frac{\partial \eta}{\partial \delta} \dot{\delta} + \frac{\partial \eta}{\partial t}$$

$$\triangleq f_\eta(x, \delta, \dot{\delta}, t) \quad (2.18)$$

The derivative of $\delta$ appears in (2.18). Of course, the input vector $\delta$ is differentiable because it is a function of the state variables of the system ($\delta$ is to be derived later).

Defining the state vector as

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} \dot{y} \\ \dot{\eta} \end{bmatrix} \in \mathbb{R}^6 \quad (2.19)$$

where $v_i \in \mathbb{R}^2$, the system (2.13) and (2.18) can be represented in a state variable form as

$$\dot{v} = \begin{bmatrix} v_2 \\ v_3 + a^*(y) + B_\eta G^* \delta - \dot{y}_r \\ f_\eta \end{bmatrix} \quad (2.20)$$

The vector function $f_\eta$ is treated as an unknown function. For the purpose of obtaining estimates of $\eta$, $\dot{y}$, $\ddot{y}$, a high-gain observer is designed (Khalil and Esfandiari 1992, Khalil, 1996, Singh et al., 2003).

Let $\hat{v} = (\hat{v}_1, \hat{v}_2, \hat{v}_3)^T \in \mathbb{R}^6$ be an estimate of $v$. In view of (2.20), the observer takes the form

$$\dot{\hat{v}}_1 = \hat{v}_2 + \epsilon^{-1}p_1(v_1 - \hat{v}_1)$$
\[
\dot{v}_2 = \dot{v}_3 + \epsilon^{-2} p_2 (\dot{v}_1 - \dot{v}_1) + a^*(x) + B_G^* \delta - \dot{y}_r
\]
\[
\dot{v}_3 = \epsilon^{-3} p_3 (\dot{v}_1 - \dot{v}_1)
\] (2.21)

where \( p_i > 0 \) and \( \epsilon > 0 \) is a small design parameter. The observer uses only the tracking error \( \dot{v}_1 = \ddot{y} \) for feedback. The observer gains \( p_i \) are chosen such that the polynomial

\[
\Delta_v(\lambda) = \lambda^3 + p_1 \lambda^2 + p_2 \lambda + p_0
\] (2.22)

is Hurwitz.

In order to examine the estimation property of the observer (2.21), let us obtain the dynamics of the observation error. For this, define the estimation error as

\[
e_i = v_i - \dot{v}_i, i = 1, 2, 3
\] (2.23)

Differentiating \( e_i \) and using (2.20) and (2.21), it easily follows that the error dynamics take the form

\[
\dot{e}_1 = e_2 - \epsilon^{-1} p_1 e_1
\]
\[
\dot{e}_2 = e_3 - \epsilon^{-2} p_2 e_1
\]
\[
\dot{e}_3 = -\epsilon^{-3} p_3 e_1 + f_\eta
\] (2.24)

Define new variables \( \xi_i \in R^2 \) given as

\[
\xi = \begin{bmatrix} 
\xi_1 \\
\xi_2 \\
\xi_3
\end{bmatrix} = \begin{bmatrix} 
\epsilon^{-2} e_1 \\
\epsilon^{-1} e_2 \\
e_3
\end{bmatrix}
\] (2.25)
Then using (2.24), the derivative of $\xi$ can be shown to be
\[
\dot{\xi} = \begin{bmatrix}
-p_1 I & I & 0 \\
-p_2 I & 0 & I \\
-p_3 I & 0 & 0
\end{bmatrix} \xi + \begin{bmatrix}
0 \\
0 \\
I
\end{bmatrix} f_n
\]
\[
\triangleq A_f \xi + \epsilon B_f f_n
\]  
(2.26)

where $I$ and $O$ denote $2 \times 2$ identity and null matrices, respectively. The system (2.26) essentially represents a fast system. Based on the theory for the stability of the singularly perturbed systems (Kokotovic, Khalil and O'Reilly, 1986, Khalil, 1992), it can be shown for sufficiently small $\epsilon$, $\xi$ converges to zero because $A_f$ is a Hurwitz matrix. This implies that $\delta(t)$ asymptotically converges to $v(t)$. Using the estimated values of states, the control law (2.14) is modified as
\[
\delta(t) = (B_0 G^*)^{-1}[-a^*(y) - \dot{v}_3 + \ddot{y}_r - K_2 \dot{\delta}_2 - K_1 \delta - K_0 x_s]
\]  
(2.27)

Substituting the modified control law (2.27) in (2.13) gives
\[
\ddot{y} = \eta - \dot{v}_3 - K_2 \dot{\delta}_2 - K_1 \delta - K_0 x_s
\]
\[
= \dot{v}_3 - K_2 (\dot{y} - e_2) - K_1 \delta - K_0 \int_0^t \ddot{y}(\tau) d\tau
\]  
(2.28)

Noting that $\dot{v}_3$ and $e_2$ asymptotically tend to zero, the error equation (2.28) asymptotically becomes (2.17). Therefore, it follows that $\ddot{y}(t) \rightarrow 0$, as $t \rightarrow \infty$.

2.3 Simulation Results

This section presents the simulation results. The submarine parameters used for computation are taken from Durnlu and Istefanopoulos (1995). Results are obtained
for the set point control with and without disturbance inputs. The initial conditions of the vehicle are \( x(0) = 0 \). The feedback gains selected are \( K_0 = 64 \times 10^{-6} \), \( K_1 = 48 \times 10^{-4} \) and \( K_2 = 12 \times 10^{-3} \). The reference depth trajectory is generated by a fourth-order filter of the form \( (s^4 + 4\lambda_cs^3 + 6\lambda_c^2s^2 + 4\lambda_c^3s + \lambda_c^4)h_r = \lambda_c^4h^* \) with the initial conditions \((h_r(0), \dot{h}_r(0), \ddot{h}_r(0), \dddot{h}_r(0)) = (0, 0, 0, 0)\) and its poles are selected at \( \lambda_c = -0.07 \). The target value of depth \( h^* \) is set to 10 ft., and the reference pitch angle trajectory \( \theta_r \) is taken to be zero degree. The constant \( \epsilon = 0.7 \) and the observer gains are \((p_1, p_2, p_3) = (3\lambda_0, 3\lambda_0^2, \lambda_0^3)\), where \( \lambda_0 = 7 \). For simplicity, the nominal vector function \( a^*(y) \) is taken to be zero. Thus it is assumed that \( L_f^2C(x) \) is completely unknown (i.e. \( \Delta a = L_f^2C(x) \)). This is rather a worse choice of uncertainty in \( L_f^2C(x) \); however this selection is made to show the robustness of the control law.

A. Set point trajectory control with parametric uncertainties without disturbance \((d=0)\)

It is assumed that the nominal matrix \( G^* = 0.8G \), that is \( \Delta G = +0.2G \). Thus the nominal \( G^* \) is 20 percent lower than the actual value of the input matrix \( G \). Of course, we have assumed that \( a^*(y) = 0 \). The closed-loop system (2.4) including the control law (2.27) and the observer (2.21) is simulated. The selected responses are shown in Figure 2.2. We observe trajectory control of the depth to the target value within 200 seconds. The pitch angle remains small (within \( 1 \times 10^{-3} \) degrees). The control magnitude is of the order of \( \delta = (10, 1.5)^T \) degrees.

Next, the uncertainty in \( G \) is assumed to be such that \( G^* = 1.2G \), and the closed loop system (2.4),(2.27), (2.21) is simulated. The selected responses are shown in Figure
2.3. Again, we observe trajectory control of the depth to the target value in about 200 seconds and the pitch angle remains small (within $6 \times 10^{-3}$ degrees). The control magnitude is of the order of $\delta = (10, 1.5)^T$ degrees.

B. Set point trajectory control with parameter uncertainties and disturbance input

To examine the robustness of the control system, simulation is done in the presence of parametric uncertainties as well as external disturbance inputs. The simulation results are presented for sea waves 3 ft. in height. The discrete-time state variable model considered by Dumlu and Istephanopulos (1995) for generating sea wave forces and moments is used. A zero order hold device is used to construct a piecewise constant disturbance input vector $(Z_{\text{wave}}(t), M_{\text{wave}}(t))$ for the purpose of simulation. The sampling interval is 0.2 seconds.

The parameter uncertainty is varied by $\pm 20$ percent from the actual values. Figure 2.4 shows selected response plots for parameter uncertainty equal to 20 percent lower than the nominal values ($G^{*} = 0.8G$) and Figure 2.5 shows the same responses for parameter uncertainty equal to 20 percent higher than the nominal values ($G^{*} = 1.2G$). Once again, we observe that the depth control is achieved within 200 seconds and the pitch angle remains small (within 0.25 degrees). The magnitude of the control is of the order of $\delta = (30, 10)$. Larger control magnitudes are required in this case to cancel the effect of the disturbance inputs.
2.4 Summary

In this chapter, a robust controller for the dive plane control of submarines using output feedback was designed. It was assumed that the parameters of the vehicle were not known precisely. The robust control system uses a high-gain observer to estimate the unknown functions and the unmeasured variables for feedback. The robust feedback linearizing controller is designed using the estimated signals for the modeling error compensation. Simulation results are presented to show that in the closed loop system, good trajectory control can be accomplished even in the presence of parametric uncertainties and sea forces.
Figure 2.1: Submersible with bow and stern hydroplanes
Figure 2.2: Set point Control \( \Delta G = +0.2G, \Delta a = L_2^2C(x), a^*(y) = 0 \): (a) Bow plane deflection (deg.) (b) Stern plane deflection (deg.) (c) depth (feet) (d) pitch angle (deg.)

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Figure 2.3: Set point Control $\Delta G = -0.2G$, $\Delta a = L_jC(x), a^*(y) = 0$: (a) Bow plane deflection (deg.) (b) Stern plane deflection (deg.) (c) depth (feet) (d) pitch angle (deg.)
Figure 2.4: Set point Control with disturbance $\Delta G = +0.2 G, \Delta a = L^2 C(x), a^*(y) = 0$: (a) Bow plane deflection (deg.) (b) Stern plane deflection (deg.) (c) depth (feet) (d) pitch angle (deg.)
Figure 2.5: Set point Control with disturbance $\Delta G = -0.2G$, $\Delta a = L^2(x), a^*(y) = 0$: (a) Bow plane deflection (deg.) (b) Stern plane deflection (deg.) (c) depth (feet) (d) pitch angle (deg.)
CHAPTER 3

MULTI-VARIABLE ADAPTIVE BACKSTEPPING CONTROL OF
SUBMERSIBLES USING SDU DECOMPOSITION

In this chapter, a multi-variable adaptive autopilot for the dive-plane control of submarines is designed. The vehicle is equipped with bow and stern hydroplanes for maneuvering. It is assumed that the system parameters are known, and the disturbance force is acting on the vehicle. Based on a back-stepping design approach, an adaptive control law is derived for the trajectory control of the depth and the pitch angle. To prevent singularity in the control law, the SDU decomposition of the high frequency gain matrix is used for the design. The advantage over the controller designed in the previous chapter is that there is no bound on the uncertainty of the high-frequency gain matrix. In the closed-loop system, asymptotic tracking of the reference depth and pitch angle trajectories is accomplished. The submersible's mathematical model used in this chapter is the same as that of Chapter 2. However the disturbance is assumed to be totally random.

This chapter is organized into four sections. The mathematical model and problem statement are given in section 3.1. The SDU decomposition and control law are derived in Section 3.2. The simulation results are given in section 3.3. The summary
is included in section 3.4.

3.1 Mathematical model

We consider the MIMO submersible model given by

\[ \dot{x} = f(x) + G_c \delta + F_a d \]

\[ y = Cx \quad (3.1) \]

where \( x = (w, Q, h, \theta)^T \). The details of the problem are given in the previous chapter.

3.2 SDU decomposition and Adaptive Control Law

The derivation of the adaptive control law is based on the back-stepping design technique [19]. Since the design of controller requires a special representation of the high frequency gain matrix \( G \in \mathbb{R}^{2 \times 2} \), we first consider its factorization

3.2.1 LDU and SDU decomposition of \( G \) matrix.

Now we consider the SDU decomposition of \( G \). For this, first we obtained the unique LDU decomposition of \( G \) [22]. The LDU decomposition of \( G \) can be shown to be

\[ G = \begin{bmatrix} 1 & 0 \\ l_0 & 1 \end{bmatrix} \begin{bmatrix} \Delta_1 & 0 \\ 0 & (\Delta_2/\Delta_1) \end{bmatrix} \begin{bmatrix} 1 & l_u \\ 0 & 1 \end{bmatrix} \]

\[ \Delta = L^* D^* U^* \quad (3.2) \]

where \( L^* \) is a unit lower and \( U^* \) is a unit upper triangular matrix, and \( D^* \) is a diagonal matrix. The leading principal minors \( \Delta_i, i = 1, 2 \), of \( G \) for the submarine model are nonzero. Let \( g_{ij}, i, j = 1, 2 \), be the elements of \( G \). Then one has \( \Delta_1 = g_{11} \)

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and $\Delta_2 = (g_{11}g_{22} - g_{12}g_{21})$. Solving (3.2) gives the elements $l_0$ and $l_u$. Now, the SDU decomposition of $G$ can be written as

$$G = S D_s U_s$$  \tag{3.3}$$

where one sets

$$S = L^* D^*(D_s)^{-1} L^{*T}$$

$$U_s = D_s^{-1}(L^{*-1})^T(D_s)U^*$$  \tag{3.4}$$

From (3.3), it easily follows that

$$D_s = \begin{bmatrix} 
\text{sgn}(\Delta_1) & 0 \\
0 & \text{sgn}(\Delta_2/\Delta_1) 
\end{bmatrix}$$  \tag{3.5}$$

The diagonal elements of $D_s$ are +1 or -1, $U_s$ is a unit upper triangular matrix given as

$$U_s = \begin{bmatrix} 
1 & u_{s1} \\
0 & 1 
\end{bmatrix}$$  \tag{3.6}$$

and $S$ is a symmetric positive definite matrix

$$S = \begin{bmatrix} 
|g_{11}| & \text{sgn}(g_{11}) g_{21} \\
\text{sgn}(g_{11}) g_{21} & \text{sgn}(g_{11}) \text{sgn}(\Delta_2) [g_{22} - g_{11}^{-1} g_{21}(g_{12} - g_{21} \text{sgn}(\Delta_2))] 
\end{bmatrix}$$  \tag{3.7}$$

The element $u_{s1}$ can be computed using (3.4). We assume that the matrices $S$ and $U_s$ have unknown elements. But for the diagonal matrix $D_s$ we make the following assumption.

**Assumption 1:** The sign of the leading principal minors $\Delta_1$ and $\Delta_2$ of $G$ are known.
This assumption is not really restrictive, and these signs of $\Delta_i$ can be obtained using some nominal parameters of the submarine model. Of course, the signs of $\Delta_i$ will remain unchanged if the parameters vary in the neighborhood of the nominal values of the parameters. It will be seen that the use of SDU decomposition of $G$ permits the design of a well defined adaptive control law.

3.2.2 Adaptive Control Law

Now the derivation of the control law, under the Assumption 1 based on a back-stepping design technique, is considered. The design is completed in two steps since the relative degree of each output $h$ and $\theta$ is two.

**Step 1:** Define $x = (x_1^T, x_2^T)^T$ where $x_1 = (h, \theta)^T$ and $x_2 = (w, Q)^T$ and let the tracking error be

$$z_1 = \begin{pmatrix} h \\ \theta \end{pmatrix} - \begin{pmatrix} h_r \\ \theta_r \end{pmatrix} = x_1 - y_r$$  \hfill (3.8)

Then using (3.8) one has

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_r = B_\theta x_2 - (U_0 \sin \theta, 0)^T - y_r$$  \hfill (3.9)

where

$$B_\theta = \begin{pmatrix} \cos \theta & 0 \\ 0 & 1 \end{pmatrix}$$

Define a change of coordinates as

$$z_2 = x_2 - \alpha_1$$  \hfill (3.10)

where $\alpha_1 \in \mathbb{R}^2$ is a stabilizing signal yet to be determined. Substituting (3.10) in
(3.9) gives

\[ \dot{z}_1 = B_0(z_2 + \alpha_1) - (U_0 \sin \theta, 0)^T - y_r \]  

(3.11)

Now \( \alpha_1 \) is chosen to regulate \( z_1 \) to the origin.

In view of (3.11) we choose the stabilization signal \( \alpha_1 \) as

\[ \alpha_1 = B_0^{-1} [-C_1 z_1 + y_r - (U_0 \sin \theta, 0)^T] \]  

(3.12)

where \( C_1 \) is a positive real number. The determinant of the matrix \( B_0 \) vanishes at \( \theta = \pm \pi/2 \); therefore the maneuvers of the vehicle with vertically aligned nose (up or down) must be avoided. Of course, for practical maneuvers, this is not a problem, and it will be seen in the next section that the dive-plane maneuvers can be performed by selecting the pitch angle reference trajectory \( \theta_r(t) \) appropriately. In the sequel, it is assumed that \( \theta(t) \neq \pm \pi/2 \). Then using (3.12) in (3.11) gives

\[ \dot{z}_1 = -C_1 z_1 + z_2 \]  

(3.13)

Consider a Lyapunov function \( V_1(z_1) \) of the form

\[ V_1(z_1) = (z_1^T z_1)/2 \]  

(3.14)

Then the derivative of \( V_1 \) along the solution of (3.13) gives

\[ \dot{V}_1 = -C_1 z_1^T z_1 + z_1^T B_0 z_2 \]  

(3.15)

Apparently, if \( z_2 = 0 \), then \( z_1 \) tends to zero as \( t \to \infty \), because \( V_1 \) is a positive definite and \( \dot{V}_1 \) is a negative definite function of the variable \( z_1 \). However, \( z_2 \) cannot be set to zero, because \( x_2 = (w, Q)^T \) is not a control input vector. In the next step, the
product term $z_1^T B \theta z_2$ of (3.15) is compensated.

**Step 2:** In the second step, control input $u_c$ is chosen to regulate $z_2$ to zero. For this, consider the derivative of $z_2$ which is given by

$$
\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1
$$

$$
= A x_p - \dot{\alpha}_1 + G u_c + F d \tag{3.16}
$$

The derivative of $\alpha_1$ is

$$
\dot{\alpha}_1 = \frac{\partial \alpha_1}{\partial \theta} Q + \frac{\partial \alpha_1}{\partial z_1} \dot{z}_1 + \frac{\partial \alpha_1}{\partial y_r} \dot{y}_r
$$

$$
= \frac{\partial \alpha_1}{\partial \theta} Q + \frac{\partial \alpha_1}{\partial z_1} (-C_1 z_1 + z_2) + \frac{\partial \alpha_1}{\partial y_r} \dot{y}_r
$$

$$
\triangleq a_\alpha(x, y_r, \dot{y}_r, \ddot{y}_r) \tag{3.17}
$$

Using the SDU decomposition of $G$, (3.16) gives

$$
\dot{z}_2 = A x_p - \dot{\alpha}_1 + S D_s U_s u_c + F d \tag{3.18}
$$

Since $U_s = I_{2 \times 2} + (U_s - I_{2 \times 2})$, (3.18) can be expressed as

$$
\dot{z}_2 = S [S^{-1} A x_p - S^{-1} a_\alpha(x, t) + D_s u_c + D_s(U_s - I_{2 \times 2}) u_c + S^{-1} F d]
$$

$$
\triangleq \dot{z}_2 = S \frac{u_s}{u_s} + D_s U_{sd} \tag{3.19}
$$

where the argument $t$ of $a_\alpha$ denotes the appearance of $y_r$ and its derivative in $a_\alpha$, and

$$
U_s - I_{2 \times 2} = \begin{bmatrix} 0 & u_{s1} \\ 0 & 0 \end{bmatrix} \triangleq U_{sd} \tag{3.20}
$$

Define the matrix of parameters

$$
P = [S^{-1} A, -S^{-1}] \in \mathbb{R}^{2 \times 5} \tag{3.21}
$$
and the vector function

\[ \Phi = \begin{bmatrix} x_p \\ a_\alpha(x, t) \end{bmatrix} \in \mathbb{R}^5 \]  

Using (3.21) and (3.22), (3.19) is written as

\[ \dot{z}_2 = S [ P \Phi + D_s u_c + D_s \tilde{U}_{sd} u_c + S^{-1} F d ] \]  

Let \( \hat{P}(t) \) and \( \tilde{U}_{sd}(t) \) be estimates of matrices \( P \) and \( U_{sd} \), respectively; where one sets

\[ \tilde{U}_{sd} = \begin{bmatrix} 0 & \tilde{u}_{s1} \\ 0 & 0 \end{bmatrix} \]

and \( \tilde{u}_{s1} \) is an estimate of \( u_{s1} \). Define the parameter errors

\[ \bar{P} = P - \hat{P}(t), \bar{U}_{sd} = U_{sd} - \tilde{U}_{sd}, \bar{u}_{s1} = u_{s1} - \tilde{u}_{s1} \]  

Then using (3.24), (3.23) gives

\[ \dot{z}_2 = S [ \bar{P}(t) \Phi + D_s \bar{U}_{sd} u_c + \bar{P} \Phi + D_s \bar{U}_{sd} u_c + D_s u_c + S^{-1} F d ] \]  

Now consider a modified Lyapunov function \( V_2 \) of the form

\[ V_2(z_1, z_2, \tilde{P}, \tilde{u}_{s1}) = V_1 + (1/2)[z_2^T S^{-1} z_2 + tr(\tilde{P} \Gamma_1 \tilde{P}(t))] \]

\[ + \gamma^{-1} |d_{s1}| \tilde{u}_{s1}^2 | \]  

where \( \Gamma_1 \) is a positive definite symmetric matrix, \( \gamma > 0 \) and \( d_{s1} \) is the diagonal element of \( D_s \) (which is \(+1\) or \(-1\)). \( V_2 \) is a positive definite function of \( z_1, z_2, \tilde{P} \) and \( \tilde{u}_{s1} \) (denoted as \( V_2 > 0 \)), because \( S^{-1} \) is a positive definite symmetric matrix.
The derivative of $V_g$ along the solution of (3.25) gives

$$
\dot{V}_2 = -C_1 z_1^T z_1 + z_1^T B_g z_2 + z_2^T [\dot{P}(t) \Phi + D_s \dot{U}_d u_e + \dot{\Phi}]
$$

$$
+ D_s \dot{U}_d u_e + D_s u_e + S^{-1} F d]
$$

$$
+ tr(\dot{P}^T \Gamma_1^{-1} \dot{P}(t)) + \gamma^{-1} |d_1| \tilde{u}_s \tilde{u}_s
$$

(3.27)

In view of (3.27), one chooses the control law of the form

$$
u_c = -\dot{U}_d u_e + D_s^{-1} [-\dot{P}(t) \Phi - B_g z_1 - C_2 z_2 + u_a]
$$

(3.28)

where $u_a = (u_{a1}, u_{a2})^T$ is an additional control signal yet to be determined, and $C_2 > 0$ is a design parameter.

Now we proceed to obtain the adaptation law. Note that

$$
z_2^T D_s \dot{U}_d u_e = z_{21} d_{s1} \tilde{u}_{s1} u_{c2}
$$

(3.29)

where $u_{ci}$ is the $i$th component of $u_c$ and $z_{21}$ is the first component of $z_2$. Substituting the control law (3.28) in (3.27) and using (3.29) gives

$$
\dot{V}_2 = -C_1 z_1^T z_1 - C_2 z_2^T z_2 + tr(\dot{P}^T z_2 \Phi^T) + tr(\dot{P}^T \Gamma_1^{-1} \dot{P}(t))
$$

$$
+ \tilde{u}_s (z_{21} d_{s1} u_{c2} + \gamma^{-1} |d_1| \tilde{u}_s) + z_2^T (S^{-1} F d + u_a)
$$

(3.30)

where the equality

$$
z_2^T \dot{P} \Phi = tr(\Phi^T \dot{P}^T z_2) = tr(\dot{P}^T z_2 \Phi^T)
$$

has been used to obtain (3.30). In view of (3.30), eliminating the unknown functions yields the adaptation law given by

$$
\dot{P} = -\dot{P} = -\Gamma_1 z_2 \Phi^T
$$
\[ \dot{u}_{s1} = -\dot{u}_{s1} = -\gamma \text{sgn}(d_{s1}) z_{21} u_{c2} \]  (3.31)

where \(d_{s1} = \text{sgn}(\Delta_1)\). Substituting the adaptation law (3.31) in (3.30) gives

\[ \dot{V}_2 = -C_1 z_1^T z_1 - C_2 z_2^T z_2 + z_2^T (u_a + S^{-1} F d) \]  (3.32)

Let \(S^{-1} F d = (\mu_{a1}, \mu_{a2})^T\). For a bounded disturbance \(d(t)\), there exist \(\mu_i^* > 0\) such that \(|\mu_{ai}| \leq \mu_i^*, i = 1, 2\). Then, using these bounds, (3.32) yields

\[ \dot{V}_2 \leq -C_1 z_1^T z_1 - C_2 z_2^T z_2 + \sum_{i=1}^{i=2} (z_{2i} u_{ai} + |z_{2i}| \mu_i^*) \]  (3.33)

where \(u_a = (u_{a1}, u_{a2})^T\).

Now the design of the auxiliary signal \(u_a\) is considered. For the compensation of the \(\mu_i\) dependent terms, one selects

\[ u_{ai} = -\text{sgn}(z_{2i}) \hat{\mu}_i(t) \]  (3.34)

where \(\hat{\mu}_i\) is an estimate of \(\mu_i^*, i = 1, 2\). Consider a modified Lyapunov function

\[ V_{2a} = V_2 + \sum_{i=1}^{i=2} \lambda_i^{-1} (\hat{\mu}_i^2 / 2) \]  (3.35)

where \(\lambda_i > 0\) and \(\hat{\mu}_i = \mu_i^* - \hat{\mu}_i\) be the parameter error. Taking the derivative of \(V_{2a}\), one finds that

\[ \dot{V}_{2a} \leq -C_1 z_1^T z_1 - C_2 z_2^T z_2 + \sum_{i=1}^{i=2} \left[ |z_{2i}| \hat{\mu}_i + \lambda_i^{-1} \hat{\mu}_i \dot{\hat{\mu}}_i \right] \]  (3.36)

Selecting the adaptation law for \(\dot{\hat{\mu}}_i\) of the form

\[ \dot{\hat{\mu}}_i = -\lambda_i |z_{2i}|, i = 1, 2 \]  (3.37)

gives

\[ \dot{V}_{2a} \leq -C_1 z_1^T z_1 - C_2 z_2^T z_2 \leq 0 \]  (3.38)
Since $\dot{V}_{2a} \leq 0$, and $V_{2a} > 0$, $V_{2a}(z_1, z_2, \tilde{P}, \tilde{u}_s, \tilde{\mu}_1, \tilde{\mu}_2) < \infty$; therefore $z_1, z_2, \tilde{p}, \tilde{u}_s, \tilde{\mu}_1, \tilde{\mu}_2 \in L^\infty[0, \infty)$ ($L^\infty[0, \infty)$ denotes the set of bounded functions). Since $y_r$ and its derivatives are bounded, boundedness of $z_1, z_2$ implies that $x \in L^\infty[0, \infty), u_c \in L^\infty[0, \infty)$ and $\dot{z}_i \in L^\infty[0, \infty)$. Integrating (3.38) gives

$$\int_0^\infty (C_1 z_1^T z_1 + C_2 z_2^T z_2) dt \leq V_{2a}(0) - V_{2a}(\infty) < V_{2a}(0)$$ (3.39)

which implies that $z_1, z_2 \in L^2[0, \infty)$ (the set of square integrable functions). Using Barbalat’s lemma [22], one concludes that $z_1, z_2$ tend to zero as $t \to \infty$ and thus $(h, \theta) \to (h_r, \theta_r)$ as $t \to \infty$. This completes the design.

The additional signal $u_a$ is used only to counteract the disturbance input, and can be set to zero, if $d = 0$, and the adaptive law $u_c$ (with $u_a = 0$) suffices to compensate for the parametric uncertainties. The control signal $u_a$ is a discontinuous function of $z_g$. However, instead, a continuous approximation of the $\text{sgn}$ function by a sat function may be used to avoid control chattering. But this approximation may result in a small tracking error, and in such a case, one may need to use $\sigma$-modification, $\epsilon$-modification or dead-zone inclusion in the adaptation laws to avoid parameter drift [22]. The simulation results of the next section show that the controller is effective without any modification of the adaptive law, as such, any modification of the adaptation law is not needed.

3.3 Simulation Results

This section presents the simulation results. The submarine parameters used for computation are taken from [5]. Results are obtained for the set point control for the
model with and without disturbance inputs. The initial conditions of the vehicle are
\[ x(0) = 0 \] and computing the nominal values of the parameters gives
\[
P = \begin{bmatrix}
-0.5356 & 33.5601 & 0.3216 & -21.7281 & 7.6470 \\
0.4359 & -68.4038 & 2.8032 & 7.6470 & -736.3794
\end{bmatrix}
\]
and \( u_{s1}(0) = -1.7127 \). The feedback gains selected are \( C_1 = 0.9 \) and \( C_2 = 4 \) and the
adaptive laws have \( \Gamma_1 = 100I_{2 \times 2}, \gamma = 1, \) and \( \lambda_1 = \lambda_2 = 1 \). A smooth reference depth
trajectory is generated by a third-order filter of the form
\[
(s^3 + 3\lambda_c s^2 + 3\lambda_c^2 s + \lambda_c^3)h_r = \lambda_c^4 h^*
\]
with the initial conditions \( (h_r(0), \dot{h}_r(0), \ddot{h}_r(0)) = (0, 0, 0) \) and its poles are selected
at -1 using \( \lambda_c = 1 \). The target value of depth \( h^* \) is set to 10 ft. For the set point
control, the reference pitch angle trajectory is taken to be zero degrees.

A. Adaptive control: parametric uncertainties, disturbance \( d = 0 \)

The closed-loop system (3.1) and (3.28) with the update law (3.31) is simulated. Since only parameter uncertainties are assumed, the signal \( u_a \) is set to zero. Initial estimates of \( P \) and \( u_{s1} \) are set to \( \hat{P}(0) = 0.6P \) and \( \hat{u}_{s1}(0) = 0.6u_{s1} \). Thus, the initial estimates \( \hat{P} \) and \( \hat{u}_{s1} \) have 40 percent lower values than the nominal values. The selected responses are shown in Figure 3.1. We observe trajectory control of the depth
to the target value in about 350 seconds. The pitch angle remains small (within 0.4
degrees). In the transient period, oscillatory responses are observed. This is natural
because the adaptive law is learning to estimate the submarine parameters. The control
magnitude is of the order of \( u_c = (5, 4)^T \) degrees.

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Now simulation is done using off-nominal higher values of parameters. Initial estimates of $P$ and $u_{s1}$ are set to $\dot{P}(0) = 1.6P$ and $\dot{u}_{s1}(0) = 1.6u_{s1}$, and the closed-loop system (3.1), (3.28) and (3.31) (with $u_a = 0$) is simulated. Thus, the initial estimates of $P$ and $u_{s1}$ have 60 percent higher values than the nominal values. The selected responses are shown in Figure 3.2. Again, we observe trajectory control of the depth to the target value in about 350 seconds and the pitch angle remains small (within 0.4 degrees). The control magnitude is of the order of $u_c = (15, 10)^T$ degrees.

B. Adaptive control: parameter uncertainties, disturbance $d \neq 0$

To examine the robustness of the control system, simulation is done in the presence of parametric uncertainties as well as external random disturbance inputs $Z_d(t)$ and $M_d(t)$. The disturbances are generated using band limited white noise that is passed through a filter whose transfer function is given by $\frac{1}{s+5}$. The magnitude of the $Z_d$ and $M_d$ disturbances are of the order of $1 \times 10^4 \text{slug ft.}/\text{sec}^2$ and $1 \times 10^4 \text{slug ft}^2/\text{sec}^2$ respectively. Here, $u_a$ given by (3.34) is synthesized. Simulation is done for the uncertainty factor of 0.6 (lower) and 1.6 (higher). The responses for $\dot{P}(0) = 0.6P$ and $\dot{u}_{s1}(0) = 0.6u_{s1}$ are shown in Figure 3.3, and Figure 3.4 presents the responses for $\dot{P}(0) = 1.6P$ and $\dot{u}_{s1}(0) = 1.6u_{s1}$. For simulation, the control surface deflections are limited to 30 degrees.

Once again, we observe that depth control is accomplished and the pitch angle is small (within 0.4 degrees). The magnitude of the control is of the order of $u_c = (30, 12)^T$ degrees. We observe that the control magnitudes are larger in this case. This is natural because now the control input must nullify the effect of the
disturbance inputs. We also observe that only the bow plane is saturating in each case. Furthermore, it is seen from Figures 3.3 and 3.4, that the adaptive law with overestimated values of the uncertain parameters \((\hat{P}(0), \hat{u}_{s1}(0))\), causes more frequent saturation of the bow plane compared to the adaptive law with underestimated parameters.

C. Nonadaptive control: parametric uncertainty, \(d = 0\)

In order to examine the necessity of the adaptive law in the presence of parametric uncertainties, simulation is done using the control law of (3.28) without using parameter adaptation. That is, the adaptation law of (3.31) now has \(\Gamma_1 = 0_{2 \times 2}\) and \(\gamma = 0\). The off-nominal values \(\hat{P}(0)\) and \(\hat{u}_{s1}(0)\) of Figure 3.2 are retained. Thus \(\hat{P}(t) = \hat{P}(0)\) and \(\hat{u}_{s1}(t) = \hat{u}_{s1}(0)\) for \(t \geq 0\) (i.e. the gains are frozen). The disturbance free case is considered; therefore, \(u_a\) is set to zero. The responses are shown in Figure 3.5. It is seen that without the parameter adaptation of (3.31), the system’s outputs diverge, and as a result, the control input diverges. This shows the effectiveness of the adaptive law in compensating the parametric uncertainties.

3.4 Summary

In this chapter, the dive plane control of submarines using an adaptive control system was considered. For the design, it was assumed that the parameters of the vehicle were not known and disturbance inputs were acting on the vehicle. Using adaptive back-stepping, a multi-variable control system was designed for the depth and pitch angle control using bow and stern planes. The SDU decomposition
of the high frequency gain matrix was used to avoid singularity in the control law. It was shown that in the closed-loop system including the adaptive law, the depth and pitch angle trajectory asymptotically converge to the reference trajectory.

Simulation results were presented which showed that depth and pitch control can be achieved even in the presence of large uncertainties in the parameters and the disturbance inputs, and that for large uncertainties, the closed loop system exhibits instability if the adaptive law is not used (i.e. if the controller gains are frozen).
Figure 3.1: Adaptive Control without disturbance \((\hat{P}(0), \hat{u}_{a1}(0)) = 0.6(P, u_{a1})\):
(a) depth (feet) (b) pitch angle (deg.) (c) Bow plane deflection (deg.) (d) Stern plane deflection (deg.)
Figure 3.2: Adaptive Control without disturbance \((\hat{P}(0), \hat{u}_{s1}(0)) = 1.6(P, u_{s1})\): (a) depth (feet) (b) pitch angle (deg.) (c) Bow plane deflection (deg.) (d) Stern plane deflection (deg.)
Figure 3.3: Adaptive Control with disturbance \( \hat{P}(0), \hat{n}_{s1}(0) = 0.6(P, n_{s1}) \): (a) depth (feet) (b) pitch angle (deg.) (c) Bow plane deflection (deg.) (d) Stern plane deflection (deg.) (e) Disturbance \( Z_d \) (slug ft/sec^2) (f) Disturbance \( M_d \) (slug ft^2/sec^2)
Figure 3.4: Adaptive Control with disturbance \((\hat{P}(0), \hat{u}_3(0)) = 1.6(P, u_3)\): (a) depth (feet) (b) pitch angle (deg.) (c) bow plane deflection (deg.) (d) stern plane deflection (deg.)
Figure 3.5: Adaptive Control without parameter adaptation $(\hat{P}(0), \hat{u}_{s1}(0)) = 0.6(P,u_{s1})$: (a) depth (feet) (b) pitch angle (deg.) (c) Bow plane deflection (deg.) (d) Stern plane deflection (deg.)
CHAPTER 4

MODEL REFERENCE ADAPTIVE CONTROL OF MIMO SUBMERSIBLES WITH UNKNOWN HIGH-FREQUENCY GAIN MATRIX

This chapter presents a model reference adaptive control (MRAC) system for the maneuvering of multi-input, multi-output (MIMO) underwater vehicles. The submarine is equipped with bow and stern hydroplane for the control in the dive plane. It is assumed that the system parameters including the high frequency gain matrix are unknown. For the purpose of control law design, the depth and the pitch angle are treated as output variables and it is assumed that only the output variables are measured for feedback. This is the main difference between the controller design in this chapter and the controller design of the previous two chapters. Based on the Lyapunov stability theory, an adaptive output feedback control is derived for the trajectory control of the depth and pitch angle. Simulation results are presented which show that in the closed-loop system, trajectory tracking is accomplished in spite of the presence of parameter uncertainties.

Section 4.1 presents the mathematical model. The SDU decomposition and model reference adaptive control law are presented in Section 4.2. Simulation results and the summary are presented in Sections 4.3 and 4.4 respectively.
4.1 Mathematical Model

We consider here the dive-plane dynamics of the submarine. Readers can refer to [2] and [5] for more details. The equations of motion along the z-axis and y-axis are given by

\[ \dot{w}(t) = \frac{Z_w'U}{Lm_3} w(t) + \frac{1}{m_3} (Z_{\theta} + m') U \dot{\theta}(t) \]
\[ + \frac{Z_{\theta} L}{m_3} \dot{\theta}(t) + \frac{Z_{\delta B} U^2}{m_3 L} \delta B(t) + \frac{Z_{\delta S} U^2}{m_3 L} \delta S(t) \]  

(4.1)

\[ \dot{\theta}(t) = \frac{M_{L}'}{L'I_2} \dot{w}(t) + \frac{M_{w}'}{L'I_2} w(t) + \frac{M_{\theta}'}{L'I_2} \dot{\theta}(t) + \frac{M_{\delta B} U^2}{L'I_2} \delta B(t) \]
\[ + \frac{M_{\delta S} U^2}{L'I_2} \delta S(t) + \frac{2mg(z_G - z_B)}{\rho L^5 I_2} \dot{\theta}(t) \]  

(4.2)

where \(w\) is the heave velocity, \(Q\) is the rotational velocity, \(h\) is the depth error, \(\theta\) is the pitch angle, and \(\delta B\) and \(\delta S\) are the hydroplane deflections in the bow and stern planes, respectively. (For notations, see [2] and [5]). In this model, we assume that the forward velocity \(U = U_0\) is a constant. We are interested in the depth control of the submarine. As such, we include the depth \((h)\) as a state variable satisfying the nonlinear differential equation

\[ \dot{h} = wc_0 \theta - U_0 \sin \theta \]  

(4.3)

Here we present a design based on the linearized model. Assuming that the pitch angle is small, (4.3) can be approximated by

\[ \dot{h} = w - U_0 \theta \]  

(4.4)
Defining the state vector \( z = (h, \theta, w, Q) \in \mathbb{R}^4 \), the equations of motion and the output vector \( y = (h, \theta) \in \mathbb{R}^2 \) are described by

\[
\dot{z} = A_z z + B_z u_c \quad y = C z
\]

where \( B_z = [O_{2 \times 2}, B_{2z}^T]^T \), \( C = [I_{2 \times 2}, O_{2 \times 2}] \in \mathbb{R}^{2 \times 4} \), and \( I \) and \( O \) indicate identity and null matrices of indicated dimensions. Computing the observability matrix \( (C^T, (C A_z)^T)^T \), one finds that it is nonsingular. Thus, the system is observable and the observability index of the system is \( \nu = 2 \). It easily follows that \( C B_z = 0 \) and \( C A_z B_z \) is nonsingular.

It will be convenient to express the system (4.4) in the observer canonical form.

Consider a state transformation

\[
x = \begin{bmatrix} I_{2 \times 2} & O_{2 \times 2} \\ -A_1 & I_{2 \times 2} \end{bmatrix} \begin{bmatrix} C \\ C A_z \end{bmatrix} z = P z
\]

where

\[
C A_z^2 \begin{pmatrix} C \\ C A_z \end{pmatrix}^{-1} = [A_0 \ A_1]
\]

Then it can be easily shown that

\[
\dot{x} = \begin{bmatrix} A_1 & I_{2 \times 2} \\ A_0 & O_{2 \times 2} \end{bmatrix} x + \begin{bmatrix} O \\ C A_z B_z \end{bmatrix} u_c \Delta = A x + B u_c
\]

\[
y = [I_{2 \times 2} \ O_{2 \times 2}] x = C x
\]

is the new representation of the system (4.5) where

\[
A = P A_z P^{-1}
\]
\[ B = PB_z = [O_{2\times 2}, B_0^T]^T \]  \hspace{1cm} (4.8)

where \( B_0 = CA_zB_z \) is a nonsingular matrix. It is assumed that the matrices \( A_0, A_1 \) and \( B_0 \) are unknown. The system transfer matrix \( G(s) \) relating the input and output can be shown to be

\[
G(s) = C(sI - A)^{-1}B = [s^2I_{2\times 2} - A_1s - A_0]^{-1}B_0 \triangleq D^{-1}(s)B_0 \hspace{1cm} (4.9)
\]

where \( s \) is the Laplace variable. The \( 2 \times 2 \) transfer matrix \( G(s) \) has the following properties:

1. \( G(s) \) does not have any transmission zeros (and therefore, \( G(s) \) is minimum phase).
2. \( G(s) \) has uniform relative degree 2, that is, \( \lim_{s \to \infty} s^2G(s) = B_0 \), where \( B_0 \) is referred to as the high-frequency gain (HFG) matrix.

Since \( G(s) \) has uniform relative degree 2, consider the reference model

\[ y_m = W_m(s)r, \quad y_m, r \in \mathbb{R}^2 \]

where the transfer function is chosen as

\[
W_m(s) = \text{diag} \left\{ \frac{1}{(s + a_1)(s + \lambda)}, \frac{1}{(s + a_2)(s + \lambda)} \right\}
\]

with \( a_1 = a_2 = a > 0 \) and \( \lambda > 0 \). Thus \( W_m(s) \) is a stable transfer matrix.

Let \( y_m(t) = (h_m(t), \theta_m(t))^T \in \mathbb{R}^2 \) be a given smooth reference depth and pitch angle trajectory. We are interested in deriving a model reference adaptive control law such that the output vector \( y(t) \) asymptotically tracks the reference depth and pitch angle trajectories in spite of the parametric uncertainties. Furthermore, control is to be synthesized using the output vector \( y(t) \).
4.2 MRAC Law for Dive-plane Maneuvers

The design of controller requires a special representation of the high frequency gain matrix $B_0 \in R^{2 \times 2}$, we first consider its factorization

4.2.1 LDU and SDU decomposition of $B_0$ matrix.

Now we consider the SDU decomposition of $B_0$. For this, first we obtained the unique LDU decomposition [22] of $B_0$. The LDU decomposition of $B_0$ can be shown to be

\[
B_0 = \begin{bmatrix} 1 & 0 \\ l_0 & 1 \end{bmatrix} \begin{bmatrix} \Delta_1 & 0 \\ 0 & (\Delta_2/\Delta_1) \end{bmatrix} \begin{bmatrix} 1 & l_u \\ 0 & 1 \end{bmatrix} = L^* D^* U^*
\]

(4.10)

where $L^*$ is a unit lower and $U^*$ is a unit upper triangular matrix, and $D^*$ is a diagonal matrix. The leading principal minors $\Delta_i, i = 1, 2,$ of $B_0$ for the submarine model are nonzero. Let $b_{ij}, i, j = 1, 2,$ be the elements of $B_0$. Then one has $\Delta_1 = b_{011}$ and $\Delta_2 = (b_{011}b_{022} - b_{012}b_{021})$. Solving (4.10) gives the elements $l_0$ and $l_u$. Now, the SDU decomposition of $B_0$ can be written as

\[
B_0 = SD_s U_s
\]

(4.11)

where one sets

\[
S = L^* D^* (D_s)^{-1} L^T
\]

\[
U_s = D_s^{-1} (L^*)^{-1} (D_s) U^*
\]

(4.12)

From (4.11), it easily follows that

\[
D_s = \begin{bmatrix} sgn(\Delta_1) & 0 \\ 0 & sgn(\Delta_2/\Delta_1) \end{bmatrix}
\]

(4.13)
The diagonal elements of $D_s$ are $+1$ or $-1$, $U_s$ is a unit upper triangular matrix given as

$$U_s = \begin{bmatrix} 1 & u_{s1} \\ 0 & 1 \end{bmatrix} \quad (4.14)$$

and $S$ is a symmetric positive definite matrix

$$S = \begin{bmatrix} |b_{011}| & \text{sgn}(b_{011})b_{021} \\ \text{sgn}(b_{011})b_{021} & \text{sgn}(b_{011})\text{sgn}(\Delta_2)[b_{022} - b_{011}^{-1}b_{012} - b_{012}^{-1}\text{sgn}(\Delta_2)] \end{bmatrix} \quad (4.15)$$

The element $u_{s1}$ can be computed using (4.11). We assume that the matrices $S$ and $U_s$ have unknown elements. But for the diagonal matrix $D_s$ we make the following assumption.

**Assumption 1:** The sign of the leading principal minors $\Delta_1$ and $\Delta_2$ of $B_0$ are known.

This assumption is not really restrictive, and these signs of $\Delta_i$ can be obtained using some nominal parameters of the submarine model. Of course, the signs of $\Delta_i$ will remain unchanged if the parameters vary in the neighborhood of the nominal values of the parameters. It will be seen that the use of SDU decomposition of $B_0$ permits the design of a well defined adaptive control law.

4.2.2 Adaptive Control Law

Now following [23], the derivation of the adaptive control law is considered. First consider the existence of a control law $u_c^*$ for matching the closed-loop transfer matrix and $W_m(s)$, when all the system parameters are known, that is,

$$y = G(s)u_c^* = W_m(s)r = y_m \quad (4.16)$$
According to [23], there exists a control law of the form

\[ u_c^* = p_1^* \omega_1 + p_2^* \omega_2 + p_3^* y + p_4^* r = p^* \omega \]  

(4.17)

which satisfies (4.16), where

\[
\begin{align*}
  p_i^* &\in R^{2 \times 2}, \omega_i \in R^2, \\
  p^* &= (p_1^*, p_2^*, p_3^*, p_4^*) \in R^{2 \times 8} \\
  \omega &= (\omega_1^T, \omega_2^T, y^T, r^T) \in R^8 \\
  p_4^* &= B_0^{-1} \\
  \omega_1 &= \Lambda^{-1}(s) u_1, \omega_2 = \Lambda^{-1}(s) y \\
  \Lambda(s) &= \lambda_0 + \lambda_1 s, \\
  \lambda_0 > 0, \lambda_1 > 0
\end{align*}
\]

For the choice of \( \lambda_i > 0 \), \( \Lambda(s) \) is a Hurwitz polynomial. The signals \( \omega_1 \) and \( \omega_2 \) are obtained by filtering the input and the output. Furthermore, it follows from [23] that the tracking error \( e(t) = y(t) - y_m(t) \) can be shown to be

\[ e = W_m(s) B_0 [u_c - p^* \omega] \]  

(4.18)

For the adaptive design, the SDU decomposition of \( B_0(=SD_s U_s) \) is introduced and (4.18) is written as

\[ e = W_m(s) SD_s U_s [u_c - p^* \omega] \]  

(4.19)

Noting that \( U_s \) is a unit upper triangular matrix, one has

\[ U_s u_c = u_c - (I - U_s) u_c = u_c + \begin{pmatrix} u_{s1} u_{c2} \\ 0 \end{pmatrix} \]  

(4.20)
Substituting (4.20) in (4.19) gives

\[ e = W_m(s)SD_s[u_c - U_s p^* \omega + (u_{s1} u_{c2}, 0)^T] \]  

(4.21)

Define

\[ U_s p^* \omega - (u_{s1} u_{c2}, 0)^T = \begin{bmatrix} (\omega^T, u_{c2}) \left( [p^*_{(1)} + p^*_{(2)} u_{s1}], -u_{s1} \right)^T \\ \omega^T p^T_{(2)} \end{bmatrix} \]

(4.22)

where \( p^*_{(i)} \) is the \( i \)th row of \( p^* \), and

\[ \Omega^T = diag \{ \Omega_1^T, \Omega_2^T \} \cdot diag \{ (\omega^T, u_{c2}), (\omega^T) \}, \Omega_1^T \in R^{1 \times 9}, \Omega_2^T \in R^{1 \times 8} \]

(4.23)

Using (4.22), the error equation takes the form

\[ e = W_m(s)SD_s u_c - \Omega^T \Theta^* \]

(4.24)

In the following, a modification of the control law is considered for yielding an adaptive law.

Following the procedure used for single-input single-output (SISO) systems, we introduce the filtered signals

\[ \zeta = L^{-1}(s) u_c \]  

(4.25)

and

\[ \xi^T = diag \{ \xi_1^T, \xi_2^T \} = L^{-1}(s) \Omega^T = diag \{ L^{-1}(s) \Omega_1^T, L^{-1}(s) \Omega_2^T \} \]

(4.26)
where \( L(s) = s + \lambda, \lambda > 0 \). Note that for this choice of \( L(s) \), \( W_m(s)L(s) \) is a strictly positive real (SPR) transfer matrix. Then the error equation can be expressed as

\[
e = W_m(s)L(s)SDsL(s)^{-1}(s)[u_c - \Omega^T\Theta^*]
\]

\[
= W_m(s)L(s)SD_s[\zeta - \xi^T\Theta^*]
\]

(4.27)

In view of (4.27), we select \( \zeta \) as

\[
\zeta = \xi^T\Theta
\]

(4.28)

where \( \Theta \) is an estimate of \( \Theta^* \). Using (4.25) and (4.28), the control input is given by

\[
u_c = L(s)\zeta = (s + \lambda)[\xi^T\Theta]
\]

\[
= \xi^T\Theta + \lambda\xi^T\Theta + \xi^T\dot{\Theta}
\]

\[
= \Omega^T\Theta + \xi^T\dot{\Theta}
\]

(4.29)

Using (4.28) in (4.27) gives

\[
e = W_m(s)L(s)SD_s\xi^T\tilde{\Theta}
\]

(4.30)

where \( \tilde{\Theta} = \Theta - \Theta^* \) is the parameter vector error.

Consider a realization of \( W_m(s)L(s)S \) of the form

\[
\dot{x}_a = A_a x_a + B_a D_s \xi^T\tilde{\Theta}
\]

\[
e = C_a x_a
\]

(4.31)

Since \( W_m(s)L(s)S \) is SPR, there exist matrices \( P_a > 0 \) and \( Q_a > 0 \) satisfying

\[
A_a^T P_a + P_a A_a = -Q_a
\]
\[ P_a B_a = C^T_a \]  
(4.32)

For a proof of stability, consider a Lyapunov function

\[ V = x_a^T P_a x_a + \bar{\Theta}^T D_a \Gamma^{-1} \bar{\Theta} \]  
(4.33)

where \( \Gamma = \text{diag}(\Gamma_1, \Gamma_2) \), \( D_a = \text{diag}(|d_{x1}| I_{6 \times 6}, |d_{x2}| I_{6 \times 6}) \) and \( \Gamma_i > 0 \). Taking the derivative along the solution of (4.31) and using (4.32) gives

\[ \dot{V} = -x_a^T Q_a x_a + 2x_a^T P_a [B_a D_a \xi \Gamma^{-\frac{1}{2}} \bar{\Theta}] + 2 \bar{\Theta}^T D_a \Gamma^{-1} \dot{\bar{\Theta}} \]
(4.34)

In order to eliminate the unknown function in (4.34), we choose the adaptation law as

\[ \hat{\Theta}_i(t) = \dot{\Theta}_i(t) = -\text{sgn}(d_{a_i}) \Gamma_i \xi_i \hat{e}_i, \quad i = 1, 2 \]  
(4.35)

Substituting (4.35) in (4.34) gives

\[ \dot{V} = -x_a^T Q_a x_a \leq 0 \]  
(4.36)

Since \( \dot{V} \leq 0 \), one has \( x_a, \Theta, \xi, e \in L_\infty \) (the set of bounded functions), and \( x_a, \xi, e \in L^2 \) (the set of square integrable functions). Furthermore, following the arguments of [19], it can be shown that all signals in the closed-loop system are bounded and \( e \to \infty \) as \( t \to \infty \).

4.3 Simulation Results

This section presents the simulation results. It is assumed that the parameters of the system are completely unknown. Results are obtained for the set point control for
the model without disturbance inputs. \( \Gamma_1 = \Gamma_2 = 1, \gamma_1 = \gamma_2 = 0.1, \lambda_0 = 0.01, \lambda_1 = 1, \)
a = 0.5 and \( \lambda = 2. \) A smooth trajectory of the form

\[ r = h^*(1 - e^{-0.02t}) \]

is chosen as the reference depth trajectory. The target value of depth \( h^* \) is set to 10 ft. For the set point control, the reference pitch angle trajectory is taken to be zero degrees.

The closed-loop system (4.5) and (4.29) with the adaptation law (4.35) is simulated. The selected responses are shown in Figure 4.1. We observe trajectory control of the depth to the target value in about 350 seconds. The pitch angle remains small (within 0.5 degrees). In the transient period, oscillatory responses are observed. This is natural because the adaptive law is learning to estimate the submarine parameters.

The control magnitude is of the order of \( u_c = (25, 11)^T \) degrees. Figure 4.2 shows the trajectories of the depth and pitch errors and Figure 4.3 shows the variation in the estimated parameter norm with time.

4.4 Summary

In this chapter, a model reference adaptive control (MRAC) system for the depth and pitch angle control of submersibles equipped with bow and stern hydroplanes was considered. It was assumed that the system parameters and high-frequency gain matrix were not known. Only the depth and pitch angle of the vehicle were measured for synthesis of the control law. It was shown that in the closed-loop system including the adaptive law, the depth and pitch angle trajectory asymptotically converge to the
reference trajectory even in the presence of large uncertainties in the parameters.
Figure 4.1: Model Reference Adaptive Control: (a) depth (feet) (b) pitch angle (deg.) (c) Bow plane deflection (deg.) (d) Stern plane deflection (deg.)

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Figure 4.2: Error Trajectory: (a) depth error (feet) (b) pitch error (deg.)
Figure 4.3: Estimated parameter norm
CHAPTER 5

INTEGRATED ADAPTIVE SLIDING MODE CONTROL OF AIRCRAFT

USING SDU DECOMPOSITION OF HIGH-FREQUENCY GAIN MATRIX

The adaptive design techniques for underwater vehicles using SDU decomposition discussed in the previous chapters can be extended to control the maneuvers of aircraft. This chapter presents an integrated adaptive sliding mode flight control system for the roll-coupled maneuvers of aircraft. It is assumed that the parameters of the aircraft as well as its high-frequency gain matrix are unknown. Based on a backstepping design approach, an adaptive variable structure control law for the trajectory control of the roll angle, angle of attack and sideslip angle using aileron, elevator and rudder is derived. The SDU decomposition of the high-frequency gain matrix is used for the derivation of a singularity free flight control law. An additional advantage of the control law lies in the choice of design parameters of SDU decomposition for shaping the response characteristics, and furthermore, existing restrictions on the uncertain portion of the input matrix for sliding mode design are relaxed. In the closed-loop system, the roll angle, angle of attack and sideslip angle trajectories asymptotically follow the reference output trajectories.

The organization of the chapter is as follows. Section 5.1 presents the math-
5.1 Aircraft Model

The equations of motion considered here are in the principal axes. The complete set of equations taken from [52] and [53] are given by (see these references for the notation and terminology)

\[
\begin{pmatrix}
\dot{\beta} \\
\dot{\alpha} \\
\dot{\phi} \\
\dot{\theta}
\end{pmatrix} = \begin{pmatrix}
\alpha_{\beta} & \alpha_{\phi} & \alpha_{\theta} \\
\beta & \phi & \theta \\
\theta_\alpha & \phi_\alpha & \theta_\phi \\
\phi_\beta & \phi_\phi & \phi_\theta
\end{pmatrix} \begin{pmatrix}
\delta a \\
\delta r \\
\delta c
\end{pmatrix}
\]

\[ (5.1) \]

Where \( \alpha_{\beta}, \alpha_{\phi}, \alpha_{\theta}, \beta, \phi, \theta, \theta_\alpha, \phi_\alpha, \phi_\phi, \phi_\theta \) are the elements of the matrices and vectors relevant to the model.

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where \( \bar{b}_a = l_b + l_{a_b} \Delta \alpha \), and \( \bar{n}_{a_b} = n_{a_b} + n_{a_b} \Delta \alpha \). The mathematical model of the airplane ignores speed changes and contains only rudimentary representation of the aerodynamic nonlinearities. These assumptions are made here for simplicity, but the design method could be applied to models with more complete aerodynamics and throttle control could be implemented for speed control.

Defining \( x_1 = (\phi, \alpha, \beta), x_2 = (p, r, q), x_3 = \theta, \) and \( x = (x_1^T, x_2^T, x_3) \in \mathbb{R}^7 \), the system (5.1) can be written as

\[
\begin{align*}
\dot{x}_1 &= f_{10}(x_1, x_3) + f_{11}(x_1) \lambda + G_1(x_1, x_3)x_2 + G_{1f}\delta \\
\dot{x}_2 &= f_2(x)w + G_2(x)\delta \\
\dot{x}_3 &= f_3(x_1, x_2)
\end{align*}
\]

(5.2)

where \( f_{10} \) and \( f_2 \) are nonlinear vector functions, \( f_{11} \) is a nonlinear \( 3 \times 2 \) matrix, \( f_3 \) is a nonlinear function, and

\[
\lambda = (z_\alpha, y_\beta)^T \in \mathbb{R}^2, w_p = (l_b, l_q, l_r, l_{\beta\alpha}, l_{\theta\alpha}, l_p, -i_1)^T \in \mathbb{R}^7,
\]

\[
w_r = (n_\beta, n_r, n_p, n_{p\alpha}, n_q, -i_3) \in \mathbb{R}^6, w_q = (m_\alpha, m_q, -m_\beta, i_2) \in \mathbb{R}^4
\]

\[
w = (w_p^T, w_r^T, w_q^T)^T \in \mathbb{R}^{17}
\]

are the vectors of parameters, and \( \delta = (\delta_{\alpha}, \delta_r, \delta_e)^T \in \mathbb{R}^3 \) is the control input vector.

The matrices \( G_1(x_1, x_3), G_{1f} \) and \( G_2(x) \) are

\[
G_1(x_1, x_3) = \begin{bmatrix}
1 & \tan \theta \cos \phi & \tan \theta \sin \phi \\
-\beta & 0 & 1 \\
\sin \alpha_0 + \Delta \alpha & \cos \alpha_0 & 0
\end{bmatrix}
\]

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\[
G_{1f} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & z_{\delta e} \\
y_{\delta a} & y_{\delta r} & 0
\end{bmatrix}
\]

\[
G_2(x) = G_2(\Delta \alpha) = \begin{bmatrix}
\tilde{I}_{\delta a} & I_{\delta r} & 0 \\
\tilde{n}_{\delta a} & n_{\delta r} & 0 \\
0 & 0 & \tilde{m}_{\delta e}
\end{bmatrix}
\]

(5.3)

We assume that the parameter vectors \( \lambda \) and \( \omega \) and the input matrix \( G_2 \) are unknown. Note that at \( \theta = \pm \pi/2, G_1(x_1, x_3) \) becomes unbounded; therefore, we shall be interested in a region \( \Omega \subset R^7 \) of the state space where

\[
\Omega = \left\{ x \in R^7 : x_3 \neq \pm \frac{\pi}{2} \right\}
\]

Let the controlled output vector be

\[
y = x_1 = (\phi, \alpha, \beta)^T
\]

(5.4)

Suppose that it is desired to track a smooth reference trajectory \( y_r = x_{1r} = (\phi_r(t), \alpha_r(t), \beta_r(t))^T \).

We are interested in designing a control law such that in the closed-loop system, the output vector \( y(t) \) asymptotically tracks the reference trajectory \( y_r(t) \) in spite of the uncertainties in the aircraft parameters.

5.2 Control Law

For the purpose of control law design, a simplified aircraft model will be used. Although aileron, rudder, and elevator produce forces, their primary effectiveness is as moment producing devices. As such, ignoring the term \( G_{1f} \delta \) and, in addition,
neglecting the $\Delta \alpha$-dependence of $G_2$, gives a simplified model

\[ \dot{x}_1 = f_{10}(x_1, x_3) + f_{11}(x_1)\lambda + G_1(x_1, x_3)x_2 \]

\[ \dot{x}_2 = f_2(x)w + B\delta \]

\[ \dot{x}_3 = f_3(x_1, x_2) \]  \hspace{1cm} (5.5)

where $B = G_2(0)$ is a constant matrix

\[
B = \begin{bmatrix}
    l_{\delta a} & l_{\delta r} & 0 \\
    n_{\delta a} & n_{\delta r} & 0 \\
    0 & 0 & \bar{m}_{\delta e}
\end{bmatrix}
\]  \hspace{1cm} (5.6)

Of course, the complete model (5.1) will be used for the closed-loop simulation later.

The adaptive control laws of [21], [39] use inversion of the estimate of the input matrix $B$, which can become singular during parameter adaptation, causing divergence in the control surface deflection. This is avoided here using the matrix decomposition of the input matrix $B$ according to [20] (also see [22]). The design is based on a backstepping method following [19] in which the SDU decomposition of $B$ is used for completing the second step of the design [22].

5.2.1 SDU decomposition

The leading principal minors $\Delta_1 = l_{\delta a}$, $\Delta_2 = l_{\delta a}n_{\delta r} - n_{\delta a}l_{\delta r}$, and $\Delta_3 = \bar{m}_{\delta e}\Delta_2$ of the matrix $B$ are nonzero. For obtaining the $SD_u U_x$ decomposition of $B$, first one obtains the $LDU$ decomposition. The $LDU$ decomposition of $B$ is given by

\[ B = LU \]  \hspace{1cm} (5.7)
where $L$ is a unit (i.e. with all diagonal elements being 1) lower triangular, $U$ is a unit upper triangular matrix and

$$D^* = \text{diag} \{d_1^*, d_2^*, d_3^*\} = \text{diag} \left\{ \Delta_1, \frac{\Delta_2}{\Delta_1}, \frac{\Delta_3}{\Delta_2} \right\} \quad \text{(5.8)}$$

Then a non unique $SDU$ decomposition of $B$ is given in [22]

$$B = S D_s U_s \quad \text{(5.9)}$$

where $S$ is a symmetric positive definite matrix, $U_s$ is a unit upper triangular matrix, and

$$D_s = \text{diag} \left\{ \text{sgn}(\Delta_1)\gamma_1, \text{sgn}(\frac{\Delta_2}{\Delta_1})\gamma_2, \text{sgn}(\frac{\Delta_3}{\Delta_2})\gamma_3 \right\} \quad \text{(5.10)}$$

where $\gamma_i > 0$, $i = 1, 2, 3$, are arbitrary real numbers. In fact, for any set of chosen $\gamma_i$, one has

$$S = LD^*D_s^{-1}L^T$$
$$U_s = D_s^{-1}L^{-T}D_sU \quad \text{(5.11)}$$

Computing $L$ and $U$ using (5.7), then substituting these in (5.11), gives the matrices $S$ and $U_s$ of the form

$$S = \begin{bmatrix}
\gamma_1^{-1}|b_{11}| & \gamma_1^{-1}\text{sgn}(b_{11})b_{21} & 0 \\
\gamma_1^{-1}\text{sgn}(b_{11})b_{21} & (b_{21}^2\gamma_1^{-1} + |\Delta_2|\gamma_2^{-1}) |b_{11}|^{-1} & 0 \\
0 & 0 & \gamma_3^{-1}|b_{33}|
\end{bmatrix}$$

$$U_s = \begin{bmatrix}
1 & u_{s1} & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} \quad \text{(5.12)}$$
where \( u_{s1} = (b_{12} - \gamma_2 \gamma_1^{-1} b_{21} \text{sgn}(\Delta_2)) b_{11}^{-1} \) and \( b_{ij} \) is the element in the \( i \)th row and \( j \)th column of matrix \( B \).

### 5.2.2 Adaptive Sliding Mode Control

In this subsection, an adaptive sliding mode control law is designed using a backstepping procedure. The design is completed in two steps. In the first step, an adaptive law is designed for the virtual control input; and this is followed by the sliding mode control law derivation.

#### Step 1:

Define new coordinates

\[
\begin{align*}
Z_1 &= (X_1 - x_{1r}) + K_0 \int_0^t (x_1 - x_{1r}) = \ddot{x}_1 + K_0 \dot{x}_s \\
Z_2 &= x_2 - x_{2d}
\end{align*}
\]  

where \( x_s \) is the integral of the tracking error \( \dot{x}_1 \) satisfying

\[
\dot{x}_s = \ddot{x}_1
\]

and \( K_0 = \text{diag}(k_{01}, k_{02}, k_{03}) \) with \( k_{0i} > 0 \). Then taking the derivative of \( z_1 \) and using (5.5) gives

\[
\dot{z}_1 = f_{10}(x_1, x_3) + f_{11}(x_1) \lambda + G_1(x_1, x_3) x_2 - \dot{x}_{1r} + K_0 \ddot{x}_1
\]

\[
= f_{10}(x_1, x_3) + f_{11}(x_1) \lambda + G_1(x_1, x_3)(z_2 + x_{2d}) - \dot{x}_{1r} + K_0 \ddot{x}_1 \tag{5.14}
\]

Let \( \dot{\lambda} \) be an estimate of \( \lambda \). In view of (5.14), one chooses the stabilizing signal \( x_{2d} \) of the form

\[
x_{2d} = G_1^{-1}(x_1, x_3) \left[ -f_{10}(x_1, x_3) - f_{11}(x_1) \lambda - C_1 z_1 + x_{1r} - K_0 \ddot{x}_1 \right] \tag{5.15}
\]
where $C_1 > 0$. Substituting (5.15) in (5.14) gives

$$
\dot{z}_1 = -C_1 z_1 + f_{11}(x_1) \tilde{\lambda} + G_1(x_1, x_3) z_2
$$

where $\tilde{\lambda} = \lambda - \hat{\lambda}$ is the parameter error.

To examine the stability property of (5.16), consider a positive definite quadratic Lyapunov function

$$
V_1(z_1) = \frac{1}{2} \left[ z_1^T z_1 + \tilde{\lambda}^T \Gamma_1 \tilde{\lambda} \right]
$$

where $\Gamma_1$ is a positive definite symmetric matrix. Its derivative along the solution of (5.16) gives

$$
\dot{V}_1 = -C_1 \|z_1\|^2 + z_1^T f_{11}(x_1) \tilde{\lambda} + z_1^T G_1(x_1, x_3) z_2 + \tilde{\lambda}^T \Gamma_1 \tilde{\lambda}
$$

In order to eliminate the unknown function of (5.18), one chooses the adaptation law of the form

$$
\dot{\lambda} = -\Delta = -\Gamma_1^{-1} f_{11}^T(x_1) z_1
$$

Substituting (5.19) in (5.18) gives

$$
\dot{V}_1 = -C_1 \|z_1\|^2 + z_1^T G_1(x_1, x_3) z_2
$$

It follows from (5.20) that $z_1$ converges to zero if $z_2$ is zero. However, $z_2$ cannot be taken to be zero, because $x_2$ is not a control input.

**Step 2:**

In step 2, the control input $\delta$ is selected so that $z_2$ converges to zero. Differentiating $z_2$ and using (5.5) gives

$$
\dot{z}_2 = f_2(x) w + B \delta - \dot{x}_{2d}
$$

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where the derivative of $x_{2d}$ is

$$
\dot{x}_{2d} = \frac{\partial x_{2d}}{\partial x_1} [f_{10}(x_1, x_3) + f_{11}(x_1)\lambda + G_1(x_1, x_3)x_2] \\
+ \frac{\partial x_{2d}}{\partial x_3} f_3(x_1, x_2) + \frac{\partial x_{2d}}{\partial x_{1r}} \dot{x}_{1r} + \frac{\partial x_{2d}}{\partial x_{1r}} \ddot{x}_{1r} \\
- \frac{\partial x_{2d}}{\partial \lambda} \Gamma_1^{-1} f_{11}(x_1)z_1 \\
\Delta h_{10}(x, \dot{\lambda}, t) + h_{11}(x_1, x_3, \dot{\lambda}, t)\lambda
$$

(5.22)

where the argument $t$ denotes the dependence of these functions on $y_r$ and its derivatives, $h_{10}$ and $h_{11}$ are known functions, and

$$
h_{11}(x_1, x_3, \dot{\lambda}, t) = \frac{\partial x_{2d}}{\partial x_1} f_{11}(x_1)
$$

(5.23)

We point out that one can derive a VSC law by setting $B = B^* + A \Delta B$ where $B^*$ is the nominal value of $B$ and $\Delta B$ represents the uncertainty in $B$ following [12] and [30], without matrix decomposition. But this method requires that $\| \Delta BB^*^{-1} \| < 1$ which restricts the allowable uncertainty in the input matrix.

Now consider a modified Lyapunov function

$$
V_2(z_1, z_2, \dot{\lambda}) = V_1 + \frac{1}{2} z_2^T S^{-1} z_2
$$

(5.24)

Differentiating $V_2$ along the solution of (5.21) gives

$$
\dot{V}_2 = -C_1 \| z_1 \|^2 + z_1^T G_1(x_1, x_3)z_2 \\
+ z_2^T S^{-1} [f_2(x)w + SD_\delta U_\delta - \dot{x}_{2d}]
$$

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Since $U_s$ is a unit upper triangular matrix, one has

$$U_s \delta = \delta + (U_s - I_{3 \times 3}) \delta$$

(5.26)

where $I_{k \times k}$ denotes an identity matrix of dimension $k$,

$$
\begin{bmatrix}
0 & u_{s1} & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\delta \\
\delta_r \\
\delta_c \\
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
\end{bmatrix}
$$

In view of (5.26), it is possible to define the control signal $\delta$ as a function of $(I_{3 \times 3} - U_s) \delta$.

No static loops appear since $\delta a$ would depend on $\delta r$, and the remaining control inputs $\delta r$ and $\delta c$ are explicitly solvable.

To this end, it will be convenient to express $S^{-1} \{f_2(x) - \dot{x}_{2d}\}$ in a linearly parametrized form. Using (5.22), one has

$$S^{-1} \{f_2(x)w - \dot{x}_{2d}\} = S^{-1} \left\{f_2(x)w - h_{10}(x, \dot{x}, t) - h_{11}(x_1, x_3, \dot{x}, t) \lambda \right\}$$

$$= \psi_a(x, \dot{x}, t) w_a$$

(5.27)

where the regressor matrix $\psi_a(x, \dot{x}, t) \in R^{3 \times m}$ is a known function, $m$ is an appropriate integer, and $w_a \in R^m$ denotes the collection of all the unknown parameters. Note that $w_a$ consists of the elements of $S^{-1}$ as well as the product of these elements with the unknown parameter vectors $\lambda$ and $w$.

Let $w_a = w_a^* + \Delta w_a$, and $u_{a1} = u_{a1}^* + \Delta u_{a1}$ where $w_a^*$ and $u_{a1}^*$ are the nominal parameters and $\Delta w_a$ and $\Delta u_{a1}$ denote the unknown portions of $w_a$ and $u_{a1}$, respectively.
Substituting (5.26) in (5.25) and using (5.27) gives

$$
\dot{V}_2 = -C_1 \|z_1\|^2 + z_2^T \left[ G_1^T(x_1, x_3)z_1 + D_\lambda \delta + (\gamma_1 sgn(\Delta_1)u_{s_1}\delta r, 0, 0)^T + \psi_a(x, \lambda, t)w_a \right]
$$

(5.28)

In view of (5.28), the control law is chosen as

$$
\delta = D_\lambda^{-1} \left[ -G_1^T(x_1, x_3)z_1 - C_2z_2 - \psi_a(x, \lambda, t)w_a^* - (\gamma_1 sgn(\Delta_1)u_{s_1}\delta r, 0, 0)^T - K sgn(z_2) \right]
$$

(5.29)

where $z_2 = (z_{21}, z_{22}, z_{23})^T$, $sgn(z_2) = [sgn(z_{21}), sgn(z_{22}), sgn(z_{23})]^T$, $K = diag(k_1, k_2, k_3)$, and $k_i > 0$. We notice from (5.29) that indeed $\delta r$ and $\delta e$ are only functions of $x, \lambda$ and $t$, and $\delta r$ can be substituted to obtain $\delta a$. Substituting (5.29) in (5.28) gives

$$
\dot{V}_2 = -C_1 \|z_1\|^2 - C_2 \|z_2\|^2 - z_2^T K sgn(z_2) + z_2^T \psi_a(x, \lambda, t)\Delta w_a + z_{21}\gamma_1 sgn(\Delta_1)\delta r \Delta u_{s_1}
$$

$$
\dot{V}_2 \leq -C_1 \|z_1\|^2 - C_2 \|z_2\|^2 + \sum_{i=1}^{3} |z_{2i}| \left[-k_i + \|\psi_{ai}\| \|\Delta w_a\| + \gamma_1 |z_{21}| |\delta r| |\Delta u_{s_1}| \right]
$$

(5.30)

where $\psi_{ai}$ is the $i$th row of the matrix $\psi_a$.

In view of (5.30), the gains $k_i$ are chosen such that

$$
k_i \geq \|\psi_{ai}(x, \lambda, t)\| \Delta w_{am} + \gamma_1 |\delta r| |\Delta u_{s_1}| + \mu_1
$$

$$
k_i \geq \|\psi_{ai}(x, \lambda, t)\| \Delta w_{am} + \mu_i, \; i = 2, 3
$$

(5.31)

where $\|\Delta w_a\| \leq \Delta w_{am}$, $|\Delta u_{s_1}| \leq \Delta u_{sm}$, and $\mu_i > 0$. Using (5.31) in (5.30) gives

$$
\dot{V}_2 \leq -C_1 \|z_1\|^2 - C_2 \|z_2\|^2 - \sum_{i=1}^{3} \mu_i |z_{2i}|
$$

(5.32)

In view of (5.32), it is concluded that $z_i(t)$ converge to zero as $t \to \infty$. This implies that $x_1 = (\phi, \alpha, \beta)^T \to x_1r = (\phi_r, \alpha_r, \beta_r)^T$, as $t \to \infty$ and, therefore, output trajectory tracking is accomplished. This completes the adaptive sliding mode controller design.
It is pointed out that the control law is well defined for arbitrary perturbations of the input matrix $B$ due to the $SDU$ decomposition of $B$. The control law (5.29) is a discontinuous function of $z_2$ and uses adaptation law (5.19) for tuning the gain $\dot{\lambda}(t)$. The derived control law is a sliding mode control law for the tracking of $x_{2d}$ by $x_2$, if the feedback term $-G_T^T(x_1, x_3)z_1$ in (5.29) is dropped. It is known that the discontinuity in the control law may cause control chattering. But this can be avoided by a continuous approximation of the sgn function.

5.3 Simulation Results

In this section, simulation results are presented for the aircraft model of [35] and [36] for flight condition 1 (FC 1) ($M = 9$, $H = 20,000$ft) and FC 2 ($M = 0.7$, $H = 0$ft) using the adaptive VSC law. The complete set of aerodynamic parameters is provided in [35] and [36]. Although, the control law has been derived for the simplified model, the nonzero parameters $y_{sa}$ and $z_{se}$ are retained to include the effect of control forces. Furthermore, $\Delta \alpha$-dependence of the input matrix $G_2(\Delta \alpha)$ is introduced for a realistic simulation, even though for the derivation of the control law, $G_2(0)$ has been used. Since the value of $y_{sa}$ is not given in [36], it is taken to be zero. Here, $\alpha_0 = 1.5$ deg and $\theta_0 = 0$. It is also assumed that the initial estimate $\hat{\lambda}(0) = 0$.

The reference roll angle trajectory $\phi_r(t)$ and reference angle of attack trajectory $\alpha_r(t)$ are generated by third-order filters of the form

$$(s^3 + 3\lambda_3 s^2 + 3\lambda_2 s + \lambda_1^3)\phi_r = \lambda_3^3 \phi^*$$
where $\phi^*$ and $\alpha^*$ are the target values for the roll angle and the angle of attack, respectively. The poles of the command generators are chosen to be at -1.5 by selecting $\lambda_r = 1.5$. The initial conditions for the command generators are $\alpha_r = 1.5$ deg and, $\phi_r(0) = \dot{\phi}_r(0) = \ddot{\phi}_r(0) = 0$. The reference sideslip angle $\beta_r(t)$ is equal to 0. The target roll angle and the angle of attack are $\phi^* = 360$ deg, and $\alpha^* = 10$ deg, respectively. Thus it is desired to roll the aircraft 360 deg and simultaneously change the angle of attack to 10 deg.

It is well known that implementation of the discontinuous VSC law can cause control chattering. For this reason, for simulation, the sgn function is replaced by a sat function given by

$$
sat(v) = \begin{cases} 
\frac{1}{\epsilon} v, & |v| \leq \epsilon \\
\text{sgn}(v), & |v| > \epsilon 
\end{cases}
$$

where $\epsilon = 0.9$, $v \in \mathbb{R}$.

The closed-loop system including the complete aircraft model (5.1) and the adaptive VSC law (5.29) is simulated. The selected gains and matrices are $\Gamma_1 = 20I_{2 \times 2}$, $C_1 = 10$, $C_2 = 20$, $K = 2I_{3 \times 3}$ and $K_0 = I_{3 \times 3}$. Although, one can use the inequalities (5.31) for the computation of the gains $k_i$, these inequalities provide only sufficient conditions for stability. Therefore, for simplicity, constant gains $k_i$, obtained by observing the simulation results, were used. The design parameters in the $SDU$ decomposition are $\gamma_1 = 0.5, \gamma_2 = 0.5, \gamma_3 = 0.1$. For simulation, first the aircraft model at FC 1 is con-
sidered. For the computation of the control law, the selected nominal parameters are \( w_a^* = 0 \in R^{32} \) and \( u_{s1}^* = 0 \in R \). Therefore, one has \( \Delta w_a = w_a \) and \( \Delta u_{s1} = u_{s1} \), where \( w_a \) and \( u_{s1} \) are the actual parameters of the aircraft at FC 1. Thus the uncertainty in the parameters is 100%. Of course, this is rather a worse choice of uncertainties, but it has been made only to show the robustness of the control law. The selected responses are shown in Figure 1. We observe trajectory control of the roll angle and the angle of attack to the target values within 5 seconds. The sideslip angle remains small (within \( 5 \times 10^{-3} \) degrees). It is found that the parameters \( \gamma_i \) used in the decomposition of the input matrix \( B \) provide flexibility in shaping the surface deflections. The control magnitude is of the order of \( \delta = (-13.1, -3.6, -7.3)^T \) degrees. The angular velocities are of the order of \( (p, q, r) = (146.5, 12, 12.8) \) deg/sec.

To examine the robustness of the adaptive VSC system, now simulation is performed for the aircraft model at FC 2. The feedback gains and initial values of the parameters \( (C_1, C_2, K_0, K, \Gamma_1, \hat{\lambda}(0)) \) for Figure 1 used at FC 1 are retained. Smooth responses are observed in this case too, as seen in Figure 2. The control magnitude is of the order of \( \delta = (-14.5, -2.5, -3.5)^T \) degrees.

5.4 Summary

In this chapter, the design of a nonlinear adaptive VSC flight control system for performing roll-coupled maneuvers was considered. It was assumed that the aircraft parameters as well as the high frequency gain matrix were unknown. The adaptive VSC law was derived based on a backstepping design approach, for tracking the roll
angle, angle of attack and sideslip angle trajectories using aileron, rudder, and elevator control surfaces. An SDU decomposition of the high frequency gain matrix was used for the derivation of singularity free control law. Simulation results obtained showed that precise nonlinear roll-coupled maneuvers of aircraft can be accomplished in the closed-loop system in spite of the uncertainty in the aerodynamic parameters. There exist several design parameters in the control law which provide flexibility in shaping the response characteristics.
Figure 5.1: Adaptive VS control of $(\phi, \alpha, \beta)$: FC 1, 100% uncertainty in $w_a$ and $u_{sl}$; a) angular velocities $(p, q, r)$ deg/sec; b) $(\alpha, \beta)$ deg; c) $(\phi, \theta)$ deg; d) control surface deflections $(\delta a, \delta r, \delta e)$ deg
Figure 5.2: Adaptive VS control of \((\phi, \alpha, \beta)\): FC 2, (control law of FC 1); a) angular velocities \((p, q, r)\) deg/sec; b) \((\alpha, \beta)\) deg; c) \((\phi, \theta)\) deg; d) control surface deflections \((\delta a, \delta r, \delta e)\) deg.
CHAPTER 6

ADAPTIVE CONTROL OF AIRCRAFT USING SDU DECOMPOSITION OF HIGH-FREQUENCY GAIN MATRIX

In the previous chapter, an integrated sliding mode control design method was used for controlling the roll-coupled maneuvers of aircraft. This requires the information on the bounds on the uncertain functions for the computation of the gain matrix $K$. In this chapter, an adaptive law is designed, which does not require these uncertainty bounds. The mathematical model of the aircraft is given in Section 6.1. In Section 6.2, an adaptive control law is derived. The simulation results and summary are presented in Sections 6.3 and 6.4 respectively.

6.1 Mathematical Model

In this chapter, the simplified aircraft model of chapter 5 given by

$$
\begin{align*}
\dot{x}_1 &= f_{10}(x_1, x_3) + f_{11}(x_1)\lambda + G_1(x_1, x_3)x_2 \\
\dot{x}_2 &= f_2(x)w + B\delta \\
\dot{x}_3 &= f_3(x_1, x_2)
\end{align*}
$$

(6.1)
is used. Here $x_1 = (\phi, \alpha, \beta), x_2 = (p, r, q), x_3 = \theta$, and $x = (x_1^T, x_2^T, x_3)$. (A detailed explanation of the parameters is given in the previous chapter.)

### 6.2 Adaptive Control Law

The adaptive design is accomplished by modifying Step 2 of the VSC design derived in the previous chapter; the derivation of Step 1 is still valid. From the previous chapter, we have

$$
\dot{z}_1 = -C_1 z_1 + f_{11}(x_1) \bar{A} + G_1(x_1, x_3) z_2
$$

(6.2)

where

$$
z_1 = (x_1 - x_{1r}) + K_0 \int_0^t (x_1 - x_{1r}) = \bar{x}_1 + K_0 x_s
$$

and

$$
\dot{z}_2 = f_2(x) w + B \delta - \dot{x}_{2d}
$$

$$
= f_2(x) w + S D_s U_s \delta - \dot{x}_{2d}
$$

(6.3)

where

$$
z_2 = x_2 - x_{2d}
$$

The stabilizing signal obtained from Step 1 is

$$
x_{2d} = G_1^{-1}(x_1, x_3) \left[ -f_{10}(x_1, x_3) - f_{11}(x_1) \bar{A} - C_1 z_1 + \dot{x}_{1r} - K_0 \dot{x}_1 \right]
$$

where $\lambda = (z_\alpha, y_\beta)^T$. The adaptive law is

$$
\dot{\lambda} = -\lambda = -\Gamma_1^{-1} f_{11}^T(x_1) z_1
$$

(6.4)
The modified Lyapunov function is

\[ V_2(z_1, z_2, \lambda) = V_1 + \frac{1}{2} z_2^T S^{-1} z_2 \]

and its derivative is found to be

\[ \dot{V}_2 = -C_1 \|z_1\|^2 + z_2^T \left[ G_1^T(x_1, x_3) z_1 + D_2 \delta + (\gamma_1 \text{sgn}(\Delta_1) u_{s1} \delta r, 0, 0)^T + \psi_a(x, \lambda, t) w_a \right] \]

(6.5)

(Readers can refer to chapter 5 for the details.)

Consider now a Lyapunov function

\[ V_a(z_1, z_2, \lambda, \tilde{w}_a, \tilde{u}_{s1}) = V_2 + \frac{1}{2} \left[ \tilde{w}_a^T \Gamma_a \tilde{w}_a + \text{sgn}(\Delta_1) \tilde{u}_{s1} \Gamma_a \right] \]

(6.6)

where \( \Gamma_a \) is a positive definite symmetric matrix, \( \Gamma_s > 0 \), \( \tilde{w}_a = w_a - \hat{w}_a \), \( \tilde{u}_{s1} = u_{s1} - \hat{u}_{s1} \), and \( \hat{w}_a \) and \( \hat{u}_{s1} \) are the estimates of \( w_a \) and \( u_{s1} \), respectively. Using (6.5), its derivative can be written as

\[ \dot{V}_a = -C_1 \|z_1\|^2 + z_2^T \left[ G_1^T(x_1, x_3) z_1 + D_2 \delta + (\gamma_1 \text{sgn}(\Delta_1) u_{s1} \delta r, 0, 0)^T + \psi_a(x, \lambda, t) w_a \right] \]

\[ + \tilde{w}_a^T \Gamma_a \dot{\tilde{w}}_a + \Gamma_s \tilde{u}_{s1} \dot{\tilde{u}}_{s1} \]

(6.7)

In view of (6.7), the adaptive control law is chosen as

\[ \delta = D_s^{-1} \left[ -G_1^T(x_1, x_3) z_1 - \psi_a(x, \lambda, t) \tilde{w}_a - C_2 z_2 - (\gamma_1 \text{sgn}(\Delta_1) \tilde{u}_{s1} \delta r, 0, 0)^T \right] \]

(6.8)

where \( C_2 > 0 \). Substituting (6.8) in (6.7) yields

\[ \dot{V}_a = -C_1 \|z_1\|^2 - C_2 \|z_2\|^2 + z_2^T \psi_a(x, \lambda, t) \tilde{w}_a + z_{21} \gamma_1 \text{sgn}(\Delta_1) \tilde{u}_{s1} \delta r + \tilde{w}_a^T \Gamma_a \dot{\tilde{w}}_a + \Gamma_s \tilde{u}_{s1} \dot{\tilde{u}}_{s1} \]

(6.9)
Now the adaptation rule can be chosen as

\[ \dot{\hat{\omega}}_a = -\hat{\omega}_a = -\Gamma_a^{-1} \psi_a^T(x, \dot{\lambda}, t)z_2 \]

\[ \dot{\hat{u}}_{s1} = -\hat{u}_{s1} = -\Gamma_{s1}^{-1} \text{sgn}(\Delta_1)\gamma_1 \delta r z_{21} \]  

(6.10)

to eliminate unknown functions in (6.9). Substituting (6.10) in (6.9) gives

\[ \dot{V}_a \leq -C_1 \|z_1\|^2 - C_2 \|z_2\|^2 \]  

(6.11)

Since \( V_a \) is a positive definite function of \( z_1, z_2, \dot{\lambda}, \hat{\omega}_a, \hat{u}_{s1} \) and \( \dot{V}_a \leq 0 \), it follows that \( z_1, z_2, \dot{\lambda}, \hat{\omega}_a, \hat{u}_{s1} \in L^\infty[0, \infty) \) (the set of bounded functions). Integrating (6.11) gives

\[ \int_0^\infty (C_1 \|z_1(t)\|^2 + C_2 \|z_2(t)\|^2)dt \leq V_a(0) - V_a(\infty) < \infty \]  

(6.12)

which implies that \( z_i \in L^2[0, \infty) \) (the set of square integrable functions). According to (6.2) and (6.3), \( z_i \in L^\infty[0, \infty) \) for \( x(t) \in \Omega \). Now invoking Barbalat’s lemma [19, 22], one concludes that \( z_i(t) \to 0 \) as \( t \to \infty \). This implies that \( (\phi, \alpha, \beta)^T \to (\phi_r, \alpha_r, \beta_r)^T \), as \( t \to \infty \), and the output tracking error converges to zero. This completes the adaptive law derivation.

6.3 Simulation Results

In this section, simulation results are presented for the aircraft model of [52] and [53] for flight condition 1 (FC 1) \((M = 9, H = 20,000\text{ft})\) and FC 2 \((M = 0.7, H = 0\text{ft})\) using the adaptive control law. The complete set of aerodynamic parameters is provided in [52] and [53]. Although, the control laws have been derived for the simplified model, the nonzero parameters \( y_{\delta a} \) and \( z_{\delta e} \) are retained to include the
effect of control forces. Furthermore, $\Delta \alpha$-dependence of the input matrix $G_2(\Delta \alpha)$ is introduced for a realistic simulation, even though for the derivation of the control laws, $G_2(0)$ has been used. Since the value of $y_{sr}$ is not given in [53], it is taken to be zero. Here, $\alpha_0 = 1.5$ deg and $\theta_0 = 0$. It is also assumed that the initial estimate $\lambda(0) = 0$.

The reference roll angle trajectory $\phi_r(t)$ and reference angle of attack trajectory $\alpha_r(t)$ are generated by third-order filters of the form

$$(s^3 + 3\lambda_r s^2 + 3\lambda_r^2 s + \lambda_r^3)\phi_r = \lambda_r^3 \phi^*$$

and

$$(s^3 + 3\lambda_r s^2 + 3\lambda_r^2 s + \lambda_r^3)\alpha_r = \lambda_r^3 \alpha^*$$

where $\phi^*$ and $\alpha^*$ are the target values for the roll angle and the angle of attack, respectively. The poles of the command generators are chosen to be at -1.5 by selecting $\lambda_r = 1.5$. The initial conditions for the command generators are $\phi_r(0) = 1.5$ deg and, $\dot{\phi}_r(0) = \ddot{\phi}_r(0) = \lambda_r(0) = \alpha_r(0) = 0$. The reference sideslip angle $\beta_r(t)$ is equal to 0. The target roll angle and the angle of attack are $\phi^* = 360$ deg, and $\alpha^* = 10$ deg, respectively. Thus it is desired to roll the aircraft 360 deg and simultaneously change the angle of attack to 10 deg.

Simulations are performed for flight conditions 1 and 2 using the adaptive control law. The gains and matrices selected for adaptive control are $\Gamma_1 = 20I_{2 \times 2}$, $\Gamma_2 = 2I_{32 \times 32}$, $\Gamma_3 = 1$, $C_1 = 4$ and $C_2 = 4$. The design parameters chosen for $SDU$ decomposition are $\gamma_1 = 1$, $\gamma_2 = 100$, $\gamma_3 = 0.2$ The initial estimate $\hat{\lambda}_a(0)$ of the parameter vector $\lambda_a$ and $\hat{u}_{s1}(0)$ are taken to be zero, where $\lambda_a$ and $u_{s1}$ are the actual parameters
of the aircraft at FC 1. The closed-loop system including the control law (6.8) and adaptation laws (6.4) and (6.10) is simulated using the aircraft model (6.1) for FC 1. The selected responses are shown in Figure 6.1. Trajectory control of the roll angle and the angle of attack to the target values is accomplished within 5 seconds, and the sideslip angle remains small (less than 0.08 deg). The control magnitude is of the order of $\delta = (-13.5, -3.7, -7.3)^T$ degrees and the peak values of the angular velocities $(p, q, r)$ are $(150, 12.2, 13)$ deg/sec. Figure 6.2 shows the variation of some of the estimated parameters with time. Here $\hat{w}_{a10}$ and $\hat{w}_{a13}$ are the estimates of $s_{111}z_\alpha$ and $s_{112}y_\beta$, respectively, where $s_{ijk}$ denotes the $jk$th element of $S^{-1}$.

In order to examine the performance of the adaptive control system at a different flight condition, simulation is performed using the aircraft model for FC 2, but the initial values of the parameters $\hat{w}_a(0), \hat{w}_{a1}(0)$ and $\hat{\lambda}(0)$ as well as the feedback gains used for FC 1 are retained. Once again, smooth responses are observed as seen in Figure 6.3. The control magnitude is of the order of $\delta = (-14.6, -2.6, -3.5)^T$ degrees. Figure 6.4 shows the variation of some of the estimated parameters with time.

6.4 Summary

In this chapter, the design of a nonlinear adaptive flight control system for performing roll-coupled maneuvers was considered. It was assumed that the aircraft parameters as well as the high frequency gain matrix were unknown. The adaptive control law was derived based on a backstepping design approach, for tracking the roll angle, angle of attack and sideslip angle trajectories using aileron, rudder, and
elevator control surfaces. An SDU decomposition of the high frequency gain matrix was used for the derivation of singularity free control law. Simulation results obtained showed that precise nonlinear roll-coupled maneuvers of aircraft can be accomplished in the closed-loop system in spite of the uncertainty in the aerodynamic parameters. There exist several design parameters in both control laws which provide flexibility in shaping the response characteristics.
Figure 6.1: Adaptive control of $(\phi, \alpha, \beta)$: FC 1, 100% uncertainty in $w_a$ and $u_{a1}$

a) angular velocities $(p, q, r)$ deg/sec
b) $(\alpha, \beta)$ deg
c) $(\phi, \theta)$ deg
d) control surface deflections $(\delta a, \delta r, \delta e)$ deg
Figure 6.2: Adaptive control of $(\phi, \alpha, \beta)$: FC 1, 100% uncertainty in $w_a$ and $u_{a1}$. a) variation in the estimated parameter $\hat{\theta}_{a10}$ b) variation in the estimated parameter $\hat{\theta}_{a13}$ c) variation in the estimated parameter $\hat{\theta}_{a1}$ d) variation in the estimated parameter vector norm $||\hat{\theta}_a||$
Figure 6.3: Adaptive control of \((\phi, \alpha, \beta)\): FC 2, (control law of FC 1); a) angular velocities \((p, q, r)\) deg/sec; b) \((\alpha, \beta)\) deg; c) \((\phi, \theta)\) deg; d) control surface deflections \((\delta a, \delta r, \delta e)\) deg
Figure 6.4: Adaptive control of $(\phi, \alpha, \beta)$: FC 2, (control law of FC1) a) variation in the estimated parameter $\hat{w}_{a10}$ b) variation in the estimated parameter $\hat{w}_{a13}$ c) variation in the estimated parameter $\hat{u}_{e1}$ d) variation in the estimated parameter vector norm $\|\hat{w}_a\|$
CHAPTER 7

CONCLUSION

In this thesis, the design of control systems for the dive plane maneuvering of submersibles and the roll-coupled maneuvers of aircraft was considered. The first part of the thesis dealt with the trajectory control of submersibles using the bow and stern hydroplanes. The second part of the thesis considered the control of roll angle, angle of attack, and sideslip angle of aircraft using aileron, rudder and elevator control inputs.

First, using nonlinear input-output (pitch angle and depth) map inversion, a robust nonlinear output feedback control law was derived for the dive-plane maneuvers of a submersible. For synthesizing the robust inverse control law, the unknown functions and unmeasurable variables are estimated using a high-gain observer. It is shown that in the closed-loop system, the asymptotic tracking of the depth and pitch angle trajectories is accomplished despite the uncertainty in the system parameters and the presence of disturbances due to sea waves.

Next, an adaptive control system for the control of depth and pitch angle trajectories for the submersible was developed. The vehicle model included nonlinear hydrodynamic forces and it was assumed that the system parameters were unknown.
It was also assumed that random disturbance forces were acting on the system. For the derivation of the controller, an adaptive backstepping procedure was used. SDU decomposition of the high-frequency gain matrix was done to prevent singularity in the control law during adaptation. Simulation results were provided which showed that in the closed-loop system, depth and pitch trajectory control was accomplished in spite of the nonlinearity and large uncertainties in the system parameters and random disturbances due to sea waves.

In chapter four, a model reference adaptive controller design was considered. It was assumed that the system parameters including the high-frequency gain matrix were unknown. For the design, it was assumed that only the output variables were measured for feedback. Simulation results showed that in the closed-loop system, trajectory tracking is accomplished in spite of parameter uncertainties.

In chapters five and six the design of two nonlinear adaptive flight control systems for performing roll-coupled maneuvers was considered. It was assumed that the aircraft parameters as well as the high frequency gain matrix were unknown. The adaptive VSC law and the adaptive control laws were derived based on a backstepping design approach, for tracking the roll angle, angle of attack and sideslip angle trajectories using aileron, rudder, and elevator control surfaces. An SDU decomposition of the high frequency gain matrix was used for the derivation of singularity free control laws. Simulation results obtained showed that precise nonlinear roll-coupled maneuvers of aircraft can be accomplished in the closed-loop system in spite of the uncertainty in the aerodynamic parameters using each of the derived control laws.
APPENDIX

SYSTEM PARAMETERS

The system parameters for simulation in chapters 2, 3 and 4 have been taken from [5]. The key vehicle parameters and hydrodynamic parameters are taken as

\[
\begin{align*}
Z_w' &= -0.0110 \quad Z_b' = -0.0075 \quad Z_{\theta}' = -0.0045 \quad Z_{\phi}' = -0.0002 \quad Z_{SB}' = -0.0025 \\
Z_{6S}' &= -0.0050 \quad M_w' = 0.0030 \quad M_b' = -0.0002 \quad M_{\theta}' = -0.0025 \quad M_{\phi}' = 0.0004 \\
M_{SB}' &= 0.0005 \quad M_{6S}' = -0.0025 \quad I_y' = 5.6867 \times 10^5 \quad C_{M1} = 0.35 \quad L = 286 ft \\
m &= 1.52 \times 10^5 \text{slugs} \quad C_{z1} = 1.28 \quad C_{z2} = 0.77 \quad \nabla = 7.6 \times 10^4 \text{ft}^3 \quad U = 8.43 \text{ft/sec} \\
\rho &= 2.0 \text{slugs/ft}^3
\end{align*}
\]

The system parameters for simulation in chapters 5 and 6 have been taken from [52]. The key vehicle parameters are
$I_x, I_y, I_z$ = Moments of inertia about principal axes ($kg\cdot m^2$)

$i_1 = (I_z - I_y)/I_z$

$i_2 = (I_z - I_x)/I_y$ = Nondimensional inertia coefficients

$i_3 = (I_y - I_x)/I_z$

$V$ = Velocity of the aircraft center of mass ($km/sec$)

$g$ = Gravitational acceleration ($m/sec^2$)

$\alpha, \beta$ = Angle of attack (rad), sideslip angle (rad)

$\theta, \phi$ = Pitch angle (rad), angle of bank (rad)

$\delta a, \delta r, \delta e$ = Aileron, rudder, and elevator deflection angles (rad)

$l$ = Rolling moment per $I_z(1/sec^2)$

$m$ = Pitching moment per $I_y(1/sec^2)$

$n$ = Yawing moment per $I_z(1/sec^2)$

$y$ = Side force (over aircraft mass and speed) (1/sec)

$z$ = Aerodynamic force along z axis (over mass and speed) (1/sec)

The values chosen for simulation in chapters 5 and 6 are $i_1 = 0.727, i_2 = 0.949, i_3 = 0.716, g/V = 0.0345$ (FC1) , 0.0412 (FC2)
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